Three Essays on Theoretical Industrial Organization

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To my daughter

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Chapter 1

Introduction

Competition has attracted increasing attention from economists, markets and governments. Because competition is an important factor affecting market efficiency and social welfare. Horizontal mergers and vertical integrations will affect markets' competition and efficiency, making mergers a focus of government attention. In many fields, such as telecommunications, technology, the Internet, and other industries, horizontal mergers have attracted more government attention because mergers between large companies will risk monopolizing the market, which is not conducive to fair market competition and will harm consumer surplus.

Chapter 2 discusses how the merging firms' before-merger market shares are related to after-merger social welfare. Pre-merger market shares serve as important indications for the likely competitive effects of merger control. Many studies have addressed the social welfare effects of price-increasing mergers. We complement the seminal work of Farrell and Shapiro (1990) by showing that there are certain types of mergers, namely runner-up mergers, which might result in negative social welfare effects despite being procompetitive. Starting with Williamson (1968), many studies have assumed a monotone relation between merger efficiencies and after-merger social welfare (Besanko and Spulber, 1993; Lagerlöf and Heidhues, 2005; Amir et al., 2009). Our results show that such a monotone relation cannot be expected necessarily in the case of runner-up mergers.

We derive necessary (sufficient) conditions for social-welfare reducing (increasing) mergers, which are consumer-surplus increasing. Our analysis singles out runner-up mergers as the only candidates for such "procompetitive" but social-welfare reducing mergers. A procompetitive runner-up merger that is social welfare decreasing is characterized by two features: first, the merging firms must have a below-average joint market share and second, the merger reduces concentration as measured by the Herfindahl-Hirschman index. Such a constellation is most likely when two relatively small firms merge in a concentrated market and realize some -but limited- merger efficiencies.

Chapter 3 takes the views onto platform competition. Nowadays, more and more research has been focused on the digital market. Several reports on the digital market, such as Crémer et al. (2020) and Scott Morton et al. (2019) have been recently released. With the development of the Internet and technology, the platform economy has become one of the main industries competing in the market. The media market in platform economics has garnered significant interest. The studies on newspaper industries (Gabszewicz et al., 2002), TV markets (Peitz and Valleti, 2008; D'Annunzio, 2017; Calvano and Polo, 2020) and superstar platforms (Carroni et al., 2023) motivate the idea of this chapter.

Inspired by the Chinese video platform market, in Chapter 3 we study the effect of vertical integration on competition between platforms when consumers can only single home. The video platform market is highly concentrated in China. Video platforms produce their own original series by vertically integrating with upstream content producers to both expand content and limit costs. However, those platforms have hiked membership fees annually, while losing money. In order to examine the price and profit discrepancies between China's video platforms before and after vertical integration, we develop a theoretical model based on the current state of these platforms. We show that platforms have an incentive to choose vertical integration but cause a prisoner's dilemma.

In two-sided market analysis, network externalities play an important role. This draws our attention back to seminal studies from the 1980s about network effects. Chapter 4 examines how market structure (as measured by the number of firms) affects market outcomes when consumption exhibits positive network effects. Building on the set-up of Katz and Shapiro (1985), we show that market efficiency always increases with more competition when products are compatible. Firms' profits can increase with the number of firms when marginal network effects are strong. When products are incompatible, market efficiency may increase or decrease depending on the shape of the network externalities function. In particular, when network effects are strong at the margin, increasing the number of competitors reduces total output. Firms' profits always decrease when the number of competitors increases.

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Chapter 2

On the Social Welfare Effects of Runner-up Mergers in Concentrated Markets

with Dragan Jovanovic and Christian Wey

2.1 Introduction

Pre-merger market shares serve as important indications for the likely competitive effects in merger control.[1] For instance, the US Horizontal Merger Guidelines (DOJ/FTC, 2010) and the EC Merger Regulation (EC, 2004) prescribe a more restrictive approach towards a merger the larger the market shares of the merger candidates. Similarly, the dominance criterion (which is used in Germany and in the EU) says that a firm holding a dominant market position[2] should not be allowed to merge with another firm within the same market.[3]

In this paper, we analyze how the merging firms' before-merger market shares are related to after-merger social welfare. Our focus is on mergers that are expected to raise consumer welfare, so that an antitrust authority applying a consumer-surplus standard should approve those merger proposals. We show that when a merger is in this sense "procompetitive", then mergers of relatively small firms are the only candidates for social-welfare reducing mergers. In contrast, mergers of relatively large firms (and, in particular, mergers involving a dominant firm) are always socially desirable. The reason is that social welfare is non-monotone in the merger efficiency level in case of mergers of relatively small firms. In contrast, social welfare is monotonically increasing when the merging firms are relatively large (or even dominant), *assuming* that the merger is consumer-surplus increasing. Because of the non-monotonicity of social welfare

[1] However, market shares as well as concentration levels do not serve as definite heuristics or sufficient conditions for approving or blocking mergers. They rather give rise to rebuttable presumptions which necessitate further analyses/evidence and cannot be used as a stand-alone rule (see DOJ/FTC, 2010, p. 3 and pp. 16-19; EU Guidelines, 2004, Section III).

[2] The OECD "Glossary of Industrial Organization Economics and Competition Law" (online available: https://www.oecd.org/regreform/sectors/2376087.pdf) states: "A dominant firm is one which accounts for a significant share of a given market and has a significantly larger market share than its next largest rival. Dominant firms are typically considered to have market shares of 40 percent or more."

[3] In the EU and Germany mergers which significantly impede effective competition, and in particular, create or strengthen a dominant position are prohibited (see ECMR Art. 2 and GWB §36, para. 1). Thus, for both Germany and the EU "creation and strengthening of dominance" constitutes a particular scenario which places higher burden on the merging parties and makes the competition authority stepping in and blocking the merger more likely.

in case of small-firms mergers, a higher level of merger efficiencies can result in negative social-welfare effects despite being beneficial to consumers.

The intuition for this result follows from the general insight that relatively small firms must have relatively high marginal production costs in an oligopolistic market equilibrium. Thus, the very existence of small firms is associated with a productive inefficiency as it would be desirable that more efficient firms take over smaller firms' market shares. If, therefore, a merger of relatively small firms leads to efficiencies (i.e., reduces the merged firm's marginal cost), then this productive inefficiency can become even larger as the merged firm takes over market shares from its (still) more efficient rivals.

Our analysis complements the seminal work of Farrell and Shapiro (1990), which examines the social welfare effects of price-increasing (or, "anticompetitive") mergers. While consumers are always harmed by price increases, outsider firms' profits can go up or down depending on the merging firms' joint market share.[4] The total external effect is unambiguously negative when the joint market share of the merging firms is sufficiently large (for instance, above 50% in a linear Cournot oligopoly model; see also Levin, 1990). The reason is that a quantity-reducing merger—which definitely harms consumers—can only be socially desirable if productive efficiency increases; that is if more efficient outsider firms expand their output after the merger. Such an output expansion can only occur if the merged firm is less efficient than at least some other outsider firms. As more efficient firms always have larger market shares than less efficient ones, the merging firms' joint market share, therefore, must not be too large in order to generate a social welfare increase.

The Farrell-Shapiro analysis provides a useful argument for a market-share based screening of anticompetitive mergers. This insight is mirrored in merger control regulations all over the world, which take an increasingly hostile stance

[4]A suppressed aspect of the Farrell-Shapiro analysis is the scource of the *assumed* profitability of the merger, which allows the authors to focus on the merger's external effect to obtain their (sufficient) condition for a social-welfare increasing merger.

on mergers the larger the merging firms' joint market share becomes. Notably, the Farrell-Shapiro analysis applies to "anticompetitive" (i.e., consumer-surplus reducing) mergers, while it somehow appears to suggest that "procompetitive" (i.e., consumer-surplus increasing) mergers are always socially desirable.

This paper challenges this presumption by showing that there are certain types of mergers, namely *runner-up mergers*, which might result in negative social-welfare effects despite being procompetitive. A runner-up merger can be defined as a merger between firms that are relatively small pre-merger and do not gain the leading market position (in terms of market shares). Our merger analysis shows that a subset of runner-up mergers may reduce social welfare even though they make consumers better off. Precisely, a social-welfare reducing runner-up merger that is procompetitive is defined by three properties. *First*, the merger must be *market-share increasing* for the merging firms to unfold a positive effect on consumer surplus.[5] *Second*, the merging firms must have a *below-average joint market share* (that is, we are dealing with a merger of relatively small firms).[6] *Third*, the merger is *concentration decreasing*; that is, the Herfindahl-Hirschman index (HHI) is smaller after the merger than before. If a procompetitive merger (i.e., one that fulfills the first property) does not meet either one of properties two and three, then the merger is social-welfare increasing.

We show our result within a homogenous-goods Cournot-oligopoly model with linear demand and constant marginal production costs. The model accounts for merger efficiencies which have to be large enough to make the merger *market-share increasing* in the first place. However, a social-welfare reducing runner-up merger can only occur if the merger efficiencies are limited, so that concentration decreases after the merger.

The merger analysis is based on a 2-step "cost-change" analysis, where in the first step the less efficient firm (i.e., the "target firm") is taken out of the market

[5]Note that any market-share increasing merger is also profitable because of the realized efficiencies.

[6]Below we show that in a homogenous-goods Cournot oligopoly the market share of the merging firms must be smaller than one-half of the average market share.

and, in the second step, the more efficient firm (i.e., the "acquirer firm") realizes merger efficiencies (i.e., a marginal cost reduction). With this approach, we can relate our merger analysis to works which analyze how a firm's marginal cost affects equilibrium outcomes under Bertrand and Cournot oligopoly competition *within* a given market structure (see the literature review below). By that we can show that our homogenous-goods Cournot analysis carries over to differentiated-goods Bertrand and Cournot oligopolies.

Our analysis implies a more nuanced approach towards mergers involving smaller firms in concentrated markets. For instance, mobile telecommunications markets have featured several mergers that involved relatively small firms and which have been cleared in recent years; namely, *Hutchinson 3G/Orange Austria* (Austria, 2012),[7] *Telefonica Deutschland/E-Plus* (Germany, 2014),[8] *T-Mobile NL/Tele2 NL* (Netherlands, 2018),[9] *and T-Mobile US/Sprint* (USA, 2020).

All those cases share several features of our model. All mergers happened in a highly concentrated market ("big-four" industries). They also have in common that each of the merging firms had a lower market share than any of the two largest competitors in the market. For instance, in the *Hutchinson 3G/Orange Austria* merger the market shares in terms of subscribers were 5-10% for *Hutchinson 3G* and 10-20% for *Orange Austria*, while the two largest competitors *Telekom Austria* and *T-Mobile* had market shares of 40-50% and 30-40%, respectively. Actually, the Commission's decision (EC/Case COMP/M.6497) states on page 34 explicitly that the transaction would combine the smallest and the second-smallest mobile network operators in Austria, with market shares of the merging firms below 25%. Clearly, the *Hutchinson 3G/Orange Austria* merger qualifies as a runner-up merger.

Similarly, in the *Telefonica Deutschland/E-Plus* case, the merging firms had market shares of 10-20% (*Telefonica Deutschland*) and 20-30% (*E-Plus*), while

[7]See Case COMP/M. 6497 - Hutchison 3G Austria/Orange Austria.
[8]See Case M. 7018 - Telefonica Deutschland/E-Plus.
[9]See Case M. 8792 - T-Mobile NL/Tele2 NL.

the two largest competitors *Deutsche Telekom* and *Vodafone* each held market shares of 30-40%. The same picture arises in the *T-Mobile NL/Tele2 NL* case. According to the Commission's decision, the merging firms had market shares of 5-10% (*Tele2 NL*) and 10-20% (*T-Mobile NL*), while the two largest competitors *KPN* and *VodafoneZiggo* had market shares of 30-40% and 20-30%, respectively. Finally, the US mobile telecommunications market also experienced a runner-up merger in 2020 with the acquisition of *Sprint* by *T-Mobile US*. Again, each of the merging firms held smaller market shares (in terms of subscribers) than each of the two largest operators *Verizon* and *AT&T*, which each had market share above one-third.[10]

While all those cases have in common that the two smallest firms merged in a "big-four" market, they also share the feature that merger efficiencies were claimed by the merging parties. Even though those claimed efficiencies were typically not confirmed by the investigating antitrust authorities,[11] one could reasonably expect that at least some of the efficiencies will materialize after the merger. In other words, all those mergers may have led to some—but limited efficiencies—, which suggests that they are candidates for social-welfare reducing runner-up mergers according to our analysis.

We contribute to the merger literature that analyzes the relation between the merging firms' market shares and after-merger market outcomes. Williamson's (1968) seminal analysis of the "welfare trade-offs" already made clear that efficiency consideration can even make a merger to monopoly socially desirable. Farrell and Shapiro (1990) derived an upper bound on the merging firms' joint market share, so that a consumer-surplus reducing merger raises social welfare (closely related results were obtained in Levin, 1990, and McAfee and Williams, 1992). The authors also show that a consumer-surplus increasing (or, price-reducing)

[10]See the online article by C. Scott Brown "T-Mobile Sprint Merger: Everything you need to know" (https://www.androidauthority.com/t-mobile-sprint-merger-plans-921398/).

[11]Claimed efficiencies have to meet three cumulative criteria (both in EU and US). In the EU, for instance, they have to be beneficial to consumers, merger specific and verifiable (see EU Guidelines, 2004, p. 13/Section VII).

merger can only occur if the merger gives rise to merger efficiencies (in the form of marginal cost reductions). Nocke and Whinston (2021) and Spiegel (2021) study how the Herfindahl-Hirschman concentration index relates to a merger's consumer-welfare effect and to the distribution of social welfare between consumers and producers, respectively.

Our analysis is also related to the analysis of merger efficiencies and the efficiency defense in merger control (for surveys, see Röller, Stennek, and Verboven, 2001; Motta, 2004).[12] Starting with Williamson (1968), this literature has assumed a monotone relation between merger efficiencies and after-merger social welfare (see Besanko and Spulber, 1993; Lagerlöf and Heidhues, 2005; Amir, Diamantoudi, and Xue, 2009). Our results show that such a monotone relation cannot be expected necessarily in case of runner-up mergers, so that the efficiency defence may turn into an "efficiency offense" (from a social welfare perspective) in those instances.

Closely related are also Cheung (1992) and Salop (2010) which examine the question how a social welfare standard would change antitrust enforcement relative to an enforcement based on a consumer surplus standard.[13] Cheung (1992) shows in his "remark" (with reference to a numerical example) that merging firms may not want to claim merger efficiencies at all, whenever the antitrust authority allows only social welfare-increasing mergers. The example shows for the case of a 3-firms Cournot oligopoly that a profitable and output-expanding merger among relatively small firms that creates some—but not too large—efficiencies can reduce social welfare. As social welfare increases in the absence of any efficiencies, the merging firms may want to abstain from claiming efficiencies at all.

[12] Efficiencies were incorporated into the US Merger Guidelines in 1997 (Section 4) and into the European Merger Guidelines in 2004 (EC Horizontal Merger Guidelines, 2004/03, Article 77). In the US, the horizontal merger guidelines of the FTC and the DOJ, as amended in April 1997, state that "the primary benefits of mergers to the economy is their potential to generate efficiencies."

[13] Whinston (2007) states that antitrust authorities' "enforcement practice in most countries (including the US and the EU) is closest to a consumer surplus standard." Recent Industrial Organization literature (e.g., Whinston and Nocke, 2010) takes the consumer surplus standard for granted.

Essentially the same point was made later on by Salop (2010) in his contribution to the question whether consumer surplus or social welfare is the proper antitrust standard.[14] Salop (2010, p. 344) mentions the possibility of a consumer-surplus increasing merger that reduces social-welfare with reference to an ad-hoc example. In his example, the merging firms each hold a market share of 5%, while the only competitor holds a market share of 90%. Salop then goes on to argue that merger-generated efficiencies that increase the merging firms' output can reduce social welfare, as this induces the more efficient competitor to produce less. Salop (2010, p. 344) concludes that such a merger "in principle could be condemned (...) [under a social welfare standard] even despite the benefits to consumers."[15]

Our analysis of merger efficiencies builds on results obtained in works that examine how a firm's marginal cost affects the oligopoly equilibrium outcome and social welfare for a *given* market structure. Lahiri and Ono (1988) and Zhao (2001) show within a homogenous-goods Cournot oligopoly that a reduction of the marginal costs of a firm holding a relatively small market share can reduce social welfare (see also Février and Linnemer, 2004, for a more general Cournot oligopoly analysis). Wang and Zhao (2007) obtain complementary results for Bertrand and Cournot competition when goods are differentiated. Related is also Salant and Shaffer (1999) which analyzes how raising the variance of marginal costs of a fixed number of Cournot competitors (while maintaining the sum) improves social welfare although it increases the Herfindahl-Hirschman index. Salant and Shaffer (1999, p. 578) interpret their result as "(...) an unmistakable reminder that increases in the HHI need not signal a decline in welfare."

[14] Salop (2010) argues in favor of using a consumer welfare standard in antitrust. A different view is expressed in Heyer (2006), who argues in favor of a social welfare standard which takes care of firms' profits.

[15] Salop (2010, p. 345) then argues in defense of the consumer welfare standard as follows: "(...) antitrust liability under the Sherman Act places no weight on competitor injury unless it is a building block to showing or inferring consumer harm. For example, in the *Brunswick* case, the Court denied standing to a competitor that complained about an injury suffered from a merger that resulted in increased competition and lower profits. (...) The Court pointed out that it would be 'inimical to the purposes' of the antitrust laws to award such damages. Antitrust laws were enacted for the 'protection of competition, not competitors.'"

We proceed as follows. In Section 2.2, we analyze runner-up mergers in a homogenous-goods Cournot oligopoly model. In Section 2.3, we show that our runner-up merger result can also occur under Bertrand and Cournot oligopoly competition when goods are differentiated. Finally, Section 2.4 concludes.

2.2 Cournot Oligopoly with Homogenous Goods

Assume a market with N firms indexed by $i = 1, ..., N$. All firms produce a homogenous good, incur constant marginal production costs, c_i, and compete à la Cournot. Inverse market demand is given by a linear function $p(Q) = A - Q$, with $Q := \sum_{i=1}^{N} q_i$, where q_i is firm i's output. We assume a parameter range such that all firms' equilibrium outputs are strictly positive in the Cournot-Nash equilibrium, which is ensured by assuming A to be sufficiently large.

The profit of firm i is given by $\pi_i = (p(Q) - c_i)\, q_i$. In the unique Cournot-Nash equilibrium, firm i's output level is given by

$$q_i = \frac{A - Nc_i + \sum_{j=1, j\neq i}^{N} c_j}{N+1} \text{ for all } i. \tag{2.1}$$

The equilibrium values for total output, consumer surplus, and firm i's profit are given by $Q = \frac{NA - \sum_{i=1}^{N} c_i}{N+1}$, $CS = \frac{1}{2}Q^2$, and $\pi_i = q_i^2$, respectively. In equilibrium, social welfare (i.e., the sum of firms' profits and consumer surplus) is given by

$$SW = \frac{1}{2}Q^2 + \sum_{i=1}^{N} q_i^2. \tag{2.2}$$

Firm i's market share is defined by $s_i := q_i/Q$ and the Herfindahl-Hirschman index by $HHI := \sum_{i=1}^{N} s_i^2$.

In the following, we analyze the social-welfare effect of a merger between two firms i and j, with $i, j \in \{1, ..., N\}$. Let $c_i \geq c_j$, with firm i being the *target* firm and firm j the *acquirer* firm. We assume that the acquirer firm's before-merger marginal cost, c_j, is reduced after the merger by σ, which stands for the *merger* *efficiencies.* Thus, the merged entity (for which we keep the acquirer-firm index j) has after-merger marginal costs, c_j^a (the superscript a stands for "after-merger"), given by $c_j^a := c_j - \sigma$, for $\sigma \in [0, c_j]$.

The Cournot equilibrium formula (2.1) gives directly the before-merger equilibrium values, q_i^b, (we indicate "before-merger" equilibrium values by the superscript b). Noting that a merger between firms i and j takes firm i out of the market (and thereby reduces the number of firms from N to $N-1$) and that firm j's after-merger marginal costs change from c_j to $c_j - \sigma$, formula (2.1) can be easily re-written to get the after-merger equilibrium values, q_i^a.

A merger between two firms can be interpreted as changing the merging firms' marginal production costs in two steps. In step 1, the marginal costs of the target firm (which is relatively inefficient) are raised to infinity (i.e., it is taken out of the market). In step 2, the acquirer firm's (or: the *merged* firm's) marginal cost is reduced by the merger efficiency, σ.

Given this "cost-change" analysis of a merger, we can relate the merger analysis to comparative static results which examine how equilibrium values change due to a marginal change of a firm's marginal cost (Zhao, 2001, and Février and Linnemer, 2004).

Lemma 2.1 (Cournot equilibrium properties). *Suppose an interior equilibrium of an N-firms Cournot oligopoly with firm-specific constant marginal costs, c_i, and a linear inverse demand function. An exogenous marginal change of firm i's marginal costs, c_i, then affects the equilibrium values as follows:*

i) Firm i's output, q_i, profits, π_i, and market share, s_i, decrease in c_i.

ii) Firm j's ($j \neq i$) output, q_j, profit, π_j, and market share, s_j, increase in c_i.

iii) The market price, p, increases in c_i, and total output, Q, as well as consumer surplus, CS, decrease in c_i.

iv) Social welfare, SW, increases (decreases) in c_i if and only if $s_i < \frac{1}{2(N+1)}$ ($s_i > \frac{1}{2(N+1)}$) (with equality holding at $s_i = \frac{1}{2(N+1)}$), while it is strictly convex in

c_i.

Proof. For parts *i)-iii)* see Zhao (2001). Part *iv)* follows from $\frac{\partial SW}{\partial c_i} = Q \cdot \frac{\partial Q}{\partial c_i} + 2 \cdot \sum_{j=1}^{N} \left[q_j \cdot \frac{\partial q_j}{\partial c_i} \right]$, which can be re-written as $\frac{\partial SW}{\partial c_i} = \left(\frac{1}{N+1} - 2s_i \right) Q$ (see Zhao, 2001, p. 466), from which we get the conditions stated in the proposition. Finally, $\frac{\partial^2 SW}{\partial c_i^2} = \left(\frac{\partial Q}{\partial c_i} \right)^2 + 2 \cdot \sum_{j=1}^{N} \left(\frac{\partial q_j}{\partial c_i} \right)^2 > 0$, so that social welfare is strictly convex in c_i.

We next perform the 2-step cost-change analysis of a merger between firms i and j, where firm j (the acquirer) has (weakly) lower marginal costs than firm i (the target).

Step 1 (increasing the marginal costs of the target firm i). A merger of two firms i and j with different technologies (such that $c_i \geq c_j$) induces the abandonment of the less efficient technology used by firm i. In other words, the target firm i is shut down, while the acquiring firm j remains active in the market. Taking firm i out of the market is equivalent to increasing the marginal costs of the target firm to infinity. The next result then follows immediately from Lemma 2.1.

Corollary 2.1 (Social welfare effect of a no-efficiency merger). *A no-efficiency merger (with $\sigma = 0$) in an N-firms Cournot oligopoly increases social welfare if and only if the target firm's market share is sufficiently small; i.e., $s_i^b < \frac{1}{2(N+1)}$ holds. Otherwise, a no-efficiency merger reduces social welfare. Finally, a no-efficiency merger always reduces consumer surplus.*

Corollary 2.1 says, absent any merger efficiencies, a takeover of a firm by a more efficient acquirer firm is socially desirable whenever the before-merger market share of the target firm is small enough. In particular, the target firm must have a market share much lower than the average (before-merger) market share, which is $\frac{1}{N}$. Raising the marginal costs of such a small target firm up to infinity (that is, taking it out of the market) raises social welfare, because any such marginal cost increase lowers the target firm's market share and therefore, makes it even more socially desirable to raise it further.

Corollary 2.1 mirrors the view that a horizontal merger appears less harmful to competition the smaller the target firm is. Conversely, if the target firm's market share (and, with that, the concentration associated with the merger) becomes large enough, then the merger is likely to harm competition significantly, in which case merger-regulations' approval conditions become increasingly restrictive. At the same time, efficiency considerations become increasingly important to counter the anticompetitive effects of the merger. The rationale behind the "efficiency defence" in merger control is that efficiencies have a positive *monotone* impact on market outcomes (Besanko and Spulber, 1983). While this reasoning is valid from a consumer-welfare view (see part *iii)* of Lemma 2.1), it may fail with regard to social welfare (see part *iv)* of Lemma 2.1). The social-welfare problem of a merger is further examined in the next step.

Step 2 (lowering the marginal costs of the acquirer firm j): The second step of the cost-change analysis of a merger between firms i and j relates to the merger efficiency, σ, realized by the acquirer firm j after the merger. To derive the social welfare effects of the merger in terms of the (observable) before-merger market shares, we focus on merger-efficiency levels that surpass the *price-fixing* efficiency level, which we denote by $\widehat{\sigma}$. That is, we focus on consumer-surplus increasing (or, procompetitive) mergers, which should be approved by an antitrust authority following a consumer-surplus standard.

At $\sigma = \widehat{\sigma}$, the merging firms' after-merger market share is just equal to their joint market share before the merger; i.e., $s_i^b + s_j^b = s_j^a(\widehat{\sigma})$.[16] For larger efficiency levels, $\sigma > \widehat{\sigma}$, the merged firm's equilibrium market share is strictly larger than the merging firms' joint before-merger market share; i.e., $s_j^a(\sigma) > s_i^b + s_j^b$.

At the price-fixing efficiency, $\widehat{\sigma}$, all firms' output levels, the equilibrium price, and consumer welfare are the same before and after the merger, whereas social welfare is strictly larger after the merger. The latter observation follows from noticing that at $\widehat{\sigma}$ the merged entity produces the same output as before but with

[16]The existence of a unique $\widehat{\sigma}$ follows from the monotonicity of a firm's output in its marginal costs (see part *i)*, Lemma 2.1).

lower marginal costs. The merging firms' profit gain is equal to the social welfare gain at $\sigma = \widehat{\sigma}$, because consumer surplus and all outsider firms' profits do not change at this point.

Given part *iv)* of Lemma 2.1, we can directly infer how a change of the merged firm's efficiency level affects social welfare for $\sigma \geq \widehat{\sigma}$; i.e., for merger efficiencies that reach beyond the merely price-fixing level. Noticing that the number of firms is reduced to $N-1$ after the merger, the marginal effect of firm j's after-merger marginal cost on social welfare is given by

$$\frac{\partial SW^a}{\partial c_j^a} = \left(\frac{1}{N} - 2s_j^a\right) Q^a. \tag{2.3}$$

Using the fact that $s_i^b + s_j^b = s_j^a(\widehat{\sigma})$ and $Q^b = Q^a(\widehat{\sigma})$ must hold at the price-fixing efficiency level $\widehat{\sigma}$, we can express (2.3) in terms of the before-merger market shares of the merging firms:

$$\left.\frac{\partial SW^a}{\partial c_j}\right|_{\sigma=\widehat{\sigma}} = \left(\frac{1}{N} - 2(s_i^b + s_j^b)\right) Q^a. \tag{2.4}$$

Note that $\frac{\partial SW^a}{\partial c_j} = -\frac{\partial SW^a}{\partial \sigma}$. From (2.4) we see that a change of the merger efficiency impacts negatively on social welfare (evaluated at the price-fixing efficiency level) if and only if $s_i^b + s_j^b < \frac{1}{2N}$ holds, while social welfare increases otherwise. We, therefore, have derived a necessary (sufficient) condition for a social-welfare reducing (increasing) merger, which makes consumer better off by reducing the market price.

