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**Credit Default Swaps and the Stability  
of the Banking Sector**

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# Credit Default Swaps and the Stability of the Banking Sector

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## Abstract

This paper considers credit default swaps (CDS) used for the transfer of credit risk within the banking sector. The banks' motive to conclude these CDS contracts is to improve the diversification of their credit risks. It is shown that these CDS reduce the stability of the banking sector in a recession. In a boom or in times characterized by a moderate economic up- or downturn, they can reduce this stability. The crucial points for these negative impacts to occur are firstly, that banks are induced to increase their investment into an illiquid, risky credit portfolio and secondly, that these CDS create a possible channel of contagion.

*JEL classification:* G21

*Keywords:* Credit Risk Transfer, Financial Stability, Contagion, Banking

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# 1 Introduction

Since the end of the 1990s the use of credit derivatives, which allow for the transfer of credit risks, has increased substantially. According to the British Bankers' Association (2006), the size of the global credit derivatives market increased from 586 billion US dollars in the year 1999 to 20,207 billion US dollars in 2006, and it is expected to expand to 33,120 billion US dollars in the year 2008. The major market participants are banks, hedge funds, and insurance companies. Although the market share of hedge funds has increased substantially during the last six years, banks still constitute the majority of market participation in both buying and selling credit protection.<sup>1</sup> The mostly traded product in the credit derivatives market is the credit default swap (CDS).<sup>2</sup> CDS are bilateral contracts where the risk seller pays a fixed periodic fee to the risk buyer in exchange for a contingent payment in case a defined credit event, such as bankruptcy of the original borrower, occurs (compensation payment).

This paper analyzes theoretically the consequences of CDS on the stability of the banking sector. We consider CDS contracts in which the risk buyer as well as the risk seller is a bank, i.e. we focus on credit risk transfer within the banking sector. The banks' motive to conclude these contracts is to improve the diversification of their credit risks. The main result of our analysis is that the consequences of these CDS contracts for the stability of the banking sector depend on the state of the macroeconomic environment. We show that these contracts reduce the stability of the banking sector in a recession, while they can reduce this stability in a boom or in times characterized by a moderate economic up- or downturn. The crucial points are that these CDS create a possible channel of contagion and that they imply an investment shifting: banks increase their investment into a risky, illiquid

<sup>1</sup>According to the British Bankers' Association (2006), in the year 2006 the banks' market share of buying credit protection was 59 %, of selling credit protection 44 %. The corresponding numbers for hedge funds were 28 % and 31 %. In the year 2000, the market share of banks was 81 % and 63 % respectively, that of hedge funds 3 % and 5 %.

<sup>2</sup>The major products in the credit derivative market are single-name credit default swaps, index trades, and synthetic collateralized debt obligations. According to British Bankers' Association (2006), the market share of single-name credit default swaps was 33 %, of full index trades 30 %, of tranching index trades 8 %, and of synthetic collateralized debt obligations 16 % in the year 2006.

credit portfolio and reduce their investment into a safe, relatively liquid asset.

It is especially the rapid growth of the credit derivatives market which has evoked an ongoing debate on the consequences of credit derivatives for financial stability. The Deutsche Bundesbank (2004) argues that on the one hand, developed and liquid credit risk transfer markets would allow for a broader diversification and more efficient price-setting which would improve the allocation of credit risk and, therefore, would foster financial stability. However, on the other hand, there would be risks involved in credit risk transfer which could have a negative impact on financial stability. These risks would result from: ineffective safeguards and inaccurate ex-ante assessments of risk/return profiles, a high concentration of intermediary services on only a small number of market participants, the possibility of using regulatory arbitrage, asymmetric information, and the interaction between credit risk transfer markets and other financial markets. Wagner (2007) argues that the new instruments for the transfer of credit risk would improve the banks' ability to sell their loans making them less vulnerable to liquidity shocks. However, this again might encourage banks to take on new risks because a higher liquidity of loans enables them to liquidate them more easily in a crisis. This effect would offset the initial positive impact on financial stability. However, Wagner and Marsh (2006) argue that particularly the transfer of credit risk from banks to non-banks would be beneficial for financial stability as it would allow for the shedding of aggregate risk which would otherwise remain within the relatively more fragile banking sector. Allen and Carletti (2006) show that risk transfer between the banking sector and the insurance sector can lead to damaging contagion of systemic risk from the insurance to the banking sector as the credit risk transfer induces insurance companies to hold the same assets as banks. If there is a crisis in the insurance sector, insurance companies will have to sell these assets forcing down the price which implies the possibility of contagion of systemic risk to the banking sector since banks use these assets to hedge their idiosyncratic liquidity risk.

This paper contributes to the literature in the following aspects. Firstly, its focus is on credit risk transfer *within* the banking sector which constitutes the largest part of the credit risk transfer market. Secondly, a measure of the banking sector's stability is explicitly modelled. (An asset buffer and a liquidity buffer are derived

which reflect the banks' shock absorbing ability. Then, is is analyzed in how far the introduction of CDS influences these buffers.) Thirdly, the paper emphasizes the importance of the state of macroeconomic environment when discussing the consequences of credit risk transfer for the stability of the banking sector.

The remainder of this paper is structured as follows. Section 2 presents the model without CDS. We derive a bank's optimal investment decision and its asset buffer as well as its liquidity buffer which reflect its shock absorbing ability. In section 3, we insert CDS into the model and analyze how these contracts influence a bank's optimal investment decision and its shock absorbing ability. Section 4 discusses the implications of our model results for the stability of the banking sector which is reflected by the banks' shock absorbing ability. Section 5 briefly summarizes the paper.

## 2 Model without CDS

### 2.1 Technology

There are three dates,  $t = 0, 1, 2$ . There is a single consumption good which serves as a numéraire. This good cannot only be consumed but it can also be invested in assets to produce future consumption. There are three types of assets: a short-term, safe asset; a long-term, safe asset; and a long-term, risky asset. The short-term, safe asset is represented by a storage technology: one unit of the consumption good at date  $t$  produces one unit of the consumption good at date  $t + 1$ . The two long-term assets can only be invested at date 0. One unit of the consumption good invested in the long-term, safe asset produces  $R > 1$  units of the good at date 2 or  $0 < r < 1$  units at date 1. Consequently, the long-term, safe asset is not completely illiquid: It can be liquidated at date 1 but only at a loss. As in Allen and Carletti (2006) the long-term, risky asset is a credit portfolio which cannot be liquidated at date 1. The investment of one unit of the consumption good into the risky credit portfolio, yields a random return  $K$  at date 2. With probability  $\alpha$  the investment succeeds and  $K = H > R$ , with probability  $1 - \alpha$  the investment fails and  $K = L < 1$ . We assume that the expected return  $E(K) = \alpha H + (1 - \alpha)L > R$ . The uncertainty is

resolved at date 1. Table 1 summarizes the returns of the different types of assets.

	Return at date 1	Return at date 2
Short-term asset	1	
Long-term, safe asset	$r < 1$	$R > 1$
Long-term, risky asset	0	$K = \begin{cases} H > R & \text{with prob. } \alpha \\ L < 1 & \text{with prob. } 1 - \alpha, \end{cases}$ where $E[K] > R$

**Table 1: Return on the Different Types of Assets** (investment at date 0: 1 unit)

## 2.2 Banking Sector and Liquidity Preferences

Consumers are uncertain about their individual consumption preferences which generates a role for banks (we will come back to this aspect below). The banking sector in our economy can be described similarly to the one in Allen and Gale (2000). The economy is divided into four ex-ante identical regions labeled  $A$ ,  $B$ ,  $C$ , and  $D$ . The regional structure can be interpreted in a variety of ways. A region can correspond to a geographical region within a country, an entire country, or a specialized sector within the banking industry. The banks' behaviour in each region can be described by a representative bank, i.e. the banking sector in our economy consists of four representative banks, labeled  $A$ ,  $B$ ,  $C$ , and  $D$ . These banks are ex-ante identical. This four-bank structure instead of a more simple two-bank structure is necessary for modelling contagion within our model framework (for details see section 3.3.4, especially footnote 19).

In each region, there is a continuum of ex-ante identical consumers. They are risk-averse and have the usual Diamond-Dybvig liquidity preferences: with probability  $\gamma$ , where  $0 < \gamma < 1$ , a consumer is an early consumer who only values consumption at date 1, with probability  $1 - \gamma$  he is a late consumer who only values consumption at date 2. His utility of consumption is represented by the function

$$U(c) = \ln c, \tag{1}$$

where  $c$  denotes the consumed quantity of the good. The uncertainty is restricted to the level of the individual consumers. At the aggregate level, we assume that the law of large numbers applies so that in each region a fraction  $\gamma$  of the depositors will turn out to be early consumers and a fraction  $1 - \gamma$  will turn out to be late consumers. The uncertainty about the individual consumption preferences generates demand for liquidity and a role for banks which have a comparative advantage of providing this liquidity. Therefore, at date 0 each consumer deposits his endowment of one unit of the consumption good in a bank. We assume that he deposits it in a bank of his region. The deposit contract allows the depositor to withdraw either  $c_1$  units of the consumption good at date 1 or  $c_2$  units of this good at date 2. The units  $c_1$  and  $c_2$  specified in the contract depend on the bank's investment decision.

