# Four Essays on Modeling and Analyzing Investor Interactions in Financial Markets, Herding Behavior, and Speculative Bubbles

**Inaugural-Dissertation**

to obtain the academic degree of Doctor of Business Administration
(doctor rerum politicarum – Dr. rer. pol.)

submitted to the

Faculty of Business Administration and Economics
Heinrich Heine University Düsseldorf

presented by

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Düsseldorf, November 6, 2024

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**Acknowledgements**

First and foremost, I would like to thank my doctoral advisor, Prof. Dr. Christoph J. Börner, for his professional and personal support at all times during the preparation of this thesis. I have considered it a privilege to be able to freely choose the topics for each project that sparked my research interest.

I would also like to thank Prof. Dr. Ulrike Neyer for her co-supervision.

Moreover, I would like to thank my mentor and co-author Prof. Dr. Ingo Hoffmann for the many engaging discussions, the fruitful collaboration, the honest feedback and the overall trusting relationship.

During my time at the chair, I have had the opportunity to meet a number of colleagues who have made this a remarkable period of time, which I have greatly enjoyed and during which I have been able to grow personally. In particular, I would like to thank our last team, consisting of Anne-Marie Ossig, Julia Schedrina, Lars M. Kürzinger, Philipp Stangor and Jonas Krettek, for the many professional and personal conversations that helped to bring this to a successful conclusion. I owe a special thanks to the latter, who, as my office mate, was an unfailing support with a great sense of humor.

Last but not least, I would like to thank my family and friends for always believing in me and supporting me in every way.

*Düsseldorf in November 2024*

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**List of Abbreviations**

| | |
|---|---|
| ADF | Augmented Dickey-Fuller (test) |
| ARIMA | Autoregressive integrated moving average |
| AR($p$) | Autoregressive model of order $p$ |
| BIC | Bayesian information criterion |
| BIS | Bank for International Settlements |
| BNTX UW | Bloomberg code for company BioNTech SE |
| BSADF | Backward SADF |
| CAPM | Capital Asset Pricing Model |
| CCI | Consumer Confidence Index |
| CEO | Chief Executive Officer |
| cf. | confer |
| Chap. | Chapter |
| const. | constant |
| Cont. | Continued |
| Corp. | Corporation |
| COVID-19 | Coronavirus disease 2019 |
| Crit. | Criterion |
| CSAD | Cross-sectional absolute deviation of returns |
| Dev. | Deviation |
| DDM | Dividend Discount Model |
| DJIA | Dow Jones Industrial Average |
| econ. | economical |
| e.g. | exempli gratia |
| EMH | Efficiency Market Hypothesis |
| EPU | Economic policy uncertainty |
| Eq. | Equation |
| et al. | et alli/aliae/alia |
| etc. | et cetera |
| EU | European Union |
| FDA | Food and Drug Administration |
| FED | Federal Reserve System |
| GDP | Gross domestic product |
| GFC | Global Financial Crisis |
| GSADF | Generalized SADF |
| HAC | Heteroscedasticity and autocorrelation consistent (standard errors) |

| | |
|---|---|
| HE | Helium |
| HT | High-tech |
| i.e. | id est |
| i.i.d | Independent and identically distributed |
| Inf. | Information |
| JB | Jarque-Bera (test) |
| LOCF | Last observation carry forward |
| L.P. | Limited Party |
| LSV | Designation of the method according to Lakonishok et al. (1992) |
| MA | Moving average |
| MAE | Mean absolute error |
| MAPE | Mean absolute percentage error |
| Max | Maximum |
| Min | Minimum |
| MSE | Mean squared error |
| NASDAQ | National Association of Securities Dealers Automated Quotations |
| Obs. | Observations |
| OLS | Ordinary least squares |
| p. | page |
| PCM | Portfolio Change Measure |
| phys. | physical |
| PSY | Designation of the method according to Phillips et al. (2015) |
| PWY | Designation of the method according to Phillips et al. (2011) |
| resp. | respectively |
| RMSE | Root mean squared error |
| SADF | Sup ADF |
| SARS-CoV-2 | Severe acute respiratory syndrome coronavirus type 2 |
| Sec. | Section |
| Sep | September |
| SIC | Standard Industrial Classification |
| S&P | Standard and Poor's |
| Std | Standard deviation |
| TS | Time series |
| US | United States |
| USD | United States dollar |
| UTC | Coordinated Universal Time |

| | |
|---|---|
| UK | United Kingdom |
| VIX | Volatility index of S&P 500 |
| Vol. | Volume |
| VOLQ | Volatility index of NASDAQ 100 |
| WHO | World Health Organization |
| 2SM | Two-state model |
| 3SM | Three-state model |

**List of Symbols**

*Latin symbols*

| | |
|---|---|
| $A$ | Abbreviation for mathematical term (auxiliary function) |
| $\mathcal{B}$ | *phys.*: magnetic field, *econ.*: news sentiment |
| $B$ | Strength of news field (absolute value of $\mathcal{B}$) |
| $B_1$ | Effective message environments due to (non-)conforming adjoining agents |
| $B_2$ | Effective message environments due to neutral adjoining agents |
| $B_t$ | Bubble dummy at time $t$ |
| $\mathrm{Binom}(\cdot)$ | Binomial distribution |
| $BSADF_t$ | BSADF test statistic at time $t$ |
| $CC_t$ | Sub-period specific dummy indicating the COVID-19 pandemic at time $t$ |
| $CSAD_t$ | Cross-sectional absolute deviation of returns a time $t$ |
| $c_B$ | *phys.*: (specific heat) capacity, *econ.*: overall system sensitivity |
| $\cosh(\cdot)$ | Cosinus hyperbolicus |
| $D$ | Region of parameter vector $(\alpha_2, \alpha_3)$ |
| $DC_t$ | Sub-period specific dummy indicating the dot-com bubble at time $t$ |
| $d$ | Index indicating daily data |
| $E$ | Internal energy |
| $\mathrm{E}_c(\cdot)$ | Cross-sectional expected value |
| $\mathrm{E}_t(\cdot)$ | Expected value at time $t$ |
| $\mathrm{E}_t^b(\cdot)$ | Biased expected value at time $t$ |
| $EPU_t$ | Economic policy uncertainty at time $t$ |
| $\exp(\cdot)$ | Exponential function |
| $F$ | Free Energy |
| $\mathrm{F}(\cdot)$ | Cumulative density function |
| $FC_t$ | Sub-period specific dummy indicating the house price boom and subsequent financial crisis at time $t$ |
| $f(\cdot)$ | Function |
| $G(\cdot)$ | General function for Itos Lemma |
| $g(\cdot)$ | General boundary condition of region $D$ |
| $gCRGDP_t$ | Growth of credit as a ratio to GDP at time $t$ |
| $gGDP_t$ | GDP growth at time $t$ |
| $gIP_t$ | Industrial production growth at time $t$ |
| $gM2_t$ | Growth of money supply M2 at time $t$ |

| | |
|---|---|
| $\mathcal{H}$ | Hamiltonian/Energy Functional |
| $H_0$ | Null hypothesis |
| $H_1$ | Alternative hypothesis |
| $H_{mt}$ | Transformed herding parameter at time $t$ |
| $h_{mt}$ | Herding parameter at time $t$ |
| $i$ | Counter or index for agents or assets |
| $\inf(\cdot)$ | Infimum |
| $J, J_{ij}$ | Coupling parameter |
| $j$ | Counter or index for agents or assets |
| $K_t$ | Concatenation factor at time $t$ |
| $k$ | *phys.*: Boltzmann constant, *econ.*: scale/unit parameter |
| | Lag order in the context of the bubble estimation method |
| $\ln(\cdot)$, $\log(\cdot)$ | Natural logarithm |
| $\log_3(\cdot)$ | Logarithm with base 3 |
| $M$ | *phys.*: magnetization, *econ.*: purchase/sale potential |
| | Supremum in the context of the Popoviciu (1935) equation |
| $m$ | Mean attitude of agents (non-)conforming |
| | Infimum in the context of the Popoviciu (1935) equation |
| $m_0$ | Mean attitude of agents in neutral position |
| $m_0^c$ | Proportion of neutral positions at critical point |
| $\text{magnitude}(\cdot)$ | Magnitude operator |
| $MW$ | Number of words in a news message |
| $\mathcal{N}(\cdot)$ | Normal distribution |
| $N$ | *phys.*: number of particles, *econ.*: number of agents and shares |
| $N_+$ | Number of agents conforming |
| $N_-$ | Number of agents non-conforming |
| $N_0$ | Number of agents holding |
| $\bar{N}_0$ | Mean neutral position of the agent system |
| $N_t$ | Number of assets in the month $t$ |
| $N_{\text{pot}}$ | Trade potential |
| $\bar{N}_{\text{pot}}$ | Average trade potential |
| $\hat{N}_{\text{pot}}$ | Estimated trade potential |
| $\hat{\bar{N}}_{\text{pot}}$ | Estimated average trade potential |
| $n!$ | Permutation of $n$ |
| $NW$ | Negative words in sentiment score |
| $\mathcal{O}(\cdot)$ | Big $O$ notation |

| | |
|---|---|
| $PW$ | Positive words in sentiment score |
| $P_t$ | Price at time $t$ |
| $\hat{P}_t$ | Estimated price at time $t$ |
| $PDR_t$ | Price-dividend ratio at time $t$ |
| $p$ | p-Value |
| $p_i$ | Occupation probabilities |
| $p_t$ | Price at time $t$ in the detailed balanced condition |
| $\text{Prob}(\cdot)$ | Probability |
| $pv_t$ | Price volatility at time $t$ |
| $PS$ | Sentiment polarity score |
| $q$ | Number of states in a Potts model |
| $\mathbb{R}$ | Real numbers |
| $R^2$ | Coefficient of determination |
| $R_{it}$ | Return on asset $i$ at time $t$ |
| $R_{mt}$ | Return on market index at time $t$ |
| $r_0$ | Minimum window size |
| $r_1$ | Start point of the rolling window |
| $r_2$ | End point of the rolling window |
| $r_e$ | (Fractional) starting points of a bubble |
| $r_f$ | (Fractional) ending points of a bubble |
| $r_t$ | Reference interest rate at time $t$ |
| $r_w$ | Size of the rolling window |
| $\hat{r}_e$ | Estimated (fractional) starting points of a bubble |
| $\hat{r}_f$ | Estimated (fractional) ending points of a bubble |
| $r_{it}$ | Excess return on asset $i$ at time $t$ |
| $r_{mt}$ | Excess return on the market at time $t$ |
| $r_{it_d}$ | Daily excess return on asset $i$, month $t$ |
| $r_{mt_d}$ | Daily excess return on the market, month $t$ |
| $\mathcal{S}$ | Entropy |
| $\bar{\mathcal{S}}$ | Average entropy |
| $S$ | Positioning in relation to the message field |
| $S_i$ | Positioning of agent $i$ in relation to the message field |
| $s_i$ | Generalized state for agent $i$ |
| $scv_{r_2}^{\beta_T}$ | Critical value |
| $SENT_t$ | Consumer sentiment at time $t$ |
| $\text{Std}_c(\cdot)$ | Cross-sectional standard deviation |

| | |
|---|---|
| $\widehat{\mathrm{Std}_c(\cdot)}$ | Estimated cross-sectional standard deviation |
| $\mathrm{sech}(\cdot)$ | Secant hyperbolicus |
| $\mathrm{sign}(\cdot)$ | Signum function |
| $\sinh(\cdot)$ | Sinus hyperbolicus |
| $\sup(\cdot)$ | Supremum |
| $\mathbb{T}$ | Finite period |
| $T$ | *phys.*: temperature, *econ.*: volatility |
| | Total sample in the context of the bubble estimation method |
| $T_{15'}$ | 15-minutes volatility |
| $T_c$ | *phys.*: critical/Curie temperature in Ising model, *econ.*: critical volatility in 2SM |
| $\tilde{T}_c$ | *phys.*: critical/Curie temperature in 3SM, *econ.*: critical volatility in 3SM |
| $t$ | Time |
| $tv_t$ | Trade volume at time $t$ |
| $\tanh(\cdot)$ | Hyperbolic tangent |
| $\mathcal{U}$ | Gross utility |
| $\bar{\mathcal{U}}$ | Average gross utility |
| $U$ | Utility |
| $\bar{U}$ | Average utility |
| $U_0$ | Ground state utility |
| $V_t$ | Market capitalization at time $t$ |
| $v_{mt}$ | Error term of the measurement equation of the state space model |
| $W_t$ | Wiener process at time $t$ |
| $\mathbf{X}$ | Vector of state variables |
| $X_t$ | Logarithmic price of a stock at time $t$ |
| $x$ | Auxiliary variable |
| $Y_t$ | Parameter vector at time $t$ |
| $y_t$ | Variable for rolling window regression at time $t$ |
| $Z$ | Canonical partition function |
| $z$ | *phys.*: coordination number, *econ.*: average number of adjoining agents |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | Regression constant in the context of the robustness test |
| $\alpha$, $\alpha_1$ | Fraction of utility contribution of neutral position |
| $\alpha_2$ | Contribution of adjacent neutral positions |

| | |
|---|---|
| $\alpha_3$ | Contribution of adjacent (non-)confirming and neutral position |
| $\alpha_4$ | Contribution of field-field interaction |
| $\alpha_{r_1,r_2}$ | Rolling window regression constant |
| $\alpha_{it}^b$ | Market model regression constant for asset $i$, month $t$ |
| $\bar{\alpha}_2$, $\bar{\alpha}_3$ | Boundary condition parameter values |
| $\beta$ | *phys.*: inverse temperature, *econ.*: inverse volatility |
| $\beta_{imt}$ | Systematic risk measure at time $t$ |
| $\beta_{imt}^b$ | Biased/observed systematic risk measure at time $t$ |
| $\hat{\beta}_{imt}^b$ | Estimated observed systematic risk measure at time $t$ |
| $\overline{\hat{\beta}_{imt}^b}$ | Mean estimated observed systematic risk measure at time $t$ |
| $\beta_{r_1,r_2}$ | Coefficient of the first lagged difference of the dependent variable |
| $\gamma_1$, $\gamma_2$, $\gamma_3$, $\gamma_4$ | Coefficients of the robustness test regression model |
| $\Delta$ | Change or difference operator |
| $\delta$ | Small changes of a quantity |
| | Frequency dependent parameter in the context of the bubble estimation method |
| $\epsilon_i$ | Individual change to the Hamiltonian by agent $i$ |
| $\epsilon_t$ | Error term of the robustness test regression model |
| $\epsilon_{it_d}$ | Market model error term using daily data $d$ for month $t$ |
| $\varepsilon_t$ | Error term of the rolling window regression at time $t$ |
| $\eta$ | *phys.*: thermal magnetic loss coefficient, *econ.*: overall system sensitivity |
| $\eta_{mt}$ | Error term of the state equation of the state space model |
| $\mu$ | *phys.*: magnetic moment/energy contribution, *econ.*: utility contribution, interpreted as: willingness to trade |
| $\hat{\mu}$ | Estimated utility contribution |
| $\mu_i$ | Individualized utility contribution |
| $\mu_M$ | Expected value of $M$ |
| $\mu_X$ | Expected value of logarithmic returns |
| $\mu_m$ | Drift term of the measurement equation of the state space model |
| $\rho$ | Correlation |
| $\rho(\cdot)$ | Distribution |
| $\sigma$ | Standard deviation |
| | Volatility |
| $\sigma_M$ | Standard deviation of $M$ |
| $\sigma_X$ | Standard deviation of logarithmic returns |

| | |
|---|---|
| $\sigma^2$ | Variance |
| $\sigma^2_{mv}$ | Variance of the error term of the measurement equation of the state space model |
| $\sigma^2_{m\eta}$ | Variance of the error term of the state equation of the state space model |
| $\phi$ | Autoregressive coefficient |
| $\phi_m$ | Autoregressive coefficient of the state space model |
| $\chi$ | *phys.*: magnetic susceptibility, *econ.*: overall system sensitivity |
| $\psi^i_{r_1,r_2}$ | Coefficient of the $k$-th lagged difference of the dependent variable |
| $\Omega$ | Microcanonical partition function |

*Other symbols*

| | |
|---|---|
| % | Per cent |
| $\sum$ | Sum |
| $\infty$ | Infinity |
| $\propto$ | Proportional to |
| $\partial$ | Partial derivative |

## 1. Introduction

When thinking of a stock exchange in the traditional sense, numerous films and other works have shaped many people's associations. Traders are often depicted in large rooms, communicating with cryptic signs and papers flying around, in ecstatic or highly intense scenes. The high tension and intensity is created by the presence of individuals interacting with each other, potentially imitating one another, and collectively sparking the mood in the room. The photograph in Figure 1, entitled Chicago Board of Trade I by Andreas Gursky, is an example of this. It depicts the crowded trading floor of the Board of Trade in Chicago, with traders from various banks standing around monitors, and papers on the floor contributing to the overall impression of frenzy and intensity. It is not about the individual situations of the traders, but about the result created by the convergence of the traders, the space, and other elements (Tate, 2024).

**Figure 1:** *Chicago Board of Trade I* by Gursky (1997). 

Although this image of financial markets is no longer found on trading floors due to technological advances and the shift of significant parts of trading to digital exchanges, it still serves as a metaphorical approach to thinking about phenomena in capital markets. In such a hectic environment, emotions can spread quickly among traders, with euphoric or panicked sentiments influencing others and leading to collective behavior such as herding (Shiller, 2015). Moreover, the frenetic pace often limits information processing, leading investors to rely more on the actions of others (Barberis and Thaler, 2003). Herding behavior is a phenomenon, where investors imitate each other (Spyrou, 2013) and trade in the same direction (Nofsinger and Sias, 1999). Although much of the trading no longer takes place on trading floors as in Figure 1, trading rooms within banks or even the internet with platforms and social media provide mechanisms through which imitation can take place (Olsen, 2011). This behavior can potentially cause asset prices to diverge increasingly from their underlying fundamental values due to imitation-driven purchases. One resulting phenomenon is a speculative bubble in the stock market, which can lead to potentially significant economic damage and crises (Jordà et al., 2015). Understanding these backgrounds and mechanisms is therefore not only of academic interest, but also of practical importance for ensuring financial market and economic stability (Phillips et al., 2015).

For this reason, this dissertation is dedicated to the study of investor interactions in financial markets. The two distinct but related concepts of herd behavior and speculative bubbles are central and are considered both separately and in combination. One of the central research questions is how herding behavior can be modeled and how this modeling can be applied in finance. The literature already contains numerous different modeling approaches, as can be seen in the review articles by Spyrou (2013); Kallinterakis and Gregoriou (2017); Komalasari et al. (2022). However, this dissertation explores the intersection between physics and economics, specifically how models from statistical physics can be applied to describe collective behavior in financial markets. Over the last half century, modeling approaches from statistical physics based on the Ising model – originally introduced as a mathematical model of ferromagnetism in statistical mechanics (Brush, 1967) – have increasingly found their way into econometrics. These modeling approaches provide a framework for analyzing interactions between investors (Sornette, 2014). Another key aspect that follows modeling and can be seen as an intermediate step towards application is the question of the transferability of variables from the physical to the economic or financial context. The transfer is challenging as the systems are initially based on different principles (Sornette, 2014; Ausloos et al., 2016) and the mathematical relationships between the variables have to be preserved. Related to this is the calibration of the innovative modeling approach. As in applied physics, the question arises in applied econometrics: How can the model parameters be determined on the basis of data, e.g. from the capital market? Within the context of this modeling approach, another question that arises is its applicability in forecasting price developments or in risk assessment.

As mentioned above, herding behavior is often cited as a trigger or driver of speculative bubbles, see for example Scharfstein and Stein (1990); Lux (1995); Avery and Zemsky (1998); DeMarzo et al. (2008); Scherbina and Schlusche (2014). Empirical studies have already pointed to the potential link between herding behavior and returns (Nofsinger and Sias, 1999; Wermers, 1999; Sias, 2004; Dasgupta et al., 2011; Singh, 2013; Brown et al., 2014; Celiker et al., 2015). Therefore, this thesis also examines whether the theoretical link between herding behavior and speculative bubbles can be observed empirically.

Due to the partly high level of abstraction of these questions and the specific research gaps addressed by the studies, reference is made to the detailed explanations of the specific research contributions of the individual studies as well as of the entire dissertation in Section 1.2.

In brief, this dissertation investigates collective phenomena in the capital market using classical econometric methods from behavioral finance and econophysics, thus contributing to a better understanding of market anomalies. Before elaborating on the contribution of the thesis in detail, a theoretical background is provided to situate the studies. After discussing the efficiency of capital markets, the current state of the behavioral finance literature, which has emerged from the market efficiency debate, is presented. This framework will then be used to contextualize the contributions of this thesis. Finally, the overall contribution of the dissertation in the context of the market efficiency debate is described and the more detailed contributions of the individual studies are presented.

## *1.1. Market Efficiency and its Limitations*

In his contribution to the discussion of market efficiency and the introduction of behavioral finance, Shleifer (2000) adopts an approach that will be followed here. Initially, the theoretical and empirical arguments in favor of the efficiency market hypothesis (EMH) are presented, followed by a juxtaposition of the theoretical and empirical challenges to the hypothesis. This approach provides a nuanced perspective on the market efficiency debate and thus allows for a better contextualization of the studies included in this dissertation.

### *1.1.1. Theoretical Base of EMH*

The EMH is a central financial model that describes how financial markets work. It has been the subject of much theoretical and empirical research and is at the heart of a long-standing academic debate on market efficiency. Although the following discussion does not follow the chronology of these studies, this perspective is also instructive. For a chronological presentation, references such as Sewell (2011) and Ramiah et al. (2015) can be considered.

Fama (1970) defines an efficient financial market as one in which asset prices always fully reflect the available information. More specifically, his definition of an efficient market involves active competition among many profit-maximizing and rational agents, each attempting to predict the future market values of individual securities using almost freely available information. Investor rationality means that securities are valued on the basis of their fundamental value, i.e. according to the net present value of future cash flows, which are discounted on a risk-adjusted basis (Shleifer, 2000). Competition among the many participants means that the actual prices of individual securities at any given time already incorporate the effects of both past and anticipated events. Due to uncertainty, it is not possible to precisely determine the intrinsic value of a security. Consequently, market participants may have different expectations about the future performance of a security, potentially leading to discrepancies between the actual price and the intrinsic value. If these discrepancies are systematic, competition will eventually neutralize them. Therefore, in an efficient market, the current price serves as the best estimate of the intrinsic value of a security. Fama (1965a,b) further deduces that successive price adjustments occur independently of each other when (new) information is immediately incorporated into market participants' expectations of intrinsic value. This reasoning underpins the random walk hypothesis, which states that stock returns are unpredictable based on past returns, at least in the short run (Samuelson, 1965; Mandelbrot, 1966).

However, the EMH holds even without the strict assumption of investor rationality. In many scenarios, markets are efficient even if not all investors are fully rational. For example, if irrational investors trade randomly and their trading strategies are uncorrelated, they are likely to cancel each other out and prices remain close to their fundamental values. The assumption of uncorrelated strategies is, however, very restrictive (Shleifer, 2000). Yet, as Friedman (1953) and Fama (1965b) show, the EMH also holds against the trading strategies of irrational investors being correlated due to arbitrage, provided that the securities have close substitutes. Arbitrage is defined as "the simultaneous purchase and sale of the same, or essentially similar, security in two different markets at advantageously different prices" (Sharpe et al., 1999). Suppose an asset is overvalued in the market relative to its fundamental value due to correlated purchases by irrational investors. Arbitrageurs would then sell the asset, or even short-sell it, while simultaneously buying other 'essentially equivalent' assets to hedge their risk. If suitable substitutes, i.e., 'essentially equivalent' assets, are available and tradable, arbitrageurs can thus make risk-less profits. By selling the asset, the price approaches the fundamental value again. If arbitrage works quickly and effectively due to the availability of substitute assets and arbitrageurs compete with each other for profits, the price of an asset can never stray far from its fundamental value. The same applies to an undervalued asset. However, the prerequisite for both scenarios is the availability of suitable substitutes (Shleifer, 2000).

Moreover, irrational investors who purchase overpriced assets generate lower returns than passive investors and arbitrageurs. Therefore, Friedman (1953) points out that irrational traders will lose money and become less wealthy until they eventually disappear from the market. If arbitrage does not immediately neutralize the influence of irrational investors, their wealth diminishes due to market forces. Hence, market efficiency can persist due to arbitrage and competitive selection (Shleifer, 2000).

#### *1.1.2. Empirical Base of EMH*

The theoretical arguments in favor of the EMH were accompanied by empirical studies that sought to test the predictions of the EMH. Shleifer (2000) divides these empirical predictions into two broad categories. First, asset prices should react quickly and correctly when new information about the value of an asset becomes available (Ramiah et al., 2015). This implies that those who perceive the information late should not be able to benefit from it. Correctness means that price adjustments in response to the news should, on average, be appropriate, i.e. neither too large nor too small as a result of overreaction or underreaction. This excludes, e.g., that neither trends nor price reversals take place after a novelty has initially been reflected in the price. The second key prediction is that prices should not react in the absence of new information. Both imply that stale information has no value or is not useful for making money, as Fama (1970) points out.

However, the empirical verification of this statement requires a concretisation of what is meant by 'making money' and 'stale information'. In the financial context, making money means generating a superior return taking into account the risk incurred. Generating a positive cash flow on average exploiting 'stale information' is therefore not evidence for market inefficiency. Instead, risk must be taken in order to achieve a return and the profit realized could be fair market compensation for the corresponding risk-taking. However, the problem with answering the question of whether it is fair compensation is that measuring the risk of a particular investment (strategy) is difficult and a model of the relationship between risk and return is needed (Shleifer, 2000). Various models exist for this purpose, such as the dividend discount model (DDM) proposed by Gordon (1962), which measures the intrinsic value of a company based on its underlying dividends. The capital asset pricing model (CAPM) by Sharpe (1964), Lintner (1965) and Mossin (1966) is an equilibrium model that focuses on the company-specific risk (beta) and the market risk premium as an indicator of the expected return on an investment. The three-factor model proposed by Fama and French (1992, 1996a,b) supplements the systematic risk factor beta in the CAPM with the factors size and book-to-market ratio. The insight by Fama (1970), that most tests of market efficiency depend on a specific risk-return model, has significantly shaped the debate.

Defining what is meant by 'stale information' leads Fama (1970) to distinguish three types of 'stale information', and thus three degrees of market efficiency. The weak form of market efficiency exists when information consists of historical prices. Under the assumption of risk neutrality, this form of market efficiency corresponds to the random walk hypothesis. In the semi-strong form of EMH, information corresponds to all publicly available information. In the strong form of information efficiency, it is not even possible to achieve superior profits based on insider information, i.e., information that is not known to every market participant. Most of the empirical evidence has focused on the weak and semi-strong forms of efficiency, as the strong form of market efficiency may not be an accurate description of reality (Fama, 1970). Studies testing the weak form of efficiency largely draw on the random walk literature, indicating that stock prices can indeed be approximated by random walks (Fama, 1965a, 1970). The tests of semi-strong market efficiency aim to examine the adjustment of stock prices to specific news events, such as announcements of annual earnings (Ball and Brown, 1968), stock splits (Fama et al., 1969), discount rate changes (Waud, 1970), or takeover bids (Keown and Pinkerton, 1981). The results of these so-called event studies are consistent with the semi-strong form of market efficiency. Thus, the empirical evidence on weak and semi-strong market efficiency has been largely confirmatory. The same is true for the second prediction of the EMH that prices do not react in the absence of new information. Scholes (1972) employs the event study methodology to examine how stock prices react to the announcement of large investors selling a significant amount of a security. Arbitrage along efficient markets would imply that no significant price changes occur because an investor's sale does not affect the fundamental value of the underlying stock. The author argues that security prices are set relative to each other, as the expected returns of assets with the same risk profile are equal. This means that the supply of stocks does not influence the price, or only very briefly, as price differences due to block sales/purchases are swiftly arbitraged away. He refers to this mechanism, where mere changes in demand for a stock should have only a minimal effect on the price, as the substitution hypothesis. If arbitrage is necessary for market efficiency, then individual securities must have close substitutes for arbitrage to work. Scholes findings indicate relatively small price reactions to block sales, which are explained by the fact that a large sale itself constitutes implicit information. Thus, this finding is consistent with his substitutions hypothesis and, consequently, the second implication of the EMH, that prices do not react to non-information (Shleifer, 2000).

After numerous empirical studies provided evidence in favor of market efficiency and rejected opportunities for earning abnormal returns by attributing them to unaccounted risks, both theoretical and empirical arguments challenging the EMH began to accumulate. Although these arguments were initially empirical in nature, similar to Shleifer (2000), for the sake of simplicity, the theoretical problems will be presented first.

*1.1.3. Theoretical challenges to the EMH*

First, investors are not fully rational. In the economic literature, this insight is associated with the concept of bounded rationality. This concept is used to describe human decision making characterized by cognitive and informational limitations (Simon, 1955; Conlisk, 1996). In relation to the financial market, Black (1986) explains that many investors react to irrelevant information that does not indicate changes in the actual value of an asset. He refers to this non-information, and the market movements that result from trading on it, as noise. In short, uninformed market participants too rarely follow passive investment strategies, which they would be expected to do according to the EMH, and instead invest actively in individual stocks, e.g., and are not sufficiently diversified (Shleifer, 2000).

Moreover, the deviations from rationality turn out to be widespread and systematic, as explained in numerous articles (Tversky and Kahneman, 1974; Kahneman and Riepe, 1998; Barberis and Thaler, 2003; Baker and Ricciardi, 2014). The following selection of behavioral biases is based on the classification of Barberis and Thaler (2003) of psychological insights into the human decision-making process that are particularly relevant to financial economics. The categorization considers biases that arise when investors form beliefs on the one hand and influence preferences on the other. In relation to belief formation, the following elements are briefly discussed: overconfidence, optimism and wishful thinking, representativeness, conservatism, belief perseverance, anchoring and availability biases. When forming their expectations, investors suffer from overconfidence and tend to overestimate their abilities and information (Ricciardi, 2008). From a practical point of view, this bias explains excessive trading and low returns, as well as the mistake of not diversifying sufficiently (Baker and Ricciardi, 2014). Another similar bias is that formed expectations tend to be optimistically skewed. For instance, optimists overestimate their talents, underestimate the probabilities of adverse events over which they have no control, and are prone to an illusion of control, where the influence of chance is underestimated. The combination of optimism and overconfidence leads people to underestimate risks, overestimate their knowledge, and overestimate their control over events (Kahneman and Riepe, 1998). Representativeness is a heuristic in which intuitive predictions are made based on the outcome that appears most representative or similar to known outcomes. Ricciardi (2008) defines heuristics as: “simple and general rules a person employs to solve a specific category of problems under conditions that involve a high degree of risk-taking behavior and uncertainty.” This heuristic disrupts the logic of statistical prediction and leads to the neglect of relevant information in the decision-making process (Kahneman and Tversky, 1973; Tversky and Kahneman, 1974; Ricciardi, 2008).

The conservatism bias refers to the tendency for beliefs to be updated in an orderly and proportional manner according to true updated figures based on Bayes' theorem, but not to a sufficient extent when new information becomes available. In the financial context, this leads to underreaction to corporate events, such as earnings announcements (Edwards, 1982; Hirshleifer, 2015). Belief perseverance refers to the persistence in maintaining a belief even when new, contradictory information becomes available. This occurs partly because individuals are inherently reluctant to seek out contradictory information and partly because they approach perceived contradictory information with considerable skepticism. A provocative example of belief perseverance in the financial market would be the maintenance of belief in the EMH even after evidence to the contrary has been presented. When this effect is even more pronounced, with contradictory information being misinterpreted to support false beliefs, it is referred to as confirmation bias (Lord et al., 1979; Barberis and Thaler, 2003). Anchoring, on the other hand, refers to the phenomenon whereby individuals' estimates are influenced by an initial reference point or anchor. When making estimates, individuals often start from an initial value, which they then adjust to arrive at a final estimate. However, different initial values lead to different estimates, indicating that the estimates are biased towards the initial values and are insufficiently adjusted (Tversky and Kahneman, 1974). The availability bias is a heuristic whereby events are evaluated based on immediate examples that come to mind and are readily available. This can lead to distortions, as not all memories are equally retrievable, and more recent events are perceived as more likely (Tversky and Kahneman, 1974; Barberis and Thaler, 2003).

The second psychological pillar concerns investor preferences, which are a crucial assumption in many economic and financial models. A typical assumption is that investors make decisions under uncertainty according to the expected utility framework. The theoretical motivation for this dates back to von Neumann (2007), which was first published in 1944. According to this framework, preferences are rational and can be represented by the expectation of a utility function if the axioms of completeness, transitivity, continuity, and independence are satisfied. However, experimental studies show that people systematically violate the expected utility theory in their decisions. A successful alternative explanatory approach that reflects these experimental findings is the 'Prospect Theory'. This approach is not a normative theory but rather aims to represent people's attitudes as simply as possible (Barberis and Thaler, 2003). According to this theory, investors evaluate outcomes relative to a reference point, which is often the status quo, and gains and losses are assessed relative to this point. Another characteristic is loss aversion, where losses are felt more intensely than gains of the same magnitude. In terms of probability weighting, people tend to weight probabilities non-linearly, resulting in the tendency to overestimate small probabilities and underestimate large ones. This results in an S-shaped value function, which is concave (diminishing marginal utility) for gains and convex (increasing marginal disutility) for losses (Kahneman and Tversky, 1979; Odean, 1998). As another example, ambiguity aversion refers to the inconsistency of preferences, where known risks are preferred to unknown risks (Ellsberg, 1961; Barberis and Thaler, 2003). There are many examples of investors behaving differently from what is expected by normative economic models. When decisions are based on heuristics rather than Bayesian rationality, this behavior is often referred to as 'investor sentiment', 'unsophisticated' or 'noise' (Black, 1986; Shleifer, 2000).

The list of behavioral biases clearly shows that deviations from the maxim of economic rationality are very likely to be systematic rather than random and therefore do not cancel each other out. This is exacerbated by the phenomenon of herding behavior, whereby investors, as social beings, tend to imitate each other and make similar buy or sell decisions as their peers (Nofsinger and Sias, 1999; Spyrou, 2013). This applies not only to retail investors, but also to professional investors. As human beings, they are themselves subject to these biases. Moreover, in the management of third-party funds, their role as agents and the associated delegation introduce further distortions that are likely to lead to deviations from rationality (Scharfstein and Stein, 1990; Lakonishok et al., 1992).

Given that investors are neither fully rational nor able to offset their irrational trading decisions due to the high systematic nature of human biases, the theoretical validity of the EMH depends on the effectiveness of arbitrage. However, one central argument of behavioral finance is that arbitrage is risky and thus limited. This is partly because 'essentially equivalent' assets, or close substitutes, are not always available. For derivative assets, this is typically not a problem. However, arbitrage does not work, for example, when the price levels of stocks or bonds are generally too high, as there are no substitute portfolios for these broad classes of securities. Regarding an overall overvaluation of stocks as a class, for instance, the arbitrageur could at most sell stocks or reduce exposure. However, this type of arbitrage is no longer risk-free due to the high positive average returns of stocks. If the arbitrageur is risk-averse, their interest in engaging in such arbitrage will be low (Shleifer, 2000). Even if individual securities have better substitutes, there is a deterrent fundamental risk because the substitutes are not necessarily perfect. For an arbitrageur, there is the risk that unexpectedly positive news will emerge about a security he has shorted, or conversely that unexpectedly negative news will emerge about a long position. Arbitrage with imperfect substitutes is therefore risky, which is why it is called 'risk arbitrage'. Even if a security has a perfect substitute, there is another significant source of risk. This arises from the possibility that irrational traders can drive prices further away from their fundamental value. Even if an arbitrageur identifies a mispricing and takes a position, the behavior of noise traders can exacerbate the mispricing and increase the arbitrageur's losses (de Long et al., 1990). If arbitrageurs are unable to maintain positions due to rising losses, arbitrage is also limited for this reason. In addition, the capital tied up in these positions, especially over longer periods, incurs opportunity costs. Consequently, the arbitrage-based theoretical argument for the EMH is also limited, even when close substitutes exist (Shleifer, 2000; Gromb and Vayanos, 2010).

By contrasting the theoretical arguments for market efficiency, it becomes clear that theory does not necessarily lead to the conclusion that markets are efficient. Empirical challenges are considered below to complete the picture.

#### *1.1.4. Empirical challenges to the EMH*

An early important empirical challenge is the excess volatility puzzle, discovered by Shiller (1981, 1997) and LeRoy and Porter (1981). Puzzles refer to problems where empirical observations do not match theoretical predictions (Sornette, 2014). This puzzle, related to stock market volatility, basically concerns the fact that stock price volatility is too high to be explained solely by the arrival of new information about future dividends. The study is based on a simple model in which prices correspond to the expected net present value of future dividends. The result has been interpreted by some as indicating that price changes occur without any fundamental justification, but rather due to sunspots or mass psychology (Shiller, 2003). This led to much criticism, suggesting that the fundamental value was misspecified, for example due to a constant discount rate (Merton, 1987; Shiller, 2003). Nevertheless, this finding was a catalyst for further criticism of the EMH. The weak form of market efficiency was challenged by, among others, the work of De Bondt and Thaler (1985). They constructed portfolios of past winners and losers and showed that past losers had higher subsequent returns over the following 1-36 months, which could not be explained by higher risk. One possible explanation is that investors extrapolate good and bad news, leading to an over- or undervaluation of stocks. Since the portfolios were constructed solely based on stale information, this calls into question the weak form of the EMH. The tendency of securities to continue their past price movements in the short term is known as momentum and has also been identified as a significant predictor (Jegadeesh and Titman, 1993). Even Fama (1991) admits that stock returns are predictable based on past returns, contrary to the conclusions of earlier studies. Subsequently, other variables have been identified that predict future returns and challenge the semi-strong form of market efficiency, such as the market-to-book ratio. This variable measures the ratio of the market value of a company's equity to its book value, with high values attributed to highly valued growth companies and low values attributed to cheaply valued companies. High values could result from the extrapolation of past good news, i.e. overreactions. Consistent with this, De Bondt and Thaler (1987); Fama and French (1992); Lakonishok et al. (1994) find that portfolios with high book-to-market ratios have generated lower returns than portfolios with lower ratios. In addition, higher ratios are associated with higher market risk (Lakonishok et al., 1994). Fama and French (1993, 1996a), however, interpret both market capitalization, i.e. the size of a company, and the book-to-market ratio as measures of fundamental risk and include these two variables as factors in the so-called three-factor model. This model posits that stocks of smaller firms or those with low market-to-book ratios are inherently riskier, as reflected in their sensitivity to size and market-to-book factors, and should therefore offer higher average returns. On the other hand, larger stocks are considered safer and thus yield lower returns. Similarly, growth stocks with high market-to-book ratios provide lower average returns because they act as hedges against market-to-book risk. The higher returns of smaller firms and lower book-to-market ratios are interpreted by Fama and French as proxies for compensated 'distress risk'. However, this interpretation is sharply criticized by Shleifer (2000), primarily due to the lack of evidence supporting its validity.

Finally, there is also empirical evidence challenging the notion that prices do not react to non-information. These findings are largely consistent with the evidence on the excess volatility of stock returns. For example, Roll (1984) examined the impact of weather news on orange futures prices and found that while there was a correlation, the variance explained by weather was too small, despite his argument that weather was the main driver. In a second study, Roll (1988) extended this idea to individual stocks and found that the variance in returns explained by aggregate economic influences, returns of other companies in the same industry and firm-specific news events was very low. The conclusion of both studies is that shocks other than news appear to move stock prices. Another phenomenon mentioned by Shleifer (2000) is the inclusion of companies in the S&P 500 Index during recomposition, as these events trigger uninformed demand shifts in the affected stocks that are not neutralized by arbitrage.

Both Shleifer (2000) and Barberis and Thaler (2003) summarize that investor sentiment and limited arbitrage are the two pillars of behavioral finance. In his review of the research, Shiller (2003) points out that while the price movements of individual stocks can indeed be explained by fundamentals, the aggregate market is less easily accounted for. Thus, market efficiency should not be expected to be so fundamentally flawed that continuous profits can be made by exploiting inefficiencies. However, insisting on market efficiency can lead to misjudging events such as speculative bubbles. Therefore, the weaknesses of the EMH should always be taken into account, especially when conducting research. Theoretical models such as the EMH have their validity as descriptions of an ideal world, but they have limitations in accurately describing real markets in their pure form. Shiller (2003) therefore suggests an eclectic approach.

After elucidating the fundamental theoretical and empirical concepts of behavioral finance, it is pertinent to examine the further development and differentiation of this research field. The following discussion will therefore focus on the recent contributions and trends in the behavioral finance literature, which have further deepened the understanding and application of these theories.

Following the publication of numerous studies on human biases and empirical findings on anomalies from the 1970s to the 1990s, as outlined above, the literature has become increasingly integrated and more widely accepted in subsequent years. This is due to the relevance of the scientific contributions and also to notable recognition, such as the Nobel Prizes awarded to Daniel Kahneman in 2002 (The Nobel Prize, 2002) and Richard H. Thaler in 2017 (The Nobel Prize, 2017). The insights from these studies have not only influenced financial market theories (Barberis and Thaler, 2003), but are also of significant relevance in several other fields such as public economics (Thaler and Sunstein, 2009), health economics (Frank, 2004; Rice, 2013), labor economics (Fehr and Falk, 2002), consumer economics (Thaler, 1980), environmental economics (Allcott, 2011), education (Bettinger and Slonim, 2007) and development economics (Banerjee and Duflo, 2011). This increasingly interdisciplinary orientation is also evident in the financial context, specifically through approaches in neuroscience (Lo and Repin, 2002), sociology (Shiller, 2015), and physics (Sornette, 2014). The integration and application of physical approaches to modeling economic relationships are referred to in the literature as econophysics (Chakraborti et al., 2011a,b). Three of the contributions in this dissertation fall within this area of research.

This cumulative thesis consists of four projects. The first three papers address an overarching project that focuses on modeling economic phenomena using approaches from statistical physics. Traditional economic theory posits that individual agents, such as investors, make rational and autonomous decisions in the capital market. However, as seen above, increasing recognition of agent imperfections has significantly influenced economic research, leading to analyses of behavioral economic factors like imitation effects (Bouchaud, 2013). Herding behavior, a related phenomenon, can arise from imitation or more broadly from interactions between different agents, and may instigate or accompany major risk events (Bekiros et al., 2017). Within this context, the field of econophysics has gained prominence by applying models from statistical physics to socioeconomic phenomena, as these models are particularly adept at capturing interaction effects among system agents (Sornette, 2014). The Ising model (Ising, 1925) is a standard model in this context, particularly due to its simplicity in representing the interaction between disorder-enhancing private information and order-enhancing social imitation (Sornette, 2014). The core objective of this overarching project is the application of an improved version of the Ising model to the capital market, where the physical model parameters and variables need to be appropriately matched to corresponding capital market metrics. In addition, the model is empirically calibrated using econometric methods and ultimately used for forecasting. The aim is to make both a theoretical and, more importantly, a practical contribution. As the physical model is essentially composed of two components, the endeavor is divided into the first two research projects to ensure a thorough treatment of each component.