Proposition 2.1 (Social welfare effect of a price-reducing merger). *Suppose that a merger reduces the equilibrium market price, and hence, increases consumer surplus; i.e., $\sigma \geq \widehat{\sigma}$ holds. Social welfare unambiguously increases after the merger if $s_i^b + s_j^b \geq \frac{1}{2N}$ holds. Otherwise, social welfare can increase or decrease with the merger depending on the efficiency level σ. If social welfare is lower after the merger than before, then the following properties of a "runner-up merger" hold:*

i) The merger occurs between relatively small firms, where the sum of the before-merger market shares of the merging firms fulfills $s_i^b + s_j^b < \frac{1}{2N}$.

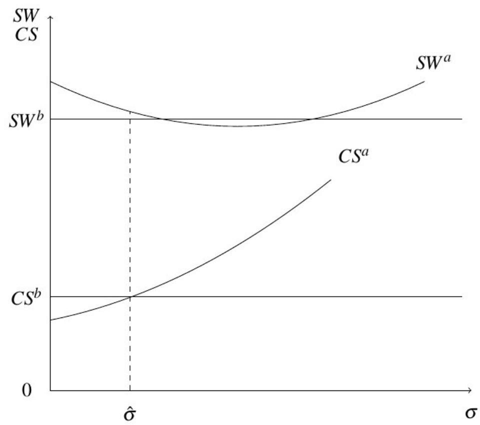
ii) Concentration is reduced after the merger; i.e., $HHI^a < HHI^b$ *holds.*

Proof. Part *i)* When $s_i^b + s_j^b < \frac{1}{2N}$ holds, then a no-efficiency merger must increase social welfare according to Corollary 1.1, because $s_i^b \leq s_j^b$, together with $s_i^b + s_j^b < \frac{1}{2N}$, implies that $s_i^b < \frac{1}{2(N+1)}$ holds. Condition $s_i^b + s_j^b < \frac{1}{2N}$ (which is implied by (2.4)) also ensures that SW^a decreases monotonically until the price-fixing efficiency level, $\widehat{\sigma}$, is reached. At that point, after-merger social welfare is strictly larger than the before-merger level. Thus, increasing the efficiency level beyond the price-fixing level is a necessary condition for a consumer-surplus increasing and social welfare-reducing merger. If, on the other hand, $s_i^b + s_j^b \geq \frac{1}{2N}$, then $SW^a > SW^b$, for all $\sigma \geq \widehat{\sigma}$, follows from the strict convexity of $SW(c_j)$ (see part *iv*), Lemma 2.1).

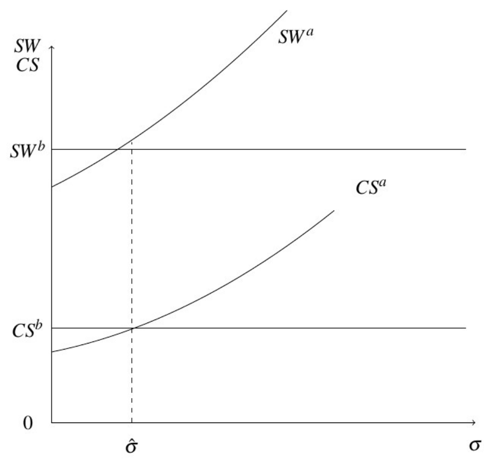
Part *ii)* We can re-write (2.2) as $SW = Q^2\left(\frac{1}{2} + HHI\right)$. We then get that $SW^a < SW^b$ can only hold if $HHI^a < HHI^b$, because $Q^a \geq Q^b$ for all $\sigma \geq \widehat{\sigma}$.

Notably, Proposition 2.1 refers to the before-merger market shares of the merging firms, a metric easily available given the relevant antitrust market is well-defined. It clearly singles out runner-up mergers as the only candidates for social-welfare reducing mergers, *given* that the merger is consumer surplus increasing. Such a procompetitive but social welfare reducing runner-up merger is characterized by two defining features. *First*, it is a merger of relatively small firms (according to part *i)* of Proposition 2.1, the combined market share is below one-half of the average market share in the respective market). *Second*, the merger induces some—but limited—merger efficiencies, so that concentration (as measured by the Herfindahl-Hirschman index) is reduced after the merger.

Figure 2.1 illustrates our result. Panel A refers to a social-welfare reducing and panel B to a social-welfare increasing merger. In both panels, the x-axis measures the merger efficiency, σ, and the y-axis stands for social and consumer welfare, respectively. The point $\widehat{\sigma}$ indicates the price-fixing merger efficiency,



(a) Panel A



(b) Panel B

Figure 2.1: Social welfare effects of a merger with efficiencies

where consumer surplus is the same before and after the merger. At this point, after-merger social welfare, SW^a, must be larger than social welfare before, SW^b. Moreover, at $\sigma = \widehat{\sigma}$, the difference between SW^a and SW^b is equal to the merging firms' profit gain from the merger.

Panel A depicts the case of a "small-firms" merger (i.e., condition $s_i^b + s_j^b < \frac{1}{2N}$ of part *i)* of Proposition 2.1 holds). In this case, after-merger social welfare is decreasing in the efficiency level at $\sigma = \widehat{\sigma}$, which implies that a no-efficiency merger must raise social welfare (i.e., $SW^a(\sigma = 0) > SW^b$).

Panel A highlights the case, where there exist merger-efficiency levels, $\sigma > \widehat{\sigma}$, such that social welfare is lower after the merger than before; notably, even though consumer surplus is increased after the merger. Increasing the merger efficiency level above the price-fixing level increases both the merging firms' profit and consumer surplus but reduces outsider firms' profits. In case of a small-firms merger, the latter effect can become large enough to outweigh the former one, so that social welfare can be lower after the merger than before. Clearly, such an outcome is only possible when the merger efficiencies do not become too large, because—otherwise—the sum of the merging firms' profit gain and the consumer surplus gain will be larger than the outsider firms' profit reductions. According to part *ii)* of Proposition 2.1, this would be the case if the HHI increases after the merger.

Panel B shows a situation, where the merging firms are large enough (i.e., $s_i^b + s_j^b \geq \frac{1}{2N}$ holds). In this case, after-merger social welfare is increasing in the efficiency level at $\sigma = \widehat{\sigma}$, which implies that after-merger social welfare remains larger than before-merger social welfare for all efficiency levels beyond the price-fixing level.[17]

Next we provide two examples of runner-up mergers, which reduce social welfare even though they meet the price-test. We are particularly interested

[17] Panel B depicts the case, where a no-efficiency merger reduces social welfare. It should be noted that the opposite can also occur under a larger firms merger. It is then also possible that social welfare after the merger is lower than before if the merger is consumer surplus reducing (i.e., when $\sigma < \widehat{\sigma}$ holds).

in the largest possible before-merger market shares of the merging firms under which such an outcome is possible within a linear Cournot oligopoly. Consider a market with a single or several (symmetric) "large" firms (with index d) and two smaller firms i (the target) and j (the acquirer), which are the merger candidates. Consider the following parameter values: $A = 1$ and $c_d = 0$, while the merging firms have strictly positive marginal costs with $c_i \geq c_j > 0$. Thus, the merging firms have strictly smaller before-merger market shares than the $N-2$ outsider firms (see the Appendix for the calculations).

Example 2.1 (merger between asymmetric firms). Consider the extreme asymmetric constellation, where the target firm's before-merger output is close to zero and the acquirer firm's output is strictly larger. That is, let $c_i \to \frac{1+c_j}{N}$, which implies $q_i^b \to 0$. It then follows that before-merger social welfare is approximately the same as after-merger social welfare when merger efficiencies are absent; i.e., $\lim_{c_i \to \frac{1+c_j}{N}} SW^b \to SW^a|_{\sigma=0}$. From part *i)* of Proposition 2.1, we know that $\frac{\partial SW^a}{\partial \sigma}\Big|_{\sigma=0} < 0$ if and only if $s_i^b + s_j^b < \frac{1}{2N}$. As $s_i^b\Big|_{c_i \to \frac{1+c_j}{N}} \to 0$, the latter condition implies that $c_j > \frac{N+1}{2N(N-1)-1}$ must hold, so that social welfare decreases in σ at the price-fixing efficiency level, which is $\widehat{\sigma} = 0$ in this example. Moreover, from $\frac{\partial SW^a}{\partial \sigma} = 0$ we get the unique root $\sigma' = c_j - \frac{N+1}{2N(N-1)-1}$, at which after-merger social welfare is minimal. Thus, for all $c_j > \frac{N+1}{2N(N-1)-1}$ there exists merger-efficiency levels $0 < \sigma < c_j - \frac{N+1}{2N(N-1)-1}$, such that social welfare is smaller after the merger than before; notably, even though consumer surplus increases for all $\sigma > 0$.

Finally, at $c_j \to \frac{N+1}{2N(N-1)-1}$, we get the largest possible before-merger market share of firm j (and hence, of the merging firms together) such that a small merger efficiency reduces social welfare. If $N = 3$, then the upper bound of the before-merger market share of the acquirer firm is $\overline{s}_j^b = \frac{1}{6} \approx 16.7\%$, so that for all $s_j^b < \overline{s}_j^b$ exists a range of merger efficiencies such that social welfare is reduced after the merger even though the merger raises consumer surplus.

Example 2.2 (merger between symmetric firms). We turn to symmetric constellations, where the merging firms have the same marginal costs, $c_i = c_j :=$

c_n, while the $N-2$ outsider firms have marginal costs of zero. Solving $SW^b - SW^a = 0$ for c_n, we get a unique threshold value, $\bar{c}_n(\sigma, N)$, such that $SW^b > SW^a$ for all $c_n > \bar{c}_n(\sigma, N)$, while consumer surplus always increases. The function $\bar{c}_n(\sigma, N)$ is u-shaped and obtains (for a given N) a minimum at σ'. Substituting σ' into $\bar{c}_n(\sigma, N)$, we get (again, for a given N) a lower bound of $c_n(\sigma', N)$ (and thus an upper bound of the before-merger market shares, $s_i^b(c_n(\sigma', N)) = s_j^b(c_n(\sigma', N))$), such that there exist an interval of values of σ, where social welfare is lower after the merger than before.

For instance, if $N = 3$, then such a range exists, if $s_i^b = s_j^b \approx 5.4\%$, so that the joint market share of the merging firms must not be larger than 10.8%. If $N = 4$, then the upper bound on the joint before-merger market share is 7.8%.

2.3 Differentiated-Goods Bertrand and Cournot Oligopolies

Extending our analysis to a differentiated-goods oligopoly is not simple because product differentiation is notoriously difficult to deal with, which is particularly true when we also have to consider firm-specific marginal costs. Product differentiation also provokes new issues; for instance, whether a merged firm wants to keep both brands or wants to abandon one of the brands and how firms' demands would change if a brand is withdrawn from the market.

Our previous analysis has shown that the comparative statics of the oligopoly equilibrium with regard to a firm's marginal cost is a building block of the runner-up merger result (Proposition 2.1). Thus, if a differentiated-goods oligopoly model exhibits the same comparative statics results as described in Lemma 1.1 for the homogenous-goods Cournot oligopoly, then this is reassuring that the runner-up merger result remains valid under differentiated-goods oligopolies.

In this regard, we refer to Wang and Zhao (2007) which analyzes the effects of a firm's marginal cost change on equilibrium outcomes in a Bertrand and Cournot oligopoly for a *given* market structure with N firms.[18] The authors consider a Bertrand-Shubik demand system (Shubik, 1980), where the demand of firm i is given by $q_i(p_1, ..., p_N) = V - p_i - \gamma(p_i - \overline{p})$. The parameter $V > 0$ is the common quantity intercept, $\overline{p} = \left(\sum_{i=1}^{N} p_i\right) \Big/ n$ is the industry average price, and $\gamma \geq 0$ is the substitutability parameter. Firms' marginal production costs are constant with $c_i \geq 0$.

The key result is that both the Bertrand oligopoly and the Cournot oligopoly react qualitatively in the same way to a small change of firm i' marginal costs as described in Lemma 2.1 for the homogenous-goods Cournot oligopoly. In particular, equilibrium social welfare is non-monotone (and strictly convex) in firm i's marginal cost in both oligopoly models (see Propositions 1 and 2 in Wang and Zhao, 2007). Precisely, social welfare under Bertrand competition increases in firm i's marginal cost if and only if firm i's marginal cost is larger than a unique threshold value (see Wang and Zhao, 2007, Proposition 1); otherwise it decreases. As firm i's market share is always inversely related to its own marginal cost, this is equivalent to the statement that social welfare is declining in firm i's market share if and only if firm i's market share is smaller than a certain (unique) threshold value, $t_W^B = t_W^B(\gamma, N)$;[19] or, formally (the superscript B stands for Bertrand competition):

$$\frac{\partial SW^B}{\partial c_i} > 0 \Leftrightarrow s_i^B < t_W^B(\gamma, N) := \frac{\gamma(N + (N-1)\gamma)}{(2N + (N-1)\gamma)(3N + (3N-1)\gamma)}. \tag{2.5}$$

An analogous result is obtained under Cournot competition (the threshold value

[18] Notably, Wang and Zhao (2007) is the only analysis—we are aware of—which provides a complete characterization of social welfare of a Bertrand oligopoly with differentiated goods and firm-specific marginal costs.

[19] The threshold value t_W^B is derived in Appendix A, part *iii)* of "Proof of Proposition 1 and Corollary 1" of Wang and Zhao (2007, p. 183). Part *iii)* of the proof refers to the derivative of social welfare with respect to firm i's marginal cost under Bertrand competition. Thus, there is a typo in the line above formula (25) on page 183: it should be "$\frac{\partial W^B}{\partial c_i} > 0 \Leftrightarrow t_i^B < t_W^B$..." rather than "$\frac{\partial \Pi^B}{\partial c_i} > 0 \Leftrightarrow t_i^B < t_W^B$...".

t_W^C is derived in the Appendix of Wang and Zhao, 2007, p. 184):

$$\frac{\partial SW^C}{\partial c_i} > 0 \Leftrightarrow s_i^C < t_W^C(\gamma, N) := \frac{\gamma(N+\gamma)}{(2N+(N+1)\gamma)(3N+2\gamma)}.$$

It is easily checked that $t_W^B, t_W^C < \frac{1}{2N}$, so that firm i's market share must be smaller than one-half of the average market share in those instances. Clearly, those results closely mirror the non-monotonicity result stated in part *iv)* of Lemma 2.1. The differentiated-goods Bertrand and Cournot oligopolies, therefore, exhibit the same social-welfare responses to a small-firms merger, we have shown in our 2-step cost-change merger analysis for the homogenous-goods Cournot oligopoly.

To show this more clearly, let us focus on a Bertrand oligopoly with $N \geq 3$ firms. Suppose firms i and j propose a merger, where $c_i \geq c_j$. The merger leads to efficiencies σ, so that the merged firm's marginal cost is $c_j^a = c_j - \sigma$. Assume that the merged firm takes out the acquired firm's brand i and produces only the acquirer firm's product j, which can be justified by brand-specific fixed costs.

A larger value of σ induces a larger output of the merged firm. Let $\widehat{\sigma}$ stand for the *market-share fixing efficiency level*, where the equilibrium market-share of the merged firm is the same as the joint market share of the merging firms before the merger; i.e., $s_i^b + s_j^b = s_j^a(\widehat{\sigma})$ holds.[20] At $\widehat{\sigma}$ after-merger social welfare is increasing in c_j^a (and hence, decreasing in the merger efficiency) according to (2.5) if and only if

$$s_i^b + s_j^b = s_j^a(\widehat{\sigma}) < t_W^B(\gamma, N-1) = \frac{\gamma((N-1)+(N-2)\gamma)}{(2(N-1)+(N-2)\gamma)(3(N-1)+(3(N-1)-1)\gamma)}, \quad (2.6)$$

where $t_W^B(\gamma, N-1)$ is the after-merger threshold value that characterizes how social welfare depends on c_j^a. It is easily checked that condition (2.6) implies that

[20]The market-share fixing efficiency level cannot be easily related to consumer welfare before and after the merger as it was the case in the homogenous-goods Cournot oligopoly. Equilibrium consumer welfare is not only a function of total output but also of each firm's squared quantities (see Wang and Zhao, 2007, eq. 10, p. 176).

the joint market share of the merging firms must be smaller than the average before-merger market share $\frac{1}{N}$. If condition (2.6) holds, we also know that an increase of firm i's marginal cost before the merger must increase social welfare (step 1 of our cost-change merger analysis). This follows from noticing that $s_i^b + s_j^b < t_W^B(\gamma, N-1)$ implies $s_i^b < t_W^B(\gamma, N)$. To show that this always holds, note first that the maximal value of s_i^b is obtained if $c_i = c_j$, so that $s_i^b = s_j^b$. Thus, $s_i^b + s_j^b < t_W^B(\gamma, N-1)$ can be re-written as $s_i^b < t_W^B(\gamma, N-1)/2$. Secondly, $t_W^B(\gamma, N-1)/2 < t_W^B(\gamma, N)$ holds for all $N \geq 3$ (see Appendix).

Thus, if the merger efficiency affects social welfare negatively (evaluated at the *market-share fixing efficiency level*, $\widehat{\sigma}$), then a no-efficiency merger is strictly social welfare increasing. Such an outcome is only possible if the merging firms' joint market share is below the average market share. Consequently, we obtain a similar situation as depicted in panel A of Figure 1 with respect to the after-merger social welfare curve. While a global comparison of social and consumer welfare before and after the merger is out of reach at a general level, those (local) results are nevertheless reassuring that market-share increasing mergers of relatively small firms remain candidates for socially undesirable runner-up mergers in differentiated-goods Bertrand and Cournot oligopolies.

2.4 Conclusion

We analyzed the social welfare effect of consumer-surplus increasing mergers, which should pass the decision screen of antitrust authorities that apply a consumer-welfare standard to evaluate merger proposals. We showed that such procompetitive mergers are only problematic from a social-welfare perspective when a runner-up merger occurs. A runner-up merger is first of all characterized by sufficiently large efficiencies which ensure that the after-merger market price is not larger than the before-merger market price (i.e., the merger is approvable for the antitrust authority). Critically, a social-welfare decreasing runner-up merger then fulfills two criteria. *First*, it is a merger between relatively small firms, where each has a below average market share. *Second*, the merger efficiencies are limited, so that market concentration as measured by the Herfindahl-Hirschman index decreases after the merger. In contrast, if a merger does not meet one of the latter criteria, then social welfare must increase after the merger. In particular, a merger of relatively large firms, which is consumer-surplus increasing is also always social welfare increasing.

The runner-up merger result becomes practically relevant when antitrust regulations would switch from a consumer-welfare standard to a social-welfare standard (see Heyer, 2006, for such a proposal).[21] Under a social-welfare standard the harm imposed by the merger on competitors has to be considered in the overall evaluation of a merger. If this harm exceeds the sum of the profit gain of the merging firms and the consumer welfare gain, then the merger had to be blocked. As the harm imposed on competitors increases with the merger-generated efficiencies, the merging firms may want to conceal any such possible merger gains (see Cheung, 1992). Thus, the antitrust agency would have to deliver the facts by claiming an "efficiency offense" of the merger proposal. According to our analysis, the efficiency offense is most likely to be critical when the merging firms have a below-average joint markets share, while efficiencies ensure that the merger is market-share increasing and tends to reduce market concentration—i.e., when a runner-up merger is at stake.

[21] Alternatively, we can also think of a combination of both standards such that a merger is only allowed if it raises consumer surplus *and* social welfare.

2.5 Appendix

In this Appendix, we deliver the calculations for the two examples presented in Section 2.2 and the claim—made in Section 2.3—that $t_W^B(\gamma, N-1)/2 < t_W^B(\gamma, N)$ holds for all $N \geq 3$.

Example 2.1 (merger between asymmetric firms). Using equation (1), we get the before-merger equilibrium values: $q_d^b = \frac{1+c_i+c_j}{N+1}$, $q_i^b = \frac{1-Nc_i+c_j}{N+1}$, $q_j^b = \frac{1-Nc_j+c_i}{N+1}$, and $Q^b = \frac{N-c_i-c_j}{N+1}$. When $c_j \longrightarrow \frac{1+c_2}{N}$, then $q_j^b \to 0$. After the merger we get $q_d^a = \frac{1+c_j-\sigma}{N}$, $q_j^a = \frac{1-(N-1)(c_j-\sigma)}{N}$, and $Q^a = \frac{N-1+\sigma-c_j}{N}$. Social welfare (see (2.2)) before and after the merger are given by

$$SW^b = \frac{1}{2}\left(\frac{N-c_i-c_j}{N+1}\right)^2 + (N-2)\left(\frac{1+c_i+c_j}{N+1}\right)^2 + \left(\frac{1-Nc_j+c_i}{N+1}\right)^2 \text{ and}$$

$$SW^a = \frac{1}{2}\left(\frac{N+\sigma-c_j-1}{N}\right)^2 + (N-2)\left(\frac{1+c_j-\sigma}{N}\right)^2 + \left(\frac{1-(N-1)(c_j-\sigma)}{N}\right)^2,$$

respectively. If $\sigma = 0$, and $c_i \longrightarrow \frac{1+c_j}{N}$ then $\lim_{c_i \to \frac{1+c_j}{N}} SW^b \to SW^a|_{\sigma=0}$, with

$$SW^a_{\sigma=0} = \frac{2N^2c_j^2 + N^2 - 2Nc_j^2 - 2Nc_j - c_j^2 - 2c_j - 1}{2N^2}.$$

Solving $\frac{\partial SW^a}{\partial \sigma} = 0$ for σ, we get the solution

$$\sigma' = c_j - \frac{N+1}{2N(N-1)-1},$$

is a global minimum because SW is strictly convex in σ. It follows that for $c_j > \frac{N+1}{2N(N-1)-1}$ there exist merger-efficiency levels $0 < \sigma < c_j - \frac{N+1}{2N(N-1)-1}$, such that social welfare is smaller after the merger than before.

Example 2.2 (merger between symmetric firms). The merging firms have the same marginal costs, $c_i = c_j := c_n$, while the $N-2$ outsider firms have marginal costs of zero. We then get $q_n^b = \frac{1-(N-1)c_n}{N+1}$, $q_d^b = \frac{1+2c_n}{N+1}$, and $Q^b = \frac{N-2c_n}{N+1}$

for the before-merger outputs and $q_n^a = \frac{1+c_n-\sigma}{N}$, $q_d^a = \frac{1-(N-1)(c_n-\sigma)}{N}$, and $Q^a = \frac{(N-1)-c_n+\sigma}{N}$ for the after-merger outputs, respectively. According to equation (2), before-merger social welfare is given by

$$SW^b = \frac{1}{2}\left(\frac{N-2c_n}{N+1}\right)^2 + 2\left(\frac{1-(N-1)c_n}{N+1}\right)^2 + (N-2)\left(\frac{1+2c_n}{N+1}\right)^2.$$

After-merger social welfare is given by

$$SW^a = \frac{1}{2}\left(\frac{(N-1)-c_n+\sigma}{N}\right)^2 + \left(\frac{1-(N-1)(c_n-\sigma)}{N}\right)^2 + (N-2)\left(\frac{1+c_n-\sigma}{N}\right)^2.$$

Let $N = 3$. Setting $SW^b - SW^a = 0$, we get two roots

$$\overline{c}_n(\sigma) = \frac{13}{38} - \frac{44}{19}\sigma + \frac{3}{19}\sqrt{308\sigma^2 + 4\sigma + 1} \text{ and} \tag{2.7}$$

$$\underline{c}_n(\sigma) = \frac{13}{38} - \frac{44}{19}\sigma - \frac{3}{19}\sqrt{308\sigma^2 + 4\sigma + 1}, \tag{2.8}$$

so that $SW^b > SW^a$ if either $c_n < \underline{c}_n(\sigma)$ or $c_n > \overline{c}_n(\sigma)$. Note that consumer surplus is larger after the merger if $Q^a - Q^b \geq 0$. For $N = 3$, this holds if $c_n > \widehat{c}_n(\sigma) := \frac{1}{2} - 2\sigma$. We can easily check that $\underline{c}_n(\sigma) < \widehat{c}_n(\sigma) < \overline{c}_n(\sigma)$ for all admissible $\sigma > 0$ (with $\widehat{c}_n(\sigma = 0) = \overline{c}_n(\sigma = 0)$). Thus, $\overline{c}_n(\sigma)$ is the relevant threshold value a social welfare decreasing merger that is price-reducing must fulfill. The threshold value $\overline{c}_n(\sigma)$ is strictly convex and obtains a unique minimum at $\sigma' = \frac{2}{77}\sqrt{11} - \frac{1}{154}$. Substituting σ' into $\overline{c}_n(\sigma)$, we get $\overline{c}_n(\sigma') = \frac{2}{77}\sqrt{11} + \frac{5}{14}$, which is the lower bound of the merging firms' marginal cost, such that social welfare decreases for all $c_n > \overline{c}_n(\sigma')$. Substituting $\overline{c}_n(\sigma')$ into the before-merger market-share formula for firms i and j we get $s_n^b(\overline{c}_n(\sigma')) \approx 5.4\%$, which is the largest possible before-merger market share of each of the merging firms such that the merger is social-welfare decreasing and price-reducing.

We proceed likewise for $N = 4$. From $SW^b - SW^a = 0$, we get two roots

$$\overline{c}_n(\sigma) = \frac{67}{321} - \frac{575}{321}\sigma + \frac{40}{321}\sqrt{322\sigma^2 + 2\sigma + 1}$$

and

$$\underline{c}_n(\sigma) = \frac{67}{321} - \frac{575}{321}\sigma - \frac{40}{321}\sqrt{322\sigma^2 + 2\sigma + 1},$$

where, again, only the larger threshold value $\overline{c}_n$ is relevant. This value obtains a minimum at $\sigma' = \frac{5}{322}\sqrt{23} - \frac{1}{322}$, which yields $\overline{c}_n(\sigma') = \frac{5}{322}\sqrt{23} + \frac{3}{14}$ and thus $s_n^b(\overline{c}_n(\sigma')) \approx 3.9\%$.

Proof of claim—made in Section 2.3—that $t_W^B(\gamma, N-1)/2 < t_W^B(\gamma, N)$ holds for all $N \geq 3$. Using the threshold value $t_W^B(\gamma, N)$ as stated in equation (2.5), we get that $t_W^B(\gamma, N-1)/2 < t_W^B(\gamma, N)$ is equivalent to

$$\frac{1}{2}\frac{\gamma((N-1)+(N-2)\gamma)}{(2(N-1)+(N-2)\gamma)(3(N-1)+(3(N-1)-1)\gamma)} < \frac{\gamma(N+(N-1)\gamma)}{(2N+(N-1)\gamma)(3N+(3N-1)\gamma)},$$

which can be rewritten as $\gamma \cdot \phi(\gamma, N) > 0$, with

$$\phi(\gamma, N) := \gamma^3\phi_1(N) + \gamma^2\phi_2(N) + \gamma\phi_3(N) + \phi_4(N),$$

where $\phi_1(N) = 3N^3 - 16N^2 + 27N - 14$, $\phi_2(N) = 12N^3 - 54N^2 + 75N - 27$, $\phi_3(N) = 15N^3 - 56N^2 + 59N - 12$, and $\phi_4(N) = 6N^3 - 18N^2 + 12N$. We show that all functions $\phi_i(N)$ for $i = 1, ..., 4$ are strictly positive for all $N \geq 3$ implying the claimed inequality. Differentiating $\phi_1(N)$ repeatedly, we get $\phi_1' = 9N^2 - 32N + 27$ and $\phi_1'' = 18N - 32 > 0$ for $N \geq 3$. We then get $\phi_1'(3) = 12$ and thus $\phi_1(3) = 4$, so that $\phi_1(N) > 0$. Differentiating $\phi_2(N)$ repeatedly, we get $\phi_2' = 36N^2 - 108N + 75$ and $\phi_2'' = 72N - 108 > 0$ for $N \geq 3$. We then get $\phi_2'(3) = 75$ and thus $\phi_2(3) = 36$, so that $\phi_2(N) > 0$. Differentiating $\phi_3(N)$ repeatedly, we get $\phi_3' = 45N^2 - 112N + 59$ and $\phi_3'' = 90N - 112 > 0$ for $N \geq 3$. We then get $\phi_3'(3) = 128$ and $\phi_3(3) = 66$, so that $\phi_3(N) > 0$. Finally, we differentiate ϕ_4 repeatedly, to get $\phi_4' = 18N^2 - 36N + 12$ and $\phi_4'' = 36N - 36 > 0$ for $N \geq 3$. We then get $\phi_4'(3) = 66$ and $\phi(3) = 36$, so that $\phi_4(N) > 0$, which proves the claim.

