Three Essays on Behavioral Targeting with Location Data and Investment Incentives

DISSERTATION

zur Erlangung des akademischen Grades

Doctor Rerum Politicarum (Dr. rer. pol.)

im Fach Volkswirtschaftslehre

eingereicht an der Wirtschaftswissenschaftlichen Fakultät
Heinrich-Heine-Universität Düsseldorf

HEINRICH HEINE
UNIVERSITÄT DÜSSELDORF

von: Tim Reiz, M.Sc.
geboren am 29.04.1990 in Köln

Erstgutachter: Prof. Dr. Alexander Rasch
Zweitgutachterin: PD Dr. Irina Baye

Abgabedatum: 24.06.2021

Dedicated to Frederik and Alexander.

Acknowledgment

I would like to thank and express my gratitude towards Irina Baye, who has supported me from my master studies on. She encouraged me to pursue doctoral studies and was always available for questions and very helpful discussions.

I would also like to thank the entire faculty of DICE and especially Alexander Rasch. I have benefited a lot from the helpful comments and cordial atmosphere as well as from the excellent infrastructure at DICE.

Finally, I would like to thank my family. They didn't necessarily make it easier, but more importantly, they made every day valuable to me.

Contents

List of Figures

List of Tables

Introduction

This thesis analyzes in three chapters profit and welfare effects of competing firms' ability to (imperfectly) recognize customers based on their past purchases and use this information (combined with consumer location data) for targeted pricing.

Modern information technologies rapidly improve the opportunities of firms and interested parties to raise customer data of various types and precision to further use it in targeted marketing. However, interestingly, while more (better) customer data clearly allow firms to extract more rents from consumers in the monopoly situation, its profit effect is not straightforward in a competitive environment. This is due to the fact that in this case (better) customer data do not only introduce a better rent-extraction opportunity for firms as in the monopoly case. Instead, it also provides the opportunity to identify and target loyal consumers of the rival more aggressively, which gives rise to the competition effect, mitigating the ability of firms to extract additional rents from consumers. Thisse and Vives (1988) were the first to demonstrate this negative effect of targeted pricing in a static Hotelling model. Fudenberg and Tirole (2000) get a similar result in a dynamic model where firms target consumers based on their behavior, hence, introducing the behavioral targeting literature, which we will also consider in this thesis, while also providing another foundation for the famous prisoner's dilemma result of competitive targeting.

Nevertheless, despite the negative profit effects of targeted pricing identified in the literature, many real-world examples from different industries actually indicate that it can be profitable to conduct personal pricing or actively implement the opportunity to track consumers' purchase history also in a competitive environment. Prominent examples are the highly successful mobile marketing campaign of Dunkin' Donuts, the behavioral data based ride-hailing pricing of Uber, or the joint loyalty programs like "Payback" or "DeutschlandCard" of the most full-range grocery stores and their subsidiaries (such as Rewe, Edeka, Real, and Penny) in the German retailing sector. Accordingly, there is evidence that in some cases practicing or implementing the opportunity to practice personal pricing based on behavioral or past purchase data is yet profitable for competing firms. This dissertation conducts profit and welfare analyzes of behavioral targeting in a variety of settings in three chapters to better identify these cases and achieve an overall better knowledge of behavioral targeting effects.

Chapter 1 analyzes the profitability and welfare effects of behavioral targeting, using the example of "mobile geo-targeting". When practicing mobile geo-targeting, firms rely on the widespread use of smartphones, which deliver not only (almost) perfect data on consumers' locations via GPS-signals, but also provide a convenient personal advertising surface to implement personal pricing via user specific discount offers. Additionally, mobile phone users clearly differ in more characteristics than location, firms might want to consider in order to target the individual even better. The responsiveness to discounts also depend on information such as age and income, which can be summarized as consumers' flexibility and (with a certain precision) inferred from their past purchase decisions. The aim of this Chapter is to investigate, how this ability to collect additional behavioral data and combine it with perfect location-based marketing influences profits and welfare in a competitive (mobile marketing) environment. Accordingly, we consider a model with consumers differing along two dimensions: their locations and flexibility. There are two firms, each selling a different brand of an otherwise homogeneous product, competing over two periods. We take as a starting point that consumer geo-locations are known to the firms. Additionally, firms can infer consumer flexibility from observing consumers' past purchases and target them respectively in the subsequent period. Finally, to concentrate on the strategic effects of behavioral customer data, we assume that consumers are myopic while firms are forward-looking. Our analysis shows that the overall profit effect of behavioral targeting can be neutral, positive or negative. Which profit effect emerges mainly depend on consumers' heterogeneity in flexibility and firms are more likely to benefit from behavioral targeting when consumers are more similar. This finding supports the observation that firms usually resort to mobile geo-targeting in markets with arguably relatively homogeneous consumers, which is the case, e.g., in the fast-food industry where Dunkin' Donuts performed its successful marketing campaign. On top of consumer heterogeneity, we find that the profit effect can also depend on firms' time preferences and firms strategically influence the quality of the inferred flexibility data by choosing the respective first-period prices, depending on the impact of data on overall profits. Interestingly, these findings are in contrast to most other results in the literature of behavioral targeting, where firms' time preferences do not influence the results and firms fail to influence the outcome strategically. We also show that firms' using behavioral data for pricing may in some cases harm customers, but their and consumers' interests are not necessarily opposed. Finally, it is worth to note that our model generates many results of related articles by determining their specific level of consumer heterogeneity in the sense of our model. Hence, our model provides a unifying framework to analyze behavioral targeting effects.

Chapter 2 intuitively extends the model via introducing an imperfect customer recognition. Hence, firms are able to identify, once again, two customer groups after the first period, however, now they can only differentiate between the own (partly) identified former customers and the foreign consumers, which include the unidentified own customers and the rival's customers of the first period. Additionally, we also introduce sophisticated consumers to the model, which in contrast to myopic consumers consider that collected customer data will be used by firms for targeting in the future. Overall, this Chapter fills the gap between the two extreme cases of perfect customer recognition (meaning usual behavior-based price discrimination) and no customer recognition (meaning uniform pricing) within the previous results and provides an analysis of the impact of an increasing recognition accuracy, while also enlarging the analysis to sophisticated consumers. We confirm our previous findings like the importance of consumer heterogeneity when considering behavioral targeting effects and the further results strengthen our knowledge on this topic. Once again, we find that the generated results nest findings of previous articles. E.g., in the special case with sophisticated and very heterogeneous consumers, which correspond to the case analyzed in Colombo (2016), we confirm that profits are u-shaped in the level of information accuracy and behavioral targeting is detrimental for firms.

In Chapter 3, we take a step back and endogenize the ability of firms to recognize customers' past purchase decisions in a standard one-dimensional but dynamic two-period Hotelling-framework à la Fudenberg and Tirole (2000). Hence, instead of analyzing the profit and welfare effects of competing firms holding a technology which enables them to practice third-degree price discrimination, we consider the investment incentives of firms which can endogeneously decide to acquire this technology before competing with the rival. In this set-up, we distinguish, once again, between the cases with all consumers being either myopic or sophisticated and show that the famous prisoner's dilemma result known since Thisse and Vives (1988) and established in the context of behavior-based price discrimination by Fudenberg and Tirole (2000) can be resolved in this model. Precisely, with myopic consumers only one of the firms invests into the screening technology, which results in higher profits for both firms compared to the outcome where both of them are able to discriminate based on consumer behavior. When consumers are sophisticated, two symmetric equilibria exist, where either both firms invest into the screening technology or do not. Respectively, in the latter equilibrium firms avoid the prisoner's dilemma problem. These findings complement our results of the previous chapters.

Chapter 1

Customer Recognition and Mobile Geo-Targeting

Co-authored with Irina Baye and Geza Sapi

1.1 Introduction

The widespread use of smartphones revolutionized marketing by providing an advertising means that allows delivering personalized commercial messages depending on a wide range of customer characteristics. One of the most profitable and novel marketing opportunities opened up by mobile devices is geo-targeting. Various apps installed on a device read the GPS signals of users and share these with affiliated advertisers and retailers who can in turn send messages with commercial offers to users. Geo-targeted mobile advertising is booming. BIA Kelsey (2015) projects location-based mobile ad revenues in the U.S. to nearly triple within four years from $6.8 billion in 2015 to $18.2 billion in 2019. Mobile phone users differ also in other characteristics than location. Clearly, age, demographics, income, profession and many other factors influence how users respond to commercial offers and discounts. Mobile marketers routinely complement geo-location data with behavioral information on customers, which can signal their responsiveness to discounts (Thumbvista, 2015).

A prominent example recognizing both geo-location and behavioral data is the highly successful mobile marketing campaign of Dunkin' Donuts. In the first quarter of 2014, Dunkin' Donuts rolled out a campaign with discounts sent to phone users "*around competitors' locations coupled with behavioral targeting to deliver coupons on mobile devices*" (Tode, 2014).[1] The campaign proved highly lucrative, with a significant share of discount recipients showing interest and redeeming the coupon.

In our paper we focus on four important features of mobile targeting, which distinguish it from traditional targeting. First, consumers' real-time locations are known to sellers. Second, location is not the only factor determining how responsive consumers are to discounts. Other factors such as age, income and occupation play a role, which are imperfectly observable to marketers. Third, sellers may infer responsiveness from observing previous purchase behavior. Finally, firms can deliver personalized offers through mobile devices to individually addressable consumers. These features of mobile technology allow firms to engage in a new form of price discrimination by charging consumers different prices depending on both their

[1] Dunkin' Donuts complemented geo-location data with external data on behavioral profiles, obtained from billions of impressions gathered through mobile devices to identify anonymous Android and Apple device IDs. The campaign delivered banner ads to targeted devices that ran in the recipient's favorite apps or on mobile web sites. These ads featured offers such as a $1 discount on a cup of coffee and $2 discount on a coffee plus sandwich meal.

real-time locations and previous purchase behavior. Our aim is to investigate how behavioral targeting combined with perfect location-based marketing affects profits and welfare in a modeling setup that matches the main features of today's mobile marketing environment. We consider a model, where consumers differ along two dimensions: their *locations* and *flexibility*. We interpret location in a physical sense, as the focus of our paper is on mobile geo-targeting.[2] Flexibility is understood as the responsiveness of consumers to discounts. There are two firms, each selling a different brand of the same product and competing over two periods. We take as a starting point that consumer geo-locations are known to the firms. Additionally they can obtain behavioral information on the flexibility of customers by observing their purchases in the first period. To concentrate on the strategic effects of customer data, we assume that consumers are myopic while firms are forward-looking.

Our main results are as follows. First, we show that combining behavioral data with geo-targeting can influence second-period profits in three different ways, depending on how strongly consumers differ in their preferences. With weakly differentiated consumers, firms always gain from additional customer data and profits of the second period are the highest when this data is most precise. With moderately differentiated consumers, profits respond differently to behavioral data (depending on its quality) and firms are better off only if data is sufficiently accurate. Finally, when consumers are strongly differentiated among each other, firms are always worse off in the second period with behavioral data of any quality. The intuition reaches back to the standard insight in the price discrimination literature distinguishing between *rent-extraction* and *competition* effects of additional customer data. More data potentially allows firms to extract more rents from consumers, but targeting may also strengthen the intensity of price competition. When consumers are weakly differentiated, each firm wants to serve most consumers close-by. This, however, induces the rival to price aggressively even absent behavioral data. Firms therefore experience only the rent-extraction effect of additional data and their profits rise. When consumers are strongly differentiated, without behavioral data firms avoid tough competition by targeting consumers of different flexibility at each location. Precisely, each firm serves only the less flexible consumers among those located closer to it. Additional customer data intensifies competition because firms compete for each group previously served by only one firm, which makes both of them worse off.

[2]Our results would apply equally if location were interpreted in *preference space*, as is often done with spatial models of product differentiation.

Finally, when consumers are moderately differentiated, the profit effect of behavioral data in the second period depends on the interplay between the rent-extraction and competition effects. In this case firms are better off only if data is sufficiently precise.

Second, we find that firms strategically influence the quality of the (revealed) behavioral data by choosing the respective first-period prices. This is due to the fact that data quality is interlinked with the distribution of firm market shares in the first period and data is most precise when on a given location firms can distinguish between two consumer groups of equal size. In addition, the value of behavioral data to the firms depends on the strength of consumer heterogeneity. When consumers are relatively homogeneous, the value of additional flexibility data is low. In this case every firm serves all consumers close-by in the initial period, such that no data about their flexibility is revealed in equilibrium. With more differentiated consumers, behavioral data boosts profits in the second period and its quality in equilibrium is higher when firms value future profits more. In contrast, with strongly differentiated consumers flexibility data intensifies competition in the second period. Consequently, firms end up with less precise information when they discount future profits less. Overall, firms influence the quality of the revealed behavioral data in a way that allows them to realize the highest profits over two periods. Precisely, when they expect the rent-extraction effect to dominate, they make sure to gather more precise information about customers. When data intensifies competition, firms strategically distort its quality downwards.

Third, we compare the overall profits with the situation where firms cannot collect behavioral data for some exogenous reasons. We first isolate cases where the profit effect of data can be assessed in a quite simple manner, as it is driven purely by the level of consumer differentiation. In particular, if consumers are relatively homogeneous, the value of behavioral data is low and firms do not collect it in equilibrium, such that the ability to observe consumers' past purchases does not influence profits. In contrast to conventional wisdom, firms are worse off when they can collect behavioral data on consumers if these are very different. In turn, behavioral data boosts the overall profits if consumers are moderately differentiated, because in this case additional data does not intensify competition dramatically. Finally, if consumer differentiation is in between these *pure* cases the effect of behavioral data on profits depends on the discount factor and is likely to be positive when firms value future profits more. In this case firms have stronger incentives to adjust their price choices in the initial period, which increases the overall profits. We also show that consumers' and firms' interests are not necessarily opposed. If consumers are moderately differentiated, both consumer surplus and profits can increase when firms are able to observe customers' past purchases leading to a higher social welfare.

1.2 Related Literature

Our paper contributes to two main strands of literature. The first strand analyzes the competition and welfare effects of firms' ability to recognize past customers and price discriminate between them and the rival's customers in the subsequent periods.[3] The second relatively new and actively developing strand analyzes competition between mobile marketers who can observe geo-locations of consumers and target them with personalized offers.

In the literature on behavior-based targeting firms adjust their prices in the first period taking into account the impact of behavioral data on their future profits. Corts (1998) proposes an elegant way how to predict the price and profit effects of firms' ability to discriminate among two customer groups. He distinguishes between two types of markets. If for a given uniform price of the rival both firms optimally charge a higher price to the same consumer group, then according to Corts such market is characterized by *best-response symmetry*. In all other cases *best-response asymmetry* applies. Corts shows that with best-response asymmetry targeted prices to both consumer groups change in the same direction relative to the uniform price. This change may be either positive or negative leading to higher or lower discrimination profits, respectively. Thisse and Vives (1988) were the first to demonstrate the negative effect of price discrimination on prices and profits leading to a prisoners' dilemma situation.[4] More recent literature showed that firms may be better off with price discrimination under best-response asymmetry.[5]

The seminal article by Fudenberg and Tirole (2000) considers a dynamic Hotelling-type duopoly model with horizontally differentiated consumer preferences and a

[3]Fudenberg and Villas-Boas (2006) provide a review of this literature.

[4]For a similar result see Shaffer and Zhang (1995), Bester and Petrakis (1996) and Liu and Serfes (2004).

[5]Precisely, the positive profit effect is demonstrated in articles starting with an asymmetric (more advantageous to one of the firms) situation (see Shaffer and Zhang, 2000 and 2002; Carroni, 2016) and in articles which assume imperfect customer data (as in Chen *et al.*, 2001; Liu and Shuai, 2016; Baye and Sapi, 2019).

market showing best-response asymmetry. In the initial period firms quote uniform prices, while in the subsequent period they can offer different prices to the former customers of each other. Under uniformly distributed consumer preferences second-period profits always remain below the level without price discrimination.[6] This is so because customer data intensifies competition: A consumer's purchase at a respective firm in the first period reveals her (relative) preference for that firm's product making the rival compete aggressively for such consumers, which creates a downward pressure for this firm's profits. Interestingly, Fudenberg and Tirole find that second-period considerations do not influence the first-period equilibrium, as a result firms end up with the lowest second-period profits possible and are worse off being able to recognize own customers.[7,8]

Esteves (2010) adopts a discrete distribution of customer preferences. In her model there are two consumer groups who prefer the product of a given firm if its price does not exceed that of the rival by more than a given amount (referred to as "the degree of consumer loyalty"). Esteves investigates both the static and dynamic (two-period) games for an equilibrium in mixed strategies and considers myopic consumers. Similar to Fudenberg and Tirole (2000), in Esteves firms are also better off in the second period when they cannot engage in behavioral price discrimination. However, second-period profit considerations do influence the prices of the first period. Firms price to soften future competition and consequently the probability of an outcome where they learn consumer preferences decreases when firms become more patient.