The first study focuses on the model component where agents or investors react to new information independently, i.e., without interacting with each other. Since the overarching goal of the study is calibration, that is, the empirical identification of parameters with capital market data, a special situation in the capital market must be identified, through which the corresponding parameters can be measured using measurement equations. This approach is similar to the methodology of physical experiments, where special experimental setups are designed to measure the relationships of interest. Unlike in physics, it is not possible to construct experiments in the context of capital markets. Instead, specific time periods that embody the desired effects must be analyzed. These cut-out experiments then allow the parameters in question to be measured. The so-called cut-out experiment in this study models the impact of sudden risk events or price jumps resulting from fundamental news. Additionally, the underlying Ising model is extended to include a third state, allowing investors to buy, sell, and hold stocks, thus better replicating the decision-making situation of real investors. The central contribution is to show how the top-down model is empirically calibrated and how this calibration is implemented. Moreover, its practical utility is demonstrated with a forecast. While not calibrating a bottom-up model (classic agent-based models), the study might contribute to the literature on model calibration, as the identified parameters might be transferred to bottom-up approaches.

The second study focuses on the model dynamics arising from interactions between agents, or investors in the capital market context. The three-state model is also used here, and it is shown how the remaining model parameters can be calibrated on an empirical basis. The empirical calibration is implemented using econometric methods. With the fully calibrated model, it can be demonstrated how short-term herding behavior can emerge during phase transitions of external state variables such as price volatility in the capital market. The model phenomenology is shown and discussed in detail through simulations. This analysis provides the insight that applying binary models to situations with three decision alternatives can result in biased predictions.

The third project differs from the methodological contributions of the first two projects, but is linked to the context of econophysical modeling and provides a theoretical contribution in terms of the transfer of variables from statistical physics to economics and financial markets. The identified analogy between the state variable temperature in spin systems and volatility in the capital market is often assumed to be proportional or is subject to ambiguity. However, a common understanding of the meaning and possible measurability of the variables is important, otherwise the practical applicability of the model is at risk. In the context of the first two papers, this third project, which can also be classified as basic theoretical research, thus ensures the interpretation and measurability of a central state variable, so that the parameterisation of the overall model gains in validity. In this project, this proportionality is derived theoretically and algebraically. The result is a derived measurement equation that can be used for empirical applications. In short, papers one to three can be seen as basic theoretical research focusing on the calibration and parameter identification of models describing reactions to news (agent-field) and individual imitation (agent-agent) in the capital market context.

The fourth study thematically builds upon the first three projects by investigating herding behavior as a driver for speculative bubbles. Anomalies such as speculative bubbles have the potential to cause significant economic damage, making it crucial to understand the factors leading to such phenomena from both a theoretical and practical perspective. Building on the few existing empirical studies investigating the driving factors of bubbles, the fourth study focuses on herding behavior as a potential driving factor, which is hypothesized by theory. However, this contribution methodologically distinguishes itself from the previous studies as it exclusively employs economic and econometric methods and features methodological objectivity in the empirical analysis by combining two established methods for calculating the input variables in different regression specifications.

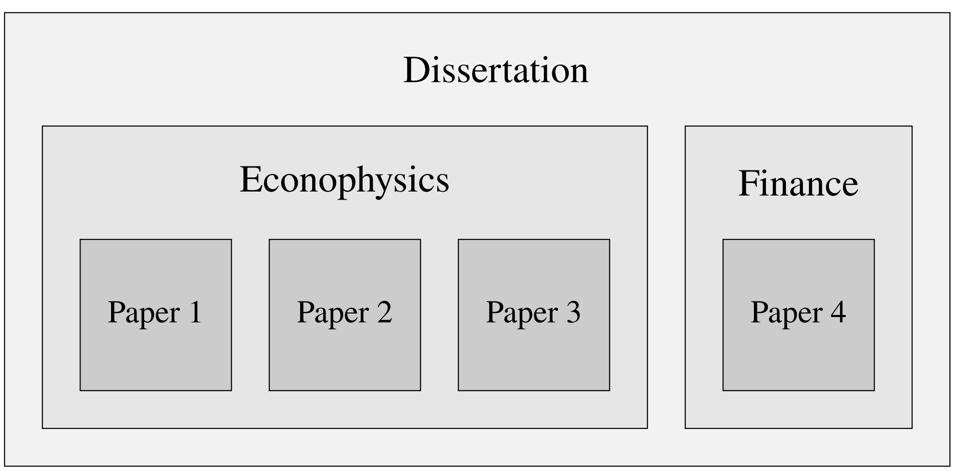


**Figure 2:** Structure of the thesis.

Contrary to the theoretical expectation that herding behavior contributes to the formation of bubbles, the results indicate the opposite: the probability of bubbles occurring decreases when herding behavior occurs in the market. However, when overreactions occur in subsets of the overall market, the probability of bubbles increases. This insight has important implications for market surveillance in the context of potential market crashes and crises following bubbles.

Figure 2 presents the structure of the thesis. The studies of the dissertation can be methodologically classified using a framework for systematizing current behavioral finance research. This framework, and the categorization of the individual contributions within it, also makes the overall contribution of the thesis more tangible. The framework developed by Sharma and Kumar (2019), shown in Figure 3, is based on two main areas: human psychological studies, which examine how psychological biases affect investors, and empirical studies, which focus on the role of investor behavior in market phenomena. Specifically, the empirical studies emphasize market anomalies, excessive and asymmetric volatility, the role of sentiment, the limits of arbitrage, and studies that integrate sentiment into a behavioral asset pricing model. Furthermore, as a future direction for the literature, the integration of these two categories is proposed, where primary and secondary data are combined.

Within this framework, all four papers in this dissertation can be classified as empirical studies that focus on the influence of investor behaviour on the capital market at different levels of abstraction. While investor biases are implicitly considered in this analysis, they are not explicitly addressed or foregrounded. Instead, what is of interest here are the patterns that emerge in the aggregate, allowing for possible inferences and explanations of market anomalies. The investor sentiment expressed therein (e.g. in the form of herd behavior) is a potential driver of price distortions, as introduced in Section 1.1. Addressing price distortions also entails studying the consequences of limited arbitrage, without explicitly foregrounding it. Thus, the two core elements of behavioral finance are implicitly addressed within the scope of this dissertation.

In terms of the contribution of the studies to the discussion of market efficiency, the econophysics studies clearly focus on modeling interactions and possibly subsequent herding behavior. Although herding behavior can be rational, in the sense of imitating better informed market participants, it is still classified as a potentially irrational phenomenon (Avery and Zemsky, 1998; Litimi et al., 2016). Thus, this modeling approach addresses the softened assumption of rationality among market participants, one pillar of behavioral finance (see above). Moreover, herding behavior is a theoretically possible driver of capital market anomalies such as speculative bubbles, where massive mispricing of assets can occur. Asset prices can deviate from their intrinsic values due to successive trading decisions by different investors, e.g. due to groupthink reinforced by the increasing interconnectedness of the internet

(Olsen, 2011), or due to direct imitation of the same trading decisions (see later chapters). The three econophysics studies do not directly address this form of anomaly but demonstrate at a micro-level how herding behavior can emerge and be modeled in the market. The issue of speculative bubbles as an anomaly and potential consequence of herding behavior is explored in the fourth paper. The central analysis focuses on the theoretically assumed connection that herding behavior drives bubbles. Thus, in an abstract sense, the dissertation addresses market (in)efficiency at the level of anomalies and attempts to find explanations for them (cf. Figure 3).

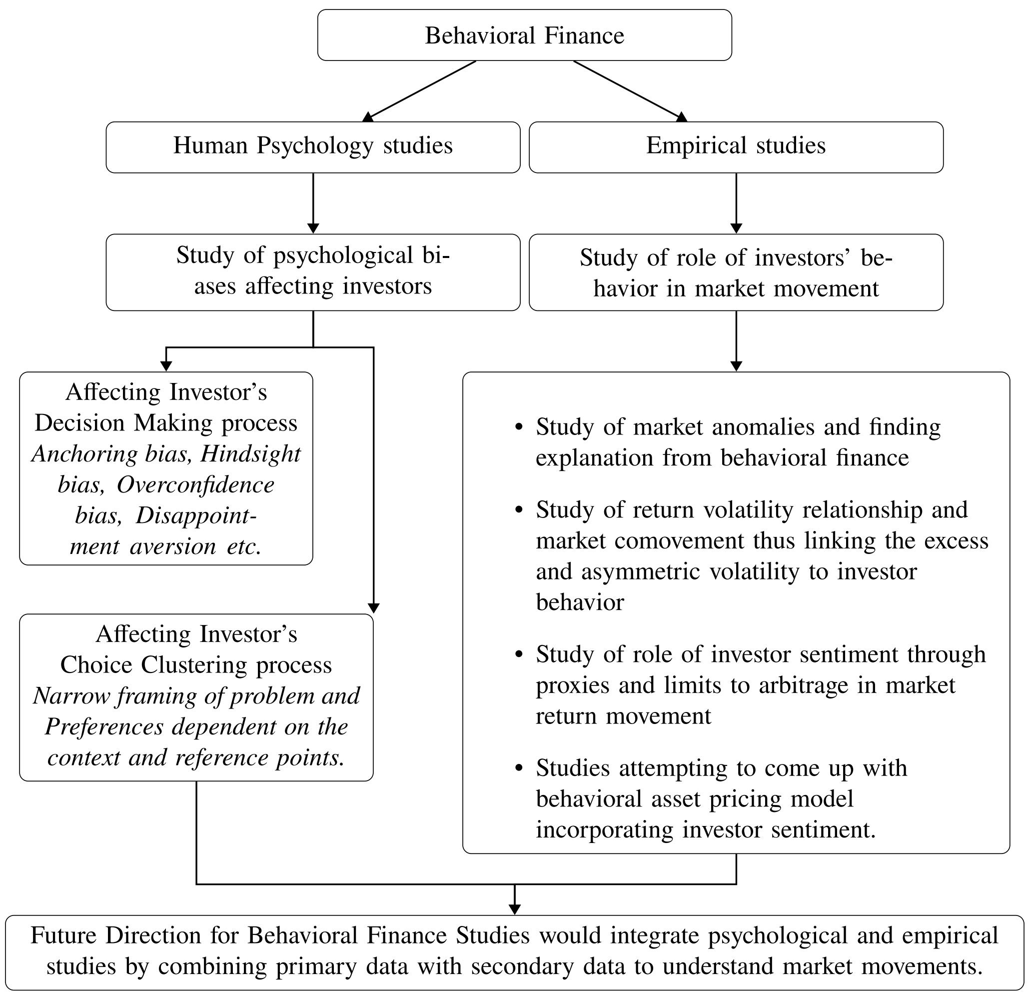


**Figure 3:** Original illustration of the behavioral finance research framework based on Sharma and Kumar (2019).

## 2. Ideal Agent System with Triplet States: Model Parameter Identification of Agent-Field Interaction

### *2.1. Abstract*

On the capital market, price movements of stock corporations can be observed independent of overall market developments as a result of company-specific news, which suggests the occurrence of a sudden risk event. In recent years, numerous concepts from statistical physics have been transferred to econometrics to model these effects and other issues, e.g., in socioeconomics. Like other studies, we extend the approaches based on the "buy" and "sell" positions of agents (investors' stance) with a third "hold" position. We develop the corresponding theory within the framework of the microcanonical and canonical ensembles for an ideal agent system and apply it to a capital market example. We thereby design a procedure to estimate the required model parameters from time series on the capital market. The aim is the appropriate modeling and the one-step-ahead assessment of the effect of a sudden risk event. From a one-step-ahead performance comparison with selected benchmark approaches, we infer that the model is well-specified and the model parameters are well determined.

**Keywords:** Agent System, Canonical Ensemble, Entropy, Partition Function, Risk Assessment, Utility Function

**JEL Classification:** C10, C46, C51

### 2.2. Introduction

Investors in individual stocks are sometimes confronted with sudden risk events that can occur due to individual company information. In addition to leaked company-internal information or (ad hoc) mandatory stock market announcements, the price-moving news can also appear in the form of unanticipated scientific publications. A current example of the latter is the shares of the pharmaceutical company BioNTech SE (ISIN US09075V1026). Political decisions, scientific studies on the vaccine produced, news about novel SARS-CoV-2 variants, and ad hoc statements from management make the price suddenly rise or fall, depending on the intensity and content of the new information.

In the phase after the information has been disseminated, the prices of the securities concerned show a dynamic that is largely decoupled from the overall market. The question arises whether such dynamics can be modeled appropriately and whether the risk event, which is a strong price movement or price jump (Föllmer, 1974; Cont and Tankov, 2004), can be assessed one step ahead. From a practical perspective, these assessments allow statements to be made about the price development depending on news that might occur. In this way, risk events can be hedged ex ante.

The main focus of this study is on the empirical calibration of a top-down Ising-based model for sudden risk events using the BioNTech SE share as an example. On the other hand, there are bottom-up agent-based model approaches that also face the unsolved problem of appropriate empirical parameter calibration (Sornette, 2014). When these models are applied and tested on real data, empirical calibration, and validation is an issue that is widely discussed in the literature on agent-based models (Werker and Brenner, 2004; LeBaron, 2006; Windrum et al., 2007; Fagiolo et al., 2007; Chen et al., 2012; Iori and Porter, 2012; Fagiolo et al., 2019). In particular, there is a need to explore how the models can be used in capital market applications, given the difficulty of empirically calibrating the models or properly choosing values for the parameters (Sornette, 2014). Although we cannot solve the problem in a generalized way, we contribute by showing how to empirically measure the model parameters of a top-down three-state model so that it can be used in practical applications. We show not only that it is possible to parameterize such a model, but also how to do so, and that it has practical utility in the application of forecasting. Thereby, we might help build the bridge between both approaches because some of the identified parameter values could possibly be used in bottom-up approaches. In addition, we develop the basic design for a forecasting procedure and compare its performance to selected benchmark approaches.

The approaches from the field of statistical physics, which in recent years have increasingly found their way into research in econometrics, appear to be a suitable framework for modeling such problems of abrupt price movements and the corresponding risk. These observations in econometrics are reminiscent of phase transitions in dynamical systems due to changing external variables. In physics, similar processes have been studied extensively and successfully in theory and practice since the early 19th century (Isihara, 1971; Landau and Lifšic, 1980; Greiner et al., 1995; Kardar, 2007), e.g., spontaneous magnetization of matter. The statistical description of agent systems in econometrics as an image of a many-body system considered in statistical physics has experienced a significant boom in recent years and emerged as a separate strand of literature that can be traced back to the "Sociodynamics" research by (Weidlich, 1971) and somewhat later to the "Sociophysics" framework of (Galam et al., 1982), among other contributions. For an overview of the econophysics literature, see, e.g., Chakraborti et al. (2011a,b); Bouchaud (2013); Sornette (2014); Schinckus (2016, 2018); Kutner et al. (2019).

The further developed methods in econometrics have been applied to a wide range of problems and particularly targeted the effects of human interaction: decision-making, voting behavior, capital market developments, etc. See, for example Kaizoji (2000), who modeled the tendency of investors to be influenced by the investment attitude of other traders, which led to regimes of bubbles and crashes. Michard and Bouchaud (2005) found imitations in three different data sources: birth rates, sales of cell phones, and the decline of applause in concert halls. Sornette and Zhou (2006) study a model in which interaction terms are reassessed continuously in time as investors are able to learn from past experiences. Borghesi and Bouchaud (2007) proposed a generic model for multiple-choice situations in the presence of herding and compared it with data from a music market experiment. Oh and Jeon (2007) studied membership dynamics in the open source software community with a spin model. Vikram and Sinha (2011) studied a model in which interaction dynamics are mediated by asset prices as a global variable accessible to every agent. Krause and Bornholdt (2012) studied the process of investors' opinion formation, and (Bouchaud, 2013) reviewed recent studies on decision models. Zhang et al. (2015) studied the volatility of financial time series with an Ising system, and Crescimanna and Di Persio (2016) proposed a variation of the Ising model to study the characteristics of stock markets. Fernandez et al. (2016) studied a three-state model and attempted to understand how social processes such as cooperation or organization happen (this list is not intended to be complete). Furthermore, in Appendix 2.6, the correspondence table (Table 5) can be used to trace in detail which sources guided us in transferring the variables from physics to econometrics.

Depending on the application in physics, various thermodynamic potentials are defined in statistical physics on the basis of partition functions, and all thermodynamic relations are derived from the latter. The different thermodynamic potentials are linked to one another via Legendre transformations, and the value of all partition functions are determined by eigenvalues of a defined functional $\mathcal{H}$ (*phys.*: Hamiltonian), i.e., the microscopic structure of the system under consideration. Hence, the various alternatives for the description of the state of the system under consideration are linked to one another. Statements that are derived in one model frame must be consistently reflected in the other frames.

In econometrics, the canonical partition function $Z$ and the free energy $F$ – as the associated thermodynamic potential – are usually considered for agent systems with binary decisions: "buy" or "sell"; "follow" or "not follow"; and "elect" or "not elect" (*phys*.: Two-State Spin-Systems) cf. Foley (1999); Marsili (1999); Bouchaud (2013) and the vast literature cited therein. Applied to the stock market, this model approach describes the macroscopic behavior of a system of $N$ agents (investors) who, through their binary decisions to "sell" or "buy" a stock, influence the price of the stock, accounting for the overall market and a flow of information. If instead a particular share and the acting agents are viewed as an isolated, closed system, decoupled from the overall market, then the microcanonical partition function $\Omega$ has to be evaluated, and the associated thermodynamic potential is the entropy $\mathcal{S}$ of the system. In what follows, we add the latter consideration to the existing stream of literature in econometrics. In addition, we tie in with existing approaches that extend the two states "buy" and "sell" with a third state "hold", such as Iori (1999); Cont and Bouchaud (2000); Takaishi (2005); Sato (2007); Vikram and Sinha (2011); Takaishi (2013) among others. The analysis of the microcanonical partition function then better reflects reality and allows for a deeper insight into the underlying dynamics, the derivation of further parameters for the description and classification of risks, and a suitable interpretation of the functional $\mathcal{H}$.

It is often discussed that $\mathcal{H}$ is related to the utility function $U$ known in economics (Marsili, 1999; Foley, 1999; Bouchaud, 2013). However, the following question arises: which utility for whom? One possible interpretation is to understand the utility from the perspective of a market observer (market analyst, researcher, investor) and to measure it in monetary units. The task of the market observer is then to describe the state of the market and to assess the effects of new information: for example, with the aim of determining the parameters of a predictive model and assessing the potential risk.

We specifically consider the ideal case in which new information reaches all agents simultaneously and instantly in a very short time, and, thus, possible risk events suddenly occur. Interactions between the agents play a subordinate role, and the system is therefore regarded as an ideal agent system. This approach allows the basic model parameters to be determined through empirical analysis. If the empirical setup is appropriate, the model parameters are determined largely free of other disruptive influences. The calculated, basic model parameters remain the same in all extended model concepts within statistical physics such that they can serve as a starting point for further, more complex models; see, e.g., Foley (1999); Marsili (1999); Anderson et al. (2001); Bouchaud (2009, 2013) for an overview.

The most important extension in such models is the additional consideration of coupled investor behaviors. Such dynamics components superpose on the ideal agent system studied here and complicate the simultaneous determination of all model parameters. The essential approach to determine the parameters appropriately is to choose special market phases in which one part of the dynamics dominates. In this paper, we focus on the part of the dynamics describing investor behavior depending on the news environment. Thus, appropriate market phases are sought to determine the model parameters of this part of the dynamics.

The examination of the procedure, and how one obtains the parameters, especially in the case of financial market problems, is not yet widespread in the literature. Experimental physicists design an experiment, conceive the experimental setup, and measure the temporal behavior of quantities of interest to determine the parameters of a theoretical model and investigate physical properties. This is more difficult to realize in econometrics. In capital market models, partial event sets can be split off from a large number of past events, described by, e.g., price developments and general conditions, which can be assigned to the phenomenon to be examined.

Therefore, in our contribution, on the one hand, we adhere to the extension of proven models by introducing the "hold" position, i.e., investors' stance to do nothing or not to change an existing equity exposure, and on the other hand, we focus on the design of the method for determining the model parameters, define the experimental setup and illustrate the method using empirical examples. For the case of an ideal agent system, we propose a way to identify this partial event set and how to determine the model parameters based on it. In addition, we show how the estimated parameters can be used to set up a one-step-ahead forecast model to assess abrupt risk events. The news field flows into the model as an external state variable and shares both technical and economic proximity to sentiment in finance literature. Sentiment is used in behavioral finance to predict asset prices (Sun et al., 2016; Gao and Yang, 2017; Renault, 2017; Pan, 2020). Therefore, with our approach, we also show an innovative method to use the sentiment scores generated from text analysis for prediction. We apply the forecast and show its performance compared to selected benchmark approaches and deduce that the model is well-specified and the model parameters well-determined. Furthermore, the ideal agent system is included as a basic model in all known model extensions. Thus, the model parameters play a fundamental role in all extensions. In this respect, our contribution in the economic context can be viewed as fundamental research on which further analysis can be built.

The remainder of the paper is structured as follows: Section 2.3 outlines the theoretical model and defines risk indicators to describe immanent risks. In Section 2.4, we describe the capital market data required to determine the external state variables. Using the external state variables, we can then estimate the model parameters. The model and key risk indicators can then be used to assess sudden risk events. We formulate the idea of a one-step-ahead forecast model and show the performance of the three-state model compared to basic benchmark approaches and a two-state model. The last Section 2.5 summarizes our findings, discusses the limitations of the concept, and presents some ideas on how to extend both the model and the method for further research topics.

### *2.3. Method*

We focus on the appropriate experimental design for the empirical determination of the parameters of the thermodynamic model for describing sudden risk events in the stock market. To this end, we consider the idealized, interaction-free theoretical model in which investors react independently of one another to new information. The parameters determined in this way are fundamental and, in accordance with the internal consistency of the thermodynamic approach, remain the same even in more complex models, for example with interactions.

In the following, we use the usual notations for describing model access; cf., e.g., Bouchaud (2009, 2013). Typically, investors are referred to as agents in generalized models. We consider a system of $N$ stocks that can be traded by a collection of agents. The model parameters are determined in Section 2.4 for a single financial asset on the basis of selected realizations in the capital market that come closest to the ideal case elaborated here. This idealization is similar to the case considered in Foley (1999, Section 6.1). Similarly, normalization also takes place here, assuming that one asset is traded by one agent. In practice, each investor will generally trade more than one stock per order. Thus, for the number of shares per trade per agent, $M \geq 1$ holds. The normalization above is an approximation and justified if the number of all stocks, typically $N \approx 10^8$ and higher (e.g., BioNTech: $N \approx 2.5 \times 10^8$), is much larger than the individual traded position $M \approx 10^2$; see also the similar discussion of Weiss domains in statistical physics (Weiss, 1907; Greiner et al., 1995).

In our transaction-based approach, each share can be bought, held, or sold. With the "hold" position, we thus expand the alternative courses of action and, overall, expand the existing model framework to include problems in the financial market. The three options "buy", "hold", and "sell" are typically called states of an agent and refer to an investor's attitude towards a stock, i.e., how he positions himself towards it, and not to the existing positions (past transactions) in his portfolio. This has the advantage that all reactions to a new piece of information are modeled, and, thus, an instantaneous trading potential can be derived for the next time step, cf. Section 2.4.4.

The three states are distinguished by the discrete variable $s_i = (-1, 0, +1)$ for each stock $i = 1, \ldots, N$. A new message $\mathcal{B}$ with a basic sentiment $\mathrm{sign}(\mathcal{B}) = (-1, 0, +1)$ – indicating "bad", "indifferent" and "good" news – and a strength $B = |\mathcal{B}|$ affects the agents and thus the shares and ultimately the events in the capital market. We concentrate on the effects that sudden new information $\mathcal{B}$ triggers, which ideally is available simultaneously to all market participants, e.g., via Bloomberg L.P., Thomson Reuters Corp. or other competitors aggregating financial and legal news. The processes by which the agents interact with one another and diffusion processes, including trends and delays (Bouchaud, 2013) for the information, are not considered here and are reserved for subsequent research on model extensions.

In the model framework considered here, each agent can be assigned a parameter $\mu_i$ that evaluates the individual change $\epsilon_i$ of $\mathcal{H}$ depending on the action $s_i$ in the external information field $\mathcal{B}$. Depending on the application, $\mu_i$ has different names in the literature: willingness to adopt, willingness to pay, or idiosyncratic judgment (Sornette and Zhou, 2006; Bouchaud, 2009, 2013; Crescimanna and Di Persio, 2016). In many studies, $\mu$ is chosen to be the same for all agents. Distributions $\rho(\mu)$ of the parameter or individual settings $\mu_i$ are considered in special extensions and applications, e.g., in sociophysics (Foley, 1999; Marsili, 1999; Anderson et al., 2001; Castellano et al., 2009). In the ideal case considered here, we initially set the parameter $\mu$ to be the same for all agents and thus follow the main stream of the literature. The focus of our empirical analysis of time series from the financial market in Section 2.4 is on the appropriate definition of the experimental setup and the determination of parameter $\mu$.

As Bouchaud (2013) notes, the decision of the individual agent depends on personal preferences, risk aversion, and framework conditions and is made given an individual utility function. In these investigations, the parameter $\mu$ models the tendency of the binary decision for or against an investment.

For a market analyst who does not know the individual circumstances of each agent and studies the level of the overall market, another interpretation of $\mu$ comes to mind. For the analyst, the parameter evaluates the change in utility that is inherent in a decision that conforms to the message. Consider a simple descriptive model: If the company news is bad and the agent decides to sell the stock, he or she behaves in accordance with the news, and the utility of the analyst increases if $\mu > 0$. The utility can thus be formulated from the perspective of the market analyst and, due to the conformity of the observation, could be interpreted as a cost reduction in the preparation of forecasts. How much the utility changes is then ascertained with the absolute value of $\mu$. In this interpretation, $\mu$ can be understood as a parameter that evaluates the conversion of new information into utility for the market analyst.

If all agents are considered, the change in the overall utility $U$ is calculated depending on a functional $\mathcal{H}$. This functional is pivotal in thermodynamics, where its minimum is considered for special thermodynamic systems. Therefore, when considering a maximum utility in econometrics, a change of sign must still be accounted for: $U = -\mathcal{H}$ (Marsili, 1999).

The functional $\mathcal{H}$ can be formulated based on the individual change $\epsilon_i$ triggered by an agent $i$:

$$\mathcal{H} = \sum_{i=1}^{N} \epsilon_i. \tag{1}$$

For the individual change $\epsilon_i$, a new discrete variable $S_i$ is introduced, which maps the fact that the individual agent behaves in conformity with the basic sentiment on company news, $S = +1$, or does not, $S = -1$. In the present case, we consider a triplet state system, and the position "hold" is also accounted for with $S = 0$, so that an indifferent investor attitude is included. The position "hold" causes a fraction $0 < \alpha < 1$ of the individual changes of the other two positions and is always rated positively regardless of the basic sentiment of the news $\mathcal{B}$. For the individual changes, we set

$$\epsilon_i = -\mu B S_i + \alpha\mu B(1 - S_i^2) \tag{2}$$

with $\mu > 0$ and $B = |\mathcal{B}|$. For $S_i = +1$, the agent conforms to the new message, e.g., "sell" in the case of "bad" company news, and $\epsilon_i$ is negative and decreases $\mathcal{H}$, i.e., increases the overall utility for the analyst or similar entity trying to understand or explain the market. A non-conforming decision ("sell" on "positive" news) increases $\mathcal{H}$ and decreases utility because the position requires explanation and involves utility-decreasing effort. The hold position consistently increases $\mathcal{H}$, since deviation from conformity creates tension that requires explanation. However, because the situation is not as difficult to explain as the non-conforming decision, the reduction in utility may be proportionately smaller. Therefore, we expect the value range for $\alpha$ to be restricted between 0 and 1. The functional $\mathcal{H}$ depends on the set $(S_1, \ldots, S_N)$ and is simply

$$\begin{aligned} \mathcal{H}(S_1, \ldots, S_N) &= -\mu B\Big(\sum_{i=1}^{N} S_i - \alpha(1 - S_i^2)\Big) \\ &= \mu B\Big(N_- + \alpha N_0 - N_+\Big). \end{aligned} \tag{3}$$

Here, $N_+$ and $N_-$ are the number of agents who conform and do not conform to the company messages, respectively, and $N_0$ is the number of agents in the "hold" position. The following applies to the occupation numbers of each state $0 \leq N_-, N_0, N_+ \leq N$ and $N = N_- + N_0 + N_+$.

The basic idea is to use the methods of statistical physics to derive a statistical model for the configuration $(N_-, N_0, N_+)$ that accounts for other macroscopic variables in addition to a particular company news item and allows inferences to be made about the number of potential buyers or sellers. The latter can then be used to estimate the impact on the price $P$ of a share.

*2.3.1. Canonical Ensemble*

In the canonical ensemble, a subsystem is considered in thermal contact with an overall system. The exchange quantity in physics is the temperature $T$. In this case, one has to calculate the canonical partition function $Z$. In finance, this means that a stock and its investors are viewed as a subsystem embedded in an overall market. One strand of the literature (Crescimanna and Di Persio, 2016) assumes that in econometrics, the role of temperature can be assigned to volatility $\sigma$. Thus, the volatility represents the exchange quantity in finance, i.e., $T = \sigma$. We share this perspective and develop this approach further in Section 2.3.2. It is customary to employ the inverse volatility (*phys.*: inverse temperature) $\beta = {}^{1}/_{kT}$ in the equations with a suitably defined constant $k$ (Kaizoji, 2000; Oh and Jeon, 2007; Krause and Bornholdt, 2012; Bouchaud, 2013). In Section 2.3.2, we present a suitable interpretation of $k$ in finance (*phys.*: Boltzmann constant).

In statistical physics, the canonical partition function is defined as the sum over all sets $(S_1, \ldots, S_N)$:

$$Z(T, B, N) = \sum_{(S_1,\ldots,S_N)} \exp\left(-\beta\mathcal{H}(S_1, \ldots, S_N)\right). \tag{4}$$

In Equation (4), we have a sum over so-called Boltzmann factors to evaluate the partition function. In statistical physics, this approach leads to the Boltzmann–Gibbs distribution specifying the probabilities of discrete states (Greiner et al., 1995; Kaizoji, 2000). There is a broad discussion in choice theory about the use of this approach in finance or generally in sociophysics or econophysics (Marsili, 1999; Kaizoji, 2000; Anderson et al., 2001). The discussion in choice theory starts from the very mathematically convenient logit rule. This rule is equivalent to the abovementioned Boltzmann–Gibbs distribution, so the well-known results of statistical physics can be used (Bouchaud, 2013).

In the simple case of noninteracting agents with Hamiltonian Equation (3), in a first step Equation (4) leads to:

$$Z(T, B, N) = [Z(T, B, 1)]^N. \tag{5}$$

Thus the canonical partition function is simply connected to the partition function for one agent. The latter follows immediately for an agent that can be in three possible states given a certain company news item:

$$Z(T, B, 1) = \exp(-\beta\mu B) + \exp(-\beta\alpha\mu B) + \exp(+\beta\mu B). \tag{6}$$

Let $x = \beta\mu B$. Then, with the partition function Equation (5), the probabilities for each state

$S = (-1, 0, +1)$ of the agent are:

$$\begin{aligned}
p_1 &= \text{Prob}(S = -1) = \frac{\exp(-x)}{\exp(-x) + \exp(-\alpha x) + \exp(+x)} \\
p_2 &= \text{Prob}(S = \phantom{-}0) = \frac{\exp(-\alpha x)}{\exp(-x) + \exp(-\alpha x) + \exp(+x)} \\
p_3 &= \text{Prob}(S = +1) = \frac{\exp(+x)}{\exp(-x) + \exp(-\alpha x) + \exp(+x)}
\end{aligned} \tag{7}$$

If we consider $N$ stocks, Equation (7) leads to the configuration:

$$\begin{aligned}
N_- &= \frac{\exp(-x)}{\exp(-x) + \exp(-\alpha x) + \exp(+x)} N \\
N_0 &= \frac{\exp(-\alpha x)}{\exp(-x) + \exp(-\alpha x) + \exp(+x)} N \\
N_+ &= \frac{\exp(+x)}{\exp(-x) + \exp(-\alpha x) + \exp(+x)} N
\end{aligned} \tag{8}$$

#### *2.3.1.1 Trade Potential*

With Equation (8), the average trade potential follows immediately:

$$\begin{aligned}
\bar{N}_{\text{pot}} &= \text{sign}(\mathcal{B}) \frac{1}{N} (N_+ - N_-) \\
&= \text{sign}(\mathcal{B}) \frac{\exp(+x) - \exp(-x)}{\exp(-x) + \exp(-\alpha x) + \exp(+x)}
\end{aligned} \tag{9}$$

The trade potential is the balance of buyers and sellers at the respective point in time and represents the variable influencing the price. Note that if all agents sell on bad news $\bar{N}_{\text{pot}} = -1$, and if all agents buy on good news, $\bar{N}_{\text{pot}} = +1$ holds. If $\mathcal{B} = 0$ or volatility becomes infinite, the trading potential is $\bar{N}_{\text{pot}} = 0$. This describes the case in which the overall system has no clear direction in terms of selling or buying.

At this point, the theoretical question arises, under which conditions does the three-state system change into a two-state system? Specifically, what is the relationship between the two models? If $x < \infty$ and it is assumed that $\alpha \to \infty$, then Equation (9) reduces to the well-known relation for a two-state system: $|\bar{N}_{\text{pot}}| \propto \tanh(x)$, cf., e.g., Greiner et al. (1995); Bouchaud (2013). This describes the case, which is not considered further here, in which the position "hold" would cause an infinitely large, negative effect related to utility. In this case, the position "hold" would not be taken. The value of $\alpha$ is expected to be between zero and one, so this case will not occur.

Since $x = 1$, i.e., $kT = \mu B$, marks a special point, we approximate the trade potential Equation (9) in an asymptotic expansion for large ($T \to 0$ resp. $B \to \infty$) and a Taylor series for small ($T \to \infty$ resp. $B \to 0$) values of $x$:

$$\bar{N}_{\text{pot}}(x) = \text{sign}(\mathcal{B}) \begin{cases} \frac{2}{3}x + \frac{2}{9}\alpha x^2 - \frac{1}{27}(\alpha^2+3)x^3 + O(x^4) & x \to 0 \\ 1 - 2\Big(\exp(-2x) - \exp(-4x) + \dots\Big) & x \to \infty \end{cases} \tag{10}$$

If we consider the leading order in the first line, the dependence of the trade potential corresponds to the well-known law noted in Bouchaud (2013), albeit with an extra factor of 2/3 that could have been guessed, because only 2 of 3 states are important for the trade potential. The extra factor was formally derived from theory and is important when estimating the true parameter $\mu$. This is because if the estimation $\hat{\mu}$ of the parameter is based only on a two-state system ("buy" and "sell"), the result is likely to be a significant underestimation of the true parameter. Figure 4 shows the exact course of the trade potential, Equation (9), and its approximations, Equation (10).

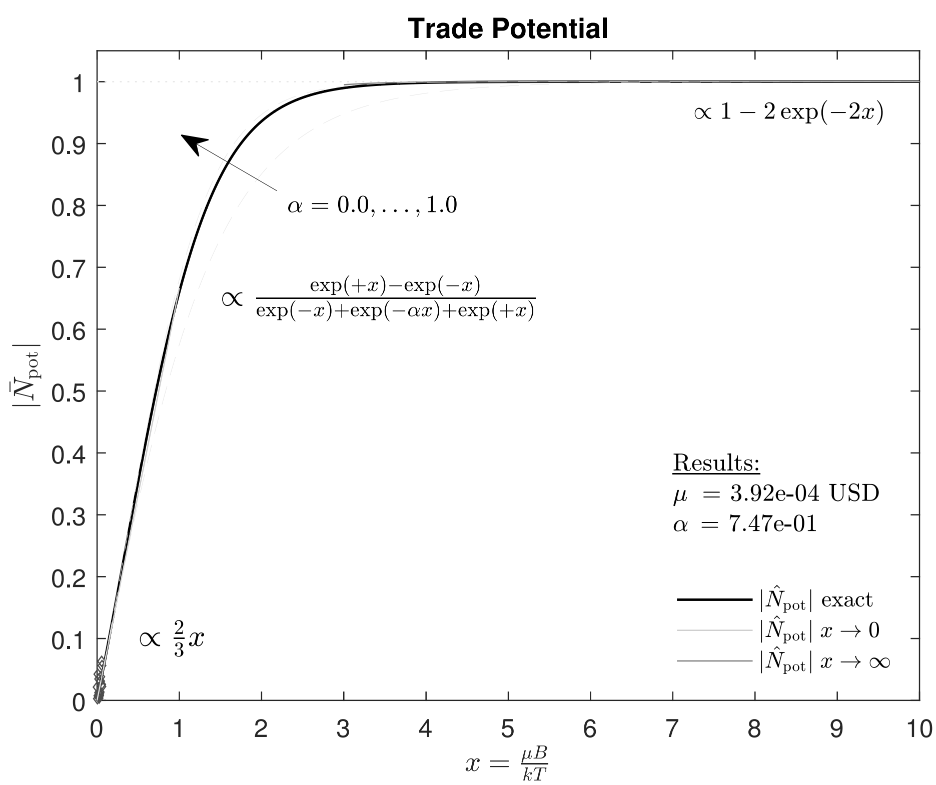


**Figure 4:** The average trade potential depending on $x$. The figure shows the exact function Equation (9) and the approximations Equation (10) for large and small $x$. Furthermore, the variation due to $\alpha$ is shown. The noted results $(\mu, \alpha)$ correspond to the model parameters determined in Table 3 for the example under consideration.

*2.3.1.2 Utility*

In a similar way, we can express the average utility in terms of $x = {}^{\mu B}/_{kT}$. Substituting Equation (8) into Equation (3) leads to:

$$\bar{U} = -\mu B \, \frac{\exp(-x) + \alpha \exp(-\alpha x) - \exp(+x)}{\exp(-x) + \quad \exp(-\alpha x) + \exp(+x)}. \tag{11}$$

The Taylor expansion for small $x$ leads to the expression:

$$\bar{U} = -\mu B \left( \frac{\alpha}{3} - \frac{2}{9}(\alpha^2 + 3)x + \frac{1}{27}\alpha(\alpha^2 - 9)x^2 + O(x^3) \right). \tag{12}$$

For $x = 0$, a basic negative average utility $\bar{U} = -\mu B \, {}^{\alpha}/_{3}$ can be observed, which results from the fact that with high volatility and low message strength, the possible states are occupied equally by the agents and the utility of conforming and not conforming to the news offset one another.

*2.3.1.3 Risk Measures*

For $x \ll 1$, i.e., the strength of the news $B \approx 0$ and a (constant) finite volatility, we can describe the behavior of the trade potential with small changes in the strength of the company message:

$$\Delta \bar{N}_{\text{pot}} = \chi \Delta B \;\; \text{with} \;\; \chi(T, B) = \frac{\partial \bar{N}_{\text{pot}}}{\partial B} = \text{sign}(\mathcal{B}) \frac{2}{3} \frac{\mu}{k} \frac{1}{T}. \tag{13}$$

The latter equation is the well-known Curie law $\chi \propto {}^{1}/_{T}$ noted in (Bouchaud, 2013, *phys.*: magnetic susceptibility) but with the extra factor ${}^{2}/_{3}$.

Similar to Equation (13), it is also possible to approximate the change in the trade potential for small changes in volatility (*phys.*: thermal magnetic loss coefficient):

$$\Delta \bar{N}_{\text{pot}} = \eta \Delta T \;\; \text{with} \;\; \eta(T, B) = \frac{\partial \bar{N}_{\text{pot}}}{\partial T} = -\text{sign}(\mathcal{B}) \frac{2}{3} \frac{\mu}{k} \frac{B}{T^2} = -\frac{2}{3} \frac{\mu}{k} \frac{\mathcal{B}}{T^2}. \tag{14}$$

In the same way, for $x \ll 1$, we can express changes in utility with a small change in volatility. Considering Equation (12) up to order $O(x)$ and $B = \text{const.}$, the change can be described as follows:

$$\Delta \bar{U} = c_B \Delta T \;\; \text{with} \;\; c_B(T, B) = \frac{\partial \bar{U}}{\partial T} = -k \frac{2}{9} (\alpha^2 + 3) \left( \frac{\mu B}{kT} \right)^2. \tag{15}$$

The coefficient $c_B$ is a kind of capacity (*phys.*: specific heat capacity). It describes the ability of the agent system to react to changes in volatility in the form of changes in utility. Since

$c_B$ is always negative, every increase in volatility leads to a loss of utility. With increasing volatility, however, the effect decreases, and the absolute change in utility becomes smaller. In the high volatility limit, $T \to \infty$, no change in utility can be determined.

Similar to the concept of duration, the coefficients $\chi$, $\eta$ and $c_B$ can be understood as risk parameters if there are sudden minor changes in the news situation or in the volatility: For small changes, they can be used to estimate the effect on the considered quantity (trade potential or utility).

*2.3.2. Microcanonical Ensemble*

The calculation and evaluation of the microcanonical partition function $\Omega$ leads to the entropy of the whole agent system (Greiner et al., 1995; Kaizoji, 2000):

$$\mathcal{S} = k \ln \Omega. \tag{16}$$

Its similarity to Shannon's term for information (Shannon, 1948) suggests that entropy should be interpreted as the information deficit of the market analyst about the microstate, i.e., the state of a single agent, associated with knowing the macroscopic variables, e.g., $\mathcal{B}$ and $T$. The greater the entropy, the less the market analyst knows about the microscopic state and the less information he or she knows about the entire agent system. Shannon himself introduced $k$ as a positive constant and assigned it the property of a unit of measurement (Shannon, 1948, p. 11). Considering the microcanonical partition function $\Omega$ in detail, we want to pursue this idea further to gain a suitable interpretation of the constant $k$ in the finance field.

