Essays on the Effects of Various Types of Heterogeneity on Business Cycle Fluctuations and Risk Sharing

DISSERTATION

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Contents

General Introduction	1
Paper I: Gender Discrimination, Inflation, and the Business Cycle	6
Paper II: Asymmetric Macroeconomic Effects of QE and Excess Reserves in a Monetary Union	a 54
Paper III: How Should Central Banks React to Household Inflation Hetero- geneity?	- 102
Paper IV: Risk Sharing Heterogeneity in the United States	141
Eidesstattliche Versicherung	168

General Introduction

Various types of heterogeneity play a vital role in the transmission of shocks and monetary policy on macroeconomic outcomes. Household heterogeneity in income, wealth, or asset holdings has a significant impact on the effectiveness of monetary policy transmission (Kaplan et al., 2018), while monetary policy itself has distributional effects with respect to income and wealth inequality (see, for instance, Furceri et al., 2018).¹ Furthermore, heterogeneity between member states of monetary unions has significant effects on business cycle fluctuations (Albonico et al., 2019) and consumption risk sharing (Kalemli-Ozcan et al., 2003; Demyanyk et al., 2007). This thesis aims to further the understanding of the impact of household and cross-state heterogeneity on business cycle fluctuations and risk sharing. In particular, four essays, henceforth referred to as papers, are presented.

The first paper, Gender Discrimination, Inflation, and the Business Cycle (co-authored with Ulrike Neyer), builds on literature suggesting that women are discriminated against in the labor market. In particular, various empirical studies provide estimates of adjusted gender wage gaps, i.e., wage gaps accounting for observable productivity measures, that are both significant and persistent over time. Our paper uses the adjusted gender wage gap as a proxy for gender discrimination and analyzes the effects of this discrimination on business cycle fluctuations and inflation dynamics. We build a New Keynesian model which includes heterogeneous households that consist of a female and a male agent. We further extend conventional New Keynesian models² by modeling unpaid household production in addition to paid labor market work and by considering gender discrimination on the firms' side. In particular, we differentiate between two types of gender discrimination: taste-based (as in Becker, 1971) and statistical discrimination (as in Phelps, 1972; Arrow, 1973). In order to examine and compare the respective impact of each type of gender discrimination on business cycle and inflation dynamics, we simulate a negative demand and an expansionary monetary policy shock. We then compare the results of an environment with taste-based or statistical discrimination with their respective non-discriminatory counterparts. We find that both types of discrimination lead to larger economic downturns after a negative demand shock. Gender discrimination implies an inefficient utilization of both female and male productivity and an inefficient intra-household working time allocation between household and labor market work. Furthermore, the transmission of

¹Ampudia et al. (2018) and Kaplan and Violante (2018) provide a detailed overview.

^{2}As in the textbooks by Galí (2015) or Walsh (2017).

expansionary monetary policy shocks on inflation is dampened by taste-based and statistical gender discrimination, as discrimination implies lower costs (wages) for firms. In addition, expansionary monetary policy shocks increase the discriminatory wage gap between women and men. Comparing the effects of the two types of discrimination, we find that taste-based discrimination implies larger macroeconomic distortions (i.e., larger dampening effects on output and inflation), while statistical discrimination leads to higher intra-household inefficiencies (i.e., a more inefficient working time allocation and larger discriminatory gender wage gaps). We conclude that introducing measures that aim to tackle gender discrimination might not only reduce inefficiencies between women and men but also serve as efficient macroeconomic stabilization tools.

The second paper, Asymmetric Macroeconomic Effects of QE and Excess Reserves in a Monetary Union (co-authored with Maximilian Horst and Ulrike Neyer), considers the heterogeneous accumulation of excess reserves in the banking sector of euro area countries caused by the specific implementation of quantitative easing (QE) by the European Central Bank (ECB). In particular, we develop a two-country New Keynesian model of a monetary union and analyze the macroeconomic effects of QE-induced heterogeneous increases in banks' excess reserves. The countries are calibrated to represent a high- and a low-liquidity euro area country (Germany and Italy, respectively). We assume that the central bank has encountered the effective lower bound on short-term interest rates and consider QE as the sole monetary policy tool. By conducting QE, the central bank lowers the long-term interest rates in the economy and increases banks' excess reserves and deposits in the two countries asymmetrically (with the high-liquidity country being affected more strongly than the low-liquidity country). These increases in excess reserves and deposits have implications for the efficacy of QE, as they cause balance sheet costs of banks (regulatory costs, for instance) to increase. In particular, after a negative demand shock, we find that conducting QE stabilizes union-wide consumer price inflation by triggering economic activity in both countries (interest rate channel). However, the efficacy of QE is dampened by increases in excess reserves and deposits and the corresponding increase in banks' balance sheet costs (reverse bank lending channel). This dampening effect is larger in the high-liquidity than in the low-liquidity country. Furthermore, we simulate a shift of QE-created deposits from the low-liquidity to the high-liquidity country due to capital flight, for instance. The increase (decrease) in deposits leads to higher (lower) balance sheet costs for banks and therefore to worse (better) credit lending conditions. Thus, the country

losing the deposits benefits, while the country gaining deposits experiences dampening effects on economic activity. We conclude that stabilizing effects of current monetary policy measures that are implemented similarly to QE, such as the Pandemic Emergency Purchase Programme (PEPP), are also dampened by the reverse bank lending channel.

The third paper aims to answer the question How Should Central Banks React to Household Inflation Heterogeneity? (co-authored with Ulrike Neyer). This paper builds on literature suggesting that significant inflation differentials exist across households. We develop a New Keynesian model with heterogeneous households that differ in their inflation experience after shocks. We allow for household heterogeneity with respect to their income, preferences, and substitution capabilities, leading low-income households to experience higher consumer price index (CPI) inflation rates than high-income households after shocks. In order to assess how central banks that aim to stabilize the economy-wide inflation rate should react to this heterogeneity in CPI inflation experiences, we assume the central bank in our model to react in three different ways: by only considering the CPI inflation rate of the low-income household. by reacting solely to the CPI inflation rate experienced by the high-income household, or by considering the average inflation rate. After a negative demand shock, a central bank that only reacts to CPI inflation of the low-income household mitigates the shock more effectively than under the other two regimes. In particular, the CPI inflation rates experienced by both households exhibit lower volatility under that regime. After a negative supply shock, a central bank that solely reacts to the CPI inflation rate of the low-income household mitigates the initial impact of the shock on the CPI inflation rates of both households more effectively. However, these inflation rates exhibit higher volatility under that regime. Generally, our findings suggest that central banks are able to stabilize the volatility of the economy-wide inflation rate more effectively when only considering the CPI inflation rate experienced by the household that is affected less by the respective shock. We conclude that discretionary reactions of central banks regarding the considered inflation rates after shocks are likely to lead to a more effective attainment of an economy-wide inflation target and to lower fluctuations of all inflation rates experienced by different households.

The fourth paper, *Risk Sharing Heterogeneity in the United States*, empirically examines heterogeneity in the insurance of consumption streams against adverse regional shocks across US states. This paper builds on literature documenting high aggregate risk sharing against output fluctuations in the United States by considering three channels of risk sharing: an income smoothing channel, federal transfers, and a consumption smoothing channel. Using a panel data set of all US states (plus Washington, DC) ranging from 1963 to 2013, I estimate aggregate and state-specific risk sharing profiles. The aggregate risk sharing profile of the United States is characterized by large contributions of the income and the consumption smoothing channel (48% and 26.6%, respectively) and a modest but significant role of federal transfers (9.4%). The estimates of the state-specific risk sharing profiles indicate considerable heterogeneity across US states along two dimensions: the extent of overall consumption insurance and the contribution of each risk sharing channel. Based on these results, I identify four distinct clusters of states. One cluster displays an insurance profile close to the aggregate risk sharing profile of the United States. The other three clusters are characterized by an insurance profile that emphasizes the contribution of one particular risk sharing channel: one cluster insures consumption significantly more through income smoothing (67.9%), one through federal transfers (17.4%), and one through consumption smoothing (53%). The paper then examines potential reasons for this heterogeneity by estimating the effects of various state characteristics on the level of overall risk sharing and the extent of consumption insurance provided by each channel. The results show that the overall level of risk sharing is positively related to lower economic activity at risk, better insurance opportunities of states, and lower shock persistence. The contribution of federal transfers is positively associated with higher volatility in unemployment rates, while consumption smoothing is negatively related to state tax and expenditure limits and higher population poverty rates.

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Paper I

Gender Discrimination, Inflation, and the Business Cycle^{*}

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Abstract

Empirical evidence suggests that women are discriminated against in the labor market. We analyze the effects of taste-based and statistical gender discrimination on business cycle and inflation dynamics by including unpaid household production, two-agent households, and discriminatory firm behavior in a tractable New Keynesian model. After a negative demand shock, we find that the economic downturn is more severe in comparison to a nondiscriminatory environment, as the shock implies an increase in the inefficient utilization of female and male productivity. Furthermore, the working time allocation between women and men becomes more inefficient. Moreover, we show that discrimination implies a lower transmission of expansionary monetary policy shocks on inflation. Overall, taste-based discrimination leads to larger macroeconomic distortions, while statistical discrimination implies higher intra-household inefficiencies.

JEL classifications: D13, D31, E32, E52, J71

Keywords: Business cycles, gender discrimination, household production, monetary policy transmission, New Keynesian models

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Contents

\mathbf{Li}	st of	Tables	8
Li	st of	Figures	8
1	Intr	roduction	9
2	ΑN	Aodel with Gender Discrimination and Household Work	12
	2.1	Households	12
	2.2	Firms	16
		2.2.1 Non-Discriminatory Environment	16
		2.2.2 Taste-Based Discrimination	18
		2.2.3 Statistical Discrimination	20
	2.3	Monetary Policy	21
	2.4	Market Clearing	21
3	Res	sults	22
	3.1	Calibration	22
	3.2	Dynamic Effects with Taste-Based Discrimination	25
		3.2.1 Discount Rate Shock	25
		3.2.2 Monetary Policy Shock	27
	3.3	Dynamic Effects with Statistical Discrimination	29
		3.3.1 Discount Rate Shock	30
		3.3.2 Monetary Policy Shock	33
4	Cor	nclusion	37
Aj	ppen	dices	38
	А	Expenditure Minimization of the Household	38
	В	Utility Maximization of the Household	40
	С	Risk Sharing	42
	D	Profit Maximization of the Firm	43
	Ε	Derivation of Marginal Costs	43
		E.1 Non-Discriminatory Environment	43
		E.2 Taste-Based Discrimination	46
		E.3 Statistical Discrimination	48
	F	Derivation of the Optimal Price	48
Re	efere	nces	52
Pι	ıblic	ations and Contribution	52

List of Tables

1	Calibration.	23
2	Steady State in Comparison to Data	24

List of Figures

1	Taste-Based Model: Impulse Responses to a Negative 1% Discount Rate Shock	
	with Persistence $\rho_Z = 0.9.$	26
2	Taste-Based Model: Impulse Responses to an Expansionary (Annual) 1% Mon-	
	etary Policy Shock with Persistence $\rho_{\nu} = 0.5.$	28
3	Statistical Model: Impulse Responses to a Negative 1% Discount Rate Shock	
	with Persistence $\rho_Z = 0.9.$	31
4	Statistical Model: Impulse Responses to an Expansionary (Annual) 1% Mone-	
	tary Policy Shock with Persistence $\rho_{\nu} = 0.5$	35

1 Introduction

Discrimination in the labor market has been at the forefront of economic research for decades. Starting with Becker (1971), who analyzes the various consequences of racial discrimination in firms, many more scholars have theoretically and empirically examined the extent and economic effects of discrimination against different groups. A considerable portion of this literature addresses gender discrimination in the labor market, captured by significant adjusted gender wage gaps (i.e., gender wage gaps controlling for productivity measures), for instance. Furthermore, systematic gender differences with respect to the time spent in paid and unpaid work can be observed: OECD (2020) data shows that women spend about 18.2% of a 24-hour day performing unpaid work while men spend half that time (9.4%). Conversely, women spend 15.1% working in the paid labor market, men 22%.¹ In principle, these differences could be explained by the preferences of women and men with regards to household and labor market work. However, studies indicate that these differences are not purely preference-driven.²

Against this background, our paper analyzes the effects of gender discrimination on business cycle fluctuations and inflation. In particular, we investigate how gender discrimination distorts the transmission of shocks into an economy by extending conventional New Keynesian models. At the household level, we introduce a female and a male agent. Furthermore, we include unpaid household work in addition to paid labor market work in order to account for the differences in the working time allocation between women and men. On the firms' side, we introduce gender discrimination into our framework, accounting for the adjusted gender wage gap. We differentiate between two types of gender discrimination: taste-based and statistical discrimination. Following Becker (1971), we conceptualize taste-based gender discrimination against women as a preference among firms to hire men over equally productive women. Moreover, we implement statistical gender discrimination as an information asymmetry between households and firms, thereby following the literature brought forward by Phelps (1972) and Arrow (1973). In particular, we specify statistical discrimination in our model by assuming that firms face greater uncertainty with respect to the productivity of women than of men:

¹Various studies underscore these averages. See, for instance, Gálves-Muñoz et al. (2011) for European countries or Sayer (2005) for the United States.

²Lewis et al. (2008) find that in Western Europe, fathers "want to work much less [and] mothers want to be employed, for the most part long part-time or full-time hours". Boye (2009) concludes that "differences between women's and men's paid working hours and housework hours are one reason why European women have lower well-being than European men have".

we assume that there are two types of women with different productivity levels, while there is only one type of men. However, firms cannot observe the individual productivities of women but rather base their decisions on average female productivity. This implies statistical discrimination against women with higher productivity levels. These extensions of common dynamic stochastic general equilibrium (DSGE) models allow us to analyze and compare the effects of different types of discriminatory behavior by firms on business cycle and inflation dynamics after demand (discount rate) as well as monetary policy shocks.

We find that in response to a negative demand shock, both taste-based and statistical gender discrimination lead to a more severe economic downturn due to an increase in the inefficient utilization of female and male productivity. The working time allocation of women and men between labor market and household production becomes even more inefficient. Quantitatively, the economy suffers more from taste-based discrimination than it does from statistical gender discrimination. However, statistical discrimination leads to quantitatively larger intra-household distortions than taste-based discrimination after negative discount rate shocks. Furthermore, we find that both types of gender discrimination weaken the transmission of expansionary monetary policy shocks on inflation. Overall, female and male wages increase too little in response to the shock, since women are discriminated against and female and male productivity is therefore utilized inefficiently. Thus, firms' marginal costs (wages) increase less compared to the non-discriminatory case. Furthermore, the adjusted gender wage gap increases after expansionary monetary policy shocks. Quantitatively, taste-based discrimination has a more dampening effect on the transmission of expansionary monetary policy shocks on inflation than statistical discrimination. However, statistical discrimination implies larger effects on the adjusted gender wage gap and on the inefficiency of the working time allocations of the households.

In our analysis, we use the adjusted gender wage gap as a proxy for discrimination against women in the labor market. Various existing studies investigate the extent of gender discrimination captured by the gender wage gap. For instance, Blau and Kahn (2017) show that the raw, unadjusted female to male wage ratio ranged from 62.1% in 1980 to 79.3% in 2010 in the United States.³ However, it is usually argued that the unadjusted gap is not a sufficient

 $^{^{3}}$ Extensive literature exists on gender wage gaps in different countries. Cebrián and Moreno (2015) estimate these gaps for Spain, Fortin et al. (2017) analyze Canada, Sweden, and the United Kingdom, Manning and Swaffield (2008) consider the United Kingdom, Francesconi and Parey (2018) and Tyrowicz et al. (2018) discuss wage gaps in Germany.

measure for potential discrimination because factors such as education or work experience may explain at least a part of the gap. Therefore, most studies report an adjusted gender wage gap, thereby taking into account productivity measures such as experience, hours worked, education, industry, occupation, or union status.⁴ Blau and Kahn (2017) find an adjusted gender wage gap of 20.6% in 1980 in the United States. They show that this gap closed to 7.6% in 1989, however this trend did not continue in the following 20 years: in 1998 the adjusted gender pay gap was still 8.6%, in 2010 8.4%. It is argued that these adjusted wage differences can at least partly be ascribed to gender discrimination.⁵

Our paper relates to the literature in the following ways. Most importantly, we contribute to the strand of literature that analyzes gender differences on a macroeconomic level. This includes the analysis of Morchio and Moser (2020) who provide estimates of output and utility gains from closing gender gaps. While static effects form the focus of their model, we specifically take a dynamic perspective. In the same vein, our paper complements studies that analyze the effects of gender discrimination on labor market $outcomes^6$ as well as work that considers the effects of female empowerment and employment on macroeconomic outcomes.⁷ Furthermore, our paper relates to work that analyzes heterogeneity across agents. This heterogeneity has been introduced into New Keynesian frameworks in recent years.⁸ However, approaches to studying gender-related topics within these frameworks are rare (exceptions include Khera, 2016; Albanesi, 2019), and scant attention has been paid to an analysis of the effects of gender discrimination on the business cycle and inflation, which is a main focus of our model.⁹ Our paper also contributes to the literature that examines the effects of monetary policy on inequality. The studies conducted by Doepke et al. (2015), Coibon et al. (2017), Ampudia et al. (2018), or Furceri et al. (2018) analyze the effects of conventional and unconventional monetary policy shocks on household inequality. However, only little attention has

 $^{^{4}}$ See Blackaby et al. (2005), Noonan et al. (2005), Blau and Kahn (2007), or Heinz et al. (2016) who find substantial differences in female and male wages even after taking these productivity measures into account.

 $^{{}^{5}}$ See, for instance, Greene and Hoffnar (1995), Noonan et al. (2005), Heinz et al. (2016), or Gharehgozli and Atal (2020). Albanesi and Olivetti (2009) relate the adjusted gender pay gap to a positive relationship between differences in female and male wages and incentive pay by introducing incentive problems in the labor market.

⁶This includes the work by Francois (1998), Albanesi and Olivetti (2009), and Gayle and Golan (2012), which endogenizes differences in female and male wages and employment or papers by Erosa et al. (2016) and Xiao (2019) who investigate gender wage gaps over the life cycle.

⁷See, for instance, Cavalcanti and Taveres (2016), Albanesi (2019), or Doepke and Tertilt (2019).

⁸See, for example, Gornemann et al. (2016), Kaplan et al. (2018), or Luetticke (2018). For a detailed overview, see Kaplan and Violante (2018).

⁹For an analysis of the relationship between business cycle fluctuations and gender unemployment gaps, see Peiró et al. (2012), Razzu and Singleton (2016), or Albanesi and Sahin (2018).

been paid to the effects on women or minorities¹⁰ and, to the best of our knowledge, none paid to the impact on the adjusted gender wage gap and the effects of gender discrimination.

The paper is organized as follows: Section 2 states the model before Section 3 analyzes the results. Section 4 concludes.

2 A Model with Gender Discrimination and Household Work

2.1 Households

The representative, infinitely-lived household k=A, B (with -k denoting the respective other household) is comprised of two agents G=F, M, a woman and a man. The share of Ahouseholds in the economy is κ , the share of B-households is $(1-\kappa)$. Depending on the type of discrimination we impose later, the two households either differ only in female labor market productivity (statistical discrimination), or are fully identical (taste-based discrimination). For each household member G, we consider a utility function that builds on the functions proposed by King et al. (1988) and Benhabib et al. (1991).¹¹ Agent G's period utility function is specified as

$$U_{G,t}^{k} = Z_{t} \left[\frac{\left(\left(C_{G,t}^{k} \right)^{b} \left(L_{G,t}^{k} \right)^{1-b} \right)^{1-\sigma}}{1-\sigma} + \Omega \right],$$
(1)

where $C_{G,t}^k$ is the composite consumption index of agent G, $L_{G,t}^k$ is the leisure of agent Gin period t, Z_t is an AR(1) discount rate shock, and Ω is a parameter ensuring that utility cannot become negative. Each agent thus gains utility from consumption and leisure; their relative importance is captured by parameter $0 \le b \le 1$. The parameter σ is defined as the inverse intertemporal elasticity of substitution.

Furthermore, agent G faces a time constraint. Normalizing the total available time of each agent to 1, we get

$$1 = N_{G,t}^k + V_{G,t}^k + L_{G,t}^k, (2)$$

¹⁰See, for instance, Carpenter and Rodgers III (2004), Braunstein and Heintz (2008), or Apergis et al. (2019). ¹¹In order to include household production in their analysis, this type of utility function is among others also used by McGrattan et al. (1997) and Gnocchi et al. (2016).

where $N_{G,t}^k$ describes the time worked in the (paid) labor market and $V_{G,t}^k$ the time spent on (unpaid) household work. Furthermore, we define the constant elasticity of substitution (CES) composite consumption index as

$$C_{G,t}^{k} \equiv \left(\gamma_{G}\left(C_{t}^{k,N}\right)^{\frac{\vartheta_{C}-1}{\vartheta_{C}}} + (1-\gamma_{G})\left(C_{t}^{k,V}\right)^{\frac{\vartheta_{C}-1}{\vartheta_{C}}}\right)^{\frac{\vartheta_{C}}{\vartheta_{C}-1}},\tag{3}$$

where $C_t^{k,N}$ is defined as a market good consumption index, $C_t^{k,V}$ as the consumption index of home-produced goods of the household, and ϑ_C denotes the elasticity of substitution between both consumption good bundles. The parameter $0 \leq \gamma_G \leq 1$ governs the agent-specific preference for market good consumption.

The market good consumption index is given by a CES (denoted by $\epsilon > 1$) function over all goods $i \in [0, 1]$ of the form

$$C_t^{k,N} \equiv \left(\int_0^1 C_{i,t}^{k,N\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}.$$

Household production uses the following CES technology:

$$C_t^{k,V} = \left(\beta \left(V_{F,t}^k\right)^{\frac{\vartheta_V - 1}{\vartheta_V}} + (1 - \beta) \left(V_{M,t}^k\right)^{\frac{\vartheta_V - 1}{\vartheta_V}}\right)^{\frac{\vartheta_V}{\vartheta_V - 1}},\tag{4}$$

with $0 \leq \beta \leq 1$ expressing potential productivity differences of men and women in home production. The parameter ϑ_V denotes the elasticity of substitution between female and male household work.

Following the literature on intra-household bargaining, such as Browning and Chiappori (1998), Browning et al. (2013), or Mohapatra and Simon (2017), the household cooperatively seeks to maximize its expected lifetime utility

$$\mathbb{E}_t \left[\sum_{\iota=0}^{\infty} \theta^{\iota} \left(\zeta_{t+\iota}^k U_{M,t+\iota}^k + (1 - \zeta_{t+\iota}^k) U_{F,t+\iota}^k \right) \right], \tag{5}$$

where the parameter $0 < \theta \leq 1$ is defined as the discount rate. The time varying parameter $\zeta_t^k = 1.5 - w_{F,t}^k / w_{M,t}^k$ captures the pareto weight on male utility, depending on the relative real wages of women and men.¹² For instance, if women and men earn the same real wage $w_{G,t}^k$, the weight on both, male and female utility is $\zeta_t^k = 0.5$. In case of wage differences, for example

 $^{^{12}}$ For empirical evidence on this property, see, for instance, Friedberg and Webb (2006).

due to discrimination, $\zeta_t^k \neq 0.5$, and women and men differ in their relative intra-household bargaining power.

The household faces the flow budget constraint

$$\int_{0}^{1} P_{i,t} C_{i,t}^{k,N} di + Q_t B_t^k \leqslant B_{t-1}^k + W_{F,t}^k N_{F,t}^k + W_{M,t}^k N_{M,t}^k + D_t^k,$$
(6)

where $P_{i,t}$ is the price of market good i, Q_t the bond price¹³, B_t^k bond holdings, $W_{G,t}^k$ the agent-specific nominal wage, and D_t^k dividends from the ownership of firms.

The household has to decide on the allocation of its consumption expenditure between the different goods, on the working time allocation (how many hours men and women work in the paid labor market and in the household), on its consumption, and on bond holdings. Expenditure minimization for each level of market consumption gives the optimal demand for good i:

$$C_{i,t}^{k,N} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} C_t^{k,N},\tag{7}$$

where P_t is defined as the price index of the economy given by $P_t \equiv \left(\int_0^1 P_{i,t}^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$.

Using equations (6) and (7), the budget constraint can be rewritten as

$$P_t C_t^{k,N} + Q_t B_t^k \leqslant B_{t-1}^k + W_{F,t}^k N_{F,t}^k + W_{M,t}^k N_{M,t}^k + D_t^k.$$
(8)

The representative household takes wages, prices for goods and bonds as well as dividends as given. It maximizes its utility given by (5) cooperatively, subject to the budget constraint (8). This maximization problem yields the following optimality conditions:

$$(1-\zeta_{t}^{k})(1-b)\frac{\left(\left(C_{F,t}^{k}\right)^{b}\left(L_{F,t}^{k}\right)^{1-b}\right)^{1-\sigma}}{L_{F,t}^{k}} = b\left(C_{t}^{k,V}\right)^{\frac{1}{\vartheta_{V}}-\frac{1}{\vartheta_{C}}}\beta\left(V_{F,t}^{k}\right)^{-\frac{1}{\vartheta_{V}}}\left(\psi_{M,t}^{k}(1-\gamma_{M})+\psi_{F,t}^{k}(1-\gamma_{F})\right),$$

$$(02)$$

$$\zeta_{t}^{k}(1-b)\frac{\left(\left(C_{M,t}^{k}\right)^{*}\left(L_{M,t}^{k}\right)^{*}\right)}{L_{M,t}^{k}} = b\left(C_{t}^{k,V}\right)^{\frac{1}{\vartheta_{V}}-\frac{1}{\vartheta_{C}}}\left(1-\beta\right)\left(V_{M,t}^{k}\right)^{-\frac{1}{\vartheta_{V}}}\left(\psi_{M,t}^{k}(1-\gamma_{M})+\psi_{F,t}^{k}(1-\gamma_{F})\right), \quad (9b)$$

$$(1-\zeta_{t}^{k})(1-b)\frac{\left(\left(C_{F,t}^{k}\right)^{b}\left(L_{F,t}^{k}\right)^{1-b}\right)^{1-\sigma}}{L_{F,t}^{k}} = b\frac{W_{F,t}^{k}}{P_{t}}\left(C_{t}^{k,N}\right)^{-\frac{1}{\vartheta_{C}}}\left(\psi_{M,t}^{k}\gamma_{M}+\psi_{F,t}^{k}\gamma_{F}\right), \quad (10a)$$

¹³Note that the relation $Q_t = 1/R_t$, with R_t being defined as the gross nominal interest rate, holds.

$$\zeta_{t}^{k}(1-b)\frac{\left(\left(C_{M,t}^{k}\right)^{b}\left(L_{M,t}^{k}\right)^{1-b}\right)^{1-\sigma}}{L_{M,t}^{k}} = b\frac{W_{M,t}^{k}}{P_{t}}\left(C_{t}^{k,N}\right)^{-\frac{1}{\vartheta_{C}}}\left(\psi_{M,t}^{k}\gamma_{M}+\psi_{F,t}^{k}\gamma_{F}\right),\qquad(10b)$$

and

$$Q_t = \theta \mathbb{E}_t \left[\Psi_{t,t+1}^k \frac{1}{\Pi_{t+1}} \right], \tag{11}$$

with $\psi_{F,t}^k \equiv (1 - \zeta_t^k) \left(\left(C_{F,t}^k \right)^b \left(L_{F,t}^k \right)^{1-b} \right)^{1-\sigma} \left(C_{F,t}^k \right)^{\frac{1-\vartheta_C}{\vartheta_C}}, \psi_{M,t}^k \equiv \zeta_t^k \left(\left(C_{M,t}^k \right)^b \left(L_{M,t}^k \right)^{1-b} \right)^{1-\sigma} \left(C_{M,t}^k \right)^{\frac{1-\vartheta_C}{\vartheta_C}}, \psi_{L,t+1}^k \equiv U_{C^{k,N},t+1}^k \Big/ U_{C^{k,N},t}^k, U_{C^{k,N},t}^k \equiv bZ_t \left(C_t^{k,N} \right)^{-\frac{1}{\vartheta_C}} \left(\psi_{M,t}^k \gamma_M + \psi_{F,t}^k \gamma_F \right), \text{ and } \Pi_{t+1} \equiv P_{t+1}/P_t$ being defined as inflation. Note that $\psi_{G,t}^k$ represents the weighted marginal utility gained by agent G from an increase in their respective consumption index (either from higher market good consumption or household production).

Equations (9a) and (9b) describe the household's optimal decision with respect to household production. Women and men equate their weighted individual marginal disutility from household work to the household's marginal utility, i.e., the weighted marginal utility of both women and men. Thus, this optimality condition equates the household's utility gain from the added value to the consumption index $C_{G,t}^k$ from one additional hour of household work to the weighted individual utility loss of foregone leisure.

Equations (10a) and (10b) describe the household's optimal decision regarding the labor market work of women and men. The weighted individual marginal disutility from working another hour in the labor market (foregone leisure) has to equal marginal utility from labor market work (higher market good consumption) of the household in optimum. Note that for a given amount of household work, women will work less in the labor market than men, when female wages are lower due to discrimination, for instance.

Finally, equation (11) represents the Euler equation describing the household's optimal intertemporal consumption-leisure decision. Note that $\theta \Psi_{t,t+1}$ depicts the stochastic discount factor between period t and t+1.

Due to the shared bonds market in the economy, we can obtain the following risk sharing condition between the two households by combining the Euler equations of both households:

$$U_{C^{k,N},t}^{k} = \varphi^{k} U_{C^{-k,N},t}^{-k}, \tag{12}$$

where $\varphi^k \equiv U_{C^{k,N},SS}^k / U_{C^{-k,N},SS}^{-k}$, with $U_{C^{k,N},SS}^k$ being the zero inflation steady state value of

the marginal utility of market good consumption. Equation (12) implies that the marginal utilities of market good consumption co-move proportionally over time.

2.2 Firms

There exists a continuum of firms indexed by $i \in [0, 1]$ that use identical technology. Each firm produces a differentiated good and supplies it on a monopolistically competitive market. Furthermore, we assume staggered price setting as suggested by Calvo (1983), i.e., only a fraction $1-\Lambda$ of firms can reset its price in each period. We differentiate between three cases: an environment in which firms do not discriminate against women, an environment with tastebased discriminatory behavior by firms, and the case of statistical discrimination.

2.2.1 Non-Discriminatory Environment

In the non-discriminatory environment, firms can perfectly observe the productivities of all women and men from both households. Furthermore, firms do not discriminatorily prefer any type of worker over another. The corresponding CES production function of a representative firm i is thus given by

$$Y_{i,t} = \left(\kappa \alpha^{A} \left(N_{i,F,t}^{A}\right)^{\frac{\vartheta_{N}-1}{\vartheta_{N}}} + (1-\kappa)\alpha^{B} \left(N_{i,F,t}^{B}\right)^{\frac{\vartheta_{N}-1}{\vartheta_{N}}} + \kappa(1-\alpha) \left(N_{i,M,t}^{A}\right)^{\frac{\vartheta_{N}-1}{\vartheta_{N}}} + (1-\kappa)(1-\alpha) \left(N_{i,M,t}^{B}\right)^{\frac{\vartheta_{N}-1}{\vartheta_{N}}}\right)^{\frac{\vartheta_{N}-1}{\vartheta_{N}-1}}, \quad (13)$$

with $0 \leq \alpha^B \leq \alpha^A \leq 1$ and $\alpha = \kappa \alpha^A + (1-\kappa)\alpha^B$ expressing the relative productivities of men and women in market production. Note that we discuss our assumptions with regards to potential productivity differences between the agents in the forthcoming subsections. The parameter ϑ_N denotes the elasticity of substitution between female and male labor market work. The real cost function of the firm is given by

$$TC_{i,t} = \kappa w_{F,t}^A N_{i,F,t}^A + (1-\kappa) w_{F,t}^B N_{i,F,t}^B + \kappa w_{M,t}^A N_{i,M,t}^A + (1-\kappa) w_{M,t}^B N_{i,M,t}^B.$$
 (14)

Accordingly, firms solve the following optimization problem

$$\max_{P_{i,t}} \mathbb{E}_t \left[\sum_{\iota=0}^{\infty} \theta^{\iota} \Lambda^{\iota} \Psi_{t,t+\iota} \left(\frac{P_{i,t}}{P_{t+\iota}} Y_{i,t+\iota|t} - TC(Y_{i,t+\iota|t}) \right) \right],$$
(15)

subject to the sequence of demand constraints

$$Y_{i,t+\iota|t} = \left(\frac{P_{i,t}}{P_{t+\iota}}\right)^{-\epsilon} Y_{t+\iota}$$

where $\theta^k \Psi_{t,t+\iota}$ is the stochastic discount factor derived in the household section, with $\Psi_{t,t+\iota} = \left(\kappa U^A_{C^{A,N},t+\iota} + (1-\kappa)U^B_{C^{B,N},t+\iota}\right) / \left(\kappa U^A_{C^{A,N},t} + (1-\kappa)U^B_{C^{B,N},t}\right)$, and $Y_{i,t+\iota|t}$ is defined as the output in period $t+\iota$ for a firm that adjusts its price in period t. Solving (15) yields the following optimality condition:

$$0 = \mathbb{E}_t \left[\sum_{\iota=0}^{\infty} \theta^{\iota} \Lambda^{\iota} \Psi_{t,t+\iota} Y_{i,t+\iota|t} \left(\frac{P_{i,t}}{P_{t+\iota}} - \mu m c_{t+\iota|t} \right) \right) \right],$$

which is the well-known solution for optimal pricing behavior in this framework, with $\mu \equiv \epsilon/(\epsilon-1)$ defined as the markup over nominal marginal costs resulting from monopolistic competition and mc_t as real marginal costs.

In the following, we take a closer look at the composition of real marginal costs. In order to determine the optimal use of the four types of labor input, $N_{i,G,t}^k$, the firm seeks to minimize total costs given by (14) for each level of $Y_{i,t}$ given by (13). Solving this cost minimization problem yields the optimality conditions

$$\frac{1-\alpha}{\alpha^A} \left(\frac{N_{i,M,t}^A}{N_{i,F,t}^A}\right)^{-\frac{1}{\vartheta_N}} = \frac{w_{M,t}^A}{w_{F,t}^A},\tag{16}$$

$$\frac{1-\alpha}{\alpha^B} \left(\frac{N_{i,M,t}^A}{N_{i,F,t}^B} \right)^{-\frac{1}{\vartheta_N}} = \frac{w_{M,t}^A}{w_{F,t}^B},\tag{17}$$

$$\left(\frac{N_{i,M,t}^A}{N_{i,M,t}^B}\right)^{-\frac{1}{\vartheta_N}} = \frac{w_{M,t}^A}{w_{M,t}^B}.$$
(18)

Firms equate the relative marginal productivities of female and male work to the ratio of their respective (perceived) costs. Using equations (13), (14), (16), (17), and (18), real marginal costs of firm i can be derived as

$$mc_{t} = \frac{\kappa w_{M,t}^{A} + \kappa w_{F,t}^{A} \left(\frac{\alpha A}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{A}}\right)^{\vartheta_{N}} + (1-\kappa) w_{M,t}^{B} \left(\frac{w_{M,t}^{A}}{w_{M,t}^{B}}\right)^{\vartheta_{N}} + (1-\kappa) w_{F,t}^{B} \left(\frac{\alpha B}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{B}}\right)^{\vartheta_{N}}}{\left[\kappa \alpha^{A} \left(\frac{\alpha A}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{A}}\right)^{\vartheta_{N}-1} + (1-\kappa) \alpha^{B} \left(\frac{\alpha B}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{B}}\right)^{\vartheta_{N}-1} + \kappa(1-\alpha) + (1-\kappa)(1-\alpha) \left(\frac{w_{M,t}^{A}}{w_{M,t}^{B}}\right)^{\vartheta_{N}-1}\right]^{\frac{\vartheta_{N}}{\vartheta_{N}-1}}}, \quad (19)$$

which shows that real marginal costs are related to the agents' real wages and potential productivity differences. Due to the symmetry of the firms, we drop the index i.

Using the solutions above, we can describe the optimal relative price $p_t^* \equiv P_t^*/P_t$, with P_t^* being defined as the optimal price of each firm that can re-optimize in period t, as

$$p_t^* = \mu \frac{x_{1,t}}{x_{2,t}},\tag{20}$$

where $x_{1,t} \equiv U_{C^N,t} Y_t m c_t + \Lambda \theta \mathbb{E}_t[\Pi_{t+1}^{\epsilon} x_{1,t+1}], x_{2,t} \equiv U_{C^N,t} Y_t + \Lambda \theta \mathbb{E}_t[\Pi_{t+1}^{\epsilon-1} x_{2,t+1}]$, and $U_{C^N,t} \equiv \kappa U^A_{C^{A,N},t} + (1-\kappa) U^B_{C^{B,N},t}$. Aggregate price dynamics can be described as

$$1 = (1 - \Lambda) p_t^{*1 - \epsilon} + \Lambda \left(\frac{1}{\Pi_t}\right)^{1 - \epsilon}.$$
(21)

Intuitively, the fraction $1-\Lambda$ of firms sets the optimal price determined by equation (20) while the fraction Λ of firms keeps the price of the previous period. The weighted average of both prices therefore determines the price level in period t.

2.2.2 Taste-Based Discrimination

In the case of taste-based discrimination by firms against women, we assume that the productivity of all women in the economy is identical, i.e., $\alpha^A = \alpha^B = \alpha$ and therefore $N_{i,F,t}^A = N_{i,F,t}^B = N_{i,F,t}$ as well as $N_{i,M,t}^A = N_{i,M,t}^B = N_{i,M,t}$. Furthermore, we will later set $\alpha = 0.5$ in this specification of the model, implying that women and men are equally productive and preferences of firms for hiring men over women are purely discriminatory. These assumptions imply the following production function

$$Y_{i,t} = \left(\alpha N_{i,F,t}^{\frac{\vartheta_N - 1}{\vartheta_N}} + (1 - \alpha) N_{i,M,t}^{\frac{\vartheta_N - 1}{\vartheta_N}}\right)^{\frac{\vartheta_N}{\vartheta_N - 1}}.$$
(22)

The real (perceived) cost function of the firm is given by

$$TC_{i,t} = w_{F,t}N_{i,F,t} + w_{M,t}N_{i,M,t} + d_F N_{i,F,t},$$
(23)

where $d_F>0$ is a real discrimination factor that applies equally to women from both households and $w_{F,t}=w_{F,t}^A=w_{F,t}^B$ as well as $w_{M,t}=w_{M,t}^A=w_{M,t}^B$. Note that the costs associated with d_F are not interpreted as resource costs but perceived costs and, therefore, represent lump-sum transfers to the households (within the dividend payments D_t^k).¹⁴ This approach to modeling taste-based discrimination was first suggested by Becker (1971) and has been used in similar manners to conceptualize discriminatory behavior by firms (see, for instance, Cavalcanti and Taveres, 2016).

Although Becker based his analysis on racial discrimination, his concepts can easily be transferred to other types of discrimination, e.g., gender discrimination. Note that Becker discusses a framework in which the extent of discrimination differs between firms. He argues that in markets where greater competition exists, discrimination is lower because less discriminatory firms have a competitive advantage in comparison to more discriminatory ones. In contrast, we assume that all firms have the same preferences and thus discriminate equally against women. This implies that no firm has a competitive advantage and discrimination does not decrease with higher competition. This assumption takes into account the empirical evidence showing the relatively constant adjusted gender wage gap over time.

Furthermore, for our analysis it is necessary to adjust Becker's definition of taste-based discrimination. He describes that this type of discrimination is a perceived "disutility caused by contact with some individuals" (Becker, 1971). This definition is not suitable for discussing gender discrimination because women and men "generally live together [...] in families," as Blau and Kahn (2007) argue. Therefore, they adjust the definition, contending that gender discrimination might arise from adapting and promoting "socially appropriate roles" rather than "the desire to maintain social distance from the discriminated group".

Naturally, taste-based discrimination by firms changes the optimal use of labor input and the composition of marginal costs. Minimizing total costs for a given level of output gives the

¹⁴A comparable approach is used by Kumhof and Wang (2019). In a different context, they also treat certain costs as lump-sum transfers for simplicity. This approach allows us to include discrimination costs in the, from the household's point of view, exogenous dividend payments D_t^k .

following optimality condition

$$\frac{1-\alpha}{\alpha} \left(\frac{N_{i,M,t}}{N_{i,F,t}}\right)^{-\frac{1}{\vartheta_N}} = \frac{w_{M,t}}{w_{F,t}+d_F}.$$
(24)

For a symmetric amount of labor $(N_{i,M,t}=N_{i,F,t})$, women earn less $(w_{M,t}>w_{F,t})$ when firms discriminate against women $(d_F>0)$. Real marginal costs can then be re-written as

$$mc_t = \frac{w_{M,t} + (w_{F,t} + d_F) \left(\frac{\alpha}{1-\alpha} \frac{w_{M,t}}{w_{F,t} + d_F}\right)^{\vartheta_N}}{\left[\alpha \left(\frac{\alpha}{1-\alpha} \frac{w_{M,t}}{w_{F,t} + d_F}\right)^{\vartheta_N - 1} + (1-\alpha)\right]^{\frac{\vartheta_N}{\vartheta_N - 1}}}.$$
(25)

The optimal price setting behavior in equation (20) as well as the overall price level in equation (21) are unaffected.

2.2.3 Statistical Discrimination

In the environment with statistical discrimination by firms against women, we assume that women from household B are less productive in the labor market than women from household $A (\alpha^A > \alpha^B)$. Men from both households are assumed to be equally productive. In order to conceptualize statistical discrimination, we assume that firms cannot observe the individual productivities of women; they cannot distinguish between women from households A and B. However, they know the distribution of women between the two households and thus consider their weighted average productivity α when optimizing. Furthermore, women from household A are as productive as men, i.e., $\alpha^A = 1 - \alpha$. This implies the following production function

$$Y_{i,t} = \left(\alpha N_{i,F,t}^{\frac{\vartheta_N - 1}{\vartheta_N}} + (1 - \alpha) N_{i,M,t}^{\frac{\vartheta_N - 1}{\vartheta_N}}\right)^{\frac{\vartheta_N}{\vartheta_N - 1}},$$
(26)

where $\alpha < 1-\alpha$ and $N_{i,F,t} = N_{i,F,t}^A = N_{i,F,t}^B$ as well as $N_{i,M,t} = N_{i,M,t}^A = N_{i,M,t}^B$. The real total cost function corresponds to

$$TC_{i,t} = w_{F,t}N_{i,F,t} + w_{M,t}N_{i,M,t},$$
(27)

with $w_{F,t} = w_{F,t}^A = w_{F,t}^B$ and $w_{M,t} = w_{M,t}^A = w_{M,t}^B$. Minimizing total costs for a given level of output gives

$$\frac{1-\alpha}{\alpha} \left(\frac{N_{i,M,t}}{N_{i,F,t}}\right)^{-\frac{1}{\vartheta_N}} = \frac{w_{M,t}}{w_{F,t}},\tag{28}$$

implying that for a symmetric amount of labor $(N_{i,M,t}=N_{i,F,t})$, women earn less due to their lower average productivity. Consequently, women who are as productive as men, i.e., women from household A, are statistically discriminated against. Marginal costs are then given by

$$mc_{t} = \frac{w_{M,t} + w_{F,t} \left(\frac{\alpha}{1-\alpha} \frac{w_{M,t}}{w_{F,t}}\right)^{\vartheta_{N}}}{\left[\alpha \left(\frac{\alpha}{1-\alpha} \frac{w_{M,t}}{w_{F,t}}\right)^{\vartheta_{N}-1} + (1-\alpha)\right]^{\frac{\vartheta_{N}}{\vartheta_{N}-1}}}.$$
(29)

In order to close the model, we state the monetary policy rule and the market clearing conditions in the following subsections.

2.3 Monetary Policy

The central bank is assumed to only target inflation. It follows a simple (log-linearized) Taylor rule of the form

$$i_t = \rho + \phi_\pi \pi_t + \nu_t, \tag{30}$$

where $i_t \equiv log(1/Q_t)$, $\rho \equiv -log(\theta)$, $\pi_t \equiv log(\Pi_t)$, and ν_t is a monetary policy shock that follows an AR(1) process. Furthermore, we assume that $\phi_{\pi} > 1$. In addition, the Fisher equation holds

$$i_t = r_t + \mathbb{E}[\pi_{t+1}],$$

where r_t is defined as the real interest rate.

2.4 Market Clearing

The economy considered consists of three markets: the bonds market, the labor market, and the goods market. Bond market clearing implies

$$B_t^k = -B_t^{-k},$$

which is the standard condition in this type of framework, implying that there is zero net supply of bonds. The labor market clears when

$$N_{G,t}^k = \int_0^1 N_{i,G,t}^k di$$

Furthermore, the goods market clearing condition is given by

$$Y_t = \kappa C_t^{A,N} + (1 - \kappa) C_t^{B,N},$$
(31)

where Y_t is an aggregate output index defined as

$$Y_t \equiv \left(\int_0^1 Y_{i,t}^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}},$$

stating that all goods produced are consumed by the households.