### 2.3 Optimal Investment

At date 0, the bank has to choose how to split its deposits investing  $x$  units into the short-term asset,  $y$  units into the long-term, safe asset, and  $u$  units into the risky credit portfolio. We assume that banks within a region compete to raise deposits so that the contract the representative bank of a region offers maximizes its depositors' expected utility. When making its investment decision, the bank must consider that a fraction  $\gamma$  of its depositors will withdraw at date 1 and a fraction  $1 - \gamma$  at date 2. At date 1, the bank uses the proceeds of the short-term asset to provide the early consumers with a level of consumption

$$c_1 = \frac{x}{\gamma}. \quad (2)$$

At date 2, the proceeds of the long-term assets are used to provide the late consumers with a level of consumption

$$c_2 = \begin{cases} \frac{yR+uL}{1-\gamma} =: c_{2,L} & \text{if } K = L \\ \frac{yR+uH}{1-\gamma} =: c_{2,H} & \text{if } K = H. \end{cases} \quad (3)$$

Consequently, the bank has to solve the optimization problem

$$E[U] = \gamma \ln c_1 + (1 - \gamma) [\alpha \ln c_{2,H} + (1 - \alpha) \ln c_{2,L}] =: f(x, y, u) \rightarrow \max \quad (4)$$

$$\text{s.t. } x + y + u = 1, \quad x, y, u \geq 0, \quad \text{and} \quad c_1 \leq c_{2,L}. \quad (5)$$



The first term on the right hand side of the objective function represents the utility of the early consumers. The second term represents the expectation of the uncertain utility of the late consumers. It is uncertain since it depends on the outcome of the investment into the risky credit portfolio. The first constraint is the bank's budget constraint at date 0. The second constraint, the non-negativity constraint, implies that beside the deposits, the bank cannot take on further liabilities. The third constraint is the incentive compatibility constraint meaning that the bank's investment decision must ensure that a bank run is avoided, i.e. even if it turns out at date 1 that the bad outcome of the credit portfolio will be realized at date 2, it must still be (weakly) optimal for the late consumers to withdraw at date 2. This constraint is due to an asymmetric information problem. The banks cannot distinguish between early and late consumers. Consequently, if  $c_1 > c_{2,L}$ , the late consumers will be better off withdrawing at date 1 which implies that all consumers pretend to be early consumers and demand withdrawal at date 1, i.e. a bank run occurs.<sup>3</sup>

When solving the maximization problem, we assume that

$$1 - \alpha > \frac{H - R}{R(H - L)}. \quad (6)$$

This assumption implies that although the expected return on the risky credit portfolio  $E[K]$  is higher than the return on safe asset  $R$ , it is neither that much higher that the bank will invest totally its long-term deposits into the risky credit portfolio nor is the expected return that much higher that without asymmetric information the bank will offer a deposit contract in which  $c_{2,L}^* < c_1^*$  (see footnote 3).<sup>4</sup> Then, we obtain the following solution of the optimization problem in which none of the

<sup>3</sup>In a situation without asymmetric information banks can distinguish between early and late depositors. Then, there is no incentive compatibility constraint, i.e. the banks can offer a contract in which  $c_{2,L} < c_1 < c_{2,H}$  without causing a bank run if the bad state  $L$  is realized.

<sup>4</sup>Equation (8) shows that the inequality constraint  $y^* \geq 0$  will be satisfied if  $1 - \alpha \geq (L(H - R))/(R(H - L))$ . Equation (11) reveals that the inequality constraint  $c_1^* \leq c_{2,L}^*$  will be satisfied if  $1 - \alpha \geq (H - R)/(R(H - L))$ .

inequality constraints is binding.<sup>5</sup>

$$x^* = \gamma, \quad (7)$$

$$y^* = \frac{(HR(1-\alpha) + LR\alpha - HL)(1-\gamma)}{(H-R)(R-L)} = \left[ 1 - \frac{R[E[K] - R]}{(H-R)(R-L)} \right] (1-\gamma), \quad (8)$$

and

$$u^* = \frac{R(L(1-\alpha) + H\alpha - R)(1-\gamma)}{(H-R)(R-L)} = \left[ \frac{R[E[K] - R]}{(H-R)(R-L)} \right] (1-\gamma) \quad (9)$$

which implies that

$$c_1^* = 1, \quad (10)$$

and

$$c_2^* = \begin{cases} \frac{(H-L)R(1-\alpha)}{H-R} =: c_{2,L}^* & \text{if } K = L \\ \frac{(H-L)R\alpha}{R-L} =: c_{2,H}^* & \text{if } K = H. \end{cases} \quad (11)$$

Concerning the short-term asset, equation (7) shows that the bank's optimal investment decision is only determined by the fraction of early consumers  $\gamma$ . Consequently, the bank's investment into the long-term assets decreases in  $\gamma$  as the equations (8) and (9) show. Furthermore, these equations reveal that the optimal allocation on the long-term, safe asset and on the long-term, risky asset depends on the (expected) returns of the two assets. The investment into the safe asset increases in  $R$  and decreases in  $E[K]$ , and vice versa for the risky asset. The levels of consumption  $c_1^*$  and  $c_2^*$  are the return payments from the bank to the consumers specified in the deposit contract. Note that the promised return payment in the second period is

<sup>5</sup>The inequality constraints  $y^* \geq 0$  and  $c_1^* \leq c_{2,L}^*$  are satisfied because of (6), the constraint  $x^* \geq 0$  is satisfied because  $\gamma > 0$  and the constraint  $u^* \geq 0$  is satisfied because  $E[K] > R$ ,  $H > R > L$ , and  $\gamma > 0$ .

state-dependent, i.e. as in Hellwig (1994) the depositors bear some risk.<sup>6</sup>

As in Allen and Gale (2000) a bank is said to be bankrupt at date 1 if it is not able to meet the demand of its depositors by liquidating all its assets which will be the case if at date 1 a shock occurs implying that  $c_2 < c_1^*$ . Then, all depositors pretend to be early consumers, and all of them demand withdrawal at date 1. For satisfying this demand, which is equal to 1, the bank has to liquidate all its assets at date 1. The liquidation value of its total assets at date 1 is  $x^* + y^*r < 1$ .<sup>7</sup> Consequently, the bank is not able to meet the demands of its depositors by liquidating all its assets and goes bankrupt. Note that as long as  $c_1^* \leq c_2$ , it is (weakly) optimal for the late consumers to withdraw at date 2, i.e. the bank does not have to liquidate all its assets at date 1, the bank is not bankrupt.

## 2.4 Shocks and Buffers

### 2.4.1 Asset Buffer

Let us assume that at date 1, one of the four banks is hit by an asset shock. There is negative information about its future asset returns in the sense that the return on the bank's credit portfolio will only be  $(L - s)$  instead of  $L$ , where  $s > 0$  is a shock that - as the liquidity shock in Allen and Gale (2000) - is assigned zero probability at date 0. This shock implies that  $c_2$  is smaller than specified in the deposit contract, i.e.

<sup>6</sup>Furthermore, it is worth mentioning that in our model  $c_1^*$  is equal to 1, contrary to Diamond and Dybvig (1983) in which the return payment specified in the deposit contract at date 1 is strictly larger than 1. In the Diamond-Dybvig model, this result motivates the role for banks as  $c_1^* > 1$  is Pareto-optimal, and this optimum cannot be achieved by a market economy, in which  $c_1 = 1$ , but by implementing a financial intermediary. However, for obtaining the result that a financial intermediary can solve the problem of an inefficient market outcome, a stronger condition than just risk averse depositors is necessary, namely that  $c \mapsto cU'(c)$  is decreasing which is interpreted as that the intertemporal elasticity of substitution must be larger than 1 (Freixas and Rochet, 2008, p. 64). The condition that  $c \mapsto cU'(c)$  is decreasing is equivalent to  $-cU'''(c)/U'(c) > 1$ . This condition is not satisfied in our model in which  $U(c) = \ln c$ , so that  $-cU'''(c)/U'(c) = 1$ . Consequently, one could argue that in our model, there is no role for banks since the optimal allocation can also be achieved by the market. However, we assume, without explicitly modelling this issue, that banks have a comparative advantage of providing the liquidity as they can monitor credits at lower costs, for example.

<sup>7</sup>At date 1, only the short-term asset and the long-term, safe asset can be liquidated. The credit portfolio is assumed to be totally illiquid so that its liquidation value at date 1 is equal to zero. Consequently, the liquidation value of a bank's total assets at date 1 is  $x^* + y^*r$ . This is less than 1 since  $x^* + y^* + u^* = 1$ ,  $u^* > 0$ , and  $r < 1$ .

$c_2 < c_{2,L}^*$ . For a bank-run not to occur,  $c_1^* = x^*/\gamma \leq c_2 = [y^*R + u^*(L - s)]/(1 - \gamma)$ , i.e.<sup>8</sup>

$$s \leq \frac{y^*R + u^*L - x^*\frac{1-\gamma}{\gamma}}{u^*}$$

which implies that the bank's asset buffer is given by

$$B^{*As} = \frac{y^*R + u^*L - x^*\frac{1-\gamma}{\gamma}}{u^*} > 0. \quad (12)$$

If  $s > B^{*As}$ , late consumers will be better off withdrawing at date 1, i.e. they will pretend to be early consumers. The bank will face a run and will go bankrupt. The buffer increases in the bank's total assets at date 2 given by  $(y^*R + u^*L)$  and decreases in the claims the late consumers will assert if they pretend to be early consumers ( $c_1^*(1 - \gamma) = (x^*/\gamma)(1 - \gamma)$ ). Note that due to (6) the asset buffer is strictly positive. Moreover, it is worth mentioning that there are no linkages to the other banks, i.e. if  $s > B^{*As}$ , the bank which is hit by the shock goes bankrupt, but there will not be a spill over of the crisis to other banks. There will be no contagion.