In finance, the partition function $\Omega$ counts the number of sets $(S_1, \ldots, S_N)$ that lead to the same, pre-given utility $U$. For the calculation, a small interval $[U - \delta U, U]$ is defined, so that only some configurations $(N_-, N_0, N_+)$ according to Equation (3) lead to a utility within the interval. If the maximum entropy principle (Jaynes, 1957; Foley, 1999) is employed, all that remains is a single configuration that meets the utility specification. Then, the number $\Omega$ of possible sets that create the configuration is counted. In the present case, with $\delta U < \alpha \mu B$, the number of sets is determined by the multinomial coefficient:

$$\Omega = \frac{N!}{N_-! N_0! N_+!}. \tag{17}$$

Using Stirling's formula, $\ln n! = n \ln n - n$, and norming with a factor $^1/_{\ln 3}$, the average entropy is

$$\bar{\mathcal{S}} = -k \sum_{i=1}^{3} p_i \log_3 p_i \tag{18}$$

with the probabilities $p_i$ defined in Equation (7). If $k = 1$, then $\bar{\mathcal{S}}$ is the entropy in a ternary numeral system with minimum $\bar{\mathcal{S}} = 0$ and maximum $\bar{\mathcal{S}} = 1$. If $p_i = 1/3$, then the agents are evenly distributed over all possible positions ("sell", "hold", and "buy"). Then, the entropy is at its maximum, and a market analyst needs one trit (trinary digit, cf. Brusentsov and Alvarez (2011)) of information to determine the exact setting of an individual agent. A similar discussion is held for the two-state systems ("buy" and "sell") with bits in Nadal et al. (1998). An obvious interpretation would be to regard $k$ as the cost of obtaining the information. The unit of measurement of $k$ would then be monetary units. If $p_i = 1$ for some $i$, then the entropy is zero, i.e., all agents are in the same state $i$, every microstate is thus the same, the market analyst does not need any further information to make statements about the position of an individual agent, and there are no costs for further information.

In this interpretation of $k$, the entropy of the entire agent system would sum up the cost of the uncertainty, i.e., the information deficit measured in trits. The constant $k$ is, therefore, not a model parameter but defines the unit of measurement.

*State Quantity T*

In statistical physics, the temperature $T$ is a state quantity that is independent of the amount of matter considered in a thermodynamic system. Analyzing the microcanonical ensemble, one finds a determining equation for the temperature. A similar approach was proposed by Marsili (1999) to define the state quantity $T$ in agent systems. We will briefly retrace the course.

According to its calculation, entropy $\mathcal{S}$, Equation (16), is a function of utility $U$ for a fixed number of agents $N$ and given news strength $B$. If a Taylor expansion is implemented with respect to $U$ and second- and higher-order terms are neglected, then Marsili (1999) finds the following relationship in analogy to statistical physics:

$$\frac{1}{T} = -\frac{\partial \mathcal{S}}{\partial U}\bigg|_{U} \tag{19}$$

Since Marsili (1999) neglects $k$ in the entropy, he comes to the interpretation that in economics, the state quantity $T$ can be considered as the price of (negative) entropy. Furthermore, he interprets that the parameter $T$ plays the role of an *internal* temperature of an agent, and it measures its deviation from rational behavior, which is recovered as $T \to 0$. This can include, for example, not behaving in conformity with the company news $\mathcal{B}$, i.e., buy on bad news, or changing positions quickly without any new information. For a stock price, this leads to an observable volatility. Furthermore, based on the observation that in many capital market-related applications, a proportionality between temperature and volatility is assumed $T \propto \sigma$

(de Mattos Neto et al., 2011; Bouchaud, 2013), it was recently shown for an Ising-based two-state model that there is a fundamental relationship between temperature and trading volatility (Börner et al., 2023c). Thus, in this interpretation, the volatility observable on the capital market is the measurand for determining the internal temperature, and in the simplest case, the measuring equation is $T = \sigma$. In the sense of the microcanonical view, a closed stock/agent system *has* a volatility. If the system is considered embedded in the overall market (canonical ensemble, Section 2.3.1), then the exchange quantity is the volatility, and the individual system *is* influenced by the volatility of the overall market.

The main qualitative effects of a changing temperature can also be observed in terms of volatility. If the volatility tends toward zero, decisions become easier even with weak new news and are primarily based on the direction of the new news. The agent system overall conforms to the company message (*phys.*: crystallization). If the volatility becomes very high, decisions with the same information become more difficult; the direction fluctuates and is not necessarily based on the new message. As in the physics of spin systems, the increasing disorder can be observed in the agent system, and no direction dominates. Galam et al. (1982) conducted a similar discussion in sociology when interpreting his model of a strike, which is based on two states ("work state" and "strike state").

If we continue to assume that the entropy and utility itself can be measured in monetary units, then $T$ becomes a unitless quantity that mediates between small changes in entropy and changes in utility in the following way: $\Delta U \propto -T\Delta\mathcal{S}$, i.e., an increase in entropy interpreted as costs for further information from the perspective of a market analyst decreases his or her utility, and the greater the volatility is, the stronger the effect.

There can be the unlikely situation that if the news is bad, the agents are predominantly in the "buy" position, or if the news is good, they are predominantly in the "sell" position. In both cases, $N_+ - N_- < 0$. For the trade potential, Equation (9), $\bar{N}_{\text{pot}} > 0$ (buyer surplus) is calculated for bad news and $\bar{N}_{\text{pot}} < 0$ (seller surplus) for good news. In any case, the entire agent system does not behave in accordance with the news situation.

Such an inversion is known in statistical physics and leads to a negative temperature being assigned to the inverse state. In general, these states are not stable, and the system relaxes within a short time (Greiner et al., 1995).

The same is assumed for the agent system considered here. An inverse state, triggered, for example, by two successive company messages with rapidly changing signs, should relax within a short time. The transition from the "inverse" to the "normal" state generally takes place via a sequence of nonequilibrium states, and methods from nonequilibrium thermodynamics are to be used for modeling, cf., e.g., the propagation of partition functions depicted in Isihara (1971). The just-described transition can only be modeled with the (suitably extended) methods described in this section if it is a sequence of quasi-steady state changes in the system.

In what follows, we are interested in a procedure to estimate the model parameters $\mu, \alpha$ and examine the "normal" case, where the entire agent system behaves in accordance with the news situation. This means that we will not further consider the above-described case of an inversion of the agent system in the following.

### *2.4. Application*

#### *2.4.1. Procedure for Determining the Model Parameters – Experimental Setup*

The model parameters $0 < \mu$ and $0 < \alpha < 1$ are to be determined on the basis of capital market data. The idealized model presented in Section 2.3 focuses on mapping special risk events for single financial assets. By construction, the model is not designed for the entire dynamic but only for special components of the dynamic in which the parameters $\mu$ and $\alpha$ essentially determine the dynamic. If there are temporary events in the capital market that correspond to the idealized specifications, the model parameters can be determined in this cut-out experiment. The following procedure is used to determine the parameters and takes the idealized conditions into account.

1. Periods of low, constant volatility $T$ with sideways price movement are sought for the single asset to be examined. Quantile specifications are used to search for such time segments. In the ideal case, the time segments found correspond to dynamic equilibria.
2. The news situation is then examined for all the time segments found, and those time segments are selected for further analysis in which a single central message $\mathcal{B}$ dominates the following period. The strength $B = |\mathcal{B}|$ and direction $\text{sign}(\mathcal{B})$ of the message are determined.
3. We set the value of $k$ to one monetary unit, compare Section 2.3.2, and calculate ${}^{B}/_{kT}$. This means that $\mu$ remains in $x = {}^{\mu B}/_{kT}$ as a variable that has yet to be determined.
4. Next, the seller or buyer surplus $\bar{N}_{\text{pot}}$ is determined from the shares traded. There are several ways to estimate $\bar{N}_{\text{pot}}$; we discuss three different approaches and suggest one for further use. A pair of measured values $({}^{B}/_{kT}, \bar{N}_{\text{pot}})$ is thus calculated for each examined time segment.
5. All pairs of measured values $({}^{B}/_{kT}, \bar{N}_{\text{pot}})$ are used to fit the curve shown in Figure 4 with Equation (9) and determine the model parameters $\mu$ and $\alpha$.

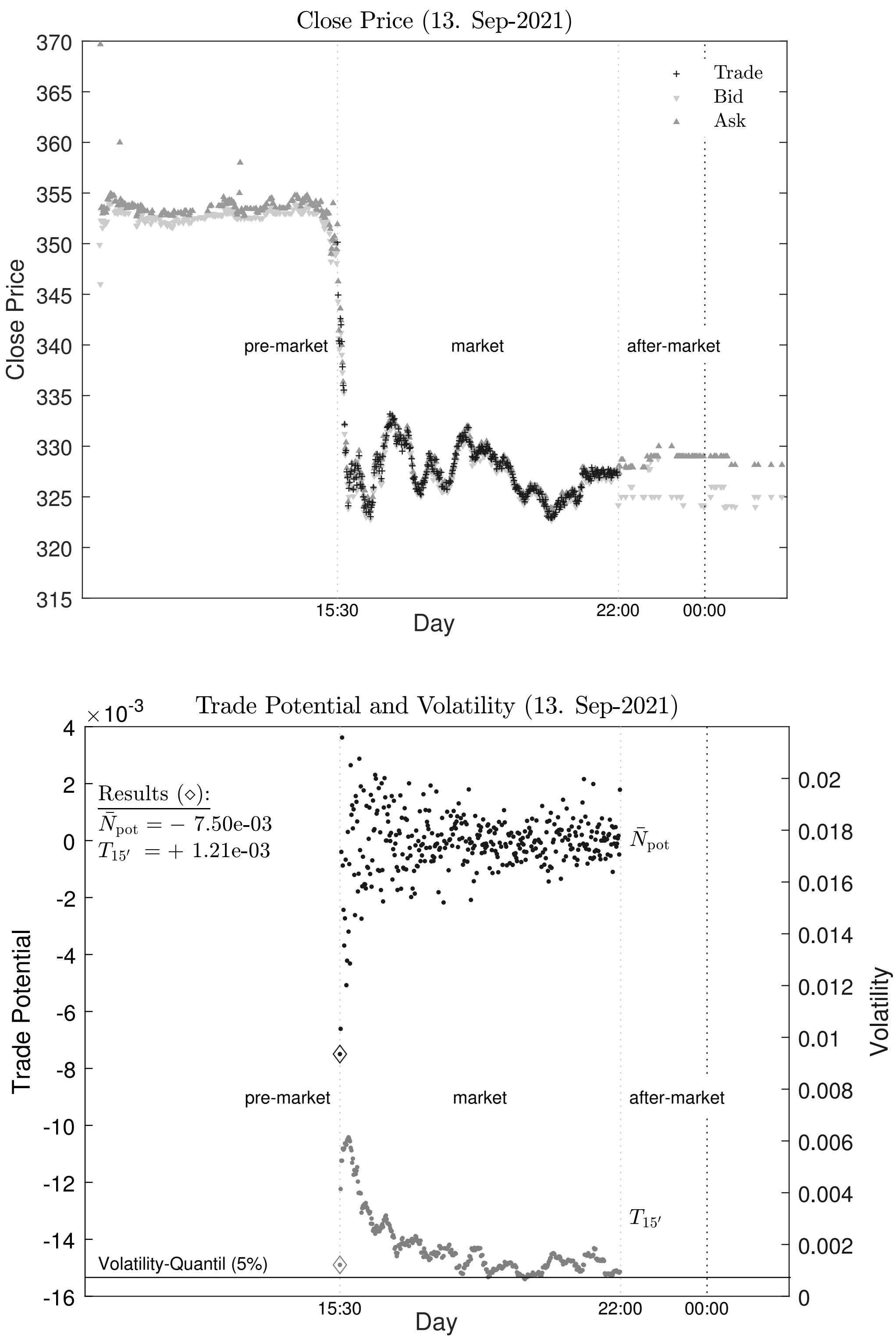


**Figure 5:** Sampled closing prices for BioNTech are in the upper panel. Trade potential and volatility are in the lower panel. Time scale: UTC+2.

### 2.4.2. *Data*

As an example, the shares of the pharmaceutical company BioNTech are examined in detail. Bloomberg trading data for this company is available from the American Stock Exchange NASDAQ for the period from 1 August 2021 to 31 January 2022 with a sample rate of 1 min (Bloomberg code: BNTX UW). The dataset is analyzed for the market session and the pre- and after-market sessions. The opening, closing, high, and low prices are available for each minute. In addition, the sales volume, sales value, and number of ticks are attached to the price data. An excerpt in Figure 5 (upper panel) shows the closing prices of the various sessions divided into bid, ask and trade for 13 September 2021.

#### 2.4.2.1 *Volatility T – Temperature*

For the statistical model in Section 2.3, the external state variable $T$ is required and must be extracted from the capital market data. The state variable $T$ was interpreted as instantaneous volatility, which is the volatility that market participants observe for the share at the moment prior to becoming aware of the news $\mathcal{B}$. This means that the volatility must be determined over a period that is as short as possible to be measured as closely as possible to the event and to include as few other effects as possible. The goal of volatility measured as close to an event as possible is thwarted by a lower limit of data points used. The limit is given by the fact that the statistical error is becoming more dominant. We successively examined shorter measurement periods of 120, 60, 30, 15, 10, and 5 min and found that below 15 min, the statistical errors dominate in determining the volatility. For the following analyses, the volatility, and, hence, the state variable $T$, are determined based on logarithmic returns over the shortest reasonable time window of 15 min. Thus, the volatility at each time $t$ is determined continuously as the standard deviation over 15 logarithmic returns $(t, t_{-1}, \ldots, t_{-14})$ in a rolling window.

#### 2.4.2.2 *Trade Potential $\bar{N}_{\text{pot}}$ – Magnetization*

Another key variable required is the effective trading potential $\bar{N}_{\text{pot}}$, which is calculated from the buyer surplus or seller surplus excluding hold positions, cf. Equation (9). Different approaches were examined to extract the trade potential from the market data. The attempt to estimate trading potential based on the bid-ask difference in turnover volume failed because the bid and ask volumes also include some older stop-loss orders, stop-buy limit orders or other types of orders that are not necessarily related to the current event. A similar problem prevents the direct use of trading volume as a proxy for the trade potential a step back in time. In the case of a price jump, older limit specifications can be processed. As a result, the trade potential attributable to the actual event may be inaccurate. If only one stock exchange is examined, it is also unclear how $N$ is to be set in Equation (9). Thus, with the data and especially the sales volume available to us, the trade potential cannot be determined with sufficient accuracy. Therefore, we use an indirect method to estimate the trade potential. Thereby, the concatenation factors $K_t = {}^{P_{t+1}}/{}_{P_t}$ of the instant price movement at a certain point in time $t$ are calculated. Then, the trade potential is estimated following Vikram and Sinha (2011):

$$\hat{\bar{N}}_{\text{pot}}(t) = \frac{K_t - 1}{K_t + 1} \tag{20}$$

With the limit $P_{t+1} \to \infty$, the trade potential tends to +1 (all agents buy on good news), and with $P_{t+1} \to 0$ the trade potential tends to $-1$ (all agents sell on bad news), given finite price $P_t > 0$. With Equation (20), the following equation is determined for the estimated price at time $t + 1$ given the price at time $t$ and the trade potential defined in Equation (9):

$$\hat{P}_{t+1} = \frac{1 + \bar{N}_{\text{pot}}}{1 - \bar{N}_{\text{pot}}} P_t \qquad \text{with} \qquad \bar{N}_{\text{pot}} = \text{sign}(\mathcal{B})\, \frac{N_+ - N_-}{N}. \tag{21}$$

An excerpt in Figure 5 (lower panel) shows the estimated trade potential and the volatility for 13 September 2021.

Vikram and Sinha (2011) offer the left part of Equation (21) as an approximated price function and find that the exact form is not critical to their results. One way to derive this equation is to find a price $p$ of the asset in the time interval $[t, t+1]$, from which the price $p_{t+1} = \exp(\bar{N}_{\text{pot}})p \approx (1 + \bar{N}_{\text{pot}})p$ is calculated in the forward direction and the price $p_t = \exp(-\bar{N}_{\text{pot}})p \approx (1 - \bar{N}_{\text{pot}})p$ in the backward direction.

The relation $p' = \exp(\bar{N}_{\text{pot}})p$ is obtained for the expected path of the stochastic process described in ((Börner et al., 2023a), Equation (5)) when $\bar{N}_{\text{pot}}$ is constant for the time interval under consideration. If $|\bar{N}_{\text{pot}}| \ll 1$, then $p' = (1+\bar{N}_{\text{pot}})p$ holds approximately. If the evolution from $p_t$ to $p_{t+1}$ is performed over the intermediate price $p$, the equation proposed by (Vikram and Sinha, 2011) is calculated. This equation has the property of shape invariance under time reversal and is suitable to describe reversible processes, i.e., $\bar{N}_{\text{pot}} \to -\bar{N}_{\text{pot}}$, in the context of quasi-stationary state changes (Greiner et al., 1995).

#### *2.4.2.3 News Sentiment $\mathcal{B}$ – The Magnetic Field*

As described above, qualified messages must first be identified. These are characterized by the fact that the price development of the company share associated with them is caused exclusively and instantly by the news itself and is, therefore, largely independent of overall market developments. If this is the case, we are closest to the case of the ideal agent system under consideration.

A Bloomberg terminal is used to analyze the news situation. This allows us to view all news at specific points in time. To ensure that the events are not distorted by another influencing variable, such as a macroeconomic shock or similar factor, we check the volatility of the BioNTech share itself and the volatilities of two indices representative of the market (S&P 500 and NASDAQ-100) in the 15 min before the events.

The idea is that the financial market "system" must be in a tranquil state, representing a dynamic equilibrium, in each case before an event happens. In our view, the stock itself and the indices are each in dynamic equilibrium or in a state of tranquility if the range of volatility in the 15 min before the event is smaller than the 95% quantile of volatility in our overall time period (August 2021 to January 2022) and does not change In experimental physics, one would measure the following: The temperature of a spin-system is low and constant, and the entire system is in dynamic equilibrium with the thermal bath (Isihara, 1971; Greiner et al., 1995). In other words, the market is in dynamic equilibrium if it is not subject to large fluctuations (negative delimitation).

Out of 24 potential events, there thus remains a sample of 18 carefully identified events, where each event is enriched by some splitter messages that make the same statement as the so-called lead headline, which is representative of the respective news package for an isolated event at a specific point in time. Once the qualified events are identified, the empirical challenge in determining $\mathcal{B}$ is to convert the qualitative information of a message into a quantitative measure. In doing so, the strength $B = |\mathcal{B}|$ on the one hand and the direction of the information $\mathrm{sign}(\mathcal{B})$ on the other hand must be determined.

We resort to a textual analysis based on a linguistic approach to evaluate text fragments and calculate sentiment scores. The approaches in the field of text analysis are manifold, and the different methods offer their individual advantages, cf., e.g., Kearney and Liu (2014); Loughran and McDonald (2016, 2020). In particular, two-dimensional approaches with "positive" and "negative" sentiments are effective at capturing different contexts (such as the economic context of business-related messages) or at dealing with linguistic peculiarities (Stangor and Kuerzinger, 2021).

However, before the messages can be evaluated, a common problem in text analysis must be considered by preprocessing the data. Since inflected words can deviate from their root word, a matching algorithm might fail to correctly assign these variations to the root word. Therefore, the words in the message must first be transformed. Linguists have proposed several approaches to address this problem (Feinerer et al., 2008). Stemming, for example, traces a word to its root by identifying and eliminating suffixes. Lemmatization, on the other hand, groups inflected words into a single group. We first transform our data using the lemmatization list (41,531 words) created by Mechura (2016), which can be accessed via the R package "textstem" by Rinker (2018). In selecting a suitable linguistic approach, we included several well-known dictionaries that are embeddable via the R package "SentimentAnalysis" of Feuerriegel and Proellochs (2021). These include Henry's Financial Dictionary (Henry, 2008), the Harvard-IV dictionary, the Loughran–McDonald Financial Dictionary (Loughran and McDonald, 2011), and the QDAP dictionary from the R package "qdapDictionaries" by Rinker (2013). The algorithm additionally performs preprocessing operations such as removing stopwords and stemming. With each dictionary, lists of positive and negative words are used, and the occurrences in the messages are counted. The sentiment scores are the netted occurrences of positive, $PW$, and negative, $NW$, words, divided by the respective number of words $MW$ in a news package to control for messages of different lengths: $\mathcal{B} = \frac{PW-NW}{MW}$. According to this definition, news sentiment $\mathcal{B}$ is measured in fractions of unit one and takes values between $-1$ and $+1$.

A major drawback of the scoring algorithm is that it does not account for negations (i.e., words such as "not"), resulting in potentially misidentified scores and sentiments. For example, the algorithm would count the phrase "not successful" as $+1$ instead of $-1$ . For this reason, we manually edit our data by multiplying the determined sentiment score of the words that are negated by $-1$. We refrain from further text transformation at this point, such as smiley recognition, since this level of sophistication is sufficient to analyze the mostly standardized text messages in finance.

The sensitivity of the sentiment analysis can be further increased if the dictionaries are supplemented with a few additional keywords appropriate to the problem. We motivate this measure with the conclusion and result of Kearney and Liu (2014); Renault (2017) that more field-specific dictionaries are needed. The addition of a few but salient keywords with high relevance in the pharmaceutical (and financial) context shows that this measure is already very effective, and there is no need to define a complete dictionary in the first approach. The supplement of suitable keywords to dictionaries through human intervention is an established procedure in machine translation. Human intervention can be found in the context of machine translation when editing results and when creating or adapting dictionaries. In practice, human intervention is used to increase the performance of machine translation (Kawasaki, 1993). The latter is driven by the fact that basic dictionaries may not be able to handle specific – industry standard – terminologies. Kawasaki (1993) deduces an "add-and-delete" strategy since, on the one hand, important words should be added to the dictionary, and on the other hand, superfluous words that do not fit the context at hand should be removed to avoid creating erroneous ambiguities. In our context of the BioNTech stock, we keep, e.g., the basic financial context of the Loughran–McDonald dictionary but add (a few strictly selected) words from the pharmaceutical (and financial) context (if they are not already included in the respective dictionary): "approval", "authorize", "complete", "gain", "protect", and "target" as positive words and "tank" and "sink" as negative words. We proceeded in the same way with the other dictionaries.

The basis of our selection of a suitable lexicon for the textual analysis is a correlation analysis between the sentiment scores based on the different dictionaries and the market reactions of the respective news packages, as shown in Table 1. High correlations are desirable, as they then suggest that the external information field is well proxied.

Even though some dictionaries are designed to cover a specific context, such as a financial context, it is still necessary to work with fundamentally insufficient recognizability of context by natural language processing (e.g., Loughran and McDonald (2016); Renault (2017)). Consequently, a single message can be evaluated with a certain score, although the message is to be understood in an opposite context, depending on the perspective to be adopted. This leads to single news items being evaluated with sentiment scores that are contrary to the direction of the market reaction. For this reason, we neutralize the signs for the correlation analysis, cf. the second row of Table 1. Since the correlation between the (news package) values neutralized by the signs and the market reactions is high, at least in two cases, we conclude that the strength $B = |\mathcal{B}|$ of a message can be well represented by a text analysis algorithm. The second row in Table 1 complements the first row in that it provides better insight into the extent to which $B = |\mathcal{B}|$ can truly be represented by the sentiment score because the correlation is not "distorted" by matching signs.

To validate our method, we draw on proven concepts in the industry. In the context of annotations by humans, an acceleration of machine learning has been achieved, which is called human-in-the-loop (Wu et al., 2021). Similarly, we validate the automated estimation of the text analysis approach by an expert survey and adjust the "wrongly" detected signs at individual news item level for consistency. The improved consistency of the data increases the correlation at the news package level for all the dictionaries, cf. the third row of Table 1. To calculate the parameters $\mu$ and $\alpha$, however, it is critical that the signs are correct, i.e., that they match the actual direction of the message. Therefore, the validation or adjustment of the signs is performed, and the correlation is increased in all cases.

**Table 1:** Correlation analysis between sentiment scores based on different dictionaries and market reactions ($p$-values in parentheses).

| | **Loughran-McDonald** | **Henry** | **Harvard-IV** | **QDAP** |
|---|---|---|---|---|
| sign unmodified | +0.31 ($2.14 \times 10^{-1}$) | +0.52 ($2.64 \times 10^{-2}$) | +0.26 ($2.93 \times 10^{-1}$) | +0.10 ($6.93 \times 10^{-1}$) |
| sign neutralized | +0.58 ($1.10 \times 10^{-2}$) | −0.07 ($7.97 \times 10^{-1}$) | −0.07 ($7.88 \times 10^{-1}$) | +0.44 ($6.82 \times 10^{-2}$) |
| sign adjusted | +0.87 ($2.36 \times 10^{-6}$) | +0.67 ($2.17 \times 10^{-3}$) | +0.68 ($1.85 \times 10^{-3}$) | +0.80 ($7.73 \times 10^{-5}$) |

The financial dictionary of Loughran and McDonald (2011) is particularly suitable for our purposes from the wide range of methods, as shown by the comparatively high correlation in Table 1, regardless of whether the signs were neutralized or adjusted. The dictionary is specially designed for the analysis of financial text and is broadly used in economic research (Chen et al., 2014; Da et al., 2015; Löffler et al., 2021; Stangor and Kuerzinger, 2021).

To assess how unambiguous a message is in each case, we use a sentiment polarity score, $PS$, as proposed by Zhang and Skiena (2010) and used similarly by Li et al. (2014). Table 2 contains the measure of $PS$ that ranges from $-1$ to $+1$ and is given by $PS = \frac{PW-NW}{PW+NW}$. The closer the score is to $-1$ or $+1$, the clearer the message is.

**Table 2:** Analyzed news and deduced key figures ($k$ = 1 USD).

| **Date** | ***Core Message*** | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Analysis | News | | Capital Market | | Risk-Measures | | | |
| | $\mathcal{B}$ | $PS$ | $T_{15'}$ | $\bar{N}_{\text{pot}}$ | $\chi$ | $\eta$ | $c_B$ | $B/kT$ |
| Units | $10^{-2}$ | $10^{-1}$ | $10^{-4}$ | $10^{-3}$ | $10^{-2}$ | $10^{-1}$ | $10^{-5}$ USD | $10^{0}$ |
| 3 August 2021 | *F.D.A. Aims to Give Final Approval to Pfizer Vaccine by Early Next Month* | | | | | | | |
| | + 6 | + 7 | + 10 | +11 | +26 | −161 | − 46 | + 62 |
| 6 August 2021 | *FDA expects to have COVID vaccine booster strategy early next month* | | | | | | | |
| | 0 | NA | + 11 | +22 | 0 | 0 | 0 | 0 |
| 20 August 2021 | *FDA Approves Pfizer-Biontech COVID-19 Vaccine* | | | | | | | |
| | +14 | +10 | + 13 | +31 | +20 | −225 | −149 | +111 |
| 10 September 2021 | *Biontech to Seek Vaccine Approval for 5–11 year olds* | | | | | | | |
| | + 8 | + 6 | + 9 | +11 | +28 | −235 | − 86 | + 84 |
| 13 September 2021 | *Covid Evidence doesn't support broad Need for Boosters* | | | | | | | |
| | − 3 | −10 | + 12 | − 8 | −22 | + 51 | − 7 | − 24 |
| 16 September 2021 | *FDA Advisers Back a Narrower Authorization for Pfizer Booster* | | | | | | | |
| | −10 | − 6 | + 13 | − 9 | −20 | +157 | − 73 | − 77 |
| 23 September 2021 | *FDA Authorizes Booster Dose of Pfizer-BioNTech COVID-19 Vaccine for Certain Populations* | | | | | | | |
| | + 8 | +10 | + 10 | + 8 | +25 | −194 | − 73 | + 78 |
| 30 September 2021 | *Pfizer/BioNTech Vaccine Antibodies Disappear* | | | | | | | |
| | − 3 | − 3 | + 14 | −42 | −18 | + 43 | − 7 | − 24 |
| 18 October 2021 | *Pfizer And AstraZeneca Vaccines Were Effective As Prior Infection, U.K. Study Finds* | | | | | | | |
| | +12 | +10 | + 8 | +23 | +34 | −510 | −273 | +150 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 20 October 2021 | *CDC: Pfizer COVID-19 Vaccine Highly Protective in 12–18 Age Group* | | | | | | | |
| | +6 | +5 | +7 | +14 | +40 | −390 | −117 | +98 |
| 4 November 2021 | *U.K. Regulator Is First to Approve Merck's COVID-19 Pill* | | | | | | | |
| | −13 | −10 | +9 | −26 | −29 | +407 | −247 | −143 |
| 5 November 2021 | *Update 2: Pfizer says antiviral pill cuts risk of severe COVID-19 by 89%* | | | | | | | |
| | −18 | −10 | +13 | −63 | −21 | +293 | −240 | −141 |
| 26 November 2021 | *Vaccine Stocks Jump Premarket Amid New Variant Fears, EU Backing* | | | | | | | |
| | +19 | +10 | +12 | +56 | +21 | −324 | −282 | +152 |
| 6 December 2021 | *Vaccine Stocks Slip as Street Weighs Omicron Variant Uncertainty* | | | | | | | |
| | −14 | −10 | +12 | −38 | −21 | +243 | −162 | −115 |
| 10 December 2021 | *Researchers in South Africa have also found a drop-off in the level of antibody protection from that vaccine versus the new strain* | | | | | | | |
| | −8 | −5 | +17 | −35 | −16 | +76 | −28 | −48 |
| 31 December 2021 | *Pfizer Vaccine Causes Myocarditis* | | | | | | | |
| | −8 | −10 | +12 | −4 | −22 | +138 | −49 | −64 |
| 10 January 2022 | *Pfizer CEO: Developing Omicron Targeted Vaccine* | | | | | | | |
| | +9 | +10 | +105 | +3 | +2 | −2 | −1 | +9 |
| 21 January 2022 | *Less-Threatening Omicron Lowers Covid Vaccine Sales Estimate* | | | | | | | |
| | −10 | −10 | +20 | −13 | −13 | +62 | −28 | −48 |

### *2.4.3. Results*

Table 2 shows our sample of 18 events used to calculate $\mu$ and $\alpha$. As mentioned above, six other events were identified, but these could not be considered further for the calculation because either trading in the BioNTech shares was not in dynamic equilibrium (strong changes in volatility indicated a transitional phase) or the overall market was not in dynamic equilibrium. The latter was determined in the same way as described above based on the volatilities of the leading indices (S&P 500, NASDAQ-100). To recap, the news headlines listed in the table are representative of the news packages as so-called lead headlines, which subsume other splitter news with the same statement. The entire database with 24 events, including all headlines and splitter news, is available upon request.

In Table 2, noteworthy findings include that the measured message strengths are very small, i.e., $|\mathcal{B}| \ll 1$, and the sometimes high value of the sentiment polarity score, $|PS| \lessapprox 1$, indicating unique messages. The 15-minutes volatilities $T_{15'}$ of BioNTech shares measured shortly before the event are of $O(10^{-3})$ and are within the 5% quantile of all volatilities measured in the observation period. The average trading potential $\bar{N}_{\text{pot}}$ determined from the market reaction using Equation (21) correlates highly ($\rho = 0.87$) with message $\mathcal{B}$. The latter must be the requirement for a suitable experimental setting for evaluating Equation (10) in linear approximation. For $k = 1$ USD, $T = T_{15'}$ and $B = |\mathcal{B}|$ the quotient ${}^{B}/_{kT}$ is calculated in the last column. The calculated risk measures $\chi, \eta, c_B$ are suitable for describing the reaction of certain output variables to small changes in input variables in a linear approximation; see Equations (13)–(15). Assume, for example, a given volatility $T$ and news situation $\mathcal{B}$. The calculated susceptibility $\chi$ can then be used to estimate the market reaction if the news situation becomes slightly worse (better). In a financial report on a specific market situation, further assessments of the sensitivity of market participants could, therefore, be possible. This aspect is not pursued further here. In the analyzed events, we observe a wide range of market situations, measured by the level of sensitivity ($\chi, \eta$ and $c_B$) to external conditions, which are worth investigating in a separate line of research.

All pairs of measured values (${}^{B}/_{kT}, \bar{N}_{\text{pot}}$) in Table 2 are used to fit the curve shown in Figure 4 with Equation (9) and determine the model parameters $\mu$ and $\alpha$.

Since one data point was always omitted for the calculation (jackknife resampling), 17 different estimates could be produced. The final result is the mean value over these estimates (jackknife estimators of the parameters) and is displayed in Table 3 along with the range. We refrain from calculating the variances here, as they would obscure the results, and instead present the ranges as error indicators. The parameter $\mu$ can be determined with sufficient accuracy. The parameter $\alpha$, however, is difficult to estimate due to the small number of data points and a missing entry with high trading potential $\bar{N}_{\text{pot}}$ and is therefore subject to high estimation errors. This high sensitivity is reflected in the broad bandwidth. However, a very high trading potential in the capital market can only be expected if a very strong information field $\mathcal{B}$, i.e., strong news, occurs or if the volatility $T$ is very low.

Considering Equations (13)–(15), it can be concluded that the risk measures $\chi$ and $\eta$ can be determined with better accuracy than $c_B$. The risk measure $c_B$ can be determined less precisely because of the direct dependency on the heavily errored variable $\alpha$ and the error superimposition of $\mu$ and $\alpha$.

**Table 3:** Results for $\mu$ and $\alpha$ along with the range of variation from the estimation.

| Parameter | Value | Range |
|---|---|---|
| $\mu$ ($10^{-4}$ USD) | 3.92 | (3.51–4.21) |
| $\alpha$ | 0.75 | (0.00–1.00) |

### 2.4.4. *Concept for a One-Step-Ahead-Forecast*

The method presented thus far focuses on modeling the dynamics that are to be expected as a result of a new message. Thus, only sudden risk events are modeled, and the full dynamics of a share/agent system are not mapped. As a result, the following prediction concept can only make statements about the short-term price development of security that is to be expected as a result of isolated news within the framework of the statistical models.

The central equations for the prediction are Equation (21) in conjunction with Equation (9). If the parameters $\mu$ and $\alpha$ have been determined – or are permanently determined and updated as part of machine learning – the volatility $T$ of the security must be determined continuously at the same time. A constant volatility over a period of time indicates that the share/agent system under consideration is in dynamic equilibrium. While determining the volatility, the news flow must be tracked using suitable information sources. From this, the share-specific messages are to be filtered and converted to a value $\mathcal{B}$ according to the method described above (Section 2.4.2.3). Then, $x = {}^{\mu|\mathcal{B}|}/_{kT}$ is calculated and from this the trade potential $\bar{N}_{\text{pot}}$ according to Equation (9). Equation (21) then supplies a prediction of a new price $\hat{P}_{t+1}$ based on the current price $P_t$.

The result is a predictor for the price one-step-ahead, and this predictor is itself subject to uncertainty and obeys a probability distribution. Therefore, the predictor $\hat{P}$ serves as an indication for the price movement in the next time step. The forecast is not exact because of the underlying distribution of the predictor. Hence, risk-free excess profits cannot be achieved.

We select several benchmark approaches to evaluate the prediction results of our three-state model (3SM) in Table 4. In addition to regular prediction benchmark approaches, which have the inherent disadvantage of not being able to process the news $\mathcal{B}$ as information in the prediction, we also compare it with the two-state model (2SM). The regular benchmark approaches include a naive approach that uses the last value before a message as the prediction value (last observation carry forward, LOCF) and the moving average (MA) with five observations smoothing. In addition, we use an ARIMA model that is well suited for estimating the next step (Siami-Namini et al., 2019). To incorporate a comparative model utilizing sentiment as information in prediction, we additionally estimate a regression model with ARIMA errors to account for the time series structure of the data (TS Regression). We employ the VOLQ, the volatility index of the NASDAQ 100, as an exogenous sentiment variable. Similar to the VIX, the volatility index of the S&P 500, is utilized as a sentiment measure and even for predicting short-term returns (Feldman, 2010; Ding et al., 2021). We leverage the VOLQ for the more narrowly focused NASDAQ 100, wherein BioNTech is included. Besides the fact that the NASDAQ 100 volatility index is better suited to predict volatility than the VIX (Corrado and Miller, 2005), the advantage is that the VOLQ is available at the same frequency as the BioNTech data, i.e., minute-by-minute, thus allowing for better comparability than, for example, with the monthly Consumer Confidence Index. Each model is fitted with all observations up to a news item and then used to predict the next value. The error indicators used for comparability are standard and include the mean absolute error (MAE), mean squared error (MSE), mean absolute percentage error (MAPE), and root mean squared error (RMSE). Based on the prediction results, we conclude that our model is well-specified and the model parameters are well-determined, as it performs better than all benchmark methods. The Shapiro–Wilk test could not be rejected ($p$-value = 0.9265), so a normally distributed forecast error (MAPE) can be assumed with $\mathcal{N}(0.026, 0.032^2)$. Thus, the forecast neither overestimates nor underestimates systematically. In addition to theoretical arguments for the use of a trinary agent system in Section 2.3.1.1, the comparison of the prediction performance of the two-state and three-state models provides a nuanced indication that the three-state model is marginally superior. However, despite the very similar performance of the 2SM and 3SM, the result confirms that our model calibration procedure works well. Fundamentally, it can be argued that the approaches from statistical physics have a strong advantage over regular prediction approaches in this application due to the processing of the news $\mathcal{B}$.

**Table 4:** Results of an (in-sample) one-step-ahead forecast using the 18 events from Table 2 with benchmark approaches and four error indicators.

| | **MAE** | **MSE** | **MAPE** | **RMSE** |
|---|---|---|---|---|
| 3SM | 7.707 | 86.987 | 0.026 | 9.327 |
| 2SM | 7.707 | 87.009 | 0.026 | 9.328 |
| LOCF | 11.079 | 217.087 | 0.036 | 14.734 |
| MA | 11.688 | 221.481 | 0.038 | 14.882 |
| ARIMA | 11.064 | 216.700 | 0.036 | 14.721 |
| TS Regression | 11.103 | 218.361 | 0.036 | 14.777 |

Depending on the new price, investor-internal reports can then be written and/or micro hedges initiated. In the case of a machine learning implementation with a continuous learning algorithm and a suitable loss function (Goodfellow et al., 2016), the process could be improved, fully automated, and operate in subminute time ranges.

## *2.5. Conclusions*

In recent decades, increasing numbers of models have been used in econometrics that have been adopted from statistical physics. The development is well advanced, and complex theoretical models are the subject of many simulative and numerical investigations. With all models, there is always the question of how to determine the model parameters. In the case of models applied to securities, in particular, the concern is how the parameters can be estimated on the basis of capital market data. We investigated this question and examined the basic model of an ideal agent system in greater detail, and developed a procedure for parameter estimation. The more complex models mentioned above, which incorporate social interactions between agents, are based on this basic model, so the methods developed for parameter estimation are also useable in advanced model approaches. In our analysis, we adhered to the extension of the commonly used binary agent system to include the hold position to a trinary agent system and derived corresponding equations to describe the dynamics, which serve as a starting point for parameter estimation. In physics, special experiments are designed to determine model parameters and to research phenomena of interest. In the field of finance, it will be difficult to construct an experiment under laboratory conditions to implement a similar approach. Here one is dependent on the abundance and correct use of data from the capital market. The art of the financial experimenter is to find the data that most closely resemble the phenomenon under study and then perform the modeling based on those data. Cut-out experiments must, therefore, be defined in which the specific research question can be analyzed. For the case of the ideal agent system with the three states "sell", "hold", and "buy", we have shown a modeling approach and paid special attention to the suitable selection of the central message for an event influencing the price of a share. In addition, we have derived and reported key risk indicators that characterize the sensitivity of the system at a specific operating point. We also examined the question of a possible forecast concept and found a way to describe the price movement triggered by a message as part of a short-term forecast. The performance comparison with selected benchmark approaches shows that our calibrated three-state model provides better predictions than regular benchmark approaches. In addition, we found nuanced advantages of the three-state model over the two-state model, which we used as another benchmark approach. Therefore, it can be concluded in principle that approaches from statistical physics have a strong advantage over regular prediction approaches due to the processing of sudden news. The use of algorithms from the field of machine learning could map the presented method and thus generate warnings or risk reports in less than a minute in the event of a risk or suggest microhedge strategies and cope with the shock of the sudden event. In connection with this, the approach chosen here must be evaluated critically since various manual interventions had to be accepted for the sake of more consistent results. Thus, our application mainly identifies limitations in the analysis of the news situation and can be further improved in this direction. These improvements depend, for example, on further developments in the field of linguistic text analysis, in particular to more precisely and reliably recognize contexts and valences. However, the results indicate that our calibration procedure works well and that the model was correctly specified, and the model parameters were well determined. In a research project based on the results presented here, the question of a suitable calibration of generalized agent systems with coupled dynamic components can be addressed and a procedure can be designed how, on the basis of capital market data from special market phases, the required coupling parameter can be estimated. Then, the complete set of model parameters of the generalized agent system is available.

## 2.6. *Appendix – Table of Correspondence*

**Table 5:** Correspondence of physical model parameters in an econometric context, substantiated with relevant literature.

| Model Parameters | | | Econometrics | Literature |
|---|---|---|---|---|
| **External Parameters** | $\mathcal{B}$ | Magnetic Field | News Sentiment | *Public Information that Affects all Agents; Investment Environment; Preference Parameter:* Bouchaud (2013); Michard and Bouchaud (2005); Sornette and Zhou (2006); Borghesi and Bouchaud (2007); Kaizoji (2000); Weidlich (1971) |
| | $T$ | Temperature | Volatility | *Noise; Irrationality; Degree of Randomness in Agents' Decisions; Collective Climate Parameter or Volatility:* Bouchaud (2013); Crescimanna and Di Persio (2016); Kaizoji (2000); Oh and Jeon (2007); Krause and Bornholdt (2012); Weidlich (1971); de Mattos Neto et al. (2011); Börner et al. (2023c) |
| | $N$ | Number of Particles | Number of Shares | *Number of Traders in buying/selling Positions:* Zhang et al. (2015) |
| **Model Parameters** | $\mu$ | Magnetic Moment | Willingness to Trade | *Willingness to adopt/buy; Idiosyncratic Judgment:* Bouchaud (2013); Michard and Bouchaud (2005); Sornette and Zhou (2006); Borghesi and Bouchaud (2007); Crescimanna and Di Persio (2016) |
| | $k$ | Boltzmann Constant | Scale/Unit Parameter | Unspecified in Literature (model endogenous) – mediates between Entropy and Energy. |
| | $\alpha$ | Energy of State $n_0$ | Energy Share of "hold"-Position | |
| **Target Parameters** | $E$ | Energy | Investors' "Utility" | *Utility Function:* Bouchaud (2013) |
| | $\mathcal{S}$ | Entropy | Entropy | *Measure of Uncertainty:* Shannon (1948); Marsili (1999) |
| | $M$ | Magnetization | Purchase/Sale Potential | *Aggregate Demand; Average Opinion; Net-Demand:* Bouchaud (2013); Michard and Bouchaud (2005); Vikram and Sinha (2011); Zhang et al. (2015) |
| | $N_{\mathrm{pot}}$ | – | Trade Potential | *Active Agents:* Fernandez et al. (2016) |
| | $\chi$ | Susceptibility | Overall System Sensitivity | *Depth parameter of the market which measures sensitivity of price fluctuation in response to changes in excess demand:* Zhang et al. (2015) |
| | $\eta$ | Thermal loss Coefficient | Overall System Sensitivity | Unspecified in Literature. Parameter which measures sensitivity of trade potential in response to changes in volatility |
| | $c_B$ | Capacity | Overall System Sensitivity | Unspecified in Literature. Parameter which measures sensitivity of utility in response to changes in volatility |

*2.7. Declaration of (Co-)Authors and Record of Accomplishments*

**Title:** Ideal Agent System with Triplet States: Model Parameter Identification of Agent-Field Interaction

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**Conferences:** Participation and presentation at 'Forschungskolloquium Finanzmärkte', June 1, 2022, Düsseldorf, Germany

**Publications:** Published on SSRN and in a special issue of *Entropy* 2023, 25(12), 1666, a single-blind peer-reviewed journal. Current status: Published. doi: `https://doi.org/10.3390/e25121666`.