3 Results

3.1 Calibration

We calibrate the model to meet certain labor market data, the specific values can be found in Table 1. In the case of **taste-based gender discrimination** by firms against women, we calibrate women and men to be ex ante identical with respect to their productivity and preferences in order to identify only the effects of taste-based gender discrimination on macroeconomic outcomes. We follow Gnocchi et al. (2016) and set $\sigma=2$, implying an intertemporal elasticity of substitution of 0.5. This value is supported by empirical findings that in general suggest values significantly lower than 1 (see, for instance, Hall (1988) or Atkeson and Ogaki (1996) for a general estimation, or Rupert et al. (2000) for estimates including household production). We calibrate the parameters b and γ_G to replicate OECD (2020) data with respect to female and male labor market work (15.1% and 22% of a 24-hour-day respectively). Furthermore, b<0.5 also ensures that the share of leisure is larger that 0.5 in steady state, which is consistent with OECD (2020) data. Note that $\gamma_G>0.5$ indicates a preference for market good consumption over home-produced goods, which tallies with the fact that the sum of the female and male labor market work share is higher than the sum of the female and male household work share in OECD (2020) data. The elasticity of substitution between market and household goods is chosen according to Rupert et al. (1995), who estimate this elasticity to be between 1.8 and 2. Several studies underscore this result, for instance, Benhabib et al. (1991) and Aguiar et al. (2013). Furthermore, we set the elasticity of substitution of female and male labor market work $\vartheta_N=4.33$, thereby following Albanesi (2019). The counterpart in household production is calibrated to match OECD (2020) data regarding the extent of male household work (9.4% of a 24-hour day). Moreover, we choose the standard parameters θ , ϵ , Λ , and ϕ_{π} as in Galí (2015).

Table 1. Cambration	Table	1:	Calibration
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	Description	Value TB	Value ST	Target/Source
		House	eholds	
κ	Share of A-households	0.5	0.5	Equal share of A- and B-households
σ	Inverse intertemporal	2	2	Intertemporal elasticity of substitution: 0.5
	elasticity of substitution			
b	Consumption preference	0.37	0.36	Steady state share of female (0.15) and
				male (0.22) labor market work of a
				24-hour-day, internally calibrated
Ω	Utility parameter	5	5	Internally calibrated
θ	Discount rate	0.99	0.99	Yearly nominal interest rate: 4%
γ_F	Female preference for	0.57	0.55	Steady state share of female (0.15) and
	market good consumption			male (0.22) labor market work of a
				24-hour-day, internally calibrated
γ_M	Male preference for	0.57	0.55	Steady state share of female (0.15) and
	market good consumption			male (0.22) labor market work of a
				24-hour-day, internally calibrated
ϑ_C	Elasticity of substitution:	1.8	1.8	Rupert et al. (1995)
	market and household goods			
ϵ	Price elasticity of demand	9	9	Steady state markup of 12.5%
β	Productivity women, household production	0.5	0.5	Equal productivity of women and men
ϑ_V	Elasticity of substitution:	16.8	16.8	Steady state share of male household work (0.09)
	female and male household work			of a 24-hour-day, internally calibrated
		Fir	rms	
α^A	Productivity A-women, market production	0.5	0.53	Equal productivity of women and men
α^B	Productivity B-women, market production	0.5	0.40	TB: Equal productivity of women and men
				ST: Adjusted gender wage gap: 5%
α	Average female productivity,	0.5	0.47	TB: Equal productivity of women and men
	market production			ST: Adjusted gender wage gap: 5%
ϑ_N	Elasticity of substitution:	4.33	4.33	Albanesi (2019)
	female and male market work			
d_F	Discrimination factor	0.06	0	Adjusted gender wage gap: 5%
Λ	Price stickiness parameter	0.75	0.75	Average price duration: 4 quarters
		Centra	ıl Bank	
ϕ_{π}	Taylor rule coefficient: inflation	1.5	1.5	Galí (2015)

Notes. TB refers to the environment with taste-based discrimination against women, ST to the environment with statistical discrimination against women.

Lastly, we calibrate d_F to receive an adjusted gender wage gap of 5% (implying a male utility weight of 55%) in steady state. As shown in Table 2, the value of the adjusted gender wage gap — and therefore all resulting effects from gender discrimination — can be interpreted as a lower bound, since the majority of studies find an adjusted gap that is significantly higher than 5%. However, calibrating the adjusted gender wage gap to be 5% in steady state ensures that we do not overstate potential effects of gender discrimination.

In the case of statistical gender discrimination by firms against women, the calibration strategy on the households' side of the model remains unchanged. On the firms' side, however, we assume that firms do not discriminate due to a preference for men over equally productive women ($d_F=0$), but consider unobservable productivity differences between women from household B and the three remaining labor market participants (women and men from household A, men from household B). We calibrate the average productivity of women, α , and thereby α^A , α^B , and $1-\alpha$, to yield an adjusted gender wage gap (i.e., the gap between the wage of household-A women and the male wage) of 5% (implying a male utility weight of 55%) in steady state.

Finally, note that due to our calibration, the female and male labor market work shares as well as the share of male household work in a 24-hour day are identical in the steady state of our model and in the data, as shown in Table 2.

Description	Value Data	Data Source	Taste-Based Model	Statistical Model
Share of female labor market work	0.151	OECD (2020)	0.151	0.151
Share of male labor market work	0.220	OECD (2020)	0.220	0.220
Share of female household work	0.182	OECD (2020)	0.223	0.223
Share of male household work	0.094	OECD (2020)	0.094	0.094
Share of female leisure	0.667	OECD (2020)	0.626	0.626
Share of male leisure	0.685	OECD (2020)	0.686	0.686
Adjusted gender wage gap	5%-12%	See Section 1	5%	5%

Table 2: Steady State in Comparison to Data.

Notes. Labor, household work, and leisure shares in relation to a 24-hour day.

While slightly overstating female household work, our model replicates that women enjoy less leisure than men due to their higher overall working time. The resulting inefficient working time allocation leads to lower output, consumption, leisure, and utility in the steady state of the model with taste-based discrimination in comparison to the non-discriminatory environment, affecting all households equally. Likewise, in the steady state of the model with statistical discrimination, the inefficient working time allocation also implies lower economic activity. However, the households are not equally affected by this type of discrimination against women. In particular, household A suffers from lower consumption, leisure, and utility, while household B benefits and enjoys inefficiently high consumption, leisure, and utility in comparison to the non-discriminatory environment.

3.2 Dynamic Effects with Taste-Based Discrimination

In the following dynamic model analysis, we will compare the impulse responses of the presented model to a negative discount rate shock and an expansionary monetary policy shock in a **non-discriminatory** and a **discriminatory environment**. The impulse responses of the **gap** are calculated as the (log-linear) difference between the discriminatory and non-discriminatory environments. Our results constitute a lower bound due to the calibration of an adjusted gender wage gap of only 5% in steady state. Note that it is not necessary to distinguish between household A and B, since we calibrate all agents to have the same productivities and preferences in the environment with taste-based discrimination (and in the related non-discriminatory case). All results are percentage deviations from the zero inflation steady state.

3.2.1 Discount Rate Shock

Figure 1 shows the impulse response functions of the model with and without taste-based gender discrimination to a negative demand shock, in particular, to a negative 1% discount rate shock.

First, consider the **non-discriminatory environment** ($d_F=0$). Due to equal preferences and productivities, the effects on women and men are symmetric. The negative discount rate shock leads to a decline in consumption and therefore in aggregate demand. The corresponding decrease in output implies lower demand for both female and male labor market work. Labor market work and wages therefore decrease, and women and men substitute household work for labor market work. However, since the household prefers market goods over home-produced goods ($\gamma_G>0.5$), the overall consumption index decreases and leisure increases. Due to the decrease in demand for market goods, firms reduce their prices, which in turn leads to lower inflation. This implies an expansionary monetary policy reaction by the central bank, decreasing the nominal and real interest rates. The initial effects of the negative discount rate shock are thus mitigated.

In the **discriminatory environment** ($d_F>0$), the economy suffers more due to an increase in the inefficient utilization of female and male productivity. After the negative discount rate shock, the demand for women decreases more than it does for men due to taste-based gender discrimination. The working time allocation becomes even more inefficient in comparison to the steady state: labor market work decreases too much for women and too little for men. This implies that the gap in marginal productivities in paid work increases even more.

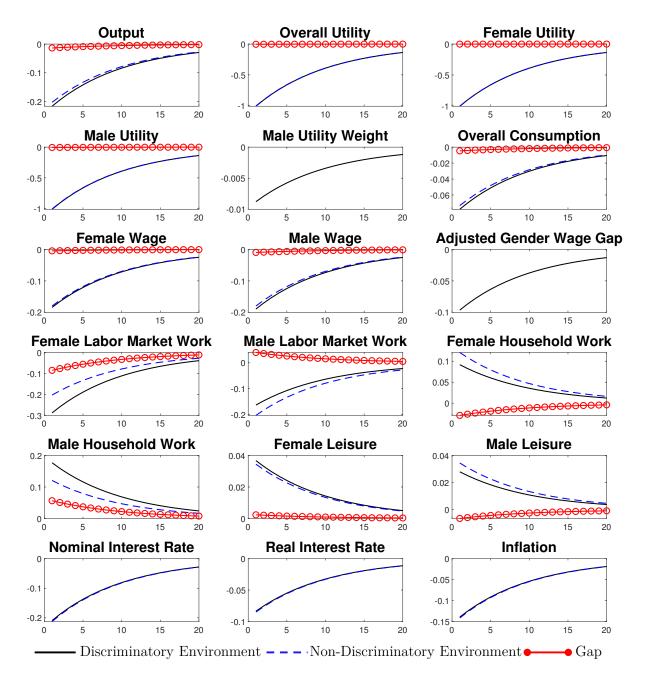


Figure 1: Taste-Based Model: Impulse Responses to a Negative 1% Discount Rate Shock with Persistence $\rho_Z = 0.9$.

Due to the increase in the inefficient utilization of female and male productivity, the eco-

nomic downturn is more severe, i.e., output decreases more in the discriminatory environment than it does in the non-discriminatory environment. In fact, the relative output loss in the discriminatory environment is 7% higher in the first period than in the non-discriminatory one.

Note that there are two opposing effects on the adjusted gender wage gap: while the lower demand for female labor market work has an increasing effect on the adjusted gender wage gap, the higher increase in marginal productivity of women relative to men has a decreasing effect. With the chosen calibration, the second effect dominates and the adjusted gender wage gap decreases slightly (implying a decrease in the weight of male utility in the household's utility function). However, both female and male wages decrease more in the discriminatory environment: female wages fall more due to discrimination against women, male wages experience a stronger drop due to lower marginal productivity caused by an inefficiently high amount of labor market work. Note that the higher intra-household bargaining power of women, in comparison to the steady state, implies that the increase in female household work is inefficiently low. Combined with the inefficiently large drop in labor market work, the increase in female leisure is inefficiently high. Conversely, male leisure increases too little due to an inefficiently high increase in male household work and the inefficiently low drop in labor market work. The interpretations of the firms' pricing behavior as well as the reaction of the central bank are qualitatively similar to the above described interpretations in the non-discriminatory environment.

3.2.2 Monetary Policy Shock

Figure 2 depicts the impulse response functions of the taste-based model to an expansionary (annual) 1% monetary policy shock. First, consider the **non-discriminatory framework** $(d_F=0)$ in which female and male responses are symmetric. The unexpected decrease in the nominal interest rate leads the real interest rate to fall as well. As a result, households have a higher incentive to consume market goods rather than to save. This in turn raises output and firms' demand for female and male work increases. Female and male wages rise as a consequence, leading women and men to decrease household work due to higher opportunity costs. Furthermore, the household decides to increase paid labor market work and to decrease leisure, i.e., the substitution effect outweighs the income effect. In terms of household utility, however, the rise in market consumption dominates the decrease in leisure and the monetary shock leads to higher utility. Due to the increase in demand, firms decide to set a higher price,

causing inflation to rise. As a result, the central bank raises the nominal interest rate, which prompts the real interest rate to rise as well. This reaction mitigates the initial effect of the expansionary monetary policy shock.

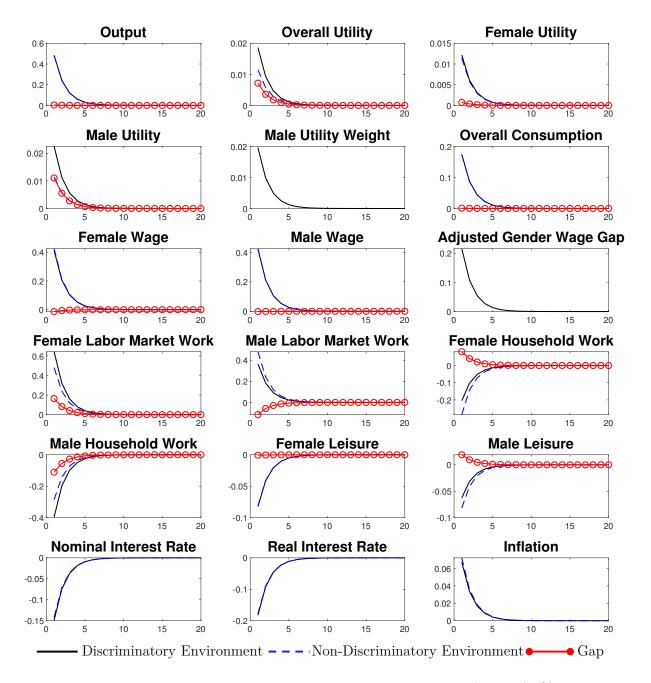


Figure 2: Taste-Based Model: Impulse Responses to an Expansionary (Annual) 1% Monetary Policy Shock with Persistence $\rho_{\nu} = 0.5$.

In the discriminatory environment $(d_F > 0)$, the transmission of the expansionary mon-

etary policy shock is dampened. After the shock, female labor market work increases more than its male counterpart due to higher marginal productivity. Rising demand implies an increase in both female and male wages. Male wages increase more than female wages due to gender discrimination against women and the larger relative decrease in marginal productivity of women in comparison to men, since women increase paid labor hours more. Thus, the adjusted gender wage gap rises, implying a higher weight of male utility in overall household utility. The higher intra-household bargaining power of men strengthens the impact of tastebased gender discrimination against women on female and male utility: female utility increases less than its male counterpart.

Note that female and male wages do not rise as much as in the non-discriminatory case. This lower overall increase in wages leads to a lower increase in marginal costs, implying a lower rise in prices and therefore a weaker transmission of the expansionary monetary policy shock on inflation. In particular, the increase in inflation is dampened by 7% in the first period.

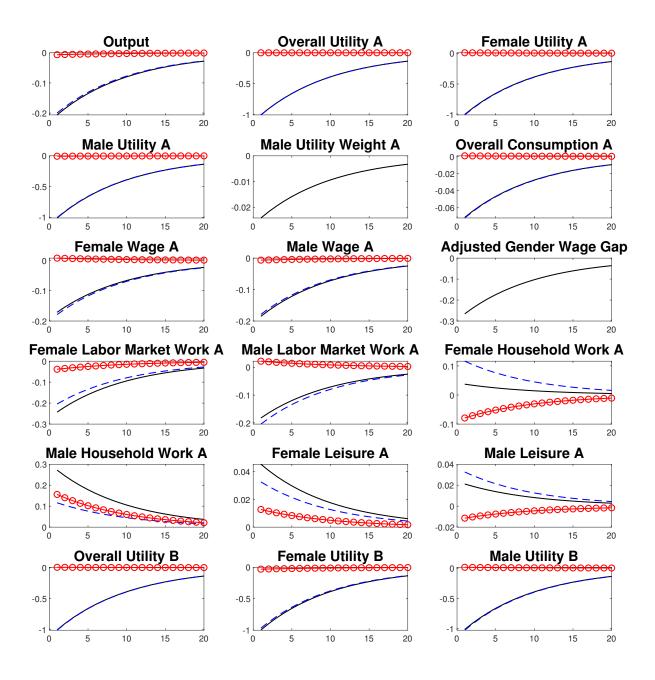
Nevertheless, the increase in wages leads men and women to reduce their household work and to work more in the paid labor market. Overall, women and men decrease leisure, the substitution effect outweighs the income effect. Household utility increases; stronger than in the non-discriminatory framework. This is caused by lower steady state market consumption of the household in the discriminatory environment than in the non-discriminatory case, implying that an additional unit of market consumption has a higher marginal utility in the former. The interpretations of the firms' pricing behavior as well as the reaction of the central bank are qualitatively similar to the corresponding interpretations in the non-discriminatory environment.

3.3 Dynamic Effects with Statistical Discrimination

In the following, we discuss the impulse responses of the model with statistical discrimination and its non-discriminatory counterpart to a negative discount rate shock and an expansionary monetary policy shock. Furthermore, we compare the results to the model responses with taste-based discrimination. Note that the productivity differences between household-B women and all other labor market participants imply different outcomes for households A and B in the non-discriminatory environment with statistical discrimination. All results are percentage deviations from the zero inflation steady state.

3.3.1 Discount Rate Shock

Figure 3 shows the impulse response functions of the model with and without statistical gender discrimination to a negative 1% discount rate shock.



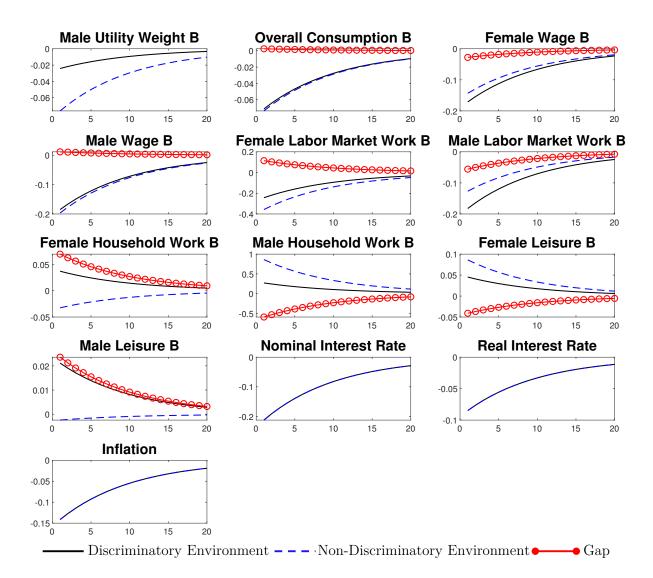


Figure 3: Statistical Model: Impulse Responses to a Negative 1% Discount Rate Shock with Persistence $\rho_Z = 0.9$.

In the **non-discriminatory environment**, i.e., in an environment where firms can perfectly observe the productivities of all agents, the impulse responses with respect to **household A** are qualitatively identical to the impulse responses in the non-discriminatory environment of the model with taste-based discrimination. Equal preferences and productivities imply symmetric effects on women and men. The negative discount rate shock leads to a decline in consumption and therefore implies lower demand for female and male labor market work. The corresponding drop in wages leads to higher household work. Overall, the consumption index decreases and

leisure increases.

Women from household B differ with respect to their productivity in market production, implying asymmetric effects on women and men of this household in the **non-discriminatory environment**. The negative discount rate shock implies a larger decrease in the demand for female labor market work than for its male counterpart due to lower female productivity. However, male wages decrease more than female wages: in steady state, women work less in the labor market than men due to their lower overall productivity. Consequently, a further drop in labor market work implies a higher increase in marginal productivity for women than for men. Female wages therefore decrease less and their utility weight in overall household utility increases. This implies that women reduce their household work and enjoy more leisure. Conversely, men increase their household work due to lower opportunity costs (wages), higher marginal productivity in household production (due to lower steady state homework hours), and the relatively lower weight on male utility. Consequently, male leisure decreases. The firms' pricing behavior and the reaction of the central bank are qualitatively similar to the responses described in the non-discriminatory environment of the model with taste-based discrimination.

In the **discriminatory environment**, i.e., an environment where firms cannot distinguish between households and thus consider the average productivity of women, the economic downturn after the negative discount rate shock is more severe than in the environment without statistical gender discrimination. This is due to an increase in the inefficient utilization of female and male productivity from both households. Firms cannot observe that women from **household A** are equally productive as all men in the economy and, therefore, base their demand for female labor market work on the average female productivity. Consequently, demand for female labor from household A decreases too much in response to the shock. Correspondingly, demand for male labor market work from household A decreases too little. Both female and male wages decrease. However, the larger increase in marginal productivity of women relative to men decreases the adjusted gender wage gap and thereby the weight on male utility in overall household utility.

The impulse responses of all variables concerning **household B** in the **discriminatory environment** are qualitatively and quantitatively identical to the responses of household A, since firms treat both households equally. However, the extent of the growing inefficiencies in comparison to the non-discriminatory environment differ between households A and B. In contrast to the response of female labor market work from household A, household-B female labor market work decreases too little in comparison to the non-discriminatory environment since firms use the average productivity of all women to determine their demand instead of the (lower) actual productivity of household-B women. Correspondingly, male labor market work from household B decreases too much since men do not need to compensate for lower female productivity in household B by working more in the labor market. Decreasing demand for female and male labor market work implies a drop in both wages. As outlined in the discussion on the impulse responses of household A, the gender wage gap decreases and therefore the weight on male utility in overall utility of household B. However, since female wages drop too much and male wages too little, the decrease in the male utility weight is inefficiently low. This implies that women increase their household work and the increase in female leisure is inefficiently low. Conversely, the relatively higher intra-bargaining power of men leads them to increase their household work less and to enjoy more leisure. The pricing behavior of the firms and the reaction of the central bank remain qualitatively unchanged.

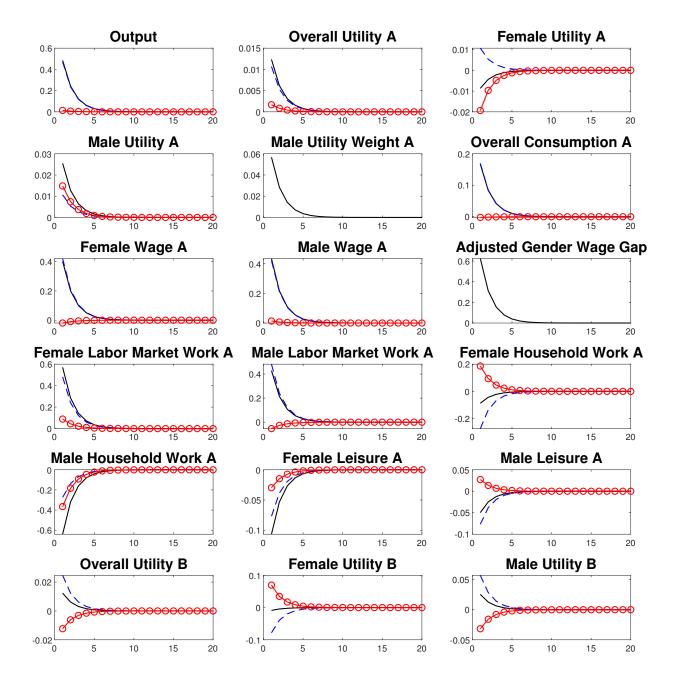
Overall, the inefficient utilization of female and male productivity from both households implies that output decreases 3% more in the discriminatory environment with statistical discrimination than in its non-discriminatory counterpart. Quantitatively, the economy suffers less from statistical discrimination than from taste-based discrimination: while taste-based gender discrimination affects all households equally, statistical discrimination has two contrasting effects. On the one hand, household A suffers from statistical discrimination and demands less market goods than in the non-discriminatory environment. On the other hand, household B benefits from statistical discrimination, implying inefficiently high market good consumption.

However, statistical discrimination implies quantitatively larger effects on the adjusted gender wage gap and on the inefficiency of the intra-household working time allocation than taste-based discrimination in comparison to their non-discriminatory environments. When unobserved, the structural productivity differences between women from household B and all other agents not only imply that the demand by firms for women with higher productivity is too low but also that their demand for less productive women is too high.

3.3.2 Monetary Policy Shock

Figure 4 depicts the impulse response functions of the model with and without statistical gender discrimination to an expansionary (annual) 1% monetary policy shock. In the **non-**

discriminatory environment, the impulse responses to the shock by **household A** tally with the corresponding responses in the non-discriminatory environment of the model with taste-based discrimination.



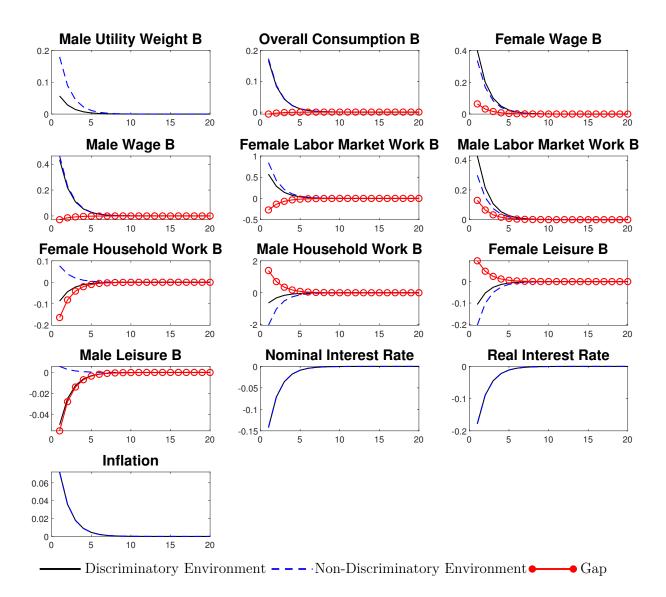


Figure 4: Statistical Model: Impulse Responses to an Expansionary (Annual) 1% Monetary Policy Shock with Persistence $\rho_{\nu} = 0.5$.

The decrease in the nominal and real interest rates leads to higher demand for market goods. Firms demand more female and male labor market work and the corresponding wages increase. Due to higher opportunity costs, women and men reduce their household work. Overall, the substitution effect outweighs the income effect and women and men from household A enjoy less leisure. Due to higher overall consumption, utility increases.

The lower productivity of women from **household B** implies different impulse responses of this household in the **non-discriminatory environment**. Female wages increase less than male wages when firms increase their demand for female and male workers, implying an increase in the weight of male utility in the household's overall utility. The higher intrahousehold bargaining power of men implies that they, in contrast to women, enjoy more leisure and reduce their household work. Consequently, male utility increases while female utility decreases. Overall, household utility rises.

Firms increase their prices in reaction to the increase in market good consumption of both households and the corresponding increase in aggregate demand. The central bank reacts to the increase in inflation by increasing the nominal interest rate, thereby mitigating the initial effects of the monetary policy shock.

In the **discriminatory environment**, the transmission of monetary policy shocks on inflation is dampened. The drop in the nominal and real interest rates leads to an increase in aggregate demand. Firms consequently demand more labor from women and men of both households. In **household A**, lower average productivity leads female wages to increase less than their male counterparts, implying an increase in the adjusted gender wage gap. As a result, the male bargaining power increases. This strengthens the effects of statistical gender discrimination on the inefficient working time allocation: men reduce their household work too much and enjoy too much leisure. Conversely, women reduce their household work too little and their leisure too much. This inefficient intra-household allocation leads to an inefficiently high increase in male utility, while female utility actually decreases in response to the shock.

In the **discriminatory environment**, the responses of **household B** and household A to the expansionary monetary policy shock are identical. However, the resulting inefficiencies differ. In particular, as firms base their demand decision for female labor on the average productivity of women instead of the lower actual productivity of household-B women, female wages increase more than in the non-discriminatory environment. Conversely, male wages rise less, implying a lower increase in the male utility weight in overall household-B utility, i.e., women lose less intra-household bargaining power. Thus, male household work decreases less and men enjoy less leisure. In contrast to the non-discriminatory environment, female household work decreases and female leisure decreases less. This implies that both the utility losses of women and the utility gains of men in household B are inefficiently low.

Due to the increase in wages of all agents, firms face higher marginal costs and increase their prices. However, the inefficient utilization of female and male labor from both households implies that inflation increases 1% less in the discriminatory environment. Quantitatively, the impact of statistical discrimination on the transmission of monetary policy shocks is lower than the effect of taste-based discrimination. This is caused by two opposing effects on firms' costs: while the wages of women and men from household A are too low, the wages of women and men from household B are too high. In household A, women are statistically discriminated against and men work too much to compensate for this discrimination, lowering their marginal productivity. In household B, women are perceived as too productive and men therefore work too little, implying a higher marginal productivity than in the non-discriminatory case.

Lastly, note that statistical discrimination implies quantitatively larger effects on the adjusted gender wage gap and on the inefficiency of the working time allocations of the households than taste-based discrimination after an expansionary monetary policy shock. The unobserved structural productivity differences between women from household B and all other agents leads to an inefficiently low (high) labor demand for relatively more (less) productive women.

4 Conclusion

While the consequences of gender discrimination especially for the labor market have been discussed and analyzed extensively over the past decades, the effects on business cycle dynamics and inflation have not yet been at the center of economic research. This paper theoretically analyzes the effects of taste-based and statistical gender discrimination against women in the labor market on the business cycle and inflation. We build a tractable New Keynesian model that includes the two types of discriminatory behavior by firms against women, two households that consist of a woman and a man, and household work in addition to a paid labor market. In order to analyze the effects of the two types of gender discrimination, we compare the model responses to a negative demand and an expansionary monetary policy shock in a nondiscriminatory environment and in a discriminatory one.

In response to a negative discount rate shock, the economic downturn is more severe in the discriminatory environment. An increase in the inefficient utilization of female and male productivity leads to a more inefficient working time allocation and lower output. Moreover, the transmission of expansionary monetary policy shocks on inflation is weakened by both types of gender discrimination. Furthermore, the expansionary monetary policy shock has distributional effects: the adjusted gender wage gap increases. Quantitatively, taste-based discrimination implies larger macroeconomic inefficiencies, while statistical discrimination leads to higher intra-household distortions and has a greater impact on the gender wage gap.

These pareto-inefficient outcomes provide a positive rationale for combating gender discrimination. In the recent past, several regulations have been passed by political institutions to tackle discriminatory behavior by firms and the continuing wage gap between women and men. For instance, in 2014 the European Union (EU) officially recommended that its member states introduce pay transparency laws. Another legislative measure to reduce gender discrimination and the gender wage gap might be to introduce quotas for the proportion of women in leading positions, for example, company boards. Empirical evidence suggests that a higher proportion of women in managerial positions implies a lower gender wage gap (see, for instance, Bryson et al., 2019). Our results show that measures which reduce discriminatory behavior by firms might not only reduce inefficiencies between women and men but also present solutions to reduce the adverse macroeconomic aspects of gender discrimination and thereby serve as efficient macroeconomic stabilization tools. Our paper also provides a basis for future research, especially with respect to the effects of monetary policy on gender inequality. To the best of our knowledge, an empirical examination of the effects of expansionary monetary policy on the *adjusted* gender wage gap has not yet been conducted. Considering the plethora of expansionary monetary policy instruments used in the recent past, empirical analyses of adjusted gender-specific effects are necessary to fully assess their economic consequences.

Appendices

A Expenditure Minimization of the Household

Household k minimizes its expenditures for any given level of consumption $(\bar{C}^{k,N})$:

$$\min_{C_{i,t}^{k,N}} \int_0^1 P_{i,t} C_{i,t}^{k,N} di,$$
(A.1)

subject to

$$\left(\int_0^1 C_{i,t}^{k,N\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}} = \bar{C}^{k,N}$$

This is equivalent to maximizing the following Lagrange function (L_t^k) with respect to the consumption of a representative good j:

$$\max_{C_{j,t}^{k,N}} L_t^k = -\int_0^1 P_{i,t} C_{i,t}^{k,N} di + \lambda_t^k \left[\left(\int_0^1 C_{i,t}^{k,N\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} - \bar{C}^{k,N} \right],$$

where λ_t^k is the Lagrange multiplier. The first order conditions are given by

$$\frac{\partial L_t^k}{\partial C_{j,t}^{k,N}} = -P_{j,t} + \lambda_t^k \left[\left(\int_0^1 C_{i,t}^{k,N\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}-1} C_{j,t}^{k,N\frac{\epsilon-1}{\epsilon}-1} \right] \stackrel{!}{=} 0, \tag{A.2}$$

$$\frac{\partial L_t^k}{\partial \lambda_t^k} \stackrel{!}{=} 0. \tag{A.3}$$

Rearranging yields

$$C_{j,t}^{k,N} = \left(\frac{P_{j,t}}{\lambda_t^k}\right)^{-\epsilon} C_t^{k,N}.$$
(A.4)

In order to obtain the expression for optimal consumption, it is necessary to solve for λ_t^k by using the constraint.

$$\begin{split} \left(\int_{0}^{1} C_{i,t}^{k,N} \frac{\frac{\epsilon-1}{\epsilon}}{d} di\right)^{\frac{\epsilon}{\epsilon_{k}}} &= \bar{C}^{k,N}, \\ \Leftrightarrow \left(\int_{0}^{1} \left[\left(\frac{P_{i,t}}{\lambda_{t}^{k}}\right)^{-\epsilon} \bar{C}^{k,N} \right]^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} &= \bar{C}^{k,N}, \\ \Leftrightarrow \int_{0}^{1} \left(\frac{P_{i,t}}{\lambda_{t}^{k}}\right)^{1-\epsilon} di = 1. \end{split}$$

The solution for λ_t^k thus is

$$\lambda_t^k = \left(\int_0^1 P_{i,t}^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}} \equiv P_t.$$

Plugging this solution into the optimal consumption decision for any good i yields

$$C_{i,t}^{k,N} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} C_t^{k,N}.$$
(A.5)

B Utility Maximization of the Household

Household k cooperatively maximizes its utility by choosing $N_{F,t}^k, N_{M,t}^k, V_{F,t}^k, V_{M,t}^k, C_t^{k,N}$, and B_t^k subject to the flow budget constraint. The Lagrange function (L_t^k) is given by:

$$L_{t}^{k} = \mathbb{E}_{t} \left[\sum_{\iota=0}^{\infty} \theta^{\iota} \left[\zeta_{t+\iota}^{k} U_{M,t+\iota}^{k} + (1 - \zeta_{t+\iota}^{k}) U_{F,t+\iota}^{k} - \lambda_{t+\iota}^{k} (P_{t+\iota} C_{t+\iota}^{k,N} + Q_{t+\iota}^{k} B_{t+\iota}^{k} - B_{t+\iota-1}^{k} - W_{F,t+\iota}^{k} N_{F,t+\iota}^{k} - W_{M,t+\iota}^{k} N_{M,t+\iota}^{k} - D_{t+\iota}^{k}) \right] \right], \quad (B.1)$$

with

$$\begin{split} U_{G,t}^{k} &= Z_{t} \left[\frac{\left(\left(C_{G,t}^{k} \right)^{b} \left(L_{G,t}^{k} \right)^{1-b} \right)^{1-\sigma}}{1-\sigma} + \Omega \right], \\ C_{G,t}^{k} &= \left(\gamma_{G} \left(C_{t}^{k,N} \right)^{\frac{\vartheta_{C}-1}{\vartheta_{C}}} + (1-\gamma_{G}) \left(C_{t}^{k,V} \right)^{\frac{\vartheta_{C}-1}{\vartheta_{C}}} \right)^{\frac{\vartheta_{C}}{\vartheta_{C}-1}}, \\ 1 &= N_{G,t}^{k} + V_{G,t}^{k} + L_{G,t}^{k}, \\ C_{t}^{k,V} &= \left(\beta \left(V_{F,t}^{k} \right)^{\frac{\vartheta_{V}-1}{\vartheta_{V}}} + (1-\beta) \left(V_{M,t}^{k} \right)^{\frac{\vartheta_{V}-1}{\vartheta_{V}}} \right)^{\frac{\vartheta_{V}-1}{\vartheta_{V}-1}}. \end{split}$$

The first order conditions are:

$$\frac{\partial L_t^k}{\partial C_t^{k,N}} = \zeta_t^k \frac{\partial U_{M,t}^k}{\partial C_{M,t}^k} \frac{\partial C_{M,t}^k}{\partial C_t^{k,N}} + (1 - \zeta_t^k) \frac{\partial U_{F,t}^k}{\partial C_{F,t}^k} \frac{\partial C_{F,t}^k}{\partial C_t^{k,N}} - \lambda_t^k P_t \stackrel{!}{=} 0, \tag{B.2}$$

$$\frac{\partial L_t^k}{\partial N_{F,t}^k} = (1 - \zeta_t^k) \frac{\partial U_{F,t}^k}{\partial L_{F,t}^k} \frac{\partial L_{F,t}^k}{\partial N_{F,t}^k} + \lambda_t^k W_{F,t}^k \stackrel{!}{=} 0, \tag{B.3}$$

$$\frac{\partial L_t^k}{\partial N_{M,t}^k} = \zeta_t^k \frac{\partial U_{M,t}^k}{\partial L_{M,t}^k} \frac{\partial L_{M,t}^k}{\partial N_{M,t}^k} + \lambda_t^k W_{M,t}^k \stackrel{!}{=} 0, \tag{B.4}$$

$$\frac{\partial L_t^k}{\partial V_{F,t}^k} = \zeta_t^k \frac{\partial U_{M,t}^k}{\partial C_{M,t}^k} \frac{\partial C_{M,t}^k}{\partial C_t^{k,V}} \frac{\partial C_t^{k,V}}{\partial V_{F,t}^k} + (1 - \zeta_t^k) \left(\frac{\partial U_{F,t}^k}{\partial C_{F,t}^k} \frac{\partial C_{F,t}^k}{\partial C_t^{k,V}} \frac{\partial C_t^{k,V}}{\partial V_{F,t}^k} + \frac{\partial U_{F,t}^k}{\partial L_{F,t}^k} \frac{\partial L_{F,t}^k}{\partial V_{F,t}^k} \right) \stackrel{!}{=} 0, \quad (B.5)$$

$$\frac{\partial L_t^k}{\partial V_{M,t}^k} = \zeta_t^k \left(\frac{\partial U_{M,t}^k}{\partial C_{M,t}^k} \frac{\partial C_{M,t}^k}{\partial C_t^{k,V}} \frac{\partial C_t^{k,V}}{\partial V_{M,t}^k} + \frac{\partial U_{M,t}^k}{\partial L_{M,t}^k} \frac{\partial L_{M,t}^k}{\partial V_{M,t}^k} \right) + (1 - \zeta_t^k) \frac{\partial U_{F,t}^k}{\partial C_{F,t}^k} \frac{\partial C_t^{k,V}}{\partial C_t^{k,V}} \frac{\partial C_t^{k,V}}{\partial V_{M,t}^k} \stackrel{!}{=} 0, \quad (B.6)$$

$$\frac{\partial L_t^k}{\partial B_t^k} = -\lambda_t^k Q_t^k + \mathbb{E}_t \left[\theta \lambda_{t+1}^k \right] \stackrel{!}{=} 0, \tag{B.7}$$

$$\frac{\partial L_t^k}{\partial \lambda_t^k} \stackrel{!}{=} 0, \tag{B.8}$$

where

$$\frac{\partial U_{G,t}^k}{\partial C_{G,t}^k} = Z_t \left(\left(C_{G,t}^k \right)^b \left(L_{G,t}^k \right)^{1-b} \right)^{-\sigma} b \left(C_{G,t}^k \right)^{b-1} \left(L_{G,t}^k \right)^{1-b}, \tag{B.9}$$

$$\frac{\partial C_{G,t}^k}{\partial C_t^{k,N}} = \left(C_{G,t}^k\right)^{\frac{1}{\vartheta_C}} \gamma_G \left(C_t^{k,N}\right)^{-\frac{1}{\vartheta_C}},\tag{B.10}$$

$$\frac{\partial U_{G,t}^k}{\partial L_{G,t}^k} = Z_t \left(\left(C_{G,t}^k \right)^b \left(L_{G,t}^k \right)^{1-b} \right)^{-\sigma} (1-b) \left(C_{G,t}^k \right)^b \left(L_{G,t}^k \right)^{-b}, \tag{B.11}$$

$$\frac{\partial L_{G,t}^k}{\partial N_{G,t}^k} = -1,\tag{B.12}$$

$$\frac{\partial C_{G,t}^k}{\partial C_t^{k,V}} = \left(C_{G,t}^k\right)^{\frac{1}{\vartheta_C}} \left(1 - \gamma_G\right) \left(C_t^{k,V}\right)^{-\frac{1}{\vartheta_C}},\tag{B.13}$$

$$\frac{\partial C_t^{k,V}}{\partial V_{F,t}^k} = \left(C_t^{k,V}\right)^{\frac{1}{\vartheta_V}} \beta \left(V_{F,t}^k\right)^{-\frac{1}{\vartheta_V}},\tag{B.14}$$

$$\frac{\partial L^k_{G,t}}{\partial V^k_{G,t}} = -1,\tag{B.15}$$

$$\frac{\partial C_t^{k,V}}{\partial V_{M,t}^k} = \left(C_t^{k,V}\right)^{\frac{1}{\vartheta_V}} \left(1-\beta\right) \left(V_{M,t}^k\right)^{-\frac{1}{\vartheta_V}}.$$
(B.16)

Combining equations (B.2)–(B.16) and rearranging, we get

$$(1 - \zeta_t^k)(1 - b) \frac{\left(\left(C_{F,t}^k\right)^b \left(L_{F,t}^k\right)^{1-b}\right)^{1-\sigma}}{L_{F,t}^k} = b\left(C_t^{k,V}\right)^{\frac{1}{\vartheta_V} - \frac{1}{\vartheta_C}} \beta\left(V_{F,t}^k\right)^{-\frac{1}{\vartheta_V}} \left(\psi_{M,t}^k(1 - \gamma_M) + \psi_{F,t}^k(1 - \gamma_F)\right),$$
(B.17a)

$$\zeta_{t}^{k}(1-b)\frac{\left(\left(C_{M,t}^{k}\right)^{b}\left(L_{M,t}^{k}\right)^{1-b}\right)^{1-\sigma}}{L_{M,t}^{k}} = b\left(C_{t}^{k,V}\right)^{\frac{1}{\vartheta_{V}}-\frac{1}{\vartheta_{C}}}\left(1-\beta\right)\left(V_{M,t}^{k}\right)^{-\frac{1}{\vartheta_{V}}}\left(\psi_{M,t}^{k}(1-\gamma_{M})+\psi_{F,t}^{k}(1-\gamma_{F})\right), \quad (B.17b)$$

$$(1-\zeta_{t}^{k})(1-b)\frac{\left(\left(C_{F,t}^{k}\right)^{b}\left(L_{F,t}^{k}\right)^{1-b}\right)^{1-b}}{L_{F,t}^{k}} = b\frac{W_{F,t}^{k}}{P_{t}}\left(C_{t}^{k,N}\right)^{-\frac{1}{\vartheta_{C}}}\left(\psi_{M,t}^{k}\gamma_{M}+\psi_{F,t}^{k}\gamma_{F}\right), \quad (B.18a)$$

$$\zeta_{t}^{k}(1-b)\frac{\left(\left(C_{M,t}^{k}\right)^{b}\left(L_{M,t}^{k}\right)^{1-b}\right)^{1-b}}{L_{M,t}^{k}} = b\frac{W_{M,t}^{k}}{P_{t}}\left(C_{t}^{k,N}\right)^{-\frac{1}{\vartheta_{C}}}\left(\psi_{M,t}^{k}\gamma_{M}+\psi_{F,t}^{k}\gamma_{F}\right), \quad (B.18b)$$

and

$$Q_t = \theta \mathbb{E}_t \left[\Psi_{t,t+1}^k \frac{1}{\Pi_{t+1}} \right].$$
(B.19)

C Risk Sharing

The Euler equation holds for both households. Thus,

$$\theta \mathbb{E}_t \left[\Psi_{t,t+1}^k \frac{1}{\Pi_{t+1}} \right] = \theta \mathbb{E}_t \left[\Psi_{t,t+1}^{-k} \frac{1}{\Pi_{t+1}} \right], \tag{C.1}$$

or

$$\mathbb{E}_{t}\left[\frac{U_{C^{k,N},t+1}^{k}}{U_{C^{k,N},t}^{k}}\right] = \mathbb{E}_{t}\left[\frac{U_{C^{-k,N},t+1}^{-k}}{U_{C^{-k,N},t}^{-k}}\right].$$
(C.2)

This relation holds in all periods, i.e.,

$$\frac{U_{C^{k,N},t}^{k}}{U_{C^{k,N},t-1}^{k}} = \frac{U_{C^{-k,N},t}^{-k}}{U_{C^{-k,N},t-1}^{-k}},$$

and

$$\label{eq:ckn} \begin{split} \frac{U^k_{C^{k,N},t-1}}{U^k_{C^{k,N},t-2}} = \frac{U^{-k}_{C^{-k,N},t-1}}{U^{-k}_{C^{-k,N},t-2}}, \\ & [\ldots]. \end{split}$$

This implies

$$U_{C^{k,N},t}^{k} = U_{C^{-k,N},t}^{-k} \frac{U_{C^{k,N},t-2}^{k}}{U_{C^{-k,N},t-2}^{-k}}$$

Continuing this procedure to the initial period, i.e., the steady state, we get

$$U_{C^{k,N},t}^{k} = \varphi^{k} U_{C^{-k,N},t}^{-k}.$$
 (C.3)

D Profit Maximization of the Firm

The firm's maximization problem is

$$\max_{P_{i,t}} \mathbb{E}_t \left[\sum_{\iota=0}^{\infty} \theta^{\iota} \Lambda^{\iota} \Psi_{t,t+\iota} \left(\frac{P_{i,t}}{P_{t+\iota}} Y_{i,t+\iota|t} - TC(Y_{i,t+\iota|t}) \right) \right], \tag{D.1}$$

subject to the sequence of demand constraints

$$Y_{i,t+\iota|t} = \left(\frac{P_{i,t}}{P_{t+\iota}}\right)^{-\epsilon} Y_{t+\iota}.$$