#### 2.4.2 Liquidity Buffer

Let us assume that at date 1, one bank is hit by a liquidity shock. The fraction of its depositors who withdraw early is  $\gamma + z$  instead of  $\gamma$ , where  $z > 0$  is a shock that is assigned zero probability at date 0. Whether the bank can absorb this liquidity shock depends on its liquidity buffer which is determined by the amount of the long-term, safe asset it can liquidate at date 1. We show that this amount is either limited by the bank's total assets or by the bank's liquid assets. If the buffer is limited by the bank's total assets, it cannot liquidate more units without causing a run, if the buffer is limited by the bank's liquid assets, it could liquidate more assets without causing a run, but it has no more liquid assets.

Considering the shock, the bank's liquidity needs at date 1 are given by  $c_1^*(\gamma + z)$ . These needs have to be covered by the proceeds of the short-term asset given by  $x^*$

<sup>8</sup>Since  $c_1^* = (x^*/\gamma) = 1$ , for  $x^*[(1 - \gamma)/\gamma]$  we could simply write  $(1 - \gamma)$ . However, abstaining from this simplification allows for a better comparison of this asset buffer with the buffer with CDS in section 3.3.2.

and of the long-term asset given by  $y^{*need}r$ , where  $y^{*need}$  denotes the units of the long-term asset the bank has to liquidate to meet the demand of all early consumers  $(\gamma + z)$  according to the deposit contract. Consequently,  $c_1^*(\gamma + z) = x^* + y^{*need}r$ , so that

$$y^{*need} = \frac{x^*z}{r\gamma} \quad (13)$$

which is equal to  $z/r$  when considering that  $c_1^* = x^*/\gamma = 1$ .<sup>9</sup> However, this liquidation of the long-term asset implies a decrease in the bank's total assets at date 2 and, therefore, in the repayment to the late consumers. The condition for a run not to occur is

$$c_1^* = \frac{x^*}{\gamma} \leq \frac{yR + u^*K}{1 - \gamma - z} = c_2 \quad (14)$$

implying that the bank must keep at least  $[x^*(1 - \gamma - z) - u^*K\gamma]/R\gamma$  units of the long-term, safe asset to prevent a run. Consequently, the amount  $y^{*crit}$  of the long-term, safe asset which can at most be liquidated at date 1 is given by

$$y^{*crit} = y^* - \frac{x^*(1 - \gamma - z) - u^*K\gamma}{R\gamma}. \quad (15)$$

Setting  $y^{*need}$  given by equation (13) equal to  $y^{*crit}$  given by equation (15) and solving for  $z$  the liquidity buffer limited by the bank's total assets is obtained. It is given by

$$B^{*Li,TA} = \frac{r \left( y^*R + u^*K - x^* \frac{(1-\gamma)}{\gamma} \right) \gamma}{x^*(R - r)}. \quad (16)$$

If  $z > B^{*Li,TA}$ , the bank which is hit by the shock, will go bankrupt, because it cannot liquidate enough assets to satisfy the demand of all early consumers  $(\gamma + z)$  without causing a bank run. Obviously, the buffer increases in the bank's total assets at date 2 given by  $(y^*R + u^*K)$  as well as in the liquidation value of the long-term asset at date 1 ( $r$ ). Furthermore, it decreases in the claims the late consumers will assert if they pretend to be early consumers ( $c_1^*(1 - \gamma) = (x^*/\gamma)(1 - \gamma)$ ).

The bank can liquidate at most  $y^{*crit}$  units of this asset without causing a run. However, the bank actually holds  $y^*$  units of this asset. Consequently, if  $y^{*crit} > y^*$ ,

<sup>9</sup>However, as with the asset buffer we abstain from this simplification because this allows for a better comparison of this liquidity buffer with the buffer with CDS in section 3.3.3.

the liquidity buffer is limited by the bank's liquid assets. This liquidity buffer is given by<sup>10</sup>

$$B^{*Li,LA} = \frac{y^* r \gamma}{x^*} = y^* r. \quad (17)$$

If  $z > B^{*Li,LA}$ , the bank which is hit by the shock, will go bankrupt, because it does not have sufficient liquid assets at its disposal to satisfy the demand of all early consumers  $(\gamma + z)$ . Obviously, this buffer decreases in the liquidation value of the total amount of the bank's relatively liquid asset  $y^* r$  and decreases in the claims of an early consumer ( $c_1^* = x^*/\gamma$ ). Using equation (15), we can further elaborate the economic meaning of equation (17). Given that  $y^{*crit} > y^*$  and using equation (15) we have that  $c_1^* = 1 < u^* K / (1 - \gamma - z)$ . This means that the return from the long-term, risky asset  $u^* K$  is enough to ensure that  $c_2$  is greater than or equal to  $c_1^*$  such that all long-term, safe assets can be liquidated at date 1 for a total amount of  $y^* r$  to try to satisfy the demand of the early consumers.

Considering that a bank's liquidity buffer can either be limited by its total assets or by its liquid assets, this buffer is given by

$$B^{*Li} = \min [B^{*Li,TA}, B^{*Li,LA}] > 0. \quad (18)$$

Note that due to assumption (6) this buffer is strictly positive.

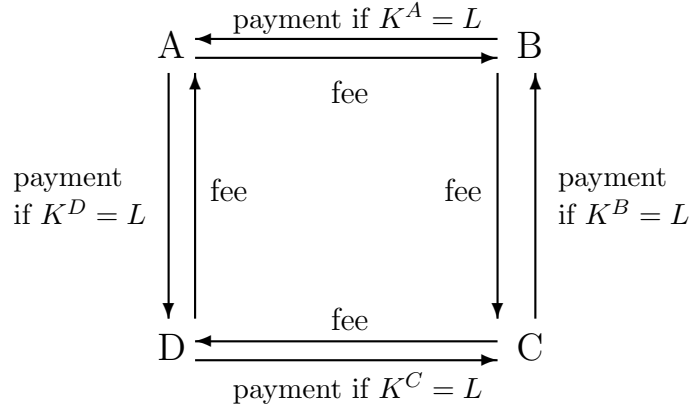
### 3 Model with CDS

#### 3.1 Credit Default Swaps

We assume that the risks of the banks' credit portfolios are perfectly correlated within a region but that they are uncorrelated between regions. Consequently, there is scope for diversification, but due to frictions which imply that the banks can conclude credit contracts only in their own region, banks cannot make use directly of this possibility of diversification. However, credit default swaps (CDS) offer a possibility to overcome these frictions. Each of the four banks  $A, B, C,$

<sup>10</sup>This liquidity buffer is obtained by setting  $y^{*need}$  given by equation (13) equal to  $y^*$  and solving for  $z$ .

and  $D$  concludes two CDS. In one CDS the bank is a risk taker, in the other it is a risk shedder as illustrated by figure 1. Let us consider bank  $A$ , for example. It concludes a CDS contract with bank  $B$  as a risk shedder, i.e. bank  $B$  agrees to make a compensation payment to bank  $A$  if a predefined credit event occurs. Furthermore, bank  $A$  also concludes a CDS contract with bank  $D$  in which it is the risk taker.



**Figure 1: Credit Default Swaps**

The CDS contracts are specified as follows: If a bank is the risk shedder, it will have to pay a fee to the risk taker at date 0, and in return, it will receive an amount of  $up$  as a compensation payment at date 2 if its credit portfolio fails, where

$$p = \begin{cases} \frac{H-L}{2} =: l & \text{if } K = L \\ 0 & \text{if } K = H. \end{cases} \quad (19)$$

This amount of a compensation payment maximizes the diversification benefits from the CDS contracts (see also page 15). Note that these CDS contracts do not provide an insurance against shocks but against the credit risk that the credit portfolio realizes  $L$  instead of  $H$ . Independently of the occurrence of a shock, the risk shedder will only receive the fixed payment  $ul$  if its portfolio fails. Obviously, an even better diversification of credit risk could be achieved if each bank concluded two CDS contracts with *all* other banks. However, for the sake of simplicity we restrict our analysis to the case in which each bank concludes a CDS contract with only two other banks.

Since all banks are assumed to be ex-ante identical, the fee is the same for all banks,

i.e. at date 0, the CDS is balance sheet neutral since each bank receives and pays a fee of the same amount. Furthermore, we assume that if a bank goes bankrupt, it will not fulfil its obligations from the CDS, i.e. it will make no compensation payment. Moreover, we assume that if a bank has to be liquidated at date 1, it will not receive any payment from the CDS contract it has concluded as a risk shedder. A compensation payment has to be paid only at date 2.

### 3.2 Optimal Investment

At date 1, there are no payments due to the CDS. Consequently, with CDS the level of consumption in the first period  $c_1$  is still given by equation (2). However, at date 2, the level of consumption does not only depend on the performance of the credit portfolio of the depositor's own bank (as without CDS), but it also depends on the performance of the credit portfolio of that bank its bank has concluded a CDS contract with as a risk taker. For a depositor of bank  $A$ , for example, the consumption at date 2 depends on the realization of  $K^A$  and on the realization of  $K^D$  so that four possible states must be distinguished:

- $LL$ : the credit portfolio of bank  $A$  realizes  $L$ , that of bank  $D$  realizes  $L$ ,
- $LH$ : the credit portfolio of bank  $A$  realizes  $L$ , that of bank  $D$  realizes  $H$ ,
- $HL$ : the credit portfolio of bank  $A$  realizes  $H$ , that of bank  $D$  realizes  $L$ ,
- $HH$ : the credit portfolio of bank  $A$  realizes  $H$ , that of bank  $D$  realizes  $H$ .