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Chapter 3

Competition among Online Streaming Platforms: To Vertically Integrate or Not?

3.1 Introduction

The proliferation of the Internet has resulted in an increasing number of individuals that view movies, series, and television shows via online streaming platforms, namely video platforms, as consumers can watch preferred video content anytime and anywhere. Audiences are no longer required to set an alarm to not miss their favorite shows because they can stream them on their mobile devices, tablets, or computers at their convenience. Moreover, these platforms provide a wider array of video content: One video platform can offer a diverse collection of TV shows, movies, and variety series to meet customers' needs. Consumers can choose from a seemingly unlimited number of episodes, movies, and shows simultaneously.

Thanks to technological improvements and the use of advanced cameras, production companies can reduce the time and effort dedicated to the filming process. Filmmakers can now complete the production of a feature film or a television series in only a few short months. Similarly, technological developments have dramatically simplified the post-production process for films and television programs, thereby considerably reducing the time and difficulties in the post-production stage. Companies like Netflix and Amazon Prime have capitalized on these innovations to ensure their platforms meet consumers' expanding needs.

Netflix has acquired more than 221 million subscribers worldwide since its inception in 1997. In recent years, Amazon Prime has emerged as one of Netflix's strongest competitors. Amazon Prime initially started as Amazon Unbox in 2006 and now has more than 200 million paying subscribers. Since 2013, both have aired internally produced dramas. These two leading video platforms have had phenomenal success. The market for video platforms is highly concentrated globally and is similar in the Chinese market. The two video platforms that contribute the most to China's overall market share are iQIYI and Tencent Video.[1]

Video platforms have transitioned from purchasing exclusive copyrights to

[1]The iQIYI online streaming service was started in April 2010, by Baidu, the company that runs China's most popular search engine. Tencent Video, launched in April 2011, is owned by Tencent, the owner of WeChat.

producing their own original series by vertically integrating with upstream content producers to both expand content and limit costs. In the past few years, Penguin Pictures, part of Tencent Video, has been working with professional film and TV teams to make high-quality content. However, despite having 124 million subscribers, Tencent Video still makes a negative profit. In the 2021 Annual Report, Tencent said that it would use "cost optimization" to cut losses at Tencent Video while maintaining its top spot. Similarly, iQIYI has several successful original series[2] and high-revenue membership fees[3] but still losing money.[4]

As early as 2014, the media reported that, in response to economic pressure, Chinese video platforms attempted to decrease the total cost by producing their original series.[5] Gong Yu, the founder and CEO of iQIYI said in a conference call about the company's financial report that they should "reduce procurement, improve the level of content production, and reduce the output of bad content" in order to achieve profitability. However, video platforms have hiked membership fees every year since 2020.[6] While video platforms claim to reduce expenses, they raise membership fees annually to make a positive profit. This phenomenon—price increases amidst cost reductions—is the motivation for this paper.

In many digital markets, platforms arrange interactions between different groups of agents (Cailaud and Jullien, 2003; Rochet and Tirole, 2003; Amstrong, 2006; Rochet and Tirole, 2006). The majority of research focuses on how plat-

[2]In June 2020, iQIYI debuted its brand, *LIGHT ON,* which primarily broadcasts suspense dramas. These original dramas consist of only 12 episodes. The second drama of *LIGHT ON, The Bad Kids,* was ranked first in viewership during the same period in June 2020 and received an 8.8 out of 10. The subsequent drama, *The Long Night,* received 9 out of 10 ratings, further enhancing iQIYI's reputation.

[3]According to iQIYI's annual report for the first quarter of 2022, the average daily number of subscribing members was 101.4 million, and membership services produced RMB 4.5 billion (USD $705.4 million) in revenue. The net income attributable to iQIYI was RMB 169.1 million (USD $26.7 million).

[4]iQIYI only achieved positive profits in the first quarter of 2022.

[5]This comes from *People.com.* Details on http://media.people.com.cn/n/2014/0725/c387044-25341867.html

[6]Tencent Video increased its monthly membership fee from RMB 15 to RMB 20 in 2021 and then to RMB 25 in April 2022. In November 2020, iQIYI made its first price adjustment in nine years, to RMB 19. The monthly subscription was further raised to RMB 22 in December 2021.

forms maximize their profit by attracting players through the introduction of cross-group externalities. Cross-group externalities are critical for developing a platform (Ambrus and Argenziano, 2009). For example, Amazon Prime has more than 26,000 movies and 2,700 TV shows[7] while Netflix provides 13,612 titles globally.[8] According to Guduodata, a total of 273 drama series were launched on various video platforms in 2022 in China. Among them, Tencent Video and iQIYI have 117 each. The amount of content establishes the network externalities to attract more viewers on the platforms.

In media markets, some researchers additionally consider the advertisers and consumers (Anderson and Coate, 2005; Peitz and Valleti, 2008; Ambrus et al., 2016). However, little attention has been paid to the supplier side of the media markets. Thus, this paper investigates content suppliers. The more content there is on the platforms, the more viewers are attracted to purchase a membership.

In this paper, we study the effect of vertical integration on competition between platforms when consumers can only single home. The following questions are addressed in this paper: What is the effect of vertical integration on the video platform market? Should platforms choose vertical integration with upstream movie producers or not? We use a two-sided market model to find that the membership fee can increase for an integrated platform when compared with the independent case. Platforms have an incentive to choose vertical integration. Platforms' profits, however, reduce after vertical integration. This analysis also reveals that consumers can watch more content following vertical integration.

Platforms compete in the market by offering differentiated content. In the baseline model, there are two groups of movie producers with differing genres. For instance, one group produces crime films, and the other produces costume movies. One streaming platform plays crime films by buying the licensed copyright from producers in the first group, while another platform buys the licensed copyright

[7]Source: JustWatch, https://www.justwatch.com/us/provider/amazon-prime-video

[8]See "Comparison of Netflix, Amazon Prime, Disney+ and HBO Max," https://studycorgi.com/comparison-of-netflix-amazon-prime-disney-and-hbo-max/.

of costume movies from the latter. Platforms strive to maximize their profits, irrespective of whether they are integrated or independent. The revenue is only from the membership fee, and the cost is the licensed fee for buying movies or production costs after a vertical merger. We further develop an advertisement model that uses the advertisers as a third party. The results are the same as the baseline model.

In the extension, we analyze an asymmetric structure with one platform with advertising, and the other without advertisements. Here, we show that the ad-free platform has more incentive to choose vertical integration. This extension suggests that after vertical integration, media platforms aim to create less heterogeneous content.

The paper is organized as follows: Section 3.2 presents a review of the related literature. Section 3.3 describes the baseline model, and Section 3.4 solves the baseline model without advertisers. Section 3.5 considers the two advertising platforms' cases. Section 3.6 develops an asymmetric model, and Section 3.7 discusses the study's limitations and conclusion.

3.2 Related Literature

Our paper relates to a large body of literature on two-sided markets, including seminal contributions by Rochet and Tirole (2003, 2006), Caillaud and Jullien (2003), and Armstrong (2006). Several recently published papers focus on multi-sided market analysis (Weyl, 2010; Correia-da-Silva et al., 2019; Tan and Zhou, 2021). These papers largely consider that platforms grow their number of users through active cross-group externalities. We contribute to the work on the supply side, focusing on content providers that sell their movies to video platforms. In doing so, we add the standard monopsony supply chain to the model.

There is substantial literature on media markets (Amstrong, 1999; Anderson and Coate, 2005; Crampes et al., 2009; Ambrus et al., 2016; Anderson et al., 2018). Most of the researchers build their model such that viewers receive negative network externalities from advertisements. Peitz and Valletti (2008) compare the competition between two platforms, pay TV and free-to-air TV, to demonstrate that the advertising intensity on free-to-air TV is higher. Free-to-air television tends to provide more similar content; pay TV immensely differentiates its content. We use a similar method to compare the competition between two platforms: the subscription platform and the subscription+ad platform. Peitz and Valletti reveal that if competition is intense or the disutility of viewing an ad is too small or too large, pay TV has better social welfare. However, they lack the ability to analyze the role of content producers to one side of the model. Thus we contribute to their work by adding content producers as one side of the model. In our model, consumers can gain positive network externalities from the content.

We further develop the argument for vertical integration in the media market. Guo et al. (2010) analyze vertical integration of content and broadband services from an economic perspective by using a game-theoretic model with a net neutrality debate. They establish that the vertically integrated broadband service provider does not have any incentive to abide by the principles of net neutrality. D'Annunzio (2017) is one of the first studies to address how vertical integration in a media market affects investments in premium content. The author focuses on exclusive provisions and program quality. This paper shows that a content creator provides premium content exclusively to a platform, regardless of whether the market is vertically integrated. However, a vertically integrated content provider has a lower incentive to produce premium content than an independent one. Thus, D'Annunzio uncovers the importance of considering the effects of vertical integration on investment choices for premium content. Carroni et al. (2023) study the impact of superstars' exclusive content on platform competition and complements' homing decisions. They find when vertical integration between superstars and platforms takes place, exclusivity might emerge less than under vertical separation, which is contrary to conventional wisdom.

This paper also incorporates the concept of network effects. A seminal paper by Katz and Shapiro (1985) develops a simple model with network externalities in an oligopoly market. Researchers combine two-sided market and network effects into the model analysis (Rochet and Tirole, 2003; Caillaud and Jullien, 2003; Parker and Van Alstyne, 2005; Armstrong, 2006). There are direct and indirect network effects considered in the platforms' economics. Some researchers argue that indirect network effects are critical for a successful platform. Ambrus and Argenziano (2009) investigate the conditions for multiple asymmetric networks and analyze the pricing decisions of firms and platform choices of consumers in a two-sided market with network externalities. The heterogeneity of consumer types can lead to asymmetric market structures. Rysman (2018) discusses the concept of the reflection problem of Manski to bear on the problem to argue that indirect network effects provide a much more natural way to address the reflection problem than direct network effects. In our model, consumers get a direct network effect from the content providers' sides.

Our paper contributes to current knowledge on vertical integration in the two-sided media market. We consider the current online streaming platforms conditions and set the supply function of content to investigate how vertical integration affects membership fees, consumer choice, and social welfare.

3.3 The Model

In the market, there are two platforms, two groups of content producers, and one set of viewers.

Platforms: There are two online streaming platforms, $i \in \{1, 2\}$, which are located at the extremes of a Hotelling line: platform 1 is located at zero and platform 2 at one. Platform 2 is dedicated to showing costume films, while platform 1 plays crime movies, so the two platforms are distinguished. Each platform charges a membership fee of m_i and pays l_i per unit of content. The profit function of each platform i is given by:

$$\pi_i = b_i m_i - F_i, \tag{3.1}$$

where b_i is the number of subscribers, and F_i is the total costs of platform i. If the platforms buy movies from upstream providers, $F_i = c_i l_i$, where c_i is the amount of content offered by platform i, and l_i is the license fee per unit of content.

Content producers: There are two groups of content producers operating in the market: V_n and $n \in \{1, 2\}$. The V_1 producers develop solely crime films and offer them exclusively to platform 1. Similarly, producers in group V_2 develop costume movies and exclusively supply platform 2. The supply function of movies for platform i is the following:

$$c_i = wl_i. \tag{3.2}$$

The marginal cost for producers of producing one movie is $f(c_i) = \frac{1}{w}c_i$.

Viewers: There is a mass one of views uniformly distributed on the interval $[0, 1]$. Viewers watch movies on only one platform. The net utility of a viewer located at a distance x from firm i is:

$$v + \theta c_i - m_i - \tau|x_i - x|, \tag{3.3}$$

where v is the gross surplus from the basic content provided by firm i, which is assumed to be large enough that all viewers can watch on a platform. θ is the network parameter from the content's side and τ is the transport cost.

The game: The timing of the game is as follows: In the first stage, both platforms simultaneously establish membership fees for viewers. They also set license fees for content producers before vertical integration or decide how many films to produce after the merger. In the second stage, viewers decide via either platform 1 or 2 to watch films, and content producers simultaneously determine whether to accept the offer from the platforms' side.

Assumption: It is assumed that $\tau > \frac{1}{3}w\theta^2$, to ensure that both platforms are active in the market after vertical integration.

Figure 3.1 represents the situation under vertically separated platforms. Initially, iQIYI and Tencent Video bought content from upstream providers. For example, iQIYI bought crime series from content producers, such as Eternity Pic-

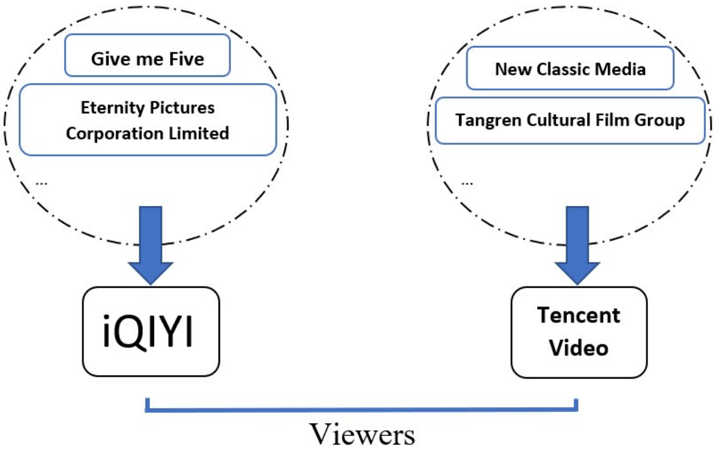

Figure 3.1: Two vertically separated platforms scenario

tures Corporation Limited and Give Me Five, which are famous for producing such series. However, producers such as New Classic Media and Chinese Entertainment Tianjin Limited film quality costume series and primarily their series to Tencent Video. This situation thus leads to a shared knowledge in the Chinese market that iQIYI is known for its crime series and Tencent Video is renowned for its costume series.

Figure 3.2 illustrates that, today, the platforms, iQIYI and Tencent Video, have their own upstream studios or production companies. For example, iQIYI has several studios to produce original video content. Similarly, Tencent Video established Tencent Penguin Pictures is a content-production company that creates series.

Figure 3.3 shows the situation in the asymmetric case. Platform 1 is vertically separated and purchases movies from upstream film providers, while platform 2 is vertically integrated. Inspired by the Chinese video market, we establish the model. In the next sections, we will discuss platform competition under different scenarios respectively.

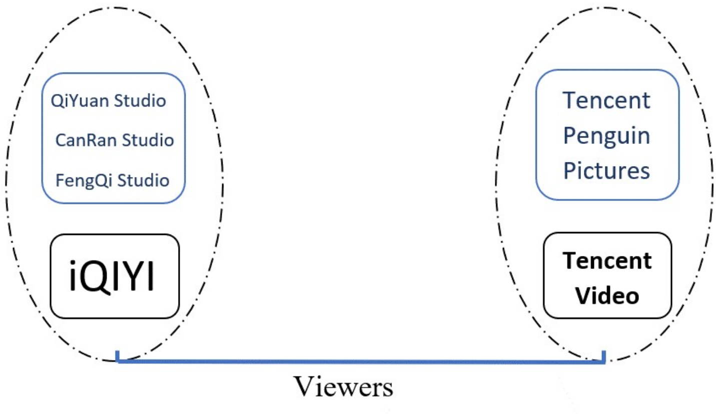

Figure 3.2: Two vertically integrated platforms scenario

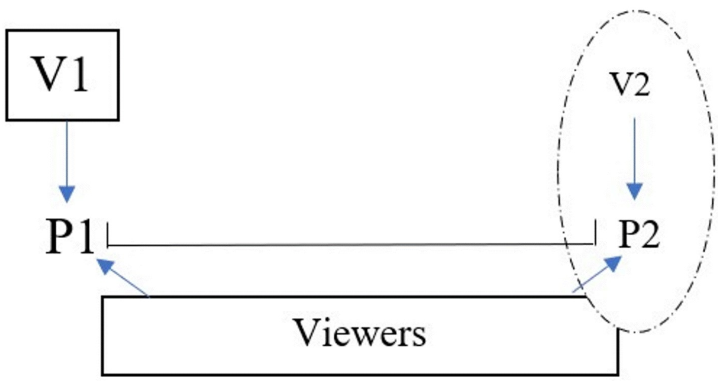

Figure 3.3: Asymmetric case

3.4 Analysis

The discussion here is undertaken in the three above mentioned scenarios: two vertically separated platforms competition, two vertically integrated platforms competition, and the asymmetric case. Note that platforms finance themselves only through viewers' membership fees.

3.4.1 Two Vertically Separated Platforms

In this subsection, we analyze the competition between the two vertically separated platforms. Viewers can only choose one platform, and content offered on one platform cannot be watched on another. Platforms' market share of users depends on the amount of content. Each platform has only one revenue stream: membership fees. In stage 2, the viewers choose platform 1 or 2 have the following different utilities according to formula (3.3):

$$u_1 = v + \theta c_1 - m_1 - \tau x \tag{3.4}$$

and

$$u_2 = v + \theta c_2 - m_2 - \tau (1 - x) . \tag{3.5}$$

We find viewer b_1, who is indifferent between choosing two platforms,

$$\theta c_1 - m_1 - \tau b_1 = \theta c_2 - m_2 - \tau (1 - b_1)$$

solving for b_1 we obtain:

$$b_1 = \frac{1}{2} + \frac{\theta (c_1 - c_2) - (m_1 - m_2)}{2\tau}. \tag{3.6}$$

Solving the number of producers based on equation (3.2), we obtain the following:

$$b_1 = \frac{1}{2} + \frac{\theta w (l_1 - l_2) - (m_1 - m_2)}{2\tau}.$$

The profit of a platform is $\pi_i = b_i m_i - c_i l_i$, and viewers are free to join the platforms. The profit function of the platforms is as follows:

$$\pi_i = \left(\frac{1}{2} + \frac{\theta w \left(l_i - l_j\right) - \left(m_i - m_j\right)}{2\tau}\right) m_i - w\left(l_i\right)^2, \; i, j = 1, 2 \; i \neq j.$$

To solve this maximization problem, Lemma 3.1 shows the results of equilibrium under two vertically separated platforms.

Lemma 3.1 (Equilibrium under two vertically separated platforms).

i) Each platform charges the same membership fee ($m_1 = m_2 = \tau$) and license fee ($l_1 = l_2 = \frac{1}{4}\theta$).

ii) Both platforms provide the same amount of content ($c_1 = c_2 = \frac{1}{4}w\theta$) in the market and share the market equally ($b_1 = b_2 = \frac{1}{2}$).

iii) The profit for both platforms is equal to $\pi_1 = \pi_2 = \frac{1}{2}\tau - \frac{1}{16}w\theta^2$.

Under equilibrium, the two platforms charge the same membership fee to consumers and the same license fee to producers, and platforms share the market equally by providing the same amount of content.

3.4.2 Two Vertically Integrated Platforms

This subsection introduces the second scenario: two vertically integrated platforms competition. Both platforms establish their own upstream film production companies. Now, the total cost for the platforms is not the license fee but the cost of producing movies. The producing cost function is as follows:

$$F_i^v = \frac{1}{2w} \cdot \left(c_i^v\right)^2.$$

This producing cost result comes from the monopsony idea. The marginal cost incurred by movie producers to produce one film is $f(c_i) = \frac{1}{w}c_i$, and the total cost to the platforms of purchasing content is $F_b = c_i \cdot l_i = \frac{1}{w}\left(c_i\right)^2$. The marginal cost to the platforms of purchasing one film is $f^{VS}(c_i) = \frac{2}{w}c_i$. After vertical integration, the marginal cost to platforms for producing one movie, we keep the

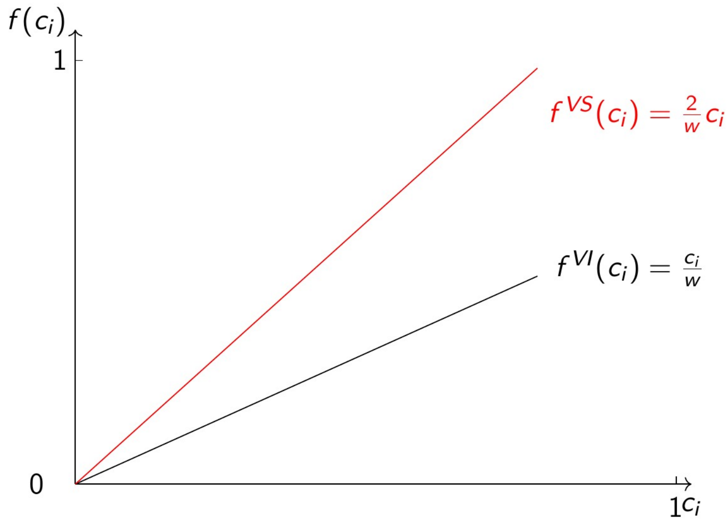

Figure 3.4: Marginal cost of vertically separated and vertically integrated platforms

same as $f^{VI}(c_i) = \frac{1}{w}c_i$. Finally, the total cost to platforms for producing content is $F_p = \int_0^{c_i} \frac{c_i}{w} dc_i = \frac{1}{2w}c_i^2$.

Figure 3.4 represents the marginal cost of the vertically separated and vertically integrated platforms. The vertically integrated platforms make their own movies, while the vertically separate platforms purchase licensed content from content producers. The marginal cost of the vertically separated platforms for each film is double that of vertically integrated platforms.

Subscribers choose platforms based on their utility. As such, the number of viewers on platform 1 is $b_1^v = \frac{1}{2} + \frac{\theta\left(c_1^v - c_2^v\right) - \left(m_1^v - m_2^v\right)}{2\tau}$, and by substituting this number into the profit function for the platforms (3.1), we obtain the profit function of vertically integrated platforms:

$$\pi_i^v = \left(\frac{1}{2} + \frac{\theta\left(c_1^v - c_2^v\right) - \left(m_1^v - m_2^v\right)}{2\tau}\right) m_i^v - \frac{1}{2w} \cdot \left(c_i^v\right)^2 \tag{3.7}$$

To solve this maximization problem, Lemma 3.2 illustrates the results of equilibrium under two vertically integrated platforms.

Lemma 3.2 (Equilibrium under two vertically integrated platforms).

i) Each platform charges the same membership fee, $m_1^v = m_2^v = \tau$.

ii) Both platforms provide the same amount of content in the market ($c_1^v = c_2^v = \frac{1}{2}w\theta$*) and share the market equally (*$b_1^v = b_2^v = \frac{1}{2}$*).*

iii) For both platforms, the profit is $\pi_1^v = \pi_2^v = \frac{1}{2}\tau - \frac{1}{8}w\theta^2$.

From Lemma 3.2, we know that the membership fees of both platforms are the same, and the platforms share the market equally by producing the same amount of content. The total costs of buying and producing content differ for streaming platforms. Comparing equilibrium under two scenarios, we obtain the following results.

Proposition 3.1 (Comparing two symmetrical cases).

i) If both platforms are vertically integrated, users pay the same membership fee and acquire access to more content.

ii) Vertical integration reduces the profitability of vertically separated platforms.

Both the vertically separated and integrated platforms have subscriptions, $m_i = m_i^v = \tau$, which indicates that the price of the service is equivalent to the transport cost. The two types of platforms acquire the same number of viewers under competition. However, vertically integrated platforms deliver more content $c_i^v > c_i$, and content is always the most effective way to market a platform. A greater amount of content makes a platform more likely to appeal to customers. Vertically integrated platforms can offer more movies to encourage people to join their services.

3.4.3 Welfare Analysis

Now, we examine two symmetrical social welfare situations. The consumer surplus is as follows:

$$CS = \int_0^{b_1} U_1 dx + \int_{b_1}^1 U_2 dx, \tag{3.8}$$

and the total social welfare is the sum of consumer surplus and profits from providers and platforms. Under the two vertically separated platforms scenario, consumer surplus equals $v + \frac{1}{4}w\theta^2 - \frac{5}{4}\tau$. The profits of platforms are: $\pi_1 = \pi_2 = \frac{1}{2}\tau - \frac{1}{16}w\theta^2$, and the sum of utility from the content producer side is $\pi_c = 2 \cdot \int_0^{\frac{w\theta}{4}} \left(\frac{\theta}{4} - \frac{l}{w}\right) dl = \frac{w\theta^2}{16}$. Total welfare is $SW = v + \frac{3}{16}w\theta^2 - \frac{1}{4}\tau$. Similarly, consumer surplus can be obtained under the two vertically integrated platforms scenario as $CS^v = v + \frac{1}{2}w\theta^2 - \frac{5}{4}\tau$. Profits of two platforms are: $\pi_1^v = \pi_2^v = \frac{1}{2}\tau - \frac{1}{8}w\theta^2$, and total social welfare is $SW^v = v + \frac{1}{4}w\theta^2 - \frac{1}{4}\tau$. Comparing these two scenarios reveals that consumer surplus and social welfare are greater under the two vertically integrated platforms scenario ($CS^v > CS$ and $SW^v > SW$). The availability of more films on both platforms enhances the consumer surplus. When platforms are fully vertically integrated, their profitability decreases. However, the increase in consumer surplus compensates for the loss in platform earnings, resulting in an increase in social welfare in the case of vertically integrated platforms.

3.4.4 Asymmetric Case

We now analyze the asymmetric case, in which one platform vertically integrates with upstream suppliers while the other remains vertically separated. In this case, we assume platform 1 is the vertically separated platform, while platform 2 integrates vertically. As such, the profit function of platform 1 is $\pi_1^s = b_1^s m_1^s - c_1^s l_1^s$ and of platform 2 is $\pi_2^s = b_2^s m_2^s - F_2^s(c_2^s)$, where $F_2^s(c_2^s) = \frac{1}{2w} \cdot (c_2^s)^2$ is the total cost of platform 2's production of all films. Group $V1$ producers create their own crime films and sell them to the downstream online streaming platform 1, and the marginal cost of production for one film is $f(c_1^s) = \frac{1}{w}c_1^s$. Producers in the other group, $V2$, are fully vertically integrated with platform 2.

As previously indicated, subscribers choose platforms based on their utility (3.4) and (3.5). The number of people watching on platform 1 is $b_1^s = \frac{1}{2} + \frac{\theta(c_1^s - c_2^s) - (m_1^s - m_2^s)}{2\tau}$. The profit function of platform 1 is calculated by substituting this number into platform 1's profit function:

$$\pi_1^s = \left(\frac{1}{2} + \frac{\theta\left(wl_1^s - c_2^s\right) - \left(m_1^s - m_2^s\right)}{2\tau}\right) m_1^s - \left(wl_1^s\right) l_1^s \tag{3.9}$$

The profit function of platform 2 is as follows:

$$\pi_2^s = \left(\frac{1}{2} + \frac{\theta\left(c_2^s - wl_1^s\right) - \left(m_2^s - m_1^s\right)}{2\tau}\right) m_2^s - \frac{1}{2w}\left(c_2^s\right)^2 \tag{3.10}$$

To solve this maximization problem, Lemma 3.3, shows the results of the equilibrium in the asymmetric case:

Lemma 3.3 (Equilibrium in the asymmetric case).

i) Platforms charge different membership fees: $m_1^s = \frac{12\tau^2 - 4w\theta^2\tau}{12\tau - 3w\theta^2}$ and $m_2^s = \frac{12\tau^2 - 2w\theta^2\tau}{12\tau - 3w\theta^2}$.

ii) Platforms provide different amount of content in the market: $c_1^s = \frac{w\theta\left(3\tau - w\theta^2\right)}{12\tau - 3w\theta^2}$ and $c_2^s = \frac{w\theta\left(6\tau - w\theta^2\right)}{12\tau - 3w\theta^2}$.

iii) Platforms have different number of consumers: $b_1^s = \frac{6\tau - 2w\theta^2}{12\tau - 3w\theta^2}$ and $b_2^s = \frac{6\tau - w\theta^2}{12\tau - 3w\theta^2}$.

iv) The profit of both platforms are $\pi_1^s = \frac{1}{9}\frac{\left(3\tau - w\theta^2\right)^2}{\left(4\tau - w\theta^2\right)^2}\left(8\tau - w\theta^2\right)$ and $\pi_2^s = \frac{1}{18}\frac{\left(6\tau - w\theta^2\right)^2}{\left(4\tau - w\theta^2\right)}$.