**Share of contributions:**

| **Contributions** | **Christoph J. Börner** | **Ingo Hoffmann** | **John H. Stiebel** |
|---|---|---|---|
| **Research design** | 10% | 45% | 45% |
| *Development of research question* | 10% | 45% | 45% |
| *Method development and specification* | 10% | 45% | 45% |
| **Research performance & analysis** | 0% | 35% | 65% |
| *Literature review and framework development* | 0% | 30% | 70% |
| *Data collection, preparation and analysis* | 0% | 10% | 90% |
| *Analysis and discussion of results* | 0% | 50% | 50% |
| *Derivation of implications and conclusions* | 0% | 50% | 50% |
| **Manuscript preparation** | 20% | 40% | 40% |
| *Final draft* | 20% | 40% | 40% |
| *Finalization* | 20% | 40% | 40% |
| **Overall contribution** | 10% | 40% | 50% |

Date, Cristoph J. Börner

Date, Ingo Hoffmann

Date, John H. Stiebel

## 3. Generalized Agent System with Triplet States: Model Parameter Identification of Agent-Agent Interaction

### *3.1. Abstract*

Interactions between different agents of a system, such as social interactions between investors in the capital market, have been studied for several years in econophysics research. Researchers in this area are concerned with the appropriate description of couplings between investors and the resulting macroscopic effects, such as herd behavior. We derive a model from econometric considerations that is similar to the model of Blume et al. (1971), which is well known in physics. We address the difficulty of empirically calibrating the parameters and take a direct approach in contrast to indirect simulation-based calibration approaches in agent-based models. A method is given that may be sufficient to determine some essential parameters of bottom-up models as well, thereby eventually helping to bridge the micro-macro problem. The model contains five parameters for which determination methods, e.g., by means of measurement equations, are presented and discussed. Empirical calibration of the model by using the capital market allows us to examine the model phenomenology in simulations as well as implications for practitioners. In addition to one-step-ahead statements, our study provides the insight that applying binary models to situations with three decision alternatives can lead to biased predictions.

**Keywords:** Agent System, Canonical Ensemble, Herding, Partition Function, Risk Assessment, Utility Function

**JEL Classification:** C10, C46, C51

*3.2. Introduction*

Traditional economic theory assumes that individual agents, as investors, make rational and independent decisions in the capital market. However, knowledge about the imperfection of agents has increasingly influenced economic research, so behavioral economic aspects such as imitation effects has been analyzed (Bouchaud, 2013). Herd behavior is a related phenomenon that can be triggered by imitations or, more generally, by couplings between different agents and may initiate or accompany significant risk events, see, e.g., Bekiros et al. (2017). In this context, one strand of literature – *Econophysics* – stands out in particular, in which models from statistical physics are applied to socioeconomic phenomena, as they are particularly suitable for modeling interaction effects between agents of a system (Sornette, 2014). The applicability of the models is versatile, and the specific design depends on the contextual use. The empirical calibration of many of these models is a complicated and unsolved problem (Sornette, 2014). Although we cannot resolve the methodological problems in the calibration of (bottom-up) agent-based models, we contribute to this research by showing how to estimate the model parameters of a three-state model based on the Ising model so that it can be used in practical applications.

We apply a model in which agents can adopt three states with respect to their decision on capital market concerns and show how the model parameters can be determined empirically. Purely from econometric considerations and not by analogy to existing models in physics, a model with rich dynamics is introduced that is suitable to model both essential characteristics of investors and observations on the overall market. Our main findings are measurement equations for the central model parameters, which are applied to capital market data of the company BioNTech SE. Based on our measurements, we deduce ranges and distribution parameters for the key parameters that might be used in other models as well. In addition, we demonstrate the short-term herd behavior included in the model phenomenology, which is equivalent to the abrupt alignment of the overall system at a critical point of an external state variable. This critical point is reached at lower values of the external state variable than a binary system may predict; this difference is due to the third state. The implication is therefore that when binary models are applied to situations with three possible states, the derived predictions are biased. Furthermore, we show how to set up a predictive model based on the empirical estimates to assess the intensity of price shocks.

Ising's model (1925), currently a well-studied standard model, has often been applied in this context, particularly because of its simplicity in representing the interacting influences of disorder enhancing private information and order enhancing social imitation (Sornette, 2014). Its use to describe social interactions dates back to Weidlich (1971) and Galam et al. (1982) and has since been applied in a wide variety of ways to socioeconomic issues, for example, in the field of tax evasion (Zaklan et al., 2008, 2009; Bazart et al., 2016; Giraldo-Barreto and Restrepo, 2021) and particularly in finance (Chowdhury and Stauffer, 1999; Kaizoji, 2000; Bornholdt, 2001; Sornette and Zhou, 2006; Zhou and Sornette, 2007; Bouchaud, 2013; Crescimanna and Di Persio, 2016). A variety of review articles provide an overview of the econophysics literature strand in general and/or highlight primarily economic issues, e.g., Chakraborti et al. (2011a,b); Bouchaud (2013); Sornette (2014); Schinckus (2016); Kutner et al. (2019); Zha et al. (2021).

We contribute to this strand of literature by adhering to the extension of the standard Ising model to include a third state. Starting from econometric considerations of the utility contribution of investor decisions, a model for a general agent system is constructed that is similar to the physical model for the $He^3$-$He^4$ phase transition of Blume et al. (1971). However, since multiple varieties of investors or multiple investment alternatives are not considered in the present case, the model studied here does not include, for example, a chemical potential that models the effect of particle number change in the Blume et al. (1971) model. In the model considered here, agents can decide between three different states, where the labeling of the states depends on the setting chosen. Within the scope of the paper, we show the application to the financial market, where the focus is on the empirical determination of the model parameter. Empirical analysis allows conclusions to be drawn not only about the value of a single parameter but also about its range. The latter leads via inequalities to an indicative determination of the variance. Thus, conclusions about a distribution for individual model parameters are possible. Furthermore, we demonstrate the rich model phenomenology (first- and second-order transitions) and thereby show how short-term herding behavior emerges and how the basic construction of a one-step-ahead forecast can be implemented.

Three-state models have already been applied to the capital market, but we distinguish ourselves from the literature because we do not use a bottom-up agent-based model but instead estimate the model parameters empirically in the market and consider all possible interactions between investors with different terms. The approaches that also use empirical data either rely on indirect calibration methods or use data to compare the properties of their models with those of real data.

In this context, a brief overview of related research directions is given below. The contents of selected, central contributions are considered, and the contribution of the present paper to the literature strand is described. Iori (1999) proposes an Ising-like model with heterogeneous interacting agents that can take three discrete values to explain several stylized facts of the financial markets. The trading activity of each agent is constrained by an initial capitalization and depends on thresholds in a probabilistic manner. The price is a function of the ratio of supply and demand, as well as the available quantity of securities. The outcomes of the model are numerically simulated with various parameter values. The model by Cont and Bouchaud (2000), which is based on percolation phenomena, shows how herd behavior can reproduce fat tails through communication between agents. Neighboring nodes form clusters based on probability and make collective investment decisions, whereby the price is influenced by excess demand. Chowdhury and Stauffer (1999) reformulate the model by Cont and Bouchaud (2000) in a way that spin values are a measure for the number of investors represented by a portfolio manager of an investment agency. The tendency of portfolio managers to influence each other is thus modeled so that bubbles and the occurrence of fat tails can be explained. Takaishi (2005, 2013) formulates a three-state model based on the Potts (1952) model, which is a generalization of the standard Ising model with $q$ possible states. The agent-based model extends the Bornholdt (2001) model and involves updating schemes for the agent states, thereby showing the main stylized facts of financial markets. In the agent-based model of Sato (2007), heterogeneous investors trade multiple currency pairs and can again choose among three trading activities based on threshold specifications. The model allows inferences about the empirical properties of the tick frequencies of currencies. Sieczka and Holyst (2008) propose a model of interacting market participants based on a generalized Ising spin model and define a price evolution as a function of the system magnetization. The model is also based on a threshold specification, according to which investors are motivated to act only when a certain magnetization in the system is overcome. Stylized facts such as the fat-tailed distribution of returns, volatility clustering, and decay of autocorrelation of returns are replicated. Murota and Inoue (2014) also extend the Ising model to a third state to explain economic crises from a microscopic perspective. In addition to the interactions and exogenous information as terms, an additional term for chemical potential is introduced, which controls how many investors actually act. The result is an update model for central, so-called dynamic hyperparameters, whose evolution is compared based on simulations and empirical data. Zubillaga et al. (2022) propose a generalized form of an opinion formation model based on Vilela et al. (2019) with three possible states in which heterogeneous agents, namely, noise traders and noise contrarians (fundamental analysis-oriented investors), interact. Capital market data are used here to test the goodness of the produced stylized properties of financial markets. Three-state models such as the Blume et al. (1971) model are also implemented in sociophysical research to study opinion formation in social networks with different characteristics (Yang, 2010; Ferri et al., 2022).

Many of the above approaches start with modeling the agents in systems, thus attributing certain properties to them and creating heterogeneous objective functions at the agent level. This is referred to in the literature as bottom-up agent-based modeling. Agents then interact with each other in simulations, and ex ante unknown macro patterns are studied as market equilibria (Schinckus, 2016). As a result of increasingly differentiated agent-based specifications, the models become very complex, and it is no longer possible to determine which dynamics are caused by which model components. This difficulty of not knowing how microlevel elements affect macrolevel equilibrium is referred to in the literature as the micro-macro problem. Furthermore, when these models are applied and tested against real data, empirical calibration and validation is an issue that is broadly discussed in the agent-based model literature (Werker and Brenner, 2004; LeBaron, 2006; Windrum et al., 2007; Fagiolo et al., 2007; Chen et al., 2012; Iori and Porter, 2012; Fagiolo et al., 2019). In particular, research is needed on how to use the models in capital market applications, as the empirical calibration of the models or the correct choice of values for the parameters is complicated (Sornette, 2014). The difficulties in determining the parameters of a bottom-up constructed model cannot be fully mitigated here. However, a way is shown that may be sufficient to be able to determine some essential parameters of bottom-up models. In the present paper, a top-down approach is taken, leading to a model from purely econometric considerations. Some components of this model can also be found in the bottom-up models described above, for example, the coupling term known from the standard Ising model. We thus address the research gap of measuring the coupling parameter empirically raised by e.g., Chang (2014) in a similar modeling exercise. If special market situations are considered, the approach examined here can be used to infer the value of the parameters largely in isolation from other influences. We show how such cut-out experiments can be designed and parameters determined.

Other three-state and agent-based models use an updating mechanism to mimic or replicate a data-generating process (Axtell and Farmer, 2022). Our model, and this is how the baseline model is intended from thermodynamics, is not a phase transition model, but an equilibrium model that describes each "frame" – or each moment – of the market as a separate state. These single "frames" of a system can be used to derive quantities that are similar to thermodynamic potentials, which are well known in physics (Isihara, 1971; Greiner et al., 1995; Bouchaud, 2013). These quantities can then be applied to the capital market as a use case. Since we use cut-out experiments to find quantities matching the thermodynamic potentials in real capital market data, we can estimate the model parameters directly from the data (Sato, 2007) and do not have to rely on indirect simulation-based approaches such as those proposed in the agent-based models literature cited above. Once the parameters are determined, they can be incorporated into a predictive model, for example, for the price of a stock. This dynamization then corresponds to the updating mechanisms in agent-based approaches.

The model to be studied in this paper follows a top-down model from econometric considerations. The approach is based on the considerations of Börner et al. (2023b), who introduced an observer and formulated the utility of a certain constellation from the observers' point of view. Thus, we operationalize our model with a utility function in which a market observer (market analyst, researcher, investor) derives utility from certain constellations of investors or investor groups, for example, regarding the need to explain a certain configuration. Basically, discrepancies between the trading decisions of neighboring investors require more explanatory effort for the market observer, which reduces the utility. Thus, the explanatory effort is equivalent to taking in information at a certain cost. Similarly, trading decisions in the same direction increase utility since no difference needs to be explained. The utility or disutility is expressed in monetary units. The result is a model that takes into account the various interaction possibilities between the agents in an additive way so that the utility is increased or decreased depending on the individual configurations. The state that the market observer assesses already represents the decisions of all investors and thus implicitly incorporates that neighboring agents have interacted with each other. An important model component that evaluates these configurations in terms of utility is already included in the standard Ising model and scales with the constant $J$. In this context, $J$ does not measure the strength of the influence from one agent on a neighboring agent but rather what influence the decisions of neighboring agents have for the utility calculus of the market observer. Thus, $J$ is not an evaluation metric for couplings but an evaluation metric for the configurations of neighboring agents in utility calculus. The parameter is therefore important for evaluating an individual configuration of investors and for describing the overall system. We show how $J$ and the other model parameters can be determined by using capital market data as part of the empirical application in Section 3.4. We focus on the correct transfer and interpretation of the parameters of the general model to the financial market, and since we follow up on Börner et al. (2023b) here and entirely determine the model parameters, we can show the extensive model dynamics by using simulations.

The remainder of the paper is structured as follows: Section 3.3 outlines the general model as well as its limits and extensions. In Section 3.4, the model is applied to the capital market. We thereby complete, on the one hand, the empirical analysis in which we empirically measure the coupling parameter $J$, and, on the other hand, we show the rich phenomenology of the model with numerical analyses. The last Section 3.5 summarizes our findings, and we present some ideas on how to extend both the model and the method for further research topics.

### *3.3. Method*

In the following, the model of a generalized agent system is developed. In this consideration, the individual agents can adopt an inner attitude toward three different actions. When using the example of a stock investor as an agent, these three actions are “buy”, “hold” and “sell”. Therefore, a generalized agent system with a triple state is developed and investigated.

The developed model is then used for the statistical description of the agent system with and without interaction effects.

The basic idea for such approaches in econometrics goes back to the early works of Weidlich (1971) and Galam et al. (1982) and has its origin in physics in the description of Ising systems (Ising, 1925; Isihara, 1971; Greiner et al., 1995). The transfer of the modeling approach from two state Ising systems to econometrics by analogy is currently standard and well-studied, e.g., Kaizoji (2000); Oh and Jeon (2007); Krause and Bornholdt (2012); Bouchaud (2013) and the extensive literature cited therein. The model approach selected here incorporates a third – neutral – state. The following model approach extends and encompasses the existing models. Recently, the model of an ideal agent system – i.e., without interaction of the agents – with three states was analyzed by Börner et al. (2023b). This model approach is used as a starting point and extended by considering interactions between the agents. As an additional model extension, the interaction of two adjoined investors resulting from the unequal states of the agents is also considered. For a better comparison and to clarify the expansion, we adopt the notation used in Börner et al. (2023b).

In physics, three state models are known and well-studied, e.g., Costabile et al. (2014); Butera and Pernici (2018), and are based on the initial, fundamental works of Blume (1966); Capel (1966) and Blume et al. (1971). A further extension of the Ising models in physics is represented by the Potts model class for $q$-states per node in a lattice (Potts, 1952). Here, the interaction of two adjoined nodes in the lattice only counts if they have the same state. To the best of our knowledge, the three-state econometric model needed here has no direct equivalent in physics, and a simple transfer by analogy cannot be made. The model is therefore to be developed from econometric considerations. The following construction of the econometric model follows rules such as those used in quantum field theory when defining the Lagrangian density, see, e.g., Peskin and Schroeder (1995) or in the definition of a Hamiltonian in statistical physics for spin lattices (Isihara, 1971; Amit, 1978; Landau and Lifšic, 1980; Greiner et al., 1995). This systematic conception leads to a model similar to the Blume et al. (1971) model. An essential difference, however, is that no formalism similar to the chemical potential in physics is considered, since here initially the number of investors or the number of shares and, more generally, the number of agents does not change (*phys.:* constant number of particles).

### *3.3.1. Utility Function*

The basis of the model is the definition of a suitable utility function $U$. When transferring approaches from physics to econometrics, the utility function is usually interpreted as the negative of the Hamiltonian known from physics: $U = -\mathcal{H}$ (Marsili, 1999; Bazart et al., 2016). Since no Hamiltonian comparable to the econometric problem at hand was found in physics, the appropriate utility function must be constructed. The starting point for this is the idea of formulating the utility function from the point of view of an observer (market observer, market analyst, researcher, investor) as done by Börner et al. (2023b), who has, e.g., the goal of describing the market and deriving a model of action for a personal investment decision as an investor (agent) in the generalized agent system. Positive utility thus results from being able to capture the market situation with little effort. Accordingly, one possible view is that the utility for the investor lies in how much or little effort in monetary units that agent has to acquire information to describe a certain state. Thus, the approach is based on the interpretation that a certain setting of the other agents (configuration) can mean an increase or a reduction in utility.

We follow Börner et al. (2023b) and define a news environment $\mathcal{B}$ (*phys.:* field). For example, the news environment for a stock investor could consist of company news on a specific issue. A message environment $\mathcal{B} = \text{sign}(\mathcal{B})B$ has a basic sentiment $\text{sign}(\mathcal{B}) = (-1, 0, +1)$ – indicating, e.g., "bad", "indifferent" and "good" news – and a strength $B = |\mathcal{B}|$ affecting the agents. Furthermore, for each agent $i = 1, \ldots, N$, a discrete variable $S_i$ is introduced, which maps the fact that the individual agent behaves in conformity with basic sentiment, $S = +1$, or not in conformity, $S = -1$. In the present case, we consider a triplet state system and a neutral position – e.g., "hold" – is also taken into account with $S = 0$, so that, e.g., an indifferent investor attitude is included.

The construction of the utility function now includes all interactions that can affect utility. On the one hand, there are interactions of the agents with the message environment (*phys.:* particle-field interaction), and on the other hand, there are interactions of the agents among themselves (*phys.:* particle-particle interaction). Depending on the respective configuration, an increase or a reduction in utility must be considered. We stick to the interpretation from Börner et al. (2023b) that an observer can be an investor and thus part of the overall system. For example, if the message field for a stock is positive and investor $j$ buys the stock ($S_j = +1$, in conformity), the observer might be satisfied with the first reason as an explanation: Agent $j$ buys the stock because $\mathcal{B}$ is positive. If investor $j$ sells the stock ($S_j = -1$, not in conformity), further information must be obtained using resources to explain the observation. In the first case, an increase in utility and, in the second case, a reduction in utility must be considered. Regardless of the message environment, the utility increases when two neighboring agents $i$ and $j$ behave in the same way and decreases when they behave differently. In the first case, the observer might be satisfied with the first reason for explanation: Investor $i$ buys the stock because investor $j$ buys the stock (and vice versa). In case of a discrepancy, the investor needs further information for the explanation, and the investor's utility decreases due to the observed configuration.

Accordingly, we can write down the following expression for the utility function:

$$
\begin{array}{rrrll}
U = & + & \mu B & \sum_i S_i & \text{Term 1} \\
 & - & \alpha_1 \mu B & \sum_i (1 - S_i^2) & \text{Term 2} \\
 & + & J & \sum_{\langle ij \rangle} S_i S_j & \text{Term 3} \\
 & + & \alpha_2 J & \sum_{\langle ij \rangle} (1 - S_i^2)(1 - S_j^2) & \text{Term 4} \\
 & - & \alpha_3 J & \sum_{\langle ij \rangle} (1 - S_i^2) S_j^2 & \text{Term 5}
\end{array}
\tag{22}
$$

The sums in Equation (22) can be further evaluated. In doing so, the individual terms (1 to 5) merge. For a better back-interpretation, especially of the parameters, the terms are not further summarized here, and Equation (22) is used as a starting point. Hence, the notation $\langle ij \rangle$ denotes the summation over adjoining agents $j = 1, \ldots, z$ of an agent $i = 1, \ldots, N$ in the agent system, with $z \leq N$, where $0 < \mu$ measures the contribution of the status "in conformity" or "not in conformity" to the utility in a given message environment. Depending on the research area, $\mu$ has different names in the literature: *willingness to adopt*, *willingness to pay* or *idiosyncratic judgment* (Sornette and Zhou, 2006; Bouchaud, 2009, 2013; Crescimanna and Di Persio, 2016). In Term 2 in Equation (22), the neutral position – e.g., "hold" in a stock investment process – is considered. In a given message environment, the neutral position causes a fraction $0 \leq \alpha_1 \leq 1$ of the effect of the other two positions and is always rated negatively regardless of the basic sentiment, $\text{sign}(\mathcal{B})$, of the news. Börner et al. (2023b) describe a method of how parameters $\mu$ and $\alpha_1$ can be determined in the case of a stock investment by using capital market data. They analyze the example of an investment in stocks of the company BioNTech SE (ISIN US09075V1026) and find the values $\mu$ and $\alpha_1$ with ranges shown in Table 7. The parameter $0 < J$ measures the contribution of a configuration with the same agent behavior to the utility, see, e.g., Bazart et al. (2016). A different valuation can be considered for the neutral position (Term 4) with $0 \leq \alpha_2$. The configuration of a neutral state to the other states is always considered with a negative contribution (Term 5), and the contribution is also related to $J$ and scaled with $0 \leq \alpha_3$.

In econometric and sociological applications, the parameters $\alpha_2$ and $\alpha_3$ should be very close to +1. Since symmetrical configurations in a system should deliver the same increase in utility, $\alpha_2$ should be approximately +1. The same applies to asymmetrical configurations. Each of these should cause about the same decrease in utility, and thus, $\alpha_3$ should be approximately +1. In our further considerations, we concentrate on exactly this case $(\alpha_2, \alpha_3) \approx (+1, +1)$ and study the system in the vicinity of these parameter values but also provide an outlook on the dependency of the system description in the case of values that deviate greatly from +1; see Section 3.3.6. The analysis of market data must provide information on how far $\alpha_2$ and $\alpha_3$ deviate from +1 in practice; see Section 3.4.4.

Considering scaling with $\alpha_2$ and $\alpha_3$, the central parameter $J$ describes the contribution of the various interactions to the utility. Parameter $J$ has different labels depending on the research area, e.g., *degree of willingness to adopt an attitude* (Weidlich, 1971); *willingness of agents to align their actions* (Cont and Bouchaud, 2000); *social pressure or imitation effects* (Michard and Bouchaud, 2005; Borghesi and Bouchaud, 2007; Bouchaud, 2013); *imitation term* (Sornette and Zhou, 2006; Crescimanna and Di Persio, 2016). In statistical physics, $J$ is the value of the so-called exchange integral, also known as the Ising constant or coupling constant in some literature (Isihara, 1971; Greiner et al., 1995). Landau and Lifšic (1980, §141 on p. 447) state that $J$ determines the energy of interaction of a pair of adjoining dipoles in a lattice. This definition, transferred to econometrics, suggests the above interpretation of measuring the contribution of a certain configuration to utility.

By adding the neutral state, Equation (22) expands the existing and well-studied model framework in econometrics. With Terms 1 and 2, the ideal agent system from Börner et al. (2023b) is taken into account, and Terms 1 and 3 include the standard Ising system with two possible states, see, e.g., Bornholdt (2001); Sornette and Zhou (2006); Harras et al. (2012); Bouchaud (2013); Vincenzo et al. (2014); Takaishi (2015); Bazart et al. (2016); Giraldo-Barreto and Restrepo (2021). Existing approaches with three state models, e.g., Iori (1999) or Murota and Inoue (2014), are also extended. In both cases, e.g., the interaction term between the neutral position and the active positions is neglected but considered in our approach. Moreover, our measured data in Sec. 3.4.2 do not suggest that we have a structure-variable system, in the sense that the coupling parameter varies discretely, as in Murota and Inoue (2014).

*Limits and Extensions*

Many extensions of the previously defined utility function Equation (22) are conceivable and at the same time show the limitations of what is considered here.

In the present case, the mainstream literature is followed, and the parameter $\mu$ is set the same for all agents. Distributions $\rho(\mu)$ of the parameter or individual settings $\mu_i$ are considered in special extensions and applications, e.g., in sociophysics (Foley, 1999; Marsili, 1999; Anderson et al., 2001; Castellano et al., 2009). We focus here on the basic dynamics of the generalized agent system and consider such extensions in later research.

The same applies to the parameter $J$. Again, we follow the mainstream and assume $J$ to be equal for all agents (Weidlich, 1971; Nadal et al., 2003; Laciana and Rovere, 2011; Bazart et al., 2016). Distributions of the parameter or individual settings $J_{ij}$ (Durlauf, 1996;

Crescimanna and Di Persio, 2016), e.g., for modeling asymmetric utility changes (*phys.:* anisotropy) depending on which agent is in a certain configuration, can be considered in later research.

We focus on a square lattice or, more generally, on hypercube structures to model the system of agents, and the different configurations are set in single and binary form, e.g., $U \sim S_i$ (Term 1) or $U \sim S_i S_j$ (Term 3). Hence, we follow the majority of the literature (Iori, 1999; Bornholdt, 2001; Takaishi, 2005; Sornette and Zhou, 2006; Sieczka and Holyst, 2008; Crescimanna and Di Persio, 2016; Zubillaga et al., 2022). Higher-order model extensions, e.g., $U \sim S_i S_j S_k$ for hexagonal lattice structures or more complex lattice structures $U \sim S_{i_1} S_{i_2} S_{i_3} \ldots S_{i_n}$, are not considered here. Thus, the evaluation of such structures and configurations remains for further research.

Finally, a sixth term $U \sim \alpha_4 B^2$ with a positive $\alpha_4$ could be considered (*phys.:* field-field interaction). This extra term models the fact that a news environment already contributes utility and that the utility increases the stronger the news is. Thus, when considering the susceptibility (Bouchaud, 2013; Zhang and Li, 2015; Börner et al., 2023b), additional terms $\chi \sim \alpha_4 B$ should appear, and $\alpha_4$ should have to be estimated by using market data.

In addition, a constant utility $U_0$ cand be added (*phys.:* ground state energy) that evaluates a specific reference state of the system of agents. In Section 3.3.3, we use Equation (28) to consider Boltzmann-Gibbs distributions (Greiner et al., 1995; Marsili, 1999; Kaizoji, 2000; Anderson et al., 2001; Bouchaud, 2013; Börner et al., 2023b) that can be interpreted as logit rule (Bouchaud, 2013). For a given message environment, i.e., $\mathcal{B}$ = const., the additional sixth term $U \sim \alpha_4 B^2$, such as $U_0$, should be constant and should continue to cancel; therefore, it is not considered further here. Research that focuses directly on utility function analysis might consider such additional terms.

### *3.3.2. Mean-Field Approximation*

The utility function Equation (22) cannot be evaluated in general for arbitrary underlying network structures of agents (*phys.:* spin-lattice) with respect to macroscopic variables, e.g., the surplus of buyers in a stock investment (*phys.:* magnetization). Special solutions of similar problems for one-dimensional (Ising, 1925) and two-dimensional (Onsager, 1944) lattice structures are known from physics. As in physics, in econometrics, solutions to higher-dimensional problems are usually determined by approximating a mean field around an agent (Weidlich, 1971; Galam et al., 1982; Brock and Durlauf, 2001; Gordon et al., 2005; Nadal et al., 2005; Michard and Bouchaud, 2005; Gordon et al., 2009; Bouchaud, 2013). The idea goes back to Weiss (1907, *phys.:* molecular field approximation) and is now part of the standard repertoire in physics (Isihara, 1971; Amit, 1978; Landau and Lifšic, 1980; Greiner et al., 1995) and econometrics, see, e.g., the research papers just cited above. The main idea of the mean-field approximation is that the state of an agent is influenced by the mean field of the states of its adjoining agents.

We follow Weiss (1907) in his original approach and apply the mean-field approximation to Terms 3, 4 and 5 of the utility function Equation (22). Therefore, the mean field of the states of the adjoining agents is approximated by the mean field of the whole agent system. This results in surcharges or deductions for Terms 1 and 2, which we specify and interpret.

At the beginning, similar to Blume et al. (1971), with $\langle x \rangle = \frac{1}{N} \sum_{i=1}^{N} x_i$, the following mean values are introduced. In the system of agents, the mean attitude ("in conformity" or "not in conformity") of the agents is $m = \langle S \rangle$, with $-1 \leq m \leq +1$, and the mean neutral position is $m_0 = 1 - \langle S^2 \rangle$, with $0 \leq m_0 \leq +1$. If $z$ denotes the average number of adjoining agents, then the utility function Equation (22) can be written in mean-field approximation:

$$U = +\mu B_1 \sum_i S_i - \alpha_1 \mu B_2 \sum_i (1 - S_i^2) \tag{23}$$

With the following effective message environments (*phys.:* effective fields):

$$B_1 = B + \frac{1}{2} \frac{J}{\mu} z\, m \tag{24}$$

$$B_2 = B - \frac{1}{2} \frac{J}{\mu} z \left( \frac{\alpha_2 + \alpha_3}{\alpha_1} m_0 - \frac{\alpha_3}{\alpha_1} \right) \tag{25}$$

If it is considered that all parameters $(\mu, J, z, \alpha_1, \alpha_2, \alpha_3)$ are positive, then according to Equations (24) and (25), the following interpretation suggests itself.

A surplus of agents in the state "in conformity" ($m > 0$) acts as an additional message, reinforcing the message environment for agents who are not in the neutral position. Depending on the empirically determined fractions $\alpha_1, \alpha_2$ and $\alpha_3$, the average of the agents in the neutral position $m_0$ has a strengthening or weakening effect on the message environment for agents who are in the neutral position.

*Limits and Extensions*

The number $z$ of adjoining agents (*phys.:* coordination number) turns out to be a significant variable. Behind this, for example, when making an investment decision is the question of how many other investors are consulted on average to secure or substantiate one's own opinion. In practice, this number should have a wide range $0 \leq z < \mathcal{O}(10^2)$.

The question of how investor networks or, more generally, information networks are structured has increasingly become the focus of economic research (Emmert-Streib et al., 2017). Some studies model the transmission of information between investors by means of direct communication (Stein, 2008; Han and Yang, 2013; Andrei and Cujean, 2017). However, an empirical approach to the question is more complicated, as direct communication between investors is rarely documented. Instead, indirect proxies, such as correlated transactions (Ozsoylev et al., 2014; Baltakys et al., 2021), are used to make statements about the characteristics of investor networks in the context of network theory (Ahern, 2017). Both small values for $z$ with six contacts on a three-dimensional square lattice (Guimaraes and Lima, 2021), e.g., in the team of a smaller investment company, and numbers of contacts in the order of $O(10^2)$ via social media are realistic due to the high level of interconnectedness on the internet. Thus, $z$ is not only an individually varying quantity but can also be modeled well by means of distributions. The Empirical Investor Network based on 2005 data from the Istanbul Stock Exchange corresponds to the order of $O(10^2)$ with a median number of 159 links (Ozsoylev et al., 2014). Studies on social networks that examine the average number of links arrive at similar figures (Ugander et al., 2011; Hampton et al., 2011). Guimaraes and Lima (2021) studied a three-dimensional Ising system in the context of the financial market and found that modeling with multiple dimensions or a higher number of neighboring investors better corresponds to the properties of empirical data. Investors with multiple connections are therefore more representative. Nevertheless, to the best of our knowledge, precise investigations on the magnitude of $z$ have not yet been carried out in econometrics. Thus, in the following, we use our own estimate of $z = 12$ to investigate the basic dynamics of the generalized agent system. This estimate is based on the rarely found, nonrepresentative references to the size of investment committees. In Section 3.3.6, we see that the dynamics are essentially determined by the product $Jz$. The latter can be determined empirically, i.e., the previously made assumption does not affect the dynamics but rather the estimate of $J$, provided $z$ is large enough, as the following suggests.

As a further extension, it is conceivable that the mean field of the states of the adjoining agents effective per agent $i$ shows fluctuations and does not correspond to the mean field of the entire agent system. Consequently, these fluctuations could be considered in the form $m_i = m + \delta m_i$. It is known from physics that as the number $z$ increases, the mean-field approximation rapidly improves and is already a very good approximation for $z \geq 12$ (Greiner et al., 1995). The influence of fluctuations diminishes in the limit $z \to \infty$ and can ultimately be ignored (Amit, 1978). However, if very small values of $z$ are to be taken into account, discrepancies between theory and practice are likely to appear.

#### *3.3.3. Canonical Partition Function and Probability of Occupancy of the States*

The canonical partition function $Z$ is to be applied and calculated if a system of agents is not to be viewed in isolation but in interaction with an environment (Isihara, 1971; Huang, 1987; Greiner et al., 1995). This is the case, for example, with investors in stocks of a company. The stocks themselves are part of an overall market, and their development is influenced by this. The interaction described above is generally described by an exchange variable $T$ (*phys.:* temperature). In other strands of literature, $T$ is also referred to as *amount of noise*, *irrationality*, *degree of randomness in agents' decisions*, or *collective climate parameter*; see, e.g., Weidlich (1971); Kaizoji (2000); Oh and Jeon (2007); Krause and Bornholdt (2012); Bouchaud (2013); Crescimanna and Di Persio (2016). In the example of a stock, as part of an overall market, most of the literature assumes that the volatility in the financial sector is this exchange variable: $T = \sigma$ (Marsili, 1999). In the following, volatility is used as a generic term for $T$.

The introduction of the quantity $\beta = 1/kT$ (*phys.:* inverse temperature) with a suitable constant $k$ (*phys.:* Boltzmann constant) simplifies the representation of the partition function $Z$ and scales the quantities in desired units (Kaizoji, 2000; Oh and Jeon, 2007; Krause and Bornholdt, 2012; Bouchaud, 2013). The constant $k$ is not a model parameter but defines the unit of measurement (Shannon, 1948) and is fixed as in Börner et al. (2023b) with $k = 1$ USD for the following analyses.

The canonical partition function is defined as the sum over all configurations $(S_1, \ldots, S_N)$:

$$Z(T, B, N) = \sum_{(S_1,\ldots,S_N)} \exp\left(\beta U(S_1, \ldots, S_N)\right). \tag{26}$$

In Equation (26), the Boltzmann factors that depend on the utility function are summed. The utility function $U$ in turn depends on the configuration $(S_1, \ldots, S_N)$. The configurations are functions of macroscopic variables, e.g., volatility $T$, so that the canonical partition function connects macroscopic quantities with microscopic states (Isihara, 1971). This approach leads to the so-called Boltzmann-Gibbs distribution specifying the probabilities of discrete configurations (Greiner et al., 1995; Marsili, 1999; Kaizoji, 2000; Anderson et al., 2001). In econometrics, this distribution corresponds to the well-known logit rule (Bouchaud, 2013).

The mean-field approximation from Section 3.3.2 led to a system of $N$ noninteracting agents, which are influenced by effective message environments $B_1$ and $B_2$. In the case of noninteracting agents, the overall system can be viewed as an ideal agent system, and the relationship $Z(T, B, N) = [Z(T, B, 1)]^N$ can be used to determine the canonical partition function (Börner et al., 2023b). Considering that each agent can be in three different states, it follows that:

$$Z(T, B, 1) = \underbrace{\exp(-\beta\mu B_1)}_{\substack{S=-1 \\ \text{non-conform} \\ \text{to } \mathcal{B}}} + \underbrace{\exp(-\beta\alpha_1\mu B_2)}_{\substack{S=0 \\ \text{neutral}}} + \underbrace{\exp(+\beta\mu B_1)}_{\substack{S=+1 \\ \text{conform} \\ \text{to } \mathcal{B}}}. \tag{27}$$

The occupation probabilities (*phys.:* Boltzmann-Gibbs distribution) for the three states $S =$

$(-1, 0, +1)$ follow immediately from this:

$$
\begin{aligned}
p_- &= \frac{\exp(-\beta\mu B_1)}{\exp(-\beta\mu B_1) + \exp(-\beta\alpha_1\mu B_2) + \exp(+\beta\mu B_1)} \\
p_0 &= \frac{\exp(-\beta\alpha_1\mu B_2)}{\exp(-\beta\mu B_1) + \exp(-\beta\alpha_1\mu B_2) + \exp(+\beta\mu B_1)} \\
p_+ &= \frac{\exp(+\beta\mu B_1)}{\exp(-\beta\mu B_1) + \exp(-\beta\alpha_1\mu B_2) + \exp(+\beta\mu B_1)}
\end{aligned}
\tag{28}
$$

Hence, the occupation numbers are:

$$
(N_- \quad N_0 \quad N_+) = (p_- \quad p_0 \quad p_+) \times N \tag{29}
$$

with $N_-$ resp. $N_+$ being the number of agents nonconforming resp. conforming to the message environment $\mathcal{B}$ and $N_0$ being the number of agents in the neutral position.

*3.3.4. Model of the generalized agent system*

In Section 3.3.2, the mean attitude $m = \langle S \rangle$ and the mean neutral position $m_0 = 1 - \langle S^2 \rangle$ of the system of agents were introduced. With Equation (29), the macroscopic state variables $(m, m_0)$ can be expressed by the occupation numbers, Equation (29), and then, by the occupation probabilities, Equation (28). For the mean attitude, this results in:

$$
\begin{aligned}
m &= \frac{N_+}{N} - \frac{N_-}{N} \\
&= p_+ - p_- \\
&= \frac{\exp(+\beta\mu B_1) - \exp(-\beta\mu B_1)}{\exp(-\beta\mu B_1) + \exp(-\beta\alpha_1\mu B_2) + \exp(+\beta\mu B_1)} \\
&= \frac{2\sinh(+\beta\mu B_1)}{2\cosh(+\beta\mu B_1) + \exp(-\beta\alpha_1\mu B_2)}.
\end{aligned}
\tag{30}
$$

Similarly, for the neutral position with $m_0 = {}^{N_0}/_{N} = p_0$:

$$
m_0 = \frac{\exp(-\beta\alpha_1\mu B_2)}{2\cosh(+\beta\mu B_1) + \exp(-\beta\alpha_1\mu B_2)}. \tag{31}
$$

With the effective message environments $B_1$ and $B_2$ defined in Equations (24) and (25).

*Findings, Limits and Extensions*

In mean-field approximation, the model of the generalized agent system with three possible states for each agent consists of the coupled, implicit system of Equations (30) and (31). If the model parameters $\mu, J, \alpha_1, \alpha_2$ and $\alpha_3$ in Equation (22) are known, the external variables $\mathcal{B}$ and $T$ are specified, and the system of equations supplies the possible macroscopic state variables $m$ and $m_0$ as solutions. The model parameters are to be estimated for a concrete problem based on a series of measurements. For the example of a stock investor, it was recently shown by Börner et al. (2023b) how $\alpha_1$ and $\mu$ can be determined from capital market data. In Section 3.4, by using the same example of a stock investment, it is shown how the remaining parameter can also be determined from capital market data. With the restriction that only the principle can be explained for $\alpha_2$ and $\alpha_3$ due to the small database.

In contrast to the standard Ising system with two possible states (Foley, 1999; Marsili, 1999; Bouchaud, 2013) and one implicit equation, an additional implicit equation must be taken into account for three states. This raises the question, under what external conditions does the solution presented here provide the solutions of the standard Ising system? This is the case when Equation (31) returns the value $m_0 = 0$ as a solution. The latter can be achieved when $\mathcal{B}$ or $\beta$ or both tend to infinity. In the case of a stock investment, this means an infinitely strong news environment or the volatility of the stock becomes zero. Both are extreme cases that are hardly observable in practice, so we can generally assume $m_0 > 0$ if a third state is possible for the agent.

For $0 < m_0 \leq 1$ Equation (31) can be rearranged, and the following relationship is calculated:

$$\exp(-\beta\alpha_1\mu B_2) = 2\frac{m_0}{1-m_0}\cosh(+\beta\mu B_1). \tag{32}$$

Equation (32) inserted into Equation (30) results in:

$$m = (1-m_0)\tanh(+\beta\mu B_1). \tag{33}$$

where $B_1$ is the effective message environment defined in Equation (24). Except for the factor $(1-m_0)$, this equation is identical to the equation of the standard Ising system, see, e.g., Greiner et al. (1995, §18, Eq. (18.24)). This also shows that in the limit case $m_0 \to 0$ described above, the solutions of the standard Ising system with two possible states for the agents are described with the model presented here. For a three-state system modeled with a two-state model, this has the dramatic consequence that predictions may be inaccurate or wrong. In Section 3.3.6, we identify and investigate a discrepancy as such an inaccuracy. Therefore, if, in practice, a third, neutral state – an investor's "hold" position – is possible for the agents, then the two-state model is presumably not suitable for description in general.

This conclusion was obtained by using the econometric model in the mean-field approximation. As an extension, it remains to be checked whether this conclusion also applies to investigations that do not take approximations into account or whether there are further consequences that are not recognizable in the context of the mean-field approximation.

### *3.3.5. Realizable Macroscopic States*

Depending on constant external variables $\mathcal{B}$ and $T$, the coupled, implicit system of Equations (30) and (31) in some cases provide a sheaf of solutions for the macroscopic variables $m$ and $m_0$. This raises the question of which solutions are realized and observable in practice.

A standard procedure in physics for evaluating possible solutions is the analysis of the free energy $F = E - T\mathcal{S}$. Here, $E$ can be calculated with the Hamiltonian $\mathcal{H}$ for a specific configuration and reflects the (internal) energy of the physical system under consideration. Furthermore, the free energy depends on the temperature $T$ and entropy $\mathcal{S}$ of the system. In a very simplified way, the following can be summarized: In a physical system left to itself, internal processes take place under constant external conditions and in exchange with a heat bath until the free energy is minimal. More directly: The system simultaneously tries to minimize its energy and maximize its entropy. For a more precise specification in physics, see Landau and Lifšic (1980); Huang (1987); Greiner et al. (1995), for example.

In econometrics, the negative utility function corresponds to energy and the volatility to temperature in physics (Marsili, 1999; Bazart et al., 2016). In the present case, the entropy of the generalized agent system is simply

$$\bar{\mathcal{S}} = -k \sum_{i=1}^{3} p_i \log_3 p_i \tag{34}$$

with the probabilities defined in Equation (28), see, e.g., Börner et al. (2023b) and the unit of measurement $k$ (Shannon, 1948). Here, entropy measures, on average, the uncertainty about an agent's state in trit (**tr**inary dig**it**, cf. Brusentsov and Alvarez (2011)) as units of information needed to specify the agent's state.