Chen and Zhang (2009) consider a market with three consumer segments, two of which are price-insensitive consumers who always purchase from the preferred firm. The third segment consists of switchers who buy from the firm with a lower price. The authors solve the model for an equilibrium in mixed strategies and assume that both firms and consumers are forward-looking. Surprisingly, profits when firms are unable to collect behavioral data are equal to those with automatic customer

[6]Fudenberg and Villas-Boas (2006) provide a detailed analysis of the uniform case of Fudenberg and Tirole (2000).

[7]This result is driven by the fact that the profits of the second period get their minimum at equal market shares, such that the optimal prices following from the first-order conditions in a one-period and dynamic models coincide.

[8]In related articles Villas-Boas (1999) and Colombo (2016) also show that firms are worse off with the ability to recognize consumers. Villas-Boas derives this result in a model with infinitely lived firms and overlapping generations of consumers, while Colombo assumes that firms can recognize only a share of their previous customers.

recognition. However, firms are better off if they must actively gather customer data. The reason is that a firm which is able to recognize its loyal consumers benefits from the rent-extraction in the second period. As a result, each firm charges a relatively high price in the first period to separate its loyal consumers from switchers, thereby softening competition.[9]

The model proposed in this paper nests the setups of Fudenberg and Tirole (2000), Esteves (2010) and Chen and Zhang (2009) as special cases. In particular, we obtain the results of Fudenberg and Tirole and Esteves when consumers are strongly differentiated. Firms are then worse off with additional customer data irrespective of the discount factor. In Esteves, with customer data firms can discriminate between two groups of consumers loyal to one of the firms. In contrast, in our analysis customer data allows to distinguish at any geo-location among two groups with different flexibility *within* the loyal consumers of a firm.[10] It follows that compared to our model, the level of consumer differentiation considered by Esteves is higher.[11] Also similar to Esteves, we show that with strongly differentiated consumers firms choose first-period prices so as to minimize the quality of the revealed customer data.

We also get the result of Chen and Zhang (2009) where firms are better off with behavioral targeting irrespective of the discount factor when we consider weakly differentiated consumers. The reason is that in Chen and Zhang firms compete for price-sensitive switchers who always buy at a firm with a lower price and are, hence, fully homogeneous in their preferences. We conclude that the level of consumer differentiation in preferences can serve as a reliable tool for predicting profit effects of targeted pricing based on customer behavioral data.[12] A further novelty of our paper compared to the previous studies is to demonstrate that this effect may also depend on the discount factor and behavioral targeting is more likely to boost profits

[9] In Chen and Zhang (2009) customer data is fully revealed in equilibrium. Precisely, the firm with a higher price in the first period identifies all of its loyal consumers, because none of them foregoes a purchase to pretend to be a switcher.

[10] In our analysis, consumers are loyal to a firm if they buy from it when both firms charge equal prices.

[11] Slightly different from Esteves (2010), in Fudenberg and Tirole (2000) with behavioral data firms can identify two consumer groups, which are only *on average* more loyal to one of them. However, this also implies a higher level of consumer heterogeneity than in our analysis.

[12] Our result that behavior-based targeting is more likely to be profitable if consumers are less differentiated, depends on the symmetry of customer data available to the firms. Shin and Sudhir (2010) show that it can be reversed when customer data is asymmetric. Precisely, in their analysis every firm can distinguish between the own low- and high-demand customers of the previous period, while the rival only knows that these consumers bought from the other firm.

if firms value future profits more. All previous articles on behavior-based price discrimination known to us found that it increases or reduces profits, irrespective of the discount factor.

We also contribute to the rapidly growing literature on oligopolistic mobile geo-targeting. Chen *et al.* (2017) consider a duopoly model with consumers located at one of the two firms' addresses as well as some consumers situated in the middle between the firms. The authors assume that consumers are differentiated along two dimensions: locations and brand preferences. Furthermore, some consumers are aware of available offers at different locations and choose among these, but incur travel costs. Chen *et al.* show that mobile targeting can increase profits compared to the uniform pricing, even in the case where traditional targeting (where consumers do not seek for the best mobile offer) does not. Unlike in Chen *et al.*, in our model firms start out with mobile geo-targeting ("traditional targeting," according to Chen *et al.*) and we analyze how the ability to collect additional behavioral data influences profits. Different from Chen *et al.*, we also vary the level of consumer heterogeneity in preferences and show that it is crucial for predicting the profit and welfare effects of combining behavior-based pricing with mobile geo-targeting.[13]

Dubé *et al.* (2017) conduct a field experiment to analyze the profitability of different mobile marketing strategies in a competitive environment. In their analysis two firms can target consumers based on both their locations and previous behavior. They show that in equilibrium firms choose to discriminate only based on consumer behavior, in which case profits increase above those with uniform prices. However, profits would be even higher if firms applied finer targeting, which relies on both behavioral and location data. Our paper differs from Dubé *et al.* in two important ways. First, we show that customer targeting based on both location and behavioral data does not necessarily increase profits and can harm firms if consumers are sufficiently differentiated.[14] Second, in our model behavioral data is generated endogenously in a dynamic setting, where firms strategically influence its quality. For instance, when consumers are rather homogeneous, in equilibrium firms do not collect any behavioral data unless they are quite patient, although they would benefit

[13]To keep our analysis tractable we do not allow consumers to strategically change their locations (to get the best mobile offer) and assume that they are targeted at their home locations. We also find this level of consumer sophistication realistic in markets where our analysis applies most.

[14]Interestingly, when commenting why geo-targeting is more profitable than behavioral targeting, Dubé *et al.* (2017) explain it through the difference in the levels of consumer heterogeneity over locations and purchase behavior (recency).

from such data in the second period.[15]

Finally, Baye and Sapi (2019) also consider today's mobile marketing data landscape, where firms can use near-perfect customer location data for targeted pricing. They analyze how firms' incentives to acquire (costlessly) data on other consumer attributes being (imperfect) signals of their flexibility depend on its quality. In our model firms collect additional data through observing consumers' purchase histories, which is not costless, because firms have to sacrifice some of their first-period profits to gain customer data of a better (worse) quality. As a result, compared to our paper, Baye and Sapi overestimate both the benefit of additional customer data to firms and its damage to consumers in mobile marketing.

1.3 The Model

There are two firms, A and B, that produce two brands of the same product at zero marginal costs and compete in prices. They are situated at the ends of a unit Hotelling line: Firm A is located at $x_A = 0$ and firm B at $x_B = 1$. There is a unit mass of consumers each with an address $x \in [0, 1]$ on the line, which describes her real physical location, as transmitted by GPS signals to retailers in mobile marketing. If a consumer does not buy at her location, she incurs linear transport costs proportional to the distance to the firm. We follow Jentzsch *et al.* (2013) and Baye and Sapi (2019) to assume that consumers differ not only in their locations, but also in transport costs per unit distance (flexibility), $t \in [\underline{t}, \overline{t}]$, where $\overline{t} > \underline{t} \geq 0$.[16,17] Transport costs are higher if t is larger. Each consumer is uniquely characterized by a pair (x, t). We assume that x and t are uniformly and independently distributed

[15] Dubé *et al.* (2017) also recognize that in a dynamic setting customer segmentation (derived from the accumulated customer data) is determined endogenously. They argue that "*an interesting direction for future research would be to explore how dynamics affect equilibrium targeting and whether firms would continue to profit from behavioral targeting.*"

[16] Different from Jentzsch *et al.* (2013) and Baye and Sapi (2019), data on consumer flexibility is generated endogenously in our model and, as we show below, firms influence strategically the quality of the revealed customer data.

[17] Esteves (2009), Liu and Shuai (2013 and 2016), Won (2017) and Chen *et al.* (2017) also consider a market, where consumer preferences are differentiated along two dimensions. However, in their analysis the strength thereof (flexibility) is the same among all consumers. In Borenstein (1985) and Armstrong (2006) consumers also differ in their transport cost parameters. Both show that firms may benefit from discrimination along this dimension of consumer preferences. In our analysis where firms are endowed with the ability to target consumers based on their locations, the profit effect of behavioral data on consumer flexibility depends on the level of the overall consumer heterogeneity and firm discount factor.

giving rise to the following density functions: $f_t = 1/(\bar{t} - \underline{t})$, $f_x = 1$ and $f_{t,x} = 1/(\bar{t} - \underline{t})$. The utility of a consumer (x, t) from buying at firm $i = \{A, B\}$ is

$$U_i(p_i(x), t, x) = v - t\,|x - x_i| - p_i(x). \tag{1.1}$$

In equation (1.1) $v > 0$ denotes the basic utility, which is assumed high enough such that the market is always covered in equilibrium. A consumer buys from the firm whose product yields higher utility.[18] Without loss of generality, we normalize $\bar{t} = 1$ and measure the level of consumer heterogeneity by the ratio of the largest to the lowest transport cost parameter: $l := \bar{t}/\underline{t} = 1/\underline{t}$, with $l \in (1, \infty)$. As we show below, parameter l plays a crucial role in our analysis. To consumers with $x < 1/2$ ($x > 1/2$) located closer to firm A (firm B) we refer as the *turf* of firm A (B). We also distinguish among consumers at the same location. Precisely, we refer to consumers with lower (higher) transport cost parameters as more (less) flexible ones.

We assume that firms observe with perfect precision the physical locations of all consumers in the market. There are two periods in the game. In the first period location is the only dimension by which firms can distinguish consumers. They issue targeted offers at the same time to all consumers, depending on their locations. Consumers at the same location will receive the same targeted offer. In the second period firms again send simultaneously targeted offers to consumers. However, this time they are able to distinguish among consumers that visited them in the first period and those that did not.[19] As a result, in the second period firms can charge (up to) two different prices at each location: one to the own past customers and the other one to those of the rival. Table 1.1 summarizes the three types of information firms can obtain in our model. We analyze how this information translates into pricing decisions in a dynamic competitive environment. We assume that firms are forward-looking while consumers are myopic, which allows us to concentrate

[18] We follow the tie-breaking rule of Thisse and Vives (1988) and assume that if a consumer is indifferent, she buys from the closer firm. If $x = 1/2$, then in the case of indifference a consumer buys from firm A.

[19] Danaher *et al.* (2015) show in a field experiment, where all consumers got coupons with the *same* discount, that both the consumer's distance to the store and her previous behavior (redemption history) determine the probability that a coupon will be redeemed by a customer. It is then consistent with these results that in our model where firms can target consumers with *personalized* coupons, they use the information on both customer locations and their purchase history to design coupons. Similarly, Luo *et al.* (2017) show in a recent field experiment that depending on consumer locations different temporal targeting strategies are needed to maximize consumer responses to mobile promotions. This also speaks for a necessity to target consumers individually depending on location.

on the strategic effects of customer data.[20] We solve for a subgame-perfect Nash equilibrium and concentrate on equilibria in pure strategies.

Type of customer data	Time obtained	Quality
Geo-location	Real-time in each period	Perfect
Own/rival's past customer	Inferred from 1-st period purchasing decisions	Perfect
Flexibility	Inferred from 1-st period purchasing decisions	Imperfect

Table 1.1: Customer data available to the firms.

1.4 Equilibrium Analysis

We start from the second period, where firms can discriminate depending on both consumer locations and their behavior.

Equilibrium analysis of the second period. As firms are symmetric, it is sufficient to analyze a single location for instance on firm A's turf. Consider an arbitrary $x < 1/2$. Let t^{α} denote the transport cost parameter such that consumers with $t \geq t^{\alpha}$ visited firm A in the previous period, while consumers with $t < t^{\alpha}$ purchased at firm B.[21] If the share of consumers at x who bought from firm B in the first period is $\alpha \in [0, 1]$, then $t^{\alpha} := \alpha + \underline{t}\,(1 - \alpha)$. We will refer to consumers with $t < t^{\alpha}$ as segment α and to those with $t \geq t^{\alpha}$ as segment $1 - \alpha$. Segment α includes the relatively flexible consumers who purchased in the first period from the firm located further away. We denote the prices of the second period to consumers on segments α and $1 - \alpha$ as p_i^{α} and $p_i^{1-\alpha}$, with $i = A, B$, respectively. Firms choose the prices so as to maximize their profits on each segment separately. Figure 1.1 depicts both segments at location x.

Consider segment α. On its own turf firm A can attract consumers with sufficiently high transport cost parameters, such that

$$U_A(p_A^{\alpha}(x), t, x) \geq U_B(p_B^{\alpha}(x), t, x) \text{ implies } t \geq t_c^{\alpha}\left(p_A^{\alpha}(x), p_B^{\alpha}(x)\right) := \tfrac{p_A^{\alpha}(x) - p_B^{\alpha}(x)}{1-2x}.$$

[20]See Esteves (2010) for a similar assumption.

[21]A standard revealed-preference argument implies that if a consumer on $x < 1/2$ with $t = \widetilde{t}$ bought from firm A (B) in the first period, then all consumers with $t > \widetilde{t}$ ($t < \widetilde{t}$) made the same choice.

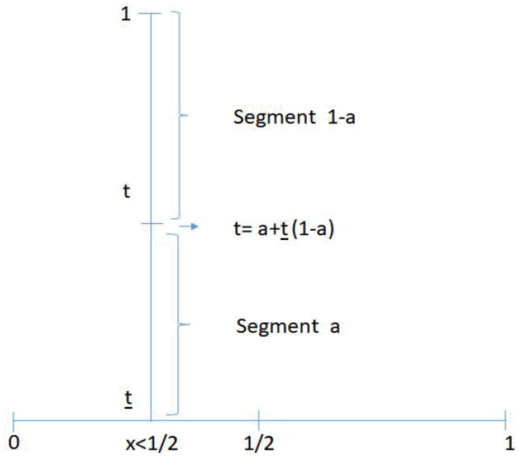


Figure 1.1: Segments α and $1-\alpha$ at some location $x < 1/2$.

Firms choose prices $p_A^\alpha(x)$ and $p_B^\alpha(x)$ to maximize their expected profits:

$$\max_{p_A^\alpha(x)} \frac{[t^\alpha - t_c^\alpha(\cdot)]p_A^\alpha(x)}{1-\underline{t}} \text{ and } \max_{p_B^\alpha(x)} \frac{[t_c^\alpha(\cdot) - \underline{t}]p_B^\alpha(x)}{1-\underline{t}}.$$

The mechanism is analogous on segment $1-\alpha$. The following Lemma describes the equilibria on each segment at a given location $x < 1/2$ depending on market shares of the first period.[22]

Lemma 1.1. *Consider an arbitrary x on the turf of firm A. The equilibrium on each segment at this location depends on the asymmetry between first-period market shares.*
i) If in the first period firm B's market share at this location was low, $\alpha \leq 1/(l-1)$, then it attracts no consumer on segment α in the second period, where firm A charges the price $p_A^\alpha(x) = \underline{t}(1-2x)$ and the price of firm B is zero. Otherwise, firm B serves the more flexible consumers on segment α, with $t < \underline{t}[\alpha(l-1)+2]/3$ at the price $p_B^\alpha(x) = \underline{t}(1-2x)[\alpha(l-1)-1]/3$, while the price of firm A is $p_A^\alpha(x) = \underline{t}(1-2x)[2\alpha(l-1)+1]/3$.
ii) If in the first period firm B's market share at this location was high, $\alpha \geq (l-2)/[2(l-1)]$, then it attracts no consumer on segment $1-\alpha$, where firm A charges the price $p_A^{1-\alpha}(x) = t^\alpha(1-2x)$ and the price of firm B is zero. Otherwise, firm B serves the more flexible consumers on segment $1-\alpha$, with $t < \underline{t}[l+1+\alpha(l-1)]/3$, and firms charge $p_A^{1-\alpha}(x) = \underline{t}(1-2x)[2l-1-\alpha(l-1)]/3$ and $p_B^{1-\alpha}(x) = \underline{t}(1-2x)[l-2-2\alpha(l-1)]/3$.

[22] All the omitted proofs are contained in the Appendix.

Remember that α denotes the share of consumers at location x that bought from firm B in the previous period. If α is small, then consumers on segment α are relatively similar in flexibility, such that firm A can attract them all without having to significantly reduce the price targeted at the least flexible consumer (with $t = t^{\alpha}$). As a result, the monopoly equilibrium emerges on segment α where firm A serves all consumers and firm B cannot do better than charging zero. Analogously, if α is large then the complementary segment $1-\alpha$ is relatively small, such that in equilibrium firm A serves all consumers there. In contrast, with large α, consumers are quite different in their preferences on segment α. In this case firm A prefers to extract rents from the less flexible consumers there and lets the rival attract the more flexible ones. The following Lemma characterizes the equilibria at any location $x < 1/2$ depending on the heterogeneity parameter, l, and the first-period market share of firm B, α.

Lemma 1.2. *Consider an arbitrary x on the turf of firm A. The equilibrium at this location depends on the asymmetry between first-period market shares and consumer heterogeneity in flexibility.*
i) If $l \leq 2$, then firm A attracts all consumers at x irrespective of α.
ii) If $2 < l < 4$, then firm A attracts all consumers at x provided firm B's first-period market share at this location was intermediate, i.e., $1/(l-1) \leq \alpha \leq (l-2)/[2(l-1)]$. Otherwise, firm A loses consumers on one of the segments.
iii) If $l \geq 4$, then irrespective of α firm A loses some consumers at x. However, it can monopolize segment α $(1-\alpha)$ provided firm B served in the first period relatively few (many) consumers at that location, with $\alpha \leq 1/(l-1)$ $(\alpha \geq (l-2)/[2(l-1)])$. For intermediate α-values , $1/(l-1) < \alpha < (l-2)/[2(l-1)]$, both firms serve consumers on both segments.