Comparing the results from Lemma 2.3, it is clear that $m_1^s < m_2^s$, $c_1^s < c_2^s$, $b_1^s < b_2^s$ and $\pi_1^s < \pi_2^s$.[9] We can thus summarize our findings in the following straightforward proposition:

Proposition 3.2 (Comparing the two platforms in the asymmetric case).

The vertically integrated platform (platform 2) plays more content, has more

[9] The difference between the profit of two platforms under this asymmetric case is $\Delta\pi^s = \pi_2^s - \pi_1^s = \frac{\left(6\tau - w\theta^2\right)^2}{18\left(4\tau - w\theta^2\right)} - \frac{1}{9}\frac{\left(3\tau - w\theta^2\right)^2}{\left(4\tau - w\theta^2\right)^2}\left(8\tau - w\theta^2\right) = \frac{1}{18}\frac{w\theta^2}{\left(4\tau - w\theta^2\right)^2}\left(w^2\theta^4 - 12w\theta^2\tau + 30\tau^2\right)$. The first two parts, $\frac{1}{18}\frac{w\theta^2}{\left(4\tau - w\theta^2\right)^2}$, are larger than zero. The last part, $w^2\theta^4 - 12w\theta^2\tau + 30\tau^2$ could be rewritten as $(w\theta^2 - 3\tau)^2 + 9\tau(3\tau - w\theta^2)$. Because of our assumption, $\frac{1}{3}w\theta^2 < \tau < 1$, this equation is larger than zero, and we could obtain $\Delta\pi^s > 0$, namely $\pi_2^s > \pi_1^s$.

users, establishes higher membership fees, and acquires more profits than the vertically separated platform (platform 1).

In this case, platform 2 outperforms the other. Although the vertically integrated platform establishes a higher membership fee, viewers are more willing to watch films on platform 2 due to the greater amount of content. More specifically, platform 2 customers must pay $2w\theta^2\frac{\tau}{12\tau-3w\theta^2}$ more than platform 1 customers, but they can enjoy $w\theta\frac{\tau}{4\tau-w\theta^2}$ more content. Viewers who switch from platform 1 to platform 2 are better off than those who remain on platform 1.

Lemma 3.4 (Network parameter effect).

i) The higher the θ, the more content and the more users and the higher the vertically integrated platform membership fee.

ii) The higher the θ, the fewer users, the lower the vertically separated platform membership fee and the higher license fee.

The vertically linked platform generates more content and can directly charge higher membership fees from the more substantial network effect it enjoys. If the network effect grows, more users will likely sign up. In other words, the greater the number of films, the greater the number of new customers on the vertically integrated platform. The platform company's primary goal is to have more people sign up for their services and pay the required subscription fees, and this revenue can be used to pay for the cost of making movies. There is an increased membership charge because the platform must pay more for a greater amount of content. This vertically separated platform wishes to utilize additional content to attract more subscribers and must therefore increase its license fee to acquire more content. However, the separate platform has fewer consumers due to less content than the integrated platform. Then the vertically separated platform will reduce the price against the competition.

The equation (3.8) can be used to obtain consumer surplus under the asym-

P2 / P1	Vertically Separated	Vertically Integrated
Vertically Separated	π, π	π^s_{VS}, π^s_{VI}
Vertically Integrated	π^s_{VI}, π^s_{VS}	π^v, π^v

Table 3.1: The profits of platforms under the baseline model

metric case:

$$CS^s = v - \frac{1}{18\left(4\tau - w\theta^2\right)^2}\left(-6w^3\theta^6 + 73w^2\theta^4\tau - 288w\theta^2\tau^2 + 360\tau^3\right).$$

When we add the total profits of both platforms and the first group of upstream producers' profits, we obtain the total welfare in this market

$$SW^s = v - \frac{1}{18\left(4\tau - w\theta^2\right)^2}\left(-4w^3\theta^6 + 35w^2\theta^4\tau - 99w\theta^2\tau^2 + 72\tau^3\right).$$

3.4.5 Summary

In this subsection, we summarize the model and compare symmetric and asymmetric cases.

Table 3.1 depicts platform profits in various scenarios where $\pi = \frac{1}{2}\tau - \frac{1}{16}w\theta^2$, $\pi^s_{VI} = \frac{1}{18}\frac{\left(6\tau - w\theta^2\right)^2}{\left(4\tau - w\theta^2\right)}$, $\pi^s_{VS} = \frac{1}{9}\frac{\left(3\tau - w\theta^2\right)^2}{\left(4\tau - w\theta^2\right)^2}\left(8\tau - w\theta^2\right)$, and $\pi^v = \frac{1}{2}\tau - \frac{1}{8}w\theta^2$. After calculating, we obtained the following: $\pi^s_{VS} < \pi^v < \pi < \pi^s_{VI}$. This equilibrium is comparable to the problem referred to in the prisoner's dilemma. Both platforms will earn π if, from the beginning, they compete on the market as vertically separate platforms. When a platform decides to integrate vertically, it cuts into the profits of platforms that compete with it. This reduction in profit makes it more likely for the platform to merge with upstream movie suppliers. When both have completed vertical integration with producers, their respective revenues will equal π^v. This profit is less than it would be if separated vertically.

Proposition 3.3 (Platforms' decision).

Platforms always have an incentive to choose vertical integration. If one plat-

form has vertical integration, another will implement the same strategy.

The unique equilibrium of this game results in two vertically integrated platforms competing in the online streaming video market. Although platforms would like to earn more, they nonetheless must choose vertical integration. The motivation for vertical integration is cost reduction, however, platforms acquire fewer profits afterward because of competition.

In Section 3.4.3, we compared the consumer surplus and social welfare under the symmetric case and found that both are greater under the two vertically integrated platforms scenario. In this section, we will compare consumer surplus and social welfare under symmetric and asymmetric circumstances. As mentioned in Section 3.4.3, consumer surplus under the two vertically separated platforms scenario is CS and CS^s in the asymmetric case. Comparing CS and CS^s shows that consumer surplus is higher under the asymmetric case, $CS^s > CS$. Under the asymmetric case, subscribers of platform 2 (the vertically integrated platform) have more content to choose from, while viewers of platform 1 have fewer options.[10] The loss of subscribers choosing platform 1 is less than the gain of consumers choosing platform 2. Each viewer on platform 1 pays less, and viewers on platform 2 pay more under the asymmetric case.[11] The gain in subscriptions from consumers on platform 1 is the same as the loss of viewers on platform 2: $\Delta m_1^s + \Delta m_2^s = 0$. However, the number of subscribers on platform 2 under the asymmetric case is more prominent than on platform 1. Moreover, although the average membership fee is higher in the asymmetric case, the gain from more content compensates for this loss, so the consumer surplus is higher in the asymmetric case compared to the two vertically separated platforms scenario.

From previous calculations, we know that $CS^v = v + \frac{1}{2}w\theta^2 - \frac{5}{4}\tau$ is the consumer surplus under the two vertically integrated platforms scenario. The difference in

[10] Because $\Delta c_2^s = c_2^s - c_2 = \frac{1}{12}\frac{w\theta}{4\tau - w\theta^2}\left(12\tau - w\theta^2\right) > 0$, and $\Delta c_1^s = c_1^s - c_1 = -w^2\frac{\theta^3}{48\tau - 12w\theta^2} < 0$, $\Delta c_1^s - \Delta c_2^s = -w\theta\frac{\tau}{4\tau - w\theta^2} < 0$.

[11] This is because $\Delta m_1^s = m_1^s - m_1 = -w\theta^2\frac{\tau}{12\tau - 3w\theta^2} < 0$, and $\Delta m_2^s = m_2^s - m_2 = w\theta^2\frac{\tau}{12\tau - 3w\theta^2} > 0$.

consumer surplus between the two vertically integrated platform scenario and the asymmetric case is $\Delta CS = CS^v - CS^s$, which is the red line of Figure 3.5. We could find that some area on the left side is still smaller than zero, which occurs if the transport cost is small enough, which would mean that the consumer surplus in the asymmetric case is larger. When the transport cost exceeds the value $\tau > \frac{3}{8}w\theta^2$, the consumer surplus in the two vertically integrated platforms scenario is larger, with $\Delta CS > 0$. Furthermore, the consumer surplus is influenced by two factors, price and network effect, which Lemma 3.5 considers.

Lemma 3.5 (Average price and network effect).

i) The average market price is higher in the asymmetric case: $\overline{m}^v < \overline{m}^s$.

ii) If $\tau \leq \frac{1}{2}w\theta^2$, the *average network effect is higher in the asymmetric case:* $\theta\overline{c}^v \leq \theta\overline{c}^s$. *If* $\tau > \frac{1}{2}w\theta^2$, the *average network effect is higher in the two vertically integrated platforms scenario:* $\theta\overline{c}^v > \theta\overline{c}^s$.

In the asymmetric case, there is a higher average price, and the network effect (the amount of content) also influences utility. In symmetric cases, the amount of content does not depend on transport cost, but it does in asymmetric cases. When τ increases, the amount of content on platform 1 (disintegrated) grows but that on platform 2 (integrated) decreases. The average network effect is higher under the two integrated platforms scenario when the transport cost is smaller than $\frac{1}{2}w\theta^2$. In the asymmetric situation, more consumers prefer a vertically integrated platform that obtains more video content than a vertically separated platform. In this way, more viewers can enjoy more films, which makes the average network effect greater in the asymmetric case. According to the asymmetric case, the more consumers and content on platform 2, the fewer consumers and content on platform 1. When there are not many differences between platforms, platform 2 has more members, even though it costlier. The gain from a more extensive network effect from content providers compensates for the loss due to higher prices, which is why more consumers would still choose platform 2.

Total social welfare under the two vertically separated platforms scenario

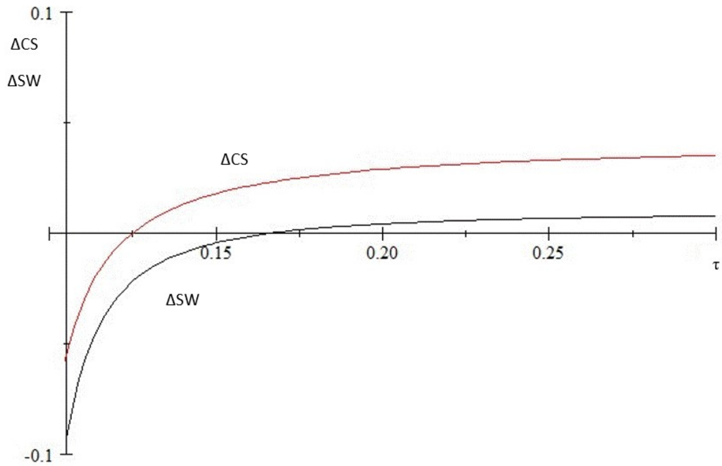

Figure 3.5: The comparison of consumer surplus and social welfare

is $SW = v + \frac{3}{16} w\theta^2 - \frac{1}{4}\tau$, which is smaller than the asymmetric case, where $SW^s > SW$. Platform 1 profits less in the asymmetric case, while platform 2 profits more.[12] However, platform 2's gain cannot compensate for platform 1's loss: $\Delta\pi^s_{VI} + \Delta\pi^s_{VS} < 0$. The license fee for producers in group $V1$ in the asymmetric case is $l^s_1 = \frac{\theta(3\tau - w\theta^2)}{12\tau - 3w\theta^2}$, which is less than the symmetric case of the two vertically separated platforms scenario, where $\Delta l^s_1 = l^s_1 - l_1 = -w\frac{\theta^3}{48\tau - 12w\theta^2} < 0$. Under the asymmetric case, the total profits of producers in $V1$ are $\pi^s_c = \frac{1}{2} w \left(\frac{\theta(3\tau - w\theta^2)}{12\tau - 3w\theta^2}\right)^2$, which is less than the symmetric case of two vertically separated platforms scenario, where$\pi^s_c < \pi_c = \frac{w\theta^2}{32}$. The overall increase in consumers' utility compensates for the loss of total profits for platforms and producers and this compensation leads to an increase in total social welfare. The difference in social welfare between the asymmetric case and the two integrated platforms scenario is shown as the black line of Figure 3.5. Similarly, as consumer surplus, sometimes $\Delta SW = SW^v - SW^s < 0$. If transportation costs more than $\frac{1}{2} w\theta^2$, social welfare is greater in the scenario of two vertically integrated platforms, $SW^v > SW^s$.

[12]Note that $\Delta\pi^s_{VS} = \pi^s_{VS} - \pi = -\frac{1}{144}\frac{w\theta^2}{(4\tau - w\theta^2)^2}\left(7w^2\theta^4 - 80w\theta^2\tau + 192\tau^2\right) < 0$, and $\Delta\pi^s_{VI} = \pi^s_{VI} - \pi = \frac{1}{144}\frac{w\theta^2}{4\tau - w\theta^2}\left(12\tau - w\theta^2\right) > 0$.

This value, $\tau > \frac{1}{2}w\theta^2$, makes both consumer surplus and social welfare higher if both platforms have vertical integration.

3.5 Model with Advertisements

Recently, Netflix began discussions on whether to include advertisements. The streaming service plans to introduce a cheaper subscription option that will be supported by advertising, targeting the US market at around USD $7–9 a month. This price represents a discount of USD $9.99 a month on the cheapest existing plan. For Netflix, subscription fees with advertisements are less expensive. However, advertising revenue is a crucial and indispensable component of China's online streaming platform market, whether the platform is iQIYI or Tencent Video. According to the reports of both platforms, during the second quarter of 2022, iQIYI's advertising revenue of RMB 1.2 billion decreased by 35%, and that of Tencent's of RMB 2.5 billion decreased by 25%. Even as a member, viewers are compelled to see advertisements, which diminishes their interest in these platforms. If a viewer is not a member, they can only watch a few shows and series. However, for consumers to view exclusive content, they must purchase a membership, despite being forced to watch advertisements.

Considering this, we now present the model with advertisers as the third component. Section 3.5.1 investigates competition among platforms that generate revenue from advertising and subscriptions that purchase content from providers. In Section 3.5.2, we endogenize the decision to integrate vertically, and in Section 3.5.3, we consider an asymmetric case with a numeric example.

In the market, there are still two different platforms, two different groups of content providers, and one set of viewers, but a group of advertisers is now added.

Platforms: The platforms are located at two extreme points on the Hotelling line and play different types of movies. Each platform imposes a membership fee of m_i^A as well as an advertising fee p_i^A and spent l_i^A on the acquisition of content from upstream producers. For each platform, the profit function with advertisers works as follows:

$$\pi_i^A = b_i^A m_i^A + a_i^A p_i^A - F_i^A, \tag{3.11}$$

where b_i^A is the number of subscribers, a_i^A is the number of advertisers, and F_i^A represents the total costs of platform i. The first two parts of the profit function are the revenue from the membership fee, while the last part is the cost of buying content from producers or producing movies.

Content producers: Two groups of providers have the same story as what we introduced in section 3.3, and the supply function is $c_i^A = wl_i^A$.

Viewers: This unit of viewers is located on the Hotelling line; they watch the movies by choosing only one platform, either 1 or 2. The utility of a viewer with preference x from watching content by choosing platform i is:

$$v + \theta c_i^A - \gamma a_i^A - m_i^A - \tau|x_i - x|, \tag{3.12}$$

where θ is the network parameter from the content side, γ is the disutility parameter from the advertising side and τ is the transport cost.

Advertisers: There is one unit mass of advertisers in the market. Each advertiser has to pay p_i^A to platform i to advertise on this platform. The inverse demand function is

$$p_i^A = k - \varphi a_i^A.$$

We assume that each advertiser only has one advertisement, and $k > \gamma$.

The game: The timing of the game is as follows. In the first stage, the platforms before vertical integration simultaneously establish membership fees for viewers, license fees for content providers, and advertising fees for advertisers. After the vertical merger, they determine membership fees and advertising fees for consumers and advertisers. In the second stage, viewers decide to watch films on either platform 1 or 2, and content producers and advertisers decide whether to accept the offer from the platform's side.

Assumptions: Transport cost is $\frac{1}{3}w\theta^2 + \frac{1}{6\varphi}\gamma^2 < \tau < 1$ to ensure that vertically integrated platforms are active in the market.

3.5.1 Equilibrium Analysis

In this section, we analyze the competition between the two platforms with advertisers. Given the utility in (3.12), an indifferent consumer b_1^A can be found through:

$$b_1^A = \frac{1}{2} + \frac{\theta(c_1^A - c_2^A) - \gamma\left(a_1^A - a_2^A\right) - \left(m_1^A - m_2^A\right)}{2\tau}.$$

Solving for the number of producers based on equation (3.2), we also obtain the following:

$$b_1^A = \frac{1}{2} + \frac{\theta w\left(l_1^A - l_2^A\right) - \gamma\left(a_1^A - a_2^A\right) - \left(m_1^A - m_2^A\right)}{2\tau} \tag{3.13}$$

Given the demand function of viewers, the profit function of platform i according to (3.11) before vertical integration is

$$\pi_i^A = b_i^A m_i^A + a_i^A p_i^A - w\left(l_i^v\right)^2,\ i,j = 1,2\ i \neq j.$$

To solve this maximization problem, Lemma 3.6 represents the results of equilibrium under two vertically separated platforms.

Lemma 3.6 (Equilibrium with advertisers under two vertically separated platforms).

i) The platforms establish the same membership fee, $m_1^A = m_2^A = \tau$, *and the same license fee,* $l_1^A = l_2^A = \frac{1}{4}\theta$.

ii) The platforms both provide the same amount of content in the market, $c_1^A = c_2^A = \frac{1}{4}w\theta$, *and share the market equally,* $b_1^A = b_2^A = \frac{1}{2}$.

iii) The platforms have the same number of advertisements, $a_1^A = a_2^A = \frac{1}{4\varphi}\left(2k - \gamma\right)$, *and establish the same advertising fee,* $p_1^A = p_2^A = \frac{1}{2}k + \frac{1}{4}\gamma$.

iv) The profit of both platforms is $\pi_1^A = \pi_2^A = \frac{1}{16\varphi}\left(4k^2 - w\varphi\theta^2 - \gamma^2 + 8\tau\varphi\right)$.

Subscribers choose platforms based on their utility. The number of viewers in platform 1 is b_1^{Av}, and if we substitute this number into the platforms' profit function (3.11), we obtain the profit function of vertically integrated platforms:

$$\pi_i^{Av} = \left(\frac{1}{2} + \frac{\theta(c_1^{Av} - c_2^{Av}) - \gamma\left(a_1^{Av} - a_2^{Av}\right) - \left(m_1^{Av} - m_2^{Av}\right)}{2\tau}\right) m_i^{Av} + ap_i^{Av} - \frac{1}{2w} \cdot \left(c_i^{Av}\right)^2 \quad (3.14)$$

To solve this maximization problem, Lemma 3.7 represents the results of equilibrium under two vertically integrated platforms.

Lemma 3.7 (Equilibrium with advertisers under two vertically integrated platforms).

i) The platforms establish the same membership fee: $m_1^{Av} = m_2^{Av} = \tau$.

ii) The platforms both provide the same amount of content in the market, $c_1^{Av} = c_2^{Av} = \frac{1}{2}w\theta$, *and share the market equally,* $b_1^{Av} = b_2^{Av} = \frac{1}{2}$.

iii) The platforms have the same number of advertisements, $a_1^{Av} = a_2^{Av} = \frac{1}{4\varphi}(2k - \gamma)$, *and establish the same advertising fee,* $p_1^{Av} = p_2^{Av} = \frac{1}{2}k + \frac{1}{4}\gamma$.

iv) The profit of both platforms is $\pi_1^{Av} = \pi_2^{Av} = \frac{1}{16\varphi}\left(4k^2 - 2w\varphi\theta^2 - \gamma^2 + 8\tau\varphi\right)$.

The comparison of the two symmetric cases with the advertisers' cases is summarized in the following proposition.

Proposition 3.4 (Comparing two symmetric advertising cases).

i) If both platforms are vertically integrated, users pay the same membership fee and acquire access to more content, and advertisers pay the same advertising fee.

ii) Vertical integration reduces the profitability of platforms.

The outcomes of Proposition 3.4 are identical to those of Proposition 3.1. In equilibrium, membership fees for subscribers and advertising fees for advertisers are identical prior to and following vertical integration. Likewise, before and after vertical integration, the cost of advertising and the number of advertisements are the same for advertisers, and the surplus of advertising does not change. Although they pay the same membership fee, subscribers can watch more content after vertical integration. Because of monopsony, the total cost of platforms is higher when they vertically integrate with upstream providers. Therefore, the profits of platforms are lower after vertical integration.

Proposition 3.5 (Comparing the model with and without advertisements).

i) The platforms establish the same membership fee: $m_i = m_i^A = \tau$, $m_i^v = m_i^{Av} = \tau$.

ii) The platforms offer the same quantity of content: $c_i = c_i^v = \frac{1}{4}w\theta$, $c_i^v = c_i^{Av} = \frac{1}{2}w\theta$.

iii) Under the model with advertising, the profit of both platforms is higher: $\pi_i < \pi_i^A$, $\pi_i^v < \pi_i^{Av}$.

iv) Advertising reduces the utility of subscribers: $u_i > u_i^A$, $u_i^v > u_i^{Av}$.

Platform profits are higher with the third side, advertisers, because they have an additional revenue source in, advertising fees. However, consumers' utility is diminished due to advertising's ineffectiveness. The membership prices and the quantity of material remain the same, but after the addition of commercials, users are forced to view advertisements, resulting in a decrease in their utility.

3.5.2 Welfare Analysis

In this subsection, we examine social welfare. We use the same method as the formula (3.8) shown in Section 3.4.3 to calculate the consumer surplus. $CS^A = v + \frac{1}{4\varphi}\left(w\varphi\theta^2 + \gamma^2 - 2k\gamma - 5\tau\varphi\right)$ is the consumer surplus across two vertically separated platforms' scenarios with advertising. The profits of the two platforms are: $\pi_p^A = \frac{1}{8\varphi}\left(4k^2 - w\varphi\theta^2 - \gamma^2 + 8\tau\varphi\right)$ and the sum of utility from the content provider side is $\pi_c^A = 2 \cdot \int_0^{\frac{w\theta}{4}} \left(\frac{\theta}{4} - \frac{l}{w}\right) dl = \frac{w\theta^2}{16}$. Total welfare is:

$$SW^A = v + \frac{1}{16\varphi}\left(8k^2 - 8k\gamma + 3w\varphi\theta^2 + 2\gamma^2 - 4\tau\varphi\right).$$

Similarly, we can obtain consumer surplus under the two vertically integrated platforms scenario, $CS^{Av} = v + \frac{1}{4\varphi}\left(2w\varphi\theta^2 + \gamma^2 - 2k\gamma - 5\tau\varphi\right)$. The total profits of the two platforms would be $\pi_p^{Av} = \frac{1}{8\varphi}\left(4k^2 - 2w\varphi\theta^2 - \gamma^2 + 8\tau\varphi\right)$, and social welfare would be

$$SW^{Av} = v + \frac{1}{8\varphi}\left(4k^2 - 4k\gamma + 2w\varphi\theta^2 + \gamma^2 - 2\tau\varphi\right).$$

A comparison of these two scenarios reveals that we can obtain results similar to those in Section 3.4.3, where consumer surplus and social welfare are greater under the two vertically integrated platforms scenario ($CS^{Av} > CS^A$ and $SW^{Av} > SW^A$). The additional content enhances the consumer surplus, but after vertical integration, platforms face low profits. Ultimately, the increase in consumer surplus compensates for the loss in platform earnings, resulting in a decrease in social welfare in the case of vertically integrated platforms.

3.5.3 Asymmetric Case

In this section, we list a numerical example because it is difficult to calculate using a general model. We assume $k = 1$ and $\varphi = 1$, and the inverse demand function of advertisers is $p_i = 1 - a_i$. The profit function for platform 1, the vertically separated platform, is

$$\pi_1^{As} = m_1^{As}\left(\frac{1}{2} + \frac{\theta\left(wl_1^{As} - c_2^{As}\right) - \gamma\left(a_1^{As} - a_2^{As}\right) - \left(m_1^{As} - m_2^{As}\right)}{2\tau}\right) + a_1^{As}\left(1 - a_1^{As}\right) - \left(wl_1^{As}\right) l_1^{As},$$

and the profit function for platform 2, which is vertically integrated, is

$$\pi_2^{As} = +m_2\left(\frac{1}{2} + \frac{\theta\left(c_2^{As} - wl_1^{As}\right) - \gamma\left(a_2^{As} - a_1^{As}\right) - \left(m_2^{As} - m_1^{As}\right)}{2\tau}\right) + a_2^{As}\left(1 - a_2^{As}\right) - \frac{1}{2w}\cdot\left(c_2^{As}\right)^2.$$

If we solve this problem, we can obtain the membership fees, $m_1^{As} < \tau < m_2^{As}$ in equilibrium. The vertically separated platform establishes lower subscription fees, which is the same result as calculated without the advertisements. Thus, the profit of vertically separated platforms in the asymmetric case is smaller than other vertically integrated platforms: $\pi_i^A < \pi_{VS}^{As} < \pi_{VI}^{As} < \pi_i^{Av}$. This game concludes with the two platforms being vertically integrated in the market. We use a numerical example to demonstrate welfare analysis. We set $\theta = \frac{2}{3}$, and $\gamma = \frac{1}{3}$, $w = 3$, while the consumer surplus is $CS^{Av} > CS^{As}$ if $\tau > \frac{5}{108}\sqrt{5} + \frac{5}{12}$. For social welfare, $SW^{Av} > SW^{As}$ if the transport cost $\tau > \frac{1}{27}\sqrt{3}\sqrt{17} + \frac{23}{54}$. Therefore, we find a similar result here with advertisers as the model without advertisers. Although platforms gain additional revenue from advertising fees, advertisements' disutility affects consumers. Ultimately, the more advertisements on a platform, the fewer the chances of consumers buying the subscription.

3.6 Extension

In Section 3.4, we found that platforms have an incentive to choose vertical integration and two vertically integrated platforms scenario is the unique equilibrium in the market. In this section, advertisements are added to one platform as additional revenue. We use a specific numerical example to demonstrate this idea.