These four states can be defined analogously for all banks. Consequently, the level of consumption at date 2 is given by

$$c_2 = \begin{cases} \frac{yR+uL}{1-\gamma} & \text{in the state } LL \\ \frac{yR+uL+ul}{1-\gamma} & \text{in the state } LH \\ \frac{yR+uH-ul}{1-\gamma} & \text{in the state } HL \\ \frac{yR+uH}{1-\gamma} & \text{in the state } HH. \end{cases} \quad (20)$$

Equation (20) shows that if the credit portfolios of both banks perform badly, compensation payments have no influence on  $c_2$ . The reason is that the bank receives a compensation payment but it also has to make one. Consequently, the net effect will



be zero. Obviously, compensation payments play no role either in case the credit portfolios of both banks perform well. Only if the performance of the two portfolios is different, compensation payments will exert an influence on  $c_2$ . Replacing  $l$  by  $(H - L)/2$  according to equation (19), gives

$$c_2 = \begin{cases} \frac{yR+uL}{1-\gamma} & =: c_{2,LL} & \text{in the state } LL \\ \frac{yR+u\frac{H+L}{2}}{1-\gamma} & =: c_{2,M} & \text{in the states } LH \text{ and } HL \\ \frac{yR+uH}{1-\gamma} & =: c_{2,HH} & \text{in the state } HH \end{cases} \quad (21)$$

which reveals that the by equation (19) defined compensation payment maximizes the diversification benefit from the CDS contracts. Independently of which of the the credit portfolios fails, i.e. independently of whether the state  $LH$  or the state  $HL$  occurs, the level of consumption at date 2 will be the same. Consequently, with CDS a bank's optimization problem becomes

$$E[U] = \gamma \ln c_1 + (1 - \gamma) \left\{ (1 - \alpha)^2 \ln c_{2,LL} + 2(1 - \alpha)\alpha \ln c_{2,M} + \alpha^2 \ln c_{2,HH} \right\} =: g(x, y, u) \rightarrow \max \quad (22)$$

s.t. the constraints given by (5). When solving this optimization problem, we neglect the inequality constraints in a first step. The necessary and sufficient (since  $g$  is concave and the constraint is linear) conditions for an optimal solution  $(\tilde{x}, \tilde{y}, \tilde{u})$  of (22) s.t. the equality constraint  $x + y + u = 1$  are

$$\frac{\gamma}{\tilde{x}} = \lambda, \quad (23)$$

$$\left[ \frac{(1 - \alpha)^2}{\tilde{c}_{2,LL}} + \frac{2(1 - \alpha)\alpha}{\tilde{c}_{2,M}} + \frac{\alpha^2}{\tilde{c}_{2,HH}} \right] R = \lambda, \quad (24)$$

$$\frac{(1 - \alpha)^2}{\tilde{c}_{2,LL}} L + \frac{2(1 - \alpha)\alpha}{\tilde{c}_{2,M}} \frac{H + L}{2} + \frac{\alpha^2}{\tilde{c}_{2,HH}} H = \lambda, \quad (25)$$

together with  $\tilde{x} + \tilde{u} + \tilde{y} = 1$ , where  $\tilde{c}_2 = c_2(\tilde{u}, \tilde{y})$  and where  $\lambda$  is the Lagrange multiplier. These optimality conditions imply  $\tilde{x} = \gamma$ .

*Proof:* See proof I in the appendix.

Since the expressions for the optimal values  $\tilde{u}$  and  $\tilde{y}$  are rather cumbersome, we restrict our analysis to showing that  $\tilde{u} > u^*$  and that  $\tilde{y} < y^*$ , i.e. that the introduction of CDS implies that the banks reduce their investment into the safe asset and

increase their investment into the credit portfolio. For showing this, we eliminate  $y$  by using the equality constraint in a first step. Then, we obtain a new objective function  $h(x, u) := g(x, 1 - x - u, u)$ , where  $h$  depends on  $c_1$  and  $c_2$  in the same way as  $g$  in (22), but the consumption  $c_2$  now depends on  $x$  and  $u$  in the following way

$$\begin{aligned} c_{2,LL} &= \frac{(1-x)R + u(L-R)}{1-\gamma}, \\ c_{2,M} &= \frac{(1-x)R + u\left(\frac{H+L}{2} - R\right)}{1-\gamma}, \\ c_{2,HH} &= \frac{(1-x)R + u(H-R)}{1-\gamma}. \end{aligned} \tag{26}$$

We compute the partial derivative  $\partial h / \partial u$  at the point  $(x^*, u^*)$ , where  $x^*$  and  $u^*$  are the optimal  $x$  and  $u$  without CDS. Using the fact that  $x^*$  and  $u^*$  satisfy (7) and (9), we obtain

$$\begin{aligned} \frac{\partial h}{\partial u}(x^*, u^*) &= (1-\alpha)\alpha \left[ (R-L) \left( \frac{1}{c_{2,LL}^*} - \frac{1}{c_{2,M}^*} \right) \right. \\ &\quad \left. + (H-R) \left( \frac{1}{c_{2,M}^*} - \frac{1}{c_{2,HH}^*} \right) \right] > 0, \end{aligned} \tag{27}$$

where  $c_2^*$  denotes the level of consumption according to (26) at the point  $(x^*, u^*)$ .

*Proof:* See proof II in the appendix.

Since we already know that for the maximizer  $(\tilde{x}, \tilde{u})$  of  $h$  it holds that  $\tilde{x} = x^* = \gamma$ , (27) implies that  $\tilde{u} > u^*$ .<sup>11</sup> This in turn means that  $\tilde{y} < y^*$ . By the preceding considerations we easily see that the solution  $(\tilde{x}, \tilde{y}, \tilde{u})$  of the optimization problem with neglected inequality constraints already satisfies the non-negativity constraints for  $x$  and  $u$ . But we cannot exclude that this solution violates the incentive compatibility constraint<sup>12</sup> or the non-negativity constraint for  $y$ . Therefore, we have also analyzed

<sup>11</sup>The strict concavity of  $h$  with respect to  $u$  implies in general that  $h(x^*, u) - h(x^*, u^*) < (u - u^*) \frac{\partial h}{\partial u}(x^*, u^*)$  for  $u \neq u^*$ , hence  $h(x^*, u) < h(x^*, u^*)$  if  $u < u^*$ .

<sup>12</sup>Even though (6) holds, so that without CDS the incentive compatibility constraint is not binding, it can become binding with CDS because of  $\tilde{c}_{2,LL} < c_{2,L}^*$ .

the Karush-Kuhn-Tucker (KKT) conditions for the optimization problem with all inequality constraints. This analysis proves that the optimal solution  $(\bar{x}, \bar{y}, \bar{u})$  for (22) including all constraints (5) still has the property  $\bar{u} > u^*$  and  $\bar{y} < y^*$ . Furthermore, this analysis shows that it can happen that  $\bar{x} < x^*$  if the incentive compatibility constraint becomes binding.

*Proof:* See proof III in the appendix.

This leads us to

*Result 1: The CDS imply an investment shifting: The banks reduce their investment into the long-term, safe asset and increase their investment into the risky credit portfolio. The short-term investment remains constant or is reduced.*

The economic interpretation of this investment shifting ( $\bar{u} > u^*$ ,  $\bar{y} < y^*$ ,  $\bar{x} \leq x^*$ ) is as follows. The CDS allow the banks to make use of diversification possibilities. The resulting reduced credit risk implies an increase in the risk averse depositors' expected marginal utility from the credit portfolio  $\partial E[U]/\partial u$  and a decrease in their expected marginal utility from the long-term, safe asset  $\partial E[U]/\partial y$ . If the incentive compatibility constraint is not binding, optimality will require the expected marginal utility from all three assets to be the same, i.e.  $\partial E[U]/\partial x = \partial E[U]/\partial y = \partial E[U]/\partial u$  (see the first order conditions given by (23) to (25)). This implies that the banks' optimal reaction to the change in the expected marginal utilities is to expand their investment into the risky credit portfolio ( $u$  increases) and to reduce their investment into the long-term, safe asset ( $y$  decreases) until  $\partial E[U]/\partial x = \partial E[U]/\partial y = \partial E[U]/\partial u$ . However, the incentive compatibility constraint may prevent a sufficient investment shifting from the safe asset to the credit portfolio. Holding  $x$  constant in that case, would imply that  $\partial E[U]/\partial y < \partial E[U]/\partial x < \partial E[U]/\partial u$ . Consequently, a further adjustment of the expected marginal utilities requires a reduction in  $x$ , since this implies a decrease in  $c_1$  which allows for a further investment shifting from the long-term, safe asset to the risky credit portfolio without violating the incentive

constraint  $c_1 \leq c_{2,L}$ .<sup>13</sup>

Comparing the consumers' expected utility without and with CDS leads us to

*Result 2: The conclusion of the CDS contracts leads to an increase in the consumers' expected utility.*

*Proof:* See proof IV in the appendix.

The reason is that the investment shifting from the long-term, safe asset to the risky credit portfolio implies an on average higher consumption in the second period since  $E(K) > R$ .

In the following section, we will analyze the consequences of the bank's changed investment behaviour for its asset buffer and its liquidity buffer. Doing this analysis one should bear in mind that the units of the numéraire the bank invests into the short-term and the two long term assets are denoted by  $x^*$ ,  $y^*$ , and  $u^*$  without CDS and by  $\bar{x}$ ,  $\bar{y}$ , and  $\bar{u}$  with CDS. Analogously, the in the deposit contract specified levels of consumption are  $c_1^*$  and  $c_2^*$  without CDS and  $\bar{c}_1$  and  $\bar{c}_2$  with CDS.