If $l \leq 2$, then independently of firm B's first-period market share consumers on both segments are quite similar in their preferences, such that in equilibrium firm A serves all consumers at x for any α. When consumers become more differentiated, with $2 < l < 4$, the optimal strategy of firm A depends on the market share of firm B in the first period. Precisely, if α takes intermediate values, then consumers have similar flexibility on each segment yielding again monopoly equilibria on both segments. Finally, for $l \geq 4$ irrespective of α on each segment firm A faces very different consumers and always loses some of them in equilibrium. Figure 1.2 provides two examples of the second-period equilibrium at some $x < 1/2$ depending on l and

α and shows each firm's demand regions on both segments.

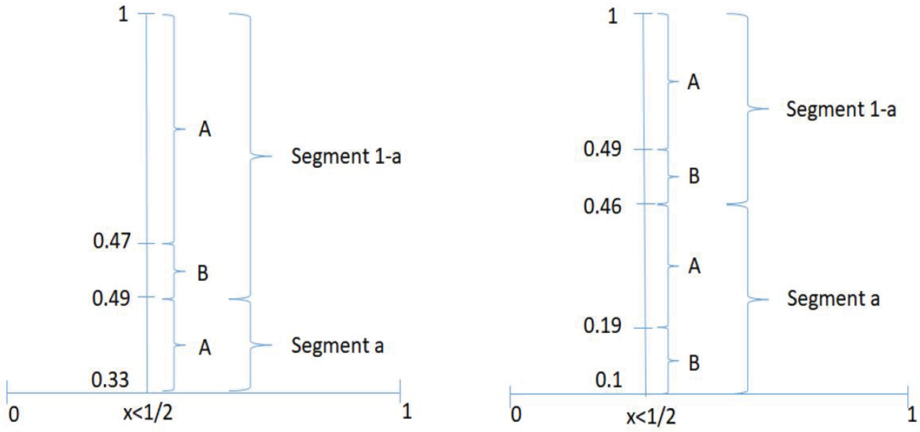


Figure 1.2: Demand regions at some $x < 1/2$ in the second period for $l = 3$ and $\alpha = 0.2$ (left) and $l = 10$ and $\alpha = 0.4$ (right).

We next analyze how total profits in the second period change with α. We assume that every firm served the share α of customers at any location on the rival's turf in the first period.[23] The following Proposition summarizes our results.

Proposition 1.1. *Assume that in the first period each firm served the share α of consumers at any location on the rival's turf. A firm's second-period profits as a function of α depend on how strongly consumers differ in their preferences.*
i) If $l \leq 2.38$, profits are an inverted U-shaped function of α. Moreover, for any $\alpha \in (0,1)$ profits are higher than at $\alpha = 0$ ($\alpha = 1$). The highest profit level is attained at $\alpha = 1/2$.
ii) If $2.38 < l < 8$, profits are given by different non-monotonic functions of l, sharing the following common features: First, there exists $\widehat{\alpha}(l)$, such that profits are lower than at $\alpha = 0$ ($\alpha = 1$) if $\alpha < \widehat{\alpha}(l)$ and are higher otherwise. Moreover, $\partial\widehat{\alpha}(l)/\partial l > 0$. Second, the highest profit level is attained at $\alpha = 1/2$ if $l < 2.8$ and at $\alpha = (9l - 16)\,/\,[8\,(l-1)]$ otherwise.
iii) If $l \geq 8$, profits are a U-shaped function of α. Moreover, for any $\alpha \in (0,1)$ profits are lower than at $\alpha = 0$ ($\alpha = 1$). The lowest profit level is attained at $\alpha = (2l - 3)\,/\,[5\,(l-1)]$.

[23] We make this assumption to derive each firm's total profits (at all locations together) in the second period in order to compare them with the similar profits from the other relevant studies mentioned above. Moreover, we demonstrate below that the first-period market share of each firm, α, in equilibrium is indeed the same at any location on the rival's turf.

The impact of combining behavioral data with geo-targeting on profits of the second period is driven by two effects: rent-extraction and competition. Precisely, with behavioral data every firm can recognize its past customers and distinguish between these and the more flexible ones who bough from the rival. It then charges higher prices to the former, which describes the rent-extraction effect. In contrast, the rival targets more aggressively exactly these consumers, to which we refer as the competition effect. The overall effect of additional data on profits depends on the interplay between these two opposing effects and is driven by the ratio l and the quality of the gained data. In the extreme cases of $\alpha = 0$ or $\alpha = 1$, flexibility data does not provide any additional information on customer preferences, because all consumers at a given location bought from the same firm in the first period. In contrast, the highest level of data accuracy is attained when the segments are of equal size, i.e., at $\alpha = 1/2$.[24] Figure 1.3 depict firm profits in the second period as a function of α for all the three cases described in Proposition 1.1 with $l = 2$, $l = 3$ and $l = 10$, respectively.

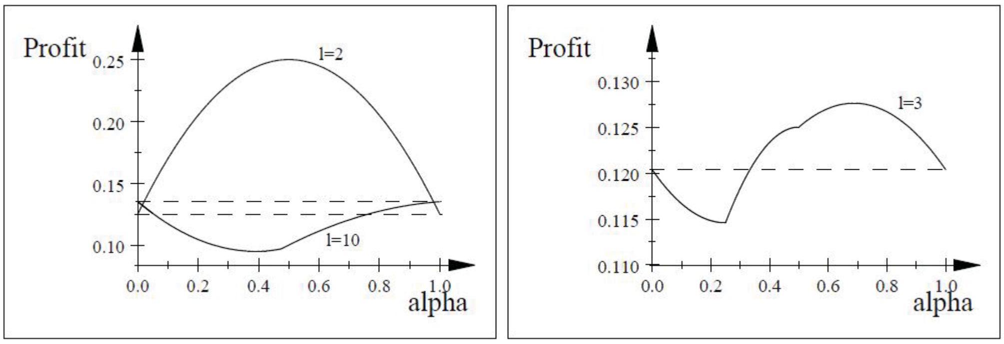


Figure 1.3: Profits of the second period depending on the market share of firm B in the first period, α, for $l = 2$ and $l = 10$ (left) and $l = 3$ (right).

According to Lemma 1.2, if $l \leq 2$, then in the second period each firm serves

[24]It is easy to see this also formally. We can say that data is most precise if it allows a firm to extract the highest rents from the consumers located closer to it on a given address for any price of the rival. The latter condition allows to abstract from the competition effect. Consider some x on the turf of firm A and let the rival's price be p_B. To serve all consumers on segments α and $1 - \alpha$ firm A has to charge prices $p_A^{\alpha}(x) = \underline{t}(1 - 2x) + p_B$ and $p_A^{1-\alpha}(x) = (\alpha + \underline{t}(1 - \alpha))(1 - 2x) + p_B$, respectively, which yield the total profit of firm A on location x: $\Pi_A(x) = p_B + (1 - 2x)\underline{t}\left[(1 - \alpha)^2 + \alpha l(1 - \alpha) + \alpha\right]$. This profit gets its maximum $\alpha = 1/2$ for any l and any p_B. Similarly, Fudenberg and Villas-Boas (2006) in their analysis of Fudenberg and Tirole (2000) also argue that data is most precise when firms can distinguish between two consumer groups of equal size.

all consumers at any location on its own turf independently of how consumers were distributed among the firms in the first period.[25] The rival cannot do better than charging zero on both segments for any α, such that the competition effect of flexibility data is absent. The remaining rent-extraction effect is in turn strongest when data precision is the highest, with $\alpha = 1/2$.

Unlike in the previous case where the changes in profits with α were driven purely by the rent-extraction effect, in the case of $2.38 < l < 8$ it is the interplay of the two effects, which determines how profits change. As α increases above $\alpha = 0$, firms get some additional data from consumers' purchase histories. This in turn boosts competition strongly: The rival decreases its prices on both segments, eroding profits. When data quality improves further, the rent-extraction effect starts to take over and profits increase. Overall, profits are the highest when data is more accurate, i.e., α takes intermediate values (close to $\alpha = 1/2$). With a further increase in α, behavioral data becomes less and less precise about the consumers who bought from the rival in the initial period decreasing its overall predictive power and profits altogether. However, profits then always remain above the level without flexibility data (at $\alpha = 1$ or $\alpha = 0$).

If $l \geq 8$, profits drop rapidly as α becomes strictly positive. As a result, although profits start recovering when α increases above a certain threshold, they never exceed the level without behavioral data (at $\alpha = 1$ or $\alpha = 0$). Interestingly, in this case profits are the lowest when data is most precise (α takes intermediate values), because competition is most intense then. The case $l \geq 8$ is similar to the result obtained by Fudenberg and Tirole (2000) for the uniformly distributed consumer preferences. This similarity is driven by the fact that their model corresponds to the case of very high l in our analysis. Indeed, behavioral customer data in Fudenberg and Tirole allows to distinguish among two consumer groups loyal (on average) to *different* firms, while in our setting at each location in the second period firms can discriminate among two consumer groups loyal to the *same* firm. We now turn to the analysis of the first period.

Equilibrium analysis of the first period. In this subsection we analyze competition in the first period where firms can discriminate only based on consumer locations and charge prices to maximize their discounted profits over two periods.

[25] While in the case of $2 < l \leq 2.38$, any firm loses some consumers at any location on its turf in equilibrium of the second period, total profits over both turfs behave in the same ways as in the case $l \leq 2$.

Similar to Fudenberg and Villas-Boas (2006) we concentrate only on equilibria in pure strategies in the first period.[26] The Proposition below summarizes our results.[27]

Proposition 1.2. *Consider an arbitrary location x on the turf of firm $i = A, B$. The subgame-perfect Nash equilibrium (in pure strategies) takes the following form:*
i) First period. In equilibrium firm i monopolizes location x only if consumers are relatively homogeneous, i.e., $l \leq h_1(\delta)$, with $h_1(0) = 2$, $h_1(1) = 1.5$ and $\partial h_1(\delta)/\partial\delta < 0$. Otherwise, in the first period firms share consumers at x, such that the more flexible of them buy at the more distant firm.
ii) Second period. In equilibrium firm i monopolizes location x if consumers are weakly differentiated, i.e., $l \leq h_2(\delta)$, with $h_2(0) = 2$, $\partial h_2(\delta)/\partial\delta > 0$ and $h_2(\delta) > h_1(\delta)$ for any $\delta > 0$. In equilibrium firm i serves all consumers on segment α, while the more flexible consumers on segment $1-\alpha$ buy at the rival provided consumers are moderately differentiated, i.e., $h_3(\delta) \leq l \leq \min\{h_4(\delta), h_5(\delta)\}$, with $h_3(0) = 2$, $h_n(0) = 5$, $\partial h_n(\delta)/\partial\delta > 0$ and $h_n(\delta) > h_3(\delta)$ for any δ, $n = 4, 5$. Finally, if consumers are strongly differentiated, i.e., $l \geq \max\{h_4(\delta), h_5(\delta)\}$, then firm i serves the less flexible consumers on both segments, while the more flexible consumers buy at the rival, with $\max\{h_4(1), h_5(1)\} = 14.13$.

The subgame-perfect Nash equilibrium in pure strategies is driven by both the level of consumer differentiation and firm discount factor. Although the relationship is intertwined, there are parameter ranges that allow unambiguous insights. In particular, if $l \leq 1.5$, then in equilibrium each firm serves all customers at any location on its turf in both periods. If $2.89 \leq l \leq 5$, then in equilibrium each firm attracts only the less flexible consumers close by in the first period, while in the following period all of them as well as the more flexible consumers on segment $1-\alpha$ buy at that firm. Finally, if $l \geq 14.13$, then in equilibrium each firm loses the more flexible consumers in the first period and also on any segment in the second period. Proposition 1.2 states that for other values of consumer differentiation in flexibility, the equilibrium depends on how strongly firms value future profits. In this case a sufficiently high discount factor leads to the monopoly outcome in the second period

[26] If $2 < l < 2,89$ or $5 < l < 14,13$ there are some values of firm discount factor, for which the equilibrium in pure strategies in the first period does not exist or two equilibria in pure strategies exist. In Proposition 1.2 we consider only those constellations of parameters l and δ, which yield the unique equilibrium prediction in pure strategies in the first period. This becomes more likely when firms are less patient because in that case the dynamic maximization function is close to the static one.

[27] In Proposition 1.2, "h" stays for the critical levels of consumer *heterogeneity*.

(on one or both segments). Figure 1.4 depicts the critical values of l (as a function of δ), which give rise to the equilibria stated in Proposition 1.2.

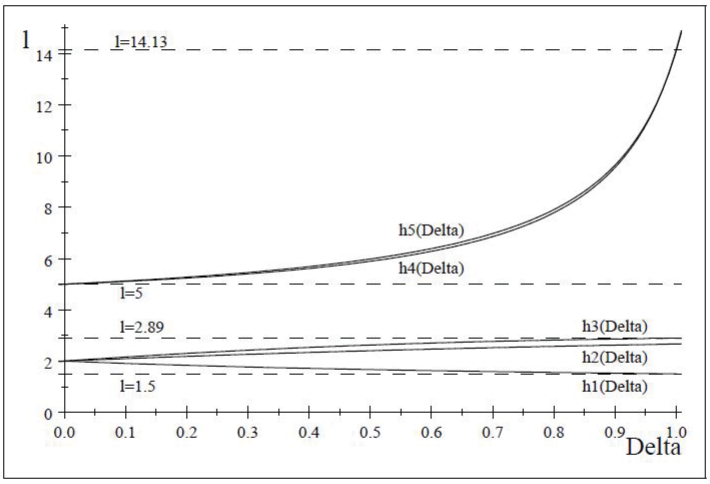


Figure 1.4: Critical values of parameter l giving rise to the equilibria stated in Proposition 1.2.

We observe from Proposition 1.2 that with increasing consumer heterogeneity (l gets larger), the equilibrium where firms lose some of the close-by consumers becomes more likely in the second period. This conclusion allows us to qualify the results of Lemma 1.2, which yields multiple equilibrium predictions for $l > 2$ depending on α. Precisely, it establishes that in equilibrium of the second period a firm serves all consumers on segment $1 - \alpha$ at any location on its turf and at the same time loses some consumers on the complementary segment if the share α is large enough. As Proposition 1.2 shows, this outcome never emerges on the equilibrium path. It is useful to recall that segment α includes those consumers at some location on a firm's turf who bought from the rival in the previous period. As none of the firms loses in the equilibrium of the initial period more than half of the consumers on its turf, this segment is relatively small, which makes it easy for a firm to monopolize it in the subsequent period relying on the acquired behavioral data.

The equilibrium of the first period also depends on how strongly consumers differ among each other. In particular, firms serve all consumers on their turfs if these are extremely similar ($l \leq h_1(\delta)$). Otherwise, they prefer to attract the less flexible consumers located close by with a correspondingly high price, allowing the flexible ones to buy at the rival. Unlike in the subsequent period, in the initial period firms additionally take into account the dynamic effect of their pricing decisions on future profits. The allocation of consumers in the first period determines the quality of information about their flexibility to be used in the future.

To understand how dynamic considerations influence pricing decisions, it is useful to start with the case of $\delta = 0$, where firms are short-sighted and fully ignore future profits. They serve all consumers on their turfs if $l \leq 2$ and lose the more flexible ones to the rival otherwise. Comparing this result with the case where firms value future profits ($\delta > 0$), we observe that the dynamic effect is absent when consumers are *relatively homogeneous* ($l \leq 1.5$): Firms monopolize any location on their turfs both with $\delta = 0$ and any $\delta > 0$. The reason is that when consumers are similar in their preferences, the value of the additional customer data is low and firms optimally prefer not to distort their pricing decisions of the first period. Note further that with $l \leq 1.5$ the dynamic effect is absent independently of δ. However, as differentiation becomes stronger, with $1.5 < l \leq 2$, discounting starts to play a role and first-period pricing decisions remain undistorted by future profits considerations only if δ is sufficiently small. Otherwise, firms sacrifice some of the short-run profits to be able to extract higher rents in the future: They lose some consumers at any location on their turfs in order to gain additional data about their preferences. Overall, the dynamic considerations play a role if consumer differentiation is sufficiently strong and enough weight is put on second-period profits in discounting.

If $l > 2$, the sharing equilibrium prevails in the first period with any $\delta \geq 0$, such that firms always gain some behavioral customer data. To understand how dynamic considerations drive pricing decisions in this case, we analyze how the distribution of consumers at a given location depends on the discount factor. We focus on the derivative of $\alpha^* (\delta)$, the market share of the rival at some location on a firm's turf in the first period, with respect to δ. If $\partial \alpha^* (\delta) / \partial \delta > 0$, we conclude that more (better) customer data is revealed in the first period when firms become more patient. Since the share $\alpha^* (\delta)$ always includes less than half of consumers at any location, a larger $\alpha^* (\delta)$ implies a more symmetric distribution of consumers between the firms and, hence, more information gained about their preferences. The following Corollary summarizes our results.