In this way, a quantity equivalent to the free energy in physics, the gross utility per agent $\bar{\mathcal{U}} = \bar{U} + T\bar{\mathcal{S}}$, can be analyzed in econometrics, where $\bar{U}$ corresponds to the average (net) utility per agent: $U = N\bar{U}$. Conjecturally, an econometric system tends toward a state with maximum gross utility. In Section 3.4.4, we examine and evaluate the solutions of the coupled, implicit system of Equations (30) and (31). Therefore, we also consider the gross utility. The results suggest that in practice, under constant external conditions, those macroscopic states can be observed where maximum utility and maximum entropy can be determined. Hence, a maximum of $\bar{\mathcal{U}}$.

*3.3.6. Measurement Equations for Parameter J.*

Börner et al. (2023b) uses the example of an investment in stocks of a company to show how the parameters $\mu$ and $\alpha_1$ can be determined by using time series of price developments. In this section, equations are derived for the empirical determination of the remaining model parameters: $J, \alpha_2$ and $\alpha_3$. This means that the interaction Terms 3, 4 and 5 in Equation (22) must be examined more closely. For this purpose, the limiting case is considered where the news environment tends toward zero from positive values: $\mathcal{B} \to 0^+$. Thus, the strength of the message environment becomes immeasurably small, $|\mathcal{B}| = B \approx 0$, and the only information that remains is the direction, here positive: $\mathrm{sign}(\mathcal{B}) = +1$. In this consideration, the three states of the agents ("conform", "neutral" and "nonconform") can still be distinguished. Let us note that the consideration defined thus far can be carried out in the same way for the limit $\mathcal{B} \to 0^-$.

To simplify the equations, a volatility $T_c = \frac{Jz}{2k}$ (*phys.:* Curie temperature) is introduced for convenience, and with $B = 0$, $m_0 > 0$ and $x = \frac{T_c}{T}m$, it follows from Equation (33):

$$\frac{1}{1-m_0}\frac{T}{T_c}\,x = \tanh(x). \tag{35}$$

As long as the prefactor $\frac{1}{1-m_0}\frac{T}{T_c}$ is greater than 1, $x = 0$ is determined as the solution of Equation (35), i.e., $m = 0$. If the prefactor is less than 1, several solutions are determined depending on the volatility $T$ and the proportion of neutral positions $m_0$. If the volatility $T \gg T_c$ is successively reduced, a critical point (*phys.:* Curie point) is exceeded, and the prefactor becomes 1, at which the macroscopic state variable $m$ of the agent system changes spontaneously (*phys.:* spontaneous magnetization). This critical point is described by a critical volatility $\tilde{T}_c$. At the critical point, a transition from $m = 0$ to $m \neq 0$ takes place (*phys.:* phase transition). Depending on $\alpha_2$ and $\alpha_3$, a defined proportion of neutral positions $m_0^c$ is observable at this critical point. From the condition that the prefactor in Equation (35) is equal to 1, the following relationship can then be established:

$$\tilde{T}_c = (1 - m_0^c)T_c. \tag{36}$$

With $m_0^c > 0$, the critical point is shifted to lower volatilities $\tilde{T}_c$ compared to those of the standard Ising system. In the latter, the critical point is described by $T_c$ and not by $\tilde{T}_c$. This means that in a system with the neutral position as a possible third state for the agents, the phase transition from $m = 0$ to $m \neq 0$ should in practice only be observable with smaller volatilities. This suggests that the agents in the neutral position have a braking effect on the overall system with respect to the occurrence of the phase transition (*phys.:* supercooling). If the model of a two-state system is incorrectly used for a three-state system, this effect cannot be observed, and the dynamics can be misinterpreted.

With Equation (36), similar to the equations for the standard Ising system in physics (Greiner et al., 1995; Kittel and McEuen, 1996), the measurement equation for $J$ or the product $Jz$ in econometrics can be defined:

$$J = \frac{2k\tilde{T}_c}{z(1-m_0^c)}. \tag{37}$$

The proportion of neutral positions $m_0^c$ is specified shortly. If this proportion is known, the parameter $J$ can be determined by measuring the critical volatility $\tilde{T}_c$ with $z$ known or guessed. Section 2.4 shows an example of how this can be done.

In the case considered here with $B = 0$, the condition $m = 0$ applies at the critical point with $T = \tilde{T}_c$. According to Equation (24), it follows that $B_1 = 0$ and thus $\cosh(0) = 1$ in Equation (31). Finally, with Equation (36) inserted in Equation (31) and after some algebraic transformations, the implicit equation for $m_0^c$ follows:

$$m_0^c = \frac{1}{1 + 2\exp\Big(A(m_0^c, \alpha_2, \alpha_3)\Big)}. \tag{38}$$

With the abbreviation:

$$A(m_0^c, \alpha_2, \alpha_3) = \frac{\alpha_3 - (\alpha_2 + \alpha_3)m_0^c}{1 - m_0^c}. \tag{39}$$

If the parameters $\alpha_2$ and $\alpha_3$ are known, e.g., from fundamental considerations of the utility function Equation (22), then the proportion of neutral positions $m_0^c$ at the critical point $T = \tilde{T}_c$ is determined as the solution to the implicit Equation (38).

Depending on $\alpha_2$ and $\alpha_3$, the implicit Equation (38) has up to three solutions. This points to the possibility of three phase transitions of the generalized agent system under consideration. Knowing $m_0^c$, Equation (36) can then be used to calculate the volatility $\tilde{T}_c$ at the critical point.

It is easy to see that for $\alpha_2 \geq 0$, Equation (38) always has at least the solution $m_0^c = 1$ with this the critical volatility $\tilde{T}_c = 0$ follows. For $\alpha_2 = 0$, there is a second solution depending on $\alpha_3$, i.e., $m_0^c = (1 + 2\exp(\alpha_3))^{-1}$. Specifically, for $\alpha_3 = 0$, this leads to $m_0^c = \frac{1}{3}$ and $\tilde{T}_c = \frac{1}{3}T_c$.

A variation of the parameters in the region $D = [0.00, 2.00]^2 \subset \mathbb{R}^2$ shows that – apart from $m_0^c = 1$ – further solutions for $m_0^c$ do not exist for all parameter combinations $(\alpha_2, \alpha_3) \in D$. However, the analyses also reveal that in subset $D' = [0.00, 1.35] \times [0.65, 2.00] \subset \mathbb{R}^2$ around $(\alpha_2, \alpha_3) = (+1, +1)$, three solutions $m_0^c$ can always be found. For the special case $\alpha_2 = 1$, considered in Section 3.4, even down to the lower limit $\bar{\alpha}_3 = 1 - \ln(2) = 0.3068528\ldots$, with

$m_0^c = 0.5$ (multiplicity two) and $\tilde{T}_c = \frac{1}{2} T_c$. Let us note that the general boundary condition $\bar{\alpha}_3 = g(\bar{\alpha}_2)$ for domain $D'$ is nonlinear and leads to different values of $\frac{1}{3} < m_0^c < \frac{3}{5}$ with multiplicity two as solutions.

For choice $(\alpha_2, \alpha_3) = (+1, +1)$ as the anchor point for further studies, Table 6 summarizes all solutions $m_0^c$ of Equation (38) and gives the corresponding factors for Equation (36) to determine the critical volatility $\tilde{T}_c$.

| Number | $m_0^c$ | $1 - m_0^c$ |
|---|---|---|
| 1 | 1.0 | 0.0 |
| 2 | 0.7281387 . . . | 0.2718612 . . . |
| 3 | 0.1882859 . . . | 0.8117141 . . . |

**Table 6:** Critical Points for $(\alpha_2, \alpha_3) = (+1, +1)$.

An example with the phase transitions noted in Table 6 is considered in Section 3.4.4. Figure 7 shows phase transition numbers 2 and 3 as a pitchfork bifurcation at the various critical volatilities in the upper image. Phase transition number 1 is at $T = 0$ and cannot be resolved graphically any further.

A potential decision criterion as to which phase transition can be observed in practice may be the gross utility $\bar{\mathcal{U}}$ introduced in Section 3.3.5. In Section 3.4.4, this is discussed in more detail.

In Section 3.3.7, the phase transitions for $m_0^c \neq 1$ are analyzed in more detail, and the question of how the parameters $\alpha_2$ and $\alpha_3$ can in principle be estimated from measured data is also addressed.

*Findings, Limits and Extensions*

In general, $\alpha_2$ and $\alpha_3$ are intrinsic parameters of the generalized agent system and reflect the relative evaluation of configurations, one of which is the neutral position, versus configurations consisting only of active positions, compare Equation (22). Thus, the parameters depend on the specific application and must be examined more closely in their context.

For investigations in other fields of application, the assumption $(\alpha_2, \alpha_3) \approx (+1, +1)$ can serve as a starting point, and the behavior described in the following sections can be observed. However, if the parameters deviate significantly from +1, the system behaves completely differently. As an example, the cases $\alpha_2 = 1$ and $\alpha_3 \to 0$ were examined in more detail in preliminary studies. As $\alpha_3$ approaches the limit $\bar{\alpha}_3 = 0.3068528$, the distance between the pitchfork bifurcations shown in Figure 7 decreases, and they push together. For $\alpha_3 = 0.50 \pm 0.01$, the curvature behavior of the right bifurcation changes significantly and is inverted. If $\alpha_3 = \bar{\alpha}_3$, both bifurcations lie indistinguishably on top of each other and form a vertex. If $\alpha_3$ is further reduced, the vertex dissolves, and the system can show a sudden noncontinuous increase in the macroscopic state variable $m$, indicating a phase transition of order one. Whether this behavior can actually be observed in practice must be clarified in individual cases by means of comprehensive, systematic investigations. For the first indication, the gross utility can also be examined here.

The identification of states that can be realized in practice and the deeper meaning of the parameters $(\alpha_2, \alpha_3)$ is revealed from the context of the application and cannot be specified here for all conceivable cases. This and a comprehensive analysis of what the previously found limits mean in each individual case is not examined further here and is reserved for subsequent studies.

*3.3.7. Phase Transition and Critical Index*

As before, $\mathcal{B} \to 0^+$ is assumed below, and the phase transition at the critical point $T = \tilde{T}_c$ for $m_0^c \neq 1$ and $(\alpha_2, \alpha_3) \approx (+1, +1)$ is examined in more detail. To do this, Equation (36) is substituted into Equation (35):

$$\frac{1 - m_0^c}{1 - m_0} \frac{T}{\tilde{T}_c} x = \tanh(x). \tag{40}$$

At the critical point, $m \approx 0$ holds. The right-hand side of Equation (40) can thus be expanded in the vicinity of $x \approx 0$ (Greiner et al., 1995): $\tanh(x) = x - \frac{1}{3}x^3 + \frac{2}{15}x^5 + \ldots$. After rearranging and factoring out the Taylor series up to the cubic order, the following expression is calculated:

$$0 = x \underbrace{\left(1 - \frac{1 - m_0^c}{1 - m_0} \frac{T}{\tilde{T}_c} - \frac{1}{3}x^2\right)}_{\overset{!}{=}0} \tag{41}$$

If $m_0 \approx m_o^c$, the following Taylor series can be determined:

$$\frac{1 - m_0^c}{1 - m_0} = 1 + \frac{m_0 - m_0^c}{1 - m_0^c} + \frac{(m_0 - m_0^c)^2}{(1 - m_0^c)^2} + \ldots \tag{42}$$

At the critical point, $\Delta m_0 = m_0 - m_0^c$ is very small, and the 0th order Taylor expansion can be used in the brackets of Equation (41). If $x = \frac{T_c}{T} m$ is back-substituted, the behavior of $m$ for $T \to \tilde{T}_c^-$ is described by the approximate equation

$$|m| = (1 - m_0^c)\sqrt{3}\frac{T}{\tilde{T}_c}\left|1 - \frac{T}{\tilde{T}_c}\right|^{\frac{1}{2}}. \tag{43}$$

From data collected near the critical point reflecting the behavior $m(T)$, both $\tilde{T}_c$ and $m_0^c$ can be estimated by using Equation (43). Apart from the prefactor $(1 - m_0^c)$, Equation (43) describing the behavior near the critical point is identical to the corresponding equation of the standard Ising system, see, e.g., Greiner et al. (1995, §18, Equation (18.28)). The critical index can be read from Equation (43) and is equal to $\frac{1}{2}$, which is also equal to the corresponding index of the standard Ising system (Isihara, 1971). Since the macroscopic variable $m$ is continuously changing at $T = \tilde{T}_c$, the phase transition is of order 2, similar to the standard Ising system (Greiner et al., 1995). Let us note that Equation (43) applies to both phase transitions specified with numbers 2 and 3 in Table 6.

*Limits and Extensions*

If Equation (43) is used to estimate $\tilde{T}_c$ and $m_0^c$ from the data and one parameter $\alpha_2$ or $\alpha_3$ is known, then the implicit Equation (38) can be used to calculate the other parameter, which also compares the discussions given in Section 3.3.6 and Section 3.4.4.

One way to determine both parameters $\alpha_2$ and $\alpha_3$ at the same time is to consider the curvatures of $m(T)$ near the critical point in more detail and compare Figures 6 and 7 for $\mathcal{B} = 0$ as an illustration of the curvature near the critical point. For this purpose, the 1st order is also considered in Equation (42) in the Taylor series. Then, from the implicit Equation (31) for $\mathcal{B} = 0$, an approximation equation for $\Delta m_0$ depending on the volatility $T$ is determined. Both can be used in Equation (41). After transformation, an additional factor (not shown here) is calculated in Equation (43), which explicitly depends on $\alpha_2$ and $\alpha_3$. Thus, the values $\tilde{T}_c, m_0^c, \alpha_2$ and $\alpha_3$ can then be estimated from data $(T_i, m_i)$ with $i = 1, \ldots, M$ collected near the critical point.

How well the two model parameters $\alpha_2$ and $\alpha_3$ can be determined from data with this outlined procedure is not investigated further and is the subject of subsequent research. The empirical investigation in Section 3.4 shows that despite a large dataset, with $M \sim O(10^5)$, only a few data points indicate a spontaneous transition $m \neq 0$; see Figure 6. Furthermore, the curvature behavior cannot be reliably determined and evaluated due to scattering. Thus, in practice, the curvature behavior in the vicinity of the critical point can probably be modeled only with significantly larger datasets.

With very large amounts of data, which reflect the curvature behavior near the critical point in great detail, Equation (43) can also be used to detect possible deviations from the critical index $\frac{1}{2}$ if this is considered a free fitting parameter. Such investigations correspond to analyses in physics that, e.g., focus on the detection of model inaccuracies due to the mean-field approximation, compare, e.g., Amit (1978); Greiner et al. (1995).

A further limit to the applicability of Equation (43) can be seen in the following. If the parameters $\alpha_2$ and $\alpha_3$ deviate significantly from $+1$, the curvature near the critical point may not be correctly modeled by Equation (43). For example, for $(\alpha_2, \alpha_3) = (+1, +0.3068528)$, the right pitchfork bifurcation in Figure 7 inverts and is not described by Equation (43) due to the constraint $T \rightarrow \tilde{T}_c^-$. Thus, in this individual case, e.g., an extended analysis based on the overall model Equation (30) and (31) is required to describe the phase transition more precisely.

### *3.4. Application*

The theoretical model from Section 3.3 is applicable to empirical examples in which a large number of involved agents can each individually choose among three alternatives. As an application, in this section, we consider the example of the financial market where $N$ shares of a single firm can be traded by a collection of agents or investors. As in Börner et al. (2023b), we normalize that investors trade one share per transaction. Each investor has the opportunity to choose among three trading options: to buy a share, to sell it or to hold it. The different states of the investors as well as the configurations of neighboring investors each influence the utility calculus of the market observer, who derives positive utility from explaining the market without additional cost or effort (cf. Section 3.3.1). As described in Section 3.3.1, the market observer can also be the investor, which is assumed for the sake of simplicity in the following. In this application, the utility is measured in USD for investors as a nonincurred cost for more information explaining a specific configuration. The unitless numbers of different configurations are included in the utility function Equation (22), so that the prefactors $\mu$ and $J$ in the utility function have the unit USD. The application of a general model to a concrete example requires the correct transfer of all model variables. Since a procedure is used in the following to estimate the model parameters empirically, the transfer is not merely abstract but shows how the variables are to be determined using capital market data. At the same time, this example provides possible guidance on how to determine model parameters in other applications. With the results and findings of Börner et al. (2023b), the experimental setup presented here to determine the parameter $J$ is the completion of the empirical application of the overall model to the capital market.

In Börner et al. (2023b), the model parameters $\alpha_1$ and $\mu$ are identified by using a design in which the couplings between agents do not operate. If the coupling parameter $J$ contained in Terms 3, 4 and 5 is to be measured, Terms 1 and 2 from Equation (22) must be "switched off" as the idiosyncratic part of the model.

After the coupling parameter $J$ is determined, the phenomenology of the model is presented by using simulations. In addition, the practical applicability of the model is discussed, showing how risk assessments can be performed with the model, for example, by estimating one-step-ahead price losses.

### 3.4.1. Determining the Model Parameter J – Experimental Set-up

Before the empirical setup and the data are presented in detail in the following, the empirical strategy is briefly outlined. If market situations are found where no unique message exists, i.e., $\mathcal{B} \approx 0$, then Terms 1 and 2 of Equation (22) become zero. Furthermore, if the capital market is still in dynamic equilibrium, i.e., volatility $T \approx$ const., for a time period, then conditions are found that allow an empirical determination of $J$. With these market conditions, the volatility $T$, critical volatility $\tilde{T}_\mathrm{c}$ and trade potential $m$ must be measured so that the measurement Equation (37) can be used to determine the parameter $J$.

The critical volatility $\tilde{T}_\mathrm{c}$ must be determined under idealized specifications by using capital market data. With the following procedure, these conditions are considered and implemented:

1. Periods of constant volatility $T$, when the market is in dynamic equilibrium, are identified by specifications regarding the rate of change of volatility. The assumption is that in dynamic equilibrium, no fundamental information leads to sharp price movements, so that $\mathcal{B} \approx 0^+$.
2. Next, the mean attitude $m$ from Equation (33) is introduced as the trading potential $\bar{N}_\mathrm{pot}$, with the message field $\mathcal{B}$ indicating whether it is a buy or sell potential. Since $\mathcal{B} \approx 0^+$ is assumed, $\bar{N}_\mathrm{pot} = m$ (buyer surplus). The empirical trading potential $\bar{N}_\mathrm{pot}$ is calculated as in Vikram and Sinha (2011).
3. Given the market conditions just described, $T$ and $m$ are determined. The goal is to find the bifurcation point in Figure 6, evaluate it, and perform the parameterization of the model from Section 3.3.
4. A recursive procedure with rolling windows is used to extract the optimal section of data points around the bifurcation point that contains the maximum bandwidth of trade potential $m$ and at the same time has the smallest window width. The average of the volatility contained in this window is identified as the critical volatility $\tilde{T}_\mathrm{c}$ (*phys.*: Curie temperature).
5. The critical volatility $\tilde{T}_\mathrm{c}$ is used to calculate the parameter $J$ according to the measurement Equation (37).

### 3.4.2. Data and Transfer of the Model Parameters

To complete the empirical research, the same example is studied as in Börner et al. (2023b). Thus, we also use the stock of the pharmaceutical company BioNTech SE as an example, accessing minute data from the American Stock Exchange NASDAQ via a Bloomberg terminal for the period from 2021-08-01 to 2022-09-30 (Bloomberg code: BNTX UW). In addition, we use minute data on the S&P 500 and NASDAQ 100 stock indices, also via the Bloomberg terminal and for the same period, to control for overall market trends. This results in a total of 91848 data points for each security.

*News Sentiment $\mathcal{B}$*

As described in Section 3.3.1, there is an external message field, or news sentiment $\mathcal{B}$, to which investors' decisions or states $S$ are aligned. This news sentiment can be company-related news that has price-influencing potential. In principle, it is possible for $\mathcal{B}$ to take values between $-1$ and $+1$ (Börner et al., 2023b), but in this section, we focus on a special case and assume that no unique message exists; therefore, the strength of the message field is positive but immeasurably small and very close to 0, i.e., $|\mathcal{B}| = B \approx 0$ and $\text{sign}(\mathcal{B}) = +1$. For example, observing a persistent, tiny positive trend in a stock might be viewed as minimally positive news. This means that the three states of an agent, "nonconform" or "conform" to the message environment and the "neutral" position, remain distinguishable. In this setting, e.g., the agent's "conform" state means that the investor is potentially buying the stock. Thus, in our example with $\mathcal{B} \approx 0^+$, the conforming state $S = +1$ corresponds to buying the stock. Similarly, $S = -1$ corresponds to selling and $S = 0$ to holding the stock. The consequence of this assumption is that Terms 1 and 2 of Equation (22) become zero, leaving Terms 3, 4, and 5 with the coupling parameter $J$ and the scaling factors $\alpha_2$ and $\alpha_3$. This already outlines how the cut-out experiment (Börner et al., 2023b) is to be designed so that the remaining model parameters $J, \alpha_2$ and $\alpha_3$ can be determined.

In general, the strength and direction of the news environment $\mathcal{B}$ can be determined by using lexical text analysis when applying the model for a generalized agent system to the example of a stock investment, see, e.g., Börner et al. (2023b). The lexical analysis of the news by using the approach of Loughran and McDonald (2011) has proven to be advantageous (Börner et al., 2023b), which provides the normalized strength of the news environment between $-1$ and $+1$ for a stock investment. In Section 3.4.4, the evaluation of the model is based on this lexical analysis concept but is focused on the solution sets for $\mathcal{B} \geq 0$. For $\mathcal{B} < 0$, the solution sets are determined analogously.

*Scaling Factors $\alpha_2$ and $\alpha_3$*

Furthermore, Section 3.3.7 explains how $\alpha_2$ and $\alpha_3$ in Equation (22) can be measured. The $\alpha$-factors scale the contribution of a given configuration of neighboring agents to the market observer's utility calculus, provided an investor with a holding position is involved. The prerequisite for the measurement is that there are enough data points at the critical point $\tilde{T}_c$ to measure the curvature of the bifurcation. Since, as seen in Figure 6, this is not the case even for the large datasets under consideration here, a fundamental derivation for the size of the alphas must suffice at this point.

Based on the model operationalization when using the utility of a market observer, it is necessary to consider how the configurations of Terms 4 and 5 in Equation (22) contribute to the utility, scaled by $\alpha_2$ and $\alpha_3$, respectively. Term 4 describes the situation where two adjacent investors are both in the hold position. Apart from the fact that both hold positions in themselves reduce the benefit, if $\mathcal{B} \neq 0$, there is no discrepancy between the two investors' positions that requires explanation. In this respect, it is appropriate that the utility is increased by two equal adjacent positions. From the observer's point of view, it should make no difference which same positions are involved in a configuration. Thus, $\alpha_2 = 1$ should be implied in the capital market so that the configurations in Terms 3 and 4 contribute equally to utility.

The same reasoning is applicable to Term 5. If one of two neighboring investors in Term 3 is in the buy position and the other in the sell position, this discrepancy, which is inherent in the configuration, reduces the utility by $J$. If Term 5, which exclusively maps configurations with discrepancies, namely, between a hold position and a buy or sell position, then it is consistent to assume that the contributions to utility are equal to those measured with Term 3. Thus, $\alpha_3 = 1$ also holds.

The fundamental considerations above suggest that for the factors, $(\alpha_2, \alpha_3) \approx (+1, +1)$ holds in practice. With a much larger amount of data, more precise measurements of the curvature behavior near the bifurcation point can be made (cf. Section 3.3.7) and thus provide information about the actual values of the $\alpha$-factors implicit in the capital market.

*Volatility $T$*

A key external state variable in physics is the temperature $T$, as introduced in Section 3.3.3, which in the financial context can be interpreted as instantaneous volatility and is observed by investors in the market, see, e.g., Börner et al. (2023b,c). Before explaining how the variable is empirically identified, intuition is needed to understand how volatility affects investors' decisions. For a given message field, volatility acts as a disturbance variable under which unambiguous decision-making becomes more difficult for investors. If volatility increases, fewer investors align themselves according to the news field. Conversely, when volatility is low or decreases, an investor's decision is subject to less uncertainty (Bouchaud, 2013; Sornette, 2014). In the situation described above with a minimal but positive news field, investors then may find it difficult to decide on a direction when volatility is high because uncertainty is high in addition to the weak news situation, and with a small group of investors, a pronounced distribution across all three possible decisions should already be observable. However, if volatility in this situation successively decreases and reaches a critical value $\tilde{T}_c$, significant price movements may occur, as investors interpret even the smallest positive news elements as a buy signal. With the observation that an adjoining investor also buys, the effect intensifies, and at this microscopic level, a coupling of the attitudes of the investors takes place. The resulting phenomenon at the macroscopic level is a spontaneous, market-effective coupling of a larger number of investors' attitudes in one direction (*phys.:* spontaneous magnetization, cf. Section 3.3.6), which can be interpreted as (short-term) herd behavior in the financial market.

To calculate the parameter $J$, the critical volatility $\tilde{T}_c$ must be identified empirically. As in Börner et al. (2023b), we measure the state variable $T$ by using log returns over a 15-minute time window. To isolate the terms containing the parameter $J$ in the overall model Equation (22), the experimental setup must be designed in such a way that the front idiosyncratic part of the model containing the ideal agent system is "switched off". This condition is reached when $\mathcal{B} \approx 0^+$ for the information field holds. Furthermore, for the empirical approach, market phases in which the market is in dynamic equilibrium must be identified (*phys.:* thermodynamic equilibrium). This is the case when no major price changes have occurred over a certain period of time and volatility remains relatively constant. Thus, it can be assumed that no fundamentally important information is present. If, at the same time, the volatility of the market index is largely constant, a momentary influence of the market volatility on the volatility of the stock under consideration can be excluded, and it can then be concluded that the system is in dynamic equilibrium. Let us note that the level of $T$ can be arbitrary, but it is important that very small volatilities are also included in the containment because only then is the effect of spontaneous coupling of investors' attitudes observable.

To find market phases with constant volatility, we determine the 15-minute volatilities $T_{15'}$ for the entire sample of the BioNTech stock and the market indices (S&P 500, NASDAQ 100) according to Börner et al. (2023b). From the rate of change of these 15-minute volatilities, we determine the standard deviation $\sigma$ as the range of variation. Finally, we use only the data points whose 15 previous data points fall within this $\sigma$-band of the rate of change. Next, the goal is to extract the critical volatility $\tilde{T}_c$ from the data. To do this, the data belonging to the bifurcation, if the data contain one, must be separated from the rest of the data scattering around the $T$-axis from the filtered sample $(T, m)$. We use a rolling window-based search procedure to find the bifurcation as well as the associated outgoing branches, as shown in Figure 6, and we take the smallest possible vertical window that includes the maximum bandwidth of the trade potential $m$. If no bifurcation is found, the smallest measurable value can be taken as an estimate for $\tilde{T}_c$. This approach has the advantage of objectively extracting a cloud of points from which $\tilde{T}_c$ can be obtained as the average of the included volatilities, see Figure 6. This selective extraction of a cloud of points neglects some points, also scattering along the $m$-axis, which could be relevant for the determination of $\tilde{T}_c$. To account for this potential error, we specify a bandwidth for $\tilde{T}_c$. The lower bound of the range is reflected by the minimum volatility, and the upper bound of the range is reflected by the same distance between $\tilde{T}_c$ and minimum volatility, but as a markup on $\tilde{T}_c$. In our application, the measured $T_c = 1.33384e - 3$ and the specified bandwidth are shown in Table 7.

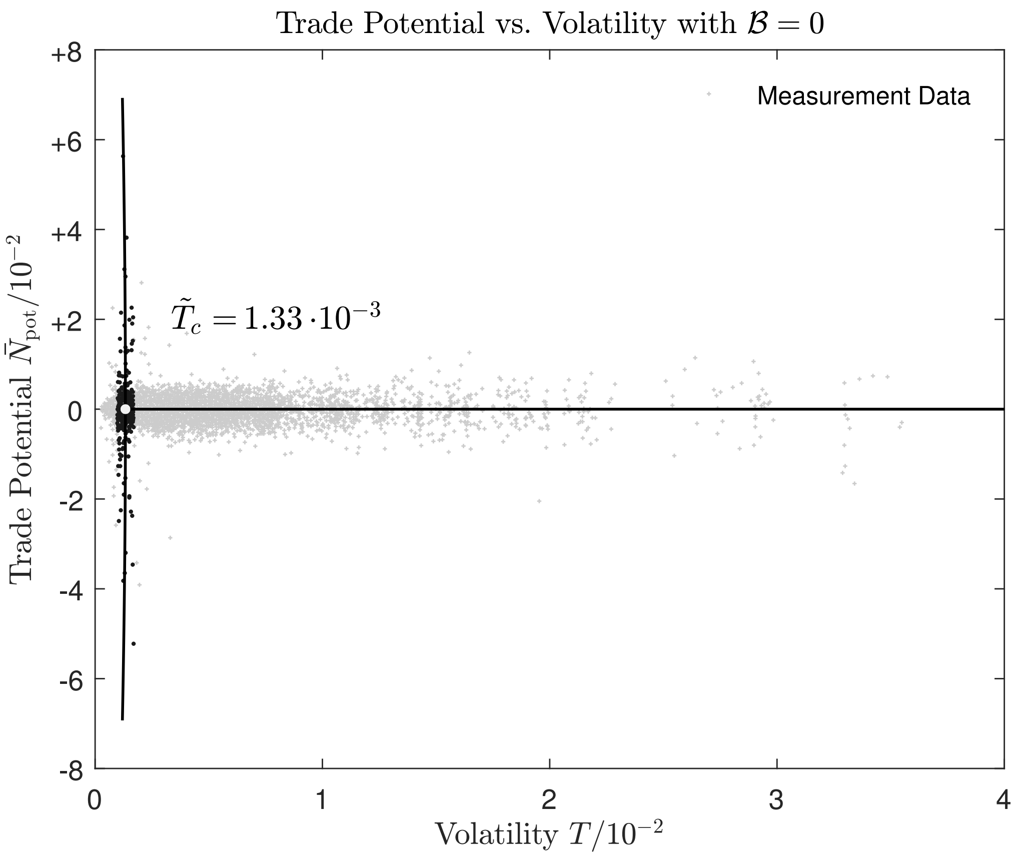


**Figure 6:** The trade potential and the respective volatilities prefiltered with the conditions are plotted. The figure shows the bifurcation around the $T$-axis, which can also be seen in the simulations in Figure 7. The peak of the bifurcation is identified as the critical volatility $\tilde{T}_c$.

*Mean Attitude $m$ and Neutral Positions $m_0$*

As introduced in Section 3.3.4, $m$ and $m_0$ as macroscopic variables indicate the mean attitude and the quantity of neutral positions, respectively. In the context of the financial market, these quantities of the system are central, as they can be interpreted as price-influencing trade potential (Börner et al., 2023b). Since the macroscopic variable $m$ does not specify a direction, it must be specified via the message field $\mathcal{B}$. Based on Equation (30), this results in the average trade potential $\bar{N}_{\text{pot}}$ with

$$\begin{aligned}\bar{N}_{\text{pot}} &= \text{sign}(\mathcal{B}) \frac{2\sinh(+\beta\mu B_1)}{2\cosh(+\beta\mu B_1) + \exp(-\beta\alpha_1\mu B_2)} \\ &= \text{sign}(\mathcal{B}) m. \end{aligned} \tag{44}$$

Given $\mathcal{B} > 0$, $\bar{N}_{\text{pot}} = m$ holds, and the trade potential is a buying potential provided $m > 0$. In empirical application, the hold positions $m_0$ can be calculated only with the coupled implicit Equations (30) and (31) if $\alpha_2$ and $\alpha_3$ are known. Assuming $\alpha_2 = \alpha_3 = 1$, there exists a fixed quantity $m_0^c = 0.1882859...$ at the critical point $\tilde{T}_{\text{c}}$, which can be calculated by using Equations (38) and (39).

For the empirical identification of the quantity $\bar{N}_{\text{pot}}$, we follow Börner et al. (2023b) and compute $\bar{N}_{\text{pot}}$ as the balance of buyer and seller potential. The number of hold positions $m_0$ can be disregarded at this point since they have no effect on price changes in the system. To extract the trade potential $\bar{N}_{\text{pot}}$ from the market data, we use an indirect method. We calculate the concatenation factors $K_t = {}^{P_{t+1}}/_{P_t}$ of the instant price movements at all time points $t$. Then, the trade potential is calculated according to Vikram and Sinha (2011) and later Börner et al. (2023b):

$$\hat{\bar{N}}_{\text{pot}}(t) = \frac{K_t - 1}{K_t + 1}. \tag{45}$$

The trade potential $\hat{\bar{N}}_{\text{pot}}$ is plotted together with the corresponding volatilities $T$ in Figure 6.

*Coupling Parameter J*

By using Equation (37), the parameter $J$ is calculated if $\tilde{T}_c$ can be obtained from the data. The remaining quantities have already been discussed. Thus, $z = 12$ was assumed for the ordinal number that indicates the average number of connected investors, $m_0^c = 0.1882859\ldots$ results from the solution of the implicit Equations (38) and (39), given that $\alpha_2 = \alpha_3 = 1$, and $k = 1$ USD is set in our application since it was introduced as a measure of cost. For $\tilde{T}_{\text{c}} = 1.33384e - 3$ according to Figure 6 we obtain $J = 2.738731e - 4$ USD. A bandwidth and a deduced standard deviation are given in Table 7.

### *3.4.3. Summary of Results*

We determine bandwidths for our estimated parameter values to account for potential estimation errors. These can be used to construct standard deviations according to $\sigma \leq \frac{1}{2}(M - m)$ given by Popoviciu (1935), where $M$ and $m$ are the supremum and infimum of the support of the underlying distribution. The sample yields a bandwidth whose limits can be used as an initial indication of the supremum or infimum. Thus, a first estimate for $\sigma$ is possible. The values and error ranges are summarized in Table 7.

| Parameter | Value | Range | Std. Dev. $\sigma$ |
|---|---|---|---|
| $\mu$ ($10^{-4}$ USD) | 3.92 | (3.51 – 4.21) | 0.35 |
| $\alpha_1$ | 0.75 | (0.00 – 1.00) | 0.50 |
| $\tilde{T}_c$ ($10^{-3}$) | 1.33 | (0.26 – 2.41) | 1.07 |
| $J$ ($10^{-4}$ USD) | 2.74 | (0.53 – 4.95) | 2.21 |
| $\alpha_2$ | 1.00 | (0.00 – 2.00) | – |
| $\alpha_3$ | 1.00 | (0.00 – 2.00) | – |

**Table 7:** Summary of the estimated parameters as well as details on the scattering in terms of the bandwidths and the standard deviation $\sigma$. Results for $\mu$ and $\alpha_1$ are taken from Börner et al. (2023b).

In addition to the modeling approach shown here, there are strands of research that deal with similar models but assume distributions of the parameters, e.g., cf. Bouchaud (2013); Crescimanna and Di Persio (2016). By specifying a value and the possible distribution of a model parameter, it might be possible to parameterize the distribution-based models mentioned above as well.

#### *3.4.4. Model Phenomenology*

With the parameters identified thus far, the solution set of Equations (30) and (31) was determined numerically as a function of the external conditions $T$ and $\mathcal{B}$. The volatility $T$ varied within an interval from 0 to slightly above $T_c$, and the news environment $\mathcal{B}$ varied within an interval from 0 to 1.

Figure 7 shows the numerically determined solution sets. The slightly gray graphs show the solution sets for the hold positions $\bar{N}_0$ as a proportion of the whole. The hold positions correspond to the neutral positions in the generalized agent system and the following applies: $\bar{N}_0 = m_0$. The black lines show the solution sets for the mean attitude $m$ in the generalized agent system and correspond here to the buyer or seller surplus. With $\text{sign}(B) = +1$, the trade potential is $\bar{N}_{\text{pot}} = m$. The bold black lines in the lower part of the upper graphic reflect the gross utility $\bar{\mathcal{U}}$ determined for each solution set. Its value can be read on the right axis.

First, there are several solution sets for the same external conditions $\mathcal{B} = \text{const.}$ depending on the setting $T = \text{const.}$ The solution that can be actually observed in practice, i.e., the macroscopic state of the system, can be deduced from fundamental econometric considerations. The analyses showed that this realized macroscopic state corresponds to a maximum of gross utility $\bar{\mathcal{U}}$, as assumed in Section 3.3.5.

We describe the solution set for decreasing volatilities $T \rightarrow 0$ using the upper graphic in Figure 7. The case where $\mathcal{B} = 0^+$ is thus described in detail. If the volatility falls below $T_c$, there is hardly any change in the system that can be observed macroscopically based on the price change of the stock. The proportion of hold positions $\bar{N}_0$ is falling, but the increasing proportions of buyers and sellers neutralize each other overall; thus, $\bar{N}_{\text{pot}} = 0$. The critical point in the standard Ising system (two-state model) is connected to $T_c$, and a macroscopically relevant phase transition is expected in this model framework; see also Section 3.3.6. However, this phase transition occurs only at a lower volatility, specifically for $T = \tilde{T}_c$. One possible interpretation is that the presence of investors in position "hold" (the third possible state) leads to a moderating effect. In the sense that the density of positions "holds" must first continue to decrease before an increased adjustment (herding) of buying behavior or selling behavior takes place, a macroscopic effect can be observed.

At point $T = \tilde{T}_c$, we observe a pitchfork bifurcation. Then, the trade potential for $T = \tilde{T}_c^-$ can have three different values: $\bar{N}_{\text{pot}} < 0$, $\bar{N}_{\text{pot}} = 0$ and $\bar{N}_{\text{pot}} > 0$. In addition, the proportion of holding positions $\bar{N}_0$ can have two values. The larger value belongs to $\bar{N}_{\text{pot}} = 0$ and the smaller one to the other two values $\bar{N}_{\text{pot}} \neq 0$. The latter two configurations are the ones observed in practice. In the literature, the configuration with $\bar{N}_{\text{pot}} = 0$ is referred to as a configuration that is not observable in econometrics and is unstable (Bouchaud, 2013). The other configurations with $\bar{N}_{\text{pot}} \neq 0$ are stable and observable in practice. This observation can be explained more precisely using the gross utility $\bar{\mathcal{U}}$: The gross utility is at its maximum for the two configurations $\bar{N}_{\text{pot}} \neq 0$, as the lower part of the upper graphic shows.

Phase transitions may occur again as volatility $T$ decreases further. According to Table 6, two further phase transitions could take place at $\tilde{T}_c = 0.2718612 \cdot T_c$ and $\tilde{T}_c = 0 \cdot T_c$. In Figure 7 (upper graph), the second phase transition at $\tilde{T}_c \approx 0.45 \cdot 10^{-3}$ is shown as the second pitchfork bifurcation. The third phase transition at $\tilde{T}_c = 0$ cannot be further resolved graphically. However, the second phase transitions should not be observable in practice if the assumption of the maximum of the gross utility applies to observable configurations. The third phase transition can only be observed under very special conditions, as the following analysis shows.

For $T = 0^+$, an interesting case can be analyzed, namely, when $\bar{N}_0$ tends toward +1 (upper branch of the slightly gray graph at the top left). For this case, i.e., $(\bar{N}_{\text{pot}}, \bar{N}_0) = (0, +1)$, the gross utility $\bar{\mathcal{U}}$ is also at its maximum and equal to the other two configurations with $(\bar{N}_{\text{pot}}, \bar{N}_0) = (\pm 1, 0)$. Thus, for $T = 0$, the system can exist in three possible pure states: All investors sell, all investors hold, or all investors buy the stock. This result may have been guessed but is derived here from theory. We show shortly that this result is a direct consequence of the assumption $\alpha_2 = 1$ and supports the latter.

The gross utility is analyzed more precisely for $T = 0$. With $T = 0$, $\bar{\mathcal{U}} = \bar{U}$ immediately applies. With $\mathcal{B} = 0$ it is calculated from Equation (23) for the configurations $(\bar{N}_{\text{pot}}, \bar{N}_0) = (\pm 1, 0)$ the gross utility $\bar{\mathcal{U}} = \frac{1}{2} J z$. For the configuration $(\bar{N}_{\text{pot}}, \bar{N}_0) = (0, +1)$ the gross utility is $\bar{\mathcal{U}} = \frac{1}{2} \alpha_2 J z$. Both gross utilities are only equal if $\alpha_2 = 1$. Thus, if all three pure states are possible from an econometric point of view at $T = 0$, then $\alpha_2 = 1$ is mandatory in this application. In this case, the maximum gross utility is $\bar{\mathcal{U}} = 1.6432 \cdot 10^{-3}$ USD and corresponds to the numerical limit value in the lower part of the upper graphic (upper bold black line on the left).

If not only $\tilde{T}_c$ can be determined from the data but also an estimator for $m_0^c$ can be determined from the curvature behavior at $\tilde{T}_c$ by using Equation (43), then, with $\alpha_2 = 1$, $\alpha_3$ can be calculated by using the implicit Equation (38). Hence, statements may be made about how the market participants implicitly assess the loss of utility through asymmetric configurations if one position in the configuration is the holding position, compare Term 5 in Equation (22). Unfortunately, modeling of the curvature behavior is not possible with the available data. Thus, the parameter $\alpha_3$ cannot be estimated here. The bifurcation on the left side that can be seen in Figure 6 does not allow any conclusions to be drawn about an inversion of the curvature near the critical point. It can therefore be assumed that $\alpha_3 > 0.5$, as in Section 3.3.6. For the simulations, it was assumed that $\alpha_3 = 1$.

Examples of solution sets for $\mathcal{B} > 0$ are shown in the lower graphic in Figure 7 and should be interpreted analogously. Here, in practice, the state $\bar{N}_{\text{pot}} > 0$ is always realized. A maximum of $\bar{\mathcal{U}}$ could also be calculated for these solutions (not shown here). There is no phase transition for the critical points observed in the case of $\mathcal{B} = 0$. The trade potential $\bar{N}_{\text{pot}}$ continuously changes its value with decreasing volatility. However, compared to the standard Ising system, a slower behavior can also be observed, with the same $\mathcal{B} \neq 0$ and the same volatility, and the trade potential is lower in comparison.

#### *3.4.5. Model Predictions with Practical Relevance*

Up to this point, we have been concerned with how models that basically replicate the properties of complex systems can be applied in practice. The necessary step of calibrating the model parameters to the underlying empirical system has been shown. The following describes a concept of how the parameterized model can be used in practice. The application concerns a one-step ahead forecast, which can be used, e.g., for short-term price forecasts.