### 3.3 Shocks, Buffers, and Contagion

#### 3.3.1 Effects on the Buffers' Determinants

The determinants of a bank's buffers are a) its total assets at date 2, b) its relatively liquid assets at date 1, and c) the claims of an early consumer. The CDS exert an influence on these determinants through three effects:

**1. Investment shifting effect** ( $\bar{y} < y^*$ ,  $\bar{u} > u^*$ ): This effect influences a bank's total assets at date 2 and its relatively liquid assets at date 1. The sign of the investment shifting effect on the total assets is state dependent. In the states  $LL$  and  $LH$ , it is negative because the return on the safe asset is higher than on the badly performing credit portfolio ( $R > L$ ). In the states  $HL$  and  $HH$ , the effect is positive

<sup>13</sup>Note that the reduction in  $x$  only implies a further adjustment of the expected marginal utilities. However, a total adjustment (so that  $\partial E[U]/\partial x = \partial E[U]/\partial y = \partial E[U]/\partial u$  as in the case in which the constraint is not binding) is not achieved (see the KKT conditions given in the appendix).

because the return on the safe asset is lower than on the well performing credit portfolio ( $H > R$ ).<sup>14</sup> The sign of the investment shifting effect on a bank's relatively liquid assets at date 1 is, independently of the state, unambiguously negative due to the reduction in  $y$ .

**2. Diversification effect:** This effect influences a bank's total assets at date 2 via the compensation payments. Consequently, this effect is state dependent. In the state  $LL$ , the effect is zero since a bank receives a compensation payment but it also has to make one by itself. Analogously, the effect is positive in the state  $LH$ , negative in the state  $HL$ , and zero in the state  $HH$ .

**3. Possible reduction in the short-term investment ( $\bar{x} \leq x^*$ ):** If the incentive compatibility constraint becomes binding, the CDS may imply a decrease in  $x$ . If the optimal  $x$  decreases, the claims of an early consumer will decrease.

### 3.3.2 Asset Buffer

Analogously to the case without CDS, a bank's asset buffer is given by<sup>15</sup>

$$\bar{B}^{As} = \begin{cases} \frac{\bar{y}R + \bar{u}L - \bar{x}^{\frac{1-\gamma}{\gamma}}}{\bar{u}} & \text{in the state } LL \\ \frac{\bar{y}R + \bar{u}L - \bar{x}^{\frac{1-\gamma}{\gamma}}}{\bar{u}} + l & \text{in the state } LH. \end{cases} \quad (28)$$

The equations (12) and (28) reveal that a bank's asset buffer without CDS as well as with CDS is determined by its total assets at date 2 and by the claims of the late consumers at date 1, i.e. if they pretend to be early consumers. Consequently, all three effects described in section 3.3.1 have to be considered when analyzing the effects of CDS on this buffer. In the state  $LL$ , the overall effect of the CDS on the buffer is unambiguously negative. The investment shifting effect is negative, the diversification effect is zero, and a possible positive effect from the reduction in the short-term asset - which will occur if the incentive compatibility constraint becomes binding - does not compensate or even overcompensate the negative effect: If the incentive compatibility constraint becomes binding, we will have  $\bar{c}_1 = \bar{c}_{2,LL}$ .

<sup>14</sup>Note that the first letter describing a state refers to the performance of the credit portfolio of the considered bank. The second letter refers to the performance of the credit portfolio of that bank the considered bank has concluded a CDS contract with as a risk taker (see also page 14).

<sup>15</sup>Note that the definition of the asset shock (see page 9) implies that the states  $HL$  and  $HH$  are irrelevant.

Consequently, in this case the asset buffer reduces to zero. However, without CDS the asset buffer is strictly positive (see page 10). In the state  $LH$ , the investment shifting from the safe asset to the risky credit portfolio also has a negative effect on the buffer. However, this negative effect may be overcompensated by two positive effects. Firstly, by the positive diversification effect and secondly, by a positive effect due to the possible reduction in  $x$  if the incentive compatibility constraint becomes binding.

This leads us to the following

*Result 3: In the state  $LL$ , the CDS reduce a bank's asset buffer. In the state  $LH$ , the effect of the CDS on the asset buffer is ambiguous.*

### 3.3.3 Liquidity Buffer

A bank can react to a liquidity shock  $z$  by liquidating units of its long-term, safe asset. As in the case without CDS, the bank must liquidate

$$\bar{y}^{need} = \frac{\bar{x}z}{r\gamma} \quad (29)$$

units in order to satisfy all early consumers  $(\gamma+z)$  according to the deposit contract. However, the liquidation of the long-term asset reduces the level of consumption of the late consumers. The condition for not a run to occur is

$$\bar{c}_1 = \frac{\bar{x}}{\gamma} \leq c_2 \quad (30)$$

where

$$c_2 = \begin{cases} \frac{yR+uL}{1-\gamma-z} & \text{in the state } LL \\ \frac{yR+u\frac{H+L}{2}}{1-\gamma-z} & \text{in the states } LH \text{ and } HL \\ \frac{yR+uH}{1-\gamma-z} & \text{in the state } HH. \end{cases} \quad (31)$$

Consequently, the bank can liquidate at most the following units of the long-term, safe asset to prevent a run:<sup>16</sup>

$$\bar{y}^{crit} = \begin{cases} \bar{y} - \frac{\bar{x}(1-\gamma-z)-\bar{u}L\gamma}{R\gamma} & \text{in the state } LL \\ \bar{y} - \frac{\bar{x}(1-\gamma-z)-\bar{u}(\frac{H+L}{2})\gamma}{R\gamma} & \text{in the states } LH \text{ and } HL \\ \bar{y} - \frac{\bar{x}(1-\gamma-z)-\bar{u}H\gamma}{R\gamma} & \text{in the state } HH \end{cases} \quad (32)$$

Setting  $\bar{y}^{need}$  given by (29) equal to  $\bar{y}^{crit}$  given by (32) and solving for  $z$  gives

$$\bar{B}^{Li,TA} = \begin{cases} \frac{r(\bar{y}R+\bar{u}L-\bar{x}\frac{1-\gamma}{\gamma})\gamma}{\bar{x}(R-r)} & \text{in the state } LL \\ \frac{r(\bar{y}R+\bar{u}(\frac{H+L}{2})-\bar{x}\frac{1-\gamma}{\gamma})\gamma}{\bar{x}(R-r)} & \text{in the states } LH \text{ and } HL \\ \frac{r(\bar{y}R+\bar{u}H-\bar{x}\frac{1-\gamma}{\gamma})\gamma}{\bar{x}(R-r)} & \text{in the state } HH \end{cases} \quad (33)$$

which is a bank's liquidity buffer limited by its total assets in the second period. If  $z > \bar{B}^{Li,TA}$ , the bank which is hit by the shock will not be able to liquidate enough assets to satisfy all early consumers according to the deposit contract without causing a bank run. The bank will go bankrupt.

As in the case without CDS, it may be that  $\bar{y}^{crit} > \bar{y}$ , i.e. the bank could liquidate more assets without causing a run, but it has no more liquid assets. Then, analogously to the case without CDS, the bank's liquidity buffer is limited by its liquid assets and it is given by

$$\bar{B}^{Li,LA} = \frac{\bar{y}r\gamma}{\bar{x}}. \quad (34)$$

Consequently, a bank's liquidity buffer is given by

$$\bar{B}^{Li} = \min [\bar{B}^{Li,TA}, \bar{B}^{Li,LA}]. \quad (35)$$

**Comparison of the Liquidity Buffers Limited by Liquid Assets:** The equations (17) and (34) show that this liquidity buffer without CDS as well as with CDS is determined by the bank's stock of its relatively liquid asset ( $y^*$ ,  $\bar{y}$ ) and by the claims of an early consumer ( $c_1^* = x^*/\gamma$ ,  $\bar{c}_1 = \bar{x}/\gamma$ ). Consequently, the first effect (investment shifting from the long-term safe asset to the credit portfolio) and the

<sup>16</sup>The critical value  $\bar{y}^{crit}$  is obtained by setting  $c_2$  given by equation (31) equal to  $\bar{c}_1 = \bar{x}/\gamma$  and solving for  $y$ .

third effect (reduction in the short-term asset) described in section 3.3.1 have to be considered when analyzing the effects of the CDS on this buffer. If the incentive compatibility constraint does not become binding so that there is no reduction in  $x$  ( $\bar{x} = x^* = \gamma$ ), the introduction of CDS will reduce this liquidity buffer in all states due to the  $y$ -reducing investment shifting. If the constraint becomes binding,<sup>17</sup> this negative effect on the buffer may be dampened due to the possible reduction in  $x$ . This possible reduction has a positive effect on the buffer because it reduces the claims of an early consumer which implies that the bank's liquidity needs are reduced. However, this positive effect does not compensate or overcompensate the negative effect due to the decrease in  $y$  since the relative change in  $y$  is strictly larger than the relative change in  $x$ , i.e. even if the constraint becomes binding, the introduction of CDS unambiguously reduces this liquidity buffer.

*Proof:* See proof V in the appendix.