The first order condition is:

$$0 \stackrel{!}{=} \mathbb{E}_t \left[\sum_{\iota=0}^{\infty} \theta^{\iota} \Lambda^{\iota} \Psi_{t,t+\iota} \left((1-\epsilon) \frac{1}{P_{t+\iota}} Y_{i,t+\iota|t} - mc(Y_{i,t+\iota|t})(-\epsilon) \frac{1}{P_{i,t}} Y_{i,t+\iota|t} \right) \right],$$

which gives the optimality condition

$$0 = \mathbb{E}_t \left[\sum_{\iota=0}^{\infty} \theta^{\iota} \Lambda^{\iota} \Psi_{t,t+\iota} Y_{i,t+\iota|t} \left(\frac{P_{i,t}}{P_{t+\iota}} - \mu mc(Y_{i,t+\iota|t}) \right) \right].$$
(D.2)

E Derivation of Marginal Costs

E.1 Non-Discriminatory Environment

In the non-discriminatory environment, the firm minimizes

$$\min_{N_{i,G,t}^{k}} \kappa w_{F,t}^{A} N_{i,F,t}^{A} + (1-\kappa) w_{F,t}^{B} N_{i,F,t}^{B} + \kappa w_{M,t}^{A} N_{i,M,t}^{A} + (1-\kappa) w_{M,t}^{B} N_{i,M,t}^{B},$$
(E.1)

subject to

$$\begin{split} \bar{Y}_i &= \left(\kappa \alpha^A \left(N_{i,F,t}^A\right)^{\frac{\vartheta_N - 1}{\vartheta_N}} + (1 - \kappa) \alpha^B \left(N_{i,F,t}^B\right)^{\frac{\vartheta_N - 1}{\vartheta_N}} \right. \\ &+ \kappa (1 - \alpha) \left(N_{i,M,t}^A\right)^{\frac{\vartheta_N - 1}{\vartheta_N}} + (1 - \kappa)(1 - \alpha) \left(N_{i,M,t}^B\right)^{\frac{\vartheta_N - 1}{\vartheta_N}} \right)^{\frac{\vartheta_N - 1}{\vartheta_N}}, \end{split}$$

where \bar{Y}_i is any given output level. The respective Lagrange function is given by

$$\max_{\substack{N_{i,G,t}^{A}\\ i,G,t}} L_{t} = -\left(\kappa w_{F,t}^{A} N_{i,F,t}^{A} + (1-\kappa) w_{F,t}^{B} N_{i,F,t}^{B} + \kappa w_{M,t}^{A} N_{i,M,t}^{A} + (1-\kappa) w_{M,t}^{B} N_{i,M,t}^{B}\right) + \lambda_{t} \left(\left(\kappa \alpha^{A} \left(N_{i,F,t}^{A}\right)^{\frac{\vartheta_{N}-1}{\vartheta_{N}}} + (1-\kappa) \alpha^{B} \left(N_{i,F,t}^{B}\right)^{\frac{\vartheta_{N}-1}{\vartheta_{N}}} + \kappa(1-\alpha) \left(N_{i,M,t}^{A}\right)^{\frac{\vartheta_{N}-1}{\vartheta_{N}}} + (1-\kappa)(1-\alpha) \left(N_{i,M,t}^{B}\right)^{\frac{\vartheta_{N}-1}{\vartheta_{N}}} - \bar{Y}_{i} \right). \tag{E.2}$$

The first order conditions are

$$\frac{\partial L_t}{\partial N^A_{i,F,t}} = -\kappa w^A_{F,t} + \lambda_t Y^{\frac{1}{\vartheta_N}}_{i,t} \kappa \alpha^A N^A_{i,F,t} - \frac{1}{\vartheta_N} \stackrel{!}{=} 0,$$
(E.3)

$$\frac{\partial L_t}{\partial N^B_{i,F,t}} = -(1-\kappa)w^B_{F,t} + \lambda_t Y^{\frac{1}{\vartheta_N}}_{i,t}(1-\kappa)\alpha^B N^B_{i,F,t} - \frac{1}{\vartheta_N} \stackrel{!}{=} 0, \tag{E.4}$$

$$\frac{\partial L_t}{\partial N^A_{i,M,t}} = -\kappa w^A_{M,t} + \lambda_t Y^{\frac{1}{\vartheta_N}}_{i,t} \kappa (1-\alpha) N^A_{i,M,t} - \frac{1}{\vartheta_N} \stackrel{!}{=} 0, \tag{E.5}$$

$$\frac{\partial L_t}{\partial N^B_{i,M,t}} = -(1-\kappa)w^B_{M,t} + \lambda_t Y^{\frac{1}{\vartheta_N}}_{i,t}(1-\kappa)(1-\alpha)N^B_{i,M,t} - \frac{1}{\vartheta_N} \stackrel{!}{=} 0,$$
(E.6)

$$\frac{\partial L_t}{\partial \lambda_t} \stackrel{!}{=} 0. \tag{E.7}$$

Dividing equation (E.5) by equations (E.3), (E.4), (E.6), respectively, gives the optimality conditions (E.5)

$$\frac{1-\alpha}{\alpha^A} \left(\frac{N_{i,M,t}^A}{N_{i,F,t}^A}\right)^{-\frac{1}{\vartheta_N}} = \frac{w_{M,t}^A}{w_{F,t}^A},\tag{E.8}$$

$$\frac{1-\alpha}{\alpha^B} \left(\frac{N_{i,M,t}^A}{N_{i,F,t}^B}\right)^{-\frac{1}{\vartheta_N}} = \frac{w_{M,t}^A}{w_{F,t}^B},\tag{E.9}$$

$$\left(\frac{N_{i,M,t}^{A}}{N_{i,M,t}^{B}}\right)^{-\frac{1}{\vartheta_{N}}} = \frac{w_{M,t}^{A}}{w_{M,t}^{B}}.$$
(E.10)

Solving for $N_{i,F,t}^A$, $N_{i,F,t}^B$, and $N_{i,M,t}^B$ yields

$$\begin{split} N_{i,F,t}^{A} &= \left(\frac{\alpha^{A}}{1-\alpha}\frac{w_{M,t}^{A}}{w_{F,t}^{A}}\right)^{\vartheta_{N}}N_{i,M,t}^{A},\\ N_{i,F,t}^{B} &= \left(\frac{\alpha^{B}}{1-\alpha}\frac{w_{M,t}^{A}}{w_{F,t}^{B}}\right)^{\vartheta_{N}}N_{i,M,t}^{A},\\ N_{i,M,t}^{B} &= \left(\frac{w_{M,t}^{A}}{w_{M,t}^{B}}\right)^{\vartheta_{N}}N_{i,M,t}^{A}. \end{split}$$

Plugging these expressions into the production function gives

$$Y_{i,t} = \left(\kappa\alpha^{A} \left(\left(\frac{\alpha^{A}}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{A}}\right)^{\vartheta_{N}} N_{i,M,t}^{A} \right)^{\frac{\vartheta_{N}-1}{\vartheta_{N}}} + (1-\kappa)\alpha^{B} \left(\left(\frac{\alpha^{B}}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{B}}\right)^{\vartheta_{N}} N_{i,M,t}^{A} \right)^{\frac{\vartheta_{N}-1}{\vartheta_{N}}} + \kappa(1-\alpha) \left(N_{i,M,t}^{A}\right)^{\frac{\vartheta_{N}-1}{\vartheta_{N}}} + (1-\kappa)(1-\alpha) \left(\left(\frac{w_{M,t}^{A}}{w_{M,t}^{B}}\right)^{\vartheta_{N}} N_{i,M,t}^{A} \right)^{\frac{\vartheta_{N}-1}{\vartheta_{N}}} \right)^{\frac{\vartheta_{N}-1}{\vartheta_{N}}}.$$

Then, we can solve for ${\cal N}^{\cal A}_{i,M,t}$

$$N_{i,M,t}^{A} = \frac{Y_{i,t}}{\left[\kappa\alpha^{A} \left(\frac{\alpha^{A}}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{A}}\right)^{\vartheta_{N}-1} + (1-\kappa)\alpha^{B} \left(\frac{\alpha^{B}}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{B}}\right)^{\vartheta_{N}-1} + \kappa(1-\alpha) + (1-\kappa)(1-\alpha) \left(\frac{w_{M,t}^{A}}{w_{M,t}^{B}}\right)^{\vartheta_{N}-1}\right]^{\frac{\vartheta_{N}}{\vartheta_{N}-1}}}$$

Accordingly, the solutions for $N_{i,F,t}^A$, $N_{i,F,t}^B$, and $N_{i,M,t}^B$ are

$$\begin{split} N_{i,F,t}^{A} &= \frac{Y_{i,t} \left(\frac{\alpha^{A}}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{A}}\right)^{\vartheta_{N}}}{\left[\kappa\alpha^{A} \left(\frac{\alpha^{A}}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{A}}\right)^{\vartheta_{N}-1} + (1-\kappa)\alpha^{B} \left(\frac{\alpha^{B}}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{B}}\right)^{\vartheta_{N}-1} + \kappa(1-\alpha) + (1-\kappa)(1-\alpha) \left(\frac{w_{M,t}^{A}}{w_{M,t}^{B}}\right)^{\vartheta_{N}-1}\right]^{\frac{\vartheta_{N}}{\vartheta_{N}-1}}, \\ N_{i,F,t}^{B} &= \frac{Y_{i,t} \left(\frac{\alpha^{B}}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{B}}\right)^{\vartheta_{N}}}{\left[\kappa\alpha^{A} \left(\frac{\alpha^{A}}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{A}}\right)^{\vartheta_{N}-1} + (1-\kappa)\alpha^{B} \left(\frac{\alpha^{B}}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{B}}\right)^{\vartheta_{N}-1} + \kappa(1-\alpha) + (1-\kappa)(1-\alpha) \left(\frac{w_{M,t}^{A}}{w_{M,t}^{B}}\right)^{\vartheta_{N}-1}\right]^{\frac{\vartheta_{N}}{\vartheta_{N}-1}}, \end{split}$$

$$N_{i,M,t}^{B} = \frac{Y_{i,t} \left(\frac{w_{M,t}^{A}}{w_{M,t}^{A}}\right)^{\vartheta_{N}}}{\left[\kappa\alpha^{A} \left(\frac{\alpha^{A}}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{A}}\right)^{\vartheta_{N}-1} + (1-\kappa)\alpha^{B} \left(\frac{\alpha^{B}}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{B}}\right)^{\vartheta_{N}-1} + \kappa(1-\alpha) + (1-\kappa)(1-\alpha) \left(\frac{w_{M,t}^{A}}{w_{M,t}^{B}}\right)^{\vartheta_{N}-1}\right]^{\frac{\vartheta_{N}}{\vartheta_{N}-1}}}.$$

Therefore, total costs are described by

$$TC(Y_{i,t}) = \frac{\kappa w_{M,t}^{A} Y_{i,t} + \kappa w_{F,t}^{A} Y_{i,t} \left(\frac{\alpha^{A}}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{A}}\right)^{\vartheta_{N}} + (1-\kappa) w_{M,t}^{B} Y_{i,t} \left(\frac{w_{M,t}^{A}}{w_{M,t}^{B}}\right)^{\vartheta_{N}} + (1-\kappa) w_{F,t}^{B} Y_{i,t} \left(\frac{\alpha^{B}}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{B}}\right)^{\vartheta_{N}}}{\left[\kappa \alpha^{A} \left(\frac{\alpha^{A}}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{B}}\right)^{\vartheta_{N}-1} + (1-\kappa) \alpha^{B} \left(\frac{\alpha^{B}}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{B}}\right)^{\vartheta_{N}-1} + \kappa(1-\alpha) + (1-\kappa)(1-\alpha) \left(\frac{w_{M,t}^{A}}{w_{M,t}^{B}}\right)^{\vartheta_{N}-1}\right]^{\frac{\vartheta_{N}}{\vartheta_{N}-1}}}$$

The first derivative with respect to $Y_{i,t}$ then gives marginal costs

$$mc_{t} = \frac{\kappa w_{M,t}^{A} + \kappa w_{F,t}^{A} \left(\frac{\alpha A}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{A}}\right)^{\vartheta_{N}} + (1-\kappa) w_{M,t}^{B} \left(\frac{w_{M,t}^{A}}{w_{M,t}^{B}}\right)^{\vartheta_{N}} + (1-\kappa) w_{F,t}^{B} \left(\frac{\alpha B}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{B}}\right)^{\vartheta_{N}}}{\left[\kappa \alpha^{A} \left(\frac{\alpha A}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{A}}\right)^{\vartheta_{N}-1} + (1-\kappa) \alpha^{B} \left(\frac{\alpha B}{1-\alpha} \frac{w_{M,t}^{A}}{w_{F,t}^{B}}\right)^{\vartheta_{N}-1} + \kappa(1-\alpha) + (1-\kappa)(1-\alpha) \left(\frac{w_{M,t}^{A}}{w_{M,t}^{B}}\right)^{\vartheta_{N}-1}\right]^{\frac{\vartheta_{N}}{\vartheta_{N}-1}}}.$$
 (E.11)

E.2 Taste-Based Discrimination

In the case of taste-based discrimination, the firm minimizes

$$\min_{N_{i,G,t}} w_{M,t} N_{i,M,t} + (w_{F,t} + d_F) N_{i,F,t},$$
(E.12)

subject to

$$\bar{Y}_i = \left(\alpha N_{i,F,t}^{\frac{\vartheta_N - 1}{\vartheta_N}} + (1 - \alpha) N_{i,M,t}^{\frac{\vartheta_N - 1}{\vartheta_N}}\right)^{\frac{\vartheta_N}{\vartheta_N - 1}},$$

where \bar{Y}_i is any given output level. The respective Lagrange function is given by

$$\max_{N_{i,G,t}} L_t = -(w_{M,t}N_{i,M,t} + (w_{F,t} + d_F)N_{i,F,t}) + \lambda_t \left[\left(\alpha N_{i,F,t}^{\frac{\vartheta_N - 1}{\vartheta_N}} + (1 - \alpha) N_{i,M,t}^{\frac{\vartheta_N - 1}{\vartheta_N}} \right)^{\frac{\vartheta_N}{\vartheta_N - 1}} - \bar{Y}_i \right].$$
(E.13)

The first order conditions are

$$\frac{\partial L_t}{\partial N_{i,M,t}} = -w_{M,t} + \lambda_t Y_{i,t}^{\frac{1}{\vartheta_N}} (1-\alpha) N_{i,M,t}^{-\frac{1}{\vartheta_N}} \stackrel{!}{=} 0, \qquad (E.14)$$

$$\frac{\partial L_t}{\partial N_{i,F,t}} = -(w_{F,t} + d_F) + \lambda_t Y_{i,t}^{\frac{1}{\vartheta_N}} \alpha N_{i,F,t}^{-\frac{1}{\vartheta_N}} \stackrel{!}{=} 0,$$
(E.15)

$$\frac{\partial L_t}{\partial \lambda_t} \stackrel{!}{=} 0. \tag{E.16}$$

Dividing equation (E.14) by equation (E.15) gives the optimality condition

$$\frac{w_{M,t}}{w_{F,t}+d_F} = \frac{1-\alpha}{\alpha} \left(\frac{N_{i,M,t}}{N_{i,F,t}}\right)^{-\frac{1}{\vartheta_N}}.$$
(E.17)

Solving for $N_{i,F,t}$ yields

$$N_{i,F,t} = \left(\frac{\alpha}{1-\alpha} \frac{w_{M,t}}{w_{F,t}+d_F}\right)^{\vartheta_N} N_{i,M,t}.$$

Plugging this into the production function gives

$$Y_{i,t} = \left(\alpha \left(\left(\frac{\alpha}{1-\alpha} \frac{w_{M,t}}{w_{F,t}+d_F}\right)^{\vartheta_N} N_{i,M,t} \right)^{\frac{\vartheta_N-1}{\vartheta_N}} + (1-\alpha) N_{i,M,t}^{\frac{\vartheta_N-1}{\vartheta_N}} \right)^{\frac{\vartheta_N}{\vartheta_N-1}}.$$

Then, we can solve for $N_{i,\boldsymbol{M},\boldsymbol{t}}$

$$N_{i,M,t} = \frac{Y_{i,t}}{\left(\alpha \left(\frac{\alpha}{1-\alpha}\frac{w_{M,t}}{w_{F,t}+d_F}\right)^{\vartheta_N-1} + (1-\alpha)\right)^{\frac{\vartheta_N}{\vartheta_N-1}}}.$$

Accordingly, the solution for $N_{i,F,t}$ is

$$N_{i,F,t} = \frac{\left(\frac{\alpha}{1-\alpha}\frac{w_{M,t}}{w_{F,t}+d_F}\right)^{\vartheta_N}Y_{i,t}}{\left(\alpha\left(\frac{\alpha}{1-\alpha}\frac{w_{M,t}}{w_{F,t}+d_F}\right)^{\vartheta_N-1} + (1-\alpha)\right)^{\frac{\vartheta_N}{\vartheta_N-1}}}.$$

Total costs are then given by

$$TC(Y_{i,t}) = \frac{w_{M,t}Y_{i,t} + (w_{F,t} + d_F)\left(\frac{\alpha}{1-\alpha}\frac{w_{M,t}}{w_{F,t}+d_F}\right)^{\vartheta_N}Y_{i,t}}{\left(\alpha\left(\frac{\alpha}{1-\alpha}\frac{w_{M,t}}{w_{F,t}+d_F}\right)^{\vartheta_N-1} + (1-\alpha)\right)^{\frac{\vartheta_N}{\vartheta_N-1}}}.$$

The first derivative with respect to $Y_{i,t}$ yields marginal costs

$$mc_{t} = \frac{w_{M,t} + (w_{F,t} + d_{F}) \left(\frac{\alpha}{1-\alpha} \frac{w_{M,t}}{w_{F,t} + d_{F}}\right)^{\vartheta_{N}}}{\left[\alpha \left(\frac{\alpha}{1-\alpha} \frac{w_{M,t}}{w_{F,t} + d_{F}}\right)^{\vartheta_{N}-1} + (1-\alpha)\right]^{\frac{\vartheta_{N}}{\vartheta_{N}-1}}}.$$
(E.18)

E.3 Statistical Discrimination

While in the case of taste-based discrimination $\alpha=1-\alpha$ and $d_F>0$, the environment with statistical discrimination is characterized by $d_F=0$ but $\alpha<1-\alpha$. Hence, marginal costs in the case of statistical discrimination are given by equation (E.18), setting $d_F=0$:

$$mc_{t} = \frac{w_{M,t} + w_{F,t} \left(\frac{\alpha}{1-\alpha} \frac{w_{M,t}}{w_{F,t}}\right)^{\vartheta_{N}}}{\left[\alpha \left(\frac{\alpha}{1-\alpha} \frac{w_{M,t}}{w_{F,t}}\right)^{\vartheta_{N}-1} + (1-\alpha)\right]^{\frac{\vartheta_{N}}{\vartheta_{N}-1}}}.$$
(E.19)

F Derivation of the Optimal Price

Start from the optimality condition of the firm given by

$$0 = \mathbb{E}_t \left[\sum_{\iota=0}^{\infty} \theta^{\iota} \Lambda^{\iota} \Psi_{t,t+\iota} Y_{i,t+\iota|t} \left(\frac{P_{i,t}}{P_{t+\iota}} - \mu m c_{t+\iota} \right) \right) \right].$$

Note that real marginal costs do not depend on the output level, as shown in Section E. Since all firms behave optimally and due to symmetry, we can define the optimal price as $P_{i,t} \equiv P_t^*$ and $p_t^* \equiv \frac{P_t^*}{P_t}$. Using that $Y_{i,t+\iota|t} = \left(\frac{P_{i,t}}{P_{t+\iota}}\right)^{-\epsilon} Y_{t+\iota}$, we can write

$$0 = \mathbb{E}_t \left[\sum_{\iota=0}^{\infty} \theta^{\iota} \Lambda^{\iota} \Psi_{t,t+\iota} \left(\frac{P_t^*}{P_{t+\iota}} \right)^{-\epsilon} Y_{t+\iota} \left(\frac{P_t^*}{P_{t+\iota}} - \mu m c_{t+\iota} \right) \right].$$

Multiplying with $\frac{P_t}{P_t}$:

$$0 = \mathbb{E}_t \left[\sum_{\iota=0}^{\infty} \theta^{\iota} \Lambda^{\iota} \Psi_{t,t+\iota} \left(\frac{p_t^*}{\Pi_{t+\iota}} \right)^{-\epsilon} Y_{t+\iota} \left(\frac{p_t^*}{\Pi_{t+\iota}} - \mu m c_{t+\iota} \right) \right],$$

$$\Leftrightarrow \mathbb{E}_t \left[\sum_{\iota=0}^{\infty} \theta^{\iota} \Lambda^{\iota} \Psi_{t,t+\iota} Y_{t+\iota} \left(\frac{p_t^*}{\Pi_{t+\iota}} \right)^{1-\epsilon} \right] = \mathbb{E}_t \left[\sum_{\iota=0}^{\infty} \theta^{\iota} \Lambda^{\iota} \Psi_{t,t+\iota} Y_{t+\iota} \mu \left(\frac{p_t^*}{\Pi_{t+\iota}} \right)^{-\epsilon} m c_{t+\iota} \right],$$

$$\Leftrightarrow p_t^* = \frac{\mathbb{E}_t \left[\sum_{\iota=0}^{\infty} \theta^{\iota} \Lambda^{\iota} \Psi_{t,t+\iota} Y_{t+\iota} \mu \Pi_{t+\iota}^{\epsilon} m c_{t+\iota} \right]}{\mathbb{E}_t \left[\sum_{\iota=0}^{\infty} \theta^{\iota} \Lambda^{\iota} \Psi_{t,t+\iota} Y_{t+\iota} \Pi_{t+\iota}^{\epsilon-1} \right]}$$

which gives

$$p_t^* = \mu \frac{x_{1,t}}{x_{2,t}}.$$
 (F.1)

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Contribution

I, Daniel Stempel, have contributed substantially to the conceptualization, development of the methodology, software, formal analysis, writing of the original draft, review and editing, and visualization of the results of this paper.

Prof. Dr. Ulrike Neyer

Paper II

Asymmetric Macroeconomic Effects of QE and Excess Reserves in a Monetary Union^{*}

Maximilian Horst Ulrike Neyer Daniel Stempel

Abstract

Large-scale asset purchases by a central bank (quantitative easing, QE) induce a strong and persistent increase in excess reserves in the banking sector. In the euro area, these excess reserves are heterogeneously distributed across the member states. This paper develops a two-country New Keynesian model—calibrated to represent a high- and a low-liquidity euro area country—to analyze the macroeconomic effects of QE, specifically considering strong and heterogeneous increases in excess reserves and deposits in a monetary union. QE triggers economic activity and increases the union-wide consumer price level after a negative preference shock. However, its efficacy is dampened by a *reverse bank lending channel* that weakens the *interest rate channel* of QE. These dampening effects are higher in the high-liquidity country. Furthermore, we show that a shock in the form of a deposit shift of QE-created deposits between the two countries, interpreted as capital flight, has negative (positive) effects for the economy of the country receiving (losing) the deposits.

JEL classifications: E51, E52, E58, F41, F45

Keywords: Unconventional monetary policy, quantitative easing (QE), monetary policy transmission, excess liquidity, credit lending, heterogeneous monetary union, New Keynesian models

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Contents

List of Tables			
List of Figures			56
1	Intr	roduction	57
2	AN	Note on the Implementation of QE in the Euro Area	59
3	Moe 3.1 3.2 3.3 3.4 3.5 3.6 3.7	del Households	62 66 67 69 70 73 75
4	Mo 4.1 4.2	del Analysis Calibration Dynamic Analysis 4.2.1 Preference Shock 4.2.2 Deposit Shift Shock	76 77 79 79 82
5	Con	nclusion	84
A]	A B C D E F G	dices Expenditure Minimization of the Household A.1 Composition of the Domestic and Foreign Composite Consumption Good A.2 Allocation between Domestic and Foreign Goods Utility Maximization of the Household	86 86 87 89 90 92 92 92 94 97
R	efere	nces	101
Ρı	ıblica	ations and Contribution	101

List of Tables

1	Calibration.	78
2	Steady State in Comparison to Data	79

List of Figures

1	Accumulation of Excess Reserves at Specific National Central Banks	61
2	Impulse Responses to a Symmetric, Negative 1% Preference Shock with Persis-	
	tence $\rho_{z,k} = 0.9.$	81
3	Impulse Responses to a Deposit Shift Shock from Country B to A with Persis-	
	tence $\rho_{\tilde{d},k} = 0.9.$	83

1 Introduction

At times when short-term monetary policy rates approach their effective lower bound, central banks may engage in quantitative easing (QE). In doing so, they buy assets at a large scale to directly lower long-term interest rates to stimulate economic activities. The Eurosystem launched its first QE program in January 2015 to address the risks of too low inflation for a too prolonged period.¹ However, large-scale asset purchases do not only decrease long-term interest rates but also create large amounts of bank reserves, implying that excess reserves in the euro area banking sector increased to unprecedented levels.² Due to the specific QE implementation, these excess reserves are distributed heterogeneously across euro area countries.

Against this background, we analyze the macroeconomic effects of QE in a monetary union within a two-country New Keynesian model, considering explicitly how it is implemented. This includes the analysis of whether the QE-induced large increases in excess reserves and their heterogenous distribution across countries are just a technical feature or whether they have real effects. We find that, by lowering long-term interest rates, QE triggers economic activities, implying that aggregate consumption and investment increase (*interest rate channel* of QE). We distinguish between two different long-term interest rates: the bond rate and the bank loan rate. Crucially, the decrease of the latter is weakened by QE-induced increases in excess reserves and deposits. In particular, these increases imply higher bank balance sheet costs, e.g., in the form of agency or regulatory costs. Consequently, bank lending, and thus the stimulating effects of QE on economic activities, are dampened (*reverse bank lending channel* of QE).³ Hence, we identify two channels of QE, an interest rate channel and a reverse bank lending channel, with the latter weakening the former. Therefore, the QE-induced increases in excess reserves and their heterogeneous distribution are not just a technical feature but

¹The term "Eurosystem" includes the institutions responsible for monetary policy in the euro area, i.e., the European Central Bank (ECB) and all euro area national central banks (NCBs). For simplicity, we use the terms ECB and Eurosystem synonymously in this paper. Note that in January 2015 the interest rate on the ECB's main refinancing operations (MROs) already amounted to .05%, the interest rate on its deposit facility was already negative at -.2%, and the interest rate on the marginal lending facility was at .3% (data source: ECB). For the respective announcement of the QE program, see European Central Bank (2015).

²Excess reserves are here defined as the sum of (i) commercial banks' current account balances at their national central bank in excess of those contributing to minimum reserve requirements, and (ii) deposits held at the ECB's overnight deposit facility. In ECB parlance this quantity is defined as "excess liquidity" since the ECB uses the term "excess reserves" to define the narrower concept of current account balances in excess of reserve requirements. We refer to excess reserves as all central bank overnight deposits beyond required reserves and hence do not distinguish between whether they are held on a current account or at the deposit facility.

³This stands in contrast to Bernanke and Gertler (1995) who introduced a bank lending channel into the literature that *reinforces* the traditional interest rate channel. Therefore, we call it *reverse* bank lending channel.

indeed have real effects. Depending on the way QE is implemented, these channels may affect monetary union members asymmetrically.

In particular, we calibrate our model to represent a high- and a low-liquidity euro area country (Germany and Italy). Thus, in steady state, excess reserves and deposits are already asymmetrically distributed between the two countries. Considering the specific QE implementation in the euro area that reinforces this heterogeneous liquidity distribution, we find that the two channels indeed have asymmetric macroeconomic effects in these countries. We analyze the model responses to two shocks: a preference shock and a deposit shift shock (sudden deposit shift between the two countries). After a symmetric, negative preference shock that implies a decrease in household consumption, the central bank reacts to the shock-induced decrease of union-wide inflation with QE. The long-term interest rates decrease, triggering economic activity and thus an increase union-wide consumer price inflation. However, the QE-induced increase in excess reserves and deposits leads to higher bank balance sheet costs, implying a dampening effect on bank lending. The interest rate channel of QE is therefore dampened by a reverse bank lending channel. These weakening effects are more pronounced in the high-liquidity country.

The deposit shift shock implies that deposits and thus (excess) reserves are moved from the low-liquidity country to the high-liquidity country, which can be interpreted as capital flight ("safe-haven-flows" or "flight-to-quality" phenomena), for instance. This increase in deposits and excess reserves leads to higher balance sheet costs for banks in the high-liquidity country. Consequently, in that country, the deposit shift has a dampening effect on economic activities. Analogously, the low-liquidity country benefits from the deposit shift.

Our paper primarily builds on three strands of literature. First, we contribute to the literature on DSGE models that include a banking sector to analyze the effects of unconventional monetary policy measures, such as QE. Respective examples are Gerali et al. (2010), Cúrdia and Woodford (2011), Gertler and Karadi (2011, 2013), Chen et al. (2012), Brunnermeier and Koby (2018), Kumhof and Wang (2019), and Wu and Zhang (2019a,b). Note that, as in Jakab and Kumhof (2019), Kumhof and Wang (2019), and Mendizábal (2020), we assume that banks create deposits endogenously by granting loans (i.e., banks provide "financing through deposit creation"). Second, our work is related to several papers that develop DSGE models to analyze monetary policy effects in monetary unions such as in Benigno (2004), Beetsma and Jensen (2005), Galí and Monacelli (2005, 2008), Ferrero (2009), Bhattarai et al. (2015), and Saraceno and Tamborini (2020). Third, our work is based on literature investigating the relationship between the implementation of QE and the creation of excess reserves. Examples include Keister and McAndrews (2009), Alvarez et al. (2017), and Baldo et al. (2017).

Our paper contributes to these strands by explicitly considering crucial technical particularities of the QE implementation in a realistically calibrated New Keynesian model of two representative euro area countries. QE is modeled more realistically compared to its presentation in other papers with respect to its aim (reducing long-term interest rates that are the relevant rates for households' consumption and for firms' investment decisions) and with respect to the technical particularities of its implementation (large increases in excess bank reserves that are heterogeneously distributed across monetary union countries). In particular, our approach to modeling QE differs from other New Keynesian models that include QE as the main monetary policy tool of central banks by considering a non-zero, positive bond rate. This depiction is more realistic with respect to the positive long-term (bond) interest rates in the euro area. To the best of our knowledge, our paper is the first to endogenously implement the development of excess reserves accompanying QE and to analyze the macroeconomic effects of this mechanical relationship in a monetary union model.

The remainder of this paper is organized as follows. Section 2 presents some notable fundamentals with regard to the implementation of QE in the euro area. In Section 3, we develop the model and derive the corresponding equilibrium. Section 4 describes the model calibration and derives and analyzes the results with regard to two different shocks. Section 5 concludes.

2 A Note on the Implementation of QE in the Euro Area

The ECB's large-scale asset purchase program (APP), commonly referred to as QE, involves four programs under which both private and public sector securities are purchased.⁴ As a consequence of the implementation of QE, aggregate excess reserves⁵ in the euro area increased from 200 billion euros in March 2015 to a temporary record high of 1.9 trillion euros in De-

⁴The APP consists of the Corporate Sector Purchase Programme (CSPP), the Public Sector Purchase Programme (PSPP), the Asset-Backed Securities Purchase Programme (ABSPP) and the Third Covered Bond Purchase Programme (CBPP3). Covering a share of more than 80% of all assets bought under the APP (until May 2020), the PSPP represents by far the largest component of the APP (European Central Bank, 2020a).

⁵For the definition of excess reserves used in this paper, see footnote 2.

cember 2018.⁶ This value has increased significantly in the aftermath of the introduction of the Pandemic Emergency Purchase Programme (PEPP) that was launched by the ECB as a reaction to the COVID-19 pandemic.⁷ The excess reserves are asymmetrically distributed across euro area countries. Since the beginning of QE, about 30% of overall excess reserves are, for example, held solely in Germany (see Figure 1). Alvarez et al. (2017) and Baldo et al. (2017) show that approximately 80-90% of total excess reserves predominantly accumulate in Germany, the Netherlands, France, Finland, and Luxembourg, whereas such holdings are much less pronounced in Italy, Portugal or Spain, for example.

Note that both an increase in excess reserves as well as a very similar heterogeneous distribution of this liquidity among euro area countries could already be observed during the financial and sovereign debt crisis (see Figure 1). However, compared to the QE period, the reason for the heterogeneous distribution during these periods was different. In particular, capital flight (so-called "safe-haven-flows" and "flight-to-quality" phenomena) from lower-rated to higher-rated euro area countries was the main provoking factor at that time (Baldo et al., 2017).

By implementing QE, each euro area national central bank purchases, inter alia, domestic government bonds according to its share in the ECB's capital key. The asset purchases are funded through the creation of reserves by the Eurosystem, implying that total excess reserves in the banking sector mechanically increase. As a consequence of the QE-induced increases in reserves, the euro area banking sector has been subjected to a structural liquidity surplus since October 2015, i.e., since then the banking sector has held so much reserves that it can cover its structural liquidity needs occurring from minimum reserve requirements and autonomous factors, such as cash withdrawals, without borrowing from the central bank.⁸

There are different reasons for the observed heterogeneous distribution of QE-created bank reserves across euro area countries. By buying assets from the non-banking sector, the Eu-

⁶Note that between March 2015 and December 2018, the average amount of monthly net asset purchases varied between 15 and 80 billion euros. Between January 2019 and October 2019, net asset purchases were stopped. In November 2019, the ECB restarted its net asset purchases at a monthly rate of 20 billion euros. In March 2020, the ECB announced additional net asset purchases of 120 billion euros in combination with the existing APP purchases until the end of 2020 as a reaction to the COVID-19 pandemic (for more detailed information, see European Central Bank (2020a)).

⁷The PEPP is implemented in the same way as the PSPP and can thus technically be viewed as a further expansion of QE. For details with regard to its introduction, its objective and its volumes, see for example European Central Bank (2020c).

⁸For detailed information with respect to the banking sector's liquidity needs and liquidity provision by the Eurosystem during different periods (*normal times, crisis times, times of too low inflation*), see e.g., Horst and Neyer (2019).

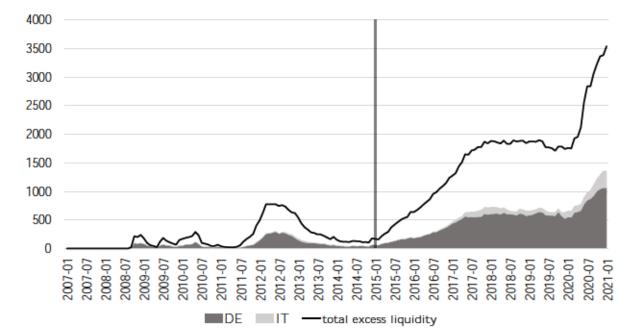


Figure 1: Excess Reserve Holdings of Selected Euro Area National Central Banks in Billion Euros (Maintenance Period Averages, Vertical Line Indicates the Launch of the QE Program). Data Source: Eurosystem.

rosystem does not only create bank reserves but also bank deposits.⁹ The individual creation of bank reserves and deposits in each country depends on the seller-type of the asset and its location. For example, if (i) a national central bank purchases assets from a domestic commercial bank, reserves in the domestic banking sector will increase. If (ii) a national central bank purchases assets from the domestic non-banking-sector (private households and private corporations), reserves and deposits in the domestic banking sector will increase. Lastly, if (iii) a national central bank purchases assets from a counterparty residing outside the respective country, reserves and bank deposits will increase in the banking sector of that euro area country in which the respective counterparty (or its bank) has its current account in order to get access to the TARGET2 system.¹⁰ Case (iii) is the main reason for the QE-induced heterogeneous distribution of reserves and deposits between euro area countries. About 80% of overall central bank asset purchases are bought outside the respective country and about 50% of overall central bank asset purchases are conducted with counterparties residing outside the euro area (see

⁹For a more profound analysis of the creation and distribution of bank reserves and deposits within the implementation of QE in the euro area, see, e.g., Baldo et al. (2017) and Horst and Neyer (2019).

¹⁰TARGET2 (Trans-European Automated Real-time Gross Settlement Express Transfer system) is the realtime gross settlement system owned and operated by the Eurosystem. It settles euro-denominated domestic and cross-border payments in central bank money continuously on an individual transaction-by-transaction basis without netting (European Central Bank, 2020f).

also Baldo et al., 2017). As those counterparties have their current accounts predominantly with commercial banks in only a few selected countries, such as Germany, France, the Netherlands, Luxembourg, and Finland (which serve as so-called financial centers or gateways), the QE-induced creation of excess reserves and deposits takes place in these countries. Thus, the majority of the excess reserves and deposits created through the QE purchases accumulates in only a few countries. This consequence of the technical particularity of the implementation of QE plays an essential role in our model setup.

3 Model

We consider a monetary union consisting of two countries indexed by $k \in \{A, B\}$, where -k denotes the respective other country. The core model framework of each country partly resembles the setup of the closed economy modeled by Gertler and Karadi (2011, 2013). In each country, there are five types of agents: households, intermediate goods firms, capital producing firms, retail firms, and banks. In both countries, each type forms a continuum of identical agents of measure unity, allowing us to consider representative agents. We denote the respective representative agent in each country by agent k. In addition, there is a union-wide central bank. Banks in each country face such large amounts of excess reserves that fulfilling reserve requirements is not a binding constraint.¹¹ In order to capture the heterogeneous distribution of this liquidity in the euro area as outlined in Section 2, we specify country A as being a high-liquidity and country B as a low-liquidity country. The model contains a nominal rigidity in the form of price stickiness as well as real rigidities in the form of consumer habit formation and capital adjustment costs. In the following, we characterize the basic ingredients of the model.

3.1 Households

The infinitely lived household k consumes, saves, and supplies labor to intermediate goods firms. Household k seeks to maximize its expected discounted lifetime utility. Its objective

¹¹Other potential liquidity requirements, such as a liquidity coverage ratio for instance, play no role in our model. Banks face such a high liquidity surplus that those requirements are not a binding constraint when granting loans.

function is

$$\max \mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[Z_\tau ln \left(C_\tau^k - \Psi_k C_{\tau-1}^k \right) - \frac{\chi_k}{1 + \varphi_k} (N_\tau^k)^{1+\varphi_k} \right] \right], \tag{1}$$

where the household draws period-t utility from consumption $C_t^k - \Psi_k C_{t-1}^k$ and period-t disutility from work N_t^k , where N_t^k denotes the number of hours worked. The variable Z_t is a preference shock¹² following an AR(1) process. The parameter Ψ_k is a habit parameter capturing consumption dynamics, χ_k determines the weight of labor disutility, and φ_k captures the inverse Frisch elasticity of labor supply.

Household k's total consumption C_t^k consists of the consumption of final goods produced in its home country $C_{k,t}^k$ and of those produced in the foreign country $C_{-k,t}^k$. Henceforth, we denote domestically produced goods as domestic goods and those produced abroad as foreign goods. The parameter σ_k can be interpreted as the share of foreign goods and $(1-\sigma_k)$ as the share of domestic goods in the household's total consumption. The respective consumption index is given by

$$C_t^k \equiv \frac{\left(C_{k,t}^k\right)^{1-\sigma_k} \left(C_{-k,t}^k\right)^{\sigma_k}}{(1-\sigma_k)^{1-\sigma_k} (\sigma_k)^{\sigma_k}},\tag{2}$$

where $C_{k,t}^k$ and $C_{-k,t}^k$ are composite goods defined by the indices

$$C_{k,t}^{k} \equiv \left(\int_{0}^{1} C_{k,t}^{k}(j)^{\frac{\epsilon_{k}-1}{\epsilon_{k}}} dj\right)^{\frac{\epsilon_{k}}{\epsilon_{k}-1}} , \qquad (3)$$

and

$$C_{-k,t}^{k} \equiv \left(\int_{0}^{1} C_{-k,t}^{k}(j)^{\frac{\epsilon_{k}-1}{\epsilon_{k}}} dj\right)^{\frac{\epsilon_{k}}{\epsilon_{k}-1}},\tag{4}$$

with $C_{k,t}^k(j)$ denoting the quantity of the domestic good j and $C_{-k,t}^k(j)$ denoting the quantity of the foreign good j consumed by household k in period t. The parameter ϵ_k represents the elasticity of substitution between differentiated goods (produced in the same country). The household's budget constraint is given by

$$\int_{0}^{1} P_{k,t}(j) C_{k,t}^{k}(j) dj + \int_{0}^{1} P_{-k,t}(j) C_{-k,t}^{k}(j) dj + B_{t}^{k} = (1+i_{t-1}) B_{t-1}^{k} + W_{k,t} N_{t}^{k} + \Upsilon_{t}^{k} .$$
(5)

 $^{^{12}}$ Other works specifying preference shocks in this fashion include Ireland (2004), Dennis (2005), and Bekaert et al. (2010).

The left-hand side (LHS) of equation (5) describes the household's nominal expenses. They include its consumption spending in countries k and -k as well as its savings in nominally riskfree bonds. The price $P_{k,t}(j)$ is the price for product j produced in country k, and $P_{-k,t}(j)$ is the price for product j produced in country -k. B_t^k represents the quantity of one-period, nominally risk-free bonds purchased in period t and maturing in t+1. Bonds purchased in period t-1 pay a long-term rate of interest, i.e., the bond rate i_{t-1} in period t. The righthand side (RHS) of equation (5) thus shows household k's nominal income. It includes its gross return on bonds, its wage earnings (with $W_{k,t}$ being the nominal wage), and exogenous (net) income Υ_t^k from the ownership of firms and banks. The budget constraint reveals that household k is connected with country -k via the consumption of goods produced in country -k and the shared bond market. Labor markets and equity incomes are separated between the two countries.

Household k faces five optimization problems: (i) the optimal composition of its domestic composite consumption good, (ii) the optimal composition of its foreign composite consumption good, (iii) the optimal allocation of its overall consumption between domestic and foreign goods, (iv) its optimal labor supply, and (v) the optimal intertemporal allocation of consumption.

Starting with the optimal composition of the domestic consumption good, household k seeks to minimize its expenditures $\int_0^1 P_{k,t}(j)C_{k,t}^k(j)dj$ for any given level of the consumption index given by equation (3). Solving this optimization problem, the household's optimal consumption of the domestic good j becomes

$$C_{k,t}^k(j) = \left(\frac{P_{k,t}(j)}{P_{k,t}}\right)^{-\epsilon_k} C_{k,t}^k , \qquad (6)$$

where $P_{k,t} \equiv \left(\int_0^1 P_{k,t}(j)^{1-\epsilon_k} dj\right)^{\frac{1}{1-\epsilon_k}}$ is a price index of the goods produced in country k. Analogously, we obtain for its optimal consumption of the foreign good j

$$C_{-k,t}^{k}(j) = \left(\frac{P_{-k,t}(j)}{P_{-k,t}}\right)^{-\epsilon_{k}} C_{-k,t}^{k} , \qquad (7)$$

where $P_{-k,t} \equiv \left(\int_0^1 P_{-k,t}(j)^{1-\epsilon_{-k}} dj\right)^{\frac{1}{1-\epsilon_{-k}}}$ is a price index for foreign goods.

In the same vein, we derive household k's optimal allocation of its overall consumption between domestic and foreign goods. The household seeks to minimize its expenditures $P_{k,t}C_{k,t}^k + P_{-k,t}C_{-k,t}^k$ for any given level of the consumption index given by equation (2). Solving this optimization problem, the optimal consumption of domestic and foreign goods are

$$C_{k,t}^{k} = (1 - \sigma_{k}) \left(\frac{P_{k,t}}{P_{k,t}^{C}}\right)^{-1} C_{t}^{k} , \qquad (8)$$

and

$$C_{-k,t}^{k} = \sigma_k \left(\frac{P_{-k,t}}{P_{k,t}^C}\right)^{-1} C_t^k , \qquad (9)$$

where $P_{k,t}^C \equiv P_{k,t}^{1-\sigma_k} P_{-k,t}^{\sigma_k}$ is the consumer price index in country k. Thus,

$$P_{k,t}C_{k,t}^{k} + P_{-k,t}C_{-k,t}^{k} = (1 - \sigma_{k})P_{k,t}^{C}C_{t}^{k} + \sigma_{k}P_{k,t}^{C}C_{t}^{k} = P_{k,t}^{C}C_{t}^{k},$$

and the budget constraint (5) becomes

$$P_{k,t}^C C_t^k + B_t^k = (1 + i_{t-1}) B_{t-1}^k + W_{k,t} N_t^k + \Upsilon_t^k .$$
(10)

In order to obtain the household's optimal labor supply and its optimal intertemporal consumption, we maximize equation (1) with respect to N_t^k , C_t^k , and B_t^k subject to equation (10). Denoting the marginal utility of consumption by

$$U_{c,t}^{k} \equiv \left(\frac{Z_{t}}{C_{t}^{k} - \Psi_{k}C_{t-1}^{k}} - \frac{\mathbb{E}_{t}\left[Z_{t+1}\right]\Psi_{k}\beta}{\mathbb{E}_{t}\left[C_{t+1}^{k}\right] - \Psi_{k}C_{t}^{k}}\right),$$

and solving the optimization problem yields the following standard first-order conditions (FOCs):

$$\chi_k(N_t^k)^{\varphi_k} = w_{k,t} U_{c,t}^k \,, \tag{11}$$

$$\beta(1+i_t) \mathbb{E}_t \left[\frac{P_{k,t}^C}{P_{k,t+1}^C} \right] \Lambda_{t,t+1}^k = 1 , \qquad (12)$$

with

$$\Lambda_{t,t+1}^{k} \equiv \mathbb{E}_{t} \left[\frac{U_{c,t+1}^{k}}{U_{c,t}^{k}} \right] \,. \tag{13}$$

Equation (11) shows that optimal labor supply requires the marginal disutility of working (LHS) to be equal to the marginal utility of working (RHS). The latter results from the additional possible consumption which is determined by the real wage $w_{k,t} \equiv W_{k,t}/P_{k,t}^C$. Equation

(12) represents the Euler equation governing optimal intertemporal consumption.