With continuous measurements of the external state variables $\mathcal{B}$ and $T$, the trading potential $m$ and $m_0$ can be calculated via Equations (30) and (31). The sign of the message field $\mathcal{B}$ provides the necessary information to interpret the trading potential as buyer or seller potential. By using Equation (45) from Vikram and Sinha (2011) and later Börner et al. (2023b), the following equation is determined for the estimated price at time $t+1$ given the price at time $t$ and the trade potential defined in Equation (44):

$$\hat{P}_{t+1} = \frac{1 + \bar{N}_{\text{pot}}}{1 - \bar{N}_{\text{pot}}} P_t \qquad \text{with} \qquad \bar{N}_{\text{pot}} = \text{sign}(\mathcal{B})\ \frac{N_+ - N_-}{N}. \tag{46}$$

Thus, the trading potential can be used to formulate an estimator for the price in the next moment, depending on the external conditions $T$ and $\mathcal{B}$. This prediction is not completely accurate but gives an initial indication of the direction of the price movement and the approximate magnitude of the possible price change. In the event of negative news, this can form the basis of a risk assessment and serve as preparation for countermeasures.

### *3.5. Conclusion*

In the last 50 years or so, several strands of research have opened up econometrics that deal with models in a strong analogy to the statistical physics of many-particle systems. While in physics, special experiments are designed and built to parameterize models, this possibility does not exist in econometrics in general. For almost all of these models, the question of parameterization arises, e.g., determining the parameters by using capital market data. The question of how to estimate parameters directly rather than indirectly in bottom-up agent-based models was addressed in this paper, and a solution for the appropriate parameterization was given. A stock investment as an example showed the practical application.

Frequently, models are found in the literature where two possible decision states (e.g., buy, sell) are considered. For many applications, however, three states (e.g., buy, hold, sell) are more appropriate. Therefore, all interaction possibilities between agents (investors) are considered and influence a utility maximizer calculus in the context of the model operationalization, where the market is explained by incurring information costs. As a main contribution, it was shown how to parameterize the developed three-state model in special cut-out experiments. Capital market data were analyzed to empirically determine the model parameters.

As part of the empirical parametrization, the variables of the general model were first transferred to the application context of the capital market. In a second step, measurement equations derived from the general model were used to specify concrete parameter values, which, in addition to the bandwidths and distribution parameters, represent key findings. In simulations when using the estimated parameter values for the shares of BioNTech SE, first- and second-order transitions were identified within the rich model phenomenology that can be used to explain short-term herd behavior in capital markets. This short-term herding behavior occurs at a critical point of volatility. Another important result in this context is that the critical point in the three-state model under consideration is reached at lower volatilities than in a binary system. Thus, predictions with binary models over three-state systems lead to biased results. Furthermore, it was shown how to implement a forecast model over short-term price changes with a one-step-ahead forecast.

Many extensions of the presented three-state model are conceivable, and a selection of extensions were discussed. All of them have in common that the parameters of the basic model determined in cut-out experiments can be used for the extensions. Even if distributions of parameters are considered in extensions, a first indication for the values of the distribution parameters can be given by the presented concept. For this purpose, a way was shown how this can be implemented. At this point, the integration and investigation of the presented model and its parameterization procedure in more complex applications, e.g., influence of social interactions in higher model degrees or the modeling of multiasset systems, is left to following, subsequent research.

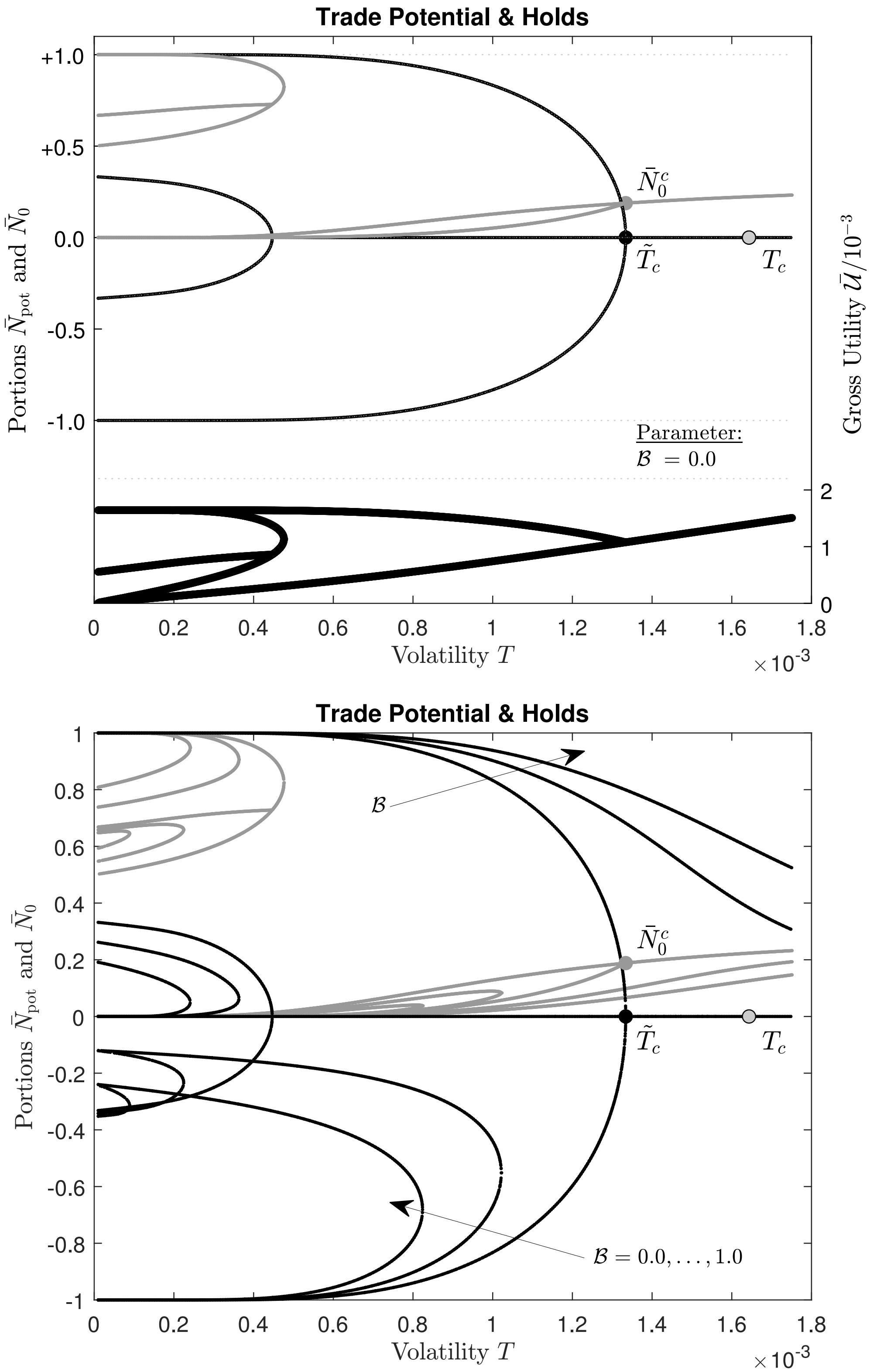


**Figure 7:** Simulation of the generalized agent system for different news environments and different volatilities for the example of a stock investment (see text for further explanations).

*3.6. Declaration of (Co-)Authors and Record of Accomplishments*

**Title:** Generalized Agent System with Triplet States: Model Parameter Identification of Agent-Agent Interaction

**Author(s):** Christoph J. Börner (Heinrich Heine University Düsseldorf)
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**Conferences:** –

**Publications:** SSRN published. Submitted to *Physica A: Statistical Mechanics and its Applications*, a single-blind peer-reviewed journal.
Current status: Under review.

**Share of contributions:**

| Contributions | Christoph J. Börner | Ingo Hoffmann | John H. Stiebel |
|---|---|---|---|
| **Research design** | 10% | 45% | 45% |
| *Development of research question* | 10% | 45% | 45% |
| *Method development and specification* | 10% | 45% | 45% |
| **Research performance & analysis** | 0% | 35% | 65% |
| *Literature review and framework development* | 0% | 30% | 70% |
| *Data collection, preparation and analysis* | 0% | 10% | 90% |
| *Analysis and discussion of results* | 0% | 50% | 50% |
| *Derivation of implications and conclusions* | 0% | 50% | 50% |
| **Manuscript preparation** | 20% | 40% | 40% |
| *Final draft* | 20% | 40% | 40% |
| *Finalization* | 20% | 40% | 40% |
| **Overall contribution** | 10% | 40% | 50% |

Date, Cristoph J. Börner

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## 4. On the Connection Between Temperature and Volatility in Ideal Agent Systems

### 4.1. Abstract

Models for spin systems known from statistical physics are applied by analogy in econometrics in the form of agent-based models. Researchers suggest that the state variable temperature $T$ corresponds to volatility $\sigma$ in capital market theory problems. To the best of our knowledge, this has not yet been theoretically derived, for example, for an ideal agent system. In the present paper, we derive the exact algebraic relation between $T$ and $\sigma$ for an ideal agent system and discuss implications and limitations.

**Keywords:** Agent System, Econophysics, Temperature, Volatility

**JEL Classification:** C10, C46, C51

### *4.2. Introduction*

In the context of econophysics, methods from statistical physics are used to model the behavior of investors (so-called agents), for example, to draw conclusions about the price movement of a stock and arrive at a one-step ahead forecast model; see, e.g., Vikram and Sinha (2011). Starting with the work of Weidlich (1971) and Galam et al. (1982), the research field of econophysics utilizing spin systems has steadily developed and branched out; see, e.g., Bouchaud (2013); Sornette (2014) for a review. The models of econophysics, based on the analogy with spin systems from statistical physics, depend on a state variable $T$ that describes the system and is called temperature in statistical physics. For temperature in statistical physics, see, e.g., Isihara (1971); Landau and Lifšic (1980); Greiner et al. (1995); Kardar (2007). In econophysics, the state variable $T$ is referred to differently in various applications: noise, irrationality, degree of randomness in agents' decisions, the collective climate parameter or volatility; see, e.g., Weidlich (1971); Kaizoji (2000); Kozuki and Fuchikami (2003); Oh and Jeon (2007); Kleinert and Chen (2007); Kozaki and Sato (2008); Krause and Bornholdt (2012); Bouchaud (2013); Crescimanna and Di Persio (2016). In models for capital markets in which temperature $T$ is identified with volatility $\sigma$, e.g., of a stock, various relationships are assumed: $T \propto \sigma^2$ (Kozuki and Fuchikami, 2003), $T = \frac{\sigma}{\sqrt{2}}$ (Kleinert and Chen, 2007), $T \propto \sigma$ (de Mattos Neto et al., 2011).

To the best of our knowledge, the relationship $T \propto \sigma$ is predominantly used in capital market theory applications (de Mattos Neto et al., 2011; Bouchaud, 2013; Börner et al., 2023b), but the exact algebraic relation does not appear to have been theoretically derived, even for an ideal agent system. This contribution is dedicated to this precise question and is intended to close the research gap reflected in the variety of algebraic equations adopted and proposed (Kozuki and Fuchikami, 2003; Kleinert and Chen, 2007; de Mattos Neto et al., 2011). The function $f : \sigma \to T$ is derived from theory. Implications are examined, and the limits that need to be considered are identified. For the simple, ideally designed example of a two-state agent system (Bouchaud, 2013) the equation $T = f(\sigma)$ is derived and further analyzed.

This study is structured as follows. In the next section, temperature is defined as state variable $T$. In Section 4.4, the example of a two-state ideal agent system is introduced. Section 4.5 is devoted to determining the central equation for deriving measurement equations. An application to the ideal agent system and a conclusion follow.

### *4.3. Definition of T*

In physics, since the revision of the international system of units, temperature $T$ has been linked to thermal energy using the Boltzmann constant $k$ (International Bureau of Weights and Measure, 2019, p. 133). If the internal energy $E = E(S, \mathbf{X})$ is described as a function of the entropy $S$ and possibly other state variables $\mathbf{X}$ and the entropy is calculated from the microcanonical partition function $\Omega$ using the equation $S = k \ln \Omega$, then

$$T := \left. \frac{\partial E}{\partial S} \right|_{\mathbf{X}=\text{const.}} \tag{47}$$

is the definition of temperature in statistical physics; see, e.g., Greiner et al. (1995).

Analogy considerations establish a connection between energy and utility in econometrics. While phyiscal systems minimize their energy, utility maximization is assumed in econometrics. Accordingly, as a first starting point for further calculations, the relation $E = -U$ is often found in the literature (Marsili, 1999; Sornette, 2014; Börner et al., 2023b).

With the relation $E = -U$, Marsili (1999) arrives at the definition of the state variable $T$ in econometrics:

$$T := - \left. \frac{\partial U}{\partial S} \right|_{\mathbf{X}=\text{const.}} \tag{48}$$

Where $\mathbf{X}$ summarizes other variables, for example, the message environment $\mathcal{B}$ in the ideal agent system (Bouchaud, 2013; Börner et al., 2023b).

If finite changes are considered with constant $\mathbf{X}$, then Equation (48) leads to the simple relation $\Delta U \propto -T \Delta S$. That is, with fixed $T$ and increasing entropy, utility decreases. Marsili (1999, p. 13) concluded that state variable $T$ "[...] *can be considered as the price of (negative) entropy*".

If and only if the definition of Equation (48) is used in econophysics, a consistent theory can be constructed based on the methods of statistical physics. The state variable $T$ is initially used as a parameter in all equations and must be identified based on the specific circumstances of the application under consideration. This is specifically shown for the example of a two-state ideal agent system.

The relationship between energy and utility, as well as Equation (48), provides a theoretical framework that offers one way in which the state variable $T$ can be interpreted in econometrics. The calculations shown in the following sections to determine $T = f(\sigma)$ focus on a simple, idealized example of a two-state agent system and are consistent with the theoretical framework (Bouchaud, 2013). Therefore, systems with interactions between the agents and more complex systems, which e.g. show segregations (Schelling, 1971, 1978) and make the analogy $E = -U$ questionable when considering an additional individual utility (Grauwin et al., 2009; Lemoy et al., 2011; Bouchaud, 2013), are not discussed here and are left to further research.

### 4.4. Two-State Ideal Agent System

A number $N$ of stock investors will be considered as an example of a system of agents. A news situation $-1 \leq \mathcal{B} \leq +1$ is given and influences the investors. For the sake of simplicity, it is assumed that each investor only acts based on the news situation $\mathcal{B}$ and only buys or sells one share at a time. Agreements or alliances between investors are excluded, so each investor acts in isolation and is uninfluenced by other investors. These systems can be referred to as systems without interactions or ideal agent systems; see, e.g., Bouchaud (2013); Börner et al. (2023b).

In the following, w.l.o.g., a positive, constant news situation with strength $|\mathcal{B}| = B$ is assumed. Investors can act in "conformity" to the news, i.e., buy if the news is positive, or in "non-conformity", i.e., sell if the news is positive. The state of an investor $i = 1, \ldots, N$ is described with $s_i = +1$ (conform) or $s_i = -1$ (non-conform). If a dynamic equilibrium is established, then there is a number $N_+$ of "conform" and a number $N_-$ of "non-conform" investors, with $N = N_+ + N_-$. If an inversion of the agent system is ruled out, i.e., the overall system acts in accordance with the positive news, the difference $N_+ - N_-$ is positive and corresponds to a buyer surplus. In econometric applications, the normalized variable $M = \frac{1}{N}(N_+ - N_-) = \frac{1}{N}\sum_i s_i$ can be referred to as trade potential (*phys.*: $\propto$ magnetization), see, e.g., Börner et al. (2023b), and can be used, e.g., for one-step ahead forecast models (Vikram and Sinha, 2011).

#### 4.4.1. Occupation Probabilities

For the two states, occupation probabilities can be specified with the help of the logit rule (Bouchaud, 2013), which corresponds to the Boltzmann-Gibbs distribution in statistical physics (Landau and Lifšic, 1980; Greiner et al., 1995).

Let $x = \frac{T_0}{T}$, and $T_0$ is calculated from all constant system parameters and constant external conditions (e.g., the constant news $B$); see, e.g., Bouchaud (2013); Börner et al. (2023b). Then, the probabilities for each state $s = (-1, +1)$ of the agent are:

$$\begin{aligned} P_- &= \mathrm{Prob}(s = -1) = \frac{\exp(-x)}{\exp(-x) + \exp(+x)} \\ P_+ &= \mathrm{Prob}(s = +1) = \frac{\exp(+x)}{\exp(-x) + \exp(+x)} \end{aligned} \tag{49}$$

with $P_- + P_+ = 1$. A similar representation of the occupation probabilities can be found in, e.g., Bouchaud (2013). Equation (49) can be derived within the framework of econophysics and is consistent with the definition given in Equation (48). Accordingly, $T$ is initially only a parameter, and the question of whether $T$ is related to the volatility of the stock remains open.

### 4.4.2. *Distribution of the Trade Potential M*

For a finite number $N < \infty$ of isolated investors (ideal agents), the occupation numbers $N_-$ and $N_+$ are stochastically dependent random variables, and thus the trade potential $M$ is also a random variable. If all investors are lined up for illustration, it becomes clear that both $N_-$ and $N_+$ follow a binomial distribution: $\mathrm{Binom}(N_-; N, P_-)$ and $\mathrm{Binom}(N_+; N, P_+)$; for a definition, see, e.g., Abramowitz (2014). This allows the expected value $\mu_M = P_+ - P_-$ and the variance $\sigma_M^2 = \frac{4}{N} P_+ P_-$ to be calculated for the random variable $M$. The derivation is almost identical to the calculation in Fließbach (2018, Chap. 2), with the difference being that the normalized variable $M$ is considered here. For large $N$, the above binomial distributions can be approximated by normal distributions (Kendall and Stuart, 1977). This means that the distribution of the random variable $M$ can also be described by a normal distribution: $M \sim \mathcal{N}(\mu_M, \sigma_M)$. The variance is proportional to $\frac{1}{N}$, so for large $N$, the relative width of the distribution tends to zero and is thus sharply localized around $\mu_M$, i.e., fluctuations in $M$ are almost no longer observed; see Fließbach (2018), and also compare Greiner et al. (1995).

### 4.5. *Stochastic Model of Market Capitalization*

Let the news situation $\mathcal{B} > 0$ be constant for a finite period $\mathbb{T}$. If this period is broken down into finite subperiods $\Delta t$, then in each subperiod $[t, t + \Delta t]$ there is a number $N_+$ of buyers and a number $N_-$ of sellers. At the beginning of the period, the last quoted price of the stock $p_t$ is known, and the potential rate of change in market capitalization ($V_t = N p_t$) that will take effect at the end of the subperiod can be inferred:

$$\begin{aligned} \Delta V_t &= p_t (N_+ - N_-) \Delta t \\ &= V_t M_t \Delta t \end{aligned} \tag{50}$$

with $M_t \sim \mathcal{N}(\mu_M, \sigma_M)$ in each subperiod. Thus, the change $\Delta V_t$ is a random variable and changes in each subperiod. The time-discrete representation of a stochastic process becomes observable. Such processes are described by stochastic differential equations; see, e.g., Hull (2018). Following technics described in Wilmott (1998, Chap. 3) and Hull (2018, Chap. 14), a stochastic differential equation with drift term ($\propto \mu_M$) and diffusion term ($\propto \sigma_M$) for the stochastic process can be written:

$$\mathrm{d}V_t = V_t \mu_M \,\mathrm{d}t + V_t \sigma_M \,\mathrm{d}W_t, \tag{51}$$

where $W_t$ denotes a Wiener process. The transformation $X_t = G(V_t, t) = \ln(V_t) - \ln(N)$ transforms to the logarithmic price $X_t$ of the stock. Using Itō's lemma with the partial derivatives $\partial_V G = \frac{1}{V}$, $\partial_{VV} G = -\frac{1}{V^2}$ and $\partial_t G = 0$, it follows that:

$$\mathrm{d}X_t = \left(\mu_M - \frac{\sigma_M^2}{2}\right)\mathrm{d}t + \sigma_M\,\mathrm{d}W_t. \tag{52}$$

Only one price is quoted on the capital market at a time. This means that there is only one logarithmic price at a time. The stochastic process for logarithmic returns (Hull, 2018, Eq. 14.17) and the process defined with Equation (52) must therefore be identical. A comparison of coefficients then provides the following relationship:

$$\mu_X = \mu_M \quad \text{and} \quad \sigma_X = \sigma_M. \tag{53}$$

Where $\mu_X$ is the expected value of the logarithmic returns and $\sigma_X$ is the standard deviation. The latter is therefore the volatility of the stock under consideration.

Note that it was assumed that the positive news situation $\mathcal{B}$ is constant for the period $\mathbb{T}$. In practice, this will only happen for a short period (a few minutes) of a daily trading session. The relationships described in Equation (53) should therefore only be observable in practice for short periods of time. If the focus is on one-step-ahead forecasts, mean values and sample variances calculated over short periods of time are to be preferred as estimators for $\mu$ and $\sigma$. Furthermore, note that Equation (52) does not capture contributions to expected value and volatility due to changes in news flow $\mathcal{B}$.

Section 4.4.2 showed that $\sigma_M \propto \frac{1}{\sqrt{N}}$. With Equation (53), this also applies to $\sigma_X$ for short periods of time with a constant news situation. The latter means that the volatility that can be observed over short periods of time decreases as the number of investors $N$ increases and in the limiting case of an infinite number of investors approaches zero. A similar phenomenon was described by Bouchaud (2013) in the context of socioeconomic issues. There, however, it related to noise levels that influence decision-making situations and scale with $\frac{1}{\sqrt{N}}$ depending on the population size.

Equation (53) provides a very general connection between the capital market quantities and the distribution parameters of the trade potential $M$. However, $|M| \leq 1$ is limited and thus also $\sigma_M$. The validity of $\sigma_X = \sigma_M$ must always be checked for large volatilities $\sigma_X$, in particular whether strongly fluctuating messages or high-frequency message changes could be the reason for the high volatility.

### 4.6. Application

In the following, the central equation $\sigma_X = \sigma_M$ for the example of the two-state ideal agent system from Section 4.4 is used to establish the connection between $T$ and the volatility $\sigma_X$:

$$\begin{aligned}\sigma_X &= \sigma_M \\ &= \sqrt{\frac{4}{N} P_+ P_-} \\ &= \sqrt{\frac{4}{N} \frac{1}{4\cosh^2(x)}} \\ &= \frac{1}{\sqrt{N}} \operatorname{sech}(x).\end{aligned} \tag{54}$$

With $x = \frac{T_0}{T}$, the measurement equation follows:

$$T = \frac{T_0}{\operatorname{sech}^{-1}(\sqrt{N}\sigma_X)}. \tag{55}$$

Where $\operatorname{sech}^{-1}(u)$ is the inverse hyperbolic secant function, and $\sigma_X$ as the volatility of the logarithmic returns is now the observable that can be measured on the capital market. Equation (55) indirectly measures the state variable $T$. For the two-state ideal agent system, it is the state variable that is consistent according to Equation (48). In further state equations for the system, this state variable determined in this way must be used to maintain consistency within the theory.

### 4.7. Conclusion

The highly simplified example of a two-state ideal agent system was studied in detail, and the relationship between the volatility $\sigma_X$ and the state variable $T$ was established. Even this simplest conceivable example shows that the relationship between $\sigma_X$ and $T$ can be nonlinear. It can be assumed that this is also the case in other applications. This consideration is not exhaustive; Equation (55) is a special measurement equation for the example, and in individual cases, the measurement equation $T = f(\sigma_X)$ must always be derived. This allows for the indirect and theory-conforming determination of the state variable $T$ from the measured volatility $\sigma_X$. If a linear relationship $T = a\sigma_X + b$ is required, then the measurement equation can be linearized around an operating point $\sigma_X^0$, i.e., considering a Taylor series up to the linear term. In practice, questions about the validity of the approximation (keyword: measuring range) then have to be answered.

**Title:** On the connection between temperature and volatility in ideal agent systems

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**Conferences:** –

**Publications:** Published on SSRN and in the *Journal of Statistical Mechanics: Theory and Experiment*, 2023, 103401, a single-blind peer-reviewed journal. Current status: Published.
doi: `https://doi.org/10.1088/1742-5468/ad002c`.

**Share of contributions:**

| **Contributions** | **Christoph J. Börner** | **Ingo Hoffmann** | **John H. Stiebel** |
|---|---|---|---|
| **Research design** | 5% | 80% | 15% |
| *Development of research question* | 10% | 70% | 20% |
| *Method development and specification* | 0% | 90% | 10% |
| **Research performance & analysis** | 5% | 70% | 25% |
| *Literature review and framework development* | 0% | 50% | 50% |
| *Data collection, preparation and analysis* | 0% | 80% | 20% |
| *Analysis and discussion of results* | 20% | 60% | 20% |
| *Derivation of implications and conclusions* | 0% | 90% | 10% |
| **Manuscript preparation** | 20% | 45% | 35% |
| *Final draft* | 20% | 50% | 30% |
| *Finalization* | 20% | 40% | 40% |
| **Overall contribution** | 10% | 65% | 25% |

Date, Cristoph J. Börner

Date, Ingo Hoffmann

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# 5. Beyond the Individual: Investigating the Interdependence of Speculative Bubbles and Herding in Financial Markets

## *5.1. Abstract*

Speculative bubbles have the potential to cause significant economic damage. It is therefore important to better understand the driving factors. This study empirically examines herding behavior as a theoretically known driver of speculative bubbles for the United States (US) stock market. First, the results suggest the presence of speculative bubbles and herding behavior within the S&P 500 stock market index. Second, it is discovered that herding behavior significantly reduces the probability of a bubble occurring. The negative influence of herding behavior can be observed for both the rate of change and for the absolute level. Analysis of the interaction effect shows that the absolute level of the herding variable moderates the rate of change. The examination of sub-hypotheses indicates that the relationship remains consistent across different industries, company sizes, sub-periods, and various time horizons, confirming the existence of the general relationship.

**Keywords:** Anomalies, Speculative Bubbles, Herding Behavior, Financial Markets, Behavioral Finance

**JEL Classification:** G10, G40, G41

## 5.2. Introduction

Anomalies such as speculative bubbles are widely documented capital market phenomena. They have the potential to cause significant economic damage (Jordà et al., 2015). Examples include the French stock market bubble of 1881–82, the US stock market bubble of 1928–29 followed by the Great Depression, and the Japanese stock market bubble of 1985–2003, all of which were followed by recessions (Brunnermeier and Schnabel, 2015). Regulators and central banks stabilize financial markets and are interested in anticipating exuberance and its driving factors (Phillips et al., 2015). The prevention of bubbles and the effectiveness of different policies in achieving this goal have been the subject of a long-standing debate (Brunnermeier and Schnabel, 2015). Moreover, investors and risk managers are exposed to potential misallocations through price distortions. Understanding the factors that lead to bubbles is therefore beneficial not only from a theoretical perspective but also from a practical one.

The existing literature addressing this question primarily examines speculative bubbles from a theoretical, often model-theoretical, perspective. There are few empirical studies investigating the driving factors behind bubbles, e.g. Wang and Chen (2019); Pan (2020); Maghyereh and Abdoh (2022). This study also addresses this research question and aims to gain a better empirical understanding of the driving factors by focusing on new variables in the analysis.

Theory suggests that herding behavior is a potential driver of asset mispricing. Empirically, herding behavior has been shown to affect asset prices in general. Several studies suggest a potential link between herding behavior and returns (Nofsinger and Sias, 1999; Wermers, 1999; Sias, 2004; Dasgupta et al., 2011; Singh, 2013; Brown et al., 2014; Celiker et al., 2015). However, herding behavior has not been empirically studied as a factor driving bubbles. Therefore, the second link is whether there is a relationship between bubbles and herding behavior as predicted by theory.

The study aims to empirically examine whether herding behavior affects stock market bubbles. This requires the identification of both variables, bubbles and herding behavior, initially. First, stock market bubbles are detected using a widely used recursive unit root test proposed by Phillips et al. (2015). Second, a dynamic herding measure proxied by market-wide herding behavior proposed by Hwang and Salmon (2004) is computed. The relationship between these input variables is then examined using various model specifications of multivariate logistic and linear regressions. Variations in the relationship across specific sub-periods, different industries, company sizes and time horizons are also considered.

The contribution of this study is the empirical examination of a theoretical hypothesis to better understand the factors driving bubbles, which is essential given the potential risks associated with bubbles. Furthermore, the empirical framework employed is innovative in its methodological objectivity, integrating two well-established methods into a logistic regression. Both methods are widely recognized and largely robust. This approach avoids the inclusion of arbitrary bubble periods as dummy variables. In addition, the use of a time-varying herd measure allows for deeper insights into the underlying herding dynamics.

The main results of the study are briefly presented. First, there is evidence of both speculative bubbles and herding behavior in the US S&P 500 stock market index over the period from 1990 to 2022. Second, it is discovered that market-wide herding behavior towards the market has a significant negative impact on the probability of a bubble occurrence. This contradicts expectations based on theoretical models but aligns with findings from other studies (Bekiros et al., 2017; Haykir and Yagli, 2022). With adverse or negative herding behavior, investors act against the market consensus rather than in its direction. This leads to systematically higher returns for subsets of the overall market than would be rationally expected. The negative influence of herding behavior is evident in both the rate of change of the variable and its absolute level. Negative rates of change, as well as negative absolute levels, favor the emergence of speculative bubbles. In addition, the examination of the interaction effect shows that the absolute level of the herding variable acts as a moderator on the rate of change. The analysis of sub-periods reveals that the relationship holds true irrespective of specific periods. Examining different industries indicates that the relationship manifests with varying degrees of significance across all sectors, thereby not highlighting clear systematic sector-specific differences. However, the most pronounced manifestations are observed in the service sector and the finance, insurance, and real estate sector. The analysis based on company size reveals that contrary to expectations, the relationship is more pronounced in larger companies. However, it is noteworthy that the relationship exists in both larger and smaller companies and that the difference is not statistically significant. Furthermore, the examination of longer time horizons indicates that herding behavior has predictive value for the probability of a bubble occurring in the medium to long term. This holds true even after a bubble has occurred, with a significant negative relationship persisting up to six months after the event.

The remainder of the paper is structured as follows: Section 5.3 establishes the theoretical foundations, introducing the concepts of bubbles and herding behavior along with the related literature. Additionally, the working hypothesis is derived from theoretical models and empirical evidence for the relationship between these two concepts. In Section 5.4, the empirical framework and results are presented and discussed. This includes the introduction of methods used to measure bubbles and herding behavior, along with an overview of the dataset. Section 5.5 tests the robustness of the main results, and finally Section 5.6 concludes.

## 5.3.1. Bubbles

### 5.3.1.1 Definition

A clear definition of the term speculative bubble is problematic, as the phenomenon of a speculative bubble is the subject of a long-standing controversy in the academic literature (O'Hara, 2008; Wöckl, 2019; Quinn and Turner, 2020). Aliber et al. (2015) take a technical perspective, according to which a bubble is characterized primarily by unsustainable patterns of price changes or cash flows. Specifically, their definition is that a bubble consists of a prolonged, between 15 and 40 months, price increase that subsequently collapses. Garber (2000) conceives of the term as a fuzzy concept that is not based on an operational definition. According to him, bubbles are the part of a price development that cannot be explained on the basis of the underlying fundamentals. Nevertheless, history shows that asset prices can be inexplicable relative to their theoretical underlying values (Brunnermeier and Oehmke, 2013; Quinn and Turner, 2020). Moreover, these phenomena are often accompanied by high price volatilities (Scheinkman and Xiong, 2003). Most of the academic research on bubbles assumes that asset prices should be linked to a true or fundamental value (O'Hara, 2008).

Therefore, in this study, a (speculative) bubble is defined as a persistent and significant deviation of the price of an asset from its fundamental value, defined as the risk-adjusted present value of all expected future cash flows (Tirole, 1985; Brooks and Katsaris, 2005; Wöckl, 2019). Additionally, the attribute "explosive" serves as a means to characterize the trajectory of bubbles. In this context, it implies that the coefficients of an autoregressive process temporarily exceed unity, as discussed in more detail in Section 5.4.3. This scenario is also referred to as having a temporarily explosive root (Phillips et al., 2015; Balcombe and Fraser, 2017). Adhering to this definition, the emergence of negative price bubbles is also feasible, signifying instances where prices fall below their theoretically justified levels (Yan et al., 2012; Scherbina and Schlusche, 2014). For instance, the loss of the illusion of control triggered by an event could lead to a panicked response among investors (Samuelson and Zeckhauser, 1988; Bracha and Weber, 2012). The illusion of control refers to the human tendency to believe that the outcome of certain events is influenced by them, even when those events are subject to chance (Langer, 1975). This response might result in market overreaction, akin to that driven by overconfidence (Daniel et al., 1998), causing prices to fall below their fundamental value.

### 5.3.1.2 Formation

Bubbles are relevant from both a research and a practical perspective because prices guide the allocation of resources in an economy, so price distortions can lead to potential incorrect investment incentives. Real estate price bubbles, e.g., could lead to inefficient new construction. In addition, the balance sheets of companies, financial institutions, and households may suffer when a bubble bursts, thus fostering an economic downturn (Brunnermeier and Oehmke, 2013). A look at historical examples also shows how bubbles can lead to economic crises, depending on the configuration of external and internal circumstances of a market or economy. See, e.g., Brunnermeier and Schnabel (2015), whose selection is based on a technical definition similar to Aliber et al. (2015). Because of the potential impact on the real economy, it is important to understand how and under which circumstances bubbles occur. The literature on bubbles is very broad and offers numerous approaches to explain the formation. Gürkaynak (2008) as well as Brunnermeier and Oehmke (2013) provide noteworthy literature reviews. Scherbina and Schlusche (2014) also survey bubble formation models and focus on behavioral models as well as rational models with incentive problems, market frictions, and non-traditional preferences. The recent survey by Wöckl (2019) categorizes bubble models broadly in rational and irrational, or behavioral models. In rational bubble models, agents are assumed to be perfectly rational with respect to expected future dividends and markets are assumed to be predominantly efficient. Most rational bubble models are built on the present value model, according to which the price of an asset consists of discounted future expected dividends. The bubble is thereby supplemented by an additional term that reflects the price development that is not justified by fundamental data (Gürkaynak, 2008). Well-known studies in this tradition include, e.g., Flood and Garber (1980); Blanchard and Watson (1982); Tirole (1985); Diba and Grossman (1988); Froot and Obstfeld (1991) among others. One implication of these models is that prices rise explosively in the build-up phase. This property is also exploited in empirical studies to identify bubbles, as discussed in more detail in Section 5.4.3. In addition to rational bubble models, there are other approaches that incorporate behavioral economic aspects into the modeling and thus soften the strong assumption of rational agents. Wöckl (2019) distinguishes between four behavioral economic models based on the fact that investors are driven by different opinions on the one hand and influenced by psychological biases on the other. In addition, Scherbina and Schlusche (2014) suggest that herding behavior is an important mechanism that can reinforce or promote bubbles.

#### *5.3.1.3 Empirical Methods*

The goal of bubble identification methods is to accurately determine when bubbles form and end using quantitative measures. The challenge is that fundamental values of securities are not observable and therefore, the determination of prices is not unambiguous. Various methods are known in the literature, for reviews see, e.g., Gürkaynak (2008); Wöckl (2019). Some types of tests are also explained below. The Variance bound test belongs to the early empirical methods and was initially designed by Shiller (1981) to criticize the EMH. Only in later applications the results of the tests were interpreted as possible bubbles (Blanchard and Watson, 1982). The test implements an upper bound for the variance of observed prices. The problem with this test is its model-related nature: testing for bubbles under the assumption that prices are adequately represented by a present value model are two hypotheses, so that the test suffers from the joint hypothesis problem. West (1987) proposed the Two-Step Test, which avoids the joint hypothesis problem by testing the model and bubble hypothesis sequentially. Another category of tests are based on statistical properties such as stationarity and cointegration. In an exemplary application of such a test, the security prices must be stationary after difference formation, if fundamentals are stationary after difference formation, under the assumption that there is no bubble (Diba and Grossman, 1988). Regime-switching tests, or Markov-switching tests, are further developments of the standard tests based on stationarity and cointegration. The advantage of these tests is that the dynamics of a bubble are not restricted to a linear process. In other words, these tests consider the periodically collapsing nature of bubbles by modeling two or more different Markov regimes. A security price can discretely switch between these regimes with certain transition probabilities (Hamilton, 1989). Similar to these tests, recursive unit root tests are also further developments of the stationarity and cointegration based tests and work with the assumption that security prices can be modeled as random walks and consequently have a unit root. Recursive means that the tests are applied sequentially to sub-samples. The approach by Phillips et al. (2011) (hereafter PWY) uses a so-called sup ADF test (SADF) based on a forward recursive series of right-sided ADF unit root tests to check a time series for bubbles. In addition, a dating strategy using a backward-looking regression technique identifies the times at which a bubble originates and ends. The approach is consistent as long as there is only one bubble in the data. If there are multiple bubbles in the sample, this procedure suffers from lower quality and can lead to inconsistent results. To address the problem of periodically collapsing bubbles, the refined version of Phillips et al. (2015) (hereafter PSY) uses flexible sub-sample sizes in the implementation of the recursive tests as an extension of the PWY approach. This improved version of the test is applied and presented in detail in Section 5.4.3.

### *5.3.2. Herding Behavior*

#### *5.3.2.1 Definition*

Herding behavior describes the phenomenon of individuals imitating the actions of other individuals or aligning their decisions with those of others (Spyrou, 2013). On the capital market, investors subsequently trade in the same direction (Nofsinger and Sias, 1999) and disregard private information to follow the current trend instead (Avery and Zemsky, 1998). Although herding behavior is widely recognized as irrational behavior that impairs market efficiency, situations exist in which herding behavior is at least similar to rational decision-making. As Litimi et al. (2016) point out, herding behavior as such cannot be strictly classified as a bias, whereas other biases, such as conformity (Hirshleifer, 2001), home bias (Feng and Seasholes, 2004), or availability bias (Kuran and Sunstein, 1999), can contribute to herding tendencies. Two general forms can be distinguished. Intentional herding means that investors choose to suppress prior judgments and instead copy actions of others. This type of imitation may be motivated by the expectation of positive externalities or "payoffs". These payoffs may be *informational* and driven by the expectation about asymmetrically distributed information, so that investors ignore private information either because other agents might be better informed or have better information processing capabilities (Devenow and Welch, 1996). Thus, by imitation, agents can gain an informational advantage in information-based models. Such behavior may foster the development of information cascades and lead to inefficient equilibria (Banerjee, 1992; Bikhchandani et al., 1992), e.g., capital market anomalies such as bubbles and collapses, increasing systemic risk. *Professional* payoffs are also subject to herd incentives and can be categorized into two types of models. Reputation-based models explain herd incentives in terms of the relative performance of fund managers compared to their peers (Scharfstein and Stein, 1990; Froot et al., 1992; Graham, 1999). In compensation-based models, managers participate in herding behavior as they are often compensated relative to a benchmark (Chevalier and Ellison, 1999; Graham, 1999; Maug and Naik, 2011). Spurious or unintentional herding occurs when investors react independently to the same set of information (Gębka and Wohar, 2013; Galariotis et al., 2015; Bekiros et al., 2017). Similar investment strategies due to similarities between investment professionals and investment styles are drivers of spurious herding behavior (Kallinterakis and Gregoriou, 2017; Kyriazis, 2020). However, as noted earlier, herding behavior is perceived as an irrational phenomenon, in part because of its potential to reduce market efficiency and induce capital market anomalies such as speculative bubbles (Lux, 1995; Olsen, 2011; Scherbina and Schlusche, 2014; Bekiros et al., 2017). Refer to literature surveys, e.g., by Hirshleifer and Hong Teoh (2003); Fenzl and Pelzmann (2012); Spyrou (2013); Kallinterakis and Gregoriou (2017); Komalasari et al. (2022) for more details about the different forms of herding behavior.

#### *5.3.2.2 Evidence for the US Stock Market*

Regardless of the methods discussed in Sections 5.3.2.3 and 5.4.2, there are various findings in the literature. Since this study is focused on the US market, the overview of empirical findings is limited to the American market. Lakonishok et al. (1992) examined the fund positions of 769 tax-exempt American funds and found no evidence of herding behavior or positive feedback trading. A similar result is reported by Grinblatt et al. (1995), who also analyzed transaction data of American investment funds and could not identify herding behavior. The studies conducted by Chang et al. (2000) and Chiang and Zheng (2010) investigated herding behavior in international financial markets but could not establish its presence in the American market. Methodologically, both studies were based on a variation of the approach introduced by Christie and Huang (1995) and confirmed their results for the American market. On the contrary, Hwang and Salmon (2004) identified herding behavior in the S&P 500 using their own methodology, both in rising and falling markets. Their findings suggest that market stress and crises contribute to restoring market equilibrium, potentially enhancing market efficiency. The studies by Sias (2004) and Choi and Sias (2009) indicate that institutional investors in the US tend to follow each other in herding behavior to derive information from previous transactions. Furthermore, Messis and Zapranis (2014) found evidence of herding behavior in the American S&P 500 and observed that herding behavior is triggered by shocks in macroeconomic variables. Finally, Bekiros et al. (2017) examined the American indices S&P 100 and DJIA and detected herding behavior in both markets. Their analysis of sub-periods revealed that herding behavior was insignificant during the global financial crisis (GFC) but prevailed in the post-crisis period. The accumulation of these studies highlights the ambiguity of the results, even within a single market. This highlights the need to re-examine the US market with a new sample and also opens up the perspective of considering herding behavior as a subject of study in the context of stock market bubbles.

#### *5.3.2.3 Empirical Methods*

The empirical literature knows several measures to identify herding behavior in market data. For review articles, see, e.g., Spyrou (2013); Kallinterakis and Gregoriou (2017). A distinction can be made between a micro level and a macro level or aggregate view.

The former uses, e.g., trading data from investors on their accounts, portfolios and transactions. An early measure of the first category is based on the LSV model of Lakonishok et al. (1992) which focuses on the micro level of investor behavior. Institutional fund data is used to identify the degree of correlation between the trading positions of different fund managers to test for herding behavior. The test statistic measures the average tendency of (pension) funds to trade (buy or sell) a respective stock at the same time. The portfolio change measure (PCM) of Grinblatt et al. (1995) is methodologically in the same vein, but additionally differentiates whether the transaction is a purchase or a sale in each case. Sias (2004) further developed this approach by testing directly for inter-temporal dependence of cross-sectional institutional demand.