The reason for the larger relative change in  $y$  than in  $x$  is the following. If the incentive compatibility constraint becomes binding,  $\bar{c}_1$  will equal  $\bar{c}_{2,LL}$  and the CDS will imply an investment shifting from the long-term, safe asset ( $\bar{y} < y^*$ ) and from the short-term asset ( $\bar{x} < x^*$ ) to the risky credit portfolio ( $\bar{u} > u^*$ ). The level of consumption in the first period  $\bar{c}_1$  is influenced negatively by the decrease in  $x$ , while the level of consumption in the second period  $\bar{c}_{2,LL}$  is influenced negatively by the decrease in  $y$ . In addition,  $\bar{c}_{2,LL}$  will be influenced positively by the increase in  $u$  if  $L > 0$ . Consequently, there must be a larger relative decrease in  $y$  than in  $x$  to maintain the equality between  $\bar{c}_1$  and  $\bar{c}_{2,LL}$ .<sup>18</sup> This leads us to

*Result 4: The CDS reduce a bank's liquidity buffer limited by its liquid assets.*

<sup>17</sup>Note that if the constraint becomes binding, the buffer can be limited by a bank's liquid assets only in the states  $LH$ ,  $HL$ , and  $HH$ . The reason is that if the constraint is binding,  $\bar{c}_{2,LL} = \bar{c}_1$ , i.e. the buffer limited by the bank's total assets at date 2 will reduce to zero in this state and, therefore, will be the minimum of the two buffers (see equation (35)).

<sup>18</sup>If  $L = 0$ , the increase in  $u$  obviously does not influence  $\bar{c}_{2,LL}$ . In this case, the relative change in  $x$  and  $y$  will be the same. However, even in this case the overall relative change in  $y$  will be larger than in  $x$ . The reason for this is that without CDS, the constraint is not binding due to assumption (6). Consequently, before the constraint becomes binding implying the relative change in both variables to be the same, there will be already a decrease in  $y$ , while  $x$  will remain constant.



**Comparison of the Liquidity Buffers Limited by Total Assets:** The equations (16) and (33) show that this liquidity buffer without CDS as well as with CDS is determined by the bank's total assets at date 2 and by the claims of the late consumers at date 1, i.e. if they pretend to be early consumers. Consequently, all three effects described in section 3.3.1 have to be considered when analyzing the effects of the CDS on this buffer. If the incentive compatibility constraint does not become binding,  $x$  will remain constant, i.e. the CDS will influence the buffer only via the investment shifting effect and the diversification effect. In the state  $LL$ , the investment shifting effect reduces the buffer, the diversification effect is zero. Consequently, the overall effect of the CDS on this buffer is negative. In the state  $LH$ , the negative effect from the investment shifting may be compensated or overcompensated by the positive diversification effect, the overall effect is ambiguous. This is also the case in the state  $HL$ , in which there is positive investment shifting effect and a negative diversification effect. However, in the state  $HH$ , the overall effect of the CDS on the buffer is positive since the investment shifting implies an increase in the buffer while the diversification effect is zero. Now let us consider what will happen to the liquidity buffer limited by the total assets if the introduction of the CDS implies that the incentive compatibility constraint becomes binding, meaning that a reduction in  $x$  may occur. Although this reduction has a positive effect on this buffer since it reduces the claims of an early consumer, the overall effect of the CDS on this buffer is qualitatively the same as in the case in which the constraint does not become binding: In the state  $LL$ , the overall effect is still unambiguously negative: If the incentive compatibility constraint becomes binding,  $\bar{c}_{2,LL} = \bar{c}_1$ , i.e. in this state the buffer reduces to zero while without CDS, there is always a positive buffer (see page 12). In the states  $LH$  and  $HL$ , the overall effect of the CDS on the buffer is still ambiguous: In the state  $LH$ , the two positive effects resulting from the diversification and the possible reduction in  $x$  may be compensated or overcompensated by the negative investment shifting effect; and in the state  $HL$ , the two positive effects due to the investment shifting and the reduction in  $x$  may be compensated or overcompensated by the negative diversification effect. And finally in the state  $HH$ , the overall effect is obviously still positive.

This leads us to

*Result 5: The CDS reduce a bank's liquidity buffer limited by its total assets in the state LL. In the states LH and HL, the effect of the CDS on this buffer is ambiguous, and in the state HH, the effect of the CDS on this buffer is positive.*

### 3.3.4 Contagion

In the model without CDS, there can be no contagion since there are no linkages between the banks. However, with CDS there is a linkage between them and contagion is possible. Let us assume that bank  $A$  is hit by an asset shock  $s$  or a liquidity shock  $z$ , and that this shock is larger than the relevant buffer so that the bank goes bankrupt. In this case, it cannot fulfill its obligations from the CDS contract it has concluded with bank  $D$  as a risk taker. This may lead to the bankruptcy of bank  $D$  if from bank  $D$ 's point of view the state  $LL$  is realized, i.e. if its own credit portfolio fails and that of bank  $C$ , too. Then, it does not receive a compensation payment although its own credit portfolio fails, but it has to make a compensation payment to bank  $C$ . Consequently, the level of consumption bank  $D$  provides to the late consumers in this state  $c_{2,LL}$  is smaller than the level specified in the deposit contract  $\bar{c}_{2,LL}$ .<sup>19</sup>

$$c_{2,LL} = \frac{\bar{y}R + \bar{u}(L - l)}{1 - \gamma} < \frac{\bar{y}R + \bar{u}L}{1 - \gamma} = \bar{c}_{2,LL}. \quad (36)$$

If this implies that that  $c_{2,LL} < \bar{c}_1$ , also bank  $D$  goes bankrupt. If then also bank  $C$  and  $B$  realize  $LL$ , all banks will go bankrupt. Consequently, if the extreme case occurs that all banks realize  $L$  and if one bank is hit by a shock it cannot absorb so that it goes bankrupt and if this implies the bankruptcy of one further bank, all

<sup>19</sup>In this context it becomes obvious why we use a four-bank structure instead of a more simple two-bank structure. If there are only two banks,  $A$  and  $D$ , their claims on each other resulting from the compensation payments would be simply offset. Bank  $D$  would not receive a compensation payment from bank  $A$  due to the bankruptcy of that bank, but then it would simply set off the compensation payment it has to make to bank  $A$ , and date-2 withdrawals at bank  $D$  are  $c_{2,LL} = \bar{c}_{2,LL} \geq \bar{c}_1$ , i.e. there will be no contagion.

other banks will also go bankrupt.<sup>20</sup> This leads us to

*Result 6: If an asset shock or a liquidity shock leads to the bankruptcy of one bank, the CDS may imply the bankruptcy of other banks. The introduction of CDS creates a possible channel of contagion.*

The intuition for this result is that the CDS imply that banks have claims on one another contingent on the outcome of their credit portfolio. If one bank fails, another bank may suffer a loss because its claim on the troubled bank reduces to zero independent of the outcome of its own credit portfolio. This loss may lead to the failure of this bank, too. In extreme cases, the crisis passes from one bank to another and captures the whole banking sector.

## 4 Stability of the Banking Sector

The stability of the banking sector reflects its shock absorbing ability.<sup>21</sup> The more stable the banking sector is, the higher is its shock absorbing ability, and the lower is the systemic risk. The systemic risk is the risk that a shock leads to a systemic crisis, i.e. to a severe impairment of the general functioning of a major part of the financial system.<sup>22</sup> In the following, we consider two types of shocks which can trigger a systemic crisis, an idiosyncratic shock and a systematic shock. An idiosyncratic shock is a shock which hits a single bank only. It will lead to a systemic crisis if the bank cannot absorb this shock and if its failure leads in a sequential fashion to the

<sup>20</sup>Note that if bank  $A$  is hit by a liquidity shock and goes bankrupt although its credit portfolio succeeds ( $H$  is realized), there can be contagion so that the banks  $D$  and  $C$  go bankrupt if they realize the state  $LL$ . However, the crisis cannot pass to bank  $B$  since this bank realizes the state  $LH$  due to the succeeding credit portfolio of bank  $A$ . This implies that bank  $B$  does not go bankrupt since in this state the level of consumption bank  $B$  provides to its late depositors  $c_{2,M}$  is smaller than the level specified in the deposit contract  $\bar{c}_{2,M}$ , but it is not smaller than the level of consumption provided to the early consumers:  $\bar{c}_1 \leq \bar{c}_{2,LL} = c_{2,M} = (\bar{y}R + \bar{u}L)/(1 - \gamma) < \bar{c}_{2,M} = (\bar{y}R + \bar{u}(L + l))/(1 - \gamma)$ .

<sup>21</sup>In its Financial Stability Reviews, the ECB defines financial stability as a condition in which the financial system - comprising of financial intermediaries, markets, and market infrastructures - is capable of withstanding shocks and the unravelling of financial imbalances, thereby mitigating the likelihood of disruptions in the financial intermediation process which are severe enough to significantly impair the allocation of savings to profitable investment opportunities.