Finally, we rewrite some identities in terms of relative prices. Defining the terms of trade of country k with country -k as $V_{-k,t}^k \equiv P_{-k,t}/P_{k,t}$, we get that

$$P_{k,t}^{C} = P_{k,t}^{1-\sigma_{k}} \left(V_{-k,t}^{k} P_{k,t} \right)^{\sigma_{k}} = P_{k,t} \left(V_{-k,t}^{k} \right)^{\sigma_{k}} , \qquad (14)$$

and

$$\Pi_{k,t}^{C} = \Pi_{k,t} \left(\frac{V_{-k,t}^{k}}{V_{-k,t-1}^{k}} \right)^{\sigma_{k}} , \qquad (15)$$

where $\Pi_{k,t}^C$ denotes consumer price inflation and $\Pi_{k,t}$ the inflation of domestic prices in country k. Due to our assumption of complete bond markets, we can obtain the following risk sharing condition using equations (12) and (13):

$$U_{c,t}^{k} = \vartheta_{k} (V_{-k,t}^{k})^{(\sigma_{k}-1)} (V_{k,t}^{-k})^{(-\sigma_{-k})} U_{c,t}^{-k} , \qquad (16)$$

where $\vartheta_k \equiv U_{c,ss}^k/U_{c,ss}^{-k}$ with $U_{c,ss}$ being the zero inflation steady state value of marginal utility of consumption. This condition implies that, adjusted for relative prices, marginal utilities of consumption of the households k and -k co-move proportionally over time.

3.2 Intermediate Goods Firms

Competitive intermediate goods firms produce goods that are solely sold to domestic retail firms. At time t, the output of a representative intermediate goods firm $Y_{m,t}^k$ is produced with capital $K_{t-1,t}^k$ and labor N_t^k . The respective production function is given by

$$Y_{m,t}^k = \left(K_{t-1,t}^k\right)^{\alpha_k} \left(N_t^k\right)^{1-\alpha_k} .$$
(17)

Intermediate goods firm k buys the capital that is productive in t from the capital producing firm in t-1, i.e., $K_{t-1,t}^k$ is the capital stock chosen and bought at real price $Q_{k,t-1}$ in period t-1and productive in t. At the end of period t, the intermediate goods firm sells the depreciated capital back to the capital producer at price $(Q_{k,t}-\delta_k)$, i.e., in t-1 they conclude a kind of repurchase agreement. The parameter δ_k is defined as the real depreciation rate.

So far, the setup closely resembles the modeling of intermediate goods firms by Gertler and Karadi (2011). However, with respect to the financing of their expenditures, we assume the following: at the end of period t, the intermediate goods firm borrows $L_{t,t+1}^k = Q_{k,t} K_{t,t+1}^k$ from bank k to buy the capital stock that is productive in t+1. The bank credits the respective amount as deposits, $L_{t,t+1}^k = D_{t,t+1}^{L,k}$, on the intermediate goods firm's bank account, i.e., as in Kumhof and Wang (2019), loans create deposits.¹³ The corresponding objective function of intermediate goods firm k is given by

$$\max \Gamma_{m,t}^{k} = mc_{k,m,t}Y_{m,t}^{k} - w_{k,t}N_{t}^{k} - \left(1 + i_{k,t-1}^{L}\right)Q_{k,t-1}K_{t-1,t}^{k} + (Q_{k,t} - \delta_{k})K_{t-1,t}^{k} .$$
(18)

Equation (18) reveals that in period t, the firm has to take into account four factors determining real profits: (i) revenues defined as the product of real marginal costs and output,¹⁴ (ii) costs of labor, (iii) interest and principal payments on the loan agreed on in period t-1, and (iv) the payoff from reselling depreciated capital to the capital producer. Solving (18) with respect to $K_{t,t+1}^k$ and N_t^k gives the following FOCs:

$$(1+i_{k,t}^{L}) Q_{k,t} = \alpha_k \mathbb{E}_t \left[mc_{k,m,t+1} \frac{Y_{m,t+1}^k}{K_{t,t+1}^k} \right] + (\mathbb{E}_t \left[Q_{k,t+1} \right] - \delta_k) , \qquad (19)$$

$$mc_{k,m,t} = \frac{w_{k,t}}{(1 - \alpha_k) \frac{Y_{m,t}^k}{N_t^k}} \,.$$
(20)

The LHS of equation (19) denotes the real marginal cost of capital in the form of credit and acquisition costs. The RHS describes the real marginal benefit of capital in the form of production revenues and the payoff from the repurchase agreement. Equation (20) shows that the real marginal costs of the intermediate goods firm in period t solely depend on the real costs of labor (i.e., the real wage), since any additional unit of output in t has to be produced using only labor input due to the lagged decision on capital input.

3.3 Capital Producing Firms

At the end of period t, the representative competitive capital producing firm k buys depreciated capital from intermediate goods firms and repairs it. Then, as in Gertler and Karadi (2011), it

 $^{^{13}}$ See Section 3.5 for details.

¹⁴Due to perfect competition, intermediate goods firms sell their products at nominal marginal costs.

sells the refurbished capital and the newly produced capital, to the intermediate goods firm.¹⁵

Therefore, gross capital produced in period t, $I_t^{gr,k}$, consists of newly created capital (net investment) I_t^k , and the refurbishment of the bought capital $\delta_k K_{t-1,t}^k$:

$$I_t^{gr,k} = I_t^k + \delta_k K_{t-1,t}^k \,. \tag{21}$$

The law of motion for capital is thus given by

$$K_{t,t+1}^k = K_{t-1,t}^k + I_t^k . (22)$$

As in Gertler and Karadi (2011), we assume that production costs per unit capital are 1 and consider capital adjustment costs (CAC) for newly produced capital. Then, the real period profit of a capital producing firm is given by

$$\Gamma_{c,t}^{k} = Q_{k,t}K_{t,t+1}^{k} - (Q_{k,t} - \delta_k)K_{t-1,t}^{k} - \delta_k K_{t-1,t}^{k} - I_t^{k} - f\left(\frac{I_t^{k} + I_{ss}}{I_{t-1}^{k} + I_{ss}}\right) \left(I_t^{k} + I_{ss}\right) , \quad (23)$$

with

$$f\left(\frac{I_t^k + I_{ss}}{I_{t-1}^k + I_{ss}}\right) = \frac{n_k}{2} \left(\frac{I_t^k + I_{ss}}{I_{t-1}^k + I_{ss}} - 1\right)^2 , \qquad (24)$$

where n_k captures the degree of capital adjustment costs and I_{ss} is steady state gross investment.¹⁶ Equation (23) shows that the real period profit is the result of: (i) the return from selling capital, (ii) the costs of buying the depreciated old capital, (iii) the costs of repairing the old capital, (iv) the costs of producing the new capital, and (v) CAC (only for new capital). Considering equations (22), (23), and (24), the objective function of the capital producing firm becomes

$$\max \mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \Lambda_{t,\tau}^k \left((Q_{k,\tau} - 1) I_{\tau}^k - \frac{n_k}{2} \left(\frac{I_{\tau}^k + I_{ss}}{I_{\tau-1}^k + I_{ss}} - 1 \right)^2 \left(I_{\tau}^k + I_{ss} \right) \right) \right] .$$
(25)

¹⁵The intermediate goods firm uses the loan-created deposits $D_{t,t+1}^{L,k}$ to pay for this capital. The capital producing firm sells these deposits at price 1 to the household in order to being able to invest. For the sake of simplicity, we neglect the general means of payment function of deposits (except for capital purchases) and focus on the bank deposit creation of bank loans (see Section 3.5).

 $^{^{16}}I_{ss}$ is included because in the zero inflation steady state net investment has to be zero since the capital stock is constant over time. This would imply a division by zero if I_{ss} were excluded.

The capital producer chooses net investment I_t^k to solve equation (25). The respective FOC is

$$Q_{k,t} = 1 + \frac{n_k}{2} \left(\frac{I_t^k + I_{ss}}{I_{t-1}^k + I_{ss}} - 1 \right)^2 + \frac{I_t^k + I_{ss}}{I_{t-1}^k + I_{ss}} n_k \left(\frac{I_t^k + I_{ss}}{I_{t-1}^k + I_{ss}} - 1 \right) \\ - \mathbb{E}_t \left[\beta \Lambda_{t,t+1}^k \left(\frac{I_{t+1}^k + I_{ss}}{I_t^k + I_{ss}} \right)^2 n_k \left(\frac{I_{t+1}^k + I_{ss}}{I_t^k + I_{ss}} - 1 \right) \right] . \quad (26)$$

The LHS shows real marginal revenues of net investment, the RHS the corresponding real marginal costs consisting of production costs as well as current and expected CAC.

3.4 Retail Firms

The representative retail firm k produces differentiated final output by aggregating intermediate goods. One unit of intermediate output is needed to produce one unit of final output. Consequently, the marginal costs of final goods firms correspond to the price of the intermediate good. Retail firm k faces demand from households in both countries. Price setting is assumed to be staggered, following Calvo (1983). Firm j chooses its price $P_{k,t}(j)$ to maximize discounted expected real profits given by

$$\max \mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \theta_k^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^k \left(\frac{P_{k,t}(j)}{P_{k,\tau}^C} Y_{\tau|t}^k(j) - TC(Y_{\tau|t}^k(j)) \right) \right] , \qquad (27)$$

subject to

$$Y_{\tau|t}^{k}(j) = \left(\frac{P_{k,t}(j)}{P_{k,\tau}}\right)^{-\epsilon_{k}} Y_{\tau}^{k} , \qquad (28)$$

where θ_k is the probability of a single producer being unable to adjust the price in a certain period. Furthermore, $\beta^{\tau-t} \Lambda_{t,\tau}^k$ denotes the stochastic discount factor, $Y_{\tau|t}^k(j)$ the output in period τ for a firm that last reset its price in t, and $TC(\cdot)$ is the real total cost function. The respective demand function, given by equation (28), depends on the relative price of the good, the heterogeneity of the goods (captured by the elasticity of substitution ϵ_k), and total aggregate demand for goods produced in country k. The FOC of the maximization problem given by equation (27) and (28) is

$$0 = \mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \theta_k^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^k Y_{\tau|t}^k(j) \left(\frac{P_{k,t}^*(j)}{P_{k,\tau}^C} - \frac{\epsilon_k}{\epsilon_k - 1} mc(Y_{\tau|t}^k(j)) \right) \right],$$
(29)

where the real marginal cost function is given by $mc(Y_{\tau|t}^k(j)) = mc_{k,m,\tau}$, and $P_{k,t}^*(j)$ is the optimal price of firm j. Since all firms that are able to reset their price choose the same one, we can drop the index j, and get

$$\frac{P_{k,t}^*}{P_{k,t}} = \frac{\epsilon_k}{\epsilon_k - 1} \frac{x_{k,1,t}}{x_{k,2,t}} , \qquad (30)$$

where

$$x_{k,1,t} \equiv U_{c,t}^k Y_t^k m c_{k,m,t} + \beta \theta_k \mathbb{E}_t \left[\Pi_{k,t,t+1}^{\epsilon_k} x_{k,1,t+1} \right] ,$$
$$x_{k,2,t} \equiv U_{c,t}^k Y_t^k \left(V_{-k,t}^k \right)^{-\sigma_k} + \beta \theta_k \mathbb{E}_t \left[\Pi_{k,t,t+1}^{\epsilon_k-1} x_{k,2,t+1} \right] .$$

Obviously, if all retail firms were able to reset their price in every period ($\theta_k=0$), they would set their optimal price as a markup over nominal marginal costs, i.e., $P_{k,t}^* = \epsilon_k/(\epsilon_k - 1)mc_{k,m,t}P_{k,t}^C$.

The overall domestic price level in country k at time t is given by

$$P_{k,t}^{1-\epsilon_k} = (1-\theta_k) (P_{k,t}^*)^{1-\epsilon_k} + \theta_k (P_{k,t-1})^{1-\epsilon_k} ,$$

i.e., a weighted average of the optimal price of the firms that can re-optimize in period t and the price level of period t-1.

3.5 Banks

Competitive bank k's assets in period t consist of one-period real loans granted at the end of period t-1, $L_{t-1,t}^k$, and real reserves R_t^k , its liabilities of real deposits D_t^k , so that its balance sheet constraint is given by

$$L_{t-1,t}^k + R_t^k = D_t^k . (31)$$

The total amount of reserves R_t^k is splitted into required reserves $R_t^{RR,k}$ and excess reserves $R_t^{ER,k}$, i.e.,

$$R_t^k = R_t^{RR,k} + R_t^{ER,k} . (32)$$

Required reserves are computed as a certain proportion r of the bank's deposits D_t^k . The required reserve ratio r is determined by the central bank. The total amount of bank k's

deposits is given by

$$D_{t}^{k} = D_{t-1,t}^{L,k} + \underbrace{\tilde{D}_{k,t} \cdot D_{k,t}^{QE}}_{:=D_{t-t}^{ex}},$$
(33)

where $D_{t-1,t}^{L,k}$ represents the amount of deposits created through credit lending and $D_{k,t}^{ex} > 0 \forall t$ denotes the amount of deposits created exogenously (from the bank's point of view) through the central bank's large scale asset purchases (QE), i.e., $D_{k,t}^{QE}$, and a potential deposit shift shock $\tilde{D}_{k,t}$. Therefore, we refer to the deposits $D_{k,t}^{ex}$ simply as exogenous deposits. In the following, we will comment on $D_{t-1,t}^{L,k}$ and $D_{k,t}^{ex}$ in more detail.

With respect to $D_{t-1,t}^{L,k}$, we assume that bank k funds only one type of activity, namely the capital goods purchases of the intermediate goods firm k. As in Kumhof and Wang (2019), the intermediate goods firm relies on bank loans to finance its capital purchases. In period t-1, bank k grants the respective loan to the intermediate goods firm. One unit of granted loans creates one unit of deposits ("financing through deposit creation"), i.e., $L_{t-1,t}^k = D_{t-1,t}^{L,k}$.¹⁷ We assume that loan-created deposits $D_{t-1,t}^{L,k}$ are credited on the intermediate goods firm k's deposit account. The intermediate goods firm transfers the newly created deposits immediately to the capital producing firm to pay for the capital good. In period t, the intermediate goods firm interest rate, i.e., the bank loan rate. The respective deposits, that are remunerated at i_{t-1} , mature. The loan $L_{t-1,t}^k$ and the deposits created through bank lending $D_{t-1,t}^{L,k}$ are extinguished.

As described in detail in Section 3.6, the bank is exposed to deposits created by the central bank's QE. Therefore, $D_{k,t}^{QE}$ evolves exogenously from the point of view of the bank. Besides the central bank's QE, a deposit shift shock $\tilde{D}_{k,t}$ may influence the bank's exogenous deposits $D_{k,t}^{ex}$. The deposit shift shock $\tilde{D}_{k,t}$ captures a shift of QE-created deposits from country k to -k which can be the result of capital flight ("safe-haven-flows" or "flight-to-quality" phenomena), for instance. In particular, $\tilde{D}_{k,t}$ depicts an AR(1) shock process—which is independent of the

¹⁷There exist two commonly known theories that describe the technical relationship between deposits and loans. In contrast to the theory of "financing through deposit creation", bank loans in the theory of "intermediation of loanable funds" reflect the intermediation of savings (or loanable funds) between non-bank savers and non-bank borrowers: banks collect deposits from one agent and lend those savings to another agent, i.e., deposits come before loans. However, our model builds on the theory of "financing through deposit creation". Banks' key function is the provision of financing through loans for a single agent that is both borrower and, at least temporarily, depositor. Banks create new deposits when granting loans. A survey of both theories can be found, for example, in Jakab and Kumhof (2019).

central bank's monetary policy—given by

$$ln\left(\tilde{D}_{A,t}\right) = \rho_{\tilde{d},A}ln\left(\tilde{D}_{A,t-1}\right) + \epsilon_{\tilde{d},t},$$
$$ln\left(\tilde{D}_{B,t}\right) = \rho_{\tilde{d},B}ln\left(\tilde{D}_{B,t-1}\right) - \frac{D_{A,ss}^{QE}}{D_{B,ss}^{QE}}\epsilon_{\tilde{d},t},$$

where $\rho_{\tilde{d},k}$ depicts the shock persistence and $\epsilon_{\tilde{d},k}$ denotes a standard normally-distributed shock. This specification ensures a one-to-one shift of QE-created deposits from country B (low-liquidity country) to country A (high-liquidity country). Consider the case that, in steady state, the deposits are equally divided between both countries. In this case $\frac{D_{A,ss}^{QE}}{D_{B,ss}^{QE}}=1$ and a 1% decrease of exogenous deposits in B leads to a 1% increase in A. However, if deposits are heterogeneously distributed between the countries, a $\frac{D_{A,ss}^{QE}}{D_{B,ss}^{QE}}\%$ decrease in $D_{B,t}^{ex}$ implies a 1% increase in $D_{A,t}^{ex}$.

In each period, each bank faces such a high liquidity surplus that fulfilling minimum reserve requirements is not a binding constraint when granting loans. Considering a one-to-one increase in QE-created deposits and reserves implies that bank k's excess reserves are given by

$$R_t^{ER,k} = D_{k,t}^{ex} - r \left(D_{k,t}^{ex} + D_{t-1,t}^{L,k} \right) , \qquad (34)$$

i.e., they correspond to the net amount of cumulated reserves created through central bank's asset purchases and/or a deposit shift shock, $D_{k,t}^{ex}$, and required minimum reserve holdings $r\left(D_{k,t}^{ex} + D_{t-1,t}^{L,k}\right)$.¹⁸

Bank loans are remunerated at the rate $i_{k,t-1}^L$, required reserves at the rate i^{RR} , and excess reserves at the rate i^{ER} , with $i^{RR} > i^{ER}$.¹⁹ The rates i^{RR} and i^{ER} are determined by the central bank. Both bonds and bank deposits are assumed to be risk-free assets, so that they are remunerated at the same rate i_{t-1} . Thus, $i_{t-1}D_t^k$ constitutes the bank's interest costs on all deposits. A key feature of our model is that the bank faces increasing marginal balance sheet costs, i.e., costs increasing disproportionately in the size of its balance sheet, given in real terms by $\frac{1}{2}v_k \left(\mathbb{E}_t[D_{t+1}^k]\right)^2$. This captures the idea of existing agency and/or regulatory

¹⁸A detailed explanation of the one-to-one increase in QE-created deposits and reserves is given in Sections 2 and 3.6.

¹⁹Note that with regard to the euro area, i^{RR} corresponds to the ECB's main refinancing rate and i^{ER} to the rate on the ECB's overnight deposit facility.

 $costs.^{20}$

In period t, bank k seeks to maximize its real expected period-(t+1) profit $\Gamma_{b,t,t+1}^k$. The bank's objective function is thus given by

$$\max \mathbb{E}_t[\Gamma_{b,t+1}^k] = i_{k,t}^L L_{t,t+1}^k + i^{RR} r \mathbb{E}_t[D_{t+1}^k] + i^{ER} \mathbb{E}_t[R_{t+1}^{ER,k}] - i_t \mathbb{E}_t[D_{t+1}^k] - \frac{1}{2} v_k \left(\mathbb{E}_t[D_{t+1}^k]\right)^2.$$

Taking all rates as given, the bank decides on its optimal loan supply to maximize this profit. Solving this optimization problem with respect to $L_{t,t+1}^k$ yields the first order condition

$$i_{k,t}^{L} + r(i^{RR} - i^{ER}) = i_t + \upsilon_k \left(\mathbb{E}_t[D_{k,t+1}^{ex}] + L_{t,t+1}^k \right) \,. \tag{35}$$

The LHS of (35) represents the bank's real marginal revenues and the RHS its real marginal costs of granting loans. Note that granting more loans does not only imply more direct interest revenues (first term) but also more indirect interest revenues (second term). The latter is the consequence of a beneficial reserve shifting: granting loans implies the creation of deposits. These deposits are subject to reserve requirements so that part of a bank's (costly) excess reserve holdings are shifted to the higher remunerated required reserve holdings.²¹ Crucially, bank costs are affected by the central bank's net asset purchases in two (opposing) ways: through interest costs i_t and through balance sheet costs $v_k \mathbb{E}_t[D_{k,t+1}^{ex}]$.

3.6 Central Bank

Monetary policy is conducted at the union level. We conceptualize the conduct of monetary policy by the central bank to closely follow the monetary policy operations of the ECB. Conventionally, the ECB implements monetary policy by setting its short-term interest rates.²² However, when these short-term interest rates reach their effective lower bound, the ECB switches to unconventional monetary policy instruments, such as QE, to directly lower long-term interest rates (resulting in a flattening yield curve), i.e., the interest rates that are relevant

²⁰Models explicitly considering balance sheet costs can, for example, also be found in Martin et al. (2013, 2016), Ennis (2018), Kumhof and Wang (2019), and Williamson (2019).

²¹With regard to the euro area, since June 2014 excess reserves have been remunerated at a negative rate, currently (February 2021) at -.5%. Neglecting the "two-tier system", this interest rate has to be paid independently of whether the liquidity is held in the ECB's overnight deposit facility or on current accounts with the Eurosystem (European Central Bank, 2019).

²²The ECB's short-term interest rates consist of (i) the rate on its one-week main refinancing operations with commercial banks, (ii) the rate on its overnight deposit facility, and (iii) the rate on its overnight marginal lending facility.

for households' consumption and firms' investment decisions (European Central Bank, 2015). However, QE does not only decrease long-term interest rates, but parallely also increases bank reserves. Furthermore, a large part of the Eurosystem's asset purchases are conducted with counterparties residing outside the euro area, which leads to a one-to-one increase in bank deposits and reserves of the banking sector in that country, where the respective counterparties have their deposit accounts with.²³

In our model, the central bank has already encountered the lower bound on short-term monetary policy rates, so that QE has become its main monetary policy tool. We do not explicitly model the asset purchases but consider the resulting increase in bank reserves. Furthermore, we consider that an increase in reserves implies a one-to-one increase in bank deposits:

$$\mathrm{d}R_t^k = \mathrm{d}D_{k,t}^{QE} \,. \tag{36}$$

This allows us to depict the monetary policy instrument QE by an increase in bank deposits $D_{k,t}^{QE}$, and to model a central bank reaction function, a kind of Taylor rule, given by

$$D_{k,t}^{QE} = \Omega_k - \iota_k \left(1 + \ln\left(\frac{1}{\beta}\right) \right) - \iota_k \phi_\pi \left(\gamma_{k,t} \pi_{k,t}^C + \gamma_{-k,t} \pi_{-k,t}^C \right) \,. \tag{37}$$

Equation (37) reveals that the central bank reacts to the weighted average of country-specific consumer price inflation rates, given by $(\gamma_{k,t}\pi_{k,t}^C+\gamma_{-k,t}\pi_{-k,t}^C)$, where $\pi_{k,t}^C \equiv ln(\Pi_{k,t}^C)$ and $\gamma_{k,t} = C_t^k / (C_t^k + C_t^{-k})$. The weights on the country-specific rates express the overall consumption level of the respective country in relation to the aggregate union consumption level. This reflects how consumer price inflation, which is relevant for the ECB's inflation target, is measured in the euro area.²⁴ Equation (37) shows that if the central bank observes a decrease in the average of consumer price inflation, it conducts QE, i.e., bank deposits $D_{k,t}^{QE}$ increase. How strongly these deposits country k increase in is determined by the parameters ι_k and ϕ_{π} . The latter represents the standard reaction coefficient of the central bank to inflation in Taylor rules. The former, a country-specific parameter, allows us to depict the country-specific QE-induced increases in bank deposits and excess liquidity (reserves) across euro area countries. The parameter Ω_k is a country-specific calibrated parameter to match the share of

 $^{^{23}}$ See Section 2 for the institutional details.

²⁴See European Central Bank (2020b) for detailed information.

QE-created deposits in the length of the bank's balance sheet.²⁵

A central bank's large asset purchases lower the longer-term interest rate. We account for this effect by modeling a negative relationship between i_t and $D_{k,t}^{QE}$:

$$1 + i_t = \frac{\Omega_k - D_{k,t}^{QE}}{\iota_k} \,. \tag{38}$$

Therefore, our model considers the simultaneous QE-induced decrease in long-term interest rates and the increase in bank reserves and bank deposits, respectively. Note that the negative relationship between i_t and $D_{k,t}^{QE}$ is a technical depiction to simplify matters. The increase in $D_{k,t}^{QE}$ and the decrease in i_t are both consequences of the implementation of QE. In reality, they occur independently of each other. New Keynesian models using QE as the central bank's monetary policy tool usually set $i_t=0$ to illustrate that the central bank has reached the lower bound on short-term interest rates. However, since i_t is the relevant interest rate for households' consumption and firms' investment decisions, it has rather a long-term characteristic and we assume that this rate is still above the lower bound, as it has actually been the case in the euro area.

In our model, all banks have a high stock of excess reserves and thus of QE-created deposits in steady state. This can be interpreted as a result of past central bank asset purchases. This allows us to also consider contractionary monetary policy. The central bank conducts monetary policy via its *net* asset purchases. If the central bank buys more assets than mature, i.e., if its net asset purchases are positive, it will conduct expansionary monetary policy. If the central bank's net asset purchases are negative, monetary policy will be contractionary (quantitative tightening). Besides conducting QE, the central bank sets the nominal interest rates on commercial banks' required and excess reserves holdings r^{RR} and r^{ER} , respectively, and determines the ratio for banks' required reserve holdings r.

3.7 Equilibrium

In order to close the model, we continue by stating the market clearing conditions. Bond market clearing implies

$$B_t^k = -B_t^{-k} , (39)$$

²⁵For more detailed information with regard to the calibrated parameters ι_k and Ω_k , see Section 4.1.

i.e., bonds are in zero net supply. Final goods are consumed by households in the union and used to adjust capital:²⁶

$$Y_t^k = C_{k,t}^k + C_{k,t}^{-k} + I_t^{gr,k} + \frac{n_k}{2} \left(\frac{I_t^k + I_{ss}}{I_{t-1}^k + I_{ss}} - 1 \right)^2 \left(I_t^k + I_{ss} \right) .$$

$$\tag{40}$$

Furthermore, all goods sold by retail firms have to be produced by intermediate goods firms, i.e.,

$$Y_{m,t}^k = Y_t^k . (41)$$

Note that the standard condition for labor market clearing with sticky prices given by

$$\left(\frac{Y_t^k}{K_{t-1,t}^{k-\alpha_k}}\right)^{\frac{1}{1-\alpha_k}} \Delta_t^k = N_t^k , \qquad (42)$$

where $\Delta_t^k \equiv \int_0^1 \left(\frac{P_{k,t}(j)}{P_{k,t}}\right)^{-\frac{\epsilon_k}{1-\alpha_k}} dj$, holds. Moreover, the market for loans clears

$$L_{t,t+1}^{k} = Q_{k,t} K_{t,t+1}^{k} . (43)$$

Lastly, the real interest rate is defined in terms of the (log-linearized) union-wide bond rate and consumer price inflation of country k (Fisher equation):

$$i_{k,t}^{real} = i_t - \mathbb{E}_t \left[\pi_{k,t+1}^C \right] \,. \tag{44}$$

4 Model Analysis

In this section, we discuss the macroeconomic consequences of a preference shock at the household level and a deposit shift shock at the bank level. Before analyzing the model responses to these shocks, we state the calibration strategy.

²⁶Note that for simplicity, as in Kumhof and Wang (2019), we assume that balance sheet costs as well as interest costs for QE-created deposits represent lump-sum transfers to the household instead of resource costs. However, our results are not affected by these assumptions.

4.1 Calibration

The calibration of our model is depicted in Table 1. As discussed in Section 2, QE asset purchases are to a large extent conducted with counterparties residing outside the euro area, implying a heterogeneous increase in excess reserves and deposits across euro area countries. Accordingly, we calibrate the model to represent Germany (as the high-liquidity country) and Italy (as the low-liquidity country) in steady state. The euro area bank balance sheet statistics officially refer to these deposits of non-euro area residents held on accounts with euro area commercial banks as "liabilities of euro area monetary financial institutions (excluding the Eurosystem) towards non-euro area residents". In our model, these deposits are captured by the variable D_k^{QE} . In relation to the length of banks' balance sheets in the respective banking sector, D_k^{QE} adds up to 9% in Germany and 2% in Italy.²⁷ We calibrate the parameter Ω_k accordingly.

In order to realistically capture the (mechanical) relationship between QE-created deposits and the bond rate i_t (ι_k in our model), we draw from the work of Urbschat and Watzka (2020), who use an event study approach to estimate the effect of QE-related press releases on bond yields. On average, German bond yields fell by 5.91 basis points (bp), while Italian bond yields dropped by 69.67 bp after APP press releases between 2014 and 2016. Naturally, these decreases can only serve as an approximation of yield changes since they only capture the impact of the announcement of QE measures while leaving out the actual purchases. However, this approach ensures that we capture the isolated effect of QE on bond yields. Alternatives, for example using actual drops in bond yields, are more likely to be prone to influences independent of the asset purchases of the ECB.

Regarding the structural parameters of the household and the firm sector, we draw from the work by Albonico et al. (2019), who build a multi-country model including Germany and Italy. They estimate certain structural parameters based on the respective economies, some of which are also used in our model specification.

The interest rates as well as the required reserve ratio set by the central bank are chosen to represent the respective values of the ECB. Note that the annual rates of the ECB have to be converted into quarterly rates due to the timing of the model.

²⁷The respective data can be found at Deutsche Bundesbank (2020) and Banca d'Italia (2020).

	Description	Value A	Value B	Target/Source
	На	Germany useholds	Italy	
β	Time preference	0.9983	0.9983	Albonico et al. (2019)
Ψ_k	Habit parameter	0.73	0.81	Albonico et al. (2019)
χ_k	Labor disutility parameter	2.62	5.98	Internally calibrated
φ_k	Inverse Frisch elasticity of labor supply	2.98	2.07	Albonico et al. (2019)
σ_k	Share of foreign goods in consumption	0.2612	0.205	Albonico et al. (2019)
ϵ_k	Price elasticity of demand	9	9	Galí (2015)
'n		Firms		
δ_K	Capital depreciation rate	0.0143	0.0136	Albonico et al. (2019)
n_k	Capital adjustment costs parameter	31	19	Albonico et al. (2019)
α_k	Partial factor elasticity of capital	0.35	0.35	Albonico et al. (2019)
θ_k	Price stickiness parameter	0.75	0.75	Galí (2015)
	—	d Central B	ank	
Ω_k	QE-created deposits in bank balance sheet	106.51	2.41	Share Germany: 9%, share Italy: 2%,
				internally calibrated
ι_k	Interdependence parameter of QE and bond rate	100.41	1.42	Drop German bond yields: 5.91 bp,
				drop Italian bond yields: 69.67 bp,
				internally calibrated
r	Required reserve ratio	0.01	0.01	ECB: minimum reserve ratio
i^{RR}	Required reserve interest rate	0	0	ECB: main refinancing rate
i^{ER}	Excess reserve interest rate	$-\frac{0.005}{4}$	$-\frac{0.005}{4}$	ECB: deposit rate
v_k	Balance sheet costs	0.000021	0.000037	Interest rate Germany: $\frac{0.0122}{4}$,
				interest rate Italy: $\frac{0.0140}{4}$,
				internally calibrated
ϕ_{π}	Inflation response Taylor rule	1.5	1.5	Galí (2015)

Table 1: Calibration.

With respect to bank costs, we calibrate balance sheet costs in a way that, given the respective ECB interest rates and the required reserve ratio, the loan interest rate matches data for average (annual) interest rates of newly granted loans to non-financial corporations in Germany and Italy provided by the European Central Bank (2020d,e). Obviously, when firms take out a loan from a bank, they do not only have to pay interest, but often additional fees. Consequently, the banks' marginal revenues (LHS of (35)) consist of more than interest payments which in turn implies higher marginal costs due to perfect competition of banks. However, as we only consider interest payments when calibrating the banks' balance sheet costs (second term on the RHS of (35)), the corresponding calibrated value of this cost factor serves as a lower bound, implying that all effects resulting from balance sheet costs also constitute a lower bound.

We now turn to a comparison of the steady state, generated by this particular calibration, with data. Table 2 shows several data points and the corresponding steady state values of our model. The steady state replicates the relative capital stock of Germany to Italy (1.24 in the data, 1.24 in the model). Furthermore, in steady state, the model fits the data for average (annual) interest rates of newly granted loans to non-financial corporations in Germany (1.22% to 1.22%) and Italy (1.40% to 1.40%). This implies that the choice of the level of balance sheet costs is reasonable. Note that, considering that our model does not capture government spending, the share of investment and consumption in GDP is slightly higher in the model than in the data, as expected.

Description	Value Data	Data Source	Value Model
Relative GDP/capita: Germany (A) to Italy (B)	1.27	OECD (2019)	1.26
Relative average (annual) salary: Germany (A) to Italy (B)	1.32	OECD (2018)	1.26
Consumption share Germany (A) in overall consumption	0.63	The World Bank (2018)	0.65
(Germany (A) + Italy (B)), Taylor rule parameter			
Relative capital stock: Germany (A) to Italy (B)	1.24	University of Groningen and University of California (2017a,b)	1.24
Investment share in GDP: Germany (A)	0.225	CEIC (2020a)	0.256
Investment share in GDP: Italy (B)	0.170	CEIC (2020c)	0.247
Consumption share in GDP: Germany (A)	0.506	CEIC (2020b)	0.744
Consumption share in GDP: Italy (B)	0.608	CEIC (2020d)	0.753
Average (annual) interest rate of new loans to corporations: Germany (A) $2017 - 2020$	1.22%	European Central Bank (2020d)	1.22%
Average (annual) interest rate of new loans to corporations: Italy (B) $2017 - 2020$	1.40%	European Central Bank (2020e)	1.40%
Share of liabilities of euro area monetary financial institutions (excluding the Eurosystem) towards non-euro area residents on banks' balance sheets: Germany (A)	9%	Deutsche Bundesbank (2020)	9%
Share of liabilities of euro area monetary financial institutions (excluding the Eurosystem) towards non-euro area residents on banks' balance sheets: Italy (B)	2%	Banca d'Italia (2020)	2%

Table 2: Steady State in Comparison to Data.

Moreover, while the model slightly understates labor income inequality between Germany and Italy (1.32 to 1.26), it closely replicates relative GDP per capita of Germany to Italy (1.27 to 1.26). In addition, the parameter relevant for weighting consumer price inflation in country A and B in the Taylor rule, $\gamma_{k,t}$, is very close to the data-equivalent in steady state (0.63 to 0.65). Lastly, as already mentioned, we calibrate the model to exactly replicate the share of liabilities of euro area monetary financial institutions (excluding the Eurosystem) towards non-euro area residents on banks' balance sheets in Germany (9%) and Italy (2%).

4.2 Dynamic Analysis

We continue by examining the model responses to a preference shock and a deposit shift shock. All results are percentage deviations from the zero inflation steady state.

4.2.1 Preference Shock

Figure 2 depicts the impulse responses of the monetary union to a symmetric negative 1% preference shock in countries A and B. The responses are qualitatively similar in both countries but differ quantitatively. The preference shock implies a decrease in the households' appreciation of consumption, formally captured by a decrease in their marginal utility for each level of consumption. Thus, consumption decreases, proportionally in domestic and foreign terms. Note that the low-liquidity country B reaches its lowest consumption slightly later due to its higher habit parameter. Furthermore, the households' marginal benefit from labor, and thus their labor supply, decreases and real wages go up initially. The demand for goods decreases, implying falling output and prices. The latter implies an expansionary monetary policy reaction. The central bank increases its net asset purchases (QE), leading to a decrease in the long-term interest rate i_t , i.e., the bond rate, and an increase in QE-created bank deposits (equation (37)). Note that, motivated by the mechanical peculiarities of QE in the euro area presented in Section 2, QE-created bank deposits increase more in the high-liquidity country A than in the low-liquidity country B.

As a consequence of this expansionary monetary policy action, there are two effects on bank costs. On the one hand, banks face lower interest costs (interest rate channel of QE), on the other hand, they have to cope with higher balance sheet costs due to the increase in deposits (reverse bank lending channel of QE). As we calibrate balance sheet costs to be rather low (see Section 4.1), ensuring that our results with respect to the negative impact of balance sheet costs on the efficacy of QE constitute a lower bound, the decrease in costs due to the lower interest rate outweighs the increase implied by higher balance sheet costs.

Consequently, bank loan supply increases, implying a decrease in the bank loan rate and higher bank lending (interest rate channel but weakened by the reverse bank lending channel). Investment and thus (one period lagged) capital increase. The increasing capital stock implies higher labor productivity. Real wages rise, leading to increasing labor and consumption. Inflation starts to increase but rather slowly, due to the price rigidities, implying that monetary policy remains expansionary, leading to further increases in the capital stock. Therefore, there are two positive effects on consumption over time: first, the shock reduction, and second, the rise in real wages due to the increase in the capital stock and thus higher labor productivity. The price rigidities imply a still expansionary monetary policy and, therefore, a further buildup of the capital stock, even when the shock itself is already completely reduced. This leads to a temporary "overshooting" (levels temporarily exceed their steady state) of real wages, consumption, and output.²⁸

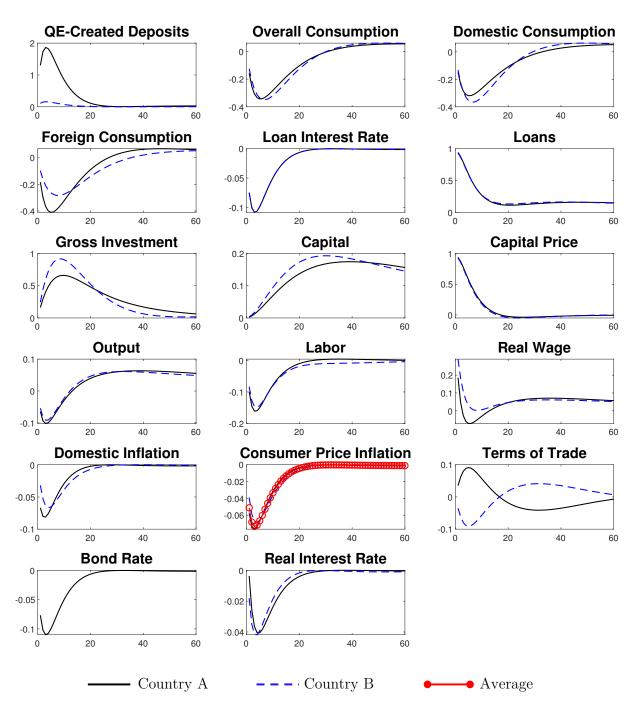


Figure 2: Impulse Responses to a Symmetric, Negative 1% Preference Shock with Persistence $\rho_{z,k}=0.9.$

²⁸This overshooting is slightly reinforced by the one-period lag between the firms' investment decision and the use of the capital in the production process.

The rigidities in the form of the CAC imply, on the one hand, that the buildup of the capital stock is impeded. Consequently, coming from negative consumption deviations, the steady state of consumption is reached later and the overshooting is dampened. However, on the other hand, the CAC also imply that the overshooting lasts longer as the reduction of the capital stock is also impeded. Note that higher CAC in the high-liquidity country A imply a lower increase in investment and capital in A than in B as well as a longer lasting overshooting.

Consequently, QE in our model works as expected of an expansionary monetary policy impulse: it triggers investment and therefore increases the capital stock, supporting output, consumption, and ultimately the consumer price level to reach steady state levels. However, the effect would be stronger if it were not for the QE-induced increase in balance sheet costs resulting from higher QE-created deposits. Balance sheet costs imply a reverse bank lending channel. The traditional bank lending channel describes a positive relationship between bank deposits and credit lending. For instance, a contractionary monetary policy impulse leads to decreasing deposits and hence to a decline in lending (Bernanke and Gertler, 1995; Kashyap and Stein, 1995). Accordingly, expansionary monetary policy, for instance QE, should increase bank deposits and credit lending. However, in our model, increasing deposits imply larger (balance sheet) costs for banks. Therefore, there is a reverse bank lending channel weakening the interest rate channel of QE. The specific implementation of QE implies a higher increase in excess reserves and QE-created deposits, and thereby also in bank balance sheet costs, in country A than in country B. Thus, the dampening effects are stronger in the high-liquidity country A, i.e., monetary policy is less effective in that country.

4.2.2 Deposit Shift Shock

Figure 3 depicts the impulse responses of the monetary union to a deposit shift shock. We simulate an approximately 12% withdrawal of QE-created deposits from low-liquidity country B. These deposits are then moved to the high-liquidity country A, increasing deposits by 2%. This shock can be interpreted as capital flight ("safe-haven-flows" or "flight-to-quality" phenomena). As described in Section 2, such a shift in deposits could be primarily observed during the financial and sovereign debt crisis. In current times, an additional deposit shift would strengthen the already existing asymmetric distribution of deposits.

The consequences of such a deposit shift shock in country A are as follows. Bank A's deposits, and thus its balance sheet costs, increase which leads to a decrease in its loan supply.

The bank loan rate increases and bank lending in country A decreases. Consequently, investment and thus the capital stock decrease, implying a lower output. The influence of the CAC are analogous to the described effects in Section (4.2.1).

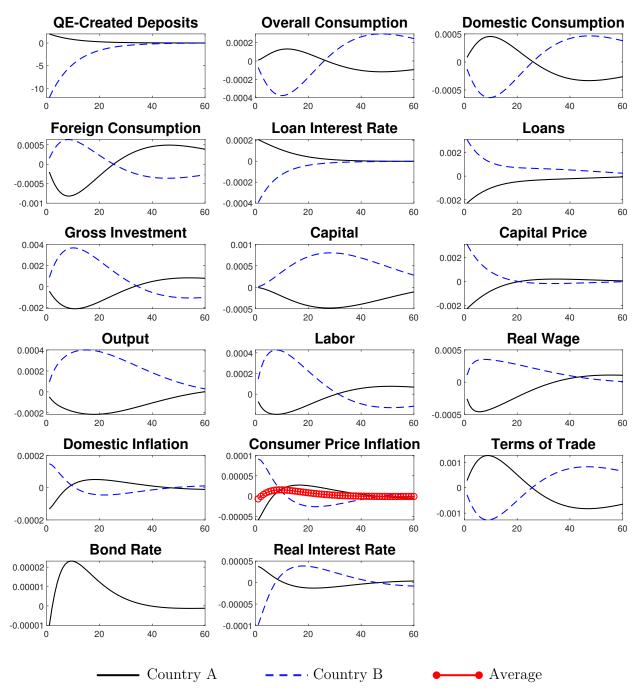


Figure 3: Impulse Responses to a Deposit Shift Shock from Country B to A with Persistence $\rho_{\tilde{d},k}=0.9.$

Labor productivity, and therefore labor demand, decrease. Real wages and labor input fall. First, the resulting lower labor costs imply decreasing prices. However, over time, higher loan and capital costs dominate and firms adjust prices upwards.

In country B, the consequences of the deposit shift are reversed. Lower bank costs imply more investment, and thus a higher capital stock and labor input, which leads to more output and initially increasing prices. As a consequence of higher prices, domestic consumption initially decreases in country B. Nevertheless, output increases due to higher investment, causing higher labor demand and wages. Over time, lower capital costs lead to a decrease in the price level, implying higher consumption of domestic goods, lower consumption of foreign goods, and an increase in the terms of trade between country B and country A over time.

Note that the monetary policy reaction is rather weak as the central bank reacts to the average consumer price inflation rate in the monetary union. As the shock becomes less relevant, so does the decrease (increase) in country A's (B's) capital stock, until it converges back to its steady state. Thus, in our model that focusses on excess reserves and does not consider potential underlying reasons for this shift, the deposit shift from country B to A negatively affects the economy of country A due to higher bank costs, implying lower investment and thus a lower capital stock, and therefore a decrease in output and consumption. Analogously, the country B economy benefits from this shock.

5 Conclusion

Since the start of the Eurosystem's QE program in March 2015, excess reserves in the euro area banking sector have persistently increased. The large quantity of excess reserves as well as its asymmetric distribution across euro area countries resulting from the specific implementation of QE has triggered a great amount of concern and debate. However, there is little analysis of whether and to what extent large quantities of excess reserves affect macroeconomic variables in different countries of a monetary union. For instance, with regard to the impact on bank lending, only little research has been conducted on whether there is a *bank lending channel* in the sense that QE-induced increases in bank reserves and deposits have a positive impact on bank lending.