Corollary 1.1 (Revelation of customer flexibility data). *Consider an arbitrary location x on the turf of firm i. The quality of the additionally revealed customer information in the first period depends on how strongly consumers differ in their preferences and firm discount factor.*

i) If consumers are relatively homogeneous, $l \leq h_1(\delta)$, no additional information is revealed in the first period independently of the discount factor, such that $\alpha^*(\delta) = 0$ for any δ.
ii) In all other cases firms obtain additional customer information on flexibility. How a higher discount factor influences its quality, depends on the intensity of consumer differentiation: If $l \lessapprox 2.64$, then $\partial\alpha^*(\delta)/\partial\delta > 0$ and the sign is opposite if $l \gtrapprox 2.67$. Finally, if $2.64 < l < 2.67$, then $\partial\alpha^*(\delta)/\partial\delta < 0$ when firms are relatively impatient and the sign is opposite otherwise.

Corollary 1.1 shows that the effect of a larger discount factor on the quality of the revealed customer data can be threefold. In particular, if l and/or δ are small so that consumers are *relatively homogeneous* ($l \leq h_1(\delta)$), then this effect is absent and firms charge the same prices yielding the same market shares as if there were no second period. As explained above, this happens because the value of customer data is low when consumers are similar and/or when firms discount away future profits. When consumers are more differentiated and/or firms put sufficient weight on future profits, dynamic considerations matter for first-period prices and market shares. Whether in this case the distribution of consumers in the first period becomes more symmetric and, hence, customer data of a better quality is revealed, depends on how future profits respond to firms holding more precise data. As we showed in Proposition 1.1, the effect of better customer data (measured by α) on second-period profits tends to be positive when consumers are more similar in their preferences and negative otherwise. In the former case firms prefer more accurate customer data when the discount factor becomes larger, so that $\alpha^*(\delta)/\partial\delta > 0$ holds if $h_1(\delta) < l \lessapprox 2.64$ (or if $2.64 < l < 2.67$ and the discount factor is large). In the latter case firms prefer less precise information, so that $\alpha^*(\delta)/\partial\delta < 0$ if $l \gtrapprox 2.67$. However, in both cases firms acquire at least some customer data even if this reduces their second-period profits. This is because with sufficient heterogeneity in flexibility ($l > h_1(\delta)$) serving all consumers on a firm's turf in the first period would require setting excessively low prices.

An important result of Esteves (2010) is that firms may avoid learning consumer preferences to prevent tense competition in the subsequent period. In particular, she shows that the probability of the sharing outcome in the first period under which consumer types are fully revealed decreases when firms become more patient. We find an analogous result with $l \gtrapprox 2.67$, in which case less precise customer data is revealed when firms value future profits more. However, our model generates also the opposite result for $h_1(\delta) < l \lessapprox 2.64$, because more accurate behavior-based targeting is likely to increase profits in this case.

We conclude that by influencing the precision of revealed customer information in the first period, firms are able to strengthen the positive and dampen the negative effect of this information on second-period profits. We now turn to the question of how overall profits change compared to the case where firms are (for some exogenous reasons) not able to collect behavioral data and therefore can only discriminate along consumer locations in the second period. The following Corollary summarizes our results, where we compare the discounted sum of profits in the subgame-perfect Nash equilibrium (in pure strategies) in both cases.

Corollary 1.2 (The profit effect of behavioral targeting). *The profit effect of combining mobile geo-targeting with behavior-based price discrimination is:*
i) neutral irrespective of the discount factor if $l \leq 1.5$,
ii) positive provided the discount factor is large enough and neutral otherwise if $1.5 < l < 2$,
iii) positive irrespective of the discount factor if $2 \leq l \lessapprox 3.07$,
iv) (weakly) positive if the discount factor is large enough and negative otherwise if $3.07 < l \lessapprox 4$,
v) negative irrespective of the discount factor if $l > 4$.

Our results demonstrate that there are *pure* cases where the profit effect of targeted pricing based on consumer purchase histories depends only on their heterogeneity in preferences. Precisely, if $l \leq 1.5$, the ability of firms to engage in behavioral targeting is neutral for their discounted profits. When consumers do not differ a lot among each other, the value of additional customer data is small and no flexibility data is revealed in equilibrium making behavioral targeting irrelevant for profits. This result is in sharp contrast with Baye and Sapi (2019), where firms are strictly better off with additional customer data when consumers are quite homogeneous in their preferences. The reason is that in Baye and Sapi additional data is costless, while in our model firms have to sacrifice some of their first-period profits to gain it. When consumer differentiation is only modest, the value of this data to the firms is low, such that they prefer not to distort their optimal prices of the first period. As a result, Baye and Sapi overestimate the positive effect of additional data on profits.

If $2 \leq l \lessapprox 3.07$ ($l > 4$), the ability of firms to collect behavioral data is beneficial (detrimental) for their discounted profits. These results are consistent with the effect of flexibility data on second-period profits, as described in Proposition 1.1. Precisely, we showed there that profits are more likely to increase above the level without flexibility information if consumers are more homogeneous. In that case price competition is intensive even without behavioral data, so that additional customer data has mainly a positive rent-extraction effect as competition cannot increase much.

If the level of consumer differentiation takes intermediate values (not covered by the *pure* cases), then the sign of the profit effect of behavioral targeting is convoluted by the discount factor. Precisely, a higher weight on future profits makes this form of price discrimination profitable. This result is also driven by the effect of flexibility data on second-period profits as stated in Proposition 1.1. We showed there that when consumer differentiation is moderate, the effect of additional flexibility data on profits is related to the share $\alpha^* (\delta)$, which in turn depends on firm discount factor. This result is novel in the literature. Previous studies attributed unambiguous profit effects to price discrimination based on purchase histories independent of the discount factor (see Fudenberg and Tirole, 2000; Chen and Zhang, 2009; Esteves, 2010).[28] We qualify these strict effects by allowing for different levels of consumer differentiation. This in turn influences the interplay between the rent-extraction and competition effects. When neither of these effects is strong enough, then the discount factor becomes the determining factor. We now turn to the analysis of how firms' ability to combine behavior-based price discrimination with geo-targeting influences consumer surplus and social welfare.[29]

Corollary 1.3 (The welfare effect of behavioral targeting). *The effect of combining mobile geo-targeting with behavior-based price discrimination on social welfare (consumer surplus) is:*
i) neutral irrespective of the discount factor if $l \leq 1.5$*,*
ii) negative if the discount factor is large enough and neutral otherwise if $1.5 < l < 2$*,*
iii) negative irrespective of the discount factor if $2 \leq l < 2.28$ *(*$2 \leq l < 2.61$*),*

[28]To make the results of Chen and Zhang (2009) comparable with ours, we need to set consumer discount factor to zero in their model. In this case firms are always better off with targeted pricing based on consumer purchase histories, irrespective of the firm discount factor (see Proposition 1.1).

[29]To keep the exposition as simple as possible, we do not mention in the Corollary a very special case of $2.28 \lessapprox l < 2.29$ ($2.61 \lessapprox l < 2.62$), where social welfare (consumer surplus) increases when the discount factor takes intermediate values and decreases otherwise.

iv) negative if the discount factor is large enough and positive otherwise if $2.28 \lessapprox l \lessapprox 2.67$ *(*$2.61 \lessapprox l \lessapprox 2.67$*),*
v) positive irrespective of the discount factor if $l > 2.67$.

Comparing the impact of the firm ability to engage in behavioral targeting on their profits and welfare, we conclude the following. If consumers are very similar in their preferences ($l \leq 1.5$), both firms serve all customers located closer to them in the first period and no flexibility data is revealed. As a result, both profits and welfare do not depend on whether firms can target consumers based on their behavior. When firms do gain flexibility data in equilibrium ($l > 1.5$), firms' and social welfare's interests are likely to be opposed. Additional customer data renders the second-period distribution of consumers more efficient, because more consumers buy from the firm located closer. This reduces transport costs and improves social welfare. However, with more homogeneous consumers (l is relatively small), firms distort first-period prices in order to obtain more flexibility data leading to a higher misalignment of consumers between the firms: The more flexible of them purchase from the firm located farther away. In this case firms benefit from behavioral data but social welfare reduces. This result is reversed when consumers become more differentiated (l is relatively large), because behavioral data in that case harms firms. They therefore consciously weaken information revelation in the first period making the distribution of consumers more efficient, because less customers buy from a farther firm. A similar pattern follows from the comparison of a change in profits and consumer surplus. From Corollaries 1.2 and 1.3 we can also conclude that firm and consumer interests are not necessarily opposed. Precisely, if $2.67 < l \lessapprox 3.07$, then profits as well as consumer surplus (social welfare) increase by adding behavioral price discrimination to geo-targeting.

As in the case of the profit effect of behavior-based price discrimination, the way how the latter influences social welfare and consumer surplus also depends on firm discount factor when consumers differ moderately in their preferences. Precisely, consumers (and the overall welfare) are more likely to gain from firms combining behavioral pricing with mobile geo-targeting when the latter discount future profits more and are, hence, more likely to be worse off.

1.5 Conclusion

This paper analyzes a model taking into account four important features of a modern mobile targeting environment. First, sellers can observe consumers' real-time locations. Second, apart from location, there are other factors influencing the responsiveness of a consumer to discounts, such as age, income and occupation. Different from location, these are imperfectly observable by marketers. Third, sellers may infer consumer responsiveness (flexibility) from the observed previous purchasing behavior of a customer. Fourth, firms can deliver personalized offers through mobile devices in a private manner based on both consumer locations and their flexibility inferred from the previous purchase decisions. Our results show that firms benefit from the ability to collect behavioral data and use it for personalized pricing in mobile geo-targeting when consumers differ moderately in their preferences. With less differentiated consumers behavior-based price discrimination is neutral for profits, while with strongly differentiated consumers it intensifies competition and reduces profits. Different from the previous studies, our results also highlight the importance of the discount factor for the profit effect of behavioral targeting, which is likely to be positive when firms are more patient. We also find that consumer and firm interests are not necessarily opposed. In particular, when customers differ modestly in their preferences both consumer surplus and profits can increase with behavioral targeting leading to a higher social welfare. Finally, we show that firms strategically influence the quality of the (revealed) consumer behavioral data so as to enable higher rents extraction in case the data allows them to do so, and reduce the profit loss if data intensifies competition.

Our results are relevant for managers and policy alike. The main managerial implication of our results is that combining behavioral marketing with geo-targeting needs very careful consideration of the market environment. We highlight the role of consumer heterogeneity and firm discount factor and derive precise conditions under which such a campaign may be profitable in a competitive landscape. The main policy message relates to consumer and privacy policy: Combining behavioral price discrimination with geo-targeting can be both beneficial and harmful for consumers. While geo-targeting has been argued to typically foster competition (e.g., Thisse and Vives, 1988), combining it with behavioral price discrimination can turn around this effect, giving scope for a careful consumer policy. For example, restricting firms in their collection of types of data, such as age and demographics, that relate to their flexibility may improve consumer outcomes when these do not differ strongly among each other. Similarly, decreasing the data retention period (a proxy for the discount factor in our model) may also benefit consumers when these are moderately differentiated.

1.6 Appendix

Proof of Lemma 1.1. As firms are symmetric, we will restrict attention to the turf of firm A. Consider some $x < 1/2$ and segment α. Maximizing the expected profit of firm A yields the best-response function, which depends on the ratio $t^\alpha/\underline{t}$. If $t^\alpha/\underline{t} \leq 2$ $(\alpha \leq 1/(l-1))$, then $p_A^\alpha(x; p_B^\alpha) = p_B^\alpha + \underline{t}(1-2x)$, such that firm A optimally serves all consumers on segment α irrespective of firm B's price. Then in equilibrium firm B charges $p_B^\alpha(x) = 0$, because it would have an incentive to deviate from any positive price. Hence, $p_A^\alpha(x) = \underline{t}(1-2x)$. If $t^\alpha/\underline{t} > 2$, then the best response of firm A takes the form:

$$p_A^\alpha(x; p_B^\alpha) = \begin{cases} p_B^\alpha + \underline{t}(1-2x) & \text{if} \quad p_B^\alpha \geq (t^\alpha - 2\underline{t})(1-2x) \\ \frac{p_B^\alpha + t^\alpha(1-2x)}{2} & \text{if} \quad p_B^\alpha < (t^\alpha - 2\underline{t})(1-2x), \end{cases} \tag{1.2}$$

such that firm A serves all consumers on segment α only if the rival's price is relatively high. Maximization of the expected profit of firm B yields the best-response function:

$$p_B^\alpha(x; p_A^\alpha) = \begin{cases} \text{any } p_B^\alpha & \text{if} \quad p_A^\alpha \leq \underline{t}(1-2x) \\ \frac{p_A^\alpha - \underline{t}(1-2x)}{2} & \text{if} \quad \underline{t}(1-2x) < p_A^\alpha < (2t^\alpha - \underline{t})(1-2x) \\ p_A^\alpha - t^\alpha(1-2x) & \text{if} \quad p_A^\alpha \geq (2t^\alpha - \underline{t})(1-2x). \end{cases} \tag{1.3}$$

Inspecting (1.2), we conclude that firm B cannot serve all consumers in equilibrium. It is straightforward to show that there are no such prices, which constitute the equilibrium, where firm A serves all consumers. Hence, only the equilibrium can exist, where both firms serves consumers. Solving (1.2) and (1.3) simultaneously, we get the prices: $p_A^\alpha(x) = \underline{t}(1-2x)[2\alpha(l-1)+1]/3$ and $p_B^\alpha(x) = \underline{t}(1-2x)[\alpha(l-1)-1]/3$. For this equilibrium to exist, it must hold that $t^\alpha/\underline{t} > 2$. In a similar way one can derive the equilibrium on the segment $1-\alpha$. Precisely, if $1/t^\alpha \leq 2$ $(\alpha \geq (l-2)/[2(l-1)])$, then in the monopoly equilibrium firm A serves all consumers, where firms charge prices: $p_A^{1-\alpha}(x) = \underline{t}[1+\alpha(l-1)](1-2x)$ and $p_B^{1-\alpha}(x) = 0$. If $1/t^\alpha > 2$, then the sharing equilibrium emerges with the prices: $p_A^\alpha(x) = \underline{t}(1-2x)[2l-1-\alpha(l-1)]/3$, $p_B^\alpha(x) = \underline{t}(1-2x)[l-2-2\alpha(l-1)]/3$. *Q.E.D.*

Proof of Lemma 1.2. Lemma 1.2 follow directly from Lemma 1.1 given the following results: $1/(l-1) > (l-2)/[2(l-1)]$ if $l < 4$, $(l-2)/[2(l-1)] > 0$

if $l > 2$, $1/(l-1) > 1$ if $l < 2$, with the opposite sign otherwise. Note that $1/(l-1) > 0$ and $(l-2)/[2(l-1)] < 1$ hold for any l. *Q.E.D.*

Proof of Proposition 1.1. Consider first some x on the turf of firm A. We start with deriving firms' profits on each segment depending on α. Consider first segment α. If $\alpha \leq 1/(l-1)$, then firm A serves all consumers and profits are

$$\begin{aligned} \frac{\Pi_A^\alpha(x|x<1/2)}{\underline{t}(1-2x)} &= \Pi_A^{\alpha,1}(l;\alpha) := \frac{t^\alpha - \underline{t}}{1-\underline{t}} = \alpha \text{ and} \\ \frac{\Pi_B^\alpha(x|x<1/2)}{\underline{t}(1-2x)} &= \Pi_B^{\alpha,1}(l;\alpha) := 0. \end{aligned}$$

If $\alpha > 1/(l-1)$, then firm A serves consumers with $t \geq \underline{t}[\alpha(l-1)+2]/3$ and profits are

$$\begin{aligned} \frac{\Pi_A^\alpha(x|x<1/2)}{\underline{t}(1-2x)} &= \Pi_A^{\alpha,2}(l;\alpha) := \left[t^\alpha - \frac{\underline{t}(\alpha(l-1)+2)}{3}\right]\frac{[2\alpha(l-1)+1]}{3(1-\underline{t})} = \frac{[2\alpha(l-1)+1]^2}{9(l-1)} \text{ and} \\ \frac{\Pi_B^\alpha(x|x<1/2)}{\underline{t}(1-2x)} &= \Pi_B^{\alpha,2}(l;\alpha) := \left[\frac{\underline{t}(\alpha(l-1)+2)}{3} - \underline{t}\right]\frac{[\alpha(l-1)-1]}{3(1-\underline{t})} = \frac{[\alpha(l-1)-1]^2}{9(l-1)}. \end{aligned}$$

Consider now segment $1-\alpha$. If $\alpha \geq (l-2)/[2(l-1)]$, then firm A gains all consumers and firms realize profits:

$$\begin{aligned} \frac{\Pi_A^{1-\alpha}(x|x<1/2)}{\underline{t}(1-2x)} &= \Pi_A^{1-\alpha,1}(l;\alpha) := \frac{(1-t^\alpha)t^\alpha}{(1-\underline{t})\underline{t}} = (1-\alpha)[1+\alpha(l-1)] \text{ and} \\ \frac{\Pi_B^{1-\alpha}(x|x<1/2)}{\underline{t}(1-2x)} &= \Pi_B^{1-\alpha,1}(l;\alpha) := 0. \end{aligned}$$

If $\alpha < (l-2)/[2(l-1)]$, firm A serves consumers with $t \geq \underline{t}[l+1+\alpha(l-1)]/3$ and firms realize profits:

$$\begin{aligned} \frac{\Pi_A^{1-\alpha}(x|x<1/2)}{\underline{t}(1-2x)} &= \Pi_A^{1-\alpha,2}(l;\alpha) := \left[1 - \frac{\underline{t}[l+1+\alpha(l-1)]}{3}\right]\frac{[2l-1-\alpha(l-1)]}{3(1-\underline{t})} = \frac{[2l-1-\alpha(l-1)]^2}{9(l-1)} \text{ and} \\ \frac{\Pi_B^{1-\alpha}(x|x<1/2)}{\underline{t}(1-2x)} &= \Pi_B^{1-\alpha,2}(l;\alpha) := \left[\frac{\underline{t}[l+1+\alpha(l-1)]}{3} - t^\alpha\right]\frac{[l-2-2\alpha(l-1)]}{3(1-\underline{t})} = \frac{[l-2-2\alpha(l-1)]^2}{9(l-1)}. \end{aligned}$$

The profits on some x on the turf of firm B can be derived in a similar way. Note now that $\int_0^{1/2}(1-2x)\,dx = \int_{1/2}^1(2x-1)\,dx = 1/4$. Using the above results, we can write down the total profits depending on l and α under the assumption that on any x on its turf in the first period every firm served consumers with $t \geq t^\alpha$.