There are two online streaming platforms in the market. Platform 1 has additional revenue from advertising, while platform 2 gains only from subscriptions. The profit functions of platforms are

$$\pi_i^e = b_i^e m^e + a_i^e p_i^e - F_i^e,$$

where $p_i^e = 1 - a_i^e$ and is the inverse demand function of advertisements, and F_i^e is the total cost of platforms. If platforms are vertically integrated, $F_i^e = \frac{1}{2w} \cdot (c_i^e)^2$, where c_i^e is the amount of content. If platform i is vertically separated from producers, the production cost is $F_i^e = c_i^e l_i^e$, where l_i^e is the license revenue of each movie from the platform's side. The supply function of movies is $c_i^e = wl_i^e$, and $w = 3$. We assume that transport costs to be $\tau > \frac{1}{2}$ to ensure that both platforms are active in the market. The marginal cost of producers for producing one film is $f(c_i^e) = \frac{1}{3}c_i^e$. Finally, the utility function of consumers is as follows:

$$v + \theta c_i^e - \gamma a_i^e - m_i^e - \tau x.$$

Assuming the network effect parameter is $\theta = \frac{2}{3}$ and the disutility from advertising is $\gamma = \frac{1}{3}$, the indifferent consumer is located at

$$b_1^e = \frac{1}{2} + \frac{\theta\left(c_1^e - c_2^e\right) - \left(m_1^e - m_2^e\right) - \gamma a_1^e}{2\tau}. \tag{3.15}$$

The profit of platform 1 is

$$\pi_1^e = \left(\frac{1}{2} + \frac{\theta\left(c_1^e - c_2^e\right) - \left(m_1^e - m_2^e\right) - \gamma a_1^e}{2\tau}\right) m_1^e + a_1^e\left(k - \varphi a_1^e\right) - F_1^e. \tag{3.16}$$

The profit of platform 2 is:

$$\pi_2^e = \left(\frac{1}{2} + \frac{\theta\left(c_2^e - c_1^e\right) - \left(m_2^e - m_1^e\right) + \gamma a_1^e}{2\tau}\right) m_2^e - F_2^e \tag{3.17}$$

There are four cases of this extension: the two platforms are vertically separated, platform 1 is vertically integrated, platform 2 is vertically integrated, and the two platforms vertically merged.

3.6.1 Equilibrium Analysis

We begin with the two platforms that are vertically separated.

Lemma 3.8.

i) The subscriptions of the platforms are $m_1^{e1} = \frac{\tau(108\tau-30)}{108\tau-25}$ and $m_2^{e1} = \frac{\tau(108\tau-20)}{108\tau-25}$.

ii) The amount of content is $c_1^{e1} = \frac{3(18\tau-5)}{108\tau-25}$ and $c_2^{e1} = \frac{3(54\tau-10)}{324\tau-75}$.

iii) The numbers of the platforms' subscribers are $b_1^{e1} = \frac{54\tau-15}{108\tau-25}$ and $b_2^{e1} =$

$\frac{54\tau-10}{108\tau-25}$.

iv) Platform 1 has $a_1^{e1} = \frac{1}{108\tau-25}(45\tau - 10)$ *advertisements and a set advertising fee at* $p_1^{e1} = \frac{1}{108\tau-25}(63\tau - 15)$.

v) The profits for both platforms are $\pi_1^{e1} = \frac{3(1944\tau^3-459\tau^2-105\tau+25)}{(108\tau-25)^2}$ and $\pi_2^{e1} = \frac{4}{3}\frac{6\tau-1}{(108\tau-25)^2}(27\tau - 5)^2$.

This scenario refers to (VS_1, VS_2), where both platforms are active in the market, but platform 1 has lower subscription fees due to advertising. Platform 1 must establish a lower price to compensate for the disutility of advertising. Otherwise, viewers will be more inclined to choose platform 2. The license fee of platform 1 is lower than the other one, which suggests that platform 1 provides less content. Platform 1 has an additional source of revenue, so it does not need to gain only from the membership fee. Therefore, this platform has less incentive to attract consumers, leading to lower license fees and less content. Due to a greater amount of content, platform 2 has more subscribers than platform 1, $b_1^{e1} < b_2^{e1}$. However, even with fewer subscribers and lower membership fees, platform 1 still gains more than platform 2, $\pi_1^{e1} > \pi_2^{e1}$. Online streaming platforms today want to include advertising in their videos because they can make more money with fewer subscribers. Lemma 3.9 represents platform 1 choosing vertical integration and platform 2 remaining independent (VI_1, VS_2).

Lemma 3.9.

i) The subscriptions of the platforms are $m_1^{e2} = \frac{\tau(108\tau-30)}{108\tau-37}$ and $m_2^{e2} = \frac{\tau(108\tau-44)}{108\tau-37}$.

ii) The amount of content is: $c_1^{e2} = \frac{(108\tau-30)}{108\tau-37}$ and $c_2^{e2} = \frac{(54\tau-22)}{108\tau-37}$.

iii) The number of subscribers of the platforms are $b_1^{e2} = \frac{54\tau-15}{108\tau-37}$ and $b_2^{e2} = \frac{54\tau-22}{108\tau-37}$.

iv) Platform 1 has $a_1^{e2} = \frac{1}{108\tau-37}(45\tau - 16)$ *advertisements and a set advertising fee at* $p_1^{e2} = \frac{1}{108\tau-37}(63\tau - 21)$.

v) The profits of both platforms are $\pi_1^{e2} = \frac{3(1944\tau^3-783\tau^2-141\tau+62)}{(108\tau-37)^2}$ and $\pi_2^{e2} = \frac{4}{3}\frac{6\tau-1}{(108\tau-37)^2}(27\tau - 11)^2$.

If platform 1, which includes advertisements, vertically merges with producers, it establishes a higher membership fee than platform 2, $m_1^{e2} > m_2^{e2}$. After vertical integration, platform 1 has a higher total cost, which is also why its subscription price is higher than before. Although the price of consumption on platform 1 is higher than that on platform 2, the number of subscribers on platform 1 is larger, $b_1^{e2} > b_2^{e2}$. The reason is that platform 1 provides more content, $c_1^{e2} > c_2^{e2}$, which makes it worthwhile for consumers to pay more. Platform 1 gains more because of a greater number of subscribers and a higher membership fee, $\pi_1^{e2} > \pi_2^{e2}$. Lemma 3.10 demonstrates platform 1 vertically separating from producers and platform 2 having vertical integration (VS_1, VI_2).

Lemma 3.10.

i) The subscriptions of the platforms are $m_1^{e3} = \frac{\tau(108\tau-54)}{108\tau-37}$ and $m_2^{e3} = \frac{\tau(108\tau-20)}{108\tau-37}$.

ii) The amount of content is: $c_1^{e3} = \frac{(54\tau-27)}{108\tau-37}$ and $c_2^{e3} = \frac{(108\tau-20)}{108\tau-37}$.

iii) The number of subscribers of platforms are $b_1^{e3} = \frac{54\tau-27}{108\tau-37}$ and $b_2^{e3} = \frac{54\tau-10}{108\tau-37}$.

iv) Platform 1 has $a_1^{e3} = \frac{1}{108\tau-37}(45\tau-14)$ *advertisements and a set advertising fee at* $p_1^{e3} = \frac{1}{108\tau-37}(63\tau-23)$.

v) The profits of both platforms are $\pi_1^{e3} = \frac{\left(5832\tau^3-3969\tau^2+513\tau+79\right)}{(108\tau-37)^2}$ and $\pi_2^{e3} = \frac{8}{3}\frac{(3\tau-1)}{(108\tau-37)^2}(27\tau-5)^2$.

Platform 2 has more subscribers under equilibrium, $b_1^{e3} < b_2^{e3}$, although platform 1 has a lower membership fee, $m_1^{e3} < m_2^{e3}$. Platform 2 has more content and no advertisements, so consumers do not need to watch advertising but can enjoy watching more movies. In this case, platform 2 also profits more, $\pi_1^{e3} < \pi_2^{e3}$, and the gain from the advertisers' side cannot compensate for the loss from subscriptions. However, the most interesting case is when the two platforms simultaneously choose to integrate vertically, (VI_1, VI_2), which Lemma 2.11 considers.

Lemma 3.11.

i) The subscriptions of platforms are $m_1^{e4} = \frac{54\tau(2\tau-1)}{108\tau-49}$ and $m_2^{e4} = \frac{4\tau(27\tau-11)}{108\tau-49}$.

ii) The amount of content are: $c_1^{e4} = \frac{(108\tau-54)}{108\tau-49}$ and $c_2^{e4} = \frac{(108\tau-44)}{108\tau-49}$.

iii) The number of subscribers of platforms are $b_1^{e4} = \frac{54\tau-27}{108\tau-49}$ and $b_2^{e4} = \frac{54\tau-22}{108\tau-49}$.

iv) Platform 1 *has* $a_1^{e4} = \frac{1}{108\tau-49}(45\tau-20)$ *advertisements and a set advertising fee at* $p_1^{e4} = \frac{1}{108\tau-49}(63\tau-29)$.

v) The profits of both platforms are $\pi_1^{e4} = \frac{\left(5832\tau^3-4941\tau^2+837\tau+94\right)}{(108\tau-49)^2}$ and $\pi_2^{e4} = 2\frac{\left(4\tau-\frac{4}{3}\right)}{(108\tau-49)^2}(27\tau-11)^2$.

If both platforms are vertically integrated, platform 2 charges higher membership fees, $m_1^{e4} < m_2^{e4}$, due to more content, $c_1^{e4} < c_2^{e4}$, and more consumers will buy memberships on platform 2, $b_1^{e4} < b_2^{e4}$. Platform 1's membership fee increases in τ, whereas m_2^{e4} decreases if $\tau < 0.6$. The profit of platform 2 is not always larger than platform 1.

Proposition 3.6. *Platform 2 makes a higher profit than platform* 1 *if* $\tau < \underline{\tau} = \frac{1}{702}\sqrt{649} + \frac{343}{702}$, *otherwise platform 1 gains more.*

Platform 2 has a higher membership fee and more subscribers, but its profit is lower than platform 1 if transportation costs are high. In this case, the revenue from advertisements plays an important role.

Figure 3.6 demonstrates the profits of the two platforms, π_1^{e4} and π_2^{e4}. The green line shows the profit of platform 1, and the red line represents the profit of platform 2. The two lines interact at $\underline{\tau}$. When $\tau < \underline{\tau}$, the difference of the platforms' profits $\Delta\pi^{e4} = \pi_1^{e4} - \pi_2^{e4} < 0$; when $\tau \geq \underline{\tau}$, this difference is $\Delta\pi^{e4} > 0$. If $\tau \leq \overline{\tau} = \frac{1}{216}\sqrt{545} + \frac{103}{216}$, the profit of platform 2 decreases with the transportation cost, otherwise, it increases. Hence, platform 2 has the potential to generate greater profits when the content offered by both platforms is either similar or very divergent. Furthermore, the higher the transport cost, the greater the profits and the more money platform 1 earns over the other platform. Platform 1, thus, possesses a strong incentive to generate unique films to enhance its distinctiveness.

When both platforms are vertically integrated, the number of advertisements decreases in τ. Platform 1 gains more from membership fees unless advertisements, because the number of viewers and the price of platform 1 increase with

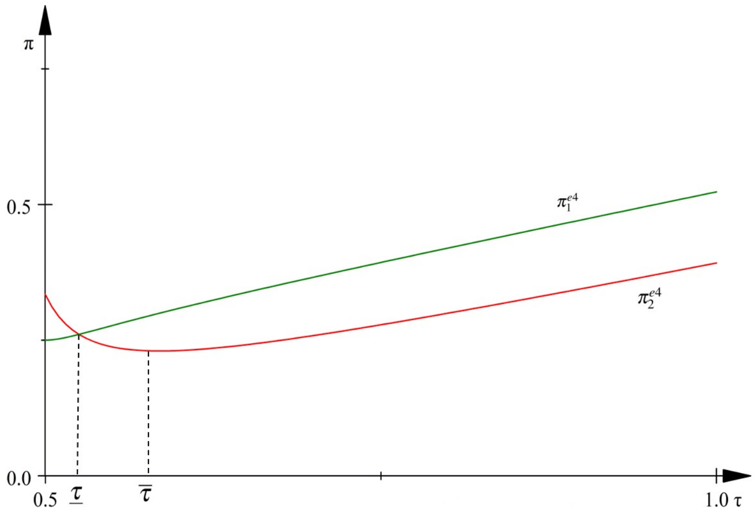

Figure 3.6: Platforms' profits under vertical integration

τ. Two platforms compete fiercely if they are relatively homogeneous. Platform 1 drives down the number of advertisements, while platform 2 only decreases the price against the competition. However, as they become increasingly heterogeneous, competition between the two platforms becomes less intense. Platform 2, therefore, raises the price without the concern of losing consumers. Consequently, the profit of platform 2 is non-monotonic.

Platform 1 can be more profitable with lower membership fees and fewer consumers if it provides unique content. Due to the additional revenue from advertisers, platform 1 can consider and produce special movies to make the two platforms more differentiated. We compare cases precisely based on membership fees. From Lemma 3.8–3.11, we know that $m_1^{e2} > m_1^{e1} > m_1^{e4} > m_1^{e3}$, and $m_2^{e3} > m_2^{e4} > m_2^{e1} > m_2^{e2}$. As such, we conclude that if platform i is the only one that engages in vertical integration, its membership fee will increase. Moreover, if platform i does not have advertisements, its membership fees will rise if it opts for vertical integration. Concerning the number of videos, platform i always provides the most if it is the only vertically integrated platform in the market and the least if its rival is the only vertically integrated platform. After vertical integration,

P2 / P1	Vertically separated	Vertically integrated
Vertically separated	$\left(\pi_1^{e1}, \pi_2^{e1}\right)$	$\left(\pi_1^{e3}, \pi_2^{e3}\right)$
Vertically integrated	$\left(\pi_1^{e2}, \pi_2^{e2}\right)$	$\left(\pi_1^{e4}, \pi_2^{e4}\right)$

Table 3.2: The profits of platforms in asymmetric market

platform 2 provides more content ($c_2^{e3} > c_2^{e4} > c_2^{e1} > c_2^{e2}$), whereas platform 1 is uncertain. If transport cost $\tau > \frac{1}{216}\sqrt{1321}+\frac{79}{216} \approx 0.53$, we obtain $c_1^{e4} > c_1^{e1}$, which indicates that under the two vertically integrated platforms scenario platform 1 would provide more content. If platform i is the only vertical merger platform on the market, it will acquire the most customers in any situation. When the rival platform j is the only vertically integrated platform, it will acquire the fewest subscribers compared to other scenarios. Ultimately, the most important thing to consider is the two platforms' profits.

Table 3.2 shows the profits in an asymmetric market. The unique equilibrium is that both platforms have vertical integration. Both platforms have an incentive to choose vertical integration. However, the profit will decrease if both platforms are vertically integrated instead of separated. Figure 3.7 shows the profit level of the platform 1 in different scenarios. Figure 3.8 illustrates the profit level of platform 2 in different scenarios.

Proposition 3.7. *If $\tau < \widetilde{\tau} \approx 0.538$, platform 2 profits more if both platforms are vertically integrated than if both are vertically separated ($\pi_2^{e4} > \pi_2^{e1}$). If $\tau \geq \widetilde{\tau}$, the opposite is ture.*

Both platforms are incentivized to integrate vertically, and if one platform merges, another will follow to achieve vertical integration. However, platform 2 has more advantages than platform 1 when choosing a vertical merger. As illustrated in Figure 3.8, if transport costs are lower than $\widetilde{\tau}$ and both platforms choose vertical integration, the profit of platform 2 is greater than if neither platform is vertically integrated. Namely, platform 2 has a stronger incentive to integrate if platforms' content is less differentiated. Comparing the profits of

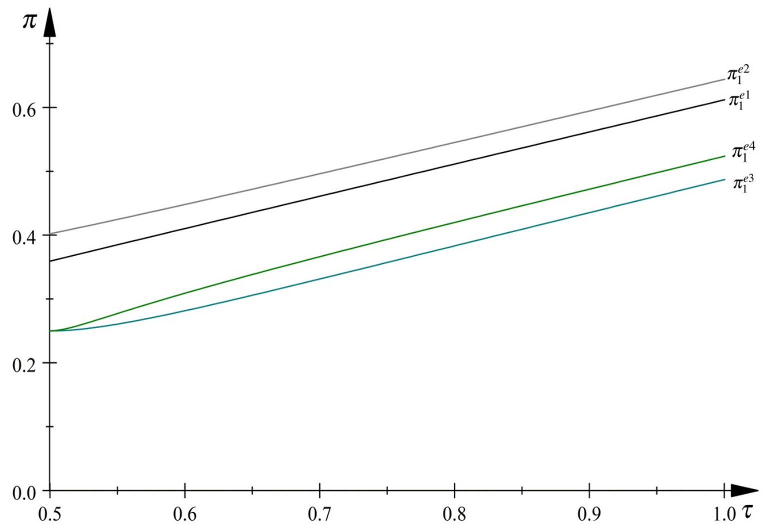

Figure 3.7: The profits level of platform 1 in different scenarios

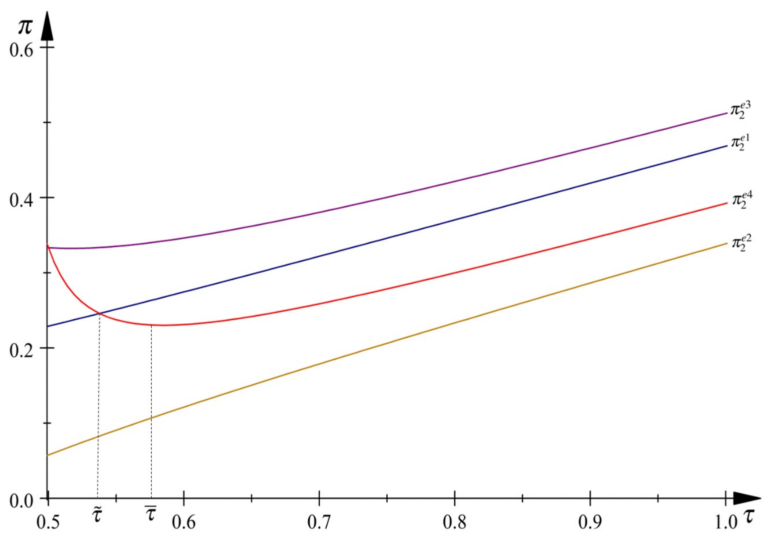

Figure 3.8: The profits level of platform 2 in different scenarios

platform 1 and platform 2, we can conclude that platforms without advertisements have more incentive to vertically integrate.

3.6.2 Welfare Analysis

In this subsection, we examine the welfare analysis of this extension model. We use formula (3.8) to calculate consumer surplus under various scenarios, and total social welfare is the sum of consumer surplus as well as producer and platform profits.

Proposition 3.8.

i) $CS^{e1} < CS^{e2} < CS^{e3} \leq CS^{e4}$ *if transport cost* $\tau \geq \tau^{CS} = 0.552$, *and* $CS^{e1} < CS^{e2} < CS^{e4} < CS^{e3}$ *otherwise.*

ii) $SW^{e1} < SW^{e3} \leq SW^{e4} < SW^{e2}$ *if transport cost* $\tau \geq \tau^{SW} = 0.9$, *and* $SW^{e1} < SW^{e4} < SW^{e3} < SW^{e2}$ *otherwise.*

From Proposition 3.8, we know that both consumer surplus and social welfare are lowest when the two platforms remain vertically separated (VS_1, VS_2). However, once one platform chooses vertical integration, consumer surplus and social welfare increase. Vertical integration under the platform with advertisements is better for social welfare. Nonetheless, if the platforms are relatively differentiated, consumers benefit more from the merger of both platforms than if only one is the vertically integrated platform $(\tau \geq \tau^{CS})$.

We can conclude that platform 1 is always less favorably positioned under (VI_1, VI_2) compared to (VS_1, VS_2), but platform 2 is not. Only when $\tau \geq \widetilde{\tau}$ does, platform 2 profit more in (VI_1, VI_2). At this transportation level $(\tau < \widetilde{\tau})$, consumer surplus and social welfare in the (VI_1, VI_2) scenario are lower than that of (VS_1, VI_2) but strictly more than in (VS_1, VS_2). Once the tranport cost $\tau \geq \tau^{CS}$, consumer surplus is the highest under (VI_1, VI_2), but the platforms are both less favorably positioned than (VS_1, VS_2). Figure 3.9 illustrates the most interesting values of transport cost: how profits and consumer surplus change

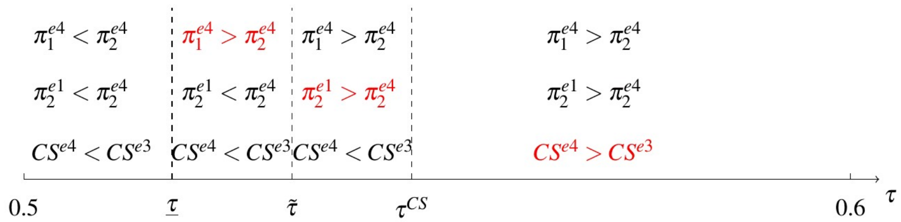

Figure 3.9: Critical values of transport cost

with transportation costs. Considering policy implementation, the antitrust authority prefers vertical integrations. Both consumer surplus and social welfare are stronger if both platforms are vertically integrated than if they are vertically separated. If both platforms have vertical integration, the antitrust authority prefers them to be more differentiated, so consumers can achieve higher utility.

3.7 Conclusion

This study demonstrates how a two-sided market model can be used to analyze the membership fee, profits of platforms, and the number of videos offered before and after vertical integration. Platforms have an incentive to vertically integrate, and our results reveal that vertical integration can increase the subscription fee. The increase in the membership fee of vertically integrated platforms, as seen in asymmetric cases (Section 3.4.4 and Section 3.5.3), is even higher when platforms introduce advertisements. With or without advertisers, two vertically integrated platforms form a unique equilibrium in the market. However, they are at a disadvantage if both choose vertical integration. In the extension model, we find that ad-free platforms have more incentive to choose vertical integration. Regarding policy interventions from the antitrust authority, this paper portrays how consumer surplus and social welfare are higher if platforms start to opt for vertical integration. When the content between video platforms is different enough, both consumer surplus and social welfare are larger under two vertical integrated platfroms scenario compared to the asymmetric case.

Currently, platforms prefer to produce high-quality content to attract consumers after a vertical merger. The lower profit after a vertical merger is likely one reason why those platforms begin to produce high-quality series: They aim to attract users by using high-quality content. However, high-quality content generally involves a higher marginal cost of production. Thus, higher costs are the primary reason for platforms to raise membership fees, even if both platforms choose vertical integration. Most platforms on the market opt for partial rather than full vertical integration. Namely, they produce original series and, at the same time, buy content from providers; one improvement to our model could be the addition of a partially integrated platform. The multi-homing model has been commonly used for many studies on the two-sided market, although we only use single homing here. Another critical factor is that platforms often opt for vertical integration and enter joint ventures with other platforms in the market. Horizontal mergers between platforms could also be harmful to the online streaming video market.

3.8 Appendix

A: Equilibrium of the model

In this part, we will show the proof of Section 3.4.

Proof of assumptions. The profit functions of two vertical separated platforms are: $\pi_i = \left(\frac{1}{2} + \frac{\theta w(l_i - l_j) - (m_i - m_j)}{2\tau}\right) m_i - (wl_i)\, l_i,\ i,j \in \{1,2\}$ and $i \neq j$. The first order conditions are: $\pi_{m_1} = \frac{1}{2\tau}(\tau - 2m_1 + m_2 + w\theta l_1 - w\theta l_2)$ and $\pi_{l_1} = \frac{1}{2}\frac{w}{\tau}(\theta m_1 - 4\tau l_1)$. Because of two variables we have to make sure that $\pi_{m_1 m_1} < 0$ and $\pi_{l_1 l_1} < 0$, and Hessian Matrix $\begin{vmatrix} \pi_{m_1 m_1} & \pi_{m_1 l_1} \\ \pi_{l_1 m_1} & \pi_{l_1 l_1} \end{vmatrix} > 0$ to examine the local maximization profits. The second order conditions $\pi_{m_1 m_1} = -\frac{1}{\tau} < 0$, and $\pi_{l_1 l_1} = -2w < 0$. $\pi_{m_1 l_1} = \pi_{l_1 m_1} = \frac{1}{2} w \frac{\theta}{\tau}$, $\begin{vmatrix} -\frac{1}{\tau} & \frac{1}{2} w \frac{\theta}{\tau} \\ \frac{1}{2} w \frac{\theta}{\tau} & -2w \end{vmatrix} = \frac{w}{4\tau^2}\left(8\tau - w\theta^2\right) > 0$, we get $\tau > \frac{w\theta^2}{8}$. Due to symmetric platforms, platform 2 has the same results in this scenario.

The profit functions of two vertical integrated platforms are:

$$\pi_i^v = \left(\frac{1}{2} + \frac{\theta\left(c_i^v - c_j^v\right) - \left(m_i^v - m_j^v\right)}{2\tau}\right) m_i^v - \frac{(c_i^v)^2}{2w}, i,j \in \{1,2\}; i \neq j.$$

The first order conditions are: $\pi_{m_1^v}^v = \frac{1}{2\tau}(\tau - 2m_1^v + m_2^v + \theta c_1^v - \theta c_2^v)$ and $\pi_{c_1^v}^v = -\frac{1}{2w\tau}(2\tau c_1^v - w\theta m_1^v)$. The second order conditions are: $\pi_{m_1^v m_1^v}^v = -\frac{1}{\tau} < 0$, $\pi_{c_1^v c_1^v}^v = -\frac{1}{w} < 0$, and $\pi_{m_1^v c_1^v}^v = \pi_{c_1^v m_1^v}^v = \frac{1}{2}\frac{\theta}{\tau}$. Then $\begin{vmatrix} -\frac{1}{\tau} & \frac{1}{2}\frac{\theta}{\tau} \\ \frac{1}{2}\frac{\theta}{\tau} & -\frac{1}{w} \end{vmatrix} = \frac{1}{4w\tau^2}\left(4\tau - w\theta^2\right)$ > 0, we have to make sure that $\tau > \frac{w\theta^2}{4}$.

In the one integrated and one separated platform scenario, the equilibria are: $m_1^s = \frac{12\tau^2 - 4w\theta^2\tau}{12\tau - 3w\theta^2}$, $m_2^s = \frac{12\tau^2 - 2w\theta^2\tau}{12\tau - 3w\theta^2}$; $c_1^s = \frac{w\theta\left(3\tau - w\theta^2\right)}{12\tau - 3w\theta^2}$, $c_2^s = \frac{w\theta\left(6\tau - w\theta^2\right)}{12\tau - 3w\theta^2}$; $b_1^s = \frac{6\tau - 2w\theta^2}{12\tau - 3w\theta^2}$, $b_2^s = \frac{6\tau - w\theta^2}{12\tau - 3w\theta^2}$; $\pi_1^s = \frac{1}{9}\frac{\left(3\tau - w\theta^2\right)^2}{(4\tau - w\theta^2)^2}\left(8\tau - w\theta^2\right)$ and $\pi_2^s = \frac{1}{18}\frac{\left(6\tau - w\theta^2\right)^2}{(4\tau - w\theta^2)}$. We have to make sure that all these solutions are larger than zero. This non-negative critical value is $\tau > \frac{1}{3}w\theta^2$.

All in all when $\tau > \frac{1}{3}w\theta^2$ we can find the local maximization solutions.