<sup>22</sup>We adopt this definition for a systemic crisis from Bandt de and Hartmann (2002).

failure of several other banks (contagion). A systematic shock is a shock which hits a large number of banks simultaneously. If this shock results in the bankruptcy of these banks, there will be a systemic crisis. Contagion plays no role in this case. In our model, the asset shock  $s$  and the liquidity shock  $z$  will be idiosyncratic shocks if only one bank ( $A$ ,  $B$ ,  $C$ , or  $D$ ) is hit by the shock. For this shock to result in a systemic crisis, two conditions have to be fulfilled: Firstly, the relevant buffer must be smaller than the shock, so that the bank which is hit by the shock goes bankrupt. Secondly, there must be contagion. In our model, the asset shock  $s$  and the liquidity shock  $z$  will be systematic shocks if all banks are hit simultaneously by the shock. For this shock to result in a systemic crisis the sufficient condition is that the relevant buffer is smaller than the shock. Contagion plays no role. In a recession, the fraction of badly performing loans is usually high, while in a boom it is usually small.<sup>23</sup> Therefore, in a stylized manner, the state  $LL$  in our model describes the situation in a recession, the state  $HH$  in a boom and the states  $LH$ , and  $HL$  describe times characterized by a moderate economic up- or downturn. Considering this, our model results lead us to

*Result 7: The CDS reduce the stability of the banking sector and thereby increase systemic risk in a recession. In a boom and in times characterized by a moderate economic up- or downturn the CDS can reduce the stability of the banking sector, i.e. they can increase systemic risk.*

We use Table 2, which summarizes the effects of CDS in different states of the macroeconomic environment, in order to elaborate on this result. In a *recession*, the CDS unambiguously reduce the banks' buffers and therefore "improve" the first condition described above for a systemic crisis to occur. Furthermore, in this state they create a channel of contagion, i.e. their introduction also implies the fulfillment of the second condition. Consequently, in a recession, the CDS increase systemic risk independent of the type of shock (idiosyncratic or systematic). In a *boom*, the CDS can reduce the banks' liquidity buffer and, therefore, they can "improve" the first condition. Consequently, in this state of the macroeconomic environment the

<sup>23</sup>For a possibility of modelling the linkage between credit defaults and the business cycle see, for example, Pennacchi (2006).

CDS can increase the risk that a systematic liquidity shock results in a systemic crisis. With regards to idiosyncratic shocks, the possible reduction in the liquidity buffer can increase systemic risk only if there are other channels of contagion such as an interbank deposit market as in Allen and Gale (2000). In times characterized by a *moderate economic up- or downturn*, the CDS can reduce the banks' asset buffer and their liquidity buffer. Furthermore, they create a possible channel of contagion (see footnote 20). Consequently, in these times they can increase the risk that a systematic shock as well as the risk that an idiosyncratic shock results in a systemic crisis.

	Recession	Moderate Economic Up- and Downturns	Boom
Asset Buffer	↓	↓ ↑	
Liquidity Buffer, Total Assets	↓	↓ ↑	↑
Liquidity Buffer, Liquid Assets	↓	↓	↓
Asset Shock, Contagion Possible	yes	no	no
Liquidity Shock, Contagion Possible	yes	yes	no

**Table 2: Effects of the CDS in Different Macroeconomic Environments**

## 5 Summary

The aim of this paper has been to analyze the consequences of CDS for the stability of the banking sector and, therefore, for systemic risk, i.e. the risk that an idiosyncratic shock or a systematic shock leads to a systemic crisis. We have considered CDS contracts which are concluded between banks in order to improve the diversification of their credit risks. We have shown that in a recession, these CDS impair the stability of the banking sector and that in a boom or in times characterized by a moderate economic up- or downturn these CDS can reduce the stability of the banking sector. There are two crucial points for our results. Firstly, the CDS

induce the banks to increase their investments into a risky, illiquid credit portfolio and to reduce their investments into a safe, relatively liquid asset. The reason for this investment shifting is that the CDS improve the diversification of the banks' credit risk. The resulting lower risk allows the banks, which seek to maximize the expected utility of their risk averse depositors, to invest more into the risky credit portfolio which has a higher expected return than the safe asset. Secondly, the CDS create a possible channel of contagion because they imply that banks have contingent claims on each other. If these claims materialize, and if one bank fails, another bank will realize a loss it may not be able to absorb. In our analysis, we have focused on CDS in which both, the protection buyer as well as the protection seller, is a bank. Currently, banks still constitute the majority of both. However, especially as protection sellers insurance companies also play an important role in the markets for credit risk transfer, and the fraction of hedge funds as protection sellers and protection buyers has increased rapidly (British Bankers' Association, 2006). The consequences for financial stability of credit risk transfer between the banking sector and the insurance sector has been analyzed by Allen and Gale (2006) and Allen and Carletti (2006), for example. The consequences for financial stability of credit risk transfer in which one counterparty is a hedge funds, should be an interesting topic for future research.

## Appendix

### ***Proof I:***

It has to be shown that  $\tilde{x} = \gamma$ . Multiplying both sides of equation (23) with  $\tilde{x}$ , of equation (24) with  $\tilde{y}$ , and of equation (25) with  $\tilde{u}$ , we obtain

$$\begin{aligned} \gamma &+ \frac{(1-\alpha)^2}{\tilde{c}_{2,LL}} \tilde{y}R + \frac{2(1-\alpha)\alpha}{\tilde{c}_{2,M}} \tilde{y}R + \frac{\alpha^2}{\tilde{c}_{2,HH}} \tilde{y}R \\ &+ \frac{(1-\alpha)^2}{\tilde{c}_{2,LL}} \tilde{u}L + \frac{2(1-\alpha)\alpha}{\tilde{c}_{2,M}} \tilde{u} \frac{H+L}{2} + \frac{\alpha^2}{\tilde{c}_{2,HH}} \tilde{u}H = \lambda(\tilde{x} + \tilde{y} + \tilde{u}) = \lambda. \end{aligned}$$

This implies that

$$\gamma + (1-\alpha)^2(1-\gamma) + 2(1-\alpha)\alpha(1-\gamma) + \alpha^2(1-\gamma) = \lambda.$$

Consequently,  $\lambda = 1$  implying that  $\tilde{x} = \gamma$  according to equation (23). ■

### ***Proof II:***

It has to be shown that

$$\begin{aligned} \frac{\partial h}{\partial u}(x^*, u^*) &= (1-\alpha)\alpha \left[ (R-L) \left( \frac{1}{c_{2,LL}^*} - \frac{1}{c_{2,M}^*} \right) \right. \\ &\quad \left. + (H-R) \left( \frac{1}{c_{2,M}^*} - \frac{1}{c_{2,HH}^*} \right) \right] > 0, \end{aligned}$$

where  $c_2^*$  denotes the level of consumption according to (26) at the point  $(x^*, u^*)$ .

We have

$$h(x, u) = \gamma \ln c_1 + (1-\gamma) \left\{ (1-\alpha)^2 \ln c_{2,LL} + 2(1-\alpha)\alpha \ln c_{2,M} + \alpha^2 \ln c_{2,HH} \right\},$$

where  $c_1$  is given by equation (2) and  $c_{2,LL}$ ,  $c_{2,M}$ ,  $c_{2,HH}$  are given by (26). Differentiating  $h$  with respect to  $u$  gives

$$\frac{\partial h}{\partial u} = \frac{(1-\alpha)^2(L-R)}{c_{2,LL}} + \frac{2(1-\alpha)\alpha \left( \frac{H+L}{2} - R \right)}{c_{2,M}} + \frac{\alpha^2(H-R)}{c_{2,HH}}. \quad (37)$$

Consequently,

$$\frac{\partial h}{\partial u}(x^*, u^*) = \frac{(1-\alpha)^2(L-R)}{c_{2,LL}^*} + \frac{2(1-\alpha)\alpha\left(\frac{H+L}{2} - R\right)}{c_{2,M}^*} + \frac{\alpha^2(H-R)}{c_{2,HH}^*}, \quad (38)$$

where  $c_2^*$  denotes the level of consumption according to (26) at the point  $(x^*, u^*)$ .

Since

$$c_{2,LL}^* = c_{2,L}^* = \frac{(H-L)R(1-\alpha)}{H-R} \quad \text{and} \quad c_{2,HH}^* = c_{2,H}^* = \frac{(H-L)R\alpha}{R-L},$$

it holds that

$$(1-\alpha)\frac{L-R}{c_{2,LL}^*} + \alpha\frac{H-R}{c_{2,HH}^*} = 0$$

and consequently,

$$\begin{aligned} (1-\alpha)^2\frac{L-R}{c_{2,LL}^*} + \alpha^2\frac{H-R}{c_{2,HH}^*} &= (1-\alpha)\frac{L-R}{c_{2,LL}^*} - \alpha(1-\alpha)\frac{L-R}{c_{2,LL}^*} \\ &\quad + \alpha\frac{H-R}{c_{2,HH}^*} - \alpha(1-\alpha)\frac{H-R}{c_{2,HH}^*} \\ &= -\alpha(1-\alpha)\left[\frac{L-R}{c_{2,LL}^*} + \frac{H-R}{c_{2,HH}^*}\right]. \end{aligned} \quad (39)$$