Consider first $l \leq 2$. The total profits of firm $i = A, B$ on both turfs are

$$\frac{4\Pi_i(l;\alpha)}{\underline{t}} = \Pi_A^{\alpha,1}(\cdot) + \Pi_A^{1-\alpha,1}(\cdot) = f_1(l;\alpha) := \alpha + (1-\alpha)[1+\alpha(l-1)].$$

Taking the derivative of $f_1(l;\alpha)$ with respect to α we get

$$\tfrac{\partial f_1(l;\alpha)}{\partial\alpha} = (1-2\alpha)(l-1),$$

such that $f_1(l;\alpha)$ is given by the inverted U-shaped function of α, which gets its maximum at $\alpha = 1/2$.

Consider now $2 < l < 4$ and $\alpha \leq (l-2)/[2(l-1)]$, then the total profits of firm i on both turfs are

$$\tfrac{4\Pi_i(l;\alpha)}{\underline{t}} = \Pi_A^{\alpha,1}(\cdot) + \Pi_A^{1-\alpha,2}(\cdot) + \Pi_B^{1-\alpha,2}(\cdot) = f_2(l;\alpha) := \alpha + \tfrac{[2l-1-\alpha(l-1)]^2}{9(l-1)} + \tfrac{[l-2-2\alpha(l-1)]^2}{9(l-1)}.$$

Taking the derivative of $f_2(l;\alpha)$ with respect to α we get

$$\tfrac{\partial f_2(l;\alpha)}{\partial\alpha} = \tfrac{10\alpha(l-1)-8l+19}{9} > 0 \text{ if } \alpha > \alpha_2 := \tfrac{8l-19}{10(l-1)}.$$

Note that $\alpha_2 \leq 0$ if $l \leq 19/8 \approx 2.38$ and $\alpha_2 < (l-2)/[2(l-1)]$ if $l < 3$. Hence, if $2 < l \leq 19/8$, then $f_2(l;\alpha)$ increases in α. If $19/8 < l < 3$, then $f_2(l;\alpha)$ decreases till $\alpha = \alpha_2$ and increases afterwards. Finally, if $3 \leq l < 4$, then $f_2(l;\alpha)$ decreases in α.

If $(l-2)/[2(l-1)] < \alpha < 1/(l-1)$, then the total profits of firm i on both turfs are

$$\tfrac{4\Pi_i(l;\alpha)}{\underline{t}} = \Pi_A^{\alpha,1}(\cdot) + \Pi_A^{1-\alpha,1}(\cdot) = f_3(l;\alpha) := \alpha + (1-\alpha)[1+\alpha(l-1)].$$

Taking the derivative of $f_3(l;\alpha)$ with respect to α we get

$$\tfrac{\partial f_3(l;\alpha)}{\partial\alpha} = (1-2\alpha)(l-1) > 0 \text{ if } \alpha < \tfrac{1}{2}.$$

Note that $(l-2)/[2(l-1)] < 1/2$ for any l and $1/(l-1) < 1/2$ if $l > 3$. Hence, if $2 < l \leq 3$, then on $(l-2)/[2(l-1)] < \alpha < 1/(l-1)$, $f_3(l;\alpha)$ increases in α till $\alpha = 1/2$ and decreases afterwards. If $3 < l < 4$, then $f_3(l;\alpha)$ increases in α on $(l-2)/[2(l-1)] < \alpha < 1/(l-1)$.

If $\alpha \geq 1/(l-1)$, then the total profits of firm i on both turfs are

$$\begin{aligned}\tfrac{4\Pi_i(l;\alpha)}{\underline{t}} &= \Pi_A^{\alpha,2}(\cdot) + \Pi_B^{\alpha,2}(\cdot) + \Pi_A^{1-\alpha,1}(\cdot)\\ &= f_4(l;\alpha) := \tfrac{[2\alpha(l-1)+1]^2}{9(l-1)} + \tfrac{[\alpha(l-1)-1]^2}{9(l-1)} + (1-\alpha)[1+\alpha(l-1)].\end{aligned}$$

Taking the derivative of $f_4(l;\alpha)$ with respect to α we get

$$\frac{\partial f_4(l;\alpha)}{\partial\alpha} = \frac{9l-8\alpha(l-1)-16}{9} > 0 \text{ if } \alpha < \alpha_4 := \frac{9l-16}{8(l-1)}.$$

Note that $\alpha_4 < 1/(l-1)$ if $l < 24/9 \approx 2.67$ and $\alpha_4 < 1$ for any $2 < l < 4$. Hence, if $2 < l < 24/9$, then $f_4(l;\alpha)$ decreases in α. If $24/9 \leq l < 4$, then $f_4(l;\alpha)$ increases till $\alpha = \alpha_4$ and decreases afterwards.

Consider finally $l \geq 4$. If $\alpha \leq 1/(l-1)$, then the total profits of firm i on both turfs are

$$\begin{aligned} \frac{4\Pi_i(l;\alpha)}{\underline{t}} &= \Pi_A^{\alpha,1}(\cdot) + \Pi_A^{1-\alpha,2}(\cdot) + \Pi_B^{1-\alpha,2}(\cdot) \\ &= f_5(l;\alpha) := \alpha + \frac{[2l-1-\alpha(l-1)]^2}{9(l-1)} + \frac{[l-2-2\alpha(l-1)]^2}{9(l-1)}. \end{aligned}$$

Taking the derivative of $f_5(l;\alpha)$ with respect to α we get

$$\frac{\partial f_5(l;\alpha)}{\partial\alpha} = \frac{10\alpha(l-1)-8l+19}{9} > 0 \text{ if } \alpha > \alpha_5 := \frac{8l-19}{10(l-1)}.$$

Note that for any $l \geq 4$ it holds that $\alpha_5 > 1/(l-1)$. Hence, $f_5(l;\alpha)$ decreases in α.

If $1/(l-1) < \alpha < (l-2)/[2(l-1)]$, then the total profits of firm i on both turfs are

$$\begin{aligned} \frac{4\Pi_i(l;\alpha)}{\underline{t}} &= \Pi_A^{\alpha,2}(\cdot) + \Pi_B^{\alpha,2}(\cdot) + \Pi_A^{1-\alpha,2}(\cdot) + \Pi_B^{1-\alpha,2}(\cdot) \\ &= f_6(l;\alpha) := \frac{[2\alpha(l-1)+1]^2}{9(l-1)} + \frac{[\alpha(l-1)-1]^2}{9(l-1)} + \frac{[2l-1-\alpha(l-1)]^2}{9(l-1)} + \frac{[l-2-2\alpha(l-1)]^2}{9(l-1)}. \end{aligned}$$

Taking the derivative of $f_6(l;\alpha)$ with respect to α we get

$$\frac{\partial f_6(l;\alpha)}{\partial\alpha} = \frac{20\alpha(l-1)-4(2l-3)}{9} > 0 \text{ if } \alpha > \alpha_6 := \frac{2l-3}{5(l-1)}.$$

Note that for any $l \geq 4$ it holds that $1/(l-1) \leq \alpha_6 \leq (l-2)/[2(l-1)]$. Hence, $f_6(l;\alpha)$ decreases till $\alpha = \alpha_6$ and increases afterwards.

If $\alpha \geq (l-2)/[2(l-1)]$, then the total profits of firm i on both turfs are

$$\begin{aligned} \frac{4\Pi_i(l;\alpha)}{\underline{t}} &= \Pi_A^{\alpha,2}(\cdot) + \Pi_B^{\alpha,2}(\cdot) + \Pi_A^{1-\alpha,1}(\cdot) \\ &= f_7(l;\alpha) := \frac{[2\alpha(l-1)+1]^2}{9(l-1)} + \frac{[\alpha(l-1)-1]^2}{9(l-1)} + (1-\alpha)[1+\alpha(l-1)]. \end{aligned}$$

Taking the derivative of $f_7(l;\alpha)$ with respect to α we get

$$\frac{\partial f_7(l;\alpha)}{\partial\alpha} = \frac{9l-16-8\alpha(l-1)}{9} > 0 \text{ if } \alpha < \alpha_7 := \frac{9l-16}{8(l-1)}.$$

Note that for any $l \geq 4$ it holds that $\alpha_7 > (l-2)/[2(l-1)]$. Moreover, $\alpha_7 > 1$ if $l > 8$, with an opposite inequality otherwise. Hence, if $4 \leq l \leq 8$, then $f_7(l;\alpha)$ increases till $\alpha = \alpha_7$ and decreases afterwards. If $l > 8$, then $f_7(l;\alpha)$ increases in α.

We can now summarize the results on the behavior of the total profits in α depending on l. *i)* If $l \leq 2.38$, then total profits are given by the inverted U-shaped function of α, which gets its maximum at $\alpha = 1/2$.

ii) If $2.38 < l < 2.67$, then total profits are given by a non-monotonic function, which gets a (local) minimum at $\alpha = (8l-19)/(10l-10)$ and a (local) maximum at $\alpha = 1/2$. This function decreases on the intervals: $[0,(8l-19)/(10l-10)]$ and $[1/2,1]$, and increases on the remaining intervals. Note that

$$\frac{4\Pi_i\left(l;\frac{1}{2}\right)}{\underline{t}} = f_3\left(l;\tfrac{1}{2}\right) = \frac{l+3}{4} > \frac{4\Pi_i(l;0)}{\underline{t}} = f_2(l;0) = \frac{5l^2-8l+5}{9(l-1)}, \text{ for any } 2.38 < l < 2.67,$$

such that $\Pi_i(l;k)$ gets a global maximum at $\alpha = 1/2$. From this and the fact that $\Pi_i(l;k)$ is a continuous function of α, we conclude there exists $(8l-19)/(10l-10) < \widehat{\alpha}(l) < 1/2$, such that $\Pi_i(l;\alpha) \geq \Pi_i(l;0)$ if $\alpha \geq \widehat{\alpha}(l)$, with an opposite inequality otherwise. $\widehat{\alpha}(l)$ is implicitly given either by the equation:

$$f_2(l;0) = \frac{5l^2-8l+5}{9(l-1)} = f_2(l;\widehat{\alpha}(l)) = \widehat{\alpha} + \frac{[2l-1-\widehat{\alpha}(l-1)]^2}{9(l-1)} + \frac{[l-2-2\widehat{\alpha}(l-1)]^2}{9(l-1)}, \tag{1.4}$$

or by the equation:

$$f_2(l;0) = \frac{5l^2-8l+5}{9(l-1)} = f_3(l;\widehat{\alpha}(l)) = \widehat{\alpha} + (1-\widehat{\alpha})[1+\widehat{\alpha}(l-1)]. \tag{1.5}$$

In the former case we get that

$$\frac{\partial\widehat{\alpha}(l)}{\partial l} = \frac{\alpha(5\alpha-8)}{8l-19-10\alpha(l-1)} > 0$$

and in the latter case we get

$$\frac{\partial\widehat{\alpha}(l)}{\partial l} = \frac{\alpha^2\left(9l^2-18l+9\right)+\alpha\left(-9l^2+18l-9\right)+5l^2-10l+3}{9(1-2\alpha)(l-1)^3} > 0,$$

because if $2.38 < l < 2.67$, $\alpha^2(9l^2-18l+9)+\alpha(-9l^2+18l-9)+5l^2-10l+3 > 0$

for any α.

iii) If $2.67 < l < 3$, the function $\Pi_i(l;k)$ decreases on: $[0,(8l-19)/(10l-10)]$, $[1/2, 1/(l-1)]$ and $[(9l-16)/(8l-8), 1]$. The comparisons show that

$$\begin{aligned}
\frac{4\Pi_i\left(l;\frac{1}{2}\right)}{\underline{t}} &= f_3\left(l;\tfrac{1}{2}\right) = \tfrac{l+3}{4} \geq \frac{4\Pi_i\left(l;\frac{9l-16}{8(l-1)}\right)}{\underline{t}} = f_4\left(l;\tfrac{9l-16}{8(l-1)}\right) = \tfrac{9l^2-16l+16}{16(l-1)} \text{ if } 2.67 < l \leq 2.8,\\
\frac{4\Pi_i\left(l;\frac{1}{l-1}\right)}{\underline{t}} &= f_3\left(l;\tfrac{1}{l-1}\right) = f_4\left(l;\tfrac{1}{l-1}\right) = \tfrac{2l-3}{l-1} > f_2(l;0) = f_4(l;1) = \tfrac{5l^2-8l+5}{9(l-1)} \text{ if } 2.67 < l < 3.
\end{aligned}$$

We make two conclusions. First, $\Pi_i(l;\alpha)$ gets the global maximum at $\alpha = 1/2$ if $l \leq 2.8$ and at $\alpha = (9l-16)/[8(l-1)]$ otherwise. Second, using the fact that $\Pi_i(l;\alpha)$ is a continuos function of α, we conclude that there exists $(8l-19)/(10l-10) < \widehat{\alpha}(l) < 1/2$, such that $\Pi_i(l;\alpha) \geq \Pi_i(l;0)$ if $\alpha \geq \widehat{\alpha}(l)$, with an opposite inequality otherwise. As in the previous case, $\widehat{\alpha}(l)$ is given by either (1.4) or (1.5). As we showed above, in both cases $\partial\widehat{\alpha}(l)/\partial l > 0$ holds.

iv) If $3 < l \leq 4$, then the function $\Pi_i(l;\alpha)$ decreases on: $[0,(l-2)/(2l-2)]$ and $[(9l-16)/(8l-8), 1]$, while increases on the remaining interval. Note that

$$f_2(l;0) = f_4(l;1) = \tfrac{5l^2-8l+5}{9(l-1)} < f_4\left(l;\tfrac{9l-16}{8(l-1)}\right) = \tfrac{9l^2-16l+16}{16(l-1)} \text{ for any } l, \tag{1.6}$$

such that $\Pi_i(l;\alpha)$ has a global maximum at $\alpha = (9l-16)/(8l-8)$. As $\Pi_i(l;\alpha)$ is a continuous function of α, we conclude that there exists $(l-2)/(2l-2) < \widehat{\alpha}(l) < (9l-16)/(8l-8)$, such that $\Pi_i(l;\alpha) \geq \Pi_i(l;0)$ if $\alpha \geq \widehat{\alpha}(l)$, with an opposite inequality otherwise. As in the previous case, $\widehat{\alpha}(l)$ is given by either (1.4) or (1.5). As we showed above, in both cases $\partial\widehat{\alpha}(l)/\partial l > 0$ holds.