Proof of Lemma 3.1. The profit of platform 1 is:

$$\pi_1 = \left(\frac{1}{2} + \frac{\theta w\,(l_1 - l_2) - (m_1 - m_2)}{2\tau}\right) m_1 - (wl_1)\, l_1.$$

We take the FOCs:

$$\begin{aligned}
\frac{\partial}{\partial m_1}\left(\left(\frac{1}{2} + \frac{\theta w\,(l_1 - l_2) - (m_1 - m_2)}{2\tau}\right) m_1 - (wl_1)\, l_1\right) &= 0, \\
\frac{\partial}{\partial l_1}\left(\left(\frac{1}{2} + \frac{\theta w\,(l_1 - l_2) - (m_1 - m_2)}{2\tau}\right) m_1 - (wl_1)\, l_1\right) &= 0.
\end{aligned}$$

We get following solutions:

$$\begin{aligned}
m_1 &= \frac{1}{2}\tau + \frac{1}{2}m_2 + \frac{1}{2}w\theta l_1 - \frac{1}{2}w\theta l_2, \\
l_1 &= \frac{1}{4}\frac{\theta}{\tau}m_1.
\end{aligned}$$

Platform 2's profit :

$$\pi_2 = \left(\frac{1}{2} + \frac{\theta w\,(l_2 - l_1) - (m_2 - m_1)}{2\tau}\right) m_2 - (wl_2)\, l_2.$$

FOCs:

$$\begin{aligned}
\frac{\partial}{\partial m_2}\left(\left(\frac{1}{2} + \frac{\theta w\,(l_2 - l_1) - (m_2 - m_1)}{2\tau}\right) m_2 - (wl_2)\, l_2\right) &= 0, \\
\frac{\partial}{\partial l_2}\left(\left(\frac{1}{2} + \frac{\theta w\,(l_2 - l_1) - (m_2 - m_1)}{2\tau}\right) m_2 - (wl_2)\, l_2\right) &= 0.
\end{aligned}$$

$$
\begin{aligned}
m_2 &= \frac{1}{2}\tau + \frac{1}{2}m_1 - \frac{1}{2}w\theta l_1 + \frac{1}{2}w\theta l_2, \\
l_2 &= \frac{1}{4}\frac{\theta}{\tau}m_2, \\
m_1 &= \frac{1}{2}\tau + \frac{1}{2}m_2 + \frac{1}{2}w\theta l_1 - \frac{1}{2}w\theta l_2, \\
l_1 &= \frac{1}{4}\frac{\theta}{\tau}m_1.
\end{aligned}
$$

Solving simultaneously we get equilibrium solutions:

$$
\begin{aligned}
m_1 &= \tau, \\
m_2 &= \tau, \\
l_1 &= \frac{1}{4}\theta, \\
l_2 &= \frac{1}{4}\theta, \\
b_1 &= b_2 = \frac{1}{2}.
\end{aligned}
$$

Plugging these into the profit functions of platforms $\pi_1 = \pi_2 = \frac{1}{2}\tau - \frac{1}{16}w\theta^2$, the profit of one group of content provider is:

$$
\Pi_{ci} = \int_0^{\frac{w\theta}{4}} \left(\frac{\theta}{4} - \frac{l}{w}\right) dl.
$$

Under standard model porviders are zero profit. Two platforms must be profitable in the markat so that $\tau > \frac{w\theta^2}{8}$.

Social Welfare: The consumer surplus is like following:

$$
CS = \int_0^{b_1} U_1 dx + \int_{b_1}^{1} U_2 dx = v + \frac{1}{4}w\theta^2 - \frac{5}{4}\tau.
$$

The profits of platforms are: $\pi_1 + \pi_2 = \tau - \frac{1}{8}w\theta^2$ and the sum of utility from content provider side is $\pi_c = 2 \cdot \left(\frac{1}{2}\frac{\theta}{4}\frac{w\theta}{4}\right) = \frac{w\theta^2}{16}$. Total welfare is: $W = CS + \pi_1 + \pi_2 + \pi_c$.

Under standard basic model we could get that:

$$CS = v + \frac{1}{4}w\theta^2 - \frac{5}{4}\tau,$$

and

$$W = v + \frac{3}{16}w\theta^2 - \frac{1}{4}\tau.$$

Proof of Lemma 3.2. The profit function of vertical integrated platforms are:

$$\begin{aligned}
\pi_1^v &= \left(\frac{1}{2} + \frac{\theta\left(c_1^v - c_2^v\right) - \left(m_1^v - m_2^v\right)}{2\tau}\right) m_1^v - \frac{\left(c_1^v\right)^2}{2w}, \\
\pi_2^v &= \left(\frac{1}{2} + \frac{\theta\left(c_2^v - c_1^v\right) - \left(m_2^v - m_1^v\right)}{2\tau}\right) m_2^v - \frac{\left(c_2^v\right)^2}{2w}.
\end{aligned}$$

We take the FOCs:

$$\begin{aligned}
\frac{\partial}{\partial m_1^v}\left(\left(\frac{1}{2} + \frac{\theta\left(c_1^v - c_2^v\right) - \left(m_1^v - m_2^v\right)}{2\tau}\right) m_1^v - \frac{\left(c_1^v\right)^2}{2w}\right) &= 0, \\
\frac{\partial}{\partial c_1^v}\left(\left(\frac{1}{2} + \frac{\theta\left(c_1^v - c_2^v\right) - \left(m_1^v - m_2^v\right)}{2\tau}\right) m_1^v - \frac{\left(c_1^v\right)^2}{2w}\right) &= 0, \\
\frac{\partial}{\partial m_2}\left(\left(\frac{1}{2} + \frac{\theta\left(c_2 - c_1\right) - \left(m_2 - m_1\right)}{2\tau}\right) m_2 - \frac{\left(c_2\right)^2}{2w}\right) &= 0, \\
\frac{\partial}{\partial c_2}\left(\left(\frac{1}{2} + \frac{\theta\left(c_2 - c_1\right) - \left(m_2 - m_1\right)}{2\tau}\right) m_2 - \frac{\left(c_2\right)^2}{2w}\right) &= 0.
\end{aligned}$$

We get following solutions:

$$\begin{aligned}
m_1 &= \frac{1}{2}\tau + \frac{1}{2}m_2 + \frac{1}{2}\theta c_1 - \frac{1}{2}\theta c_2, \\
c_1 &= \frac{1}{2}w\frac{\theta}{\tau}m_1, \\
m_2 &= \frac{1}{2}\tau + \frac{1}{2}m_1 - \frac{1}{2}\theta c_1 + \frac{1}{2}\theta c_2, \\
c_2 &= \frac{1}{2}w\frac{\theta}{\tau}m_2.
\end{aligned}$$

We solve FOCs for equations could get:

$$\begin{aligned} c_1^v &= \frac{1}{2}w\theta, \\ c_2^v &= \frac{1}{2}w\theta, \\ m_1^v &= \tau, \\ m_2^v &= \tau, \\ b_1^v &= b_2^v = \frac{1}{2}. \end{aligned}$$

Profits of two platforms are: $\pi_1^v = \pi_2^v = \frac{1}{2}\tau - \frac{1}{8}w\theta^2$, cosumer surplus is

$$CS^v = \int_0^{\frac{1}{2}} \left(v + \theta\left(\frac{1}{2}w\theta\right) - \tau - \tau x\right) dx + \int_{\frac{1}{2}}^1 \left(v + \theta\left(\frac{1}{2}w\theta\right) - \tau - \tau(1-x)\right) dx = v + \frac{1}{2}w\theta^2 - \frac{5}{4}\tau,$$

and total social welfare is

$$W^v = CS^v + \pi_1^v + \pi_2^v = v + \frac{1}{4}w\theta^2 - \frac{1}{4}\tau.$$

Proof of Lemma 3.3. The profit function of platform 1 under asymmetric case: $\pi_1^s = \left(\frac{1}{2} + \frac{\theta(wl_1 - c_2) - (m_1 - m_2)}{2\tau}\right) m_1 - (wl_1)\, l_1$, then we take first order conditions:

$$\begin{aligned} \frac{\partial}{\partial m_1}\left(\left(\frac{1}{2} + \frac{\theta(wl_1 - c_2) - (m_1 - m_2)}{2\tau}\right) m_1 - (wl_1)\, l_1\right) &= 0, \\ \frac{\partial}{\partial l_1}\left(\left(\frac{1}{2} + \frac{\theta(wl_1 - c_2) - (m_1 - m_2)}{2\tau}\right) m_1 - (wl_1)\, l_1\right) &= 0. \end{aligned}$$

The profit function of platform 2 is:

$$\pi_2 = \left(\frac{1}{2} + \frac{\theta(c_2 - wl_1) - (m_2 - m_1)}{2\tau}\right) m_2 - \frac{c_2^2}{2w},$$

take first order conditions we get:

$$\begin{aligned}
\frac{\partial}{\partial m_2}\left(\left(\frac{1}{2}+\frac{\theta\left(c_2-wl_1\right)-\left(m_2-m_1\right)}{2\tau}\right)m_2-\frac{c_2^2}{2w}\right) &= 0,\\
\frac{\partial}{\partial c_2}\left(\left(\frac{1}{2}+\frac{\theta\left(c_2-wl_1\right)-\left(m_2-m_1\right)}{2\tau}\right)m_2-\frac{c_2^2}{2w}\right) &= 0.
\end{aligned}$$

Sloving first order conditions:

$$\begin{aligned}
m_2 &= \frac{1}{2}\tau+\frac{1}{2}m_1+\frac{1}{2}\theta c_2-\frac{1}{2}w\theta l_1,\\
m_1 &= \frac{1}{2}\tau+\frac{1}{2}m_2-\frac{1}{2}\theta c_2+\frac{1}{2}w\theta l_1,\\
l_1 &= \frac{1}{4}\frac{\theta}{\tau}m_1,\\
c_2 &= \frac{1}{2}w\frac{\theta}{\tau}m_2.
\end{aligned}$$

we could get quilibrium solutions under vertical integration:

$$\begin{aligned}
l_1^s &= \frac{\theta\left(3\tau-w\theta^2\right)}{12\tau-3w\theta^2},\\
c_2^s &= \frac{w\theta\left(6\tau-w\theta^2\right)}{12\tau-3w\theta^2},\\
m_1^s &= \frac{4\tau\left(3\tau-w\theta^2\right)}{12\tau-3w\theta^2},\\
m_2^s &= \frac{2\tau\left(6\tau-w\theta^2\right)}{12\tau-3w\theta^2},\\
b_1^s &= \frac{6\tau-2w\theta^2}{12\tau-3w\theta^2},\\
b_2^s &= \frac{6\tau-w\theta^2}{12\tau-3w\theta^2}.
\end{aligned}$$

We could calculate the profit of platforms:

$$\begin{aligned}\pi_1^s &= b_1^s m_1^s - c_1^s l_1^s = \frac{1}{9}\frac{\left(3\tau - w\theta^2\right)^2}{\left(4\tau - w\theta^2\right)^2}\left(8\tau - w\theta^2\right)\\ \pi_2^s &= b_2^s m_2^s - \frac{\left(c_2^s\right)^2}{2w} = \frac{1}{18\left(4\tau - w\theta^2\right)}\left(6\tau - w\theta^2\right)^2.\end{aligned}$$

The profit of upstream providers: $\pi_c^s = \frac{1}{2}w\left(\frac{\theta\left(3\tau - w\theta^2\right)}{12\tau - 3w\theta^2}\right)^2$ and consumer surplus is

$$\begin{aligned}CS^s &= \int_0^{b_1^s} U_1^s dx + \int_{b_1^s}^1 U_2^s dx\\ &= v - \frac{1}{18\left(4\tau - w\theta^2\right)^2}\left(-6w^3\theta^6 + 73w^2\theta^4\tau - 288w\theta^2\tau^2 + 360\tau^3\right).\end{aligned}$$

The Total welfare is:

$$\begin{aligned}W^s &= CS^s + \pi_1^s + \pi_2^s + \pi_c^s\\ &= v - \frac{1}{18\left(4\tau - w\theta^2\right)^2}\left(-4w^3\theta^6 + 35w^2\theta^4\tau - 99w\theta^2\tau^2 + 72\tau^3\right).\end{aligned}$$

When we compare welfare we get:

$$\begin{aligned}\Delta CS &= CS^v - CS^s\\ &= \frac{1}{36}w\frac{\theta^2}{\left(4\tau - w\theta^2\right)^2}\left(6w^2\theta^4 - 43w\theta^2\tau + 72\tau^2\right).\end{aligned}$$

When we take first derivative of τ:

$$\begin{aligned}\Delta CS_\tau &= \frac{\partial}{\partial\tau}\left(\frac{1}{36}w\frac{\theta^2}{\left(4\tau - w\theta^2\right)^2}\left(6w^2\theta^4 - 43w\theta^2\tau + 72\tau^2\right)\right)\\ &= \frac{1}{36}w^2\frac{\theta^4}{\left(4\tau - w\theta^2\right)^3}\left(28\tau - 5w\theta^2\right) > 0,\end{aligned}$$

second derivative:

$$\Delta CS_{\tau\tau} = \frac{\partial}{\partial\tau}\left(\frac{1}{36}w^2\frac{\theta^4}{(4\tau - w\theta^2)^3}\left(28\tau - 5w\theta^2\right)\right)$$
$$= -\frac{8}{9}w^2\frac{\theta^4}{(4\tau - w\theta^2)^4}\left(7\tau - w\theta^2\right) < 0.$$

Therefore, ΔCS is a concave function if $\tau > \frac{1}{3}w\theta^2$. When $\tau > \frac{3}{8}w\theta^2$, $\Delta CS > 0 \Rightarrow CS^v > CS^s$.

Proof of Lemma 3.5. Average price and network effect: The average price when both platforms have vertical berger is $\overline{m}^v = \tau$. Average price under asymmetric: $\overline{m}^s = b_1^s m_1^s + b_2^s m_2^s = \frac{2}{9}\frac{\tau}{(4\tau - w\theta^2)^2}\left(5w^2\theta^4 - 36w\theta^2\tau + 72\tau^2\right)$. The difference of average price level is $\Delta\overline{m} = \overline{m}^v - \overline{m}^s = -\frac{1}{9}w^2\theta^4\frac{\tau}{(4\tau - w\theta^2)^2} < 0$. The average network effect for each consumer under asymmetric case is: $\theta\overline{c}^s = \theta\left(b_1^s c_1^s + b_2^s c_2^s\right) = \frac{1}{3}w\frac{\theta^2}{(4\tau - w\theta^2)^2}\left(w^2\theta^4 - 8w\theta^2\tau + 18\tau^2\right)$ and the average network effect if both are integrated platforms: $\theta\overline{c}^v = \frac{1}{2}w\theta^2$.
The difference is: $\Delta\theta\overline{c} = \theta\left(\overline{c}^v - \overline{c}^s\right) = \frac{1}{6}w\frac{\theta^2}{(4\tau - w\theta^2)^2}\left(w^2\theta^4 - 8w\theta^2\tau + 12\tau^2\right)$. Because $\tau > \frac{1}{3}w\theta^2$, only one root should be taken into account, $\frac{1}{2}w\theta^2$. Because $\Delta\theta\overline{c}_\tau > 0$ and $\Delta\theta\overline{c}_{\tau\tau} < 0$, this is a concave function, when $\tau > \frac{1}{2}w\theta^2$, $\Delta\theta\overline{c} > 0$, which is equivalent to $\theta\overline{c}^v > \theta\overline{c}^s$.

B: Equilibrium of the model with advertisments

In this part, we will show the proof of section 3.5.

Proof of assumption: The profit function of vertical separated platform is $\pi_1^A = a_1\left(k - \varphi a_1\right) + m_1\left(\frac{1}{2} + \frac{\theta w(l_1 - l_2) - \gamma(a_1 - a_2) - (m_1 - m_2)}{2\tau}\right) - \left(wl_1\right)l_1$ and first-order conditions are:

$$\begin{aligned}
\pi_{l_1}^A &= \frac{1}{2}\frac{w}{\tau}\left(\theta m_1 - 4\tau l_1\right), \\
\pi_{m_1}^A &= \frac{1}{2\tau}\left(\tau - 2m_1 + m_2 - \gamma a_1 + \gamma a_2 + w\theta l_1 - w\theta l_2\right), \\
\pi_{a_1}^A &= -\frac{1}{2\tau}\left(\gamma m_1 - 2k\tau + 4\tau\varphi a_1\right).
\end{aligned}$$

To make sure we can find the local maximization solutions we have to know the second order conditions: $\pi^A_{m_1m_1} = -\frac{1}{\tau} < 0$, $\pi^A_{a_1a_1} = -2\varphi < 0$, $\pi^A_{l_1l_1} = -2w < 0$, and $\pi^A_{m_1a_1} = \pi^A_{a_1m_1} = -\frac{1}{2\tau}\gamma$, $\pi^A_{m_1l_1} = \pi^A_{l_1m_1} = \frac{1}{2}w\frac{\theta}{\tau}$, $\pi^A_{a_1l_1} = \pi^A_{l_1a_1} = 0$. Then $\begin{vmatrix} \pi^A_{m_1m_1} & \pi^A_{m_1a_1} \\ \pi^A_{a_1m_1} & \pi^A_{a_1a_1} \end{vmatrix} = \begin{vmatrix} -\frac{1}{\tau} & -\frac{1}{2\tau}\gamma \\ -\frac{1}{2\tau}\gamma & -2\varphi \end{vmatrix} = \frac{1}{4\tau^2}\left(8\tau\varphi - \gamma^2\right) > 0 \Rightarrow \tau > \frac{\gamma^2}{8\varphi}$.

Additionally, $\begin{vmatrix} \pi^A_{m_1m_1} & \pi^A_{m_1l_1} \\ \pi^A_{l_1m_1} & \pi^A_{l_1l_1} \end{vmatrix} = \begin{vmatrix} -\frac{1}{\tau} & \frac{1}{2}w\frac{\theta}{\tau} \\ \frac{1}{2}w\frac{\theta}{\tau} & -2w \end{vmatrix} = -\frac{1}{4\tau^2}\left(w^2\theta^2 - 8w\tau\right) > 0 \Rightarrow \tau > \frac{w\theta^2}{8}$. Moreover, $\begin{vmatrix} \pi^A_{m_1m_1} & \pi^A_{m_1a_1} & \pi^A_{m_1l_1} \\ \pi^A_{a_1m_1} & \pi^A_{a_1a_1} & \pi^A_{a_1l_1} \\ \pi^A_{l_1m_1} & \pi^A_{l_1a_1} & \pi^A_{l_1l_1} \end{vmatrix} = \begin{vmatrix} -\frac{1}{\tau} & -\frac{1}{2\tau}\gamma & \frac{1}{2}w\frac{\theta}{\tau} \\ -\frac{1}{2\tau}\gamma & -2\varphi & 0 \\ \frac{1}{2}w\frac{\theta}{\tau} & 0 & -2w \end{vmatrix} = \frac{1}{2\tau^2}\left(\varphi w^2\theta^2 + w\gamma^2 - 8\tau\varphi w\right) < 0 \Rightarrow \tau > \frac{\theta^2}{8\varphi}(\varphi w + 1)$.

The profit of vertical integrated platform is:

$$\pi^{Av}_2 = m_2\left(\frac{1}{2} + \frac{\theta\left(c_2 - c_1\right) - \gamma\left(a_2 - a_1\right) - \left(m_2 - m_1\right)}{2\tau}\right) + a_2\left(k - \varphi a_2\right) - \frac{c_2^2}{2w},$$

and the first order conditions are:

$$\begin{aligned}
\pi^{Av}_{m_2} &= \frac{1}{2\tau}\left(\tau + m_1 - 2m_2 - \theta c_1 + \gamma a_1 + \theta c_2 - \gamma a_2\right), \\
\pi^{Av}_{c_2} &= -\frac{1}{2w\tau}\left(2\tau c_2 - w\theta m_2\right), \\
\pi^{Av}_{a_2} &= -\frac{1}{2\tau}\left(\gamma m_2 - 2k\tau + 4\tau\varphi a_2\right).
\end{aligned}$$

Second order conditions are: $\pi^{Av}_{m_2m_2} = -\frac{1}{\tau} < 0$, $\pi^{Av}_{c_2c_2} = -\frac{1}{w} < 0$, $\pi^{Av}_{a_2a_2} = -2\varphi < 0$, $\pi^{Av}_{m_2c_2} = \pi^{Av}_{c_2m_2} = \frac{1}{2}\frac{\theta}{\tau}$, $\pi^{Av}_{m_2a_2} = \pi^{Av}_{a_2m_2} = -\frac{1}{2\tau}\gamma$; $\pi^{Av}_{c_2a_2} = \pi^{Av}_{a_2c_2} = 0$. Then $\begin{vmatrix} \pi^{Av}_{m_2m_2} & \pi^{Av}_{m_2c_2} \\ \pi^{Av}_{c_2m_2} & \pi^{Av}_{c_2c_2} \end{vmatrix} = \begin{vmatrix} -\frac{1}{\tau} & \frac{1}{2}\frac{\theta}{\tau} \\ \frac{1}{2}\frac{\theta}{\tau} & -\frac{1}{w} \end{vmatrix} = \frac{1}{4w\tau^2}\left(4\tau - w\theta^2\right) > 0 \Rightarrow \tau > \frac{w\theta^2}{4}$;

$\begin{vmatrix} \pi^A_{m_2m_2} & \pi^A_{m_2a_2} & \pi^A_{m_2c_2} \\ \pi^A_{a_2m_2} & \pi^A_{a_2a_2} & \pi^A_{a_2c_2} \\ \pi^A_{c_2m_2} & \pi^A_{c_2a_2} & \pi^A_{c_2c_2} \end{vmatrix} = \begin{vmatrix} -\frac{1}{\tau} & -\frac{1}{2\tau}\gamma & \frac{1}{2}\frac{\theta}{\tau} \\ -\frac{1}{2\tau}\gamma & -2\varphi & 0 \\ \frac{1}{2}\frac{\theta}{\tau} & 0 & -\frac{1}{w} \end{vmatrix} = \frac{1}{4w\tau^2}\left(2w\varphi\theta^2 + \gamma^2 - 8\tau\varphi\right) <$ $0 \Rightarrow \tau > \frac{1}{8\varphi}\left(\gamma^2 + 2w\theta^2\varphi\right)$. However in the asymmetric case we need to make sure all equilibria are larger than zero. This non-negative critical value is $\tau > \frac{\gamma^2}{6\varphi} + \frac{w\theta^2}{3}$.

Proof of Lemma 3.6. The profit function of platform 1 under symmetric case: two vertically separated platforms:

$$\pi_i^A = a_i^A\left(k - \varphi a_i^A\right) + m_i^A\left(\frac{1}{2} + \frac{\theta w\left(l_i^A - l_j^A\right) - \gamma\left(a_i^A - a_j^A\right) - \left(m_i^A - m_j^A\right)}{2\tau}\right) - \left(wl_i^A\right)l_i^A, i, j \in \{1,2\}; i \neq j.$$

then we solve first order conditions: $\frac{\partial \pi_i^A}{\partial m_i^A} = 0$, $\frac{\partial \pi_i^A}{\partial a_i^A} = 0$, and $\frac{\partial \pi_i^A}{\partial l_i^A} = 0$ and can get:

$$\begin{aligned}
m_1^A &= \frac{1}{2}\tau + \frac{1}{2}m_2^A - \frac{1}{2}\gamma a_1^A + \frac{1}{2}\gamma a_2^A + \frac{1}{2}w\theta l_1^A - \frac{1}{2}w\theta l_2^A, \\
a_1^A &= -\frac{1}{4\tau\varphi}\left(-2k\tau + \gamma m_1^A\right), \\
l_1^A &= \frac{1}{4}\frac{\theta}{\tau}m_1^A, \\
m_2^A &= \frac{1}{2}\tau + \frac{1}{2}m_1^A + \frac{1}{2}\gamma a_1^A - \frac{1}{2}\gamma a_2^A - \frac{1}{2}w\theta l_1^A + \frac{1}{2}w\theta l_2^A, \\
a_2^A &= -\frac{1}{4\tau\varphi}\left(-2k\tau + \gamma m_2^A\right), \\
l_2^A &= \frac{1}{4}\frac{\theta}{\tau}m_2^A.
\end{aligned}$$

Then the solutions are: $a_1^A = a_2^A = \frac{1}{4\varphi}(2k - \gamma), l_1^A = l_2^A = \frac{1}{4}\theta, m_1^A = m_2^A = \tau$.

Proof of Lemma 3.7. The profit function of Platform 1 under symmetric case: two vertically separated platforms:

$$\pi_i^{Av} = m_i^{Av}\left(\frac{1}{2} + \frac{\theta\left(c_i^{Av} - c_j^{Av}\right) - \gamma\left(a_i^{Av} - a_j^{Av}\right) - \left(m_i^{Av} - m_j^{Av}\right)}{2\tau}\right) + a_i^{Av}\left(k - \varphi a_i^{Av}\right) - \frac{\left(c_i^{Av}\right)^2}{2w},$$

then we solve first order conditions: $\frac{\partial \pi_i^{Av}}{\partial m_i^{Av}} = 0$, $\frac{\partial \pi_i^{Av}}{\partial a_i^{Av}} = 0$, and $\frac{\partial \pi_i^{Av}}{\partial l_i^{Av}} = 0$ and can get:

$$\begin{aligned}
m_1^{Av} &= \frac{1}{2}\tau + \frac{1}{2}m_2^{Av} + \frac{1}{2}\theta c_1^{Av} - \frac{1}{2}\gamma a_1^{Av} - \frac{1}{2}\theta c_2^{Av} + \frac{1}{2}\gamma a_2^{Av}, \\
a_1^{Av} &= -\frac{1}{4\tau\varphi}\left(-2k\tau + \gamma m_1^{Av}\right), \\
c_1^{Av} &= \frac{1}{2}w\frac{\theta}{\tau}m_1^{Av}, \\
m_2^{Av} &= \frac{1}{2}\tau + \frac{1}{2}m_1^{Av} - \frac{1}{2}\theta c_1^{Av} + \frac{1}{2}\gamma a_1^{Av} + \frac{1}{2}\theta c_2^{Av} - \frac{1}{2}\gamma a_2^{Av}, \\
a_2^{Av} &= -\frac{1}{4\tau\varphi}\left(-2k\tau + \gamma m_2^{Av}\right), \\
c_2^{Av} &= \frac{1}{2}w\frac{\theta}{\tau}m_2^{Av}.
\end{aligned}$$

The solutions are: $a_1^{Av} = a_2^{Av} = \frac{1}{4\varphi}\left(2k - \gamma\right), c_1^{Av} = c_2^{Av} = \frac{1}{2}w\theta, m_1^{Av} = m_2^{Av} = \tau.$

C: Assumption of the extension

In this part, we will show the proof of section 3.6.

Proof of assumption. In the first senario e1, For platform 1, from first order conditions we know that: $\pi_{m_1} = -\frac{1}{6\tau}\left(a_1 - 3\tau - 6l_1 + 6l_2 + 6m_1 - 3m_2\right);$

$\pi_{l_1} = \frac{1}{\tau}(m_1 - 6\tau l_1)$; $\pi_a = -\frac{1}{6\tau}(m_1 - 6\tau + 12\tau a_1)$. Second order conditions:

$$\begin{aligned}
\pi_{m_1m_1} &= \frac{\partial}{\partial m_1}\left(-\frac{1}{6\tau}(a_1 - 3\tau - 6l_1 + 6l_2 + 6m_1 - 3m_2)\right) = -\frac{1}{\tau},\\
\pi_{m_1a_1} &= \pi_{a_1m_1} = \frac{\partial}{\partial a_1}\left(-\frac{1}{6\tau}(a_1 - 3\tau - 6l_1 + 6l_2 + 6m_1 - 3m_2)\right) = -\frac{1}{6\tau},\\
\pi_{m_1l_1} &= \pi_{l_1m_1} = \frac{\partial}{\partial l_1}\left(-\frac{1}{6\tau}(a_1 - 3\tau - 6l_1 + 6l_2 + 6m_1 - 3m_2)\right) = \frac{1}{\tau},\\
\pi_{a_1a_1} &= \frac{\partial}{\partial a_1}\left(-\frac{1}{6\tau}(m_1 - 6\tau + 12\tau a_1)\right) = -2,\\
\pi_{a_1l_1} &= \pi_{l_1a_1} = \frac{\partial}{\partial l_1}\left(-\frac{1}{6\tau}(m_1 - 6\tau + 12\tau a_1)\right) = 0,\\
\pi_{l_1l_1} &= \frac{\partial}{\partial l_1}\left(\frac{1}{\tau}(m_1 - 6\tau l_1)\right) = -6.
\end{aligned}$$

Then $\begin{vmatrix} \pi_{mm} & \pi_{ma} \\ \pi_{am} & \pi_{aa} \end{vmatrix} > 0 \Rightarrow \begin{vmatrix} -\frac{1}{\tau} & -\frac{1}{6\tau} \\ -\frac{1}{6\tau} & -2 \end{vmatrix} = \frac{1}{36\tau^2}(72\tau - 1)$,
and $\begin{vmatrix} \pi_{mm} & \pi_{ma} & \pi_{ml} \\ \pi_{am} & \pi_{aa} & \pi_{al} \\ \pi_{lm} & \pi_{la} & \pi_{ll} \end{vmatrix} = \begin{vmatrix} -\frac{1}{\tau} & -\frac{1}{6\tau} & \frac{1}{\tau} \\ -\frac{1}{6\tau} & -2 & 0 \\ \frac{1}{\tau} & 0 & -6 \end{vmatrix} = -\frac{1}{6\tau^2}(72\tau - 13) < 0$, Solution is: $\tau > \frac{13}{72} = 0.180\,56$. All in all $\tau > \frac{13}{72}$ we get maximization solution.