From (38) and (39) we obtain

$$\begin{aligned} \frac{\partial h}{\partial u}(x^*, u^*) &= \alpha(1-\alpha)\left[\frac{H+L-2R}{c_{2,M}^*} - \frac{L-R}{c_{2,LL}^*} - \frac{H-R}{c_{2,HH}^*}\right] \\ &= \alpha(1-\alpha)\left[(R-L)\left(\frac{1}{c_{2,LL}^*} - \frac{1}{c_{2,M}^*}\right) + (H-R)\left(\frac{1}{c_{2,M}^*} - \frac{1}{c_{2,HH}^*}\right)\right], \end{aligned}$$

where the expression on the right hand side is strictly positive since  $L < R < H$ ,

$$c_{2,LL}^* < c_{2,M}^* < c_{2,HH}^*. \quad \blacksquare$$



**Proof III:**

The KKT conditions for a maximizer  $(\bar{x}, \bar{y}, \bar{u})$  of (22) s.t. all constraints (5), being necessary and sufficient since the objective  $g$  is concave and all constraints are linear (for details about KKT conditions see e.g. Sun and Yuan (2006), Theorem 8.2.7, Corollary 8.2.9 and Theorem 8.2.11), are the following

$$\frac{\gamma}{\bar{x}} + \mu_x - \frac{\nu}{\gamma} = \lambda, \quad (40)$$

$$\left[ \frac{(1-\alpha)^2}{\bar{c}_{2,LL}} + \frac{2(1-\alpha)\alpha}{\bar{c}_{2,M}} + \frac{\alpha^2}{\bar{c}_{2,HH}} \right] R + \mu_y + \frac{\nu R}{1-\gamma} = \lambda, \quad (41)$$

$$\frac{(1-\alpha)^2}{\bar{c}_{2,LL}} L + \frac{2(1-\alpha)\alpha}{\bar{c}_{2,M}} \left( \frac{H+L}{2} \right) + \frac{\alpha^2}{\bar{c}_{2,HH}} H + \mu_u + \frac{\nu L}{1-\gamma} = \lambda, \quad (42)$$

$$\mu_x, \mu_y, \mu_u, \nu \geq 0, \quad (43)$$

$$\mu_x \bar{x} = \mu_y \bar{y} = \mu_u \bar{u} = \nu \left( \frac{R\bar{y} + L\bar{u}}{1-\gamma} - \frac{\bar{x}}{\gamma} \right) = 0, \quad (44)$$

$$\bar{x} + \bar{y} + \bar{u} = 1, \quad \bar{x}, \bar{y}, \bar{u} \geq 0, \quad \frac{R\bar{y} + L\bar{u}}{1-\gamma} - \frac{\bar{x}}{\gamma} \geq 0. \quad (45)$$

Multiplying both sides of equation (40) with  $\bar{x}$ , of equation (41) with  $\bar{y}$ , and of equation (42) with  $\bar{u}$ , adding the 3 equations and regarding (44) and  $\bar{x} + \bar{y} + \bar{u} = 1$  we obtain again  $\lambda = 1$ .

If we assume that  $\bar{x} > \gamma$ , then (40) implies  $\mu_x > \frac{\nu}{\gamma} \geq 0$ , i.e.  $\bar{x} = 0$  due to (44), a contradiction. Consequently it must hold  $\bar{x} \leq \gamma$ . Moreover, if  $\bar{x} < \gamma$  then (40) implies  $\frac{\nu}{\gamma} > \mu_x \geq 0$ , hence  $\frac{R\bar{y} + L\bar{u}}{1-\gamma} - \frac{\bar{x}}{\gamma} = 0$ , i.e. the incentive compatibility constraint must be binding.

It remains to show that  $\bar{y} < y^*$ , since then we also have

$$\bar{u} = 1 - \bar{x} - \bar{y} > 1 - x^* - y^* = u^*.$$

We first consider the case that the incentive compatibility constraint becomes binding, i.e. that it holds

$$\frac{R\bar{y} + L\bar{u}}{1 - \gamma} = \frac{\bar{x}}{\gamma}.$$

After inserting  $\bar{u} = 1 - \bar{x} - \bar{y}$  into this equation and some rearrangement we obtain

$$(R - L)\bar{y} = \left( \frac{1 - \gamma}{\gamma} + L \right) \bar{x} - L.$$

On the other hand, due to (6) we obtained

$$\frac{Ry^* + Lu^*}{1 - \gamma} > \frac{x^*}{\gamma}$$

implying

$$(R - L)y^* > \left( \frac{1 - \gamma}{\gamma} + L \right) x^* - L.$$

Because of  $\bar{x} \leq x^*$  we can conclude

$$(R - L)\bar{y} = \left( \frac{1 - \gamma}{\gamma} + L \right) \bar{x} - L \leq \left( \frac{1 - \gamma}{\gamma} + L \right) x^* - L < (R - L)y^*,$$

i.e.  $\bar{y} < y^*$ .

If the incentive compatibility constraint does not become binding then we have  $\bar{x} = x^* = \gamma$  and we can argue that it must hold  $\bar{u} > u^*$  and  $\bar{y} < y^*$  in the same way like we did it for  $\tilde{u}$  and  $\tilde{y}$  in the case where the inequality constraints have been neglected. ■

#### ***Proof IV:***

It has to be shown that

$$\max g(x, y, u) > \max f(x, y, u).$$

Let  $(x^*, y^*, u^*)$  be the solution of the optimization problem without CDS, i.e. the maximizer of  $f$  and  $c_{2,LL}^*$ ,  $c_{2,LH}^*$ ,  $c_{2,HL}^*$  and  $c_{2,LL}^*$  the consumption at *this* point with

CDS. Since then,  $c_{2,LL}^* = c_{2,L}^*$  and  $c_{2,HH}^* = c_{2,H}^*$  we have

$$\begin{aligned} g(x^*, y^*, u^*) - f(x^*, y^*, u^*) \\ = (1 - \gamma)\alpha(1 - \alpha)[2 \ln c_{2,M}^* - \ln c_{2,LL}^* - \ln c_{2,HH}^*]. \end{aligned}$$

Furthermore, we have

$$c_{2,LL}^* + \frac{u^*l}{1 - \gamma} = c_{2,M}^* < c_{2,HH}^* = c_{2,M}^* + \frac{u^*l}{1 - \gamma}.$$

Setting  $a := \frac{u^*l}{1 - \gamma}$ , we obtain

$$\begin{aligned} g(x^*, y^*, u^*) - f(x^*, y^*, u^*) = (1 - \gamma)\alpha(1 - \alpha) \Big\{ [\ln(c_{2,LL}^* + a) - \ln c_{2,LL}^*] \\ - [\ln(c_{2,M}^* + a) - \ln c_{2,M}^*] \Big\} > 0 \end{aligned}$$

by the strict concavity of  $\ln$ . Consequently,

$$\max g(x, y, u) \geq g(x^*, y^*, u^*) > f(x^*, y^*, u^*) = \max f(x, y, u)$$

what had to be shown. ■

### ***Proof V:***

It has to be shown that

$$\bar{B}^{Li,LA} < B^{*Li,LA} \tag{46}$$

even if the incentive compatibility constraint becomes binding.

If the constraint becomes binding

$$\frac{R\bar{y} + L\bar{u}}{1 - \gamma} = \frac{\bar{x}}{\gamma}$$

holds implying

$$\bar{B}^{Li,LA} = \frac{\bar{y}r\gamma}{\bar{x}} = \frac{r}{R} \left( 1 - \gamma - \frac{\gamma L\bar{u}}{\bar{x}} \right).$$

Since due to assumption (6) without CDS the incentive compatibility constraint is not binding we have

$$\frac{Ry^* + Lu^*}{1 - \gamma} > \frac{x^*}{\gamma}$$

implying

$$B^{*Li,LA} = \frac{y^* r \gamma}{x^*} > \frac{r}{R} \left( 1 - \gamma - \frac{\gamma Lu^*}{x^*} \right).$$

From  $\bar{u} > u^*$  and  $\bar{x} \leq x^*$  we can conclude that

$$\bar{B}^{Li,LA} = \frac{r}{R} \left( 1 - \gamma - \frac{\gamma L \bar{u}}{\bar{x}} \right) < \frac{r}{R} \left( 1 - \gamma - \frac{\gamma Lu^*}{x^*} \right) < B^{*Li,LA}$$

what had to be shown. ■

## References

- ALLEN, F., AND E. CARLETTI (2006): “Credit Risk Transfer and Contagion,” *Journal of Monetary Economics*, 53, 80–111.
- ALLEN, F., AND D. GALE (2000): “Financial Contagion,” *Journal of Political Economy*, 108, 1–33.
- (2006): “Systemic Risk and Regulation,” Working Paper No. 05-24 (January 2006), Financial Institutions Center, Wharton School, University of Pennsylvania.
- BANDT DE, O., AND P. HARTMANN (2002): “Systemic Risk in Banking: A Survey,” in *Financial Crises, Contagion, and the Lender of Last Resort*, ed. by C. Goodhart, and G. Illing, chap. 14, pp. 249–297. Oxford University Press, Oxford.
- BRITISH BANKERS’ ASSOCIATION (2006): “Credit Derivatives Report 2006,” London.
- DEUTSCHE BUNDESBANK (2004): “Credit Risk Transfer Instruments: Their Use by German Banks and Aspects of Financial Stability,” *Monthly Report (April 2004)*, pp. 27–44.
- DIAMOND, D. W., AND P. H. DYBVIG (1983): “Bank Runs, Deposit Insurance, and Liquidity,” *Journal of Political Economy*, 91, 401–419.
- FREIXAS, X., AND J.-C. ROCHET (2008): *Microeconomics of Banking*. MIT Press, Cambridge (Mass.), 2 edn.
- HELLWIG, M. (1994): “Liquidity provision, banking, and the allocation of interest rate risk,” *European Economic Review*, 38, 1363–1389.
- PENNACCHI, G. (2006): “Deposit Insurance, Bank Regulation, and Financial System Risks,” *Journal of Monetary Economics*, 53, 1–30.
- SUN, W., AND Y. YUAN (2006): *Optimization Theory and Methods. Nonlinear Programming*. Springer, Berlin.
- WAGNER, W. (2007): “The Liquidity of Bank Assets and Banking Stability,” *Journal of Banking and Finance*, 31, 121–139.
- WAGNER, W., AND I. MARSH (2006): “Credit Risk Transfer and Financial Sector Stability,” *Journal of Financial Stability*, 2, 173–193.

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