At the macro level, the methods are primarily based on easily accessible market data, such as prices and trading volumes (Kallinterakis and Gregoriou, 2017). Christie and Huang (1995) proposed a model based on the relationship between cross-sectional dispersion of returns and extreme market returns. Thus, they measure herding behavior towards the market under the assumption that investors are more likely to neglect their own information in favor of market opinion in times of market stress. When herding takes place, the dispersion of individual asset returns is smaller than a rational asset price model would predict. Therefore, a negative coefficient indicates herding behavior when the dispersion measure is regressed on extreme market returns. The approach of Chang et al. (2000) is similar, but in addition to the linear relationship between the cross-sectional return dispersion and the market return, non-linearities are considered in the regression specification. Hwang and Salmon (2004) measure herding behavior using the deviation of the observed cross-sectional dispersion of the factor sensitivities of individual assets from the equilibrium beliefs derived from the capital asset pricing model (CAPM). This approach is implemented and discussed in detail in Section 5.4.2.

### *5.3.3. Bubbles due to Herding Behavior*

#### *5.3.3.1 Theoretical Contributions*

Numerous models and studies exist which show how herding behavior can lead to price distortions. Avery and Zemsky (1998) propose a model that takes several dimensions of uncertainty in the financial market into account. There is uncertainty about the future value of an asset and uncertainty about the information of other market participants. The authors show that herding behavior can occur and lead to bubbles when uncertainty is high and the information of market participants is not perfect. Scherbina and Schlusche (2014) list several incentives for why money managers participate in herding behavior. For example, managers may be forced to invest in high-sentiment stocks, thereby perpetuating bubbles, knowing that mutual fund investors will move their money into the best-performing funds, as suggested by Frazzini and Lamont (2008). Managers therefore have incentives to follow their peers when their performance lags behind. Shiller (2002) argues that money managers' limited resources imply that not every investment opportunity can be conclusively evaluated. Observing peers' trading decisions might lead to the assumption that it was made on the basis of more extensive assessments so that managers may decide to imitate. In the model of Scharfstein and Stein (1990), competing agents of two different types behave in a manner consistent with Keynes' assertion that "it is better to fail conventionally than to succeed unconventionally" (Keynes, 1936). Under certain circumstances, the decisions of the first mover are adapted irrespective of private information, since the feared reputational damage is greater if they fail alone. In addition, the fact that money managers are often evaluated based on their relative performance to an appropriate benchmark and compensated accordingly reinforces the case to follow other managers (Lux, 1995). DeMarzo et al. (2008) show that retail investors, like professional investors, are also affected by incentives to imitate. In their model, agents' utility depends on their relative wealth compared to other agents. Therefore, they choose to invest in bubbles to avoid falling behind the relative wealth of peers when prices rise. The price distortion thereby grows over time and increases the incentive to herd. The authors identify herding behavior as a key component in driving bubbles and thus characterize bubbles as a social phenomenon. Bikhchandani et al. (1992) explain localized herding behavior in a general environment with sequential decisions and social contagion, where individuals stochastically choose to adopt a prior decision of other investors. Chang (2014) supposes a model with heterogeneous interacting agents and finds that herding behavior arises naturally when there are strong social interactions among investors. Moreover, a bubble can be perpetuated by herding behavior in the presence of strong exogenous social interactions. Steiger and Pelster (2020) use an experimental test to show that social interaction increases the likelihood of bubbles occurring. Interestingly, the effect is stronger in natural face-to-face situations than in typical social media-like interactions. The agent-based model by Harras and Sornette (2011) shows how bubbles can arise due to a social feedback mechanism that transforms a streak of positive news into a transient collective herding regime. Schaal and Taschereau-Dumouchel (2023) propose a herding model in which technology-driven boom-bust cycles arise due to investors' overoptimism. Thoma (2013) shows that the likelihood of a bubble occurring is higher when the herding propensity of heterogeneous agents increases due to bad advice. Hott (2009) proposes a model in which herding behavior leads to bubbles without investors being exposed to speculative incentives. Instead, market participants follow so-called mood investors who overestimate the level of information in the market. The positive feedback effect ultimately results in price distortions.

#### *5.3.3.2 Empirical Evidence*

The theoretical contributions suggest quite clearly that herding behavior, among other factors, can trigger and drive financial market bubbles. The empirical evidence is less clear. First, studies show that asset returns are influenced by herding behavior, cf. e.g., Nofsinger and Sias (1999); Sias (2004); Singh (2013); Celiker et al. (2015). Chen and Demirer (2018) study the Taiwanese stock market using different herding measures and find that a high level of herding yields higher subsequent returns. The findings indicate that the level of herding could act as a systematic driver of returns. Other studies demonstrate the relationship between volatility and herding behavior. Hoitash and Krishnan (2008) argue that herding behavior might lead to excessive volatility. Venezia et al. (2011) find in a four-year data set of customer transactions of an Israeli bank a tendency toward herding behavior among both professional and amateur investors, although it is more pronounced among the latter. Moreover, they observe that herding behavior correlates with stock price volatility and even conditions it in terms of Granger causality. Tan et al. (2008) find that herding behavior is more pronounced in a rising market and also relate this to high trading volume and excessive volatility. Other studies have also commented on this, but some go further by exploring the link between herding behavior and bubbles. Litimi et al. (2016) argue that excessive trading volume can thus further drive herding behavior and lead to potential abnormal price increases. They study the US stock market during four major periods of turbulence using dummy variables in a modified version of the regression model by Chang et al. (2000), the Black Monday (1987), the dot-com bubble (1997–2000), the downturn afterwards (2002) and the GFC (2008). The Granger causality test results suggest that herding behaviour and trading volume have an inhibitory effect on overall market volatility in large markets which is contrasted with other research that focuses on more concentrated markets. Moreover, the study shows that herding behavior is a crucial factor in the formation of bubbles in some sectors, but not all. This confirms the theoretical prediction that growing bubbles incentivise temporary collective herding behavior. Singh (2013) studies the trading behavior of institutional investors during the internet bubble and crash of 1998-2001. The study shows that institutional herding is associated with positive abnormal returns, while negative abnormal returns occur when herding behavior ceases. This suggests that institutional herding may have caused temporary price pressure and contributed to a price bubble. Bekiros et al. (2017) employ a cross-sectional absolute deviation approach in the spirit of Chang et al. (2000) and quantile regressions in US markets over varying time periods. They examine the impact of the GFC (2007–2009) on herding behavior and find a dynamic pattern in which herding behavior was strong and significant in the pre-crisis period, i.e. the build-up phase of the housing bubble. During and after the crisis, the intensity increasingly weakened. The authors also find a positive and significant correlation between herding behavior and market volatility. Chmura et al. (2022) study herding behavior in an experimental financial market with a feature reminiscent of social trading platforms. Thereby, the trading decisions of investors who have the largest wealth gains on a scoreboard are imitated. In this particular setting, the authors find that herding behavior is associated with less asset mispricing. Haykir and Yagli (2022) examine the cryptocurrency market for bubbles using the identification method of Phillips et al. (2015) during 15 months of the COVID-19 period. In addition, they investigate whether herding behavior occurs during bubbles by including dummies for bubbles of each cryptocurrency in the regression model of Chang et al. (2000). They find that herding behavior in the overall market declines when a particular cryptocurrency has a bubble, suggesting that bubbles cannot be explained by herding behavior. In addition, they calculate herding dummies using a time-varying measure based on 30-day rolling window regressions according to Bouri et al. (2019). Examining the reverse relationship, they find in a logistic regression that only bubbles in a few cryptocurrencies have an impact on herding behavior.

The mixed empirical results necessitate further research that takes a different methodological approach while aiming at a general investigation of the relationship between bubbles and herding behavior. In contrast to other studies, this study aims at methodological objectivity by combining two well-established methods in logistic and other multivariate regressions which are accepted in the literature and largely robust. This approach avoids the arbitrary inclusion of bubble periods as dummy variables. In addition, unlike the studies conducted with Chang et al. (2000), a time-varying measure is used, providing a deeper insight into herding dynamics. Based on the theoretical starting point, the relationship is examined. This is based on the following working hypothesis:

$H_0$: *Bubbles are not driven or triggered by herding behavior.*

### *5.4. Empirical Analysis*

#### *5.4.1. Data*

The daily stock market variables for all constituents of the S&P 500 stock index, including closing prices, market capitalizations, dividend yields, and trading volumes, representing the US market from 1990-01-01 to 2022-12-31 are obtained from Refinitiv Eikon. Changes in index composition are taken into account on an annual basis to avoid survivorship bias. A market capitalization-weighted market index is calculated to ensure greater consistency when comparing herding behaviour with the occurrence of bubbles.

Trade volume and return volatility are included as control variables as the theoretical models of Scheinkman and Xiong (2003) and Mei et al. (2009) provide evidence that bubbles are associated with increasing trading volume and higher volatility, while Wang and Chen (2019) empirically confirm these variables as drivers of bubbles. Wang and Chen (2019) additionally note that monetary policy and credit have an impact, so the growth rate of money supply M2, a reference interest rate and the growth of credit relative to GDP are included. The interest rate, proxied by the 3-month US government bond yield, reflects the macroeconomic situation and influences both the cost of borrowing and investment behaviors (Pan, 2020). As bubbles are often accompanied by economic expansion (Scherbina and Schlusche, 2014), economic growth is additionally included depicting the macroeconomic situation. The variable is proxied by the growth of industrial production (Pan, 2020) and GDP. Pan (2020) also shows that investor sentiment can be used to predict bubble probabilities, highlighting the importance of investor sentiment in explaining bubbles. For this reason, a sentiment variable proxied by the survey-based US Consumer Confidence Index (CCI) by the Conference Board is included. Moreover, Bouri et al. (2019) demonstrate that economic uncertainty significantly increases the probability of herding in cryptocurrencies using a probit model. Enoksen et al. (2020) find that the occurrence of bubbles can also be explained by economic uncertainty. Therefore, uncertainty is also included here, measured by the news-based economic policy uncertainty ($EPU_t$) index of the US by Baker et al. (2016). This complements the consideration of return volatility, as volatility can also be seen as an indicator of uncertainty and bubbles can be accompanied by higher volatility (Avery and Zemsky, 1998).

Most of the control variables are also obtained from Refinitiv Eikon, such as trade volume ($tv_t$), economic growth ($gGDP_t$), industrial production growth ($gIP_t$), growth of money supply M2 ($gM2_t$), the reference interest rate ($r_t$), consumer sentiment ($SENT_t$) and the price-dividend ratio ($PDR_t$). The growth of credit as a ratio to GDP ($gCRGDP_t$) is from the Bank for International Settlements (BIS) data base. Monthly volatility ($pv_t$) is calculated as standard deviation over daily log returns of the market index for each month. Most variables are available in monthly frequency, except for $gGDP_t$ and $gCRGDP_t$. However, as outlined in Section 5.4.2, the main drawback of the method utilized subsequently, which was proposed by Hwang and Salmon (2004), is the reduction in data points compared to the methodologies introduced by Christie and Huang (1995) and Chang et al. (2000). Therefore, quarterly control variables are converted to monthly data when feasible to avoid losing even more observations in the subsequent regression analysis. The quarterly GDP data are transformed from quarterly to monthly data using the R package “tempdisagg” proposed by Sax and Steiner (2013) for temporal disaggregation of time series. In this case, the standard Denton-Cholette method (Dagum and Cholette, 2006) applies a simple interpolation that satisfies the temporal additivity constraint, which makes sense here since the quarterly values of GDP are split between the individual months, as the quarterly values of GDP are a sum of the monthly output generated. The quarterly data for credit relative to GDP are in turn transformed using linear interpolation, since the level of credit or money supply remains at the quarterly level each month.

Table 8 summarizes the descriptive statistics of the main variables. As can be seen, the results of the Jarque-Bera (JB) test suggest that all variables are significantly different from a normal distribution. The results of the augmented Dickey-Fuller (ADF) test indicate that six of the variables are stationary, while trading volume, interest rate, and sentiment become stationary after taking the first-order differences. The variable $h_{mt}$ calculated in Section 5.4.2 fluctuates around 0 in the long run and therefore has no drift and no trend as suggested by Hwang and Salmon (2004). Thus, the ADF test specification excludes trend and drift for $h_{mt}$. This is important because the power of the ADF test suffers when irrelevant parameters are included in the specification (Enders, 2015). In addition, a correlation analysis indicated that there are no multicollinearity problems. All variance inflation factors are below 2.5. The results are available upon request.

**Table 8:** Descriptive statistics.

| | Mean | Std | Max | Min | JB test | ADF test |
|---|---|---|---|---|---|---|
| $h_{mt}$ | 0.045 | 0.290 | 0.692 | −0.747 | 10.312*** | −2.888*** |
| $pv_t$ | 0.015 | 0.009 | 0.063 | 0.003 | 421.34*** | −3.954** |
| $tv_t$ | 15662.51 | 10978.68 | 54458.48 | 0.000 | 53.153*** | −2.295 |
| $gGDP_t$ | 0.002 | 0.005 | 0.049 | −0.054 | 58155*** | −8.295*** |
| $gIP_t$ | 0.001 | 0.011 | 0.063 | −0.144 | 129380*** | −6.658*** |
| $gM2_t$ | 0.005 | 0.007 | 0.063 | −0.015 | 3144.8*** | −4.541*** |
| $gCRGDP_t$ | 0.001 | 0.003 | 0.013 | −0.009 | 158.31*** | −3.450** |
| $r_t$ (%) | 2.583 | 2.272 | 8.094 | −0.010 | 31.365*** | −2.754 |
| $SENT_t$ | 95.166 | 26.523 | 1.447 | 25.300 | 12.543*** | −1.916 |
| $EPU_t$ | 126.276 | 60.251 | 503.963 | 44.783 | 1224.9*** | −3.810** |

*Note:* Descriptive statistics for all variables relevant to the subsequent regression analysis. Significance levels of 1%, 5% and 10% are indicated by ***, ** and *, respectively.

### *5.4.2. Detection of Herding Behavior*

In the subsequent analysis of the relationship between bubbles and herding behavior in Section 5.4.4, where both variables are merged as inputs, it is preferable, for the sake of consistency, to calculate both input variables on a common data base. Therefore, a market-wide measure of herding is more suitable here than a transaction-based one. The advantage of the method by Hwang and Salmon (2004) is that it provides a visual and continuous measure that can be used as an explanatory variable in regression. Moreover, in the realm of market-wide herding measures, Demirer et al. (2010) confirm that the method of Hwang and Salmon (2004) is equally effective as the method of Chang et al. (2000). For these reasons, herding behavior is calculated using the approach by Hwang and Salmon (2004) for the following analysis.

The CAPM (Sharpe, 1964; Lintner, 1965; Mossin, 1966) is used to define the risk-return equilibrium relationship of equities and is expressed as:

$$\mathrm{E}_t(r_{it}) = \beta_{imt}\mathrm{E}_t(r_{mt}), \tag{56}$$

where $r_{it}$ and $r_{mt}$ are the excess returns on asset $i$ and the market at time $t$, respectively. $\beta_{imt}$ is the systematic risk measure, and $\mathrm{E}_t(\cdot)$ is conditional expectation at time $t$. The concept of market-wide herding behavior differs from the common definition of investors following each other by imitating trading activities, because here investors follow market views about a market index, for example. Both notions of herding behavior are important because both potentially lead to mispricing of individual assets as equilibrium beliefs are suppressed. It is assumed that investors first form expectations of market returns and then, conditional on these, consider the value of an individual asset. In this view, herding behavior leads to a distorted risk-return relationship, so that CAPM betas for individual assets deviate from their unbiased equilibrium values. The conditional expectation on the excess return of asset $i$ and its beta at time $t$ are denoted by $E_t^b(r_{it})$ and $\beta_{imt}^b$, respectively. According to the CAPM, if the market rises significantly, then for an asset with $\beta_{imt} > 1$, $E_t(r_{it}) > E_t(r_{mt})$. Herding behavior in the direction of the market performance, however, leads to selling of the asset, which causes the price to fall, so that $0 < E_t^b(r_{it}) < E_t(r_{it})$ and correspondingly $1 < \beta_{imt}^b < \beta_{imt}$. When the market falls significantly, $E_t(r_{it}) < E_t(r_{mt})$ holds for the same asset according to the CAPM. Here, however, herding behavior in the direction of the market performance causes the asset to be bought and the price to rise, so that $E_t(r_{it}) < E_t^b(r_{it}) < 0$ and accordingly $1 < \beta_{imt}^b < \beta_{imt}$. For an asset with $\beta_{imt} < 1$, the inverse relationship holds such that $1 > \beta_{imt}^b > \beta_{imt}$. An asset with $\beta_{imt} = 1$ is immune to herding behavior. Consequently, herding behavior towards the performance of the market portfolio reduces the cross-sectional dispersion of individual betas. In the extreme case where the expected returns of all assets become equal to the expected return of the market portfolio, all individual betas would be equal to one and the cross-sectional dispersion would be zero. The opposite form of herding behavior exists as well, adverse herding behavior, where betas > 1 become even larger and betas < 1 become even smaller. Individual returns are then more sensitive to stocks with high betas and less sensitive to stocks with low betas. Adverse herding behavior represents mean reversion and must exist because there must be a systematic mechanism by which the long-run equilibrium beta can be regained.

Therefore, instead of the equilibrium relationship in Equation (56), in the presence of herding Hwang and Salmon (2004) assume the following relationship:

$$\frac{E_t^b(r_{it})}{E_t(r_{mt})} = \beta_{imt}^b = \beta_{imt} - h_{mt}(\beta_{imt} - 1), \tag{57}$$

where $h_{mt}$ is a latent herding parameter that changes over time. If $h_{mt} = 0$, then no herding behavior exists and $\beta_{imt} = \beta_{imt}^b$. On the other hand, if $h_{mt} = 1$, then $\beta_{imt}^b = 1$, which is the beta of the market portfolio, so the expected excess return of the asset will be equal to that of the market portfolio. In general, if $0 < h_{mt} < 1$, there exists a degree of herding behavior whose magnitude is given by $h_{mt}$. As described, adverse herding behavior also exists, so $h_{mt} < 0$ is allowed. The level of herding behavior is calculated across all assets, as market-wide herding behavior is examined here, thus eliminating idiosyncratic effects in any individual $\beta_{imt}^b$. It is assumed that Equation (57) holds for all assets in the market. Therefore, the cross-sectional standard deviation is calculated according to

$$
\begin{aligned}
\mathrm{Std}_c(\beta^b_{imt}) &= \sqrt{\mathrm{E}_c((\beta_{imt} - h_{mt}(\beta_{imt} - 1) - 1)^2)} \\
&= \sqrt{\mathrm{E}_c((\beta_{imt} - 1)^2)}(1 - h_{mt}) \\
&= \mathrm{Std}_c(\beta_{imt})(1 - h_{mt}),
\end{aligned}
\tag{58}
$$

where $\mathrm{E}_c(\cdot)$ and $\mathrm{Std}_c(\cdot)$ are the cross-sectional expectation and standard deviation, respectively. The impact of idiosyncratic changes in $\beta_{imt}$ is minimized by calculating $\mathrm{Std}_c(\beta_{imt})$ using a large number of assets, while $\mathrm{Std}_c(\beta_{imt})$ is allowed to be stochastic to monitor movements in the equilibrium beta. It is assumed that $\mathrm{Std}_c(\beta_{imt})$ does not change significantly, so that systematic movements in $\mathrm{Std}_c(\beta^b_{imt})$ can be explained by changes in $h_{mt}$. By taking the logarithm of Equation (58), $h_{mt}$ can be extracted in the following:

$$
\log[\mathrm{Std}_c(\beta^b_{imt})] = \log[\mathrm{Std}_c(\beta_{imt})] + \log(1 - h_{mt}). \tag{59}
$$

Defining $\mu_m = \mathrm{E}(\log[\mathrm{Std}_c(\beta_{imt})])$, the above assumption on $\mathrm{Std}_c(\beta_{imt})$ yields:

$$
\log[\mathrm{Std}_c(\beta_{imt})] = \mu_m + v_{mt}, \tag{60}
$$

where $v_{mt} \sim i.i.d.(0, \sigma^2_{mv})$ is assumed. The parameter $H_{mt}$ is assumed to be dynamic and modeled as an AR(1) process:

$$
\log[\mathrm{Std}_c(\beta^b_{imt})] = \mu_m + H_{mt} + v_{mt}, \tag{61}
$$

$$
H_{mt} = \phi_m H_{mt-1} + \eta_{mt}, \tag{62}
$$

where $H_{mt} = \log(1 - h_{mt})$ and $\eta_{mt} \sim i.i.d.(0, \sigma^2_{m\eta})$. This results in a state-space model that can be estimated with a Kalman filter, where Equations (61) and (62) are the measurement equation and state equation. The focus is on the process of the latent variable $H_{mt}$. If $\sigma^2_{m\eta} = 0$ there is no herding behavior and $H_{mt} = 0$ for all $t$. In contrast, a significant value for $\sigma^2_{m\eta}$ indicates the existence of herding behavior while this interpretation is supported by a significant $\phi$. Since $H_{mt}$ is assumed to be stationary, $|\phi_m| \leq 1$ is required.

To calculate $\mathrm{Std}_c(\beta^b_{imt})$, first the market model betas are estimated over monthly intervals with daily return data using OLS according to:

$$
r_{it_d} = \alpha^b_{it} + \beta^b_{imt} r_{mt_d} + \epsilon_{it_d}, \tag{63}
$$

where $t_d$ indicates daily data $d$ for the given month $t$. Using the estimated betas for each sample stock $i$ and month $t$, $\hat{\beta}^b_{imt}$, the monthly time-series of cross-sectional standard deviation of the betas is calculated as follows:

$$\widehat{\mathrm{Std}_c(\hat{\beta}^b_{imt})} = \sqrt{\frac{\sum_{i=1}^{N_t} (\hat{\beta}^b_{imt} - \overline{\hat{\beta}^b_{imt}})^2}{N_t}}, \tag{64}$$

where $\overline{\hat{\beta}^b_{imt}} = \frac{1}{N_t} \sum_{i=1}^{N_t} \hat{\beta}^b_{imt}$ and $N_t$ is the number of assets in the month $t$.

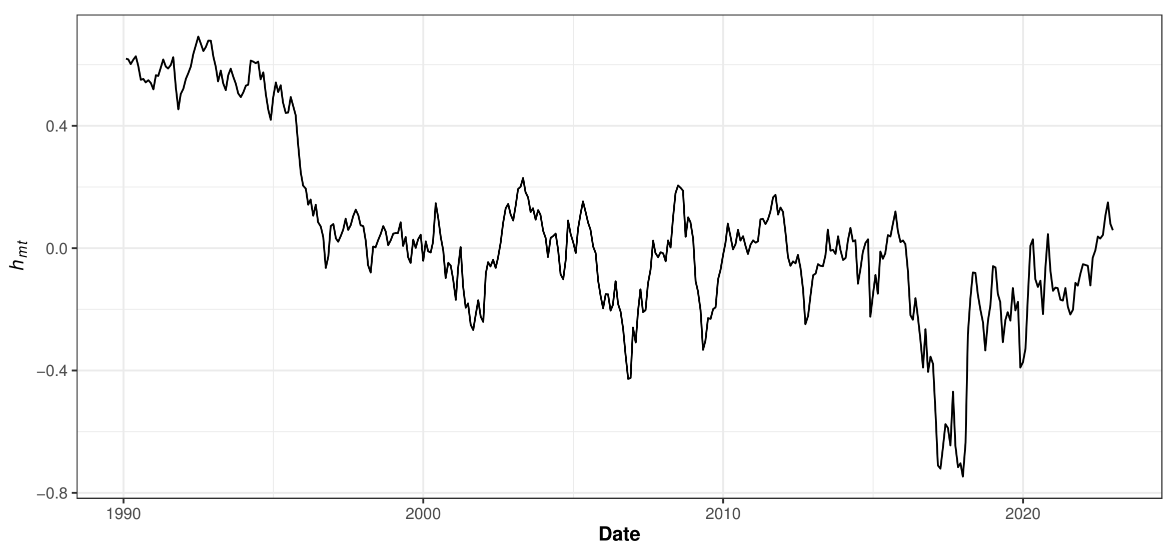
**Table 9:** Results of the state-space model.

| | $\mu_m$ | $\sigma_{mv}$ | $\phi_m$ | $\sigma_{m\eta}$ |
|---|---|---|---|---|
| S&P 500 | $-0.751^{***}$ | $0.0564^{***}$ | $0.978^{***}$ | $0.005^{***}$ |
| | (0.162) | (0.005) | (0.010) | (0.001) |

*Note:* The table shows the results of estimating the state-space model from Equation (61). Standard errors are given in parentheses. Significance levels of 1%, 5% and 10% are indicated by $^{***}$, $^{**}$ and $^{*}$, respectively.

Results for the state-space model estimation are displayed in Table 9. The variable $H_{mt}$ exhibits strong persistence with a high and statistically significant $\phi_m$. Additionally, $\sigma^2_{m\eta}$ is highly significant at the 1% level, indicating the presence of herding behavior towards the S&P 500 market index. The herding measure $h_{mt} = 1 - \exp(H_{mt})$ is displayed in Figure 8

**Figure 8:** The herding measure $h_{mt}$ for the S&P 500 based on the state-space model specified in Equations (61) and (62).




for the US market. Initially, the highest values of $h_{mt}$ are less than 1, reaching a maximum of 0.69 in the positive range and $-0.75$ in the negative range. Overall, $h_{mt}$ moves around its long-term average value of zero, similar to Hwang and Salmon (2004). In 1995, however, there is a significant drop in its level, which causes $h_{mt}$ to hover around zero in 1996, with minor fluctuations. Between 2000 and 2015, a significant downward movement is observed, marking the beginning of various cycles of herding and adverse herding behavior towards the market portfolio, some of which have strong characteristics. From 2016 to 2018, $h_{mt}$ experiences a sharp decline, characterized by a period of adverse herding behavior. Towards the end of the sample period, around 2020, $h_{mt}$ shows an upward trend, reaching higher levels by 2022.

*5.4.3. Detection of Bubbles*

The method of PWY (2011) and the refined version of PSY (2015) offer a widely acknowledged generalized sup ADF (GSADF) test that has been used multiple times and is considered as largely robust, cf. e.g., Phillips et al. (2015); Cheung et al. (2015); Escobari et al. (2017); Corbet et al. (2018); Gomez-Gonzalez et al. (2018); Hu and Oxley (2018a,b); Pan (2020); Hudepohl et al. (2021). Therefore, the improved test is utilized to identify bubbles in the US stock market. However, for the application of the PSY (2015) test procedure, the GSADF test statistic must first be explained. The GSADF test takes repeated ADF test regressions on sub-samples of the data while varying the start and end points of the sub-samples. The sample of a rolling regression starts at $r_1$ and ends at $r_2$, where $r_1$ and $r_2$ are each fractions of the total sample $T$ and $r_w$ indicates the size of the window, since $r_2 = r_1 + r_w$ and $r_w > 0$. The start points $r_1$ vary in the range from 0 to $r_2 - r_0$, and the end points $r_2$ vary from $r_0$ to 1. The GSADF statistic is the largest ADF statistic over all feasible sub-samples and is specified by:

$$GSADF(r_0) = \sup_{r_2 \in [0, r_1]; r_1 \in [0, r_2 - r_0]} \{ADF_{r_1}^{r_2}\}. \tag{65}$$

The following regression is used to perform the test:

$$y_t = \alpha_{r_1,r_2} + \beta_{r_1,r_2} y_{t-1} + \sum_{i=1}^{k} \psi_{r_1,r_2}^{i} \Delta y_{t-i} + \varepsilon_t, \tag{66}$$

where $y_t$ is the variable of interest at time $t$, $k$ indicates the lag order, and $\Delta$ is the difference operator. For the error term $\varepsilon_t \sim i.i.d.(0, \sigma^2)$ is assumed. If the null hypothesis is rejected because the test statistic exceeds a critical value, this is considered an indicator of explosive behavior, which is interpreted as a bubble. The null and alternative hypotheses of the test are:

$H_0 : \beta_{r_1,r_2} = 1$ (no bubble)
$H_1 : \beta_{r_1,r_2} > 1$ (explosive bubble).

To determine the period of a bubble, the backward SADF (BSADF) method of PSY (2015) is used. Right-tailed ADF tests are applied to the backward expanding sample sequences using

$$BSADF_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} \{ADF_{r_2}(r_1)\}. \tag{67}$$

The (fractional) starting and ending points (i.e., $r_e$ and $r_f$) of a bubble are the first chronological observation whose BSADF test statistic exceeds a corresponding critical value, and the first observation of the BSADF test statistic that falls below the critical value after $\lfloor T\hat{r}_e \rfloor + \delta \log(T)$, respectively. The parameter $\delta$ is determined based on the frequency of the data. The effect of this specification is that the duration of bubbles should exceed a minimum period to avoid interpreting short-lived outliers as bubbles. Formally, the points are given by:

$$\hat{r}_e = \inf_{r_2 \in [r_0, 1]} \{r_2 : BSADF_{r_2}(r_0) > scv_{r_2}^{\beta_T}\}, \tag{68}$$

$$\hat{r}_f = \inf_{r_2 \in [\hat{r}_e + \delta \log(T)/T, 1]} \{r_2 : BSADF_{r_2}(r_0) < scv_{r_2}^{\beta_T}\}, \tag{69}$$

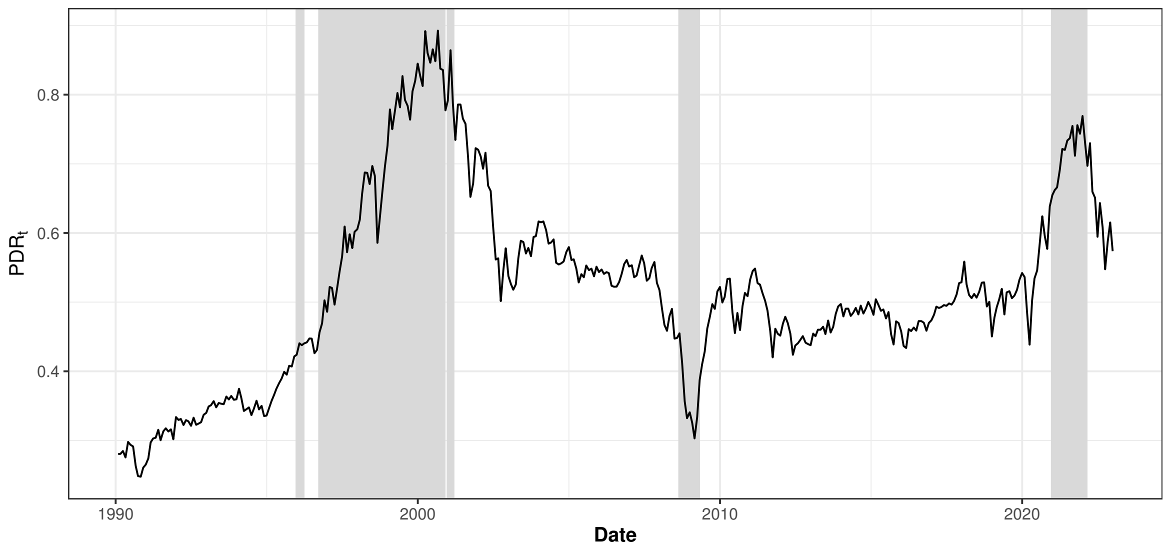
where $scv_{r_2}^{\beta_T}$ is the $100(1 - \beta_T)\%$ critical value of the sup ADF statistic based on $\lfloor T\hat{r}_2 \rfloor$ observations. The minimum sub-sample size $r_0$ is recommended to follow $r_0 = 0.01 + 1.8\sqrt{T}$ so that sufficient observations are used for initial calculation. In addition, a bootstrapping procedure following Phillips and Shi (2020) is implemented to mitigate the potential influence of conditional heteroskedasticity in asset returns (Harvey et al., 2016) and to address the multiplicity issue in recursive testing (Shi et al., 2020).

The procedure is applied to the price-dividend ratio of the market index and the optimal lag $k$ length is selected based on the Bayesian information criterion (BIC) with a maximum lag order of six. The result is a dummy variable, indicating a bubble at time $t$ with a value of 1, so that:

$$B_t = \begin{cases} 1, & \text{if } BSADF_t \geq scv_{r_2}^{\beta_T} \\ 0, & \text{if } BSADF_t \leq scv_{r_2}^{\beta_T} \end{cases} \tag{70}$$

Figure 9 shows the identified explosive periods for the S&P 500 (shaded areas), as well as the path of the price-dividend ratio. The explosive market phases encompass bubble and crisis periods, depending on whether there is an overall increase or decrease in the price-dividend ratio (Phillips et al., 2015). In the context of the analysis, we consider both of these explosive

**Figure 9:** The price-dividend ratio ($PDR_t$) with shaded areas indicating exuberant bubble periods.




phases as bubbles, as a sharp decline in the price-dividend ratio leads to undervaluation, which can be interpreted as a negative bubble (Yan et al., 2012; Scherbina and Schlusche, 2014).

Three significant bubble periods have been identified using the method proposed by Phillips et al. (2015). The first and longest bubble period commences in January 1996 and, with the exception of two brief interruptions, extends until February 2001. This period coincides with the dot-com period and pertains to the late 1990s stock market boom, during which numerous internet companies were founded. Both private and institutional investors invested heavily in technology stocks, encouraged by media coverage of the boom and expectations of strong productivity improvements due to technological progress (Kindleberger, 2005; Brunnermeier and Schnabel, 2015). The second explosive phase in the sample spans seven months, from September 2008 to April 2009, marking the collapse of the accumulated house price boom, coinciding with the insolvency of investment bank Lehman Brothers in the same month, which triggered the GFC Brunnermeier and Schnabel (2015). This explosive period is not a classic bubble but rather a negative bubble due to the undervaluations caused by the collapse. The third period runs from January 2021 to February 2022, coinciding with the time of the COVID-19 pandemic. As the COVID-19 virus rapidly spread globally and the S&P 500 lost over 30% of its value in March 2020, the Federal Reserve (FED), like other central banks, implemented loose monetary policies. Monetary (and fiscal) instruments to support the US economy, such as lowering the federal funds rate, reviving quantitative easing programs, among others (Haas et al., 2020), may have contributed to the rapid recovery of the financial markets, thus causing the third explosive period in this sample.

*5.4.4. Regression Analysis*

*5.4.4.1 Baseline Analysis*

To analyze the influence of herding on bubbles the following binary regression specifications are employed:

$$\text{Prob}(B_t = 1) = \text{F}(Y_t\beta) \tag{71}$$

where Prob$(\cdot)$ represents the probability of a bubble at time $t$. F$(\cdot)$ is the cumulative density function, and $Y_t$ is the parameter vector containing predictors, including the herding measure and control variables, described in Section 5.4.1. For the logit model, F$(\cdot)$ is the cumulative density function of the logistic distribution, and for the probit model, F$(\cdot)$ is the cumulative density function of the standard normal distribution. Additionally, a linear regression specification based on the BSADF test statistic is included as it provides guidance to which variables are statistically significant (Cameron and Trivedi, 2005, p. 471) and offers a more comfortable interpretation of the estimated coefficients as predicted probabilities.

Table 10 displays the results of the baseline regressions to test $H_0$. The variable $h_{mt}$ is included as a rate of change, as this makes it more responsive. The results in columns (1), (3) and (5) demonstrate a significant negative influence of the change of herding denoted as $\Delta h_{mt}$ on the probability of speculative bubbles to occur. The coefficient's significance level varies based on the selected model specification, being observed at the 10% significance level in the logit and probit model specification and at the 5% level in the OLS specification.

Notably, the negative coefficient sign for $\Delta h_{mt}$ raises curiosity, suggesting that herding towards markets may not inherently contribute to bubble formation, which is contrary to theory or educated expectation. Instead, this negative correlation implies that adverse herding could serve as a predictive factor for the emergence of speculative bubbles or exuberant behavior. Since $h_{mt}$ has been included as a differentiated variable in the regression and is therefore more sensitive, the probability of bubble occurrence decreases as herding behavior increases, and conversely, the probability of bubble occurrence increases as herding behavior decreases.

If $h_{mt}$ decreases from a positive level towards zero, then adverse herding behavior will occur to restore the long term equilibrium. If $h_{mt}$ is absolutely negative, this is also adverse herding, and the effect from there back to equilibrium should then be normal herding towards the market. However, the same mechanical process always occurs with adverse herding behavior: the betas develop away from the market instead of aligning with it. In the positive absolute range of herding behavior ($h_{mt} > 0$) this means that equilibrium is restored, whereas in the negative absolute range ($h_{mt} < 0$) the magnitudes become even stronger, beyond equilibrium (Hwang and Salmon, 2004).

**Table 10:** Results of the baseline regressions.

| | $B_t$ | | $B_t$ | | $BSADF_t$ | |
|---|---|---|---|---|---|---|
| | Logit | | Probit | | OLS | |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| $\Delta h_{mt}$ | $-5.938^{*}$ | $-8.862^{***}$ | $-3.301^{*}$ | $-4.974^{***}$ | $-3.452^{**}$ | $-5.453^{***}$ |
| | (3.411) | (3.090) | (1.869) | (1.682) | (1.506) | (1.172) |
| magnitude$(h_{mt})$ | | $-0.296^{*}$ | | $-0.174^{**}$ | | $-0.156^{**}$ |
| | | (0.158) | | (0.086) | | (0.070) |
| $\Delta h_{mt} \times$ magnitude$(h_{mt})$ | | $-1.428^{***}$ | | $-0.817^{***}$ | | $-0.750^{***}$ |
| | | (0.454) | | (0.242) | | (0.222) |
| $\log(pv_t)$ | $1.767^{**}$ | $2.612^{***}$ | $1.018^{**}$ | $1.496^{***}$ | $1.142^{***}$ | $1.587^{***}$ |
| | (0.792) | (0.893) | (0.445) | (0.495) | (0.377) | (0.304) |
| $\Delta \log(tv_t)$ | $0.615^{***}$ | $0.597^{***}$ | $0.348^{***}$ | $0.336^{***}$ | $0.149^{**}$ | $0.119^{*}$ |
| | (0.151) | (0.173) | (0.087) | (0.094) | (0.063) | (0.067) |
| $gGDP_t$ | $89.492^{***}$ | $100.815^{***}$ | $54.592^{***}$ | $60.982^{***}$ | $48.876^{***}$ | $48.582^{***}$ |
| | (20.363) | (30.480) | (12.857) | (18.958) | (17.510) | (17.790) |
| $IP_t$ | −9.394 | −3.043 | −5.954 | −2.806 | −3.635 | −2.218 |
| | (15.687) | (23.463) | (8.815) | (13.489) | (6.159) | (4.483) |
| $gM2_t$ | $82.028^{***}$ | $75.175^{**}$ | $45.621^{***}$ | $41.467^{**}$ | $28.644^{**}$ | $28.073^{**}$ |
| | (22.207) | (36.714) | (12.745) | (20.577) | (12.729) | (12.714) |
| $gCRGDP_t$ | −93.222 | −118.404 | −55.992 | −69.754 | 18.174 | 10.411 |
| | (184.768) | (184.031) | (106.505) | (105.806) | (93.567) | (85.729) |
| $\Delta r_t$ (%) | −0.333 | −0.099 | −0.227 | −0.114 | −0.249 | −0.122 |
| | (0.848) | (0.836) | (0.490) | (0.478) | (0.445) | (0.403) |
| $\Delta SENT_t$ | −0.001 | 0.001 | −0.002 | −0.00004 | 0.001 | 0.001 |
| | (0.015) | (0.021) | (0.008) | (0.011) | (0.006) | (0.007) |
| $\log(EPU_t)$ | $-1.862^{**}$ | $-2.495^{**}$ | $-1.042^{**}$ | $-1.392^{***}$ | −0.741 | $-0.987^{**}$ |
| | (0.818) | (0.975) | (0.458) | (0.536) | (0.450) | (0.431) |
| Constant | $14.548^{**}$ | $21.129^{***}$ | $8.227^{**}$ | $11.911^{***}$ | $8.325^{**}$ | $11.409^{***}$ |
| | (6.585) | (7.766) | (3.730) | (4.310) | (3.395) | (2.964) |
| Observations | 337 | 337 | 337 | 337 | 337 | 337 |
| Log Likelihood | −142.140 | −135.886 | −141.893 | −135.835 | | |
| Akaike Inf. Crit. | 306.28 | 297.77 | 305.79 | 297.67 | | |
| $R^2$ | 0.160 | 0.197 | 0.162 | 0.197 | 0.221 | 0.262 |
| F Statistic | | | | | $9.218^{***}$ | $9.529^{***}$ |

*Note:* Results of the baseline regressions with heteroscedasticity and autocorrelation consistent (HAC) standard errors, given in parentheses among other regression statistics. Columns (1), (3) and (5) investigate the relationship with $\Delta h_{mt}$ only. Columns (2), (4) and (6) additionally include the magnitude of $h_{mt}$, denoted by magnitude$(h_{mt})$, as well as the interaction term. Significance levels of 1%, 5% and 10% are indicated by $^{***}$, $^{**}$ and $^{*}$, respectively..

This circumstance of adverse herding potentially reflects a shift in investor sentiments, favoring a prevailing perspective or a specific subset of assets. Consequently, this dynamic could lead to pronounced price fluctuations, surpassing the equilibrium threshold, thereby accentuating the dispersion in market returns (Bekiros et al., 2017). This interpretation gains relevance within the sample context, notably during the dot-com bubble when the exaggerated market behavior primarily affected technology firms – an identifiable subgroup within the S&P 500 index – so that the returns on these shares have risen more than in equilibrium. To rigorously validate this assumption, a more nuanced investigation involving sub-periods and industry affiliations becomes imperative.

Thus, the interpretation of the baseline regression initially suggests that adverse herding behavior increases the probability of bubbles occurring, which makes sense given the mechanics of the method, regardless of the level. Therefore, the next question pertains to whether the level of $h_{mt}$ has an impact, meaning whether a high value of $h_{mt}$ exerts a stronger influence. Furthermore, the interaction is of interest, specifically whether decreasing herding behavior in the positive ($h_{mt} > 0$) or negative ($h_{mt} < 0$) range has a more pronounced effect.