Against this background, our paper develops a two-country New Keynesian model to analyze the macroeconomic effects of QE, explicitly considering the QE-induced heterogeneous increases in excess reserves and deposits in a monetary union. The model is calibrated for Germany and Italy to represent a high- and a low-liquidity euro area country. Hereby, we capture the consequences of the specific implementation of QE in the euro area, i.e., the resulting large amount of excess reserves in the banking sector, as well as its heterogeneous distribution across euro area countries. These consequences have important implications for our model as banks are exposed to balance sheet costs, i.e., costs related to the size of their balance sheet (for instance, in the form of agency or regulatory costs). We introduce QE as the central bank's monetary policy tool. Conducting QE decreases long-term interest rates, but, in addition, also implies an increase in banks' excess reserve and deposit holdings.

Analyzing the model responses to a negative preference shock in both countries, we find that QE, as an expansionary monetary policy tool, works as expected: the QE-induced decrease in long-term interest rates implies an increase in consumption and bank loan-financed investment. As a consequence, output, employment, and prices rise (interest rate channel of QE). However, the effects of this expansionary monetary policy reaction to the shock are weakened by QE-induced increases in excess reserves and deposits, implying increasing (balance sheet) costs for banks and, therefore, a smaller decrease in the bank loan rate and thus a lower increase in bank loan-financed investment. Consequently, the interest rate channel is dampened by a reverse bank lending channel. The dampening effects are more pronounced in the high-liquidity country.

With respect to the ECB's reaction to the COVID-19 pandemic, one can conclude the following from our model. One measure of the ECB in response to the pandemic was the introduction of the Pandemic Emergency Purchase Programme (PEPP). While the PEPP has a dual objective, i.e., creating financial conditions (low interest rates) to stabilize the economy and mitigating the pandemic-induced malfunctioning of financial markets (Schnabel, 2020), its implementation is similar to the implementation of the APP introduced in 2015. Therefore, the stabilizing effects on the economy of the PEPP through an interest rate channel are also weakened by a reverse bank lending channel.

Our model suggests that central banks should consider that QE-induced increases in excess reserves and deposits may dampen the stimulating and stabilizing effects of this monetary policy measure on the economy. In particular, it should be taken into consideration that these dampening effects may differ across countries due to the asymmetric distribution of excess reserves and bank deposits as a consequence of the specific technical implementation of QE in the euro area. Moreover, optimal monetary policy within the given institutional framework may differ when these effects are taken into account.

Appendices

A Expenditure Minimization of the Household

A.1 Composition of the Domestic and Foreign Composite Consumption Good

The household minimizes its expenditures for any given level of domestic consumption:

$$\min_{C_{k,t}^k(j)} \int_0^1 P_{k,t}(j) C_{k,t}^k(j) dj,$$
(A.1)

subject to

$$\left(\int_0^1 C_{k,t}^k(j)^{\frac{\epsilon_k-1}{\epsilon_k}} dj\right)^{\frac{\epsilon_k}{\epsilon_k-1}} = \bar{C}_k^k.$$

This is equivalent to maximizing the following Lagrange function with respect to the consumption of a representative good i:

$$\max_{C_{k,t}^k(i)} L_t^k = -\int_0^1 P_{k,t}(j) C_{k,t}^k(j) dj + \lambda_{k,t} \left[\left(\int_0^1 C_{k,t}^k(j)^{\frac{\epsilon_k - 1}{\epsilon_k}} dj \right)^{\frac{\epsilon_k}{\epsilon_k - 1}} - \bar{C}_k^k \right].$$

The first order conditions are given by

$$\frac{\partial L_t^k}{\partial C_{k,t}^k(i)} = -P_{k,t}(i) + \lambda_{k,t} \left[\left(\int_0^1 C_{k,t}^k(j)^{\frac{\epsilon_k - 1}{\epsilon_k}} dj \right)^{\frac{\epsilon_k - 1}{\epsilon_k - 1} - 1} C_{k,t}^k(i)^{\frac{\epsilon_k - 1}{\epsilon_k} - 1} \right] \stackrel{!}{=} 0, \tag{A.2}$$

$$\frac{\partial L_t^k}{\partial \lambda_{k,t}} \stackrel{!}{=} 0. \tag{A.3}$$

Rearranging yields

$$0 = -P_{k,t}(i) + \lambda_{k,t} \left[\left(\int_0^1 C_{k,t}^k(j)^{\frac{\epsilon_k - 1}{\epsilon_k}} di \right)^{\frac{1}{\epsilon_k - 1}} C_{k,t}^k(i)^{-\frac{1}{\epsilon_k}} \right],$$

$$\Leftrightarrow C_{k,t}^k(i) = \left(\frac{P_{k,t}(i)}{\lambda_{k,t}}\right)^{-\epsilon_k} C_{k,t}^k.$$
(A.4)

In order to obtain the expression for optimal consumption, it is necessary to solve for the lagrange multiplier $\lambda_{k,t}$ by using the constraint.

$$\left(\int_{0}^{1} C_{k,t}^{k}(j)^{\frac{\epsilon_{k}-1}{\epsilon_{k}}} dj\right)^{\frac{\epsilon_{k}}{\epsilon_{k}-1}} = \bar{C}_{k}^{k},$$
$$\Leftrightarrow \left(\int_{0}^{1} \left[\left(\frac{P_{k,t}(j)}{\lambda_{k,t}}\right)^{-\epsilon_{k}} \bar{C}_{k}^{k} \right]^{\frac{\epsilon_{k}-1}{\epsilon_{k}}} dj \right)^{\frac{\epsilon_{k}}{\epsilon_{k}-1}} = \bar{C}_{k}^{k},$$
$$\Leftrightarrow \int_{0}^{1} \left(\frac{P_{k,t}(j)}{\lambda_{k,t}}\right)^{1-\epsilon_{k}} dj = 1.$$

Thus, the solution for $\lambda_{k,t}$ is

$$\lambda_{k,t} = \left(\int_0^1 P_{k,t}(j)^{1-\epsilon_k} dj\right)^{\frac{1}{1-\epsilon_k}} \equiv P_{k,t}.$$

Plugging this solution into the optimal consumption decision for any domestic good j yields

$$C_{k,t}^k(j) = \left(\frac{P_{k,t}(j)}{P_{k,t}}\right)^{-\epsilon_k} C_{k,t}^k.$$
(A.5)

Symmetrically, the optimal consumption for any foreign good j is

$$C_{-k,t}^{k}(j) = \left(\frac{P_{-k,t}(j)}{P_{-k,t}}\right)^{-\epsilon_{k}} C_{-k,t}^{k}.$$
(A.6)

A.2 Allocation between Domestic and Foreign Goods

The household minimizes its expenditures for any level of overall consumption:

$$\min_{C_{k,t}^k, C_{-k,t}^k} P_{k,t} C_{k,t}^k + P_{-k,t} C_{-k,t}^k, \tag{A.7}$$

subject to

$$\frac{\left(C_{k,t}^k\right)^{1-\sigma_k} \left(C_{-k,t}^k\right)^{\sigma_k}}{(1-\sigma_k)^{1-\sigma_k} (\sigma_k)^{\sigma_k}} = \bar{C}^k.$$

This is equivalent to maximizing the following Lagrange function with respect to the domestic and foreign consumption level:

$$L_{t}^{k} = -P_{k,t}C_{k,t}^{k} - P_{-k,t}C_{-k,t}^{k} + \lambda_{k,t} \left(\frac{\left(C_{k,t}^{k}\right)^{1-\sigma_{k}} \left(C_{-k,t}^{k}\right)^{\sigma_{k}}}{(1-\sigma_{k})^{1-\sigma_{k}} (\sigma_{k})^{\sigma_{k}}} - \bar{C}^{k} \right).$$

The first order conditions are

$$\frac{\partial L_t^k}{\partial C_{k,t}^k} = -P_{k,t} + \lambda_{k,t} \left(\frac{(1 - \sigma_k) \left(C_{k,t}^k \right)^{-\sigma_k} \left(C_{-k,t}^k \right)^{\sigma_k}}{(1 - \sigma_k)^{1 - \sigma_k} (\sigma_k)^{\sigma_k}} \right) \stackrel{!}{=} 0, \tag{A.8}$$

$$\frac{\partial L_t^k}{\partial C_{-k,t}^k} = -P_{-k,t} + \lambda_{k,t} \left(\frac{\left(C_{k,t}^k \right)^{1-\sigma_k} \sigma_k \left(C_{-k,t}^k \right)^{\sigma_k - 1}}{(1 - \sigma_k)^{1-\sigma_k} (\sigma_k)^{\sigma_k}} \right) \stackrel{!}{=} 0, \tag{A.9}$$

$$\frac{\partial L_t^k}{\partial \lambda_{k,t}} \stackrel{!}{=} 0. \tag{A.10}$$

Rearranging yields

$$P_{k,t} = \lambda_{k,t} (1 - \sigma_k) \left(\frac{\left(C_{k,t}^k\right)^{-\sigma_k} \left(C_{-k,t}^k\right)^{\sigma_k}}{(1 - \sigma_k)^{1 - \sigma_k} (\sigma_k)^{\sigma_k}} \frac{C_{k,t}^k}{C_{k,t}^k} \right),$$
$$P_{-k,t} = \lambda_{k,t} \sigma_k \left(\frac{\left(C_{k,t}^k\right)^{1 - \sigma_k} \left(C_{-k,t}^k\right)^{\sigma_k - 1}}{(1 - \sigma_k)^{1 - \sigma_k} (\sigma_k)^{\sigma_k}} \frac{C_{-k,t}^k}{C_{-k,t}^k} \right),$$

which can be rewritten as

$$C_{k,t}^{k} = (1 - \sigma_k) \left(\frac{P_{k,t}}{\lambda_{k,t}}\right)^{-1} C_t^k, \qquad (A.11)$$

$$C_{-k,t}^{k} = \sigma_k \left(\frac{P_{-k,t}}{\lambda_{k,t}}\right)^{-1} C_t^k.$$
(A.12)

Plugging into the constraint gives

$$\frac{\left(\left(1-\sigma_k\right)\left(\frac{P_{k,t}}{\lambda_{k,t}}\right)^{-1}\bar{C}^k\right)^{1-\sigma_k}\left(\sigma_k\left(\frac{P_{-k,t}}{\lambda_{k,t}}\right)^{-1}\bar{C}^k\right)^{\sigma_k}}{(1-\sigma_k)^{1-\sigma_k}(\sigma_k)^{\sigma_k}}=\bar{C}^k.$$

Clearly,

$$\lambda_{k,t} = P_{k,t}^{1-\sigma_k} P_{-k,t}^{\sigma_k} \equiv P_{k,t}^C,$$

and the optimal allocation of consumption expenditures between domestic and foreign consumption is given by

$$C_{k,t}^{k} = (1 - \sigma_{k}) \left(\frac{P_{k,t}}{P_{k,t}^{C}}\right)^{-1} C_{t}^{k},$$
(A.13)

$$C_{-k,t}^{k} = \sigma_k \left(\frac{P_{-k,t}}{P_{k,t}^C}\right)^{-1} C_t^k.$$
(A.14)

B Utility Maximization of the Household

The household seeks to maximize expected lifetime utility:

$$\max_{C_t^k, N_t^k, B_t^k} \mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[Z_\tau ln \left(C_\tau^k - \Psi_k C_{\tau-1}^k \right) - \frac{\chi_k}{1 + \varphi_k} (N_\tau^k)^{1+\varphi_k} \right] \right], \tag{B.1}$$

subject to

$$P_{k,t}^C C_t^k + B_t^k = (1 + i_{t-1}) B_{t-1}^k + W_{k,t} N_t^k + \Upsilon_t^k.$$

The Lagrange function is

$$L_{t}^{k} = \mathbb{E}_{t} \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[Z_{\tau} ln \left(C_{\tau}^{k} - \Psi_{k} C_{\tau-1}^{k} \right) - \frac{\chi_{k}}{1 + \varphi_{k}} (N_{\tau}^{k})^{1+\varphi_{k}} - \lambda_{k,\tau} \left(P_{k,\tau}^{C} C_{\tau}^{k} + B_{\tau}^{k} - (1 + i_{\tau-1}) B_{\tau-1}^{k} - W_{k,\tau} N_{\tau}^{k} - \Upsilon_{\tau}^{k} \right) \right] \right].$$
(B.2)

The first order conditions are given by

$$\frac{\partial L_t^k}{\partial C_t^k} = U_{c,t}^k - \lambda_{k,t} P_{k,t}^C \stackrel{!}{=} 0, \tag{B.3}$$

$$\frac{\partial L_t^k}{\partial N_t^k} = -\chi_k \left(N_t^k \right)^{\varphi_k} + \lambda_{k,t} W_{k,t} \stackrel{!}{=} 0, \tag{B.4}$$

$$\frac{\partial L_t^k}{\partial B_t^k} = -\lambda_{k,t} + \beta (1+i_t) \mathbb{E}_t \left[\lambda_{k,t+1} \right] \stackrel{!}{=} 0, \tag{B.5}$$

$$\frac{\partial L_t^k}{\partial \lambda_{k,t}} \stackrel{!}{=} 0, \tag{B.6}$$

with

$$U_{c,t}^{k} \equiv \left(\frac{Z_{t}}{C_{t}^{k} - \Psi_{k}C_{t-1}^{k}} - \frac{\mathbb{E}_{t}\left[Z_{t+1}\right]\Psi_{k}\beta}{\mathbb{E}_{t}\left[C_{t+1}^{k}\right] - \Psi_{k}C_{t}^{k}}\right).$$

Plugging (B.3) into (B.4) and (B.5) gives

$$\chi_k \left(N_t^k \right)^{\varphi_k} = U_{c,t}^k \frac{W_{k,t}}{P_{k,t}^C},\tag{B.7}$$

$$\beta(1+i_t) \mathbb{E}_t \left[\frac{P_{k,t}^C}{P_{k,t+1}^C} \right] \Lambda_{t,t+1}^k = 1,$$
(B.8)

with

$$\Lambda_{t,t+1}^k \equiv \mathbb{E}_t \left[\frac{U_{c,t+1}^k}{U_{c,t}^k} \right].$$

C Risk Sharing

The Euler equation holds in both countries at all times. Thus,

$$\frac{P_{k,t}^{C}}{\mathbb{E}_{t}\left[P_{k,t+1}^{C}\right]}\frac{\mathbb{E}_{t}\left[U_{c,t+1}^{k}\right]}{U_{c,t}^{k}} = \frac{P_{-k,t}^{C}}{\mathbb{E}_{t}\left[P_{-k,t+1}^{C}\right]}\frac{\mathbb{E}_{t}\left[U_{c,t+1}^{-k}\right]}{U_{c,t}^{-k}},$$

or, using the definition of the terms of trade,

$$\frac{P_{k,t}\left(V_{-k,t}^{k}\right)^{\sigma_{k}}}{\mathbb{E}_{t}\left[P_{k,t+1}\left(V_{-k,t+1}^{k}\right)^{\sigma_{k}}\right]}\frac{\mathbb{E}_{t}\left[U_{c,t+1}^{k}\right]}{U_{c,t}^{k}} = \frac{P_{-k,t}\left(V_{k,t}^{-k}\right)^{\sigma_{-k}}}{\mathbb{E}_{t}\left[P_{-k,t+1}\left(V_{k,t+1}^{-k}\right)^{\sigma_{-k}}\right]}\frac{\mathbb{E}_{t}\left[U_{c,t+1}^{-k}\right]}{U_{c,t}^{-k}}.$$

This relation holds in all periods, i.e.,

$$\frac{P_{k,t-1}\left(V_{-k,t-1}^{k}\right)^{\sigma_{k}}}{P_{k,t}\left(V_{-k,t}^{k}\right)^{\sigma_{k}}}\frac{U_{c,t}^{k}}{U_{c,t-1}^{k}} = \frac{P_{-k,t-1}\left(V_{k,t-1}^{-k}\right)^{\sigma_{-k}}}{P_{-k,t}\left(V_{k,t}^{-k}\right)^{\sigma_{-k}}}\frac{U_{c,t}^{-k}}{U_{c,t-1}^{-k}}$$

This condition can be rearranged in the following way:

$$\begin{split} \frac{U_{c,t}^{k}}{U_{c,t}^{-k}} &= \frac{P_{k,t}}{P_{-k,t}} \frac{P_{-k,t-1}}{P_{k,t-1}} \frac{\left(V_{k,t-1}^{-k}\right)^{\sigma_{-k}}}{\left(V_{-k,t-1}^{k}\right)^{\sigma_{k}}} \frac{\left(V_{-k,t}^{k}\right)^{\sigma_{k}}}{\left(V_{-k,t}^{-k}\right)^{\sigma_{-k}}} \frac{U_{c,t-1}^{k}}{U_{c,t-1}^{-k}},\\ \Leftrightarrow \frac{U_{c,t}^{k}}{U_{c,t}^{-k}} &= \left(V_{-k,t}^{k}\right)^{-1} V_{-k,t-1}^{k} \frac{\left(V_{-k,t-1}^{-k}\right)^{\sigma_{-k}}}{\left(V_{-k,t-1}^{k}\right)^{\sigma_{k}}} \frac{\left(V_{-k,t}^{k}\right)^{\sigma_{k}}}{\left(V_{-k,t}^{-k}\right)^{\sigma_{-k}}} \frac{U_{c,t-1}^{k}}{U_{c,t-1}^{-k}},\\ \Leftrightarrow \frac{U_{c,t}^{k}}{U_{c,t}^{-k}} &= \left(V_{-k,t}^{k}\right)^{\sigma_{k}-1} \left(V_{-k,t-1}^{k}\right)^{1-\sigma_{k}} \left(V_{k,t}^{-k}\right)^{-\sigma_{-k}} \left(V_{k,t-1}^{-k}\right)^{\sigma_{-k}} \frac{U_{c,t-1}^{k}}{U_{c,t-1}^{-k}}. \end{split}$$

In the previous period:

$$\frac{U_{c,t-1}^k}{U_{c,t-1}^{-k}} = \left(V_{-k,t-1}^k\right)^{\sigma_k - 1} \left(V_{-k,t-2}^k\right)^{1 - \sigma_k} \left(V_{k,t-1}^{-k}\right)^{-\sigma_{-k}} \left(V_{k,t-2}^{-k}\right)^{\sigma_{-k}} \frac{U_{c,t-2}^k}{U_{c,t-2}^{-k}}$$

Recursively,

$$\frac{U_{c,t}^{k}}{U_{c,t}^{-k}} = \left(V_{-k,t}^{k}\right)^{\sigma_{k}-1} \left(V_{-k,t-2}^{k}\right)^{1-\sigma_{k}} \left(V_{k,t}^{-k}\right)^{-\sigma_{-k}} \left(V_{k,t-2}^{-k}\right)^{\sigma_{-k}} \frac{U_{c,t-2}^{k}}{U_{c,t-2}^{-k}}.$$

Continuing this procedure to the initial period, i.e., the steady state, we get

$$\frac{U_{c,t}^{k}}{U_{c,t}^{-k}} = \left(V_{-k,t}^{k}\right)^{\sigma_{k}-1} \left(V_{-k,ss}^{k}\right)^{1-\sigma_{k}} \left(V_{k,t}^{-k}\right)^{-\sigma_{-k}} \left(V_{k,ss}^{-k}\right)^{\sigma_{-k}} \frac{U_{c,ss}^{k}}{U_{c,ss}^{-k}},$$

with $V_{-k,ss}^k = V_{k,ss}^{-k} = 1$. Rearranging yields:

$$U_{c,t}^{k} = \vartheta_{k} (V_{-k,t}^{k})^{(\sigma_{k}-1)} (V_{k,t}^{-k})^{(-\sigma_{-k})} U_{c,t}^{-k}.$$
 (C.1)

D Profit Maximization of the Intermediate Goods Firm

The competitive intermediate goods firm maximizes its period profit:

$$\max_{N_t^k, K_{t,t+1}^k} \Gamma_{m,t}^k = mc_{k,m,t} Y_{m,t}^k - w_{k,t} N_t^k - \left(1 + i_{k,t-1}^L\right) Q_{k,t-1} K_{t-1,t}^k + (Q_{k,t} - \delta_k) K_{t-1,t}^k, \quad (D.1)$$

with

$$Y_{m,t}^k = \left(K_{t-1,t}^k\right)^{\alpha_k} \left(N_t^k\right)^{1-\alpha_k}.$$

The first order conditions are given by

$$\frac{\partial \Gamma_{m,t}^k}{\partial N_t^k} = mc_{k,m,t}(1-\alpha_k)\frac{Y_{m,t}^k}{N_t^k} - w_{k,t} \stackrel{!}{=} 0, \tag{D.2}$$

$$\frac{\partial \Gamma_{m,t+1}^k}{\partial K_{t,t+1}^k} = \mathbb{E}_t \left[mc_{k,m,t+1} \alpha_k \frac{Y_{m,t+1}^k}{K_{t,t+1}^k} \right] - \left(1 + i_{k,t}^L \right) Q_{k,t} + \mathbb{E}_t \left[Q_{k,t+1} \right] - \delta_k \stackrel{!}{=} 0. \tag{D.3}$$

Rearranging yields:

$$mc_{k,m,t} = \frac{w_{k,t}}{(1 - \alpha_k)\frac{Y_{m,t}^k}{N_t^k}},$$
 (D.4)

$$(1+i_{k,t}^{L}) Q_{k,t} = \alpha_k \mathbb{E}_t \left[mc_{k,m,t+1} \frac{Y_{m,t+1}^k}{K_{t,t+1}^k} \right] + (\mathbb{E}_t \left[Q_{k,t+1} \right] - \delta_k).$$
(D.5)

E Profit Maximization of the Capital Producing Firm

The real period profit of a capital producing firm is given by

$$\Gamma_{c,t}^{k} = Q_{k,t}K_{t,t+1}^{k} - (Q_{k,t} - \delta_k)K_{t-1,t}^{k} - \delta_k K_{t-1,t}^{k} - I_t^{k} - f\left(\frac{I_t^{k} + I_{ss}}{I_{t-1}^{k} + I_{ss}}\right) \left(I_t^{k} + I_{ss}\right), \quad (E.1)$$

$$\Leftrightarrow \Gamma_{c,t}^{k} = Q_{k,t}K_{t,t+1}^{k} - Q_{k,t}K_{t-1,t}^{k} + \delta_{k}K_{t-1,t}^{k} - \delta_{k}K_{t-1,t}^{k} - I_{t}^{k} - I_{t}^{k} - f\left(\frac{I_{t}^{k} + I_{ss}}{I_{t-1}^{k} + I_{ss}}\right)\left(I_{t}^{k} + I_{ss}\right),$$

$$\Leftrightarrow \Gamma_{c,t}^{k} = Q_{k,t} (K_{t,t+1}^{k} - K_{t-1,t}^{k}) - I_{t}^{k} - f \left(\frac{I_{t}^{k} + I_{ss}}{I_{t-1}^{k} + I_{ss}}\right) \left(I_{t}^{k} + I_{ss}\right),$$

$$\Rightarrow \Gamma_{c,t}^{k} = Q_{k,t} I_{t}^{k} - I_{t}^{k} - f \left(\frac{I_{t}^{k} + I_{ss}}{I_{t-1}^{k} + I_{ss}}\right) \left(I_{t}^{k} + I_{ss}\right),$$

$$\Rightarrow \Gamma_{c,t}^{k} = (Q_{k,t} - 1) I_{t}^{k} - f \left(\frac{I_{t}^{k} + I_{ss}}{I_{t-1}^{k} + I_{ss}}\right) \left(I_{t}^{k} + I_{ss}\right),$$

with capital adjustment costs (CAC) given by:

$$f\left(\frac{I_{t}^{k}+I_{ss}}{I_{t-1}^{k}+I_{ss}}\right) = \frac{n_{k}}{2} \left(\frac{I_{t}^{k}+I_{ss}}{I_{t-1}^{k}+I_{ss}}-1\right)^{2}.$$

The objective function of the capital producing firm thus becomes

$$\max_{I_{t}^{k}} \mathbb{E}_{t} \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \Lambda_{t,\tau}^{k} \left((Q_{k,\tau} - 1) I_{\tau}^{k} - \frac{n_{k}}{2} \left(\frac{I_{\tau}^{k} + I_{ss}}{I_{\tau-1}^{k} + I_{ss}} - 1 \right)^{2} \left(I_{\tau}^{k} + I_{ss} \right) \right) \right].$$
(E.2)

Capital producers choose net investment I^k_t to maximize their discounted expected profits. The respective FOC is

$$Q_{k,t} - 1 - \frac{n_k}{2} \left(\frac{I_t^k + I_{ss}}{I_{t-1}^k + I_{ss}} - 1 \right)^2 + 2\frac{n_k}{2} \frac{1}{I_{t-1}^k + I_{ss}} \left(\frac{I_t^k + I_{ss}}{I_{t-1}^k + I_{ss}} - 1 \right) \left(I_t^k + I_{ss} \right) \\ - \mathbb{E}_t \left[\beta \Lambda_{t,t+1}^k \left(\frac{I_{t+1}^k + I_{ss}}{I_t^k + I_{ss}} \right)^2 n_k \left(\frac{I_{t+1}^k + I_{ss}}{I_t^k + I_{ss}} - 1 \right) \right] \stackrel{!}{=} 0,$$

$$\Leftrightarrow Q_{k,t} = 1 + \frac{n_k}{2} \left(\frac{I_t^k + I_{ss}}{I_{t-1}^k + I_{ss}} - 1 \right)^2 + \frac{I_t^k + I_{ss}}{I_{t-1}^k + I_{ss}} n_k \left(\frac{I_t^k + I_{ss}}{I_{t-1}^k + I_{ss}} - 1 \right) - \mathbb{E}_t \left[\beta \Lambda_{t,t+1}^k \left(\frac{I_{t+1}^k + I_{ss}}{I_t^k + I_{ss}} \right)^2 n_k \left(\frac{I_{t+1}^k + I_{ss}}{I_t^k + I_{ss}} - 1 \right) \right].$$
(E.3)

F Profit Maximization of the Retail Firm

Firm j chooses its price $P_{k,t}(j)$ to maximize discounted expected real profits:

$$\max_{P_{k,t}(j)} \mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \theta_k^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^k \left(\frac{P_{k,t}(j)}{P_{k,\tau}^C} Y_{k,\tau|t}(j) - TC(Y_{k,\tau|t}(j)) \right) \right] , \tag{F.1}$$

subject to

$$\begin{split} Y_{k,\tau|t}(j) &= \left(\frac{P_{k,t}(j)}{P_{k,\tau}}\right)^{-\epsilon_k} \left(C_{k,\tau}^k + C_{k,\tau}^{-k}\right) \\ &= \left(\frac{P_{k,t}(j)}{P_{k,\tau}}\right)^{-\epsilon_k} Y_{k,\tau}, \end{split}$$

$$\Rightarrow \max_{P_{k,t}(j)} \mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \theta_k^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^k \left(P_{k,t}(j) \frac{1}{P_{k,\tau}^C} \left(P_{k,t}(j) \right)^{-\epsilon_k} \left(\frac{1}{P_{k,\tau}} \right)^{-\epsilon_k} Y_{k,\tau} - TC \left(\left(\frac{P_{k,t}(j)}{P_{k,\tau}} \right)^{-\epsilon_k} Y_{k,\tau} \right) \right) \right],$$

$$\Leftrightarrow \max_{P_{k,t}(j)} \mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \theta_k^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^k \left(\frac{1}{P_{k,\tau}^C} \left(P_{k,t}(j) \right)^{1-\epsilon_k} \left(\frac{1}{P_{k,\tau}} \right)^{-\epsilon_k} Y_{k,\tau} - TC \left(\left(\frac{P_{k,t}(j)}{P_{k,\tau}} \right)^{-\epsilon_k} Y_{k,\tau} \right) \right) \right].$$

First oder condition:

$$\begin{split} \mathbb{E}_{t} \left[\sum_{\tau=t}^{\infty} \theta_{k}^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^{k} \left((1-\epsilon_{k}) \frac{1}{P_{k,\tau}^{C}} \left(P_{k,t}^{*}(j) \right)^{-\epsilon_{k}} \left(\frac{1}{P_{k,\tau}} \right)^{-\epsilon_{k}} Y_{k,\tau} - (-\epsilon_{k}) \left(P_{k,t}^{*}(j) \right)^{-\epsilon_{k}-1} \left(\frac{1}{P_{k,\tau}} \right)^{-\epsilon_{k}} Y_{k,\tau} mc(Y_{k,\tau|t}(j)) \right) \right] \stackrel{!}{=} 0, \\ \Leftrightarrow \mathbb{E}_{t} \left[\sum_{\tau=t}^{\infty} \theta_{k}^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^{k} \left((1-\epsilon_{k}) \frac{1}{P_{k,\tau}^{C}} \left(\frac{P_{k,t}^{*}(j)}{P_{k,\tau}} \right)^{-\epsilon_{k}} Y_{k,\tau} - (-\epsilon_{k}) \left(P_{k,t}^{*}(j) \right)^{-\epsilon_{k}-1} \left(\frac{1}{P_{k,\tau}} \right)^{-\epsilon_{k}} Y_{k,\tau} mc(Y_{k,\tau|t}(j)) \right) \right] = 0, \\ \Leftrightarrow \mathbb{E}_{t} \left[\sum_{\tau=t}^{\infty} \theta_{k}^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^{k} \left((1-\epsilon_{k}) \frac{1}{P_{k,\tau}^{C}} Y_{k,\tau|t}(j) - (-\epsilon_{k}) \left(P_{k,t}^{*}(j) \right)^{-\epsilon_{k}} \left(P_{k,t}^{*}(j) \right)^{-1} \left(\frac{1}{P_{k,\tau}} \right)^{-\epsilon_{k}} Y_{k,\tau} mc(Y_{k,\tau|t}(j)) \right) \right] = 0, \\ \Leftrightarrow \mathbb{E}_{t} \left[\sum_{\tau=t}^{\infty} \theta_{k}^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^{k} \left((1-\epsilon_{k}) \frac{Y_{k,\tau|t}(j)}{P_{k,\tau}^{C}} + \epsilon_{k} \left(P_{k,t}^{*}(j) \right)^{-1} Y_{k,\tau|t}(j) mc(Y_{k,\tau|t}(j)) \right) \right] = 0, \end{aligned}$$

$$\Leftrightarrow \mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \theta_k^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^k Y_{k,\tau|t}(j) \left((1-\epsilon_k) \frac{1}{P_{k,\tau}^C} + \epsilon_k \frac{1}{P_{k,t}^*(j)} mc(Y_{k,\tau|t}(j)) \right) \right] = 0,$$

$$\Leftrightarrow \mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \theta_k^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^k Y_{k,\tau|t}(j) \left(\frac{P_{k,t}^*(j)}{P_{k,\tau}^C} - \frac{\epsilon_k}{\epsilon_k - 1} mc(Y_{k,\tau|t}(j)) \right) \right] = 0,$$

$$h$$

with

$$mc(Y_{k,\tau|t}(j)) = mc_{k,m,\tau} = P_{k,m,\tau}.$$

Derive the optimal price $P_{k,t}^*(j)$ of firm j:

$$\mathbb{E}_{t}\left[\sum_{\tau=t}^{\infty}\theta_{k}^{\tau-t}\beta^{\tau-t}\Lambda_{t,\tau}^{k}Y_{k,\tau|t}(j)\frac{P_{k,t}^{*}(j)}{P_{k,\tau}^{C}}\right] = \frac{\epsilon_{k}}{\epsilon_{k}-1}\mathbb{E}_{t}\left[\sum_{\tau=t}^{\infty}\theta_{k}^{\tau-t}\beta^{\tau-t}\Lambda_{t,\tau}^{k}Y_{k,\tau|t}(j)mc_{k,m,\tau}\right],$$
with $Y_{k,\tau|t}(j) = \left(\frac{P_{k,t}^{*}(j)}{P_{k,\tau}^{C}}\right)^{-\epsilon_{k}}Y_{k,\tau}$:
$$\mathbb{E}_{t}\left[\sum_{\tau=t}^{\infty}\theta_{k}^{\tau-t}\beta^{\tau-t}\Lambda_{t,\tau}^{k}\left(\frac{P_{k,t}^{*}(j)}{P_{k,\tau}^{C}}\right)^{-\epsilon_{k}}Y_{k,\tau}\right] = \epsilon_{k} \mathbb{E}_{t}\left[\sum_{\tau=t}^{\infty}\theta_{k}^{\tau-t}\beta^{\tau-t}\Lambda_{t,\tau}^{k}\left(\frac{P_{k,t}^{*}(j)}{P_{k,\tau}^{C}}\right)^{-\epsilon_{k}}Y_{k,\tau}\right]$$

$$\mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \theta_k^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^k \left(\frac{P_{k,t}^*(j)}{P_{k,\tau}} \right)^{-\epsilon_k} Y_{k,\tau} \frac{P_{k,t}^*(j)}{P_{k,\tau}^C} \right] = \frac{\epsilon_k}{\epsilon_k - 1} \mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \theta_k^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^k \left(\frac{P_{k,t}^*(j)}{P_{k,\tau}} \right)^{-\epsilon_k} Y_{k,\tau} m c_{k,m,\tau} \right].$$

Solving for $P_{k,t}^*(j)$ yields:

$$1 = \frac{\epsilon_k}{\epsilon_k - 1} \frac{\mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \theta_k^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^k \left(\frac{P_{k,t}^*(j)}{P_{k,\tau}} \right)^{-\epsilon_k} Y_{k,\tau} m c_{k,m,\tau} \right]}{\mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \theta_k^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^k \left(\frac{P_{k,t}^*(j)}{P_{k,\tau}} \right)^{-\epsilon_k} Y_{k,\tau} \frac{P_{k,t}^*(j)}{P_{k,\tau}^C} \right]},$$

$$\Leftrightarrow 1 = \frac{\epsilon_k}{\epsilon_k - 1} \frac{\mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \theta_k^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^k \left(P_{k,t}^*(j) \right)^{-\epsilon_k} \left(P_{k,\tau} \right)^{\epsilon_k} Y_{k,\tau} m c_{k,m,\tau} \right]}{\mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \theta_k^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^k \left(P_{k,t}^*(j) \right)^{1-\epsilon_k} \left(P_{k,\tau} \right)^{\epsilon_k} Y_{k,\tau} \left(P_{k,\tau}^C \right)^{-1} \right]},$$

$$\Leftrightarrow 1 = \frac{\epsilon_k}{\epsilon_k - 1} \frac{\mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \theta_k^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^k \left(P_{k,\tau} \right)^{\epsilon_k} Y_{k,\tau} m c_{k,m,\tau} \right]}{\mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \theta_k^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^k P_{k,t}^*(j) \left(P_{k,\tau} \right)^{\epsilon_k} Y_{k,\tau} \left(P_{k,\tau}^C \right)^{-1} \right]},$$

$$\Leftrightarrow P_{k,t}^*(j) = \frac{\epsilon_k}{\epsilon_k - 1} \frac{\mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \theta_k^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^k (P_{k,\tau})^{\epsilon_k} Y_{k,\tau} m c_{k,m,\tau} \right]}{\mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \theta_k^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^k (P_{k,\tau})^{\epsilon_k} Y_{k,\tau} \left(P_{k,\tau}^C \right)^{-1} \right]},$$

with $P_{k,\tau}^C(j) = P_{k,\tau} \left(V_{-k,\tau}^k \right)^{\sigma_k}$:

$$P_{k,t}^{*}(j) = \frac{\epsilon_{k}}{\epsilon_{k}-1} \frac{\mathbb{E}_{t}\left[\sum_{\tau=t}^{\infty} \theta_{k}^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^{k} \left(P_{k,\tau}\right)^{\epsilon_{k}} Y_{k,\tau} m c_{k,m,\tau}\right]}{\mathbb{E}_{t}\left[\sum_{\tau=t}^{\infty} \theta_{k}^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^{k} \left(P_{k,\tau}\right)^{\epsilon_{k}} Y_{k,\tau} \left(P_{k,\tau}\right)^{-1} \left(V_{-k,\tau}^{k}\right)^{-\sigma_{k}}\right]},$$

Multiplying with $\frac{1}{P_{k,t}} = \frac{P_{k,t}^{\epsilon_k - 1}}{P_{k,t}^{\epsilon_k}}$:

$$P_{k,t}^{*}(j) = \frac{\epsilon_{k}}{\epsilon_{k}-1} \frac{\mathbb{E}_{t} \left[\sum_{\tau=t}^{\infty} \theta_{k}^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^{k} \left(P_{k,\tau}\right)^{\epsilon_{k}} Y_{k,\tau} m c_{k,m,\tau}\right]}{\mathbb{E}_{t} \left[\sum_{\tau=t}^{\infty} \theta_{k}^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^{k} \left(P_{k,\tau}\right)^{\epsilon_{k}-1} Y_{k,\tau} \left(V_{-k,\tau}^{k}\right)^{-\sigma_{k}}\right]},$$

$$\Leftrightarrow \frac{P_{k,t}^{*}(j)}{P_{k,t}} = \frac{\epsilon_{k}}{\epsilon_{k}-1} \frac{\mathbb{E}_{t} \left[\sum_{\tau=t}^{\infty} \theta_{k}^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^{k} \left(\Pi_{k,\tau}\right)^{\epsilon_{k}} Y_{k,\tau} m c_{k,m,\tau}\right]}{\mathbb{E}_{t} \left[\sum_{\tau=t}^{\infty} \theta_{k}^{\tau-t} \beta^{\tau-t} \Lambda_{t,\tau}^{k} \left(\Pi_{k,\tau}\right)^{\epsilon_{k}-1} Y_{k,\tau} \left(V_{-k,\tau}^{k}\right)^{-\sigma_{k}}\right]}.$$

Re-write expression as

$$\frac{P_{k,t}^{*}(j)}{P_{k,t}} = \frac{\epsilon_k}{\epsilon_k - 1} \frac{x_{k,1,t}}{x_{k,2,t}},$$
(F.2)

where

$$x_{k,1,t} \equiv \left[\frac{Z_t}{C_t^k - \Psi_k C_{t-1}^k} - \frac{\mathbb{E}_t[Z_{t+1}]\Psi_k\beta}{\mathbb{E}_t[C_{t+1}^k] - \Psi_k C_t^k}\right] Y_{k,t} \ mc_{k,m,t} + \beta \theta_k \,\mathbb{E}_t \left[(\Pi_{k,t,t+1})^{\epsilon_k} \ x_{k,1,t+1} \right],$$

$$x_{k,2,t} \equiv \left[\frac{Z_t}{C_t^k - \Psi_k C_{t-1}^k} - \frac{\mathbb{E}_t[Z_{t+1}]\Psi_k \beta}{\mathbb{E}_t[C_{t+1}^k] - \Psi_k C_t^k}\right] Y_{k,t} \left(V_{-k,t}^k\right)^{-\sigma_k} + \beta \theta_k \mathbb{E}_t \left[(\Pi_{k,t,t+1})^{\epsilon_k - 1} x_{k,2,t+1}\right],$$

$$\Leftrightarrow x_{k,1,t} \equiv U_{c,t}^k Y_{k,t} m c_{k,m,t} + \beta \theta_k \mathbb{E}_t \left[(\Pi_{k,t,t+1})^{\epsilon_k} x_{k,1,t+1} \right],$$

$$\Leftrightarrow x_{k,2,t} \equiv U_{c,t}^k Y_{k,t} \left(V_{-k,t}^k \right)^{-\sigma_k} + \beta \theta_k \mathbb{E}_t \left[(\Pi_{k,t,t+1})^{\epsilon_k - 1} x_{k,2,t+1} \right].$$

G Profit Maximization of the Bank

The bank's objective function is given by:

$$\max \mathbb{E}_{t}[\Gamma_{b,t+1}^{k}] = i_{k,t}^{L} L_{t,t+1}^{k} + i^{RR} r \mathbb{E}_{t}[D_{t+1}^{k}] + i^{ER} \mathbb{E}_{t}[R_{t+1}^{ER,k}] - i_{t} \mathbb{E}_{t}[D_{t+1}^{k}] - \frac{1}{2} \upsilon_{k} \left(\mathbb{E}_{t}[D_{t+1}^{k}]\right)^{2},$$
(G.1)

with
$$D_{t+1}^{k} = D_{k,t+1}^{ex} + D_{t,t+1}^{L,k}, D_{t,t+1}^{L,k} = L_{t,t+1}^{k}$$
, and $R_{t+1}^{ER,k} = D_{k,t+1}^{ex} - r\left(D_{k,t+1}^{ex} + D_{t,t+1}^{L,k}\right)$. Hence,

$$\max_{L_{t,t+1}^{k}} \mathbb{E}[\Gamma_{b,t+1}^{k}] = i_{k,t}^{L} L_{t,t+1}^{k} + i^{RR} r \mathbb{E}_{t} \left[D_{k,t+1}^{ex} + L_{t,t+1}^{k}\right] + i^{ER} \mathbb{E}_{t} \left[D_{k,t+1}^{ex} - r\left(D_{k,t+1}^{ex} + L_{t,t+1}^{k}\right)\right] - i_{t} \mathbb{E}_{t} \left[D_{k,t+1}^{ex} + L_{t,t+1}^{k}\right] - \frac{1}{2} v_{k} \left(\mathbb{E}_{t}[D_{k,t+1}^{ex} + L_{t,t+1}^{k}]\right)^{2}$$
(G.2)

The first order condition is given by

$$\frac{\partial \mathbb{E}_t[\Gamma_{b,t+1}^k]}{\partial L_{t,t+1}^k} = i_{k,t}^L + i^{RR}r - i^{ER}r - i_t - \upsilon_k \left(\mathbb{E}_t[D_{k,t+1}^{ex}] + L_{t,t+1}^k \right) \stackrel{!}{=} 0,$$

$$\Leftrightarrow i_{k,t}^{L} + r(i^{RR} - i^{ER}) = i_t + v_k \left(\mathbb{E}_t[D_{k,t+1}^{ex}] + L_{t,t+1}^k \right).$$
(G.3)

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Contribution

I, Daniel Stempel, have contributed substantially to the conceptualization, development of the methodology, software, formal analysis, writing of the original draft, review and editing, and visualization of the results of this paper.

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Paper III

How Should Central Banks React to Household Inflation Heterogeneity?*

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Abstract

Empirical evidence suggests that considerable differentials in inflation rates exist across households. This paper investigates how central banks should react to household inflation heterogeneity in a tractable New Keynesian model. We include two households that differ in their consumer price inflation rates after adverse shocks. The central bank reacts to either an average of the households' consumer price inflation rates or their individual rates, respectively. After a negative demand shock, the consumer price inflation rates of both households diverge less from their steady states when the central bank only considers the individual inflation rate of the household experiencing the higher inflation rate. Furthermore, output fluctuates less under that regime. After a negative supply shock, a central bank only considering the household experiencing the higher inflation rate mitigates the immediate effects of the shock on both consumer price inflation rates more effectively. Our results imply that central banks, which react discretionarily to differing inflation experiences in an economy, lead to a more efficient attainment of an economy-wide inflation target and to lower fluctuations of all inflation rates.

JEL classifications: E31, E32, E52

Keywords: Business cycles, inflation, inequality, household heterogeneity, New Keynesian models

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Contents

Li	st of	Tables	104
List of Figures		104	
1	Intr	roduction	105
2	ΑN	Nodel with Household Inflation Heterogeneity	107
	2.1	Households	107
	2.2	Firms	110
		2.2.1 Firm 1	110
		2.2.2 Firm 2	111
	2.3	Monetary Policy	113
	2.4	Market Clearing	113
	2.5	Aggregate Dynamics	114
3	\mathbf{Res}	sults	117
	3.1	Calibration	117
	3.2	Dynamic Analysis	118
		3.2.1 Demand Shock	118
		3.2.2 Cost-Push Shock	121
4	Con	nclusion	123
\mathbf{A}	ppen	dices	124
	Ā	Expenditure Minimization of the Household	124
		A.1 Composition of the Essential and Non-Essential Composite Consumption	
		Good	124
		A.2 Allocation between Essential and Non-Essential Goods	126
	В	Consumption Expenditures in the Budget Constraint	128
	\mathbf{C}	Utility Maximization of the Household	128
	D	Risk Sharing	129
	\mathbf{E}	Profit Maximization of the Firm	130
	\mathbf{F}	Log-Linearization	131
		F.1 Dynamic IS Equation	
		F.2 New Keynesian Philips Curve	132
		F.3 Marginal Costs	135
		F.4 Consumer Price Inflation	137
		F.5 Overall Output	137
R	efere	nces	137
Publications and Contribution			

List of Tables

1	Calibration.	117
2	0.5% Demand Shock Volatilities.	120
3	1% Cost-Push Shock Volatilities	123

List of Figures

1	Impulse Responses to a Negative 0.5% Demand Shock with Persistence $\rho_Z = 0.9.119$
2	Impulse Responses to a Positive 1% Cost-Push Shock with Persistence $\rho_A = 0.9$. 121

1 Introduction

For central banks, an accurate measure of inflation is vital in order to appropriately implement monetary policy. However, commonly used consumer price indices (CPI) hide substantial heterogeneity across households, depending on various household characteristics. For instance, studies show that households with lower income experience considerably higher inflation rates than households with higher income (see Gürer and Weichenrieder (2020), for instance).

Against this background, this paper analyzes how central banks that aim to stabilize the economy-wide inflation rate should react to household inflation heterogeneity. We introduce two households into a tractable New Keynesian model: a low- and a high-income household, with the low-income household experiencing higher CPI inflation after adverse shocks. In our model, the central bank is assumed to follow a Taylor rule considering either only the CPI inflation rate of one of the households or a weighted average of both CPI inflation rates, respectively. We find that household inflation heterogeneity, and therefore the weight the central bank assigns to the respective CPI inflation rates, has significant effects on the model outcomes. After a negative demand shock, a central bank that only reacts to the inflation rate experienced by the low-income household mitigates the impact of the shock more effectively. The CPI inflation rates of both households and output exhibit lower volatility under that regime. After a negative supply shock, a central bank that only considers CPI inflation of the low-income household mitigates the initial impact of the shock on CPI inflation of both households more effectively. However, these inflation rates as well as output exhibit higher volatility under that regime. These results are generalizable and do not depend on income differences but rather only on inflation differentials across households. In particular, we find that central banks are able to stabilize the volatility of the economy-wide inflation rate more effectively after demand and supply shocks when only considering the household whose CPI inflation rate is less affected by these shocks.