v) If $4 < l < 8$, then the function $\Pi_i(l;\alpha)$ decreases on: $[0,(2l-3)/(5l-5)]$ and $[(9l-16)/(8l-8), 1]$, while increases on the remaining interval. Due to (1.6), $\Pi_i(l;\alpha)$ has a global maximum at $\alpha = (9l-16)/(8l-8)$. As $\Pi_i(l;\alpha)$ is a continuous function of α, we conclude that there exists $(2l-3)/(5l-5) < \widehat{\alpha}(l) < (9l-16)/(8l-8)$, such that $\Pi_i(l;\alpha) \geq \Pi_i(l;0)$ if $\alpha \geq \widehat{\alpha}(l)$, with an opposite inequality otherwise. $\widehat{\alpha}(l)$ is implicitly given either by the equation:

$$f_5(l;0) = \tfrac{5l^2-8l+5}{9(l-1)} = f_6(l;\widehat{\alpha}(l)) = \tfrac{[2\widehat{\alpha}(l-1)+1]^2}{9(l-1)} + \tfrac{[\widehat{\alpha}(l-1)-1]^2}{9(l-1)} + \tfrac{[2l-1-\widehat{\alpha}(l-1)]^2}{9(l-1)} + \tfrac{[l-2-2\widehat{\alpha}(l-1)]^2}{9(l-1)},$$

or by the equation:

$$f_5(l;0) = \tfrac{5l^2-8l+5}{9(l-1)} = f_7(l;\widehat{\alpha}(l)) = \tfrac{[2\widehat{\alpha}(l-1)+1]^2}{9(l-1)} + \tfrac{[\widehat{\alpha}(l-1)-1]^2}{9(l-1)} + (1-\widehat{\alpha})\left[1+\widehat{\alpha}(l-1)\right].$$

In the former case we get that

$$\begin{aligned}
\frac{\partial\widehat{\alpha}(l)}{\partial l} &= -\frac{\left(-5l^2+10l-5\right)[\alpha-\alpha_1(l)][\alpha-\alpha_2(l)]}{2(l-1)^2[2l-3-\alpha(5l-5)]}, \text{ where} \\
\alpha_1(l) &= \frac{-\left(4l^2-8l+4\right)+2(l-1)\sqrt{4l^2-8l+9}}{2(-5l^2+10l-5)} \text{ and } \alpha_2(l) = \frac{-\left(4l^2-8l+4\right)-2(l-1)\sqrt{4l^2-8l+9}}{2(-5l^2+10l-5)}.
\end{aligned}$$

Note that as $f_6(l;\alpha)$ is defined on $1/(l-1) < \alpha < (l-2)/[2(l-1)]$, while $\alpha_1(l) < 1/(l-1)$ and $\alpha_2(l) > (l-2)/[2(l-1)]$ for any $4 < l < 8$, then $\alpha - \alpha_1(l) > 0$ and $\alpha - \alpha_2(l) < 0$. Finally, as $-5l^2 + 10l - 5 < 0$ for any $4 < l < 8$ and $\widehat{\alpha}(l) > (2l-3)/(5l-5)$, we conclude that $\partial\widehat{\alpha}(l)/\partial l > 0$. In the latter case we get

$$\frac{\partial\widehat{\alpha}(l)}{\partial l} = -\frac{4(\alpha-1)\left(\alpha-\frac{5}{4}\right)}{8\alpha(l-1)-(9l-16)} > 0 \text{ as } \widehat{\alpha}(l) < \frac{9l-16}{8\alpha(l-1)}.$$

vi) If $l \geq 8$, then $\Pi_i(l;\alpha)$ is a U–shaped function, which gets its (global) minimum at $\alpha = (2l-3)/(5l-5)$. *Q.E.D.*

Proof of Proposition 1.2. Consider some x on the turf of firm A. Using the results of Lemma 1.2 and the notation from the proof of Proposition 1.1, we can write down second-period profits at x depending on l and α. If $l \leq 2$, then firm A gains all consumers at x independently of α, such that profits of firm $i = A, B$ at x, $\Pi_i(x|\, x < 1/2)$, are

$$\begin{aligned}
\frac{\Pi_A(x|x<1/2)}{\underline{t}(1-2x)} &= \Pi_A^{\alpha,1}(l;\alpha) + \Pi_A^{1-\alpha,1}(l;\alpha) = \alpha + (1-\alpha)[1+\alpha(l-1)], \quad (1.7) \\
\frac{\Pi_B(x|x<1/2)}{\underline{t}(1-2x)} &= \Pi_B^{\alpha,1}(l;\alpha) + \Pi_B^{1-\alpha,1}(l;\alpha) = 0.
\end{aligned}$$

Consider now $2 < l < 4$, in which case second-period profits at x depend on α. If $\alpha \leq (l-2)/[2(l-1)]$, then firm A gains all consumers on α and loses some consumers on $1-\alpha$, such that profits are

$$\begin{aligned}
\frac{\Pi_A(x|x<1/2)}{\underline{t}(1-2x)} &= \Pi_A^{\alpha,1}(l;\alpha) + \Pi_A^{1-\alpha,2}(l;\alpha) = \alpha + \frac{[2l-1-\alpha(l-1)]^2}{9(l-1)}, \quad (1.8) \\
\frac{\Pi_B(x|x<1/2)}{\underline{t}(1-2x)} &= \Pi_B^{\alpha,1}(l;\alpha) + \Pi_B^{1-\alpha,2}(l;\alpha) = \frac{[l-2-2\alpha(l-1)]^2}{9(l-1)}.
\end{aligned}$$

If $(l-2)/[2(l-1)] \leq \alpha < 1/(l-1)$, then firm A serves all consumers on both segments, and profits are given by (1.7). If $\alpha \geq 1/(l-1)$, then firm A loses consumers on α, and profits are

$$\begin{aligned}
\frac{\Pi_A(x|x<1/2)}{\underline{t}(1-2x)} &= \Pi_A^{\alpha,2}(l;\alpha) + \Pi_A^{1-\alpha,1}(l;\alpha) = \frac{[2\alpha(l-1)+1]^2}{9(l-1)} + (1-\alpha)[1+\alpha(l-1)], \quad (1.9) \\
\frac{\Pi_B(x|x<1/2)}{\underline{t}(1-2x)} &= \Pi_B^{\alpha,2}(l;\alpha) + \Pi_B^{1-\alpha,1}(l;\alpha) = \frac{[\alpha(l-1)-1]^2}{9(l-1)}.
\end{aligned}$$

Consider finally $l \geq 4$. If $\alpha \leq 1/(l-1)$, then firm A loses consumers on $1-\alpha$, and profits are given by (1.8). If $1/(l-1) < \alpha < (l-2)/[2(l-1)]$, then firm A loses consumers on both segments, and firms realize profits:

$$\begin{aligned} \tfrac{\Pi_A(x|x<1/2)}{\underline{t}(1-2x)} &= \Pi_A^{\alpha,2}(l;\alpha) + \Pi_A^{1-\alpha,2}(l;\alpha) = \tfrac{[2\alpha(l-1)+1]^2}{9(l-1)} + \tfrac{[2l-1-\alpha(l-1)]^2}{9(l-1)}, \quad (1.10) \\ \tfrac{\Pi_B(x|x<1/2)}{\underline{t}(1-2x)} &= \Pi_B^{\alpha,2}(l;\alpha) + \Pi_B^{1-\alpha,2}(l;\alpha) = \tfrac{[\alpha(l-1)-1]^2}{9(l-1)} + \tfrac{[l-2-2\alpha(l-1)]^2}{9(l-1)}. \end{aligned}$$

If $\alpha \geq (l-2)/[2(l-1)]$, then firm A loses consumers on α, and profits are given by (1.9).

We introduce now a new notation for the (adjusted) price of firm $i = A, B$ on some $x < 1/2$:

$$p_i^x := \tfrac{p_A(x)}{(1-2x)\underline{t}}.$$

At any $x < 1/2$ those consumers buy at firm A who have relatively high transport costs:

$$\begin{aligned} t &\geq t^\alpha(p_A(x), p_B(x)) = \alpha + \underline{t}(1-\alpha), \text{ where} \qquad (1.11) \\ t^\alpha(\cdot) &= \tfrac{p_A(x)-p_B(x)}{1-2x} = (p_A^x - p_B^x)\,\underline{t}, \end{aligned}$$

from where we can derive α as follows

$$\alpha = \tfrac{p_A(x)-p_B(x)}{(1-2x)\underline{t}(l-1)} - \tfrac{1}{l-1} = \tfrac{p_A^x - p_B^x - 1}{l-1}. \qquad (1.12)$$

Note next that if $\underline{t} \leq t^\alpha(\cdot) \leq 1$, then the profit of firm A at $x < 1/2$ in the first period is

$$\begin{aligned} \left[1 - \tfrac{p_A(x)-p_B(x)}{1-2x}\right] \tfrac{p_A(x)}{1-\underline{t}} &= \underline{t}(1-2x)\left[l - \tfrac{p_A(x)-p_B(x)}{(1-2x)\underline{t}}\right] \tfrac{p_A(x)}{\underline{t}(1-2x)(l-1)} \\ &= \tfrac{\underline{t}(1-2x)\left(l-p_A^x+p_B^x\right)p_A^x}{l-1}. \end{aligned}$$

Similarly, the profit of firm B at $x < 1/2$ in the first period is

$$\left[\tfrac{p_A(x)-p_B(x)}{1-2x} - \underline{t}\right] \tfrac{p_B(x)}{1-\underline{t}} = \tfrac{\underline{t}(1-2x)\left(p_A^x-p_B^x-1\right)p_B^x}{(l-1)}.$$

To derive the optimal prices of the first period, we will consider the discounted sum of each firm's profits over two periods, multiplied by $(l-1)$ and divided by $\underline{t}(1-2x)$.

Part 1. Consider first $l \leq 2$, in which case firm A chooses p_A^x to maximize the profits:

$$\begin{aligned} & \left(l - p_A^x + p_B^x\right) p_A^x + \delta \left(l - 1\right) \left[\alpha + \left(1 - \alpha\right) \left(1 + \alpha \left(l - 1\right)\right)\right] \\ = \; & \left(l - p_A^x + p_B^x\right) p_A^x + \delta \left[p_A^x - p_B^x - 1 + \left(l - p_A^x + p_B^x\right) \left(p_A^x - p_B^x\right)\right]. \end{aligned} \tag{1.13}$$

Firm B chooses p_B^x to maximize the profits:

$$\left(p_A^x - p_B^x - 1\right) p_B^x. \tag{1.14}$$

Solving firms' first-order conditions we arrive at the prices:

$$p_A^{x*} = \tfrac{2l(1+\delta)-1}{2\delta+3} \text{ and } p_B^{x*} = \tfrac{l-2-\delta(1-l)}{2\delta+3}. \tag{1.15}$$

Note that second-order conditions are also fulfilled. For the prices (1.15) to constitute the equilibrium, it must hold that $\underline{t} < t^{\alpha}\left(\left(1-2x\right)\underline{t}p_A^{x*}, \left(1-2x\right)\underline{t}p_B^{x*}\right) \leq 1$, which yields the condition:

$$\tfrac{1}{1+l} < \tfrac{1+\delta}{3+2\delta} \leq \tfrac{l}{1+l}. \tag{1.16}$$

Note that $l/\left(1+l\right) > 0.5$ for any l and $\left(1+\delta\right)/\left(3+2\delta\right) \leq 0.4$ for any δ, such that the right-hand side of (1.16) is fulfilled for any δ and any l. The left-hand side of (1.16) is fulfilled if

$$l > \bar{l}_1\left(\delta\right) := \tfrac{2+\delta}{1+\delta}. \tag{1.17}$$

It holds that $1.5 \leq \bar{l}_1\left(\delta\right) \leq 2$ for any δ and $\partial\bar{l}\left(\delta\right)/\partial\delta < 0$. Note finally that if (1.17) holds, then $p_A^{x*} > 0$ and $p_B^{x*} > 0$.

If $l \leq \bar{l}_1\left(\delta\right)$, then the monopoly equilibrium emerges, where firm A serves all consumers at x. In this equilibrium firm A charges the highest price at which it can gain all consumers:

$$p_A\left(x, p_B\left(x\right)\right) = p_B\left(x\right) + \underline{t}\left(1-2x\right). \tag{1.18}$$

It follows from (1.18) that $p_B^*\left(x\right) = 0$, because firm B would have an incentive to deviate downwards from any positive price. Hence,

$$p_A^{x*}\left(x\right) = \underline{t}\left(1-2x\right) \text{ and } p_B^{x*}\left(x\right) = 0. \tag{1.19}$$

For the prices (1.19) to constitute the equilibrium, none of the firms should have an incentive to deviate. Precisely, firm A should not have an incentive to increase its price: the derivative of (1.13) evaluated at $p_A^x = \underline{t}(1-2x)$ and $p_B^x = 0$ must be non-positive, which yields the condition $l \leq \bar{l}_1(\delta)$, which is the opposite of (1.17).

Part 2. Consider now $2 \leq l \leq 4$, in which case second-period profits are given by different functions depending on α.

i) Consider first $\alpha \leq (l-2)/[2(l-1)]$. The profits of the second period are then given by (1.8). Firm A chooses p_A^x to maximize the profits:

$$\begin{aligned} & (l - p_A^x + p_B^x)\, p_A^x + \tfrac{\delta}{9}\left[9\alpha(l-1) + (2l-1-\alpha(l-1))^2\right] \\ = \; & (l - p_A^x + p_B^x)\, p_A^x + \tfrac{\delta}{9}\left[9(p_A^x - p_B^x - 1) + (2l - p_A^x + p_B^x)^2\right]. \end{aligned} \tag{1.20}$$

Firm B chooses p_B^x to maximize the profits:

$$(p_A^x - p_B^x - 1)\, p_B^x + \tfrac{\delta[l-2-2\alpha(l-1)]^2}{9} = (p_A^x - p_B^x - 1)\, p_B^x + \tfrac{\delta\left(l-2p_A^x+2p_B^x\right)^2}{9}. \tag{1.21}$$

We first show that no monopoly equilibrium (where firm A serves all consumers at x) in the first period exists. Assume that there is such an equilibrium. In this equilibrium firm A will charge the price $p_A^x = p_B^x + 1$. Indeed, at a higher price firm A does not serve all consumers at x and at any lower price firm A realizes lower profits (first-period profits decrease, while second-period profits do not change). Firm B does not have an incentive to increase its price, because both the first-period and second-period profits do not change. However, one has to exclude the incentive of firm B to decrease its price. Similarly, one has to exclude the incentive of firm A to increase its price. Taking the derivatives of (1.20) and (1.21) with respect to p_A^x and p_B^x and evaluating them at $p_A^x = p_B^x + 1$, yields the following inequalities, respectively:

$$l + \tfrac{11}{9}\delta - \tfrac{4}{9}l\delta - 2 \leq p_B^x \text{ and } \tfrac{4}{9}l\delta - \tfrac{8}{9}\delta \geq p_B^x. \tag{1.22}$$

There exists p_B^x, which satisfies both inequalities in (1.22), only if

$$l \leq \tfrac{18-19\delta}{9-8\delta}. \tag{1.23}$$

Note that for any δ and any $l \geq 2$ (except for $\delta = 0$ and $l = 2$, which is covered by $l \leq \bar{l}_1(\delta)$ in *Part 1*), (1.23) does not hold, which proves that in the first period only the sharing equilibrium can exist.