Columns (2), (4) and (6) in Table 10 underscore the significance of the negative coefficient $\Delta h_{mt}$. Moreover, the significance of $\Delta h_{mt}$ is robust to the amount of lags $k$ considered in calculating the bubble periods. In terms of odds, if $\Delta h_{mt}$ decreases by 10% points, the odds of a bubble increase by factor 2.462 for the coefficient in column (2) of Table 10. Additionally, the variable magnitude$(h_{mt})$ has been introduced to approximate the magnitude of the herding parameter. This numerical variable is centered around zero and spans from $-1$ to $+1$ in tenths, covering the entire spectrum of $h_{mt}$. When choosing the step size, steps smaller than 0.1 appear overly detailed, whereas steps larger than 0.2 are too coarse. However, the results are robust for step sizes of 0.1 or 0.2. The negative coefficient of magnitude$(h_{mt})$ suggests that positive values of $h_{mt}$ do not contribute to the probability of bubble occurrences; quite the opposite, negative absolute values are again conducive to the emergence of bubbles. The significant interaction term reinforces this interpretation in all specifications, but also reveals complex dynamics. The interaction term $\Delta h_{mt} \times$ magnitude$(h_{mt})$ additionally suggests that the main effect of $\Delta h_{mt}$ also depends on the specific absolute level of the herding parameter. For instance, the effect of $\Delta h_{mt}$ on the probability of bubble occurrence is stronger at very high positive absolute values of magnitude$(h_{mt})$ compared to high negative values. Specifically, the odds of a bubble occurring increase by a factor of 1.030 for a 10%-point reduction in $\Delta h_{mt}$ at a high negative absolute level (magnitude$(h_{mt}) = -6$). At a high positive absolute level (magnitude$(h_{mt}) = +6$), the odds for bubbles increase by a factor of 5.715 for a 10%-point reduction in $\Delta h_{mt}$. For this reason, magnitude$(h_{mt})$ serves as a moderating variable, as the effect of $\Delta h_{mt}$ varies depending on specific values of magnitude$(h_{mt})$. However, on one hand, the decrease in herding behavior (adverse herding) promotes the occurrence of bubbles, and on the other hand, a negative absolute level also contributes to their emergence, as the main effect of magnitude($h_{mt}$) is negative as well. As a result, bubbles do not emerge when investors follow the market but rather when exaggerations occur in certain assets within the overall market. In response to $H_0$, it can be stated that herd behavior towards the market does not contribute to the formation of bubbles. On the other hand, the reversion process and absolute negative or adverse herding increase the probability.

Although the overall approach is methodologically objective, careful econometric choices have to be made, e.g. when the monthly market index is calculated by aggregating daily data. During this aggregation process, the question arises as to which value best represents the monthly observation. Possible options are the closing value and the average value within a month. An analysis of the regression results based on both alternatives has shown the robustness of the results to this arbitrary choice, with no significant effect observed. Detailed results are available upon request.

In addition, the interpretation of the control variables is also of interest. Both volatility and trading volume exhibit a positive and significant influence on the probability of bubble occurrence. These findings align with theoretical models suggesting that bubbles are often accompanied by high trading volumes and occur in periods of elevated volatility (Scheinkman and Xiong, 2003; Mei et al., 2009). The positive coefficient for $gGDP_t$ further confirms that bubbles tend to emerge during economic expansions, consistent with Scherbina and Schlusche (2014). Similarly, the positive coefficient for $gM2_t$ suggests that looser monetary policy contributes to a higher likelihood of bubble occurrences, in line with the findings of Wang and Chen (2019). The present findings on economic uncertainty contrast with those of Enoksen et al. (2020), who studied bubbles in cryptocurrencies. In this context, the negative and significant coefficient implies that bubbles are more likely to occur in economically stable environments.

#### *5.4.4.2 Sub-period Analysis*

As motivated above, the presumption arises that a subset of the S&P 500, such as technology companies during the dot-com bubble, exhibited higher returns than the overall average. Therefore, the next step is to examine whether the herding parameter shows any particularities during the sub-periods. The sample includes three different characteristic periods with exuberant periods. Two of them have a positive bubble, while the period surrounding the GFC is a negative bubble. As described above, the negative sign of the coefficient on $\Delta h_{mt}$ seems quite reasonable, especially in the context of the dot-com bubble, as the technology stocks included in the S&P 500 experienced a significant boom and achieved higher returns than the market average. The sample exhibits a limited number of bubble periods, prompting the need for a careful consideration of sub-periods. To ensure that the unique characteristics of each period are adequately captured, the sub-periods are implemented as dummies and categorized as follows: The first sub-period corresponds to the dot-com period, spanning from 1995 to 2001 according to Brunnermeier and Schnabel (2015). The second sub-period encompasses the house price boom and the GFC, extending from 2003 to 2010, as defined by Brunnermeier and Schnabel (2015). The final period covers the exuberance during the COVID-19 pandemic, from March 2020 to 2022, as the World Health Organization (WHO) declared a global health emergency and classified the outbreak of the virus as a pandemic on 11 March 2020 (World Health Organization, 2023).

Table 11 provides further insight into the sub-periods. In column (1), all three dummies for the sub-periods are included, whereas columns (2) to (4) each include only one sub-period. The interactions of the sub-periods with the herding parameter exhibit ambiguous significances depending on the respective model specification. Only the third sub-period, inclusive of COVID-19, shows a significant difference at least at a 10% significance level. In the context of the dot-com bubble, for example, it could be that individual assets initially benefited from higher returns, but subsequently, the enthusiasm spilled over into the overall market. Anderson et al. (2010) support this conjecture by showing that the dot-com bubble was widespread in the US stock market and affected other sectors besides the technology sector. Hatipoglu and Uyar (2012) provide similar evidence of bubble contagion between the US stock market and emerging markets. However, the conclusion of this analysis is that no clear sub-period-specific characteristics can be identified. It can be inferred that the relationship is not specific to sub-periods but is universally applicable and remains more or less constant. This, on one hand, reinforces the overall relationship identified earlier. On the other hand, this analysis does not provide clear evidence as to whether the assumption that subsets of the overall market with above-average returns are actually driving the relationship, or whether it is indeed a phenomenon that applies to the overall market.

**Table 11:** Logistic regression results for sub-periods.

| | $B_t$ | | | |
|---|---|---|---|---|
| | Combined (1) | Dot-com (2) | GFC (3) | COVID-19 (4) |
| $\Delta h_{mt}$ | $-15.670^{**}$ | $-8.382$ | $-8.487^{***}$ | $-12.025^{***}$ |
| | (7.595) | (7.160) | (3.060) | (2.133) |
| magnitude$(h_{mt})$ | $-0.634^{**}$ | $-0.503^{**}$ | $-0.484^{***}$ | $-0.323^{**}$ |
| | (0.305) | (0.213) | (0.179) | (0.160) |
| $\Delta h_{mt} \times$ magnitude$(h_{mt})$ | $-3.233^{*}$ | $-1.178$ | $-1.369^{**}$ | $-1.730^{***}$ |
| | (1.824) | (0.948) | (0.591) | (0.539) |
| $DC_t$ | $21.057^{***}$ | $4.450^{***}$ | | |
| | (1.346) | (1.247) | | |
| $FC_t$ | $16.409^{***}$ | | $-2.950^{***}$ | |
| | (0.360) | | (1.003) | |
| $CC_t$ | $20.359^{***}$ | | | $3.148^{***}$ |
| | (1.018) | | | (1.076) |
| $\Delta h_{mt} \times DC_t$ | 15.812 | 9.481 | | |
| | (11.936) | (6.292) | | |
| $\Delta h_{mt} \times FC_t$ | $-8.634$ | | $-13.225^{***}$ | |
| | (7.141) | | (4.518) | |
| $\Delta h_{mt} \times CC_t$ | $13.465^{*}$ | | | $7.041^{**}$ |
| | (7.261) | | | (2.947) |
| $\log(pv_t)$ | $2.814^{***}$ | $2.369^{**}$ | $3.302^{***}$ | $3.098^{***}$ |
| | (0.861) | (1.045) | (0.976) | (0.706) |
| $\Delta \log(tv_t)$ | $0.499^{**}$ | $0.494^{***}$ | $0.529^{***}$ | $0.643^{***}$ |
| | (0.231) | (0.182) | (0.106) | (0.214) |
| $gGDP_t$ | 56.855 | $91.202^{*}$ | $97.565^{**}$ | $84.132^{***}$ |
| | (56.975) | (52.286) | (38.704) | (28.506) |
| $gIP_t$ | $-10.749$ | $-18.437$ | $-7.780$ | $-3.616$ |
| | (23.904) | (36.072) | (23.317) | (12.353) |
| $gM2_t$ | 59.630 | 61.322 | $66.790^{**}$ | $67.424^{**}$ |
| | (40.018) | (41.773) | (30.654) | (32.174) |
| $gCRGDP_t$ | $-340.644^{**}$ | $-429.492^{***}$ | $-157.893$ | $-73.565$ |
| | (140.250) | (133.298) | (179.003) | (163.556) |

| | | | | |
|---|---|---|---|---|
| $\Delta r_t$ (%) | 0.036 | 1.575 | −0.562 | −0.936 |
| | (1.196) | (0.984) | (1.094) | (0.934) |
| $\Delta SENT_t$ | −0.024 | −0.018 | −0.012 | −0.003 |
| | (0.042) | (0.024) | (0.030) | (0.025) |
| $\log(EPU_t)$ | −3.174** | −1.134 | −3.531*** | −3.979*** |
| | (1.561) | (1.447) | (1.045) | (1.053) |
| Constant | 6.412 | 11.972 | 29.540*** | 29.830*** |
| | (9.642) | (10.206) | (8.274) | (6.672) |
| Observations | 337 | 337 | 337 | 337 |
| Log Likelihood | −72.150 | −90.297 | −117.058 | −124.407 |
| Akaike Inf. Crit. | 182.3 | 210.59 | 264.12 | 278.81 |
| $R^2$ | 0.574 | 0.467 | 0.308 | 0.265 |

*Note:* Results of the logistic regression with heteroscedasticity and autocorrelation consistent (HAC) standard errors, given in parentheses. Columns (1) to (4) show the results of the sub-period analysis, where (1) includes all sup-period-specific dummies. Columns (1) to (3) each include only one sup-period-specific dummy. $DC_t$, $FC_t$ and $CC_t$ represent the dot-com bubble, the house price boom and subsequent financial crisis as well as the COVID-19 pandemic. Significance levels of 1%, 5% and 10% are indicated by ***, ** and *, respectively.

#### *5.4.4.3 Long-term Analysis*

Given that bubbles are typically not short-term phenomena and instead can develop over an extended period, an investigation into the extent to which herding behavior can be utilized for the long-term prediction of bubbles presents itself as an interesting question. As can be seen in Table 12, the coefficient of $\Delta h_{mt}$ is significant for the next three months as well as the sixth month in the future, indicating that herding behavior predicts the medium- to long-term probability of a bubble occurring. The interaction with the absolute level of herding behavior also points in the same direction as in the baseline regression and is significant even in the fourth month. The perspective on what happens after a bubble has occurred is also interesting in the context of this horizon analysis. For instance, it could be the case that months after a bubble has occurred, herding behavior remains at a low level. As shown in Table 13, adverse herding behavior remains consistently significant six months after a bubble has occurred. This applies to both the rate of change $\Delta h_{mt}$ and the level of herding behavior magnitude$(h_{mt})$. In terms of the level, this means that herding behavior tends to remain medium- to long-term negative after a bubble has occurred.

**Table 12:** Logistic regression results for longer horizons in the future.

| | $B_{t+1}$ | $B_{t+2}$ | $B_{t+3}$ | $B_{t+4}$ | $B_{t+5}$ | $B_{t+6}$ |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| $\Delta h_{mt}$ | $-8.493^{***}$ | $-9.570^{***}$ | $-8.221^{***}$ | $-5.702^{*}$ | $-3.503$ | $-5.944^{**}$ |
| | (2.864) | (2.315) | (2.828) | (3.360) | (2.709) | (3.023) |
| magnitude$(h_{mt})$ | $-0.217$ | $-0.175$ | $-0.152$ | $-0.115$ | $-0.101$ | $-0.132$ |
| | (0.149) | (0.148) | (0.162) | (0.160) | (0.163) | (0.173) |
| $\Delta h_{mt} \times$ magnitude$(h_{mt})$ | $-1.453^{***}$ | $-1.916^{***}$ | $-2.051^{***}$ | $-1.382^{**}$ | $-0.308$ | $-0.751$ |
| | (0.452) | (0.494) | (0.697) | (0.606) | (0.699) | (0.733) |
| $\log(pv_t)$ | $2.100^{**}$ | $1.949^{**}$ | $1.910^{**}$ | $1.700^{*}$ | $1.591^{*}$ | $1.871^{**}$ |
| | (0.876) | (0.860) | (0.974) | (0.954) | (0.903) | (0.916) |
| $\Delta\log(tv_t)$ | $0.358^{***}$ | $0.386^{***}$ | $0.311^{**}$ | 0.195 | 0.256 | $0.371^{***}$ |
| | (0.134) | (0.133) | (0.142) | (0.186) | (0.164) | (0.129) |
| $gGDP_t$ | $91.462^{***}$ | $75.716^{**}$ | $66.405^{*}$ | $70.042^{*}$ | $104.861^{**}$ | $253.989^{***}$ |
| | (33.085) | (33.863) | (40.354) | (38.653) | (49.936) | (74.502) |
| $gIP_t$ | $-3.194$ | 3.494 | 8.136 | 9.681 | 9.814 | $-7.712$ |
| | (17.915) | (16.677) | (14.799) | (19.986) | (21.713) | (26.652) |
| $gM2_t$ | $82.319^{***}$ | $76.507^{***}$ | $64.429^{**}$ | $50.432^{*}$ | $42.206^{*}$ | $61.498^{***}$ |
| | (27.062) | (29.153) | (26.702) | (26.037) | (25.029) | (22.849) |
| $gCRGDP_t$ | $-95.577$ | $-36.953$ | 1.411 | 26.075 | 42.036 | 58.257 |
| | (163.941) | (152.426) | (147.293) | (138.967) | (142.079) | (144.580) |
| $\Delta r_t$ (%) | $-0.175$ | 0.086 | $-0.113$ | $-0.054$ | $-0.050$ | 0.438 |
| | (0.863) | (1.013) | (0.817) | (0.818) | (0.919) | (1.158) |
| $\Delta SENT_t$ | 0.001 | 0.013 | 0.016 | 0.015 | 0.006 | 0.019 |
| | (0.020) | (0.017) | (0.016) | (0.013) | (0.016) | (0.014) |
| $\log(EPU_t)$ | $-2.157^{**}$ | $-1.803^{**}$ | $-1.742^{*}$ | $-1.592$ | $-1.708$ | $-1.832$ |
| | (0.942) | (0.897) | (1.056) | (1.159) | (1.284) | (1.383) |
| Constant | $17.316^{**}$ | $15.037^{**}$ | $14.640^{*}$ | 13.084 | 13.088 | 14.370 |
| | (7.449) | (7.041) | (8.227) | (8.539) | (8.849) | (9.288) |
| Observations | 338 | 338 | 338 | 337 | 336 | 336 |
| Log Likelihood | $-142.331$ | $-145.066$ | $-146.507$ | $-149.712$ | $-148.191$ | $-137.016$ |
| Akaike Inf. Crit. | 310.66 | 316.13 | 319.01 | 325.42 | 322.38 | 300.03 |
| $R^2$ | 0.160 | 0.144 | 0.136 | 0.115 | 0.123 | 0.189 |

*Note:* Results of the logistic regression with heteroscedasticity and autocorrelation consistent (HAC) standard errors, given in parentheses. Columns (1) through (6) respectively indicate the results for predictive capability when the bubble occurs in one to six months. Significance levels of 1%, 5% and 10% are indicated by $^{***}$, $^{**}$ and $^{*}$, respectively.

**Table 13:** Logistic regression results for bubble persistence.

| | $B_{t-1}$ | $B_{t-2}$ | $B_{t-3}$ | $B_{t-4}$ | $B_{t-5}$ | $B_{t-6}$ |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| $\Delta h_{mt}$ | $-7.036^{**}$ | $-10.490^{***}$ | $-10.609^{**}$ | $-8.830^{**}$ | $-6.784^{*}$ | $-7.290^{*}$ |
| | (3.046) | (3.260) | (4.584) | (3.616) | (3.686) | (3.741) |
| magnitude$(h_{mt})$ | $-0.310^{**}$ | $-0.380^{**}$ | $-0.399^{**}$ | $-0.423^{***}$ | $-0.420^{***}$ | $-0.433^{***}$ |
| | (0.148) | (0.152) | (0.156) | (0.153) | (0.153) | (0.161) |
| $\Delta h_{mt} \times$ magnitude$(h_{mt})$ | −0.763 | $-1.451^{**}$ | $-1.346^{**}$ | $-1.027^{*}$ | −1.102 | −0.720 |
| | (0.792) | (0.702) | (0.626) | (0.556) | (0.732) | (0.538) |
| $\log(pv_t)$ | $2.664^{***}$ | $3.085^{***}$ | $3.001^{***}$ | $2.992^{***}$ | $2.989^{***}$ | $3.125^{***}$ |
| | (0.831) | (0.913) | (1.037) | (0.908) | (0.923) | (0.999) |
| $\Delta\log(tv_t)$ | $0.317^{***}$ | $0.390^{*}$ | $0.438^{***}$ | 0.058 | −0.003 | −0.075 |
| | (0.107) | (0.209) | (0.153) | (0.107) | (0.134) | (0.099) |
| $gGDP_t$ | $89.715^{***}$ | $90.412^{**}$ | $86.960^{**}$ | 61.567 | 55.144 | 57.822 |
| | (29.927) | (38.882) | (42.589) | (40.266) | (36.834) | (37.640) |
| $gIP_t$ | 9.799 | 15.239 | 13.447 | $27.082^{*}$ | 28.264 | 31.389 |
| | (27.280) | (21.815) | (18.244) | (15.754) | (21.352) | (25.677) |
| $gM2_t$ | $68.001^{**}$ | $84.081^{***}$ | 37.048 | −21.089 | 4.884 | −7.812 |
| | (27.675) | (24.530) | (23.469) | (23.219) | (16.435) | (17.369) |
| $gCRGDP_t$ | −105.902 | −73.758 | −25.879 | 25.845 | 40.684 | 36.124 |
| | (185.058) | (208.843) | (192.254) | (181.628) | (177.060) | (175.568) |
| $\Delta r_t$ (%) | −0.129 | 0.878 | 0.818 | 0.628 | 0.849 | 0.802 |
| | (0.653) | (1.044) | (0.794) | (0.777) | (0.755) | (0.783) |
| $\Delta SENT_t$ | 0.013 | 0.037 | −0.005 | −0.012 | −0.001 | 0.006 |
| | (0.020) | (0.027) | (0.028) | (0.027) | (0.031) | (0.035) |
| $\log(EPU_t)$ | $-2.534^{***}$ | $-2.615^{***}$ | $-2.444^{***}$ | $-2.115^{**}$ | $-2.062^{**}$ | $-1.962^{**}$ |
| | (0.850) | (0.821) | (0.913) | (0.895) | (0.892) | (0.917) |
| Constant | $21.566^{***}$ | $23.607^{***}$ | $22.676^{***}$ | $21.377^{***}$ | $21.024^{***}$ | $21.223^{***}$ |
| | (6.829) | (6.748) | (7.905) | (7.218) | (7.123) | (7.584) |
| Observations | 336 | 335 | 334 | 333 | 332 | 331 |
| Log Likelihood | −136.270 | −129.309 | −133.023 | −135.262 | −135.923 | −135.526 |
| Akaike Inf. Crit. | 298.54 | 284.62 | 292.05 | 296.52 | 297.85 | 297.05 |
| $R^2$ | 0.194 | 0.234 | 0.211 | 0.196 | 0.198 | 0.205 |

*Note:* Results of the logistic regression with heteroscedasticity and autocorrelation consistent (HAC) standard errors, given in parentheses. Columns (1) through (6) respectively indicate the results for the persistence of herding behavior when a bubble has occurred one to six months earlier. Significance levels of 1%, 5% and 10% are indicated by $^{***}$, $^{**}$ and $^{*}$, respectively.

#### *5.4.4.4 Industry Analysis*

Herding behavior also varies across sectors and industries, as various studies have shown. Consequently, the relationship between bubbles and herding behavior may also vary significantly within different sectors. Analogous to the findings of Voronkova and Bohl (2005), it is plausible to assume that this relationship is more pronounced in certain sectors, e.g. those characterized by a higher degree of uncertainty about earnings and future cash flows. Again, existing studies are based on different methodologies. Therefore, a brief reference to the methods mentioned in Section 5.3.2.3 will be made at this point. Through a micro level transaction-based analysis of the Polish market, Voronkova and Bohl (2005) showed that herding behavior is more pronounced in sectors such as computer services, banking and metal production. Similarly, Zhou and Lai (2009) employed a transaction-based herding measure and discovered that herding behavior is less prevalent in sectors such as industrial goods, information technology, properties and construction, and utilities. Demirer et al. (2010) apply macro level market-wide herding measures by Chang et al. (2000) and Hwang and Salmon (2004) and find herding in all investigated sectors. Gavriilidis et al. (2013) conducted an investigation into transaction-based institutional herding behavior. They found that Spanish fund managers engaged in significant herding in industries like consumer services and industrials. Conversely, they found less evidence of herding in industries such as basic materials, financials and consumer goods. Gębka and Wohar (2013) employed market-wide herding measures and discovered herding behavior in sector-specific indices across countries. Notably, they found evidence of herding in sectors such as basic materials, consumer services, and oil and gas. Surprisingly, the patterns of herding were less pronounced in industries with high information asymmetries like information technology and financials. This finding aligns with the results from Zhou and Lai (2009). Celiker et al. (2015) also identified institutional investors engaging in herding within specific industries. However, they argued that this behavior does not significantly deviate industry valuations from their fundamental values.

To examine whether there are sector-specific differences in the relationship between bubbles and herding behavior, it is necessary to construct sector-specific sub-indices of the S&P 500. The sectoral classification is based on the first two digits of the SIC industry classification, as shown in Table 14. Initially, it was suspected that during the dot-com bubble, higher returns were associated with a subset of technology companies in the S&P 500. As no distinct period-specific characteristics could be identified, industry analysis might provide more insights. The industry analysis is therefore extended to include a high-tech (HT) category. Following the approach of Ljungqvist and Wilhelm (2003) and Loughran and Ritter (2004), this category is composed of companies with SIC codes 3571, 3572, 3575, 3577, 3578 (computer hardware), 3661, 3663, 3669 (communication equipment), 3674 (electronics), 3812 (navigational equipment), 3823, 3825, 3826, 3827, 3829 (measuring and control equipment), 4899 (communication services) and 7370, 7371, 7372, 7373, 7374, 7375, 7378 and 7379 (software).

**Table 14:** General SIC code classifications based on first two digits.

| Division | Title | Major Group |
|---|---|---|
| A | Agriculture, Forestry, Fishing | 01-09 |
| **B** | **Mining** | **10-14** |
| C | Construction | 15-17 |
| **D** | **Manufacturing** | **20-39** |
| **E** | **Transportation & Public Utilities** | **40-49** |
| F | Wholesale Trade | 50-51 |
| **G** | **Retail Trade** | **52-59** |
| **H** | **Finance, Insurance, Real Estate** | **60-67** |
| **I** | **Services** | **70-89** |
| J | Public Administration | 91-99 |

The herding parameters themselves for industries Mining, Transportation & Public Utilities and Finance, Insurance and Real Estate are significant at the 1% level, for the industries High-Tech, Manufacturing, Retail Trade and Services at the 5% level, and insignificant for the remaining industries. The examination of the constructive relationship between bubbles and herding behavior is conducted solely for those industries in which the herding parameter is significant. The corresponding industries are marked in bold in Table 14. The results are presented in Table 15.

The analysis of industries reveals that the relationship between bubbles and herding behavior is present in all industries, with varying magnitudes and significance levels in the growth rate, level or interaction of both of the parameters affiliated with $h_{mt}$. The results are, therefore, mixed and not straightforward to interpret. Nevertheless, based on the (inverted) odds ratios and average marginal effects, it can be highlighted that the association is most pronounced in the Services industry when considering $\Delta h_{mt}$. The strongest effect in terms of magnitude($h_{mt}$) is observed in the sector Finance, Insurance and Real Estate, confirming previous findings. Once again, it is noteworthy that concerning the high-tech industry, only the interaction effect is significant, and the main parameters are not. This is consistent with the finding that no particular specifics were identified for the dot-com period compared to the baseline. This suggests that spill-over effects ensured that not only individual assets but the entire market was affected during the dot-com bubble. However, due to the varying significances regarding the herding-related parameters, it can be concluded that no clear systematic sector-specific differences were found, strengthening the significance of the relationship across the baseline.

**Table 15:** Logistic regression results by industry.

| | $B_t^{\mathrm{HT}}$ | $B_t^{\mathrm{B}}$ | $B_t^{\mathrm{D}}$ | $B_t^{\mathrm{E}}$ | $B_t^{\mathrm{G}}$ | $B_t^{\mathrm{H}}$ | $B_t^{\mathrm{I}}$ |
|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| $\Delta h_{mt}$ | −3.836 | −3.908$^{***}$ | −5.205$^{*}$ | −8.938$^{***}$ | −7.525$^{**}$ | 83.817 | −14.521$^{*}$ |
| | (3.287) | (1.090) | (2.944) | (2.359) | (3.742) | (113.196) | (7.448) |
| magnitude($h_{mt}$) | −0.064 | −0.108 | 0.107 | −0.112 | −0.274 | −1.639$^{***}$ | −0.050 |
| | (0.229) | (0.151) | (0.419) | (0.123) | (0.231) | (0.506) | (0.146) |
| $\Delta h_{mt} \times$ magnitude($h_{mt}$) | −1.739$^{**}$ | −0.502$^{**}$ | −0.155 | −0.346 | 1.093 | −15.911 | −1.553 |
| | (0.848) | (0.236) | (1.264) | (0.676) | (1.642) | (17.576) | (1.514) |
| $\log(pv_t)$ | 1.906$^{**}$ | 0.706 | 2.609$^{***}$ | 1.715$^{***}$ | 3.183$^{***}$ | 2.560$^{***}$ | 1.697$^{*}$ |
| | (0.772) | (0.909) | (0.755) | (0.479) | (0.845) | (0.490) | (0.910) |
| $\Delta \log(tv_t)$ | 0.175 | 0.477$^{***}$ | 0.370$^{**}$ | 0.343 | 0.252 | 0.276 | 0.280$^{**}$ |
| | (0.165) | (0.164) | (0.160) | (0.479) | (0.211) | (0.316) | (0.116) |
| $gGDP_t$ | 205.872$^{***}$ | 30.441 | 98.442$^{***}$ | 100.404$^{***}$ | 155.054$^{***}$ | −42.245 | 240.185$^{***}$ |
| | (76.276) | (49.701) | (35.152) | (20.753) | (40.417) | (68.077) | (78.944) |
| $gIP_t$ | 40.132 | 47.505 | 26.983 | −0.125 | 15.695 | 34.319 | 18.139 |
| | (40.182) | (35.087) | (21.031) | (18.390) | (29.272) | (26.859) | (28.638) |
| $gM2_t$ | 31.676 | −1.063 | 76.059$^{***}$ | 51.903$^{*}$ | 68.712$^{***}$ | 48.073$^{*}$ | 17.645 |
| | (28.538) | (32.589) | (27.254) | (31.415) | (18.888) | (25.733) | (31.117) |
| $gCRGDP_t$ | −34.162 | 152.443 | −181.547 | 503.651$^{***}$ | 126.383 | 129.944 | −36.326 |
| | (125.589) | (196.283) | (162.856) | (104.865) | (116.677) | (180.641) | (119.419) |
| $\Delta r_t$ (%) | 0.290 | 0.471 | 0.003 | 1.067 | −1.196 | 0.165 | −0.059 |
| | (0.971) | (2.073) | (0.889) | (1.924) | (1.220) | (1.213) | (0.814) |
| $\Delta SENT_t$ | 0.014 | 0.004 | 0.002 | 0.033 | 0.044 | −0.039 | 0.005 |
| | (0.015) | (0.025) | (0.020) | (0.049) | (0.029) | (0.059) | (0.015) |
| $\log(EPU_t)$ | −1.230 | −3.663$^{***}$ | −3.285$^{***}$ | −2.030$^{***}$ | 0.018 | −3.234$^{***}$ | −1.182 |
| | (1.030) | (0.988) | (0.925) | (0.600) | (0.888) | (0.770) | (1.105) |
| Constant | 12.149$^{*}$ | 16.958$^{***}$ | 25.423$^{***}$ | 11.619$^{***}$ | 10.864$^{*}$ | 33.469$^{***}$ | 11.340 |
| | (6.594) | (6.529) | (5.628) | (3.549) | (6.581) | (7.665) | (8.328) |
| Observations | 337 | 337 | 337 | 337 | 337 | 337 | 337 |
| Log Likelihood | −147.284 | −85.064 | −127.326 | −30.579 | −87.672 | −48.656 | −168.184 |
| Akaike Inf. Crit. | 320.57 | 196.13 | 280.65 | 87.159 | 201.34 | 123.31 | 362.37 |
| $R^2$ | 0.201 | 0.242 | 0.271 | 0.191 | 0.308 | 0.277 | 0.147 |

*Note:* Results of the logistic regression with heteroscedasticity and autocorrelation consistent (HAC) standard errors, given in parentheses. The columns (1) through (7) display results for the High-Tech (HT) industry and other industries as listed in Table 14, each corresponding to the superscripts of the bubble dummies. Significance levels of 1%, 5% and 10% are indicated by $^{***}$, $^{**}$ and $^{*}$, respectively.

#### 5.4.4.5 *Company Size Analysis*

Another recurring theme in the literature is the influence of firm size on the intensity of herding behavior. Therefore, in this research context, it is explored whether a company's size influences the relationship between bubbles and herding behavior. The underlying theory suggests that due to higher information risks associated with small-capitalization stocks, observing the trading behavior of other investors can be informative. Hence, herding behavior might be more pronounced in smaller companies (Ferreruela et al., 2022).

For instance, Wermers (1999) examines the trading behavior of institutional investors based on transaction data and found that mutual funds tend to exhibit more herding behavior in small-capitalized stocks. Sias (2004), Wylie (2005) and Hung et al. (2010) also find that institutional investors are more likely to herd in smaller capitalization securities. Chang et al. (2000) conduct market capitalization-based portfolio tests to measure herding behavior and find that it is present for all sizes of companies, not just large or small capitalization stocks. Using a market-wide herding measure, Benkraiem et al. (2021) find that herding behavior is more prevalent among small- and medium-sized companies in normal times than in crisis periods. This phenomenon is attributed to the assumption that during crises all investors face the same information problems and are less likely to engage in herding behavior because it is assumed that no one has superior information.

To examine the relationship of interest among firms of different sizes, it is essential to divide the overall index into indices consisting of large and small companies. The size of the companies is proxied by market capitalization. The large-cap index comprises companies from the S&P 500 whose market capitalization falls within the upper quartile (75th percentile). Similarly, the small-cap index includes companies whose market capitalization falls within the lower quartile (25th percentile). Both size-based indices are re-weighted annually. This approach has been used in a similar context by Jacobs et al. (2010).

The herding parameters themselves for both large and small companies are significant at the 1% level in both cases, as seen in Table 9 for the overall index. Table 16 shows the results of the analysis of company size in the context of the relationship between herding behavior and bubbles. The coefficients in column (4) pertaining to small companies, while considering the absolute magnitude of $h_{mt}$, are notably more significant than the coefficients for large enterprises in column (2). The average marginal effects and the marginal effects at the mean for the coefficients associated with herding behavior are absolutely greater (larger negative magnitude) for large companies compared to small ones. They imply that the probability of bubbles occurring due to adverse herding behavior is higher for large companies than for small companies. The same conclusions can be drawn from a comparison with the directly interpretable coefficients of a linear regression. The marginal effects and linear regression

**Table 16:** Logistic regression results by company size.

| | $B_t^{\text{Large}}$ | | $B_t^{\text{Small}}$ | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| $\Delta h_{mt}$ | $-6.047^{**}$ | $-5.560^{*}$ | $-2.701^{**}$ | $-4.539^{***}$ |
| | (2.409) | (2.910) | (1.172) | (1.429) |
| magnitude$(h_{mt})$ | | $-0.361^{*}$ | | $-0.367^{***}$ |
| | | (0.190) | | (0.099) |
| $\Delta h_{mt} \times$ magnitude$(h_{mt})$ | | $-0.383$ | | $-0.415^{**}$ |
| | | (0.815) | | (0.202) |
| $\log(pv_t)$ | $1.860^{**}$ | $2.435^{***}$ | 0.340 | $2.174^{**}$ |
| | (0.809) | (0.876) | (0.742) | (0.917) |
| $\Delta \log(tv_t)$ | $0.426^{**}$ | $0.356^{*}$ | 0.101 | 0.279 |
| | (0.179) | (0.190) | (0.536) | (0.591) |
| $gGDP_t$ | $94.008^{**}$ | $103.343^{**}$ | $-78.551^{*}$ | $-94.532^{**}$ |
| | (43.713) | (51.986) | (41.051) | (42.495) |
| $gIP_t$ | 7.533 | 11.377 | 4.493 | 0.780 |
| | (20.097) | (20.432) | (26.710) | (18.451) |
| $gM2_t$ | $72.240^{**}$ | $74.529^{**}$ | $-16.890$ | $-26.404$ |
| | (28.405) | (33.166) | (32.179) | (45.604) |
| $gCRGDP_t$ | $-79.793$ | $-38.761$ | $-216.580$ | $-263.124^{**}$ |
| | (119.037) | (121.912) | (151.823) | (125.129) |
| $\Delta r_t$ (%) | $-0.088$ | 0.105 | $-1.709^{***}$ | $-1.220$ |
| | (0.961) | (0.926) | (0.638) | (0.830) |
| $\Delta SENT_t$ | 0.013 | 0.006 | $-0.078^{**}$ | $-0.085^{*}$ |
| | (0.014) | (0.020) | (0.034) | (0.046) |
| $\log(EPU_t)$ | $-1.694$ | $-1.905$ | $-1.595^{*}$ | $-3.116^{***}$ |
| | (1.258) | (1.295) | (0.896) | (0.757) |
| Constant | $14.680^{*}$ | $18.036^{**}$ | $6.453^{*}$ | $20.512^{***}$ |
| | (8.604) | (9.184) | (3.482) | (4.600) |
| Observations | 337 | 337 | 337 | 337 |
| Log Likelihood | $-155.840$ | $-149.230$ | $-74.935$ | $-64.115$ |
| Akaike Inf. Crit. | 333.68 | 324.46 | 171.87 | 154.23 |
| $R^2$ | 0.170 | 0.206 | 0.107 | 0.236 |

*Note:* Results of the logistic regression with heteroscedasticity and autocorrelation consistent (HAC) standard errors, given in parentheses. Columns (1) and (2) represent the large companies and are proxied by the upper quartile of market capitalization, re-weighted annually. Columns (3) and (4) represent the small companies and are proxied by the lower quartile of market capitalization, re-weighted annually. The superscripts of the bubble dummies indicate the company size indices. Significance levels of 1%, 5% and 10% are indicated by $^{***}$, $^{**}$ and $^{*}$, respectively.

results are available on request. A consideration of inverted odds of all parameters associated with herding behavior leads to the same conclusion. For example, the odds of a bubble occurring increases by a factor of 1.744 if $\Delta h_{mt}$ falls by 10% points for large firms, while the analogous change in odds for small firms is only 1.574, based on columns (2) and (4) in Table 16. Only the inverse odds for magnitude$(h_{mt})$ are slightly higher for small firms. However, when comparing the confidence intervals of the coefficients, the differences are not significant. Interpretatively, they are so close that the effect of the degree of adverse herding on the occurrence of bubbles is roughly the same for large and small firms. The results, therefore, suggest that both large and small companies can contribute to the emergence of bubbles through adverse herding behavior, even though it is surprising that the relationship is not more pronounced for small companies. Conversely, there are also studies demonstrating investor herding behavior towards larger stocks (Wylie, 2005; Kremer and Nautz, 2013). An explanation for this phenomenon is offered by Walter and Moritz Weber (2006), who argue that institutional investors have incentives to mirror changes in the composition of a typically blue-chip benchmark index within their portfolios to minimize tracking error. They refer to this type of herding behavior as "benchmark herding".

### *5.5. Robustness Analysis*

Given that different methods of measuring herding behavior, as outlined in Section 5.3, can produce divergent or inconclusive results, the literature is followed and another widely used measure is implemented at this point. The method proposed by Chang et al. (2000) has already been mentioned in Section 5.3.2.3 and in the context of other empirical studies. Its main advantage is that no data points are lost in the estimation of monthly factors.

Chang et al. (2000) generalise the approach of Christie and Huang (1995) and also build on the CAPM, so that linearity between the individual stock returns and the market return is assumed. However, if the relationship is non-linear and the individual stock returns converge to the market, this is interpreted as evidence for the existence of herding behavior. The model is defined as

$$CSAD_t = \alpha + \gamma_1 |R_{mt}| + \gamma_2 R_{mt}^2 + \epsilon_t, \tag{72}$$

where $R_{mt}$ is the return of the market index at time $t$. The dispersion of returns is proxied by the cross-sectional absolute deviation of returns (CSAD) and is defined as:

$$CSAD_t = \frac{1}{N} \sum_{i=1}^{N} |R_{it} - R_{mt}|, \tag{73}$$

where $R_{it}$ are the individual stock returns at time $t$ and $N$ is the number of assets at time $t$. The non-linearity is represented by the coefficient $\gamma_2$. If the coefficient is negative and significant, then there is herding behavior towards the market, i.e. the CSAD is lower than a rational asset-pricing model would imply. If the coefficient is positive and significant, this indicates adverse herding behavior, which means that the CSAD has risen above the rationally expected level (Gębka and Wohar, 2013; Klein, 2013; Bekiros et al., 2017). In addition, as in Haykir and Yagli (2022), the bubble dummy $B_t$ is included in the model in Equation (72) to examine the relationship between herding behavior and bubbles:

$$CSAD_t = \alpha + \gamma_1 |R_{mt}| + \gamma_2 R_{mt}^2 + \gamma_3 B_t + \gamma_4 R_{mt}^2 \times B_t + \epsilon_t. \tag{74}$$

The results of the robustness check are shown in Table 17. The significant and positive coefficient for $R_{mt}^2$ suggests adverse herding behavior in the sample. This initially confirms the finding in the main part that herding behavior exists in the sample. The positive coefficient for $B_t$ indicates that the CSAD are higher during bubbles. In other words, this suggests that higher returns occur during bubbles, supporting the main finding that subsets of the market benefit from adverse herding behavior. The significant interaction term also suggests that a different regime applies during bubbles compared to when no bubble exists. Due to the negative sign, there is less adverse herding behavior during a bubble.

**Table 17:** Results of the robustness analysis.

| Model | Constant | $\|R_{mt}\|$ | $R_{mt}^2$ | $B_t$ | $R_{mt}^2 \times B_t$ | Obs. | $R^2$ |
|---|---|---|---|---|---|---|---|
| (1) | 0.009*** | 0.467*** | 2.225*** | | | 8574 | 0.635 |
| | (0.0002) | (0.028) | (0.589) | | | | |
| (2) | 0.009*** | 0.432*** | 2.905*** | 0.002*** | −3.023*** | 8319 | 0.629 |
| | (0.0002) | (0.024) | (0.498) | (0.0005) | (0.477) | | |

*Note:* Results of the linear regression with heteroscedasticity and autocorrelation consistent (HAC) standard errors, given in parentheses. Row (1) and (2) represent the models from Equations (72) and (74) respectively. Significance levels of 1%, 5% and 10% are indicated by ***, ** and *, respectively.

### *5.6. Conclusion*

The aim was to empirically investigate the relationship between speculative bubbles and herding behavior. To achieve this, the two variables had to be measured separately at first. To this end, a number of different approaches were presented, from which the methods ultimately used were selected. Using the method proposed by Hwang and Salmon (2004), based on the cross-sectional standard deviation of betas, herding behavior was identified in the American stock market for the period from 1990 to 2022. By employing the recursive unit root test by Phillips et al. (2015), three bubble periods were identified within this sample. The relationship between the two variables was then examined in different specifications of mainly binary regression models.

Contrary to the initial assumption that herding behavior towards the market favors the emergence of speculative bubbles, the results suggest the opposite. In this respect, the results would confirm the findings of Haykir and Yagli (2022), who found that herding decreases during bubbles in cryptocurrencies. However, the results of this paper go further, as the adverse process of herding behavior increases the probability of bubbles to occur. Looking at the mechanism of the herding approach used when adverse herding behavior occurs, it is found that the cross-sectional standard deviation of the factor sensitivities exceeds the standard deviation in the CAPM equilibrium. This implies that bubbles are more likely to occur when subsets of the overall market systematically generate higher returns than would be rationally expected. The negative influence of herding behavior is evident in both the rate of change and the absolute level. In addition, the interaction effect between the two variables indicates that the rate of change is moderated by the absolute level, i.e. the effect of the rate of change varies in strength at different absolute levels. The examination of the sub-hypotheses revealed that the relationship holds regardless of industry, company size, sub-periods and different time horizons, as no clear differences were found. Although isolated differences can be identified, such as slightly larger absolute coefficients for larger companies, the results of these sub-hypotheses can be interpreted as confirming the existence of the general relationship.

However, the study also needs to be critically reflected upon. As described in Section 5.3.2.2, the empirical evidence on herding behavior in the American market is inconclusive, which is also related to the diversity of methods. A robustness check was conducted to address the issue that the relationship holds regardless of the method. The alternative method confirms the results of the main analysis, showing that adverse herding behavior continues to exist in the US market. Additionally, cross-sectional absolute deviations are higher during bubbles. Nevertheless, investigating the relationship with additional methods could be a possible subject for further research. However, the empirical approach chosen here is characterized by methodological objectivity, as the methods themselves do not require arbitrary calibrations.

*5.7. Declaration of (Co-)Authors and Record of Accomplishments*

**Title:** Beyond the Individual: Investigating the Interdependence of Speculative Bubbles and Herding in Financial Markets

**Author(s):** John H. Stiebel (Heinrich Heine University Düsseldorf)

**Conferences:** Participation and presentation at 'Forschungskolloquium Finanzmärkte', May 3rd, 2023, Düsseldorf, Germany

Participation and presentation at 'HVB Doctoral Colloquium', February 2nd, 2024, hosted by University Münster, Germany

**Publications:** SSRN published. Submitted to *Cogent Economics & Finance*, a single-blind peer-reviewed journal. Current status: Revise & resubmit.

**Share of contributions:**

| **Contributions** | **John H. Stiebel** |
|---|---|
| **Research design** | 100% |
| *Development of research question* | 100% |
| *Method development and specification* | 100% |
| **Research performance & analysis** | 100% |
| *Literature review and framework development* | 100% |
| *Data collection, preparation and analysis* | 100% |
| *Analysis and discussion of results* | 100% |
| *Derivation of implications and conclusions* | 100% |
| **Manuscript preparation** | 100% |
| *Final draft* | 100% |
| *Finalization* | 100% |
| **Overall contribution** | 100% |

Date, John H. Stiebel

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**Statutory declaration**

I, John Henrik Stiebel, swear that I am writing this dissertation independently and without inadmissible outside help, taking into account the ‘principles for ensuring good scientific practice at the Heinrich Heine University Düsseldorf’.

Düsseldorf, November 6, 2024

John H. Stiebel