For platform 2, from first order conditions we know:

$$\begin{aligned}
\pi_{m_2} &= \frac{1}{6\tau}(3\tau + a_1 - 6l_1 + 6l_2 + 3m_1 - 6m_2),\\
\pi_{l_2} &= \frac{1}{\tau}(m_2 - 6\tau l_2).
\end{aligned}$$

Second-order conditions:

$$\begin{aligned}
\pi_{m_2m_2} &= \frac{\partial}{\partial m_2}\left(\frac{1}{6\tau}(3\tau + a_1 - 6l_1 + 6l_2 + 3m_1 - 6m_2)\right) = -\frac{1}{\tau},\\
\pi_{m_2l_2} &= \pi_{l_2m_2} = \frac{\partial}{\partial l_2}\left(\frac{1}{6\tau}(3\tau + a_1 - 6l_1 + 6l_2 + 3m_1 - 6m_2)\right) = \frac{1}{\tau},\\
\pi_{l_2l_2} &= \frac{\partial}{\partial l_2}\left(\frac{1}{\tau}(m_2 - 6\tau l_2)\right) = -6.
\end{aligned}$$

The maximization solution exists if: $\begin{vmatrix} \pi_{m_2m_2} & \pi_{m_2l_2} \\ \pi_{l_2m_2} & \pi_{l_2l_2} \end{vmatrix} > 0 \Rightarrow \begin{vmatrix} -\frac{1}{\tau} & \frac{1}{\tau} \\ \frac{1}{\tau} & -6 \end{vmatrix} = \frac{1}{\tau^2}(6\tau - 1) > 0$. Therefore, maximization profits exist if $\tau > \frac{1}{6}$.

After both of them are vertical integrated, the first order conditions of platform 1 are: $\pi_{m_1} = -\frac{1}{6\tau}(a_1 - 3\tau - 2c_1 + 6l_2 + 6m_1 - 3m_2)$, $\pi_{c_1} = \frac{1}{3\tau}(m_1 - \tau c_1)$, $\pi_{a_1} = -\frac{1}{6\tau}(m_1 - 6\tau + 12\tau a_1)$. The second order conditions are:

$$\begin{aligned}
\pi_{m_1m_1} &= \frac{\partial}{\partial m_1}\left(-\frac{1}{6\tau}(a_1 - 3\tau - 2c_1 + 6l_2 + 6m_1 - 3m_2)\right) = -\frac{1}{\tau}, \\
\pi_{m_1c_1} &= \pi_{c_1m_1} = \frac{\partial}{\partial c_1}\left(-\frac{1}{6\tau}(a_1 - 3\tau - 2c_1 + 6l_2 + 6m_1 - 3m_2)\right) = \frac{1}{3\tau}, \\
\pi_{m_1a_1} &= \pi_{a_1m_1} = \frac{\partial}{\partial a_1}\left(-\frac{1}{6\tau}(a_1 - 3\tau - 2c_1 + 6l_2 + 6m_1 - 3m_2)\right) = -\frac{1}{6\tau}, \\
\pi_{a_1a_1} &= \frac{\partial}{\partial a_1}\left(-\frac{1}{6\tau}(m_1 - 6\tau + 12\tau a_1)\right) = -2, \\
\pi_{a_1c_1} &= \pi_{c_1a_1} = \frac{\partial}{\partial c_1}\left(-\frac{1}{6\tau}(m_1 - 6\tau + 12\tau a_1)\right) = 0, \\
\pi_{c_1c_1} &= \frac{\partial}{\partial c_1}\left(\frac{1}{3\tau}(m_1 - \tau c_1)\right) = -\frac{1}{3}.
\end{aligned}$$

The maximization exists when $\begin{vmatrix} \pi_{m_1m_1} & \pi_{m_1a_1} \\ \pi_{a_1m_1} & \pi_{a_1a_1} \end{vmatrix} = \begin{vmatrix} -\frac{1}{\tau} & -\frac{1}{6\tau} \\ -\frac{1}{6\tau} & -2 \end{vmatrix} = \frac{1}{36\tau^2}(72\tau - 1) > 0 \Rightarrow \tau > \frac{1}{72}$ and $\begin{vmatrix} \pi_{m_1m_1} & \pi_{m_1a_1} & \pi_{m_1c_1} \\ \pi_{a_1m_1} & \pi_{a_1a_1} & \pi_{a_1c_1} \\ \pi_{c_1m_1} & \pi_{c_1a_1} & \pi_{c_1c_1} \end{vmatrix} = \begin{vmatrix} -\frac{1}{\tau} & -\frac{1}{6\tau} & \frac{1}{3\tau} \\ -\frac{1}{6\tau} & -2 & 0 \\ \frac{1}{3\tau} & 0 & -\frac{1}{3} \end{vmatrix} = -\frac{1}{108\tau^2}(72\tau - 25) < 0 \Rightarrow \tau > \frac{25}{72}$. For platform 2, from first order condition: $\pi_{m_2} = \frac{1}{6\tau}(3\tau + a_1 + 2c_2 - 6l_1 + 3m_1 - 6m_2)$ and $\pi_{c_2} = \frac{1}{3\tau}(m_2 - \tau c_2)$. The second order conditions are:

$$\begin{aligned}
\pi_{m_2m_2} &= \frac{\partial}{\partial m_2}\left(\frac{1}{6\tau}\left(3\tau + a_1 + 2c_2 - 6l_1 + 3m_1 - 6m_2\right)\right) = -\frac{1}{\tau}, \\
\pi_{m_2c_2} &= \pi_{c_2m_2} = \frac{\partial}{\partial c_2}\left(\frac{1}{6\tau}\left(3\tau + a_1 + 2c_2 - 6l_1 + 3m_1 - 6m_2\right)\right) = \frac{1}{3\tau}, \\
\pi_{c_2c_2} &= \frac{\partial}{\partial c_2}\left(\frac{1}{3\tau}\left(m_2 - \tau c_2\right)\right) = -\frac{1}{3}.
\end{aligned}$$

We have to make sure that: $\begin{vmatrix} \pi_{m_2m_2} & \pi_{m_2c_2} \\ \pi_{c_2m_2} & \pi_{c_2c_2} \end{vmatrix} = \begin{vmatrix} -\frac{1}{\tau} & \frac{1}{3\tau} \\ \frac{1}{3\tau} & -\frac{1}{3} \end{vmatrix} = \frac{1}{9\tau^2}\left(3\tau - 1\right) > 0 \Rightarrow \tau > \frac{1}{3}$.

In the scenario e3, we have our binding that $\tau > \frac{1}{2}$ to make sure that platform 1's profit is positive. Therefore, our assumption in the extension section is $\tau > \frac{1}{2}$.

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Chapter 4

Entry in Markets with Network Effects

with Fang Liu

4.1 Introduction

Markets with network effects have attracted a lot of interest since the 1980s, renewed interest in platform competition, and recently again with the emergence of data-driven network effects. In such markets, consumers obtain extra utility from the number of other consumers buying the product besides the stand-alone value of the good. In other words, consumers' willingness to pay may increase as the quantity sold. One of the main concerns in the market with network effects is that firms are not able to make credible commitments to their productions.

In the seminal paper by Katz and Shapiro (1985), one of the most famous papers in the Industrial Organization we suppose, the authors developed a simple model with network externalities in oligopoly markets. With consumers' expectations being fulfilled in equilibrium, they find that a monopolist's profit may be lower than that in a duopoly market if only symmetric equilibrium is considered. Economides (1996) shows that firms may be incentivized to invite competitors into the market when network effect is sufficiently strong, since a monopolist cannot make a credible commitment on a higher production level. The paper of Economides (1996) implicitly assumes that firms produce perfectly compatible products. Besides inviting more competitors into the market, Etziony and Weiss (2010) introduces a new remedy of pre-producing as an alternative way to make credible commitments and consequently influence consumers' beliefs. Similar phenomena may also be observed in two-sided markets. Tan and Zhou (2020) show with a discrete choice model that the equilibrium profit of each competing platform may increase with a high number of competitors in the market. Prüfer and Schottmüller (2021) show that when firms compete with big data, data-driven indirect network externalities may cause market tipping towards monopoly and the dominant firm can leverage its market position to a connected market. We analyze the relation between competitive intensity (as measured by the number of firms) and firms' profits when products exhibit positive network effects under both compatibility and incompatibility. With a relatively general demand function, we show that the degree of firms' incentive to invite competitors depends on the influence of the network effect on total demand relative to the price effect. For a given price, when the influence of network effect on total demand increases, firms will have a higher incentive to invite competitors into the market.

We proceed as follows. In Section 4.2, we introduce the model and analyze the equilibrium in Section 4.3. In Section 4.4, we examine how total output, total welfare, and firms' profits change in the number of competitors under compatibility and incompatibility. In Section 4.5, we extend our analysis to the general demand function form. Finally, Section 4.6 concludes.

4.2 The Model

Consider a Cournot oligopoly with n firms, indexed by $i = 1, ..., n$. Production costs are set at zero. The inverse demand function for firm i is

$$p_i = A + v(y_i^e) - z,$$

with $z := \sum_{i=1}^{n} x_i$ and where y_i^e is consumers' prediction of the size of the network with which firm i is associated. We consider two regimes: perfect compatibility (C) and perfect incompatibility (I). For regime C we have $y_i^e = \sum_{i=1}^{n} x_i^e$ for all i and for regime I we get $y_i^e = x_i^e$ for all i.

We invoke the following assumption for the network effects function $v(y_i^e)$: i) $v(0) = 0$, ii) $v' > 0$, $v'' < 0$, and $\lim_{y\to\infty} v'(y) = 0$. These assumptions guarantee an interior solution because each firm's maximization problem is globally concave.

Definition. *In a Fulfilled Expectations Cournot Equilibrium (FECE) each firm chooses its output level under the assumptions that:*
(a)*consumers' expectations about the sizes of the networks* $(y_1^e, ..., y_n^e)$*, are given; and*
(b) *the actual output level of the other firms,* $\sum_{j\neq i} x_j \equiv x_{-i}$ *, is fixed.*
(c) *Firms' optimal quantity choices are consistent with consumers' expectations*

about firms' network sizes.

Assumption (a) means that a firm's quantity decision has no effect on consumers' expectations. Assumption (b) is the standard Nash equilibrium requirement. Assumption (c) says that expectations must be fulfilled in equilibrium.

We find the FECE in three steps: First, we fix the consumers' expectations $(y_1^e, y_2^e, ..., y_n^e)$. Second, each firm maximizes its profit for given quantity choices of the other firms and for given expectations. This gives n first-order conditions. Third, we solve the system of first-order conditions by requiring that initial expectations are fulfilled.

4.3 Analysis

Firm i's profits are

$$\pi_i = (A + v(y_i^e) - z)x_i$$

which gives the first-order condition

$$A + v(y_i^e) - z^* - x_i^* = 0$$

which implies

$$x_i^* = A + v(y_i^e) - \sum_{j=1}^{n} x_j^* = A + v(y_i^e) - z^*$$

and hence

$$\pi^* = (x_i^*)^2$$

for all $i = 1, ..., n$. Solving the equation system gives

$$x_i^* = \frac{A + nv(y_i^e) - \sum_{j \neq i} v(y_j^e)}{n+1} \text{ for all } i.$$

In a FECE the expeced network size of firm i ($y_i^e = y_i^*$) is the equilibrium network size y^*.

Compatibility (C). Compatibility implies that all firms are symmetric: they have all the same expected network sizes which is the entire network:

$$y_i^e = z^e = \sum_{i=1}^{n} x_i^e \text{ for all } i.$$

Hence, each firms' equilibrium quantity must be the same:

$$x_i^* = \frac{A + v(z^e)}{n+1}.$$

Adding up $\sum_i x_i^* = z^c$ yields

$$\begin{aligned} z^c &= n \cdot \frac{A + v(z^e)}{n+1} \text{ or} \\ \frac{n+1}{n} \cdot z^c &= A + v(z^e). \end{aligned} \tag{4.1}$$

Condition (4.1) defines the symmetric equilibrium outcome under compatibility implicitly. Let z^c be the solution, then each firm produces the same

$$x_i^c = \frac{z^c}{n}.$$

Incompatibility (I). When goods are incompatible, we have

$$y_i^e = x_i^e$$

for all i. We focus on the symmetric equilibrium, where all n firms produce the same quantity, so that each firm's network has the same size. Taking $x_i^* = \frac{z^I}{n}$ and adding up all

$$x_i^* = \frac{A + nv\left(\frac{z^I}{n}\right) - (n-1)v\left(\frac{z^I}{n}\right)}{n+1}$$

gives

$$\frac{(n+1)}{n} \cdot z^I = A + v\left(\frac{z^I}{n}\right)$$

which has a unique solution z^I. Note that in an interior solution $v' < \frac{n+1}{n}$ must always holds. Moreover, $z^I < z^c$ must be true.

4.4 Competitive Intensity and Market Outcomes

Compatible products. In a standard Cournot oligopoly model, increasing the number of firms will reduce each firm's profit. This may not longer be true in the presence of positive network effects when goods are compatible.

Proposition 4.1. *Total output and total welfare always increase in the number of firms. A firm's profit increases in the number of Cournot competitors under compatibility, whenever* $1 < v' < \frac{n+1}{n}$ *holds; otherwise it decreases.*

Proof. Equilibrium profit is $\pi_i^* = (z^c/n)^2$, where z^c solves $\frac{n+1}{n} \cdot z^c = A + v(z^c)$ or

$$(n+1)z^c - nA - nv(z^c) = 0. \tag{4.2}$$

Noting that $z^c = z^c(n)$, we calculate the derivative of (4.2) with respect to n to get

$$(n+1)\frac{dz^c}{dn} + z^c - A - v(z^c) - nv' \cdot \frac{dz^c}{dn} = 0.$$

Solving for $\frac{dz^c}{dn}$ we get

$$\frac{dz^c}{dn} = \frac{A - z^c + v(z^c)}{n + 1 - nv'}$$

or let $A + v(z^c) = (n+1)z^c/n$ we get

$$\frac{dz^c}{dn} = \frac{z^c}{n(n+1-nv')}$$

which is positive for $v' < \frac{n+1}{n}$ and holds always in equilibrium (otherwise $\frac{n+1}{n} \cdot z < A + v(z)$ holds with $z < z^c$, see figure). Thus total output (and hence also total welfare) always increase in the number of firms.

We next have to show that the total effect of a change of n increases equilibrium profits; i.e.:

$$\frac{d\pi_i^*}{dn} = \frac{\partial \pi_i^*}{\partial n} + \frac{\partial \pi_i^*}{\partial z^c} \cdot \frac{dz^c}{dn} > 0. \tag{4.3}$$

For the partial derivatives, we get $\frac{\partial \pi_i^*}{\partial n} = -2 \cdot \frac{z^c}{n^3}$ and $\frac{\partial \pi_i^*}{\partial z^c} = 2 \cdot \frac{z^c}{n^2}$. Plugging all derivatives into (4.3) we get the condition

$$\frac{d\pi_i^*}{dn} = -2 \cdot \frac{(z^c)^2}{n^3} + 2 \cdot \frac{z^c}{n^2} \cdot \frac{z^c}{n(n+1-nv')} > 0$$

or

$$\frac{1}{n+1-nv'} > 1,$$

which can only be true if $1 < v' < \frac{n+1}{n}$. The interval $(1, \frac{n+1}{n})$ is non-empty for $n < \infty$, which ensures the possibility that firms' profits increase with the number of competitors. □

When incumbent firms benefit from more competition, then they have an incentive to invite entry into the market which was already worked out by Economides (1996).[1] The market is more competitive when more enter the market, but this leads to more output in the market. The network effect will increase along with the demand, and consumers' willingness to pay is higher. Then the entry benefits both consumers and firms.

Incompatible products. When goods are incompatible total output can decrease depending on the marginal network value, which is contrary to standard comparative static results in Cournot oligopoly. Firms' profits always decrease when competitive intensity becomes stronger.

Proposition 4.2. *Total output and total welfare decrease (increase) in the number of firms whenever* $1 < v' < \frac{n+1}{n}$ $(v' < 1)$ holds. *A firm's profit always decreases in the number of Cournot competitors under incompatibility.*

[1]However, the *Proposition 5* in Economides (1996) states that a firm's profits increase in n if $\frac{n-1}{n} < v' < \frac{n+1}{n}$. The lower bound is obviously wrong because for $n = 1$, v' can become arbitrarily close to zero.

Proof. Equilibrium profit is $\pi_i^* = (z^I/n)^2$, where z^I solves $\frac{n+1}{n} \cdot z^I = A + v\left(\frac{z^I}{n}\right)$ or

$$(n+1)z^I - nA - nv\left(\frac{z^I}{n}\right) = 0. \tag{4.4}$$

Noting that $z^I = z^I(n)$, we calculate the derivative of (4.4) with respect to n to get

$$(n+1)\frac{dz^I}{dn} + z^I - A - v\left(\frac{z^I}{n}\right) - nv' \cdot \left[\frac{dz^I}{dn} \cdot \frac{1}{n} - \frac{z^I}{n^2}\right] = 0.$$

Solving for $\frac{dz^I}{dn}$ we get

$$\frac{dz^I}{dn} = \frac{A - z^I + v\left(\frac{z^I}{n}\right) - v'\frac{z^I}{n}}{n+1-v'}$$

or (using $A + v\left(\frac{z^I}{n}\right) = \frac{n+1}{n}z^I$)

$$\frac{dz^I}{dn} = \frac{z^I}{n}\frac{1-v'}{n+1-v'}.$$

It then follows that $\frac{dz^I}{dn} > 0$ if $v' < 1$ and that $\frac{dz^I}{dn} < 0$ if $1 < v' < \frac{n+1}{n}$. We next show that profits must decrease in n. We have to calculate

$$\frac{d\pi_i^*}{dn} = \frac{\partial \pi_i^*}{\partial n} + \frac{\partial \pi_i^*}{\partial z^c} \cdot \frac{dz^I}{dn} \tag{4.5}$$

with $\frac{\partial \pi_i^*}{\partial n} = -2 \cdot \frac{z^I}{n^3} < 0$ and $\frac{\partial \pi_i^I}{\partial z^c} = 2 \cdot \frac{z^c}{n^2} > 0$. The derivative (4.5) can only be positive if $v' < 1$. Substituting the derivatives we get

$$\frac{d\pi_i^*}{dn} = -2 \cdot \frac{z^I}{n^3} + 2 \cdot \frac{z^c}{n^2} \cdot \frac{z^I}{n}\frac{1-v'}{n+1-v'} > 0 \text{ or}$$
$$\frac{1-v'}{n+1-v'} > 1 \text{ or}$$
$$\frac{1}{n+1-v'} > \frac{1}{1-v'}.$$

which can never hold because $n > 0$. □

4.5 The Model with General Demand Function

From the analysis above, it is obvious that when the network effect is sufficiently strong, firms find it optimal to invite competitors into the market to credibly expand the network size. In this section, we analyze the model with an implicit demand function and provide a more general condition under which firms have an incentive to invite competitors. We show that the results we find in the previous section prevail qualitatively even with more general demand function, namely, firms' profits increase with the number of firms in the market when the network effect is strong enough to outweigh competition effect. Consider now the following inverse demand function

$$p_i = P(z, v(y_i^e)). \tag{4.6}$$

The market price is a function of total output z and that of the network effects $v(y_i^e)$. We invoke the following assumptions for the inverse demand function:

- $P(z, v(y_i^e))$ is differentiable at least twice in $v(y_i^e)$. $\frac{\partial P}{\partial z} < 0$ and $\underset{z \to \infty}{lim} P = 0$. This makes sure that infinite total output is not possible in the market.

- $\frac{\partial P}{\partial v(\cdot)} > 0$ $\forall z$, namely the network effect function only shifts the demand curve upwards.[2]

Note that $v(y_i^e)$ also satisfies the assumptions made for the network effect function in Section 4.2. We still analyze the model under perfect compatibility and perfect incompatibility.

[2]Here, the network effect function works similarly to the quality level in Spence (1975), in which a monopolist optimally the quality of the product. In this model, the demand function is $P(x, Q)$, where x is the total quantity and Q is the quality level chosen by the monopolist. We can interpret the network effect in our setting in a such way that the quality of the products is represented by the number of consumers buying the products. When $\frac{\partial^2 P}{\partial z \partial v} = 0$, the network effect function simply shifts the demand curve parallel outward. In this case, consumers value network size in the same way. For a given level of quantity, when $\frac{\partial^2 P}{\partial z \partial v} < 0$, the marginal consumers value the network effect more and are willing to pay more than the marginal consumer. In other words, the slope of the demand curve decreases in $v(\cdot)$, which means that the demand curve becomes more inelastic than that without a network effect. When $\frac{\partial^2 P}{\partial z \partial v} > 0$, the situation is reversed. Note that firms maximize their profit by considering the utility of the marginal consumer.

4.5.1 Compatible Products

Firm i's profit is

$$\pi_i = P(z, v(y_i^e)) \cdot x_i. \tag{4.7}$$

Taking the first-order condition we get

$$\frac{\partial \pi_i}{\partial x_i} = P(z, v(y_i^e)) + x_i \cdot \frac{\partial P(z, v(y_i^e))}{\partial z} = 0.$$

We focus on symmetric equilibria with $x^* = \frac{z^*}{n}$. Assume that firms are symmetric and consumers' expectations are fulfilled in equilibrium. The equilibrium quantity x^* satisfies

$$F := \left.\frac{\partial \pi_i}{\partial x_i}\right|_{x^*} = P(z^*, v(z^*)) + x^* \cdot \frac{\partial P(z^*, v(z^*))}{\partial (z^*)} = 0 \tag{4.8}$$

Lemma 4.1. *The fulfilled expectation equilibrium is locally stable if* $\frac{v'(P_v + x^* \cdot P_{zv})}{(n+1)P_z + P_{zz} \cdot n \cdot x^*} > -\frac{1}{n}$ *holds.*

Proof. Similar as in Economides (1996), a fulfilled expectation equilibrium is locally stable if $\frac{\partial z^*}{\partial y_i^e} < 1$ holds in equilibrium. We then get

$$\begin{aligned} \frac{\partial z^*}{\partial y_e^i} &= n \cdot \frac{\partial x^*}{\partial y_i^e} \\ &= -n \cdot \frac{\frac{\partial F}{\partial y_i^e}}{\frac{\partial F}{\partial x^*}} \\ &= -n \cdot \frac{v'\,(P_v + x^* \cdot P_{zv})}{(n+1)\,P_z + P_{zz} \cdot n \cdot x^*} < 1 \end{aligned}$$

We define $P_v = \frac{\partial P(z^*, v(y_i^e))}{\partial v}$, $P_{zv} = \frac{\partial^2 P(z^*, v(y_i^e))}{\partial z^* \partial v}$, $P_z = \frac{\partial P(z^*, v(y_i^e))}{\partial z^*}$, and $P_{zz} = \frac{\partial^2 P(z^*, v(y_i^e))}{(\partial z^*)^2}$. Hence the condition state in the Lemma.

$$\frac{(P_v + x^* \cdot P_{zv}) \cdot v'}{(n+1)\,P_z + P_{zz} \cdot n \cdot x^*} > -\frac{1}{n} \tag{4.9}$$

The profit under equilibrium of firm i is given by

$$\pi_i|_{y_e^i=z^*} = -(x^*(n))^2 \cdot \frac{\partial P(z^*, v(y_e^i))}{\partial z^*}. \tag{4.10}$$

□

Now we analyze in detail the relationship between firms' profit and network effects. Firms' profit increases in the number of firms in the market if $2P_z + 2P_v \cdot v' + xP_{zz} + xP_{zv} \cdot v'$ is positive. [3] For simplicity, we assume that the demand function without network effect is linear, namely $P_{zz} = 0$.

Proposition 4.3. *A firm's profit increases in the number of competitors when products are compatible, whenever* $1 - \frac{xP_{zv} \cdot v'}{2|P_z|} < \frac{P_v \cdot v'}{|P_z|} < \frac{n+1}{n} - \frac{xP_{zv} \cdot v'}{|P_z|}$ *holds, otherwise it decreases.*

Proof in Appendix

We need to guarantee $2 - \frac{P_v \cdot v'}{|P_z|} < \frac{n+1}{n}$ to achieve Proposition 3. Hence, $\frac{P_v \cdot v'}{|P_z|} > \frac{n-1}{n} \geq 0$, and $P_v > 0$ from our assumptions, therefore, $v' > 0$ should be fullfiled. From Proposition 4.3 we can rewrite as $|P_z| - \frac{xP_{zv} \cdot v'}{2} < P_v \cdot v' < \frac{n+1}{n} \cdot |P_z| - xP_{zv} \cdot v'$. We have to analyze the relationship between the marginal network effect $(P_v \cdot v')$ and the competition effect $(|P_z|)$, which affects the firms' profits whether increase with higher n. Taking derivative of $P_v \cdot v'$ with respect to z we get the change of marginal value of network effect along the demand curve:

$$\partial \frac{P_v \cdot v'}{|P_z|} / \partial z = \frac{1}{|P_z|} \cdot P_{zv} v'. \tag{4.11}$$

The sign of (4.11) depends on that of P_{zv}, which represents how marginal consumer values marginal network effect. If $P_{zv} = 0$, we are in the case analyzed in Section 4.3, in which the function of network effect shifts the demand curve parallel upwards. When $P_{zv} \neq 0$, we consider the following four cases:

[3] $\frac{d\pi_i}{dn} = x^2 P_z \frac{2P_z + 2P_v v' + xP_{zz} + xP_{zv} v'}{P_z + nP_z + nP_v v' + nxP_{zz} + nxP_{zv} v'}$, the denominator is negative, and P_z is negative. Therefore, $\frac{d\pi_i}{dn}$ is positive when $2P_z + 2P_v v' + xP_{zz} + xP_{zv} v'$ is positive. Details in Appendix.