In the sharing equilibrium first-order conditions have to be fulfilled. Solving them simultaneously, we arrive at the prices:

$$\begin{aligned} p_A^{x*} &= \tfrac{54l+60\delta-36l\delta-24\delta^2+8l\delta^2-27}{81-30\delta}, \\ p_B^{x*} &= \tfrac{27l+33\delta-12l\delta-24\delta^2+8l\delta^2-54}{81-30\delta}, \text{ which yield} \end{aligned} \tag{1.24}$$

$$\alpha\left(p_A^{x*}, p_B^{x*}\right) = \tfrac{9l+19\delta-8l\delta-18}{(l-1)(27-10\delta)}. \tag{1.25}$$

Note that second-order conditions are fulfilled, and for any δ and any $l \geq 2$ it holds that $\alpha\left(p_A^{x*}, p_B^{x*}\right) \geq 0$. Imposing $\alpha\left(p_A^{x*}, p_B^{x*}\right) \leq (l-2) / [2(l-1)]$, we arrive at

$$l \geq \bar{l}_2(\delta) := \tfrac{6(1+\delta)}{3+2\delta}.$$

Note that $\partial \bar{l}_2(\delta) / \partial \delta > 0$, $\bar{l}_2(0) = 2$ and $\bar{l}_2(1) = 2.4$. Note that $p_A^{x*} \geq 0$ stated in (1.24) requires

$$l \geq f_1(\delta) := \tfrac{24\delta^2-60\delta+27}{2(4\delta^2-18\delta+27)},$$

which is true for any δ and any l, because for any δ it holds that $f_1(\delta) < 1$. Finally, $p_B^{x*} \geq 0$ stated in (1.24) requires

$$l \geq f_2(\delta) := \tfrac{24\delta^2-33\delta+54}{8\delta^2-12\delta+27},$$

which is true for any δ and any $l \geq 2$, because for any δ it holds that $f_2(\delta) \leq 2$.

ii) Consider now $(l-2) / [2(l-1)] \leq \alpha < 1/(l-1)$. In this case the (adjusted) profits over two periods are given by (1.13) and (1.14), which yields the equilibrium prices (1.15) and the share of firm B in the first period:

$$\alpha\left(p_A^{x*}, p_B^{x*}\right) = \tfrac{p_A^{x*}-p_B^{x*}-1}{l-1} = \tfrac{l(1+\delta)-(2+\delta)}{(3+2\delta)(l-1)}.$$

The condition $\alpha\left(p_A^{x*}, p_B^{x*}\right) < 1/(l-1)$ requires $l < (5+3\delta)/(1+\delta)$, which is fulfilled for any δ, because $(5+3\delta)/(1+\delta) \geq 4$ for any δ. The condition $\alpha\left(p_A^{x*}, p_B^{x*}\right) \geq (l-2)/[2(l-1)]$ requires $l \leq \bar{l}_3(\delta) := 2(1+\delta)$. Note that $\partial \bar{l}_3(\delta)/\partial\delta > 0$, $\bar{l}_3(0) = 2$ and $\bar{l}_3(1) = 4$. Note finally that for any $2 \leq l \leq 4$ and any δ, it holds that $p_A^{x*} > 0$ and $p_B^{x*} \geq 0$.

iii) Consider finally $\alpha \geq 1/(l-1)$, in which case the profits of the second period are given by (1.9). Then firm A chooses p_A^x to maximize the profits:

$$\begin{aligned} & \left(l-p_A^x+p_B^x\right)p_A^x+\tfrac{\delta[2\alpha(l-1)+1]^2}{9}+\delta\left(1-\alpha\right)\left(l-1\right)\left[1+\alpha\left(l-1\right)\right] \quad (1.26) \\ = & \left(l-p_A^x+p_B^x\right)p_A^x+\tfrac{\delta\left(2p_A^x-2p_B^x-1\right)^2}{9}+\delta\left(l-p_A^x+p_B^x\right)\left(p_A^x-p_B^x\right). \end{aligned}$$

Firm B chooses p_B^x to maximize the profits:

$$\begin{aligned} & \left(p_A^x-p_B^x-1\right)p_B^x+\tfrac{\delta[\alpha(l-1)-1]^2}{9} \quad (1.27) \\ = & \left(p_A^x-p_B^x-1\right)p_B^x+\tfrac{\delta\left(p_A^x-p_B^x-2\right)^2}{9}. \end{aligned}$$

Solving simultaneously first-order conditions of the firms, we arrive at the prices:

$$\begin{aligned} p_A^{x*} &= -\tfrac{-54l+42\delta-48l\delta-16\delta^2+6l\delta^2+27}{24\delta+81}, \quad (1.28) \\ p_B^{x*} &= -\tfrac{-27l+18\delta-21l\delta-16\delta^2+6l\delta^2+54}{24\delta+81}, \end{aligned}$$

which yield the share of firm B in the first period:

$$\alpha\left(p_A^{x*},p_B^{x*}\right)=\tfrac{p_A^{x*}-p_B^{x*}-1}{l-1}=\tfrac{9l(1+\delta)-16\delta-18}{(27+8\delta)(l-1)}. \quad (1.29)$$

Note that for any $l \geq 2$ and δ, $p_A^{x*}>0$ and $p_B^{x*} \geq 0$ hold. The condition $\alpha(\cdot) \geq 1/(l-1)$ requires that

$$l \geq \bar{l}_4(\delta) := \tfrac{15+8\delta}{3(1+\delta)}.$$

Note that $\partial\bar{l}_4(\delta)/\partial\delta < 0$ and $\bar{l}_4(\delta) \leq 4$ if $\delta \geq 0.75$. The condition $\alpha(\cdot) \leq 1$ requires that

$$l \geq \tfrac{9-8\delta}{18-\delta}. \quad (1.30)$$

Since the the right-hand side of (1.30) is for any δ smaller than 1, then $\alpha(\cdot) \leq 1$ holds for any δ and any $l \geq 2$. Finally, it can be checked that second-order conditions are fulfilled.

Combining the results from the analysis of the cases *i)*, *ii)* and *iii)*, we conclude that depending on l and δ we have either one, two or three candidate equilibria. At the next step we have to find the equilibrium for any l and δ.

1) Consider first $\bar{l}_2(\delta) \leq l \leq \min\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\}$, which yield two candidate equilibria, (1.15) and (1.24).

1.a) Consider first the candidate equilibrium (1.24). It is straightforward that none of the firms has an incentive to deviate on $\alpha(\cdot) \leq (l-2)/[2(l-1)]$. We consider two other deviations by each of the firms.

Deviation on $(l-2)/[2(l-1)] \leq \alpha(\cdot) \leq 1/(l-1)$**.**

i) The incentives of firm A. If firm A deviates, then it realizes the profits (1.13). Maximizing (1.13) with respect to p_A^x and keeping p_B^x at p_B^{x*} given in (1.24), yields the deviation price:

$$p_A^{x,dev}\left(p_B^{x*}\right) = \frac{108l+6\delta+93l\delta+12\delta^2-48\delta^3-46l\delta^2+16l\delta^3-54}{-60\delta^2+102\delta+162}, \tag{1.31}$$

and the market share of firm B in the first period:

$$\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right), p_B^{x*}\right) = -\frac{54\delta-54l-63l\delta-54\delta^2+38l\delta^2+108}{6(l-1)(27-10\delta)(1+\delta)}.$$

Note that $p_A^{x,dev}\left(p_B^{x*}\right) > 0$ for any δ and any l. Imposing the requirement that

$$\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right), p_B^{x*}\right) \geq \frac{l-2}{2(l-1)}, \tag{1.32}$$

we arrive at the constraint:

$$l \leq \frac{48\delta-6\delta^2+54}{8\delta^2-12\delta+27}. \tag{1.33}$$

Note that for any δ it holds that

$$\frac{48\delta-6\delta^2+54}{-12\delta+8\delta^2+27} > \min\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\},$$

such that for any l with $\bar{l}_2(\delta) \leq l \leq \min\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\}$, (1.33) holds and, hence, (1.32) is fulfilled. Imposing the requirement

$$\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right), p_B^{x*}\right) \leq \frac{1}{l-1} \tag{1.34}$$

we arrive at the constraint:

$$l \leq \frac{156\delta-114\delta^2+270}{63\delta-38\delta^2+54}. \tag{1.35}$$

Note that for any δ it holds that

$$\frac{156\delta-114\delta^2+270}{63\delta-38\delta^2+54} > \min\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\},$$

such that for any l with $\bar{l}_2(\delta) \leq l \leq \min\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\}$, (1.35) holds and, hence, (1.34) is fulfilled. It follows that the price (1.31) is indeed the optimal deviation price of firm A on $(l-2)/[2(l-1)] \leq \alpha(\cdot) \leq 1/(l-1)$. The difference between firm A's equilibrium and deviation profits is

$$f_1(l,\delta) := \frac{\delta\left[l^2\left(-100\delta^3+372\delta^2+531\delta+2268\right)-l\left(-120\delta^3+276\delta^2+6552\delta+6156\right)-360\delta^3+2520\delta^2+6120\delta+3240\right]}{36(\delta+1)(10\delta-27)^2}.$$

The function $f_1(l,\delta)$ opens upwards for any δ and has two roots:

$$\begin{aligned} l_1(\delta) &:= \frac{-120\delta^3+276\delta^2+6552\delta+6156-36(27-10\delta)(1+\delta)\sqrt{(\delta+1)(9-\delta)}}{2(-100\delta^3+372\delta^2+531\delta+2268)} \text{ and} \\ l_2(\delta) &:= \frac{-120\delta^3+276\delta^2+6552\delta+6156+36(27-10\delta)(1+\delta)\sqrt{(\delta+1)(9-\delta)}}{2(-100\delta^3+372\delta^2+531\delta+2268)}. \end{aligned} \tag{1.36}$$

For any δ it holds: $l_1(\delta) < \bar{l}_2(\delta)$ and $\bar{l}_2(\delta) \leq l_2(\delta) \leq \min\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\}$. Hence, for any l with $\bar{l}_2(\delta) \leq l \leq \min\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\}$ we have that $f_1(l,\delta) \geq 0$ if $l \geq l_2(\delta)$, while $f_1(l,\delta) < 0$ if $l < l_2(\delta)$. In the former case firm A does not have an incentive to deviate from p_A^{x*} in (1.24) and does in the latter.

ii) The incentives of firm B. If firm B deviates, then its profits are given by (1.14). Maximizing (1.14) with respect to p_B^x and keeping p_A^x at p_A^{x*} in (1.24), yields the deviation price:

$$p_B^{x,dev}\left(p_A^{x*}\right) = \frac{27l+45\delta-18l\delta-12\delta^2+4l\delta^2-54}{81-30\delta},$$

and the market share of firm B in the first period:

$$\alpha\left(p_A^{x*}, p_B^{x,dev}\left(p_A^{x*}\right)\right) = \frac{27l+45\delta-18l\delta-12\delta^2+4l\delta^2-54}{(l-1)(81-30\delta)}.$$

Note that for any δ and any $l \geq 2$ it holds that $p_B^{x,dev}\left(p_A^{x*}\right) \geq 0$. The comparison shows that

$$\alpha\left(p_A^{x*}, p_B^{x,dev}\left(p_A^{x*}\right)\right) - \frac{l-2}{2(l-1)} = \frac{(9-4\delta)(6\delta-3l-2l\delta+6)}{6(27-10\delta)(l-1)}. \tag{1.37}$$

The right-hand side of (1.37) is non-negative if

$$l \leq \bar{l}_2(\delta) = \frac{6(1+\delta)}{3+2\delta}. \tag{1.38}$$

As we consider $l \geq \bar{l}_2(\delta)$, the optimal deviation price of firm B follows from $\alpha\left(p_A^{x*}, p_B^{x,dev}\left(p_A^{x*}\right)\right) = (l-2)/[2(l-1)]$. Then from the continuity of firm B's profits we conclude that it does not have an incentive to deviate.

Deviation on $\alpha(\cdot) \geq 1/(l-1)$.

i) The incentives of firm A. If firm A deviates, then its profit is given by (1.26). Keeping p_B^x at p_B^{x*} in (1.24) and taking the derivative of (1.26) with respect to p_A^x yields the deviation price of firm A:

$$p_A^{x,dev}\left(p_B^{x*}\right) = -\frac{567\delta - 972l - 621l\delta - 234\delta^2 + 240\delta^3 + 318l\delta^2 - 80l\delta^3 + 486}{-300\delta^2 + 270\delta + 1458}. \tag{1.39}$$

Note that for any δ and $l \geq 2$ it holds that $p_A^{x,dev}\left(p_B^{x*}\right) > 0$. The prices $p_A^{x,dev}\left(p_B^{x*}\right)$ and p_B^{x*} yield the market share of firm B in the first period:

$$\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right), p_B^{x*}\right) = \frac{297\delta - 162l - 189l\delta - 212\delta^2 + 114l\delta^2 + 324}{(100\delta^2 - 90\delta - 486)(l-1)}.$$

Note that for any δ and any $l \geq 2$ it holds that $\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right), p_B^{x*}\right) < 1$. The other comparison shows that

$$\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right), p_B^{x*}\right) - \frac{1}{l-1} = -\frac{3\left(129\delta - 54l - 63l\delta - 104\delta^2 + 38l\delta^2 + 270\right)}{2(l-1)(-50\delta^2 + 45\delta + 243)}. \tag{1.40}$$

The right-hand side of (1.40) is non-negative if

$$l \geq \bar{l}_5(\delta) := \frac{129\delta - 104\delta^2 + 270}{63\delta - 38\delta^2 + 54}. \tag{1.41}$$

Solving $\bar{l}_5(\delta) = \bar{l}_3(\delta)$, we get $\delta \approx 0.89$. If $\delta < 0.89$, then $\bar{l}_5(\delta) > \min\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\}$, such that (1.41) does not hold and $\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right), p_B^{x*}\right) < 1/(l-1)$.

Consider first $\delta < 0.89$ and $\delta \geq 0.89$ with $l \leq \bar{l}_5(\delta)$. The optimal deviation price of firm A follows from $\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right), p_B^{x*}\right) = 1/(l-1)$, which yields

$$p_A^{x,dev}\left(p_B^{x*}\right) = \frac{27\delta - 27l + 12l\delta + 24\delta^2 - 8l\delta^2 - 108}{30\delta - 81}.$$

Note that for any l and any δ, it holds that $p_A^{x,dev}\left(p_B^{x*}\right) > 0$. Then the difference between the equilibrium and deviation profits of firm A is

$$\frac{l^2\left(112\delta^3 - 480\delta^2 + 513\delta + 243\right) - l\left(712\delta^3 - 2874\delta^2 + 2322\delta + 2430\right) + 1053\delta^3 - 3807\delta^2 + 1215\delta + 6075}{3(10\delta - 27)^2}. \tag{1.42}$$

The numerator of (1.42) is a quadratic function with respect to l with a non-positive discriminant (for any δ), such that it does not have roots. As this function opens upwards (for any δ), it takes only positive values. Hence, firm A does not have an incentive to deviate if $\delta < 0.89$ or if $\delta \geq 0.89$ with $l \leq \bar{l}_5(\delta)$ hold.

Consider now $\delta \geq 0.89$ with $\bar{l}_5(\delta) < l < \min\{\bar{l}_3(\delta), \bar{l}_4(\delta)\}$, in which case (1.41) holds and (1.39) is the optimal deviation price of firm A. The difference between the equilibrium and deviation profits of firm A is

$$\frac{\delta\left[-l^2\left(2092\delta^3-8796\delta^2+6615\delta-2916\right)+l\left(9472\delta^3-36876\delta^2+10530\delta+11664\right)-11388\delta^3+42264\delta^2+5805\delta-43740\right]}{12(5\delta+9)(10\delta-27)^2}. \tag{1.43}$$

The function in the parentheses of (1.43) is quadratic in l, opens upwards for any δ and has two roots, one of which is negative for any δ, while the other root is smaller than $\bar{l}_5(\delta)$ for any δ. Hence, for any $\bar{l}_5(\delta) < l < \min\{\bar{l}_3(\delta), \bar{l}_4(\delta)\}$ and $\delta \geq 0.89$, the function in the parentheses in (1.43) is positive, such that firm A does not have an incentive to deviate.

ii) The incentives of firm B. Given the rival's price, p_A^{x*} in (1.24), there exists $p_B^{x,dev} \geq 0$, which yields the share of firm B in the first period satisfying $\alpha(\cdot) \geq 1/(l-1)$ only if

$$l \geq l_5(\delta) := \frac{-120\delta+24\delta^2+189}{-36\delta+8\delta^2+54}. \tag{1.44}$$

In the following we will restrict attention to l satisfying (1.44), because only in that case firm B can deviate. Keeping p_A^x at p_A^{x*} in (1.24) and taking the derivative of (1.27) with respect to p_B^x yields the deviation price of firm B:

$$p_B^{x,dev}\left(p_A^{x*}\right) = \frac{243l+594\delta-216l\delta-228\delta^2+24\delta^3+72l\delta^2-8l\delta^3-486}{30\delta^2-351\delta+729}.$$

Note that $p_B^{x,dev}\left(p_A^{x*}\right) > 0$ for any δ and $l > 2$. The prices $p_B^{x,dev}\left(p_A^{x*}\right)$ and p_A^{x*} yield the market share of firm B in the first period:

$$\alpha\left(p_A^{x*}, p_B^{x,dev}\left(p_A^{x*}\right)\right) = \frac{p_A^{x*}-p_B^{x,dev}\left(p_A^{x*}\right)-1}{l-1} = \frac{81l+108\delta-54l\delta-26\delta^2+12l\delta^2-162}{(l-1)(10\delta^2-117\delta+243)}. \tag{1.45}$$

Note that for any l and δ it holds that $\alpha\left(p_A^{x*}, p_B^{x,dev}\left(p_A^{x*}\right)\right) < 1$. The other comparison shows that

$$\alpha\left(p_A^{x*}, p_B^{x,dev}\left(p_A^{x*}\right)\right) - \frac{1}{l-1} = \frac{3\left(27l+75\delta-18l\delta-12\delta^2+4l\delta^2-135\right)}{(l-1)(10\delta^2-117\delta+243)}. \tag{1.46}$$

The right-hand side of (1.46) is non-negative if

$$l \geq \frac{135-75\delta+12\delta^2}{27-18\delta+4\delta^2}. \tag{1.47}$$

Note that the right-hand side of (1.47) is for any δ larger than 4, such that (1.47) does not hold and $\alpha\left(p_A^{x*}, p_B^{x,dev}\left(p_A^{x*}\right)\right) < 1/\left(l-1\right)$ stated in (1.45). Hence, the optimal deviation price of firm B is such that $\alpha\left(\cdot\right) = 1/\left(l-1\right)$, which yields

$$p_B^{x,dev}\left(p_A^{x*}\right) = \frac{54l+120\delta-36l\delta-24\delta^2+8l\delta^2-189}{81-30\delta}. \tag{1.48}$$

Note that for any $l > l_5\left(\delta\right)$, $p_B^{x,dev}\left(p_A^{x*}\right) > 0$. Then the optimal deviation price of firm B is given by (1.48) and the difference between the equilibrium and deviation profits is

$$-\frac{l^2\left(52\delta^3-204\delta^2+297\delta-243\right)-l\left(352\delta^3-1608\delta^2+2862\delta-2430\right)+588\delta^3-3123\delta^2+6642\delta-6075}{3(10\delta-27)^2}. \tag{1.49}$$

Note that the numerator of (1.49) is a quadratic function with respect to l, which opens downwards (for any δ). As its discriminant is non-positive (for any δ), for any δ and l it takes only negative values, such that the whole term in (1.49) is positive for any δ and l and firm B does not have an incentive to deviate.