Moreover, our results have considerable monetary policy implications. Discretionary reactions of central banks likely lead to lower fluctuations of economy-wide inflation rates after shocks. In particular, it seems sensible for central banks to consider a range of inflation rates experienced in an economy. Depending on the type of shock, the central bank could then choose to react to specific inflation rates in order to reach its economy-wide inflation target more effectively and stabilize all inflation rates in the economy. Considering the Taylor rule in our model, this discretion implies a central bank that is able to choose the weight of the household-specific inflation rates depending on the type of shock.

Our paper relates to the literature in the following ways. It connects to the strand of literature investigating the relationship between inflation and income inequality, such as Al-Marhubi (1997), or Albanesi (2007). Our paper further complements work that empirically investigates inflation differentials between households and that relates these differentials to certain household characteristics. In particular, this includes studies showing that households with lower income experience higher inflation rates than households with higher income, such as Hobijn et al. (2009), Kaplan and Schulhofer-Wohl (2017), Jaravel (2019), Gürer and Weichenrieder (2020), or Argente and Lee (2021). As shown by Hobijn et al. (2009), Portillo et al. (2016), and Gürer and Weichenrieder (2020), this property can be ascribed to the fact that low-income households spend a higher share of their income on essential goods (like food, electricity, gas, or rent), as these goods exhibit above-average inflation. In addition, there is evidence that high-income households can substitute goods more effectively (Gürer and Weichenrieder, 2020; Argente and Lee, 2021), contributing to the inflation differential. Our paper also relates to theoretical literature examining the various effects of inflation differentials. Most of this work focuses on regional inflation differentials within currency unions (Canzoneri et al., 2006; Duarte and Wolman, 2008), in particular on the European monetary union (Angeloni and Ehrmann, 2007; Andrés et al., 2008; Rabanal, 2009). Lastly, our paper links to work that analyzes the effects of various types of household heterogeneity in New Keynesian models. In particular, this includes studies examining income and wealth inequality, such as Gornemann et al. (2016), Kaplan et al. (2018), or Luetticke (2018).¹ We contribute to these strands of the literature by theoretically examining how central banks should react to inflation differentials across households, thereby analyzing the effects of household inflation heterogeneity on business cycle fluctuations.

The paper is organized as follows: Section 2 states the model before Section 3 describes the model responses to a demand and a supply shock. Section 4 concludes.

¹For a comprehensive overview, see Kaplan and Violante (2018).

2 A Model with Household Inflation Heterogeneity

2.1 Households

There exist two households, k=L,H. We will calibrate L to be the household with lower income and H to be the household with higher income. The share of household L is denoted by κ , the share of household H by $1-\kappa$. The period utility function of household k is given by

$$U_{t}^{k} = \frac{\left(C_{t}^{k}\right)^{1-\sigma_{k}}}{1-\sigma_{k}} - \frac{\left(N_{t}^{k}\right)^{1+\varphi_{k}}}{1+\varphi_{k}},\tag{1}$$

where σ_k is the inverse intertemporal elasticity of substitution, N_t^k denotes labor supply, φ_k the inverse Frisch elasticity of labor supply, and C_t^k is defined as a constant elasticity of substitution (CES) index given by

$$C_{t}^{k} \equiv \left(\gamma_{k}^{\frac{1}{\vartheta_{C}^{k}}} \left(C_{1,t}^{k} - C_{1}^{*}\right)^{\frac{\vartheta_{C}^{k} - 1}{\vartheta_{C}^{k}}} + \left(1 - \gamma_{k}\right)^{\frac{1}{\vartheta_{C}^{k}}} Z_{t}^{\frac{1}{\vartheta_{C}^{k}}} \left(C_{2,t}^{k}\right)^{\frac{\vartheta_{C}^{k} - 1}{\vartheta_{C}^{k}}}\right)^{\frac{\vartheta_{C}^{k}}{\vartheta_{C}^{k} - 1}},$$
(2)

similar to Rabanal (2009). The parameter γ_k determines the household-specific share of type 1 goods, presented by the consumption index $C_{1,t}^k$, in the overall consumption index. We interpret type 1 goods as essential goods (such as food, gas, or rent) with a subsistence level of C_1^* that has to be met at all times. We further assume that households always have enough income to finance this subsistence level. $C_{2,t}^k$ denotes the consumption index of type 2 goods, i.e., non-essential goods. The parameter ϑ_C^k is defined as the elasticity of substitution between the two types of goods and Z_t is an AR(1) demand shock affecting solely non-essential goods. This property tallies with the results of empirical analyses, showing that households decrease non-essential good consumption rather than essential good consumption after adverse economic shocks (see Kamakura and Yuxing Du (2012) and Loxton et al. (2020), for instance). Both indices, $C_{h,t}^k$ with h=1, 2, are CES functions over all goods $i \in [o, s]$ and $j \in [s, 1]$, with s being the share of firms producing good 1 in the economy, given by

$$C_{1,t}^{k} \equiv \left(\int_{0}^{s} C_{i,t}^{k} \frac{\epsilon-1}{\epsilon} di\right)^{\frac{\epsilon}{\epsilon-1}},\tag{3}$$

$$C_{2,t}^{k} \equiv \left(\int_{s}^{1} C_{j,t}^{k} \frac{\epsilon-1}{\epsilon} dj\right)^{\frac{\epsilon}{\epsilon-1}},\tag{4}$$

with ϵ denoting the elasticity of substitution between the varieties.

With respect to its consumption, the household chooses its optimal consumption of individual goods within each type, its optimal consumption of good types, and its optimal overall consumption level. The optimal consumption of the individual goods of each type is given by

$$C_{i,t}^{k} = \left(\frac{P_{i,t}}{P_{1,t}}\right)^{-\epsilon} C_{1,t}^{k},\tag{5}$$

$$C_{j,t}^{k} = \left(\frac{P_{j,t}}{P_{2,t}}\right)^{-\epsilon} C_{2,t}^{k},\tag{6}$$

with $P_{1,t} \equiv \left(\int_0^s P_{i,t}^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$ and $P_{2,t} \equiv \left(\int_s^1 P_{j,t}^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$ being the overall price indices of good 1 and good 2, respectively.² Optimal consumption of each variety negatively depends on the relative price of the good and the overall level of consumption of the good type.

The optimal consumption of the each good type is given by

$$C_{1,t}^{k} = \left(V_{1,t}^{C,k}\right)^{-\vartheta_{C}^{k}} \gamma_{k} C_{t}^{k} + C_{1}^{*},$$
(7)

$$C_{2,t}^{k} = \left(V_{2,t}^{C,k}\right)^{-\vartheta_{C}^{k}} (1-\gamma_{k}) Z_{t} C_{t}^{k}, \qquad (8)$$

where $V_{h,t}^{C,k} \equiv \frac{P_{h,t}}{P_t^{C,k}}$ and $P_t^{C,k} \equiv \left(\gamma_k P_{1,t}^{1-\vartheta_C^k} + (1-\gamma_k)Z_t P_{2,t}^{1-\vartheta_C^k}\right)^{\frac{1}{1-\vartheta_C^k}}$ is defined as the household-specific CPI. In general, optimal consumption of each good type depends on its relative price and overall consumption. In addition, the optimal level of good 1 consumption is determined by the subsistence level C_1^* , and the optimal level of good 2 consumption is affected by the demand shock.

The household maximizes its expected discounted lifetime utility with respect to its overall ²We denote type h goods as good h in the following.

consumption level, labor, and bond holdings:

$$\mathbb{E}_t \left[\sum_{\iota=0}^{\infty} \beta^{\iota} U_{t+\iota}^k \right],\tag{9}$$

subject to the budget constraint

$$P_t^{C,k}C_t^k + P_{1,t}C_1^* + Q_t B_t^k = B_{t-1}^k + W_t^k N_t^k + D_t^k,$$
(10)

where B_t^k are one-period, nominally risk-free bonds purchased in period t at price Q_t , W_t^k is the nominal wage, and D_t^k are dividends from the ownership of firms. The optimality conditions are given by

$$\left(N_t^k\right)^{\varphi_k} = w_t^k \left(C_t^k\right)^{-\sigma_k},\tag{11}$$

$$Q_t = \beta \mathbb{E}_t \left[\Lambda_{t,t+1}^k \frac{1}{\Pi_{t+1}^{C,k}} \right], \tag{12}$$

where $w_t^k \equiv \frac{W_t^k}{P_t^{C,k}}$ is defined as the real wage, $\beta \Lambda_{t,t+1}^k \equiv \beta \left(\frac{C_{t+1}^k}{C_t^k}\right)^{-\sigma_k}$ as the stochastic discount factor, and $\Pi_{t+1}^{C,k} \equiv \frac{P_{t+1}^{C,k}}{P_t^{C,k}}$ as CPI inflation. Equation (11) describes the optimal labor supply of household k, equating the marginal disutility from working to its marginal utility. Equation (12) is the Euler equation governing intertemporal consumption.

Due to the shared bond market, we can obtain the following risk sharing conditions between the two households by combining (12) for each household k, with -k denoting the respective other household:

$$\left(C_t^k\right)^{-\sigma_k} = \left(C_t^{-k}\right)^{-\sigma_k} \Phi^k \frac{P_t^{C,k}}{P_t^{C,-k}},\tag{13}$$

with $\Phi^k \equiv \frac{C_{SS}^{k} - \sigma_k}{C_{SS}^{-k} - \sigma_{-k}}$, where the subscript *SS* denotes the zero inflation steady state of a variable. Equation (13) implies that consumption of both households co-moves proportionally over time.

2.2 Firms

There are two types of firms in the economy: type 1 firms producing good 1 and type 2 firms producing good 2.³ We assume perfectly separated labor markets, with household L working in firm 1 and household H working in firm 2.⁴ Following Calvo (1983), we assume that only a fraction $1-\lambda_h$ of firms can reset their price in each period, independently from the last adjustment.

2.2.1 Firm 1

Firm 1 produces with a simple production function given by

$$Y_{i,t} = \left(N_{i,t}^L\right)^{1-\alpha_1},$$
(14)

where α_1 is the output elasticity labor, governing the marginal productivity of labor from household L. The firm's real total cost function is given by

$$TC_{i,t} = w_t^L N_{i,t}^L A_t, (15)$$

where A_t is an AR(1) cost-push shock. The firm maximizes its expected discounted stream of profits

$$\mathbb{E}_{t}\left[\sum_{\iota=0}^{\infty}\beta^{\iota}\Lambda_{t,t+\iota}^{L}\lambda_{1}^{\iota}\left(\frac{P_{i,t}}{P_{t+\iota}^{C,L}}Y_{i,t+\iota|t} - TC\left(Y_{i,t+\iota|t}\right)\right)\right],\tag{16}$$

subject to

$$Y_{i,t+\iota|t} = \left(\frac{P_{i,t}}{P_{1,t+\iota}}\right)^{-\epsilon} Y_{1,t+\iota},\tag{17}$$

where $Y_{i,t+\iota|t}$ is defined as the output in period $t+\iota$ for a firm that adjusts its price in period t, with $Y_{1,t+\iota}$ denoting the economy-wide output of good 1. The optimality condition is

$$0 \stackrel{!}{=} \mathbb{E}_{t} \left[\sum_{\iota=0}^{\infty} \beta^{\iota} \Lambda_{t,t+\iota}^{L} \lambda_{1}^{\iota} Y_{i,t+\iota|t} \left(\frac{P_{i,t}}{P_{t+\iota}^{C,L}} - \mu mc\left(Y_{i,t+\iota|t}\right) \right) \right], \tag{18}$$

³We denote type h firms as firm h in the following.

⁴Note that, for the sake of simplicity, we assume that household L owns firm 1 and household H owns firm 2.

with $\mu \equiv \frac{\epsilon}{\epsilon-1}$ and $mc(Y_{i,t}) = \frac{1}{1-\alpha_1} w_t^L A_t Y_{i,t}^{\frac{\alpha_1}{1-\alpha_1}}$ being defined as real marginal costs of firm *i*. The optimal price is equal for all firms that are able to adjust, due to symmetry. It is given by

$$\left(p_{1,t}^{*}\right)^{1+\frac{\epsilon\alpha_{1}}{1-\alpha_{1}}} = \mu\left(V_{1,t}^{C,L}\right)^{-1}\frac{b_{1,t}}{d_{1,t}},\tag{19}$$

where the auxiliary variables are defined as

$$b_{1,t} \equiv \left(C_t^L\right)^{-\sigma_L} Y_{1,t} m c_{1,t} + \beta \lambda_1 \mathbb{E}_t \left[\Pi_{1,t+1}^{\frac{\epsilon}{1-\alpha_1}} b_{1,t+1}\right],$$
$$d_{1,t} \equiv \left(C_t^L\right)^{-\sigma_L} Y_{1,t} + \beta \lambda_1 \mathbb{E}_t \left[\Pi_{1,t+1}^{\epsilon} \left(\Pi_{t+1}^{C,L}\right)^{-1} d_{1,t+1}\right],$$

and $p_{1,t}^* \equiv \frac{P_{1,t}^*}{P_{1,t}}$. The variable $mc_{1,t}$ denotes the economy-wide real marginal costs of good 1 and $\Pi_{1,t+1} \equiv \frac{P_{1,t+1}}{P_{1,t}}$ is defined as inflation of good 1. Aggregate price dynamics are given by

$$1 = (1 - \lambda_1) \left(p_{1,t}^* \right)^{1-\epsilon} + \lambda_1 \left(\frac{1}{\Pi_{1,t}} \right)^{1-\epsilon}.$$
 (20)

The overall price level is a weighted average of the price set by firms that are able to adjust their prices in t (given by equation (19)) and the remaining share λ_1 of firms that keep the price of the previous period.

2.2.2 Firm 2

As for firm 1, we assume a simple production function for firm 2 given by

$$Y_{j,t} = \left(N_{j,t}^{H}\right)^{1-\alpha_{2}},\tag{21}$$

where α_2 is the output elasticity labor of firm 2, determining the marginal productivity of labor from household *H*. The firm's real total cost function is given by

$$TC_{j,t} = w_t^H N_{j,t}^H.$$
(22)

Note that firm 2 does not face cost-push shocks. The firm maximizes its expected discounted stream of profits

$$\mathbb{E}_{t}\left[\sum_{\iota=0}^{\infty}\beta^{\iota}\Lambda_{t,t+\iota}^{H}\lambda_{2}^{\iota}\left(\frac{P_{j,t}}{P_{t+\iota}^{C,H}}Y_{j,t+\iota|t} - TC\left(Y_{j,t+\iota|t}\right)\right)\right],\tag{23}$$

subject to

$$Y_{j,t+\iota|t} = \left(\frac{P_{j,t}}{P_{2,t+\iota}}\right)^{-\epsilon} Y_{2,t+\iota},\tag{24}$$

with $Y_{2,t+\iota}$ denoting the economy-wide output of good 2. The optimality condition is

$$0 \stackrel{!}{=} \mathbb{E}_t \left[\sum_{\iota=0}^{\infty} \beta^{\iota} \Lambda^H_{t,t+\iota} \lambda^{\iota}_2 Y_{j,t+\iota|t} \left(\frac{P_{j,t}}{P^{C,H}_{t+\iota}} - \mu mc\left(Y_{j,t+\iota|t}\right) \right) \right], \tag{25}$$

with $mc(Y_{j,t}) = \frac{1}{1-\alpha_2} w_t^H Y_{j,t}^{\frac{\alpha_2}{1-\alpha_2}}$ being defined as real marginal costs of firm j. The optimal price is given by

$$\left(p_{2,t}^{*}\right)^{1+\frac{\epsilon\alpha_{2}}{1-\alpha_{2}}} = \mu\left(V_{2,t}^{C,H}\right)^{-1}\frac{b_{2,t}}{d_{2,t}},\tag{26}$$

where the auxiliary variables are defined as

$$b_{2,t} \equiv (C_t^H)^{-\sigma_H} Y_{2,t} m c_{2,t} + \beta \lambda_2 \mathbb{E}_t \left[\Pi_{2,t+1}^{\frac{\epsilon}{1-\alpha_2}} b_{2,t+1} \right],$$
$$d_{2,t} \equiv (C_t^H)^{-\sigma_H} Y_{2,t} + \beta \lambda_2 \mathbb{E}_t \left[\Pi_{2,t+1}^{\epsilon} \left(\Pi_{t+1}^{C,H} \right)^{-1} d_{2,t+1} \right],$$

and $p_{2,t}^* \equiv \frac{P_{2,t}^*}{P_{2,t}}$. The variable $mc_{2,t}$ denotes the economy-wide real marginal costs of good 2 and $\Pi_{2,t+1} \equiv \frac{P_{2,t+1}}{P_{2,t}}$ is defined as inflation of good 2. Aggregate price dynamics are defined as

$$1 = (1 - \lambda_2) \left(p_{2,t}^* \right)^{1-\epsilon} + \lambda_2 \left(\frac{1}{\Pi_{2,t}} \right)^{1-\epsilon}.$$
 (27)

2.3 Monetary Policy

We assume that the central bank wants to stabilize economy-wide inflation. The central bank follows a Taylor rule given by

$$i_t = \rho + \phi_\pi \left(\delta_\pi \pi_t^{C,L} + (1 - \delta_\pi) \, \pi_t^{C,H} \right), \tag{28}$$

where $i_t \equiv log\left(\frac{1}{Q_t}\right)$, $\rho \equiv log\left(\frac{1}{\beta}\right)$, and $\pi_t^{C,k} \equiv log\left(\Pi_t^{C,k}\right)$. The parameter $\phi_{\pi} > 1$ denotes the reaction coefficient of the central bank to the weighted (with $\delta_{\pi} \in [0,1]$) CPI inflation rates of households L and H. The parameter δ_{π} is of particular importance for our analysis. If $\delta_{\pi} = \kappa$, the central bank reacts to the average, economy-wide inflation rate given by

$$\pi_t^C = \kappa \pi_t^{C,L} + (1 - \kappa) \pi_t^{C,H}.$$
(29)

However, we additionally consider $\delta_{\pi} \neq \kappa$, i.e., the central bank reacts more strongly to the CPI inflation rate of either household H ($\delta_{\pi} < \kappa$) or L ($\delta_{\pi} > \kappa$) than suggested by the economy-wide inflation rate.

Furthermore, the Fisher equation holds for each household

$$i_t = r_t^k + \mathbb{E}_t \left[\pi_{t+1}^{C,k} \right]. \tag{30}$$

2.4 Market Clearing

Bonds markets clear

$$B_t^k = -B_t^{-k}, (31)$$

as well as labor markets

$$N_t^L = \int_0^s N_{i,t}^L di \ , \ N_t^H = \int_s^1 N_{j,t}^H dj.$$
(32)

Finally, goods markets clear for both goods

$$Y_{1,t} = \kappa C_{1,t}^{L} + (1-\kappa)C_{1,t}^{H} , \quad Y_{2,t} = \kappa C_{2,t}^{L} + (1-\kappa)C_{2,t}^{H},$$
(33)

and overall production is given by

$$Y_t = sY_{1,t} + (1-s)Y_{2,t}.$$
(34)

2.5 Aggregate Dynamics

In log-linear fashion, with \hat{x} being defined as the log-linear deviation of variable X from its steady state and $x \equiv log(X)$, the dynamic IS equation is given by

$$\hat{c}_t^k = \mathbb{E}_t \left[\hat{c}_{t+1}^k \right] - \frac{1}{\sigma_k} \left(\hat{i}_t - \mathbb{E}_t \left[\hat{\pi}_{t+1}^{C,k} \right] \right), \qquad (35)$$

implying that consumption in period t depends positively on expected consumption in t+1 representing consumption smoothing and negatively on the real interest rate due to a lower incentive to consume.

For each firm h, a sort of New Keynesian Phillips curve relating the inflation rate of good h to marginal costs, relative prices, and future inflation can be derived as

$$\hat{\pi}_{h,t} = \Psi_h \left(\hat{mc}_{h,t} - \hat{v}_{h,t}^C \right) + \beta \mathbb{E}_t \left[\hat{\pi}_{h,t+1} \right], \tag{36}$$

with $\Psi_h \equiv (1 - \beta \lambda_h) \frac{1 - \lambda_h}{\lambda_h} \frac{1 - \alpha_h}{1 - \alpha_h + \epsilon \alpha_h}$, $\hat{v}_{1,t}^C \equiv \hat{v}_{1,t}^{C,L}$, $\hat{v}_{2,t}^C \equiv \hat{v}_{2,t}^{C,H}$, and where

$$\hat{mc}_{1,t} = \frac{(\alpha_1 + \varphi_L)g_{L,1}\kappa_{\overline{l_{L,1}}}^1\gamma_L + \sigma_L(1 - \alpha_1)}{1 - \alpha_1}\hat{c}_t^L + \frac{(\alpha_1 + \varphi_L)g_{H,1}(1 - \kappa)\frac{1}{l_{H,1}}\gamma_H}{1 - \alpha_1}\hat{c}_t^H - \frac{(\alpha_1 + \varphi_L)g_{L,1}\kappa_{\overline{l_{L,1}}}^1\gamma_L\vartheta_C^L}{1 - \alpha_1}\hat{v}_{1,t}^{C,L} - \frac{(\alpha_1 + \varphi_L)g_{H,1}(1 - \kappa)\frac{1}{l_{H,1}}\gamma_H\vartheta_C^H}{1 - \alpha_1}\hat{v}_{1,t}^{C,H} + a_t, \quad (37)$$

and

$$\hat{mc}_{2,t} = \frac{(\alpha_2 + \varphi_H)g_{L,2\kappa}}{1 - \alpha_2}\hat{c}_t^L + \frac{(\alpha_2 + \varphi_H)g_{H,2}(1 - \kappa) + \sigma_H(1 - \alpha_2)}{1 - \alpha_2}\hat{c}_t^H \\
- \frac{(\alpha_2 + \varphi_H)g_{L,2\kappa}\vartheta_C^L}{1 - \alpha_2}\hat{v}_{2,t}^{C,L} - \frac{(\alpha_2 + \varphi_H)g_{H,2}(1 - \kappa)\vartheta_C^H}{1 - \alpha_2}\hat{v}_{2,t}^{C,H} + \frac{(\alpha_2 + \varphi_H)(\kappa g_{L,2} + (1 - \kappa)g_{H,2})}{1 - \alpha_2}z_t,$$
(38)

where $g_{k,h} \equiv \frac{C_{h,SS}^k}{Y_{h,SS}}$, $l_{k,h} \equiv \frac{C_{h,SS}^k}{C_{SS}^k}$, and the relative price $\hat{v}_{h,t}^{C,k} = \hat{p}_{h,t} - \hat{p}_t^{C,k}$ can be rewritten in terms

of inflation rates as

$$\hat{v}_{h,t}^{C,k} - \hat{v}_{h,t-1}^{C,k} = \hat{\pi}_{h,t} - \hat{\pi}_t^{C,k}.$$
(39)

Equations (36)–(38) imply that the inflation rate of firm h positively depends on the consumption of the respective good by each household, since higher consumption leads to higher demand for labor by firms which in turn increases wages (i.e, costs). Furthermore, inflation of firm h negatively depends on the relative price of good h with respect to the CPI of households L and H. Consider, for instance, an increase in the CPI of household k, while the price of good h remains unchanged. In this case, the relative price of good h decreases and its demand increases. This implies an increase in output and labor demand by firm h, leading to higher wages, i.e., higher marginal costs.

The described impact of consumption and relative prices positively depends on φ_k , governing the convexity of the utility function in labor, as a higher disutility of labor necessitates higher increases in wages and thereby marginal costs (see equation (11)). Furthermore, the impact of the relative prices is strengthened by larger values of ϑ_C^k due to a corresponding higher importance of the relative price of a good for its demand (see equations (7) and (8)). More pronounced changes in demand lead to larger changes in marginal costs. Naturally, marginal costs and thereby inflation of good 1 positively depend on the cost-push shock.

Finally, inflation of good 2 depends positively on the demand shock. Consider, for instance, a negative demand shock: the decrease in demand for good 2 leads to lower labor demand by firm 2, implying lower wages and marginal costs.

Solving equation (36) forward, we get

$$\hat{\pi}_{h,t} = \Psi_h \sum_{\iota=0}^{\infty} \beta^{\iota} \mathbb{E}_t \left[\hat{mc}_{h,t+\iota} - \hat{v}_{h,t+\iota}^C \right].$$
(40)

Equation (40) reveals that inflation in period t depends on current and (discounted) future changes in marginal costs, as firms that can adjust their prices consider that they might not be able to do so in the future. Furthermore, inflation negatively depends on current and (discounted) future changes in the relative price, implying that inflation of the individual firm co-moves with the CPI inflation rate. Consider, for instance, an increase in the CPI: in that case, firm h is also able to set a higher price without losing demand. CPI inflation follows

$$\hat{\pi}_{t}^{C,k} = \gamma_{k}\hat{\pi}_{1,t} + (1-\gamma_{k})\hat{\pi}_{2,t} + \frac{1-\gamma_{k}}{1-\vartheta_{C}^{k}}\Delta z_{t},$$
(41)

where $\Delta z_t \equiv z_t - z_{t-1}$. CPI inflation of each household is a weighted average of the inflation rates of both firms and further depends positively on the demand shock.

Finally, aggregate output is given by

$$\hat{y}_{t} = \left(m_{1}\kappa g_{L,1}\frac{1}{l_{L,1}}\gamma_{L} + m_{2}\kappa g_{L,2}\right)\hat{c}_{t}^{L} + \left(m_{1}(1-\kappa)g_{H,1}\frac{1}{l_{H,1}}\gamma_{H} + m_{2}(1-\kappa)g_{H,2}\right)\hat{c}_{t}^{H} - \left(m_{1}\kappa g_{L,1}\frac{1}{l_{L,1}}\gamma_{L}\vartheta_{C}^{L}\right)\hat{v}_{1,t}^{C,L} - \left(m_{2}\kappa g_{L,2}\vartheta_{C}^{L}\right)\hat{v}_{2,t}^{C,L} - \left(m_{1}(1-\kappa)g_{H,1}\frac{1}{l_{H,1}}\gamma_{H}\vartheta_{C}^{H}\right)\hat{v}_{1,t}^{C,H} - \left(m_{2}(1-\kappa)g_{H,2}\vartheta_{C}^{H}\right)\hat{v}_{2,t}^{C,H} + \left(\kappa g_{L,2} + (1-\kappa)g_{H,2}\right)m_{2}z_{t}, \quad (42)$$

where $m_1 \equiv \frac{sY_{1,SS}}{Y_{SS}}$ and $m_2 \equiv \frac{(1-s)Y_{2,SS}}{Y_{SS}}$. Equation (42) reveals that overall output depends positively on the overall consumption of both households and negatively on all relative prices. The first line of the equation shows that higher consumption increases output of each firm and thereby overall output. The weighted sum multiplying \hat{c}_t^k corresponds to the share of a change in overall consumption that translates into a change in the consumption of good 1 and 2. An increase in the relative price leads to lower output of each firm and, consequently, to lower overall output. The strength of this effect positively depends on the share of the respective good in consumption and output as well as on ϑ_C^k , as a higher elasticity of substitution between good 1 and 2 leads to a higher relevance of the relative price for the consumption of the good (equations (7) and (8)). These effects are symmetric for the low-income (second line of equation (42)) and the high-income household (third line). Lastly, a negative demand shock leads to a decrease in overall output due to lower demand for good 2, as displayed in the fourth line of equation (42).

3 Results

3.1 Calibration

Table 1 shows the calibration of the model. We calibrate household H to be the household with higher income. Accordingly, we set $\vartheta_C^L < \vartheta_C^H$ in order to reflect that households with higher income can substitute goods more effectively (Gürer and Weichenrieder, 2020; Argente and Lee, 2021). The values are chosen to represent data retrieved from the United States Department of Agriculture (2012).

	Description	Value		Target/Source				
Households								
		L	Η					
κ	Share of households	0.5	0.5	Equal share of L and H households				
σ_k	Inverse intertemporal	2.5	1.5	Average intertemporal elasticity of				
	elasticity of substitution			substitution: 0.53				
φ_k	Inverse Frisch elasticity	5	5	Frisch elasticity of labor supply: 0.2				
	of labor supply							
γ_k	Weight of good 1	0.57	0.46	$\frac{C_{1,SS}^L}{C_{1,SS}^L + C_{2,SS}^L} = 0.65, \ \frac{C_{1,SS}^H}{C_{1,SS}^H + C_{2,SS}^H} = 0.5,$				
	in overall consumption			internally calibrated				
ϑ^k_C	Elasticity of substitution	0.15	0.5	Larger substitution capabilities of household ${\cal H}$				
	between good 1 and 2							
C_1^*	Subsistence level of good 1	0.2	0.2	$\frac{C_{1,SS}^L}{C_{1,SS}^L + C_{2,SS}^L} = 0.65, \ \frac{C_{1,SS}^H}{C_{1,SS}^H + C_{2,SS}^H} = 0.5,$				
				internally calibrated				
ϵ	Price elasticity of demand	9	9	Steady state markup: 12.5%				
β	Discount rate	0.99	0.99	Yearly nominal interest rate: 4%				
			Fi	irms				
		1	2					
s	Share of firm 1	0.5	0.5	Equal share of firms				
α_h	Output elasticity labor	0.5	0.33	Higher income of household H				
λ_h	Calvo parameter	0.6	0.8	Higher flexibility of good 1 prices				
Central Bank								
ϕ_{π}	Taylor rule coefficient	1.5		Galí (2015)				
δ_{π}	CPI inflation weight	0; 0.5; 1		Analysis parameter				

Table 1:	Calibration.
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We set the average intertemporal elasticity of substitution to an empirically plausible value of 0.53 (see Hall, 1988; Atkeson and Ogaki, 1996; Rupert et al., 2000; Gnocchi et al., 2016). Note that we set $\sigma_L > \sigma_H$, taking into account the fact that households with lower income exhibit a lower intertemporal elasticity of substitution.⁵ We set $\varphi_k=5$, leading to a Frisch elasticity of labor supply of 0.2, which is in line with the findings of Chetty et al. (2012) or Peterman (2015), for instance. We calibrate γ_k and C_1^* to match the relative consumption of good 1 and 2 in steady state, as presented in Gürer and Weichenrieder (2020). In particular, Gürer and Weichenrieder (2020) find that low-income households spend roughly 65% of their consumption expenditures on goods with above-average CPI inflation, while that share amounts to about 50% for high-income households.⁶ The remaining standard household parameters are chosen as in Galí (2015).

On the firms' side, we follow Kaplan et al. (2018) by setting α_2 to 0.33. We continue by choosing $\alpha_1 > \alpha_2$, implying lower productivity of household L and thereby lower income of that household. In order to account for the fact that food prices are more flexible and volatile than non-food prices (Portillo et al., 2016), we set $\lambda_1 < \lambda_2$, since we assume good 1 to be the essential good which includes food, for instance. Lastly, we solve the model with three different weights on CPI inflation of household L in the Taylor rule: 0, 0.5, and 1. The central bank considers only the low-income household ($\delta_{\pi}=1$), only the high-income household ($\delta_{\pi}=0$), or a weighted average of both households ($\delta_{\pi}=0.5$).

3.2 Dynamic Analysis

3.2.1 Demand Shock

Figure 1 shows the impulse responses of the model (as percentage deviations from the zero inflation steady state) to a negative 0.5% demand shock on non-essential goods for the three monetary policy regimes. In general, i.e., independently from the regime of the central bank, the effects of the demand shock are as follows:

The shock implies that both households decrease their consumption of the non-essential good 2. This lower demand leads to a lower output and a decrease in inflation of non-essential goods. All CPI inflation rates decrease.⁷ The decrease is larger for household H than for household L, as the high-income household spends a higher share of its income on non-essential goods. This result tallies with the fact that low-income households experience higher inflation

⁵For a comprehensive overview of empirical studies on this property, see Havranek et al. (2015).

⁶Note that in Gürer and Weichenrieder (2020), these values correspond to the lowest and highest income decile. Our results remain qualitatively unchanged when considering a lower difference between the households' consumption shares spent on goods with above-average CPI inflation.

⁷Note that the strong initial decrease in the CPI inflation rates is due to the relationship between $\hat{\pi}_t^{C,k}$ and Δz_t , as derived in equation (41).

rates than high-income households (see Section 1).⁸ Note that the decrease in CPI inflation implies downward pressure on the prices of essential goods as the CPI decreases and essential goods become relatively more expensive (see equation (40)). The central bank reacts to the decrease in CPI inflation by decreasing the nominal interest rate. The resulting drop in the real interest rate incentivizes the consumption of both goods. This implies that the displayed decrease of good 2 output is already mitigated and the output of good 1 even increases due to the expansionary monetary policy reaction. Furthermore, the decrease in inflation of both essential and non-essential goods caused by the demand shock is mitigated, as higher demand due to lower interest rates leads firms to adjust their prices upwards.

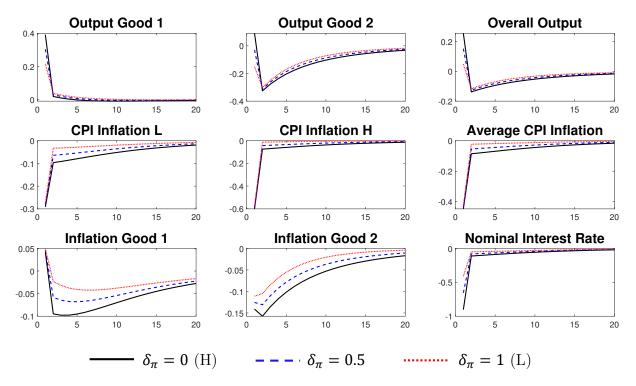


Figure 1: Impulse Responses to a Negative 0.5% Demand Shock with Persistence $\rho_Z = 0.9$.

Upon examining the effects of the different central bank regimes, we find that the weight on the respective CPI inflation rates has a significant impact on the model outcomes. Overall, the higher the weight on the CPI inflation rate of the high-income household is, the more expansionary the central bank reacts as this household experiences a stronger drop in its CPI inflation rate. However, the central bank reaches its goal of economy-wide consumer price

⁸Note that in case of a positive demand shock, the consumer price inflation rate of household H is larger than the one of household L. However, the results of our analysis remain unchanged.

stability most efficiently when only considering the low-income household (i.e., the household experiencing higher CPI inflation): the CPI inflation rates of both households—and thereby also the economy-wide, average CPI inflation rate—diverge less from their steady states when the central bank only reacts to household L, as the inflation rates of good 1 and 2 fluctuate less. Since household L's CPI inflation rate drops less, the nominal interest rate decreases less and households shift less consumption from the future into the initial period, implying higher demand for goods over time. Therefore, the incentive to increase consumption is lower and output of both goods increases less. This implies a lower initial increase in marginal costs. However, firms do not only consider current but also future marginal costs when setting their price (see equation (40)). After the initial shock period, marginal costs are consistently higher the larger δ_{π} is, as consumption for both goods is higher the larger δ_{π} is. Therefore, the deviations of all inflation rates from their steady states are lower in every period.

This result is further underscored by Table 2, which displays the volatilities of model variables under the different Taylor rules. All variables fluctuate less when only the CPI inflation rate of the low-income household is considered. These results are driven by decreasing fluctuations of the nominal interest rate when δ_{π} increases: the less expansionary reaction of the central bank results in a smaller increase in the nominal interest rate between the initial and the subsequent period, i.e., the nominal interest rate displays lower volatility. This leads households to shift less consumption from the future into the initial period and consume more over time.

			Volatility	
Variable	Description	$\delta_{\pi} = 0 (\mathrm{H})$	$\delta_{\pi} = 0.5$	$\delta_{\pi} = 1 \ (L)$
\hat{c}_t^L	Overall consumption L	0.448	0.394	0.351
\hat{c}^H_t	Overall consumption H	1.184	1.129	1.087
$\hat{y}_{1,t}$	Output good 1	0.394	0.307	0.222
$\hat{y}_{2,t}$	Output good 2	0.652	0.592	0.564
\hat{y}_t	Overall output	0.391	0.299	0.227
$\hat{\pi}_t^{C,L}$	CPI inflation L	0.385	0.330	0.288
$\hat{\pi}_t^{C,H}$	CPI inflation H	0.636	0.601	0.583
$\hat{\pi}^C_t$	Average CPI inflation	0.497	0.458	0.432
$\hat{\pi}_{1,t}$	Inflation good 1	0.310	0.230	0.154
$\hat{\pi}_{2,t}$	Inflation good 2	0.351	0.279	0.211

Table 2: 0.5% Demand Shock Volatilities.

Notes. All variables are deviations from their zero inflation steady state.

Hence, consumption and output exhibit less volatility, and thereby also the inflation rates of essential and non-essential goods, the more the central bank weights the CPI inflation rate of the low-income household. This further implies less volatility of both CPI inflation rates.

3.2.2 Cost-Push Shock

Figure 2 shows the impulse responses of the model (as percentage deviations from the zero inflation steady state) to a positive 1% cost-push shock on essential goods for the three monetary policy regimes. Again, we start with a general description of the effects of the shock on the model variables, independently of the central bank's regime.

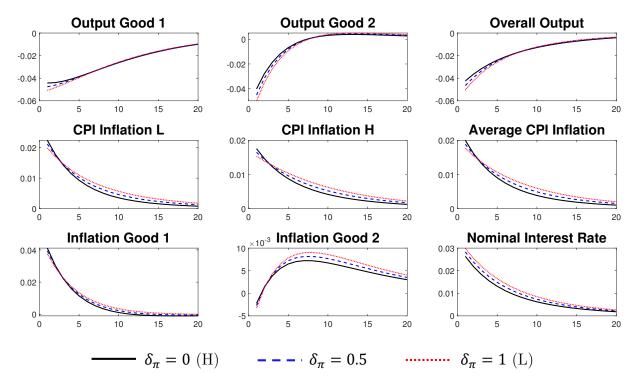


Figure 2: Impulse Responses to a Positive 1% Cost-Push Shock with Persistence $\rho_A = 0.9$.

The increase in marginal costs prompts firm 1 to increase its price, causing households to consume less of the essential good 1. In addition, CPI inflation of both households increases. The low-income household is affected more strongly than the high-income household, as the low-income household spends a higher share of its income on essential goods. The central bank increases the nominal interest rate in order to mitigate the effects of the shock on CPI inflation. The resulting increase in the real interest rate incentivizes households to save rather than to consume. Hence, consumption (and thereby output) of both goods decreases. This effect strengthens the decrease of good 1 output caused by the shock—the typical problem for monetary policy when facing supply shocks. Furthermore, there are two opposing effects on the inflation rate of non-essential goods: the increase in the CPI of both households allows firm 2 to increase its price, while the decrease in demand implies downward pressure on prices. After the initial period, the first effect dominates and inflation of non-essential goods increases.

Moreover, when examining the impact of the three monetary policy regimes, the impulse responses again show that the weight assigned to the respective CPI inflation rates significantly affects the model outcomes. In particular, when only considering the CPI inflation rate of the low-income household, the central bank manages to mitigate the effect of the cost-push shock on all inflation rates more effectively in the initial period: the inflation rates of essential and non-essential goods as well as the CPI inflation rates of both households are lower under this regime. However, all inflation rates deviate more from their steady states over time. The stronger contractionary monetary policy reaction under that regime leads households to shift more consumption from the initial period into the future. The inflation rates of essential and non-essential goods—and therefore also the CPI inflation rates—respond accordingly: in the initial shock period, all inflation rates are lower due to the stronger contractionary monetary policy reaction. However, over time, higher demand for goods implied by the consumption shift leads to higher marginal costs for both types of firms and therefore to higher prices and larger deviations of all inflation rates from their steady states. Hence, the central bank faces a trade-off between mitigating the initial impact of the shock (and therefore only considering the more strongly affected low-income household's CPI inflation rate) and stabilizing inflation rates over time (only considering the less affected high-income household's CPI inflation rate).

This result is further underscored when examining the volatilities of the model variables. As displayed in Table 3, a higher weight on the CPI inflation rate of the low-income household stabilizes the inflation rate in the affected sector (i.e., good 1) but leads all other variables to fluctuate more. This is caused by the increasing strength of the contractionary monetary policy reaction when δ_{π} is higher: consumption (and thereby output) decreases more in the initial period.

		Volatility		
Variable	Description	$\delta_{\pi} = 0 (\mathrm{H})$	$\delta_{\pi} = 0.5$	$\delta_{\pi} = 1 \ (L)$
$\frac{\hat{c}_t^L}{\hat{c}_t^H}$	Overall consumption L	0.078	0.080	0.084
\hat{c}_t^H	Overall consumption H	0.106	0.115	0.117
$\hat{y}_{1,t}$	Output good 1	0.130	0.133	0.135
$\hat{y}_{2,t}$	Output good 2	0.056	0.063	0.070
\hat{y}_t	Overall output	0.087	0.091	0.096
$\hat{\pi}_t^{C,L}$	CPI inflation L	0.038	0.039	0.040
$\hat{\pi}_t^{C,H}$	CPI inflation H	0.033	0.034	0.036
$\hat{\pi}^C_t$	Average CPI inflation	0.035	0.037	0.038
$\hat{\pi}_{1,t}$	Inflation good 1	0.060	0.060	0.059
$\hat{\pi}_{2,t}$	Inflation good 2	0.025	0.028	0.031

Table 3: 1% Cost-Push Shock Volatilities.

Notes. All variables are deviations from their zero inflation steady state.

Over time, as the shock fades, demand for goods increases again, moving back towards the steady state. This increase is larger the higher δ_{π} is, since the initial decrease in output is larger as a consequence of the stronger increase in the nominal interest rate in this case. This implies that households have more of an incentive to postpone consumption to future periods, implying higher levels and fluctuations in consumption over time. Therefore, the CPI inflation rates as well as output also fluctuate more the higher δ_{π} is.

4 Conclusion

Inflation differentials across households are a well-documented phenomenon. For instance, low-income households experience higher inflation rates than households with higher income. This paper examines how central banks that aim to stabilize the economy-wide inflation rate should react to this household inflation heterogeneity. In particular, we incorporate a lowand a high-income household in a New Keynesian model, with the low-income household experiencing higher inflation after adverse shocks. The central bank in our model reacts to either the individual CPI inflation rate of one of the households or to the weighted average of both rates. We find that the weight that the central bank assigns to the inflation rates experienced by the households significantly affects model outcomes. After a negative demand shock, a central bank that only takes into account CPI inflation of the low-income household leads to lower volatility of all model variables. After a negative supply shock, a central bank that only considers the inflation experience of the low-income household mitigates the initial effects of the shock on inflation more effectively, while allowing for larger overall volatility in the economy. Generally, the central bank manages to stabilize the volatility of the economywide inflation rate more effectively after demand and supply shocks when only considering the household whose CPI inflation rate is less affected by these shocks.

These findings raise important questions with respect to the implementation of monetary policy. In particular, reacting to the average inflation rate experienced by households in the economy might lead to larger fluctuations in inflation rates and output in comparison to reacting to specific inflation rates. This should be taken into account when determining optimal monetary policy to reach the economy-wide inflation target in response to shocks. For instance, it seems sensible for central banks to consider a range of inflation rates experienced in an economy, specifically after shocks that lead to a deviation of the economy-wide inflation rate from its target. This allows for the central bank to react discretionarily to the differing inflation experiences: depending on the type of shock, the central bank could choose to react to specific inflation rates in order to reach its economy-wide inflation target more effectively and stabilize all inflation rates in the economy. As an example, consider the Taylor rule in our model: it would be at the discretion of the central bank to choose the weight of the household-specific inflation rates depending on the type of shock.

Finally, our paper builds a basis for future research. Specifically, we consider shocks that affect households symmetrically. An investigation of the effects of asymmetric, householdspecific shocks seems interesting to further our understanding of the macroeconomic effects of household inflation heterogeneity.