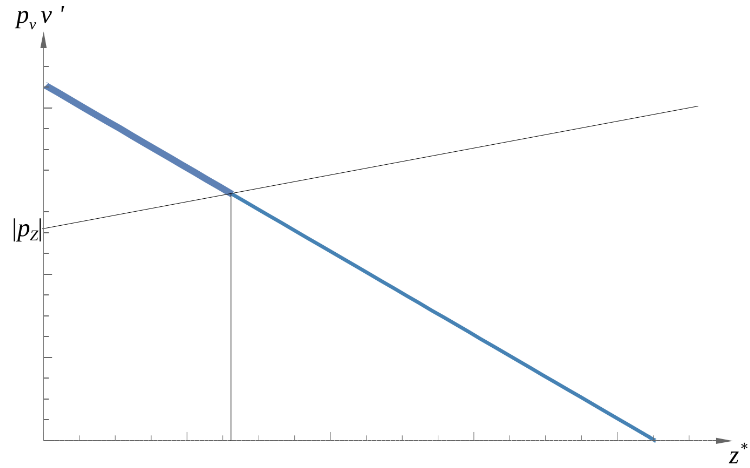

Figure 4.1: The marginal consumer to the marginal network effect is negative

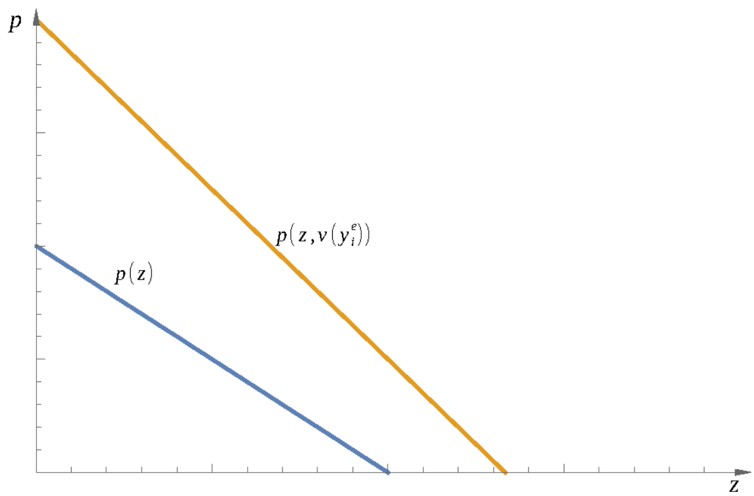

Figure 4.2: The example of demand curve when the marginal consumer to the marginal network effect is negative

Case 4.1: $P_{zv} < 0$ as shown in Figure 4.1. In this case $P_{zv} < 0$, which is equivalent to $\frac{P_v \cdot v'}{|P_z|} > 1$. In this case, the marginal consumer values network effect less than the average consumer. In other words, if a firm's profit increases in n, the speed of increase becomes slower and slower. For example, as the two demand functions shown in Figure 4.2. The lower demand curve represents the market without network effect while the upper one represents that with network effect. It is obvious that the difference in consumer prices becomes smaller and smaller for a given quantity. Interestingly, we find out that if the competition effect ($|P_z|$) is fixed, a firm's profit increases with the number of firms if and only if the equilibrium quantity is sufficiently small, as shown in the bold area in Figure 4.2. This reminds us of some markets, for example, industries of luxury goods, in which consumers' interest in the product is maximized when the network size they are facing is not too large. When the equilibrium quantity becomes too high, firms face way more intense competition such that the competition effect exceeds the network effect and firms are worse off. If we consider now the relation between firms' profits and the number of firms in the market, we find out that firms' profits are maximized when $\frac{P_v \cdot v'}{|P_z|} = 1$, namely when competition effect and network effect are identical.

Now let us compare the firms' incentives to the socially optimal level. We know from the analysis above that consumer surplus as well as social welfare always increases with more firms. However, firms will invite competitors into the market only if the competition effect does not exceed the network effect. When the number of firms becomes larger, the influence of the competition effect becomes more important in firms' profits and finally firms' incentive of inviting entry vanishes when firms fully exploit consumers' additional willingness to pay due to the network effect.

Case 4.2: $P_{zv} > 0$ as shown in Figure 4.3. In this case, firms have an incentive to invite competitors only if the equilibrium output is sufficiently large (the bold part in Figure 4.3). Figure 4.4 illustrates an example of demand curves. To put it differently, firms' profits reach their minimum point when the competition

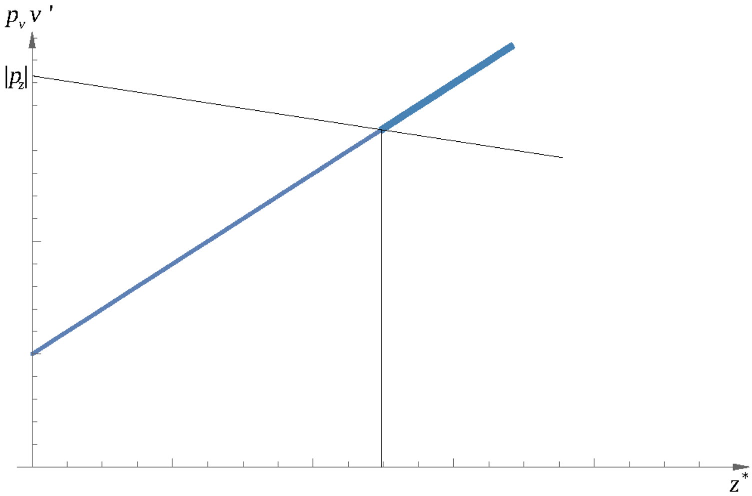

Figure 4.3: The marginal consumer to the marginal network effect is positive

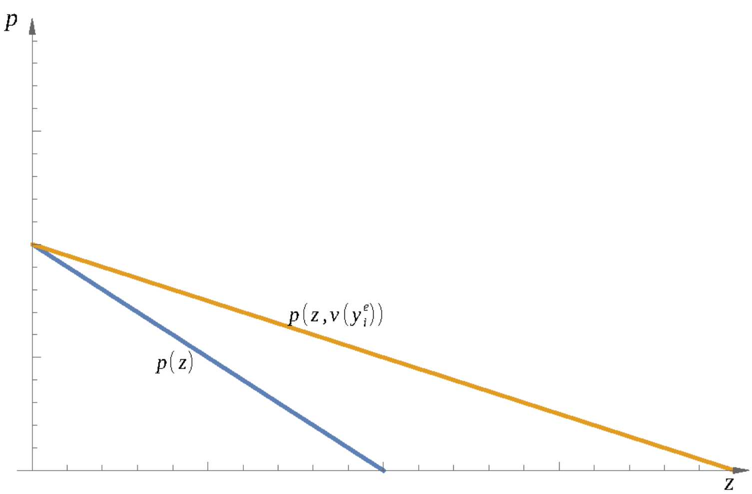

Figure 4.4: The example of demand curve when the marginal consumer to the marginal network effect is positive

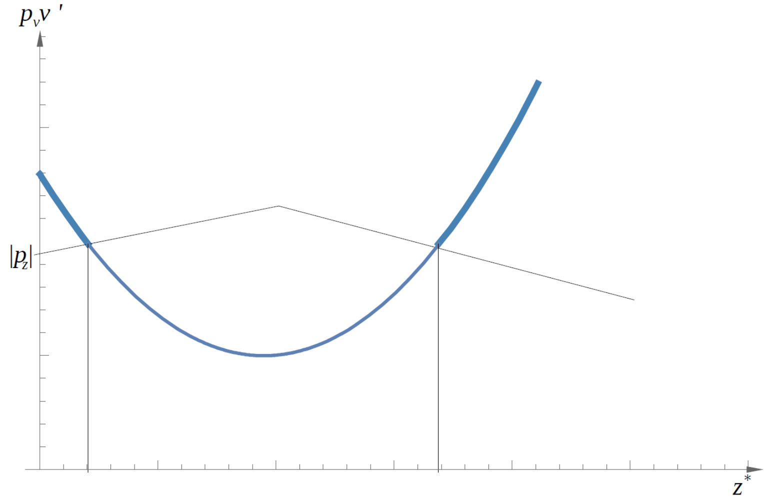

Figure 4.5: The marginal consumer to the marginal network effect is convex

effect and network effect are identical. In reality, some consumers may find the product more attractive when there are already large numbers of consumers in the market. For example, some fractions of consumers are less informed about the characteristics (quality, functions and so on) of the product. So they choose to take the market size of this product as a signal. When they observe more users of this product, they would expect that there will be even more users entering the market and therefore are willing to pay more. From the perspective of firms, they find it more difficult to attract consumers with higher values of the product and they engage in more intense competition in this market fragment. When more consumers with a relatively low value of the product enter the market, network effect dominants and firms enjoy a higher profit from more competitors.

Case 4.3: P_{zv} is convex as shown in Figure 4.5. Similar to before, the lower curve of Figure 4.6 is the basic market demand curve without network effect while the upper one is the demand curve with network effect and now it is convex. Figure 4.5 shows that consumers are willing to pay the least when the equilibrium quantity lies relatively in the middle of the demand curve. This means that, theoretically there is a such market in which firms' profits increase with more

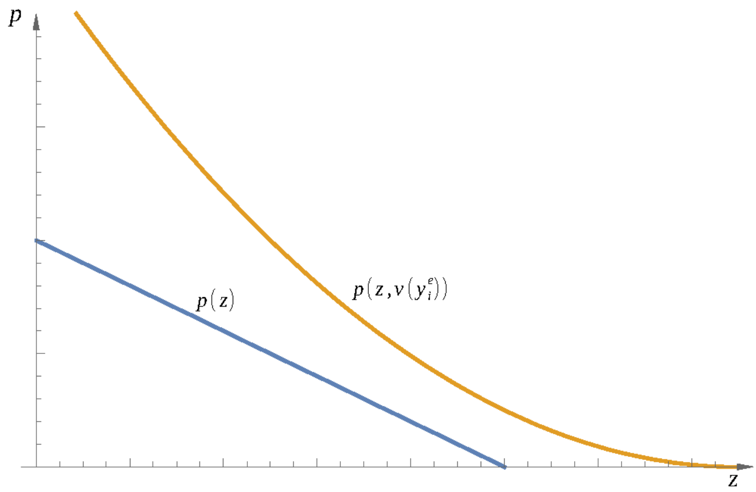

Figure 4.6: The example of demand curve when the marginal consumer to the marginal network effect is convex

competitors when they realize an extremely small or extremely large amount of output (as shown in the bold part in Figure 4.6).

Case 4.4: P_{zv} is concave as shown in Figure 4.7. In this case, we have a market in which consumers' willingness to pay is maximized when the equilibrium quantity is neither very large nor very small (see Figure 4.8 as an example). Theoretically, there exists such a market with network externalities in which firms have an incentive to invite competitors when they realize non-extreme output levels in equilibrium (as shown in the bold part in Figure 4.7).

From the analysis above, especially from **Case 4.1** and **Case 4.2**, we can find out that firms' incentive to invite competitors depends on the level of total quantity in equilibrium. For example in **Case 4.1**, if firms manage to conduct cost-reducing innovation and realize higher total output in equilibrium, firms would not find it profitable to invite competitors but rather prefer to produce by themselves. In **Case 4.2**, things go the opposite. When the equilibrium output becomes smaller resulting from, for example, a negative demand shock or a sudden increase of input materials, firms' profits decrease with a higher number of firms.

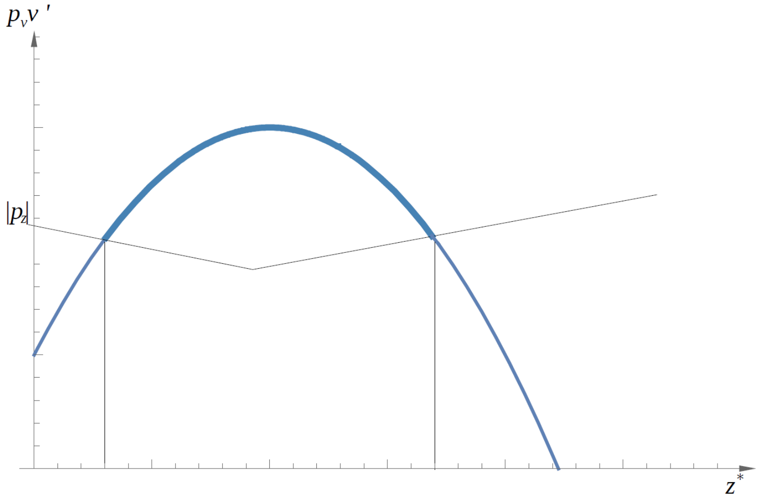

Figure 4.7: The marginal consumer to the marginal network effect is concave

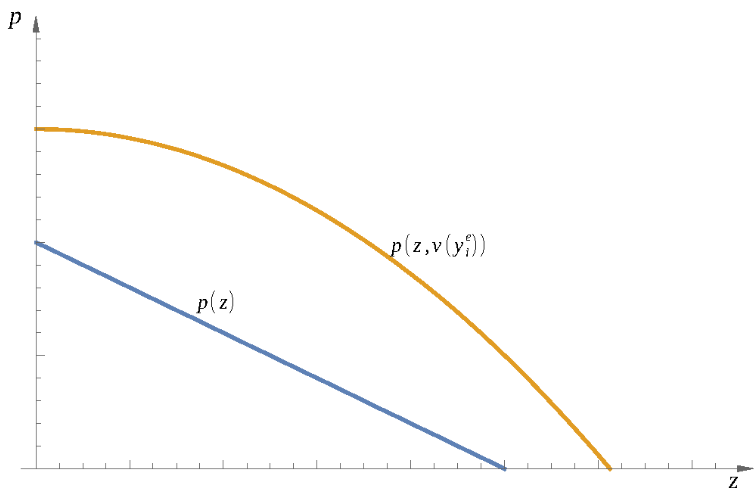

Figure 4.8: The example of demand curve when the marginal consumer to the marginal network effect is concave

We can find more complicated scenarios in **Case 4.3** and **Case 4.4** theoretically.

We have analyzed firms' incentives down to the details. Next, we analyzed the influence of the number of firms on total output and social welfare.

Proposition 4.4. *Total output and social welfare always increase in the number of competitors in the market when products are compatible.*

Proof. We take the first derivative of (4.8) with respect to n and obtain

$$\frac{\partial z^*}{\partial n} = \frac{z^* P_z}{n\,(n+1)\,P_z + P_v v' + z^* P_{zv} v'} > 0. \tag{4.12}$$

The numerator of (4.12) is negative since $P_z < 0$. The denominator is also negative, since the condition of a locally stable equilibrium must hold here. Therefore total output increases with higher n.

Sum up firms' profits and consumer surplus we get social welfare which can be calculated as

$$SW(n) = \int_0^{z^*(n)} P dz. \tag{4.13}$$

Taking the first derivative of (4.13) with respect to n we obtain

$$\frac{\partial SW}{\partial n} = P^* \cdot \frac{\partial z^*}{\partial n},$$

which is always positive, since $\frac{\partial z^*}{\partial n} > 0$ always holds. □

4.5.2 Incompatible Products

Proposition 4.5. *A firm's profit decreases in the number of competitors under incompatible.*

Proof. The equilibrium quantity x^* now satisfies

$$G := \left.\frac{\partial \pi_i}{\partial x_i}\right|_{x^*} = P(z^*, v(x^*)) + x^* \cdot P_z = 0. \tag{4.14}$$

The effect of n on firm i's profit is given by

$$\frac{d\pi_i}{dn} = -\frac{\partial x^*}{\partial n}\left(2x^* P_z + (x^*)^2 P_{zv} v^{'}\right).$$

Similar as under the case of compatible products, we need to find out the sign of $\frac{\partial x^*}{\partial n}$.

$$\frac{dG}{dn} = \frac{\partial x^*}{\partial n}\left((n+1) P_z + P_v + x^* P_{zv} v^{'}\right) + x^* P_z.$$

Solving for $\frac{\partial x^*}{\partial n}$ we obtain

$$\frac{\partial x^*}{\partial n} = -\frac{x^* P_z}{(n+1) P_z + P_v v' + x^* P_{zv} v'}.$$

Therefore, the effect of n on firm i's profit is given by

$$\frac{d\pi_i}{dn} = (x^*)^2 P_z \frac{2P_z + x^* P_{zv} v^{'}}{(n+1) P_z + P_v v' + x^* P_{zv} v'}.$$

The denominator is always negative because the condition for a locally stable equilibrium in (4.9) must hold. In the numerator $2P_z + x^* P_{zv} v^{'}$ is always negative. Therefore, with incompatible products, firms' profits always decrease with a higher number of firms in the market. □

Proposition 4.6. *Total output and total welfare decrease (increase) in the number of firms whenever* $|P_z| < P_v v^{'} + \frac{z}{n} P_{zv} v^{'}$ *(*$|P_z| > P_v v^{'} + \frac{z}{n} P_{zv} v^{'}$*) holds.*

Proof. We take the first derivative of (4.14) with respect to n and obtain

$$\frac{dz^*}{dn} = \frac{z^*\left(P_z + P_v v^{'} + \frac{z^*}{n} P_{zv} v^{'}\right)}{n(n+1) P_z + nP_v v^{'} + z^* P_{zv} v^{'}}. \tag{4.15}$$

Similar as the case under compatibility, the denominator of (4.15) is negative, since the condition of a locally stable equilibrium must be true. The numerator is negative (positive) if $|P_z| < P_v v^{'} + \frac{z}{n} P_{zv} v^{'}$ ($|P_z| > P_v v^{'} + \frac{z}{n} P_{zv} v^{'}$).

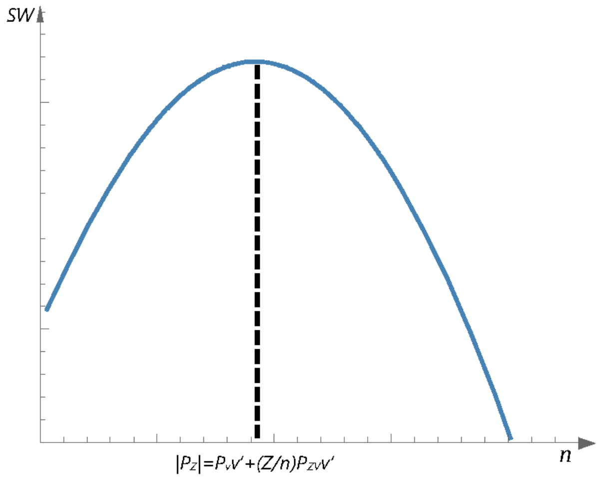

Figure 4.9: The number of firms affects the social welfare in incompatibility case

Given the equilibrium quantity z^*, we get social welfare under incompatibility

$$SW(n) = \int_0^{z^*(n)} P dz. \tag{4.16}$$

Taking the derivative of (4.16) with respect to n gives

$$\frac{\partial SW}{\partial n} = P^* \cdot \frac{\partial z^*}{\partial n}.$$

We find out that the sign of $\frac{\partial SW}{\partial n}$ depends on the sigh of $\frac{dz^*}{dn}$ and we obtain the same condition as that from (4.16), namely social welfare decrease (increase) in number of firms if $|P_z| < P_v v' + \frac{z}{n} P_{zv} v'$ ($|P_z| > P_v v' + \frac{z}{n} P_{zv} v'$) holds. □

Figure 4.9 shows how the number of firms affects the social welfare in incompatibility case. We can find that social welfare does not always increase in the number of competitors. When products are incompatible, firms' individual output in equilibrium always decreases with more competitors. This is because consumer cannot enjoy the network size of the whole market and their beliefs of network size are restricted by the individual output level. Even when the network effect is sufficiently strong, firms cannot increase their individual output with more entries. Therefore, consumers' willingness to pay decreases and the market shrinks as the number of firms increases. That is the reason why total output and social welfare can decrease when competition becomes more intense.

4.6 Conclusion

Our results are interesting for the regulation of the market with network effects. We show when products are compatible, social welfare is always higher with more competitors in the market. The profits of firms can increase when marginal network effects are sufficiently strong. When products are incompatible, we found that market efficiency may increase or decrease with a higher number of firms depending on the shape of the network externalities function. In particular, when network effects are strong at the margin, then increasing the number of competitors reduces total output. Firms are always worse off when the number of competitors in the market increases.

4.7 Appendix

In this part we show the proof of Lemma 3.1. Here we need to note that $F := \frac{\partial \pi_i}{\partial x_i}\Big|_{x^*} = P(z^*, v(y_i^e)) + x^* \cdot \frac{\partial P(z^*, v(y_i^e))}{\partial (z^*)} = 0$, $z^* = nx^*$. Because of Implicit Theorem

$$n \cdot \frac{\partial x^*}{\partial y_e^i} = -n \cdot \frac{\frac{\partial F}{\partial y_i^e}}{\frac{\partial F}{\partial x^*}}. \tag{4.17}$$

$\frac{\partial F}{\partial y_e^i} = \frac{\partial P(z^*, v(y_i^e))}{\partial v} \cdot \frac{\partial v}{\partial y_e^i} + x^* \cdot \frac{\partial^2 P(z^*, v(y_i^e))}{\partial z^* \partial v} \cdot \frac{\partial v}{\partial y_e^i} = v' \left(\frac{\partial P(z^*, v(y_i^e))}{\partial v} + x^* \cdot \frac{\partial^2 P(z^*, v(y_i^e))}{\partial z^* \partial v} \right)$, and $\frac{\partial F}{\partial x^*} = \frac{\partial P(z^*, v(y_i^e))}{\partial z^*} \cdot n + \frac{\partial P(z^*, v(y_i^e))}{\partial z^*} + \frac{\partial^2 P(z^*, v(y_i^e))}{(\partial z^*)^2} \cdot n \cdot x^* = (n+1) \frac{\partial P(z^*, v(y_i^e))}{\partial z^*} + \frac{\partial^2 P(z^*, v(y_i^e))}{(\partial z^*)^2} \cdot n \cdot x^*$. Then we could rewrite equation (4.17) as:

$$\frac{\partial z^*}{\partial y_e^i} = -n \cdot \frac{P_v \cdot v' + x^* \cdot P_{zv} \cdot v'}{(n+1) P_z + P_{zz} \cdot n \cdot x^*},$$

and this solution should be smaller than one.

Proof of Proposition 3.3. The detailed proof of Proposition 3.3 is as follows. The profit function is $\pi_i|_{y_i^e = z^*} = -(x^*(n))^2 \cdot \frac{\partial P(z^*, v(y_i^e))}{\partial z^*}$, and when we take respect to n gives

$$\frac{d\pi_i}{dn} = -(2x^*(n) \cdot \frac{\partial x^*(n)}{\partial n} \cdot P_1 + x^3 P_{zz} + nx^2 \frac{dx^*(n)}{dn} P_{zz} + x^3 P_{zv} \cdot v' + nx^2 \frac{dx^*(n)}{dn} P_{zv} \cdot v'),$$

where $\frac{\partial z^*(n)}{\partial n} = x^*(n) + n\frac{dx^*(n)}{dn}$, hence we can rewrite it as

$$\frac{d\pi_i}{dn} = -\frac{\partial x^*(n)}{\partial n} \left(2xP_z + nx^2 \left(P_{zz} + P_{zv} \cdot v' \right) \right) - x^3 \left(P_{zz} + P_{zv} \cdot v' \right).$$

When we calculate for $\frac{\partial x^*(n)}{\partial n}$, note that here $P_v = \frac{\partial P(z^*, v(z^*))}{\partial v}$, $P_{zv} = \frac{\partial^2 P(z^*, v(z^*))}{\partial z^* \partial v}$, $P_z = \frac{\partial P(z^*, v(z^*))}{\partial z^*}$, and $P_{zz} = \frac{\partial^2 P(z^*, v(z^*))}{(\partial z^*)^2}$. Because $F = P(z^*, v(z^*)) + x^* \cdot$

$\frac{\partial P(z^*,v(z^*))}{\partial(z^*)} = 0$, we can take F with restpect to n. Therefore, we canget that

$$P_z \cdot \frac{\partial z^*(n)}{\partial n} + P_v \cdot v^{'} \cdot \frac{\partial z^*(n)}{\partial n} + \frac{\partial x}{\partial n} \cdot P_z + x\left(P_{zz} \cdot \frac{\partial z^*(n)}{\partial n} + P_{zv} \cdot v^{'} \cdot \frac{\partial z^*(n)}{\partial n}\right) = 0.$$

Plug $\frac{\partial z^*(n)}{\partial n} = x^*(n) + n\frac{dx^*(n)}{dn}$ into the equation, we then get:

$$\frac{\partial x^*(n)}{\partial n} = -\frac{x^2\left(P_{zz} + P_{zv} \cdot v^{'}\right) + x\left(P_z + P_v \cdot v^{'}\right)}{n\left(P_z + P_v \cdot v'\right) + P_z + xn\left(P_{zz} + P_{zv} \cdot v'\right)}.$$

When we plug into the profit function with respect to n:

$$\frac{d\pi_i}{dn} = -\frac{\partial x^*(n)}{\partial n}\left(2xP_z + nx^2\left(P_{zz} + P_{zv} \cdot v^{'}\right)\right) - x^3\left(P_{zz} + P_{zv} \cdot v^{'}\right)$$

Then we get the result:

$$\frac{d\pi_i}{dn} = x^2 P_z \frac{2P_z + 2P_v v^{'} + xP_{zz} + xP_{zv}v^{'}}{P_z + nP_z + nP_v v' + nxP_{zz} + nxP_{zv}v'}.$$

From Lemma 1 we know the denominator is negative, $\frac{d\pi_i}{dn}$ is positive if the numerator $2P_z + 2P_v v^{'} + xP_{zz} + xP_{zv}v^{'} > 0$.

Using Lemma 1 $\frac{P_v v^{'} + x \cdot P_{zv}v^{'}}{(n+1)P_z + P_{zz} \cdot n \cdot x} > -\frac{1}{n}$, and assuming $P_{zz} = 0$, then

$$\frac{d\pi_i}{dn} = x^2 P_z \frac{2P_z + 2P_v v^{'} + xP_{zv}v^{'}}{P_z + nP_z + nP_v v' + nxP_{zv}v'}.$$

According to our Lemma 1 we can get: $\frac{P_v v^{'} + x \cdot P_{zv}v^{'}}{|P_z|} < \frac{n+1}{n}$, denominator is negative for sure, P_z is negative, then $\frac{d\pi_i}{dn}$ is larger than zero when $2P_z + 2P_v v^{'} + xP_{zv}v^{'} > 0$. We get $\frac{P_v v^{'} + x \cdot P_{zv}v^{'}}{|P_z|} > 2 + \frac{P_v}{P_z}$. Hence, we get the result for Proposition 3.

4.8 References

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Chapter 5

Conclusion

In my dissertation, I discuss the competition among three different markets using theoretical methods.

In Chapter 2, we analyzed the social welfare effect of consumer-surplus increasing mergers, which should pass the decision screen of antitrust authorities that apply a consumer-welfare standard to evaluate merger proposals. We show that such procompetitive mergers are only problematic from a social-welfare perspective when a runner-up merger occurs. A social-welfare decreasing runner-up merger fulfills two criteria. First, it is a merger between relatively small firms. Second, the merger efficiencies are limited. In contrast, a merger of relatively large firms, which is consumer-surplus increasing and also always social welfare increasing. According to our analysis, the efficiency offense is most likely critical when the merging firms have a below-average joint market share, while efficiencies ensure that the merger's market share increases and tends to reduce market concentration—i.e., when a runner-up merger is at stake.

In Chapter 3, we demonstrate how a two-sided market model can be used to analyze the membership fee, profits of platforms, and the number of videos offered before and after vertical integration. Platforms have an incentive to integrate vertically, and our results reveal that vertical integration of only one platform can increase the subscription fee. When the vertically integrated platform competes with another vertically separated platform in the market, the integrated platform sets a higher price compared with the two vertically separated platforms' competition scenario. Two platforms will end up with a fully vertical integration scenario. In the extension, we discuss the competition between the subscription platform and the subscription+ad platform. The results mainly show that the subscription platform has more incentive to choose vertical integration. The welfare analysis implies that platforms need to provide relatively different content to ensure consumer surplus and social welfare both increase.

In Chapter 4, we analyze the relationship between competitive intensity and firms' profits when products exhibit positive network effects under both compatibility and incompatibility. We show that for a given price, when the influence of network effects on total demand increases, firms will have a higher incentive to invite competitors into the market. When products are compatible, social welfare is always higher with more competitors in the market. The profits of firms can increase when marginal network effects are sufficiently strong.