Appendices

A Expenditure Minimization of the Household

A.1 Composition of the Essential and Non-Essential Composite Consumption Good

Household k minimizes its expenditures for any given level of essential good consumption (\bar{C}_1^k) :

$$\min_{C_{i,t}^{k}} \int_{0}^{s} P_{i,t} C_{i,t}^{k} di,$$
(A.1)

subject to

$$\left(\int_0^s C_{i,t}^k \frac{\frac{\epsilon-1}{\epsilon}}{\epsilon} di\right)^{\frac{\epsilon}{\epsilon-1}} = \bar{C}_1^k.$$

This is equivalent to maximizing the following Lagrange function (L_t^k) with respect to the consumption of a representative good a:

$$\max_{C_{a,t}^k} L_t^k = -\int_0^s P_{i,t} C_{i,t}^k di + \lambda_t^k \left[\left(\int_0^s C_{i,t}^k \frac{\epsilon - 1}{\epsilon} di \right)^{\frac{\epsilon}{\epsilon - 1}} - \bar{C}_1^k \right],$$

where λ_t^k is the Lagrange multiplier. The first order conditions are given by

$$\frac{\partial L_t^k}{\partial C_{a,t}^k} = -P_{a,t} + \lambda_t^k \left[\left(\int_0^s C_{i,t}^k \frac{\epsilon - 1}{\epsilon} di \right)^{\frac{\epsilon}{\epsilon - 1} - 1} C_{a,t}^k \frac{\epsilon - 1}{\epsilon} - 1 \right] \stackrel{!}{=} 0, \tag{A.2}$$

$$\frac{\partial L_t^k}{\partial \lambda_t^k} \stackrel{!}{=} 0. \tag{A.3}$$

Rearranging yields

$$C_{a,t}^{k} = \left(\frac{P_{a,t}}{\lambda_{t}^{k}}\right)^{-\epsilon} C_{1,t}^{k}.$$
(A.4)

In order to obtain the expression for optimal consumption, it is necessary to solve for λ_t^k by using the constraint.

$$\begin{split} \left(\int_0^s C_{i,t}^k \frac{\epsilon-1}{\epsilon} di\right)^{\frac{\epsilon}{\epsilon_k}} &= \bar{C}_1^k, \\ \Leftrightarrow \left(\int_0^s \left[\left(\frac{P_{i,t}}{\lambda_t^k}\right)^{-\epsilon} \bar{C}_1^k\right]^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}} &= \bar{C}_1^k, \\ \Leftrightarrow \int_0^s \left(\frac{P_{i,t}}{\lambda_t^k}\right)^{1-\epsilon} di &= 1. \end{split}$$

The solution for λ_t^k thus is

$$\lambda_t^k = \left(\int_0^s P_{i,t}^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}} \equiv P_{1,t}.$$

Plugging this solution into the optimal consumption decision for any essential good i yields

$$C_{i,t}^{k} = \left(\frac{P_{i,t}}{P_{1,t}}\right)^{-\epsilon} C_{1,t}^{k}.$$
(A.5)

Due to symmetry, the optimal consumption decision for any non-essential good j is given by

$$C_{j,t}^{k} = \left(\frac{P_{j,t}}{P_{2,t}}\right)^{-\epsilon} C_{2,t}^{k}.$$
 (A.6)

A.2 Allocation between Essential and Non-Essential Goods

Household k minimizes its expenditures for any given level of overall consumption (\bar{C}^k) :

$$\min_{C_{1,t}^k, C_{2,t}^k} P_{1,t} C_{1,t}^k + P_{2,t} C_{2,t}^k, \tag{A.7}$$

subject to

$$\bar{C}^{k} = \left(\gamma_{k}^{\frac{1}{\vartheta_{C}^{k}}} \left(C_{1,t}^{k} - C_{1}^{*}\right)^{\frac{\vartheta_{C}^{k} - 1}{\vartheta_{C}^{k}}} + \left(1 - \gamma_{k}\right)^{\frac{1}{\vartheta_{C}^{k}}} Z_{t}^{\frac{1}{\vartheta_{C}^{k}}} \left(C_{2,t}^{k}\right)^{\frac{\vartheta_{C}^{k} - 1}{\vartheta_{C}^{k}}}\right)^{\frac{\vartheta_{C}^{k} - 1}{\vartheta_{C}^{k} - 1}}$$

This is equivalent to maximizing the following Lagrange function (L_t^k) :

$$\max_{C_{1,t}^{k}, C_{2,t}^{k}} L_{t}^{k} = -\left(P_{1,t}C_{1,t}^{k} + P_{2,t}C_{2,t}^{k}\right) + \lambda_{t}^{k} \left[\left(\gamma_{k}^{\frac{1}{\vartheta_{C}^{k}}} \left(C_{1,t}^{k} - C_{1}^{*}\right)^{\frac{\vartheta_{C}^{k}-1}{\vartheta_{C}^{k}}} + \left(1 - \gamma_{k}\right)^{\frac{1}{\vartheta_{C}^{k}}} Z_{t}^{\frac{1}{\vartheta_{C}^{k}}} \left(C_{2,t}^{k}\right)^{\frac{\vartheta_{C}^{k}-1}{\vartheta_{C}^{k}}} - \bar{C}^{k} \right],$$

where λ_t^k is the Lagrange multiplier. The first order conditions are given by

$$\frac{\partial L_t^k}{\partial C_{1,t}^k} = -P_{1,t} + \lambda_t^k \left(C_t^k\right)^{\frac{1}{\vartheta_C^k}} \gamma_k^{\frac{1}{\vartheta_C^k}} \left(C_{1,t}^k - C_1^*\right)^{-\frac{1}{\vartheta_C^k}} \stackrel{!}{=} 0, \tag{A.8}$$

$$\frac{\partial L_t^k}{\partial C_{2,t}^k} = -P_{2,t} + \lambda_t^k \left(C_t^k \right)^{\frac{1}{\vartheta_C^k}} \left(1 - \gamma_k \right)^{\frac{1}{\vartheta_C^k}} Z_t^{\frac{1}{\vartheta_C^k}} \left(C_{2,t}^k \right)^{-\frac{1}{\vartheta_C^k}} \stackrel{!}{=} 0, \tag{A.9}$$

$$\frac{\partial L_t^k}{\partial \lambda_t^k} \stackrel{!}{=} 0. \tag{A.10}$$

,

Rearranging yields

$$C_{1,t}^{k} = \left(\frac{P_{1,t}}{\lambda_t^k}\right)^{-\vartheta_C^k} \gamma_k C_t^k + C_1^*, \tag{A.11}$$

$$C_{2,t}^{k} = \left(\frac{P_{2,t}}{\lambda_{t}^{k}}\right)^{-\vartheta_{C}^{k}} (1 - \gamma_{k}) Z_{t} C_{t}^{k}.$$
(A.12)

In order to obtain the expression for optimal consumption, it is necessary to solve for λ_t^k by using the constraint.

$$\begin{split} \bar{C}^{k} &= \left(\gamma_{k}^{\frac{1}{\vartheta_{C}^{k}}} \left(\left(\frac{P_{1,t}}{\lambda_{t}^{k}} \right)^{-\vartheta_{C}^{k}} \gamma_{k} \bar{C}^{k} + C_{1}^{*} - C_{1}^{*} \right)^{\frac{\vartheta_{C}^{k} - 1}{\vartheta_{C}^{k}}} + (1 - \gamma_{k})^{\frac{1}{\vartheta_{C}^{k}}} Z_{t}^{\frac{1}{\vartheta_{C}^{k}}} \left(\left(\frac{P_{2,t}}{\lambda_{t}^{k}} \right)^{-\vartheta_{C}^{k}} (1 - \gamma_{k}) Z_{t} \bar{C}^{k} \right)^{\frac{\vartheta_{C}^{k} - 1}{\vartheta_{C}^{k}}} \right)^{\frac{\vartheta_{C}^{k} - 1}{\vartheta_{C}^{k}}} \\ &\Leftrightarrow 1 = \left(\gamma_{k} P_{1,t}^{1 - \vartheta_{C}^{k}} \left(\lambda_{t}^{k} \right)^{\vartheta_{C}^{k} - 1} + (1 - \gamma_{k}) Z_{t} P_{2,t}^{1 - \vartheta_{C}^{k}} \left(\lambda_{t}^{k} \right)^{\vartheta_{C}^{k} - 1} \right)^{\frac{\vartheta_{C}^{k} - 1}{\vartheta_{C}^{k}}}. \end{split}$$

The solution for λ_t^k thus is

$$\lambda_t^k = \left(\gamma_k P_{1,t}^{1-\vartheta_C^k} + (1-\gamma_k) Z_t P_{2,t}^{1-\vartheta_C^k}\right)^{\frac{1}{1-\vartheta_C^k}} \equiv P_t^{C,k}.$$

Plugging this solution into the optimal consumption decision for essential and non-essential goods yields

$$C_{1,t}^{k} = \left(\frac{P_{1,t}}{P_{t}^{C,k}}\right)^{-\vartheta_{C}^{k}} \gamma_{k} C_{t}^{k} + C_{1}^{*},$$
(A.13)

$$C_{2,t}^{k} = \left(\frac{P_{2,t}}{P_{t}^{C,k}}\right)^{-\vartheta_{C}^{k}} (1-\gamma_{k}) Z_{t} C_{t}^{k}.$$
(A.14)

B Consumption Expenditures in the Budget Constraint

In general, consumption expenditures are given by

$$\int_0^s P_{i,t} C_{i,t}^k di + \int_s^1 P_{j,t} C_{j,t}^k dj$$

Plugging in equations (A.5), (A.6), (A.13), and (A.14) gives

$$\begin{split} \int_{0}^{s} P_{i,t} \left(\frac{P_{i,t}}{P_{1,t}}\right)^{-\epsilon} \left(\left(\frac{P_{1,t}}{P_{t}^{C,k}}\right)^{-\vartheta_{C}^{k}} \gamma_{k} C_{t}^{k} + C_{1}^{*} \right) di + \int_{s}^{1} P_{j,t} \left(\frac{P_{j,t}}{P_{2,t}}\right)^{-\epsilon} \left(\frac{P_{2,t}}{P_{t}^{C,k}}\right)^{-\vartheta_{C}^{k}} (1 - \gamma_{k}) Z_{t} C_{t}^{k} dj, \\ \Leftrightarrow P_{1,t}^{1-\vartheta_{C}} \left(P_{t}^{C,k}\right)^{\vartheta_{C}} \gamma_{k} C_{t}^{k} + P_{1,t} C_{1}^{*} + P_{2,t}^{1-\vartheta_{C}} \left(P_{t}^{C,k}\right)^{\vartheta_{C}} (1 - \gamma_{k}) Z_{t} C_{t}^{k}, \\ \Leftrightarrow \left(\gamma_{k} P_{1,t}^{1-\vartheta_{C}} + (1 - \gamma_{k}) Z_{t} P_{2,t}^{1-\vartheta_{C}}\right) \left(P_{t}^{C,k}\right)^{\vartheta_{C}} C_{t}^{k} + P_{1,t} C_{1}^{*}, \end{split}$$

which implies

$$\int_{0}^{s} P_{i,t}C_{i,t}^{k}di + \int_{s}^{1} P_{j,t}C_{j,t}^{k}dj = P_{t}^{C,k}C_{t}^{k} + P_{1,t}C_{1}^{*}.$$
(B.1)

C Utility Maximization of the Household

The household seeks to maximize expected lifetime utility:

$$\max_{C_t^k, N_t^k, B_t^k} \mathbb{E}_t \left[\sum_{\iota=0}^{\infty} \beta^{\iota} \left(\frac{\left(C_t^k\right)^{1-\sigma_k}}{1-\sigma_k} - \frac{\left(N_t^k\right)^{1+\varphi_k}}{1+\varphi_k} \right) \right],$$
(C.1)

subject to

$$P_t^{C,k}C_t^k + P_{1,t}C_1^* + Q_tB_t^k = B_{t-1}^k + W_t^kN_t^k + D_t^k.$$

The Lagrange function is

$$L_{t}^{k} = \mathbb{E}_{t} \left[\sum_{\iota=0}^{\infty} \beta^{\iota} \left(\frac{\left(C_{t}^{k}\right)^{1-\sigma_{k}}}{1-\sigma_{k}} - \frac{\left(N_{t}^{k}\right)^{1+\varphi_{k}}}{1+\varphi_{k}} - \lambda_{t+\iota}^{k} \left(P_{t+\iota}^{C,k}C_{t+\iota}^{k} + P_{1,t+\iota}C_{1}^{*} + Q_{t+\iota}B_{t+\iota}^{k} - B_{t-1+\iota}^{k} - W_{t+\iota}^{k}N_{t+\iota}^{k} - D_{t+\iota}^{k} \right) \right] \right]. \quad (C.2)$$

The first order conditions are

$$\frac{\partial L_t^k}{\partial C_t^k} = \left(C_t^k\right)^{-\sigma_k} - \lambda_t^k P_t^{C,k} \stackrel{!}{=} 0, \tag{C.3}$$

$$\frac{\partial L_t^k}{\partial N_t^k} = -\left(N_t^k\right)^{\varphi_k} + \lambda_t^k W_t^k \stackrel{!}{=} 0, \tag{C.4}$$

$$\frac{\partial L_t^k}{\partial B_t^k} = -\lambda_t^k Q_t + \beta \mathbb{E}_t \left[\lambda_{t+1}^k \right] \stackrel{!}{=} 0, \tag{C.5}$$

$$\frac{\partial L_t^k}{\partial \lambda_t^k} \stackrel{!}{=} 0. \tag{C.6}$$

Plugging (C.3) into (C.4) and (C.5) gives

$$\left(N_t^k\right)^{\varphi_k} = w_t^k \left(C_t^k\right)^{-\sigma_k},\tag{C.7}$$

$$Q_t = \beta \mathbb{E}_t \left[\Lambda_{t,t+1}^k \frac{1}{\Pi_{t+1}^{C,k}} \right].$$
(C.8)

D Risk Sharing

The Euler equation holds for both households at all times. Thus,

$$\left(\frac{C_t^k}{C_{t-1}^k}\right)^{-\sigma_k} \frac{P_{t-1}^{C,k}}{P_t^{C,k}} = \left(\frac{C_t^{-k}}{C_{t-1}^{-k}}\right)^{-\sigma_{-k}} \frac{P_{t-1}^{C,-k}}{P_t^{C,-k}}.$$

Rearranging yields

$$\left(C_{t}^{k}\right)^{-\sigma_{k}} = \left(C_{t}^{-k}\right)^{-\sigma_{-k}} \frac{\left(C_{t-1}^{k}\right)^{-\sigma_{k}}}{\left(C_{t-1}^{-k}\right)^{-\sigma_{-k}}} \frac{P_{t-1}^{C,-k}}{P_{t}^{C,-k}} \frac{P_{t}^{C,k}}{P_{t-1}^{C,k}}.$$

Hence,

$$\left(C_{t-1}^{k}\right)^{-\sigma_{k}} = \left(C_{t-1}^{-k}\right)^{-\sigma_{-k}} \frac{\left(C_{t-2}^{k}\right)^{-\sigma_{k}}}{\left(C_{t-2}^{-k}\right)^{-\sigma_{-k}}} \frac{P_{t-2}^{C,-k}}{P_{t-1}^{C,-k}} \frac{P_{t-1}^{C,k}}{P_{t-2}^{C,-k}}.$$

Plugging in gives

$$\left(C_{t}^{k}\right)^{-\sigma_{k}} = \left(C_{t}^{-k}\right)^{-\sigma_{-k}} \frac{\left(C_{t-2}^{k}\right)^{-\sigma_{k}}}{\left(C_{t-2}^{-k}\right)^{-\sigma_{-k}}} \frac{P_{t-2}^{C,-k}}{P_{t-2}^{C,k}} \frac{P_{t}^{C,k}}{P_{t}^{C,-k}}.$$

Continuing this procedure to the initial period, i.e., the steady state, implies

$$\left(C_t^k\right)^{-\sigma_k} = \left(C_t^{-k}\right)^{-\sigma_k} \Phi^k \frac{P_t^{C,k}}{P_t^{C,-k}}.$$
(D.1)

E Profit Maximization of the Firm

Essential good firm i chooses its price $P_{i,t}$ to maximize discounted expected real profits:

$$\max_{P_{i,t}} \mathbb{E}_t \left[\sum_{\iota=0}^{\infty} \beta^{\iota} \Lambda_{t,t+\iota}^L \lambda_1^{\iota} \left(\frac{P_{i,t}}{P_{t+\iota}^{C,L}} Y_{i,t+\iota|t} - TC\left(Y_{i,t+\iota|t}\right) \right) \right] , \qquad (E.1)$$

subject to

$$Y_{i,t+\iota|t} = \left(\frac{P_{i,t}}{P_{1,t+\iota}}\right)^{-\epsilon} Y_{1,t+\iota},$$
$$\Rightarrow \mathbb{E}_t \left[\sum_{\iota=0}^{\infty} \beta^{\iota} \Lambda_{t,t+\iota}^L \lambda_1^{\iota} \left(\frac{P_{i,t}}{P_{t+\iota}^{C,L}} \left(\frac{P_{i,t}}{P_{1,t+\iota}}\right)^{-\epsilon} Y_{1,t+\iota} - TC\left(Y_{i,t+\iota|t}\right)\right)\right].$$

First oder condition:

$$\mathbb{E}_{t}\left[\sum_{\iota=0}^{\infty}\beta^{\iota}\Lambda_{t,t+\iota}^{L}\lambda_{1}^{\iota}\left((1-\epsilon)\frac{1}{P_{t+\iota}^{C,L}}\left(\frac{P_{i,t}}{P_{1,t+\iota}}\right)^{-\epsilon}Y_{1,t+\iota}-(-\epsilon)\left(\frac{P_{i,t}}{P_{1,t+\iota}}\right)^{-\epsilon}Y_{1,t+\iota}\frac{1}{P_{i,t}}mc\left(Y_{i,t+\iota|t}\right)\right)\right]\stackrel{!}{=}0,$$

$$\Leftrightarrow \mathbb{E}_{t} \left[\sum_{\iota=0}^{\infty} \beta^{\iota} \Lambda_{t,t+\iota}^{L} \lambda_{1}^{\iota} Y_{i,t+\iota|t} \left(\frac{P_{i,t}}{P_{t+\iota}^{C,L}} - \mu mc\left(Y_{i,t+\iota|t}\right) \right) \right] \stackrel{!}{=} 0.$$
(E.2)

Solving for $P_{i,t}$:

$$\mathbb{E}_{t}\left[\sum_{\iota=0}^{\infty}\beta^{\iota}\Lambda_{t,t+\iota}^{L}\lambda_{1}^{\iota}Y_{1,t+\iota}P_{1,t+\iota}^{\epsilon}\left(P_{i,t}^{-1-\epsilon}\left(P_{t+\iota}^{C,L}\right)^{-1}-\mu P_{i,t}^{-\frac{\epsilon}{1-\alpha_{1}}}P_{1,t+\iota}^{\frac{\epsilon\alpha_{1}}{1-\alpha_{1}}}mc_{1,t}\right)\right]=0,$$

$$\Leftrightarrow\mathbb{E}_{t}\left[\sum_{\iota=0}^{\infty}\beta^{\iota}\Lambda_{t,t+\iota}^{L}\lambda_{1}^{\iota}Y_{1,t+\iota}P_{1,t+\iota}^{\epsilon}\left(P_{t+\iota}^{C,L}\right)^{-1}\right]P_{i,t}^{-1+\frac{\epsilon\alpha_{1}}{1-\alpha_{1}}}=\mu\mathbb{E}_{t}\left[\sum_{\iota=0}^{\infty}\beta^{\iota}\Lambda_{t,t+\iota}^{L}\lambda_{1}^{\iota}Y_{1,t+\iota}P_{1,t+\iota}^{\frac{\epsilon}{1-\alpha_{1}}}mc_{1,t}\right].$$

Noting that $\left(\frac{1}{P_{1,t}}\right)^{1+\frac{\epsilon\alpha_1}{1-\alpha_1}} = \left(\frac{1}{P_{1,t}}\right)^{\frac{\epsilon}{1-\alpha_1}} \frac{1}{P_{1,t}} P_{1,t}^{\epsilon}$ and rearranging gives $\left(p_t^*\right)^{1+\frac{\epsilon\alpha_1}{1-\alpha_1}} = \mu \frac{\mathbb{E}_t \left[\sum_{\iota=0}^{\infty} \beta^{\iota} \Lambda_{t,t+\iota}^L \lambda_1^{\iota} Y_{1,t+\iota} \Pi_{1,t+\iota}^{\frac{\epsilon}{1-\alpha_1}} mc_{1,t}\right]}{\mathbb{E}_t \left[\sum_{\iota=0}^{\infty} \beta^{\iota} \Lambda_{t,t+\iota}^L \lambda_1^{\iota} Y_{1,t+\iota} \Pi_{1,t+\iota}^{\epsilon} \left(\Pi_{t+\iota}^{C,L}\right)^{-1} \frac{P_{1,t}}{P_t^{C,L}}\right]},$

which implies

$$\left(p_{1,t}^*\right)^{1+\frac{\epsilon\alpha_1}{1-\alpha_1}} = \mu \left(V_{1,t}^{C,L}\right)^{-1} \frac{b_{1,t}}{d_{1,t}}.$$
(E.3)

Due to symmetry of essential and non-essential goods firms:

$$\left(p_{2,t}^*\right)^{1+\frac{\epsilon\alpha_2}{1-\alpha_2}} = \mu \left(V_{2,t}^{C,H}\right)^{-1} \frac{b_{2,t}}{d_{2,t}}.$$
(E.4)

F Log-Linearization

All equations are log-linearized around the zero inflation steady state. Without loss of generality, all prices and shocks are assumed to be of unity in steady state.

F.1 Dynamic IS Equation

Use the Euler equation and log-linearize:

$$1 = R_t \beta \mathbb{E}_t \left[\Lambda_{t,t+1}^k \frac{1}{\prod_{t+1}^{C,k}} \right],$$

where $R_t = \frac{1}{Q_t}$. Note that $X_t = X_{SS} e^{\hat{x}_t}$ and rewrite

$$1 = R_{SS} e^{\hat{i}_t} \beta \mathbb{E}_t \left[\left(\frac{C_{SS}^k e^{\hat{c}_{t+1}^k}}{C_{SS}^k e^{\hat{c}_t^k}} \right)^{-\sigma_k} \frac{1}{\prod_{SS}^{C,k} e^{\hat{\pi}_{t+1}^{C,k}}} \right],$$

with $R_{SS}\beta = \prod_{SS}^{C,k} = 1$ and $e^{\hat{x}_t} \approx 1 + \hat{x}_t$:

$$1 - \sigma_k \hat{c}_t^k = 1 - \sigma_k \mathbb{E}_t \left[\hat{c}_{t+1}^k \right] - \mathbb{E}_t \left[\hat{\pi}_{t+1}^{C,k} \right] + \hat{i}_t.$$

Rearranging gives the dynamic IS equation

$$\hat{c}_t^k = \mathbb{E}_t \left[\hat{c}_{t+1}^k \right] - \frac{1}{\sigma_k} \left(\hat{i}_t - \mathbb{E}_t \left[\hat{\pi}_{t+1}^{C,k} \right] \right).$$
(F.1)

F.2 New Keynesian Philips Curve

Take (natural) logs on both sides of equations (E.3) and (E.4):

$$\left(1 + \frac{\epsilon \alpha_1}{1 - \alpha_1}\right) ln\left(p_{1,t}^*\right) = ln\left(\mu\right) - ln\left(V_{1,t}^{C,L}\right) + ln\left(b_{1,t}\right) - ln\left(d_{1,t}\right),$$
$$\left(1 + \frac{\epsilon \alpha_2}{1 - \alpha_2}\right) ln\left(p_{2,t}^*\right) = ln\left(\mu\right) - ln\left(V_{2,t}^{C,H}\right) + ln\left(b_{2,t}\right) - ln\left(d_{2,t}\right).$$

In steady state:

$$\left(1 + \frac{\epsilon \alpha_1}{1 - \alpha_1}\right) ln\left(p_{1,SS}^*\right) = ln\left(\mu\right) - ln\left(V_{1,SS}^{C,L}\right) + ln\left(b_{1,SS}\right) - ln\left(d_{1,SS}\right),$$
$$\left(1 + \frac{\epsilon \alpha_2}{1 - \alpha_2}\right) ln\left(p_{2,SS}^*\right) = ln\left(\mu\right) - ln\left(V_{2,SS}^{C,H}\right) + ln\left(b_{2,SS}\right) - ln\left(d_{2,SS}\right).$$

Subtracting gives

$$\left(1 + \frac{\epsilon \alpha_1}{1 - \alpha_1}\right)\hat{p}_{1,t}^* = \hat{b}_{1,t} - \hat{d}_{1,t} - \hat{v}_{1,t}^{C,L},\tag{F.2}$$

$$\left(1 + \frac{\epsilon \alpha_2}{1 - \alpha_2}\right)\hat{p}_{2,t}^* = \hat{b}_{2,t} - \hat{d}_{2,t} - \hat{v}_{2,t}^{C,H}.$$
(F.3)

Now, log-linearize the auxiliary variables utilizing a first order Taylor approximation around the steady state. As an example, take $b_{1,t}$:

$$ln (b_{1,SS}) + \frac{1}{b_{1,SS}} (b_{1,t} - b_{1,SS}) \approx ln (b_{1,SS}) + \frac{1}{b_{1,SS}} \left(-\sigma_L \left(C_{SS}^L \right)^{-\sigma_L - 1} Y_{1,SS} mc_{1,SS} \left(C_t^L - C_{SS}^L \right) \right) \\ + \left(C_{SS}^L \right)^{-\sigma_L} mc_{1,SS} \left(Y_{1,t} - Y_{1,SS} \right) + \left(C_{SS}^L \right)^{-\sigma_L} Y_{1,SS} \left(mc_{1,t} - mc_{1,SS} \right) \\ + \beta \lambda_1 \mathbb{E}_t \left[\frac{\epsilon}{1 - \alpha_1} \Pi_{1,SS} \frac{\epsilon}{1 - \alpha_1}^{-1} b_{1,SS} \left(\Pi_{1,t+1} - \Pi_{1,SS} \right) \right] + \beta \lambda_1 \mathbb{E}_t \left[\Pi_{1,SS} \frac{\epsilon}{1 - \alpha_1} \left(b_{1,t+1} - b_{1,SS} \right) \right] \right).$$
(F.4)

Noting that $\frac{X_t - X_{SS}}{X_{SS}} = \hat{x}_t$ and $X_t - X_{SS} = \frac{X_t - X_{SS}}{X_{SS}} X_{SS}$:

$$b_{1,SS}\hat{b}_{1,t} = \left(C_{SS}^L\right)^{-\sigma_L} Y_{1,SS}mc_{1,SS} \left(-\sigma_L \hat{c}_t^L + \hat{y}_{1,t} + \hat{m}c_{1,t}\right) + \beta \lambda_1 b_{1,SS} \mathbb{E}_t \left[\frac{\epsilon}{1-\alpha_1} \hat{\pi}_{1,t+1} + \hat{b}_{1,t+1}\right].$$
(F.5)

Analogously, for $b_{2,t}$, $d_{1,t}$, and $d_{2,t}$:

$$b_{2,SS}\hat{b}_{2,t} = \left(C_{SS}^{H}\right)^{-\sigma_{H}}Y_{2,SS}mc_{2,SS}\left(-\sigma_{H}\hat{c}_{t}^{H} + \hat{y}_{2,t} + \hat{m}c_{2,t}\right) + \beta\lambda_{2}b_{2,SS}\mathbb{E}_{t}\left[\frac{\epsilon}{1-\alpha_{2}}\hat{\pi}_{2,t+1} + \hat{b}_{2,t+1}\right],$$
(F.6)

$$d_{1,SS}\hat{d}_{1,t} = \left(C_{SS}^{L}\right)^{-\sigma_{L}} Y_{1,SS} \left(-\sigma_{L}\hat{c}_{t}^{L} + \hat{y}_{1,t}\right) + \beta\lambda_{1}d_{1,SS} \mathbb{E}_{t} \left[\epsilon\hat{\pi}_{1,t+1} - \hat{\pi}_{t+1}^{C,L} + \hat{d}_{1,t+1}\right], \quad (F.7)$$

$$d_{2,SS}\hat{d}_{2,t} = \left(C_{SS}^{H}\right)^{-\sigma_{H}} Y_{2,SS} \left(-\sigma_{H}\hat{c}_{t}^{H} + \hat{y}_{2,t}\right) + \beta\lambda_{2}d_{2,SS} \mathbb{E}_{t} \left[\epsilon\hat{\pi}_{2,t+1} - \hat{\pi}_{t+1}^{C,H} + \hat{d}_{2,t+1}\right].$$
(F.8)

The price level development can be approximated as

$$0 = (1 - \lambda_h) (1 - \epsilon) \left(p_{h,SS}^* \right)^{-\epsilon} \left(p_{h,t}^* - p_{h,SS}^* \right) - \lambda_h (1 - \epsilon) \pi_{h,SS}^{\epsilon-2} \left(\pi_{h,t} - \pi_{h,SS} \right),$$

$$\Leftrightarrow 0 = (1 - \lambda_h) \hat{p}_{h,t}^* - \lambda_h \hat{\pi}_{h,t},$$

$$\Leftrightarrow \hat{p}_{h,t}^* = \frac{\lambda_h}{1 - \lambda_h} \hat{\pi}_{h,t}.$$
 (F.9)

Plugging equation (F.9) into equations (F.2) and (F.3) gives

$$\left(1 + \frac{\epsilon \alpha_1}{1 - \alpha_1}\right) \frac{\lambda_1}{1 - \lambda_1} \hat{\pi}_{1,t} = \hat{b}_{1,t} - \hat{d}_{1,t} - \hat{v}_{1,t}^{C,L},$$
$$\left(1 + \frac{\epsilon \alpha_2}{1 - \alpha_2}\right) \frac{\lambda_2}{1 - \lambda_2} \hat{\pi}_{2,t} = \hat{b}_{2,t} - \hat{d}_{2,t} - \hat{v}_{2,t}^{C,H}.$$

Then, calculate $\hat{b}_{h,t} - \hat{d}_{h,t}$ using equations (F.5)–(F.8). As an example, consider firm 1. The steady state values of the auxiliary variables are given by

$$b_{1,SS} = \frac{\left(C_{SS}^{L}\right)^{-\sigma_{L}} Y_{1,SS}mc_{1,SS}}{1 - \beta\lambda_{1}},$$
$$d_{1,SS} = \frac{\left(C_{SS}^{L}\right)^{-\sigma_{L}} Y_{1,SS}}{1 - \beta\lambda_{1}}.$$

Plugging in and rearranging:

$$\hat{b}_{1,t} = (1 - \beta \lambda_1) \left(-\sigma_L \hat{c}_t^L + \hat{y}_{1,t} + \hat{m} \hat{c}_{1,t} \right) + \beta \lambda_1 \mathbb{E}_t \left[\frac{\epsilon}{1 - \alpha_1} \hat{\pi}_{1,t+1} + \hat{b}_{1,t+1} \right],$$
$$\hat{d}_{1,t} = (1 - \beta \lambda_1) \left(-\sigma_L \hat{c}_t^L + \hat{y}_{1,t} \right) + \beta \lambda_1 \mathbb{E}_t \left[\epsilon \hat{\pi}_{1,t+1} - \hat{\pi}_{t+1}^{C,L} + \hat{d}_{1,t+1} \right].$$

Thus,

$$\hat{b}_{1,t} - \hat{d}_{1,t} = (1 - \beta \lambda_1) \, \hat{mc}_{1,t} + \beta \lambda_1 \, \mathbb{E}_t \left[\frac{\epsilon \alpha_1}{1 - \alpha_1} \hat{\pi}_{1,t+1} + \hat{\pi}_{t+1}^{C,L} + \hat{b}_{1,t+1} - \hat{d}_{1,t+1} \right],$$

with

$$\hat{b}_{1,t+1} - \hat{d}_{1,t+1} = \left(1 + \frac{\epsilon \alpha_1}{1 - \alpha_1}\right) \frac{\lambda_1}{1 - \lambda_1} \hat{\pi}_{1,t+1} + \hat{v}_{1,t+1}^{C,L},$$

and

$$\hat{v}_{1,t+1}^{C,L} = \hat{\pi}_{1,t+1} - \hat{\pi}_{t+1}^{C,L} + \hat{v}_{1,t}^{C,L}.$$

$$\Rightarrow \left(1 + \frac{\epsilon \alpha_1}{1 - \alpha_1}\right) \frac{\lambda_1}{1 - \lambda_1} \hat{\pi}_{1,t} = (1 - \beta \lambda_1) \, \hat{m}_{c_{1,t}} + \beta \lambda_1 \, \mathbb{E}_t \left[\frac{\epsilon \alpha_1}{1 - \alpha_1} \hat{\pi}_{1,t+1} + \left(1 + \frac{\epsilon \alpha_1}{1 - \alpha_1}\right) \frac{\lambda_1}{1 - \lambda_1} \hat{\pi}_{1,t+1} + \hat{\pi}_{1,t+1} + \hat{v}_{1,t}^{C,L}\right] - \hat{v}_{1,t}^{C,L},$$

$$\Leftrightarrow \left(1 + \frac{\epsilon \alpha_1}{1 - \alpha_1}\right) \frac{\lambda_1}{1 - \lambda_1} \hat{\pi}_{1,t} = (1 - \beta \lambda_1) \left(\hat{m}_{c_{1,t}} - \hat{v}_{1,t}^{C,L}\right) + \beta \lambda_1 \, \mathbb{E}_t \left[\left(1 + \frac{\epsilon \alpha_1}{1 - \alpha_1}\right) \frac{1}{1 - \lambda_1} \hat{\pi}_{1,t+1}\right].$$

Solving for $\hat{\pi}_{1,t}$ yields

$$\hat{\pi}_{1,t} = \Psi_1 \left(\hat{mc}_{1,t} - \hat{v}_{1,t}^{C,L} \right) + \beta \mathbb{E}_t \left[\hat{\pi}_{1,t+1} \right].$$
(F.10)

Due to symmetry, the general New Keynesian Philips curve can be stated as

$$\hat{\pi}_{h,t} = \Psi_h \left(\hat{mc}_{h,t} - \hat{v}_{h,t}^C \right) + \beta \mathbb{E}_t \left[\hat{\pi}_{h,t+1} \right].$$
(F.11)

Solving forward:

$$\mathbb{E}_t \left[\hat{\pi}_{h,t+1} \right] = \Psi_h \mathbb{E}_t \left[\hat{mc}_{h,t} - \hat{v}_{h,t}^C \right],$$
(...),

$$\Rightarrow \hat{\pi}_{h,t} = \Psi_h \sum_{\iota=0}^{\infty} \beta^{\iota} \mathbb{E}_t \left[\hat{mc}_{h,t+\iota} - \hat{v}_{h,t+\iota}^C \right].$$
(F.12)

F.3 Marginal Costs

First, consider log-linearized marginal costs of firm 1:

$$\hat{mc}_{1,t} = \hat{w}_t^L + \frac{\alpha_1}{1 - \alpha_1} \hat{y}_{1,t} + a_t, \qquad (F.13)$$

where, derived from the optimal labor supply condition of the households and the production function of firm 1,

$$\varphi_k \hat{n}_t^k = \hat{w}_t^k - \sigma_k \hat{c}_t^k, \tag{F.14}$$

$$\hat{y}_{1,t} = (1 - \alpha_1)\hat{n}_t^L.$$
 (F.15)

The market clearing condition of essential goods can be approximated as

$$ln\left(Y_{1,SS}\right) + \frac{1}{Y_{1,SS}}\left(Y_{1,t} - Y_{1,SS}\right) = ln\left(Y_{1,SS}\right) + \frac{1}{Y_{1,SS}}\left(\kappa\left(C_{1,t}^{L} - C_{1,SS}^{L}\right) + (1-\kappa)\left(C_{1,t}^{H} - C_{1,SS}^{H}\right)\right),$$

$$\Leftrightarrow \hat{y}_{1,t} = g_{L,1} \kappa \hat{c}_{1,t}^L + g_{H,1} (1-\kappa) \hat{c}_{1,t}^H.$$
(F.16)

Combining equations (F.13)-(F.16) gives

$$\hat{mc}_{1,t} = \frac{\varphi_L + \alpha_1}{1 - \alpha_1} \left(g_{L,1} \kappa \hat{c}_{1,t}^L + g_{H,1} (1 - \kappa) \hat{c}_{1,t}^H \right) + \sigma_L \hat{c}_t^L + a_t.$$
(F.17)

Demand for essential goods by household k can be approximated as

$$\hat{c}_{1,t}^{k} = \frac{1}{l_{k,1}} \gamma_k \left(\hat{c}_t^k - \vartheta_C^k \hat{v}_{1,t}^{C,k} \right).$$
(F.18)

Combining equations (F.17) and (F.18) gives

$$\hat{mc}_{1,t} = \frac{(\alpha_1 + \varphi_L)g_{L,1}\kappa \frac{1}{l_{L,1}}\gamma_L + \sigma_L(1 - \alpha_1)}{1 - \alpha_1}\hat{c}_t^L + \frac{(\alpha_1 + \varphi_L)g_{H,1}(1 - \kappa)\frac{1}{l_{H,1}}\gamma_H}{1 - \alpha_1}\hat{c}_t^H - \frac{(\alpha_1 + \varphi_L)g_{L,1}\kappa \frac{1}{l_{L,1}}\gamma_L\vartheta_C^L}{1 - \alpha_1}\hat{v}_{1,t}^{C,L} - \frac{(\alpha_1 + \varphi_L)g_{H,1}(1 - \kappa)\frac{1}{l_{H,1}}\gamma_H\vartheta_C^H}{1 - \alpha_1}\hat{v}_{1,t}^{C,H} + a_t. \quad (F.19)$$

For non-essential goods firms, $a_t=0 \forall t$, and the following market clearing condition holds

$$\hat{y}_{2,t} = g_{L,2}\kappa\hat{c}_{2,t}^L + g_{H,2}(1-\kappa)\hat{c}_{2,t}^H.$$
(F.20)

Hence,

$$\hat{mc}_{2,t} = \frac{\varphi_H + \alpha_2}{1 - \alpha_2} \left(g_{L,2} \kappa \hat{c}_{2,t}^L + g_{H,2} (1 - \kappa) \hat{c}_{2,t}^H \right) + \sigma_H \hat{c}_t^H.$$
(F.21)

Furthermore, non-essential goods are subject to demand shocks but do not have a subsistence level. The demand for non-essential goods can therefore be approximated as

$$\hat{c}_{2,t}^k = -\vartheta_C^k \hat{v}_{2,t}^{C,k} + z_t + \hat{c}_t^k.$$
(F.22)

Combining equations (F.21) and (F.22) gives

$$\begin{split} \hat{mc}_{2,t} &= \frac{(\alpha_2 + \varphi_H)g_{L,2}\kappa}{1 - \alpha_2}\hat{c}_t^L + \frac{(\alpha_2 + \varphi_H)g_{H,2}(1 - \kappa) + \sigma_H(1 - \alpha_2)}{1 - \alpha_2}\hat{c}_t^H \\ &- \frac{(\alpha_2 + \varphi_H)g_{L,2}\kappa\vartheta_C^L}{1 - \alpha_2}\hat{v}_{2,t}^{C,L} - \frac{(\alpha_2 + \varphi_H)g_{H,2}(1 - \kappa)\vartheta_C^H}{1 - \alpha_2}\hat{v}_{2,t}^{C,H} + \frac{(\alpha_2 + \varphi_H)(\kappa g_{L,2} + (1 - \kappa)g_{H,2})}{1 - \alpha_2}z_t. \end{split}$$
(F.23)

F.4 Consumer Price Inflation

A first order Taylor approximation of the consumer price level of household k is given by

$$\begin{split} \ln\left(P_{SS}^{C,k}\right) + \frac{1}{P_{SS}^{C,k}} \left(P_{t}^{C,k} - P_{SS}^{C,k}\right) &= \ln\left(P_{SS}^{C,k}\right) + \frac{1}{P_{SS}^{C,k}} \left(\left(P_{SS}^{C,k}\right)^{\vartheta_{C}^{k}} \gamma_{k} P_{1,SS}^{-\vartheta_{C}^{k}} \left(P_{1,t} - P_{1,SS}\right) \right. \\ &+ \left(P_{SS}^{C,k}\right)^{\vartheta_{C}^{k}} \frac{1 - \gamma_{k}}{1 - \vartheta_{C}^{k}} P_{2,SS}^{1 - \vartheta_{C}^{k}} \left(Z_{t} - Z_{SS}\right) + \left(P_{SS}^{C,k}\right)^{\vartheta_{C}^{k}} \left(1 - \gamma_{k}\right) P_{2,SS}^{-\vartheta_{C}^{k}} Z_{SS} \left(P_{2,t} - P_{2,SS}\right)\right), \\ &\Leftrightarrow \hat{p}_{t}^{C,k} = \gamma_{k} \hat{p}_{1,t} + (1 - \gamma_{k}) \hat{p}_{2,t} + \frac{1 - \gamma_{k}}{1 - \vartheta_{C}^{k}} z_{t}. \end{split}$$

In t-1:

$$\hat{p}_{t-1}^{C,k} = \gamma_k \hat{p}_{1,t-1} + (1 - \gamma_k) \hat{p}_{2,t-1} + \frac{1 - \gamma_k}{1 - \vartheta_C^k} z_{t-1}.$$

Thus, in terms of inflation rates,

$$\hat{\pi}_{t}^{C,k} = \gamma_{k}\hat{\pi}_{1,t} + (1-\gamma_{k})\hat{\pi}_{2,t} + \frac{1-\gamma_{k}}{1-\vartheta_{C}^{k}}\Delta z_{t}.$$
(F.24)

F.5 Overall Output

The log-linear version of overall output is given by

$$\hat{y}_t = m_1 \hat{y}_{1,t} + m_2 \hat{y}_{2,t}. \tag{F.25}$$

Plugging in equations (F.16), (F.18), (F.20), and (F.22) gives

$$\hat{y}_{t} = \left(m_{1}\kappa g_{L,1}\frac{1}{l_{L,1}}\gamma_{L} + m_{2}\kappa g_{L,2}\right)\hat{c}_{t}^{L} + \left(m_{1}(1-\kappa)g_{H,1}\frac{1}{l_{H,1}}\gamma_{H} + m_{2}(1-\kappa)g_{H,2}\right)\hat{c}_{t}^{H} - \left(m_{1}\kappa g_{L,1}\frac{1}{l_{L,1}}\gamma_{L}\vartheta_{C}^{L}\right)\hat{v}_{1,t}^{C,L} - \left(m_{2}\kappa g_{L,2}\vartheta_{C}^{L}\right)\hat{v}_{2,t}^{C,L} - \left(m_{1}(1-\kappa)g_{H,1}\frac{1}{l_{H,1}}\gamma_{H}\vartheta_{C}^{H}\right)\hat{v}_{1,t}^{C,H} - \left(m_{2}(1-\kappa)g_{H,2}\vartheta_{C}^{H}\right)\hat{v}_{2,t}^{C,H} + \left(\kappa g_{L,2} + (1-\kappa)g_{H,2}\right)m_{2}z_{t}.$$
 (F.26)

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Contribution

I, Daniel Stempel, have contributed substantially to the conceptualization, development of the methodology, software, formal analysis, writing of the original draft, review and editing, and visualization of the results of this paper.

Prof. Dr. Ulrike Neyer

Paper IV

Risk Sharing Heterogeneity in the United States^{*}

Daniel Stempel

Abstract

Several studies document high risk sharing against output fluctuations in the United States. Building on these studies, this paper documents substantial heterogeneity in interstate risk sharing between US states. Using a panel data set ranging from 1963 to 2013, aggregate and state-specific risk sharing profiles are estimated. Moreover, four distinct clusters of states, each characterized by a unique risk sharing profile emphasizing one specific consumption insurance channel, are derived. This paper then shows that this heterogeneity in insurance levels and profiles is related to differences in state characteristics, such as the composition of state output, insurance opportunities, vulnerability to idiosyncratic shocks, and the capacity to finance countercyclical policies.

JEL classifications: E21, F15, F45

Keywords: Risk sharing, consumption insurance, heterogeneity, monetary union

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Contents

Li	st of	Tables	143	
\mathbf{Li}	st of	Figures	143	
1	Intr	roduction	144	
2	Me	asuring Aggregate Risk Sharing	145	
	2.1	Insurance Channels and Estimation Strategy		
	2.2	Data		
	2.3	Results	148	
3	Mea	asuring Heterogeneous Risk Sharing	150	
	3.1	Estimation Strategy		
	3.2	Results		
	3.3	Risk Sharing Clusters	151	
4	Determinants of Risk Sharing Heterogeneity			
	4.1	Estimation Strategy	153	
	4.2	State Characteristics and Data	154	
	4.3	Results	155	
5	Cor	clusion	157	
\mathbf{A}	ppen	dices	158	
	А	Aggregate Data Construction	158	
	В	Aggregate Risk Sharing per Decade	160	
	С	Sate-Specific Estimation	160	
	D	K-Means Clustering	162	
	Е	Determinants of Risk Sharing Heterogeneity – Data Construction	163	
R	efere	nces	166	
Publications			167	

List of Tables

1	Aggregate Risk Sharing in the United States.	148
2	State-Specific Risk Sharing Summary Statistics.	151
3	Risk Sharing Clusters.	152
4	Determinants of Risk Sharing Heterogeneity in the United States	156
A.1	Aggregate Data Construction.	158
B.1	Aggregate Risk Sharing per Decade.	160
C.1	State-Specific Risk Sharing in the United States.	161
E.1	Gross State Product Composition - Data Construction.	163
E.2	Public Revenue and Spending Restrictions.	164

List of Figures

1	Time Variation in Insurance Channel Estimates of a 10-Year Rolling Window	
	Estimation of Equations (4) – (7). \ldots \ldots \ldots \ldots \ldots \ldots	149
2	State-Specific Consumption Insurance $1 - \beta_U \dots \dots \dots \dots \dots \dots \dots \dots \dots$	150
3	Cluster Composition and Insurance Profiles	153

1 Introduction

Risk sharing refers to the notion that individuals attempt to insure their consumption streams against adverse regional economic events. Insurance takes place across regions through various mechanisms. In a monetary union, the understanding of these mechanisms is essential to mitigate the vulnerability of regions to economic shocks when nominal price adjustments are not possible.

Literature building on the seminal contribution of Asdrubali et al. (1996) has developed a methodology to quantify the sources of insurance across regions. In particular, Asdrubali et al. (1996) propose a variance decomposition of regional output growth to estimate the contribution of various risk sharing channels to consumption insurance. Several studies utilize this framework to investigate risk sharing in the United States (see, for instance, Mélitz and Zumer, 1999), the euro area (e.g., Cimadomo et al., 2018), or OECD countries (such as Sørensen and Yosha, 1998). Moreover, the methodology has been augmented to examine the effects of several different variables on the extent of risk sharing. For instance, Kalemli-Ozcan et al. (2003) investigate the effects of industrial specialization, Demyanyk et al. (2007) the impact of banking deregulation, and Sørensen et al. (2007) the consequences of home bias in debt and equity holdings. This paper contributes to this literature by estimating to which extent and why regions differ in insurance profiles. Using available panel data of the United States from 1963 to 2013, I report estimates for risk sharing heterogeneity between US states and further analyze potential determinants of this heterogeneity which have not yet been examined in the related literature. To the best of my knowledge, this paper is also the first to report risk sharing heterogeneity based on state-specific estimations of insurance profiles.

Following the aforementioned literature, three channels of risk sharing are considered: an *income smoothing* channel, *federal transfers*, and a *consumption smoothing* channel. The empirical results derived in this paper point to large but imperfect consumption insurance. Income and consumption smoothing play a decisive role, as they insulate 48% and 26.6% of state consumption against regional output fluctuations, respectively. Federal transfers across states also play a significant but less vital role, contributing 9.4%. These estimates provide an *aggregate* insurance profile.

In order to document the diversity of insurance profiles across US states, I augment the methodology provided by Asdrubali et al. (1996) and estimate state-specific risk sharing profiles. Based on these estimates, this paper reports distinct clusters of states, each with a unique insurance profile. In particular, the analysis documents that states differ substantially along two dimensions: the magnitude of consumption insurance and the contribution of each risk sharing channel. The state-specific analysis shows that overall insurance ranges from 68.1% to full insurance. Grouping states based on their individual risk sharing profile, four distinctive clusters can be identified. One cluster displays an insurance profile similar to the aggregate average profile. The other clusters are characterized by an insurance profile that emphasizes one specific risk sharing channel: one cluster insures significantly more through income smoothing (67.9%), one through federal transfers (17.4%), and one through consumption smoothing (53%).

I then investigate state observables which might determine these distinctive profiles. The paper shows that overall risk sharing is positively associated with lower economic activity at risk, better insurance opportunities, and lower shock persistence. Furthermore, the contribution of federal transfers is positively associated with higher unemployment rate volatility and consumption smoothing is negatively associated with state tax and expenditure limits and higher population poverty rates.

The paper is organized as follows. Section 2 discusses aggregate risk sharing channels and introduces the application to the United States. Section 3 then investigates the heterogeneity of risk sharing profiles between US states, and Section 4 relates the observed insurance heterogeneity to different state characteristics. Section 5 concludes.

2 Measuring Aggregate Risk Sharing

2.1 Insurance Channels and Estimation Strategy

Asdrubali et al. (1996) develop a methodology to identify and quantify inter-regional insurance channels. Consider the following decomposition of gross state product gsp for a state i at time t:

$$gsp_{it} = \frac{gsp_{it}}{si_{it}} \frac{si_{it}}{dsi_{it}} \frac{dsi_{it}}{c_{it}} c_{it}, \tag{1}$$

where si is defined as state income, dsi as disposable state income, and c as state consumption. From this expression, one can retrieve the following three channels that contribute to

insulating consumption against gsp fluctuations.

Income flows. While gsp measures goods and services produced within the geographical boundaries of a state, si includes income from non-domestic financial investment, e.g., dividend, interest, and rental payments across states. Ex ante, these returns from diversified capital holdings might buffer variations in gsp.

Federal transfers. The difference between *si* and *dsi* reflects interstate public net transfers, i.e., it refers to the extent of the insurance provided by federal taxes and transfers.

Consumption smoothing. Ex post, private and public state residents can save or dissave on credit markets to adjust consumption c to variations in income.

As an illustration, assume that changes in si perfectly offset changes in gsp and c is constant over time. In this example, income flows provide perfect insurance to consumption against fluctuations in state output.

The empirical estimation of the contribution of each channel relies on a decomposition of the cross-sectional variance in gsp given by equation (1). Omitting i and t,

$$var (\Delta log(gsp)) = cov (\Delta log(gsp), \Delta log(gsp) - \Delta log(si)) + cov (\Delta log(gsp), \Delta log(si) - \Delta log(dsi)) + cov (\Delta log(gsp), \Delta log(dsi) - \Delta log(c)) + cov (\Delta log(gsp), \Delta log(c)).$$

$$(2)$$

Dividing each side by the variance of $(\log) gsp$ growth yields

$$1 = \beta_I + \beta_F + \beta_C + \beta_U. \tag{3}$$

In this expression, β_U is the *unsmoothed* share of *gsp* variations which translate into consumption fluctuations: perfect insurance corresponds to $\beta_U=0$. The remaining coefficients are associated with the insurance contribution of income flows (β_I), federal transfers (β_F), and consumption smoothing (β_C). These coefficients are estimated by running panel regressions. Following Asdrubali et al. (1996), I estimate:

$$\Delta \log\left(gsp_{i,t}\right) - \Delta \log\left(si_{i,t}\right) = \mu_{I,t} + \beta_I \Delta \log\left(gsp_{i,t}\right) + u_{i,I,t},\tag{4}$$

$$\Delta \log(si_{i,t}) - \Delta \log(dsi_{i,t}) = \mu_{F,t} + \beta_F \Delta \log(gsp_{i,t}) + u_{i,F,t}, \tag{5}$$

$$\Delta log \left(dsi_{i,t} \right) - \Delta log \left(c_{i,t} \right) = \mu_{C,t} + \beta_C \Delta log \left(gsp_{i,t} \right) + u_{i,C,t},\tag{6}$$

$$\Delta log(c_{i,t}) = \mu_{U,t} + \beta_U \Delta log(gsp_{i,t}) + u_{i,U,t}.$$
(7)

where $\mu_{z,t}$ are time fixed effects¹, $u_{i,z,t}$ an error term, and $z \in \{I, F, C, U\}$ the respective risk sharing channel. Formally, β_z is the elasticity of an insurance channel (left-hand side) to variations in regional income² (right-hand side). Importantly, time fixed effects eliminate aggregate fluctuations, so that coefficients capture the regional consumption insurance to regional shocks. Idiosyncratic regional fluctuations account for around 50% of the total fluctuations in state output.³

2.2 Data

The estimation of equations (4)–(7) relies on a panel data set of gsp, si, dsi, and c for each US state (plus Washington, DC), at annual frequency, covering 1963-2013. For this analysis, I merge two data sets: Asdrubali et al. (1996) provide the data for 1963-1998, data from Alcidi et al. (2017) is used for the remaining time period 1999-2013. Both rely on the same data construction procedure suggested by Asdrubali et al. (1996). A detailed overview of this method can be found in Table A.1 in the appendix. Note that when discussing the results, I will show that merging the data sets is valid and does not bias the results. In short, the relevant panel variables are constructed as follows.

Gross state product is defined as the value added of all industries at the state level. *State income* measures the sum of personal and public income. Personal income includes, for instance, wages, supplements, or dividend income. Public income consists of non-personal tax

and interest income, minus public transfers.

³Formally, the regression

$$\Delta log\left(gsp_{i,t}\right) = \mu_t + u_{i,t},$$

filtering aggregate shocks from variations in state output, is associated with an R^2 of 0.49.

¹Note that the structure of the equations implies that time fixed effects sum up to 0, i.e., $\sum_{z} \mu_{z,t} = 0 \forall t$.

²Note that gsp is regarded as exogenous, as in Asdrubali et al. (1996). Thus, the forthcoming results should be interpreted as statistical rather than causal relationships. Asdrubali and Kim (2004) address this issue in more detail and endogenize the output process. Overall, their results are broadly in line with the literature assuming exogenous output processes.

Disposable state income is defined as state income plus federal transfers to private individuals and (state or local) governments. Federal (non-)personal taxes are deducted.

State consumption measures the sum of private and public consumption at the state level.

2.3 Results

Table 1 displays the estimates of equations (4)–(7). Column 2 shows that aggregate consumption insurance is imperfect but high: $1-\beta_u=84.1\%$ of gsp fluctuations do not translate into consumption fluctuations. Income and consumption smoothing channels provide the largest buffers against gsp fluctuations, while federal transfers across states contribute around 10%.⁴

In order to ensure the validity of merging the data sets, I additionally report separate results for the corresponding time frames in columns 3 and 4. Clearly, the results do not differ significantly between data sets, neither in terms of the level of the insurance contribution of each channel, nor in terms of (clustering-robust) standard errors.⁵

	1963-2013	1963 - 1998	1999-2013
Income Smoothing (β_I)	0.480***	0.482^{***}	0.472^{***}
medine smoothing (p_I)	(0.06)	(0.08)	(0.06)
Federal Transfers (β_F)	0.094^{***}	0.096^{***}	0.089^{***}
Federal framsiers (β_F)	(0.01)	(0.01)	(0.02)
Consumption Smoothing (β_C)	0.266^{***}	0.268^{***}	0.258^{***}
Consumption Smoothing (ρ_C)	(0.06)	(0.08)	(0.06)
Unsmoothed (β_U)	0.159^{***}	0.153^{***}	0.181^{***}
Unshibothed (βU)	(0.03)	(0.04)	(0.04)

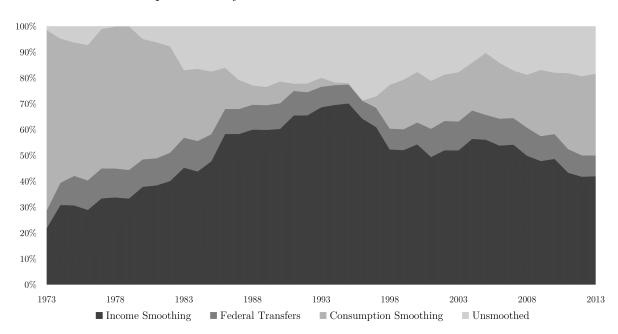
Table 1: Aggregate Risk Sharing in the United States.

Notes. The second column refers to the estimation results for all periods. Columns 3 and 4 report the estimates for the periods associated with each data set. Clustering-robust standard errors in parentheses. *p < 0.1, **p < 0.05, ***p < 0.01.

Note that the contribution of each insurance channel is not constant over time. As Figure 1 reports, there is substantial time variation when estimating risk sharing per year. Determination of the four coefficients relies on the estimation of equations (4)–(7) on a 10-year

⁴These results are broadly in line with Asdrubali et al. (1996), who report a share of 39% income smoothing, 13% federal transfers, and 23% consumption smoothing between 1963 and 1990. They also tally with Alcidi et al. (2017), who report 47%, 8%, and 27%, respectively, between 1998 and 2013.

⁵Clustering-robust standard errors adjust for the 51 regions in the sample (50 states plus Washington, DC) and account for autocorrelation and heteroscedasticity.



rolling window, i.e., on an estimation of the equations for each year from 1973 onwards, using observations from the previous 10 years.

Figure 1: Time Variation in Insurance Channel Estimates of a 10-Year Rolling Window Estimation of Equations (4) - (7).

The results show an increasing role of income smoothing, a constant modest contribution of federal transfers (close to 10%), and strong variations in consumption smoothing. The unsmoothed share stabilizes around 20%.⁶ These results tally with a similar analysis conducted by Asdrubali et al. (1996).

⁶Note that estimating aggregate risk sharing per decade confirms these results (see Table B.1 in the appendix).

3 Measuring Heterogeneous Risk Sharing

3.1 Estimation Strategy

In order to shed light on potential heterogeneity between US states, I augment the system of equations (4)-(7) with state dummy variables to derive state-specific insurance profiles:

$$\Delta \log\left(gsp_{i,t}\right) - \Delta \log\left(si_{i,t}\right) = \mu_{I,t} + \beta_I \Delta \log\left(gsp_{i,t}\right) + \sum_j \theta_{i,j} \beta_{j,I} \Delta \log\left(gsp_{j,t}\right) + u_{i,I,t},\tag{8}$$

$$\Delta \log\left(si_{i,t}\right) - \Delta \log\left(dsi_{i,t}\right) = \mu_{F,t} + \beta_F \Delta \log\left(gsp_{i,t}\right) + \sum_j \theta_{i,j}\beta_{j,F} \Delta \log\left(gsp_{j,t}\right) + u_{i,F,t},\tag{9}$$

$$\Delta \log\left(dsi_{i,t}\right) - \Delta \log\left(c_{i,t}\right) = \mu_{C,t} + \beta_C \Delta \log\left(gsp_{i,t}\right) + \sum_j \theta_{i,j}\beta_{j,C} \Delta \log\left(gsp_{j,t}\right) + u_{i,C,t},\tag{10}$$

$$\Delta log(c_{i,t}) = \mu_{U,t} + \beta_U \Delta log(gsp_{i,t}) + \sum_j \theta_{i,j} \beta_{j,U} \Delta log(gsp_{j,t}) + u_{i,U,t}, \qquad (11)$$

where β_z is the risk sharing coefficient for the first state in the panel, i.e., Alabama, and $\theta_{i,j}$ is a dummy variable equal to 1 if i=j. In that case, $\beta_z + \beta_{i,z}$ is the risk sharing contribution of channel z to state *i*'s consumption insurance.

3.2 Results

The estimation results suggest that there exists substantial heterogeneity in total insurance and large diversity of insurance profiles across US states.

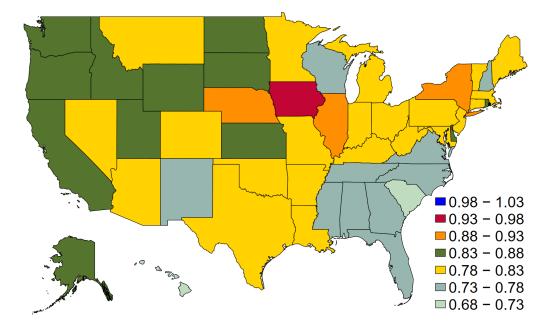


Figure 2: State-Specific Consumption Insurance $1 - \beta_U$.

The detailed results for each state can be found in Table C.1 in the appendix. Figure 2 reports the share of total consumption insurance for each state. Estimates range from a low of 68.1% in Hawaii to full insurance in Washington, DC.

Furthermore, there exists substantial heterogeneity with respect to the extent to which each risk sharing channel contributes to the insurance profile of individual states. Table 2 reports key statistics for all channels. In particular, income smoothing contributes to 60.5%of consumption insurance in Alaska and only to 27.5% in North Dakota. Federal transfers vary from 14.5% in Michigan to 6.1% in Washington, DC. Finally, consumption smoothing contributes to only 10.7% of insurance against *gsp* fluctuations in Hawaii but to 49.6% in North Dakota.

Table 2: State-Specific Risk Sharing Summary Statistics.

	Minimum	Maximum	Average	Median	SD
Income Smoothing (β_I)	0.275	0.605	0.441	0.432	0.049
Federal Transfers (β_F)	0.061	0.145	0.109	0.110	0.016
Consumption Smoothing (β_C)	0.107	0.496	0.268	0.260	0.068
Unsmoothed (β_U)	-0.012	0.319	0.182	0.188	0.054

Notes. This table reports summary statistics of the state-specific insurance profiles estimated using equations (8) - (11). SD stands for standard deviation.

3.3 Risk Sharing Clusters

In order to identify representative risk sharing profiles, I use a k-means clustering procedure based on the state-specific insurance profiles. The clustering method allocates states into N clusters $\{c_j\}_{j=1}^N$ by minimizing the sum of squared differences within clusters:⁷

$$\min\sum_{j=1}^{N}\sum_{i\in c_j} dist(\gamma_j, \beta_i)^2,$$
(12)

where $\gamma_j = \{\gamma_{z,j}\}$ is the set of average risk sharing coefficients $\gamma_{z,j} = \frac{1}{\operatorname{card}(c_j)} \sum_{i \in c_j} \beta_{z,i}$ within each cluster. Once states have been allocated into different clusters, I run the panel regressions outlined in equations (4)–(7) for each cluster to retrieve their respective risk sharing profiles.

I identify four distinct clusters, each characterized by a unique risk sharing profile.⁸ Table 3

⁷See Appendix D for more details on the implemented algorithm.

⁸Note that Washington, DC, is left out of the analysis because it has a unique insurance profile (see Table C.1 in the appendix for details).

reports the cluster-specific insurance profiles and associated economic and demographic statistics. Clusters 1 to 3 are characterized by an insurance profile which emphasizes one specific channel: income smoothing (67.9% in Cluster 1), federal transfers (17.4% in Cluster 2), and consumption smoothing (53% in Cluster 3). Note that each of these clusters differs from all other clusters in their emphasized dimension at the 99% level. Cluster 4 gathers states with insurance profiles closest to the average profile reported in Table 1. Note that about 40% of states differ from the average risk sharing profile, constituting to roughly 30% of US gdpand population in 2013. Lastly, a measure for cluster compactness is reported, showing that the clustering method is successful in reducing the variance of state-specific insurance profiles within each cluster in comparison to the total variance across these profiles.

Cluster 1 – Income Smoothing Cluster				
Insurance Profil	Descriptive Statistics			
Income Smoothing (β_I)	0.679^{***} (0.02)	Number of states	5	
Federal Transfers (β_F)	0.067^{***} (0.01)	Population share	3.09%	
Consumption Smoothing (β_C)	0.146^{**} (0.06)	gsp share	3.13%	
Unsmoothed (β_U)	0.109^{**} (0.05)	Cluster compactness	63.51%	
Cluster 2	2 – Federal Tra	nsfer Cluster		
Insurance Profil	e	Descriptive Statis	stics	
Income Smoothing (β_I)	0.394^{***} (0.04)	Number of states	10	
Federal Transfers (β_F)	0.174^{***} (0.01)	Population share	23.03%	
Consumption Smoothing (β_C)	0.129^{**} (0.06)	gsp share	23.61%	
Unsmoothed (β_U)	0.303^{***} (0.06)	Cluster compactness	9.32%	
Cluster 3 – C	Consumption Sn	noothing Cluster		
Insurance Profil	e	Descriptive Statis	stics	
Income Smoothing (β_I)	0.245^{***} (0.07)	Number of states	4	
Federal Transfers (β_F)	0.086^{***} (0.02)	Population share	2.06%	
Consumption Smoothing (β_C)	0.530^{***} (0.05)	gsp share	2.19%	
Unsmoothed (β_U)	0.139^{***} (0.03)	Cluster compactness	60.74%	
Clust	ter 4 – Average	e Cluster		
Insurance Profil	Descriptive Statis	stics		
Income Smoothing (β_I)	0.484^{***} (0.04)	Number of states	31	
Federal Transfers (β_F)	0.098^{***} (0.01)	Population share	71.63%	
Consumption Smoothing (β_C)	0.204^{***} (0.03)	gsp share	68.99%	
Unsmoothed (β_U)	0.213^{***} (0.05)	Cluster compactness	34.34%	

Table 3: Risk Sharing Clusters.

Notes. Population share and gsp share refer to the relative population size and economic weight of each cluster in 2013. Cluster compactness refers to the variance of state-specific insurance profiles within each cluster relative to the total variance across clusters. Clustering-robust standard errors in parentheses. *p < 0.1, **p < 0.05, ***p < 0.01.

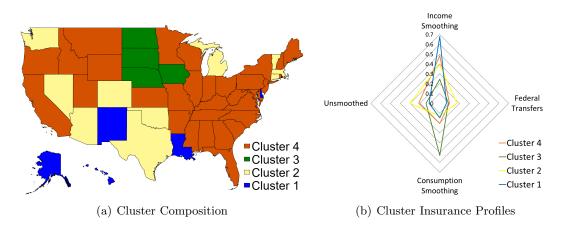


Figure 3: Cluster Composition and Insurance Profiles.

Figure 3 depicts the composition of each cluster and their insurance profiles graphically. Note that more details regarding the cluster composition can be found in Table C.1 in the appendix. This illustration further underscores the extent of heterogeneity between clusters and US states. Naturally, the results raise the questions of where this heterogeneity stems from and what accounts for the diversity of insurance profiles between states.

4 Determinants of Risk Sharing Heterogeneity

4.1 Estimation Strategy

In order to identify characteristics that are associated with state-specific insurance profiles, I follow Demyanyk et al. (2007) and Sørensen et al. (2007) by introducing an interaction term into equations (4)–(7). As an illustration, to assess how the overall insurance level is sensitive to variations in variable $x_{i,t}$, I estimate

$$\Delta log(c_{i,t}) = \mu_{U,t} + (\beta_U + \vartheta_U x_{i,t}) \Delta log(gsp_{i,t}) + u_{i,U,t},$$
(13)

where β_U is the average unsmoothed share and ϑ_U is the component associated with higher realizations of $x_{i,t}$, i.e., the sensitivity parameter.

Depending on the analyzed variable, the specification of $x_{i,t}$ can take two different forms:

$$x_{i,t} = \zeta_{i,t} - \bar{\zeta}_t,\tag{14}$$

$$x_{i,t} = D_{i,t} = \begin{cases} 1 & \text{if state i meets a certain condition in year t} \\ 0 & \text{otherwise.} \end{cases}$$
(15)

Equation (14) implies that any continuous variable $\zeta_{i,t}$ is corrected by the mean over all states $\overline{\zeta}_t$. The impact of binary state characteristics are measured by using dummy variables $D_{i,t}$ as defined in equation (15). I estimate ten relations of variables with risk sharing on all four dimensions, nine by using equation (14), one by using equation (15).

4.2 State Characteristics and Data

In this section, I briefly describe the ten considered state characteristics by commenting on the expected relationship between each variable and state insurance profiles as well as on the data (see Appendix E for details).

Composition of gsp – share of manufacturing sector. The level of risk sharing might be sensitive to the sectoral composition of gsp. For instance, one might expect that states with a relatively high manufacturing share have lower overall risk sharing due to the declining dynamism of manufacturing, i.e., $\vartheta_U > 0$. This hypothesis implies that states with a higher share of economic activity at risk have lower insurance capacities. In terms of data construction, the manufacturing sector share is defined as the value added by this sector at the state level for the entire time frame.

Composition of gsp – share of service sector. In contrast to the previous hypothesis, I expect $\vartheta_U < 0$ due to the continuing increase in the importance of the service sector. The service sector share is defined as the value added by this sector at the state level for all years. Table E.1 in the appendix delivers further details.

Correlation of gsp growth with US gdp growth. States whose output processes are negatively associated with the aggregate output process potentially have better diversification opportunities. Thus, I expect $\vartheta_U > 0$, i.e., states in which the relationship is particularly negative are characterized by higher overall insurance. Both gsp and gdp are defined as the value added of all industries for all periods.

Autocorrelation of gsp growth. Following Blundell et al. (2008), who find that consumption insurance against permanent income shocks is lower than against transitory shocks at the

or

household level, I expect states with higher autocorrelation of gsp to have a lower overall insurance level ($\vartheta_U > 0$). The autocorrelation ρ for each state is retrieved by running the following simple estimation:

$$\Delta log \left(gsp_{i,t} \right) = \rho_i \Delta log \left(gsp_{i,t-1} \right) + \epsilon_{i,t},$$

where $\epsilon_{i,t}$ is the error term.

Unemployment rate volatility. High state unemployment rate volatility implies a stronger reaction of the state's unemployment rate to shocks. Thus, I expect $\vartheta_F > 0$, i.e., federal insurance mechanisms (like unemployment benefits, for instance) play a more vital role when the relative unemployment rate volatility is high. The volatility is calculated on the basis of average yearly unemployment rates at the state level between 1976 and 2013.⁹

Poverty rate level. A state's poverty rate level is a potential indicator of the capacity of individuals to react ex post to idiosyncratic shocks. Hence, I expect $\vartheta_C < 0$, implying that a higher poverty rate limits state residents in their consumption smoothing capacity. The estimation is based on the yearly poverty rate at the state level between 1995 and 2013.¹⁰

Public revenue and spending restrictions. Similar to the hypothesis for the impact of the poverty rate at the individual level, public revenue and spending restrictions might constrain states in reacting ex post to shocks. Between 1978 and 2006, 31 states introduced either a revenue limit (tieing state revenue to some index, for instance, inflation), an expenditure limit (tieing state expenditures to similar types of indices), or limited appropriations to a percentage of revenue estimates (tieing appropriations to a revenue forecast). A detailed overview can be found in Table E.2 in the appendix.

4.3 Results

Using the structure given by equation (15) to estimate the sensitivity of states' risk sharing profiles to the introduction of public revenue and spending restrictions and equation (14) for all other variables, I estimate the sensitivity parameter for each state characteristic as illustrated by equation (13) for all four channels. The findings are presented in Table 4.

States where manufacturing contributes to a higher share of output have a higher unsmoothed share: an increase in the relative share of manufacturing in gsp by 1 percentage point decreases the consumption insurance level by 0.261 percentage points. This supports the

⁹Note that the US Department of Labor only publishes state unemployment rates from 1976 onwards.

¹⁰Note that the US Census Bureau only publishes state poverty rates from 1995 onwards.

hypothesis that states with a higher share of economic activity at risk have lower insurance capacities. Correspondingly, the sensitivity parameter of the unsmoothed share with respect to the service sector is negative. However, this estimate is not significantly different from 0. Moreover, the overall level of risk sharing is positively associated with higher negative correlations of gsp growth with US gdp growth: a decrease of the relative correlation by 0.1 increases the overall level of insurance by 1.25 percentage points. The results suggest that this higher level is achieved through both higher income and consumption smoothing. However, the estimates for these two coefficients are not significantly different from 0. Moreover, when shocks to state output are more persistent, insurance opportunities decrease and the overall insurance level is lower. The loss in insurance capacities primarily results from a decrease in consumption smoothing.

Variable	ϑ_I	ϑ_F	ϑ_C	$artheta_U$
Composition of gsp				
Manufacturing	-0.101	0.054	-0.214	0.261^{**}
Manufacturing	(0.13)	(0.03)	(0.18)	(0.12)
Services	-0.255^{**}	0.146^{***}	0.234	-0.124
Services	(0.12)	(0.04)	(0.16)	(0.12)
Correlation of <i>an</i> with US <i>adv</i>	-0.068	0.052***	-0.109	0.125**
Correlation of gsp with US gdp	(0.13)	(0.02)	(0.14)	(0.06)
Autocorrelation of <i>acn</i> growth	0.064	0.012	-0.165^{***}	0.089**
Autocorrelation of gsp growth	(0.06)	(0.01)	(0.06)	(0.04)
Unemployment rate volatility	0.017	0.015**	-0.019	-0.013
Onemployment rate volatility	(0.02)	(0.007)	(0.01)	(0.01)
Poverty rate level	1.251***	-0.164	-1.302^{**}	0.215
Toverty rate level	(0.38)	(0.22)	(0.62)	(0.42)
Public revenue and spending restrictions				
All limits	0.088^{*}	-0.002	-0.083^{**}	-0.003
All lillings	(0.05)	(0.01)	(0.04)	(0.02)
Limited appropriations	0.043	-0.024^{***}	-0.050	0.031
Emitted appropriations	(0.03)	(0.01)	(0.04)	(0.03)
Revenue limit	-0.018	0.027^{**}	-0.007	-0.002
	(0.03)	(0.01)	(0.05)	(0.03)
Expenditure limit	0.098^{*}	-0.003	-0.085^{**}	-0.011
	(0.06)	(0.01)	(0.04)	(0.02)

Table 4: Determinants of Risk Sharing Heterogeneity in the United States.

Notes. Data gathered and constructed as described in Section E in the appendix. Clustering-robust standard errors in parentheses. *p < 0.1, **p < 0.05, ***p < 0.01.

Furthermore, we find that risk sharing through federal transfers is positively associated with

higher unemployment rate volatility, i.e., with unemployment rates that are very sensitive to shocks. An increase in relative volatility by 1 is associated with an increase in insurance through federal transfers of 1.5 percentage points.

Lastly, the results suggest that consumption smoothing is negatively associated with tax or expenditure limits for states and higher poverty rates. Tax and expenditure limits constrain states in financing countercyclical policies, higher poverty rates reflect low opportunities for individuals to do so. Overall, the introduction of a public revenue or spending restriction decreases a state's capacity to react ex post to idiosyncratic shocks by 8.3 percentage points. Interestingly, this effect is driven by states that introduced expenditure limits rather than revenue limits or states that limited appropriations. At the individual level, a relative increase in the poverty rate by 1 percentage point decreases consumption smoothing by 1.3 percentage points.

5 Conclusion

This paper presents novel findings on substantial risk sharing heterogeneity between US states. In particular, by estimating state-specific risk sharing profiles and identifying four unique clusters, I show that states differ along two dimensions: the extent of overall insurance and the contribution of each risk sharing channel. Potential determinants of this heterogeneity are shown to be the composition of *gsp*, insurance opportunities of states, vulnerability to idiosyncratic shocks, or the capacity to finance countercyclical policies (by both individuals and states). Clearly, this is not an extensive list. There is a multitude of other state or individual characteristics that might play a role in explaining risk sharing heterogeneity. Moreover, this paper invites to further deepen the understanding of heterogeneity in risk sharing. Naturally, the examination of insurance heterogeneity is not constrained to analyses between states but extends to investigations at the county or individual level. The analysis can also be extended by using dynamic econometric models (as in Asdrubali and Kim, 2004) to estimate risk sharing heterogeneity. While it seems intuitive that the presented results qualitatively apply to this dynamic perspective, empirical evidence is necessary to further underscore the relevance of heterogeneity in risk sharing.

Appendices

A Aggregate Data Construction

Category	Sources
Gross State Product	Bureau of Economic Analysis (bea)
State Income	
State Personal Income	bea
+ Federal Non-personal Taxes and Contributions	US Budget and Government Finances
+ State and Local Non-personal Taxes	Government Finances and bea
+ Interest on State and Local Funds	Government Fiances
– Direct Transfers (Federal and State)	bea
where	
Federal Non-personal Taxes and Contributions =	
Federal Corporate Income Taxes	United States Budget
+ Tobacco Taxes	United States Budget
+ Miscellaneous Taxes and Other Excise Taxes	United States Budget
+ Social Security Contributions	United States Budget
+ Unemployment Insurance Taxes	Government Finances
and where	
State and Local Non-personal Taxes $=$	
State and Local Tax Revenue	Government Finances
- State and Local Personal Taxes	bea
and where	
Interest on State and Local Funds =	
Interest on Insurance Trust Funds	Government Finances
+ Interest on State Miscellaneous Funds	Government Finances
+ Interest on Local Insurance Trust Funds	Government Finances
+ Interest on Local Miscellaneous Funds	Government Finances

Table A.1: Aggregate Data Construction.

- Interest on State Unemployment Deposits at the Treasury Government Finances

State Income

+ Federal Grants to State Governments	United States Statistical Abstract
+ Federal Transfers to Individuals	bea and US Statistical Abstract
– Federal Non-personal Taxes and Contributions	US Budget and Government Finances
– Federal Personal Taxes	bea

where

Federal Transfers to Individuals =	
OASDI Payments	bea
+ Railroad Retirement and Disability Payments	bea
+ Federal Civilian Employee Retirement Payments	bea
+ Military Retirement Payments	bea
+ Workers' Compensation	bea
+ Supplemental Social Security	bea
+ Food Stamps	bea
+ Other Federal Income Maintenance	bea
+ Unemployment Insurance Benefits	bea
+ Veterans Benefits	bea
+ Federal Education and Training Payments	bea
+ Federal Payments to Nonprofit Institutions	bea
+ Total Medical Payments	bea
- Medicaid Payments	United States Statistical Abstract

 $State \ Consumption$

- State and Local Transfers

Retail Sales (Rescaled) (1963-1996),	Sales & Marketing Management (1963-1996),
Private Consumption (1997-2013)	bea (1997-2013)
+ State and Local Government Consumption	Government Finances
where	
State and Local Government Consumption $=$	
State and Local Government Expenditure	Government Finances

where

State and Local Transfers $=$	
Direct Transfers	bea
- Federal Direct Transfers	bea

Notes. Construction of data as in Asdrubali et al. (1996).

B Aggregate Risk Sharing per Decade

	1963-1970	1971-1980	1981-1990	1991-2000	2001-2010	2004-2013
Income Creathing (2)	0.296^{***}	0.379***	0.603***	0.543^{***}	0.487^{***}	0.419^{***}
Income Smoothing (β_I)	(0.04)	(0.07)	(0.11)	(0.06)	(0.05)	(0.06)
Fodoral Transford (β_{-})	0.061^{***}	0.106^{***}	0.100***	0.083^{***}	0.096^{***}	0.080^{***}
Federal Transfers (β_F)	(0.02)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)
Consumption Smoothing (β_C)	0.343^{***}	0.466^{***}	0.084	0.196^{**}	0.237^{***}	0.317^{***}
	(0.09)	(0.12)	(0.07)	(0.09)	(0.07)	(0.07)
Unsmoothed (β_U)	0.300^{***}	0.05	0.214***	0.177^{***}	0.180^{***}	0.183^{***}
	(0.09)	(0.05)	(0.06)	(0.06)	(0.03)	(0.05)

Table B.1: Aggregate Risk Sharing per Decade.

Notes. Estimates using equations (4) – (7) by decades. Clustering-robust standard errors in parentheses. *p < 0.1, **p < 0.05, ***p < 0.01.

As Table B.1 reports, there is substantial time variation when estimating risk sharing per decade. The contribution of the income smoothing channel increases over time, federal transfers contribute close to 10% in most subperiods, and consumption smoothing varies strongly between 8.4% and 46.6%. The unsmoothed share stabilizes around 20% from 1981 onwards.

C Sate-Specific Estimation

Table C.1 displays the state-specific estimation results. Note that, due to the introduction of state dummies, standard errors are high and most deviations from the first state in the panel (Alabama) are not statistically significant. Nevertheless, the results indicate substantial heterogeneity. Therefore, the cluster analysis is conducted. The estimation of the risk sharing profiles of these clusters display heterogeneity, which is also highly statistically significant.

	19	064-2013	I	1	
State	β_I	β_F	β_C	β_U	Cluster
Alabama	0.443 (0.04)	0.104 (0.01)	0.232(0.08)	0.221 (0.07)	4
Alaska	0.605^{***} (0.04)	0.073^{**} (0.02)	$0.166\ (0.08)$	0.157 (0.07)	1
Arizona	0.437 (0.06)	0.122 (0.02)	0.262(0.10)	0.180 (0.09)	2
Arkansas	$0.399\ (0.05)$	0.108 (0.02)	$0.316\ (0.10)$	0.176(0.08)	4
California	0.459(0.06)	0.124 (0.02)	0.281 (0.10)	0.137(0.09)	4
Colorado	0.413(0.06)	0.130 (0.02)	$0.271 \ (0.10)$	0.186(0.09)	2
Connecticut	0.432(0.05)	0.124 (0.02)	$0.256\ (0.10)$	0.188(0.09)	2
Delaware	0.571^{**} (0.05)	0.079(0.02)	0.186(0.10)	0.164(0.09)	1
Dist. of Col.	0.521 (0.05)	0.061^{**} (0.02)	0.431^{**} (0.09)	-0.012^{***} (0.08)	
Florida	0.402 (0.06)	0.114 (0.02)	0.259(0.10)	0.225(0.09)	4
Georgia	0.430(0.05)	0.115(0.02)	0.223(0.10)	0.232(0.09)	4
Hawaii	0.481 (0.05)	0.092(0.02)	0.107(0.10)	0.319(0.09)	1
Idaho	0.436(0.06)	0.113(0.02)	0.313(0.10)	0.139(0.09)	4
Illinois	0.450 (0.06)	0.120(0.02)	0.310(0.11)	0.120 (0.09)	4
Indiana	0.472(0.06)	0.104 (0.02)	0.237~(0.10)	0.187(0.09)	4
Iowa	0.419(0.05)	0.098(0.02)	0.419^{*} (0.10)	0.064^{*} (0.09)	3
Kansas	0.443 (0.05)	0.112 (0.02)	0.296(0.10)	0.150(0.09)	4
Kentucky	0.422(0.06)	0.110 (0.02)	0.260(0.10)	0.208 (0.09)	4
Louisiana	0.516(0.04)	0.098 (0.02)	0.195(0.09)	0.190 (0.08)	1
Maine	0.424 (0.06)	0.107 (0.02)	0.262 (0.10	0.207 (0.09)	4
Maryland	0.455 (0.06)	0.100 (0.02)	0.254(0.10)	0.191(0.09)	4
Massachusetts	0.437 (0.05)	0.125(0.02)	$0.241 \ (0.10)$	0.197(0.09)	2
Michigan	0.429(0.06)	0.145^{**} (0.02)	0.234(0.10)	0.192 (0.09)	2
Minnesota	0.412 (0.05)	0.111 (0.02)	0.288(0.10)	0.188 (0.09)	4
Mississippi	0.422(0.05)	0.095 (0.02)	0.259(0.10)	0.225(0.09)	4
Missouri	0.455 (0.06)	0.102 (0.02)	0.269(0.10)	0.174 (0.09)	4
Montana	0.434(0.06)	0.113 (0.02)	0.278(0.10)	0.175 (0.09)	4
Nebraska	0.418(0.05)	0.093 (0.02)	0.373(0.10)	0.116 (0.09)	3
Nevada	0.430(0.06)	$0.139^{*} (0.02)$	0.244(0.10)	0.187(0.09)	2
New Hampshire	0.424 (0.05)	0.113 (0.02)	0.215(0.10)	0.249 (0.09)	4
New Jersey	0.448 (0.06)	0.111 (0.02)	0.246 (0.10)	0.195 (0.09)	4
New Mexico	0.529 (0.05)	0.093 (0.02)	0.150 (0.10)	0.228 (0.09)	1
New York	0.467 (0.06)	0.101 (0.02)	0.318 (0.10)	0.114 (0.09)	4
North Carolina	0.402 (0.06)	0.111 (0.02)	0.228 (0.10)	0.258 (0.09)	4

Table C.1: State-Specific Risk Sharing in the United States.

North Dakota	0.275^{***} (0.05)	0.101 (0.02)	0.496^{***} (0.09)	0.128(0.08)	3
Ohio	0.418 (0.06)	0.112 (0.02)	0.277(0.10)	0.194(0.09)	4
Oklahoma	0.429 (0.05)	$0.135^{*} (0.02)$	0.233(0.09)	0.203 (0.08)	2
Oregon	0.453(0.06)	0.116 (0.02)	0.301 (0.10)	0.130(0.09)	4
Pennsylvania	0.432 (0.06)	0.107(0.02)	0.263(0.10)	0.198(0.09)	4
Rhode Island	0.442 (0.06)	0.105(0.02)	0.292(0.10)	0.161(0.09)	4
South Carolina	0.420 (0.05)	0.106 (0.02)	0.179(0.10)	0.295(0.09)	4
South Dakota	0.355^{*} (0.05)	0.092 (0.02)	0.421^{**} (0.09)	0.131(0.08)	3
Tennessee	0.419 (0.06)	0.101 (0.02)	0.252(0.10)	0.228(0.09)	4
Texas	0.457(0.05)	0.121 (0.02)	0.234(0.10)	0.188(0.08)	2
Utah	0.458 (0.06)	0.122 (0.02)	0.272(0.10)	0.148(0.09)	4
Vermont	0.409 (0.06)	0.121 (0.02)	0.267(0.10)	0.202(0.09)	2
Virginia	0.419 (0.05)	0.108 (0.02)	0.233(0.10)	0.239(0.09)	4
Washington	0.423 (0.06)	$0.138^{*} (0.02)$	0.300 (0.10)	0.139(0.09)	2
West Virginia	0.428 (0.06)	0.112 (0.02)	0.247(0.10)	0.213(0.09)	4
Wisconsin	0.417 (0.06)	0.109 (0.02)	0.242(0.10)	0.231(0.09)	4
Wyoming	0.500 (0.05)	0.098 (0.02)	$0.276\ (0.09)$	0.125(0.08)	4

Notes. Standard errors in parentheses. Significance in terms of deviations from value of first state in panel (Alabama). *p < 0.1, *p < 0.05, ***p < 0.01.

D K-Means Clustering

The k-means clustering algorithm performs the following steps:

(1.) pick arbitrary sets of average risk sharing coefficients γ_j

(2.) assign each state i to a cluster j as to minimize the associated increase in variance

(3.) given the allocation, compute the cluster mean γ_i

Repeat (2.) and (3.) until there is no reassignment of states across clusters that further reduces the objective function. Importantly, note that the procedure is implemented on standardized coefficients $\mathbb{E}[\beta]=0$ and $Var[\beta]=1$ to eliminate sorting weighted by relative size of insurance channels.

E Determinants of Risk Sharing Heterogeneity – Data Construction

Table E.1: Gr	coss State	Product	Composition -	- Data	Construction.
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Category	Sources
Manufacturing Sector	bea
Service Sector	
From 1963-1996:	
Services	bea
+ Retail Trade	bea
+ Wholesale Trade	bea
+ Transportation and Public Utilities	bea
From 1997-2013:	
Private Services Producing Industries	bea
- Finance, Insurance, Real Estate, Rental, and Leasing	bea

Notes. Data construction of manufacturing and service sector in order to get a consistent measure for the considered time frame.

Composition of gsp – manufacturing and services. The bea publishes the composition of gross state product for the whole considered time frame. In 1997, the measure of gsp was changed and consequently also the way components within gsp were reported. While there are no changes to the measures of the manufacturing sector, the composition of the reported service sector changes. Up to 1997, the bea reports a component of gsp called "service". After 1997, however, this measure changes to "private service-providing industries". In order to have a consistent measure, I add "retail trade", "wholesale trade, and transportation" and "public utilities" to "services" between 1963 and 1996. From 1997 to 2013, "finance, insurance, or real estate services" are subtracted from the "private service-providing industries" measure. This ensures that a consistent measure over the entire time frame is used. Table E.1 summarizes the construction process.

Correlation of gsp with US gdp. The data for gsp and gdp are published by the bea for the entire considered time period.

Autocorrelation of gsp growth. Again, gsp data is drawn from the bea for all years.

Unemployment rate volatility. The state specific unemployment rates are published by the US Department of Labor from 1976 onwards on a monthly basis. I calculate and use the average unemployment rate for every year. The overall US unemployment rate is taken from the Cur-

rent Population Survey, also using the average for every year.

Public revenue and spending restrictions. State tax and expenditure limits are published by the National Conference of State Legislatures (2010). Following their definition, states can operate under traditional limits or other limitations. Traditional limits include revenue limits (tieing state revenue to some index, for instance, inflation), expenditure limits (tieing state expenditures to similar types of indices), appropriations limited to a percentage of revenue estimates (tieing appropriations to a revenue forecast), or Hybrids (combining different aspects of the limits mentioned before). Other tax and expenditure limitations include voter approval requirements (implying that tax increases require voter approval) or supermajority requirements (implying a certain threshold of votes in the responsible government branches). Table E.2 shows in which year a state adopted a certain limit, if it was introduced.

State	Appropriations	Revenue	Expenditure
Alabama	-	-	-
Alaska	-	-	1982
Arizona	-	-	1978
Arkansas	-	-	-
California	-	-	1979
Colorado	-	-	1991
Connecticut	-	-	1991
Delaware	1978	-	-
Dist. of Col.	-	-	-
Florida	-	1994	-
Georgia	-	-	-
Hawaii	-	-	1978
Idaho	-	-	1980
Illinois	-	-	-
Indiana	-	-	2002
Iowa	1992	-	-

Table E.2: Public Revenue and Spending Restrictions.

		1 1	
Kansas	-	-	-
Kentucky	-	-	-
Louisiana	-	-	1993
Maine	-	-	2005
Maryland	-	-	-
Massachusetts	-	1986	-
Michigan	-	1978	-
Minnesota	-	-	-
Mississippi	1982	-	-
Missouri	-	1980	-
Montana	-	-	1981^{*}
Nebraska	-	-	-
Nevada	-	-	1979
New Hampshire	-	-	-
New Jersey	-	-	1990
New Mexico	-	-	-
New York	-	-	-
North Carolina	-	-	1991
North Dakota	-	-	-
Ohio	-	-	2006
Oklahoma	-	-	1985
Oregon	-	2000	-
Pennsylvania	-	-	-
Rhode Island	1992	-	-
South Carolina	-	-	1980
South Dakota	-	-	-
Tennessee	-	-	1978
Texas	-	-	1978
Utah	-	-	1989
Vermont	-	-	-
Virginia	-	-	-

Washington	-	-	1993
West Virginia	-	-	-
Wisconsin	-	-	2001
Wyoming	-	-	-

Notes. Appropriations, revenue, and expenditure denote the year in which that type of tax or expenditure limit has been introduced in a state, respectively. Cell with "-" indicate that a state has not introduced such a limit in the given time frame.* Montana introduced an expenditure limit only between 1981 and 2004.

Poverty rate level. The state specific and overall US poverty rate levels from 1995 onwards are taken from the Federal Reserve Bank of St. Louis, drawing from data of the US Census Bureau.

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Eidesstattliche Versicherung

Ich, Daniel Stempel, versichere an Eides statt, dass die vorliegende Dissertation von mir selbstständig und ohne unzulässige fremde Hilfe unter Beachtung der "Grundsätze zur Sicherung guter wissenschaftlicher Praxis an der Heinrich-Heine-Universität Düsseldorf" erstellt worden ist.

Düsseldorf, der 07. Dezember 2021

Daniel Stempel