Conclusion from 1.a). We conclude that on $\bar{l}_2\left(\delta\right) \leq l \leq \min\left\{\bar{l}_3\left(\delta\right), \bar{l}_4\left(\delta\right)\right\}$ the equilibrium (1.24) exists if $l \geq l_2\left(\delta\right)$, with

$$l_2\left(\delta\right) = \frac{-120\delta^3+276\delta^2+6552\delta+6156+36(27-10\delta)(1+\delta)\sqrt{(\delta+1)(9-\delta)}}{2(-100\delta^3+372\delta^2+531\delta+2268)}.$$

1.b) Consider now the equilibrium (1.15).

Deviation on $\alpha \leq \left(l-2\right)/\left[2\left(l-1\right)\right]$.

i) The incentives of firm A. If firm A deviates, then its profits are given by (1.20). Maximizing (1.20) with respect to p_A^x and keeping p_B^x at p_B^{x*} in (1.15) yields the deviation price of firm A:

$$p_A^{x,dev}\left(p_B^{x*}\right) = \frac{36l+22\delta+13l\delta+20\delta^2-10l\delta^2-18}{-4\delta^2+30\delta+54}, \tag{1.50}$$

such that the market share of firm B in the first period is

$$\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right), p_B^{x*}\right) = \frac{p_A^{x,dev}\left(p_B^{x*}\right)-p_B^{x*}-1}{l-1} = \frac{18l+6\delta-3l\delta+22\delta^2-8l\delta^2-36}{(l-1)(54-4\delta^2+30\delta)}.$$

Note that for any l and any δ, $p_A^{x,dev}\left(p_B^{x*}\right)$ in (1.50) is positive. The comparison shows that

$$\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right),p_B^{x*}\right)-\tfrac{l-2}{2(l-1)}=\tfrac{3\left(12\delta-3l-6l\delta+6\delta^2-2l\delta^2+6\right)}{2(l-1)\left(-2\delta^2+15\delta+27\right)}. \tag{1.51}$$

Note that for any $l\geq\bar{l}_2(\delta)$ it holds that

$$l>\tfrac{12\delta+6\delta^2+6}{6\delta+2\delta^2+3}, \tag{1.52}$$

such that the right-hand side of (1.51) is negative.
Note next that $\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right),p_B^{x*}\right)\geq 0$ if

$$l\geq\tfrac{36-6\delta-22\delta^2}{18-3\delta-8\delta^2}. \tag{1.53}$$

As any $l\geq 2$ also satisfies (1.53), $\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right),p_B^{x*}\right)\geq 0$ holds. We conclude that (1.50) is the optimal deviation price of firm A.

Given (1.50), the difference between the equilibrium and the deviation profits of firm A is

$$\begin{aligned}
& -\tfrac{\delta f_2(l,\delta)}{4(9-\delta)(2\delta+3)^2},\text{ with}\\
f_2(l,\delta)\ :\ & =l^2\left(4\delta^3+24\delta^2+49\delta+28\right)-l\left(24\delta^3+124\delta^2+176\delta+76\right)\\
& +40\delta^3+120\delta^2+120\delta+40.
\end{aligned}$$

The function $f_2(l,\delta)$ opens upwards (for any δ) and has two roots:

$$\begin{aligned}
l_3(\delta)\ :\ & =\tfrac{24\delta^3+124\delta^2+176\delta+76-4(\delta+1)(2\delta+3)\sqrt{(\delta+1)(9-\delta)}}{2(4\delta^3+24\delta^2+49\delta+28)},\\
l_4(\delta)\ :\ & =\tfrac{24\delta^3+124\delta^2+176\delta+76+4(\delta+1)(2\delta+3)\sqrt{(\delta+1)(9-\delta)}}{2(4\delta^3+24\delta^2+49\delta+28)}.
\end{aligned} \tag{1.54}$$

For any δ it holds that $l_3(\delta)<2$ and $\bar{l}_2(\delta)\leq l_4(\delta)\leq\min\left\{\bar{l}_3(\delta),\bar{l}_4(\delta)\right\}$. It then follows that for any l such that $\bar{l}_2(\delta)\leq l\leq\min\left\{\bar{l}_3(\delta),\bar{l}_4(\delta)\right\}$, firm A does not have an incentive to deviate if $l\leq l_4(\delta)$ and deviates otherwise.

ii) The incentives of firm B. If firm B deviates, then its profits are given by (1.21). Maximizing (1.21) with respect to p_B^x and keeping p_A^x at p_A^{x*} in (1.15), yields the deviation price of firm B:

$$p_B^{x,dev}\left(p_A^{x*}\right)=-\tfrac{5\delta-9l-7l\delta+4l\delta^2+18}{-8\delta^2+6\delta+27}.$$

Note that $p_B^{x,dev}\left(p_A^{x*}\right) \geq 0$ for any δ and any $l \geq 2$. This deviation price yields the market share of firm B in the first period:

$$\alpha\left(p_A^{x*}, p_B^{x,dev}\left(p_A^{x*}\right)\right) = \frac{p_A^{x*} - p_B^{x,dev}\left(p_A^{x*}\right) - 1}{l-1} = \frac{9l + 3\delta + 3l\delta + 8\delta^2 - 4l\delta^2 - 18}{(l-1)(-8\delta^2 + 6\delta + 27)}.$$

The comparison shows that

$$\alpha\left(p_A^{x*}, p_B^{x,dev}\left(p_A^{x*}\right)\right) - \frac{l-2}{2(l-1)} = \frac{9(2\delta - l + 2)}{2(l-1)(-8\delta^2 + 6\delta + 27)}. \tag{1.55}$$

The right-hand side of (1.55) is non-positive if $l \geq \bar{l}_3(\delta) = 2(1+\delta)$. Hence, for any $\bar{l}_2(\delta) \leq l \leq \min\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\}$, the optimal deviation price of firm B follows from $\alpha(\cdot) = (l-2) / [2(l-1)]$. Then from the continuity of firm B's profits we conclude that for any l and δ firm B does not have an incentive to deviate.

Deviation on $\alpha \geq 1/(l-1)$.

i) The incentives of firm A. If firm A deviates, it realizes the profit (1.26). Keeping p_B^x at p_B^{x*} in (1.15) and taking the derivative of (1.26) with respect to p_A^x yields the deviation price of firm A:

$$p_A^{x,dev}\left(p_B^{x*}\right) = -\frac{41\delta - 36l - 64l\delta + 18\delta^2 - 28l\delta^2 + 18}{20\delta^2 + 66\delta + 54}, \tag{1.56}$$

which is non-negative if

$$l \geq \frac{41\delta + 18\delta^2 + 18}{64\delta + 28\delta^2 + 36}. \tag{1.57}$$

Note that for any δ the right-hand side of (1.57) is smaller than 1, such that (1.57) is fulfilled as strict inequality for any δ and l. The price (1.56) yields the market share of firm B in the first period:

$$\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right), p_B^{x*}\right) = \frac{p_A^{x,dev}\left(p_B^{x*}\right) - p_B^{x*} - 1}{l-1} = -\frac{69\delta - 18l - 36l\delta + 28\delta^2 - 18l\delta^2 + 36}{(l-1)(20\delta^2 + 66\delta + 54)}. \tag{1.58}$$

The comparison shows that

$$\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right), p_B^{x*}\right) - \frac{1}{l-1} = -\frac{3\left(45\delta - 6l - 12l\delta + 16\delta^2 - 6l\delta^2 + 30\right)}{2(l-1)(10\delta^2 + 33\delta + 27)}. \tag{1.59}$$

The right-hand side of (1.59) is non-negative if

$$l \geq \frac{45\delta + 16\delta^2 + 30}{12\delta + 6\delta^2 + 6}. \tag{1.60}$$

We showed above that on $\bar{l}_2(\delta) \leq l \leq \min\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\}$, the equilibrium (1.15) exists if $l \leq l_4(\delta)$, the latter being stated in (1.54). The comparison shows that for any δ, the right-hand side of (1.60) is larger than $l_4(\delta)$, such that (1.60) is not fulfilled for any δ and the right-hand side of (1.59) is negative. Hence, the optimal deviation price of firm A follows from $\alpha\left(p_A^{x,dev}(p_B^{x*}), p_B^{x*}\right) = 1/(l-1)$. Then from the continuity of firm A's profits, we conclude that firm A does not have an incentive to deviate.

ii) The incentives of firm B. Keeping p_A^x at p_A^{x*} in (1.15) and requiring that $\alpha \geq 1/(l-1)$ yields the restriction on firm B's deviation price:

$$p_B^{x,dev} \leq -\tfrac{4\delta-2l-2l\delta+7}{2\delta+3}. \tag{1.61}$$

There exists $p_B^{x,dev} \geq 0$, which satisfies (1.61) if $l \geq (4\delta+7)/(2\delta+2)$. We showed above that on $\bar{l}_2(\delta) \leq l \leq \min\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\}$, the equilibrium (1.15) exists if $l \leq l_4(\delta)$. Note that for any δ it holds that $(4\delta+7)/(2\delta+2) > l_4(\delta)$, which implies that firm B cannot deviate.

Conclusion from 1.b). We conclude that on $\bar{l}_2(\delta) \leq l \leq \min\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\}$, the equilibrium (1.15) exists if $l \leq l_4(\delta)$, with

$$l_4(\delta) = \tfrac{24\delta^3+124\delta^2+176\delta+76+4(\delta+1)(2\delta+3)\sqrt{(\delta+1)(9-\delta)}}{2(4\delta^3+24\delta^2+49\delta+28)}.$$

2) Consider now $2 \leq l \leq \bar{l}_2(\delta)$, which yields the unique candidate equilibrium (1.15).

Deviation on $\alpha \leq (l-2)/[2(l-1)]$.

i) The incentives of firm A. Refer to the analysis in *1.b).* For any δ it holds that

$$2 \leq \tfrac{12\delta+6\delta^2+6}{6\delta+2\delta^2+3} \leq \bar{l}_2(\delta).$$

For any

$$\tfrac{12\delta+6\delta^2+6}{6\delta+2\delta^2+3} \leq l \leq \bar{l}_2(\delta)$$

the analysis in *1.b)* applies, such that firm A does not have an incentive to deviate. Consider now

$$2 \leq l \leq \tfrac{12\delta+6\delta^2+6}{6\delta+2\delta^2+3},$$

in which case the optimal deviation price of firm A follows from

$$\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right), p_B^{x*}\right) = \tfrac{l-2}{2(l-1)}.$$

Then from the continuity of firm A's profits, we conclude that firm A does not have an incentive to deviate either.

ii) The incentives of firm B. The analysis in *1.b)* allows us to conclude that firm B does not have an incentive to deviate.

Deviation on $\alpha \geq 1/(l-1)$. The analysis in *1.b)* allows us to conclude that neither firm A, nor firm B have an incentive to deviate.

Conclusion from 2). We conclude that on $2 \leq l \leq \bar{l}_2(\delta)$ the unique equilibrium is (1.15).

3) Consider now $\bar{l}_3(\delta) \leq l \leq \min\left\{4, \bar{l}_4(\delta)\right\}$, which yields the unique candidate equilibrium (1.24).

Deviation on $(l-2)/[2(l-1)] \leq \alpha(\cdot) \leq 1/(l-1)$.

i) The incentives of firm A. We introduce a new notation:

$$l_6(\delta) := \tfrac{48\delta - 6\delta^2 + 54}{8\delta^2 - 12\delta + 27}.$$

Consider first $\bar{l}_3(\delta) \leq l \leq \min\left\{l_6(\delta), \bar{l}_4(\delta)\right\}$. From the analysis in *1.a)* it follows that firm A does not have an incentive to deviate. Consider now $l_6(\delta) \leq l \leq \min\left\{\bar{l}_4(\delta), 4\right\}$. From the analysis in *1.a)* we conclude that the optimal deviation price of firm A follows from $(l-2)/[2(l-1)] = \alpha\left(p_A^{x,dev}\left(p_B^{x*}\right), p_B^{x*}\right)$. As firm A's profits are continuous, we conclude that firm A does not have an incentive to deviate.

ii) The incentives of firm B. From the analysis in *1.a)* we conclude that firm B does not have an incentive to deviate.

Deviation on $\alpha(\cdot) \geq 1/(l-1)$.

i) The incentives of firm A. We first consider $\bar{l}_3(\delta) \leq l \leq \min\left\{\bar{l}_5(\delta), 4\right\}$, with $\bar{l}_5(\delta)$ being defined in (1.41). From the analysis in *1.a)* it follows that firm A does not have an incentive to deviate. Consider now $\bar{l}_5(\delta) \leq l \leq \min\left\{\bar{l}_4(\delta), 4\right\}$. From the analysis in *1.a)* it follows that firm A does not have an incentive to deviate either.

ii) The incentives of firm B. From the analysis in *1.a)* it follows that firm B does not have an incentive to deviate.

Conclusion from 3). We conclude that on $\bar{l}_3(\delta) \leq l \leq \min\left\{4, \bar{l}_4(\delta)\right\}$ the unique equilibrium is (1.24).

4) Consider now $\max\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\} \leq l \leq 4$, which yields two candidate equilibria, (1.24) and (1.28).

4.a) Consider first the candidate equilibrium (1.24).

Deviation on $\alpha(\cdot) \geq 1/(l-1)$.

i) The incentives of firm A. If firm A deviates, its profit is given by (1.26). Keeping p_B^x at p_B^{x*} in (1.24) and taking the derivative of (1.26) with respect to p_A^x yields the deviation price of firm A:

$$p_A^{x,dev}\left(p_B^{x*}\right) = -\frac{-l\left(80\delta^3-318\delta^2+621\delta+972\right)+240\delta^3-234\delta^2+567\delta+486}{270\delta-300\delta^2+1458}, \tag{1.62}$$

which is non-negative if

$$l \geq \frac{567\delta-234\delta^2+240\delta^3+486}{621\delta-318\delta^2+80\delta^3+972}. \tag{1.63}$$

Since the right-hand side of (1.63) is smaller than 1 for any δ, then (1.63) holds for any l. Using p_B^{x*} in (1.24) and $p_A^{x,dev}\left(p_B^{x*}\right)$ in (1.62) we calculate the market share of firm B in the first period:

$$\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right), p_B^{x*}\right) = -\frac{297\delta-162l-189l\delta-212\delta^2+114l\delta^2+324}{(l-1)(486-100\delta^2+90\delta)}.$$

The comparison shows that $\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right), p_B^{x*}\right) \geq 1/(l-1)$ if

$$l \geq \frac{129\delta-104\delta^2+270}{63\delta-38\delta^2+54},$$

which is fulfilled for any δ and any $\max\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\} \leq l \leq 4$. The other comparison shows that $\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right), p_B^{x*}\right) \leq 1$ if

$$l \geq \frac{-207\delta+112\delta^2+162}{-99\delta+14\delta^2+324}. \tag{1.64}$$

Since the right-hand side of (1.64) is smaller than 1 for any δ, then (1.64) holds for any l and any δ. Hence, (1.62) is the optimal deviation price of firm A, which yields the following difference between the equilibrium and the deviation profits:

$$\frac{\delta\left[-l^2\left(2092\delta^3-8796\delta^2+6615\delta-2916\right)+l\left(9472\delta^3-36876\delta^2+10530\delta+11664\right)-11388\delta^3+42264\delta^2+5805\delta-43740\right]}{12(5\delta+9)(10\delta-27)^2}. \tag{1.65}$$

The function in the brackets in (1.65) is quadratic in l. It opens upwards (for any δ) and has two roots:

$$\frac{-\left(9472\delta^3-36\,876\delta^2+10\,530\delta+11\,664\right)+4(27-10\delta)\sqrt{-(3485\delta^4-16\,032\delta^3+801\delta^2+42\,930\delta-55\,404)}}{-2(2092\delta^3-8796\delta^2+6615\delta-2916)},$$
$$\frac{-\left(9472\delta^3-36\,876\delta^2+10\,530\delta+11\,664\right)-4(27-10\delta)\sqrt{-(3485\delta^4-16\,032\delta^3+801\delta^2+42\,930\delta-55\,404)}}{-2(2092\delta^3-8796\delta^2+6615\delta-2916)}.$$

Both of these roots are smaller than $\max\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\}$ for any δ. Hence, for any δ and any $\max\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\} \leq l \leq 4$, the expression in (1.65) is non-negative, such that firm A does not have an incentive to deviate.

ii) The incentives of firm B. If firm B deviates, then its profit is given by (1.27). Keeping p_A^x at p_A^{x*} in (1.24) and taking the derivative of (1.27) with respect to p_B^x yields the deviation price of firm B:

$$p_B^{x,dev}\left(p_A^{x*}\right) = \frac{243l+594\delta-216l\delta-228\delta^2+24\delta^3+72l\delta^2-8l\delta^3-486}{30\delta^2-351\delta+729}, \tag{1.66}$$

which is non-negative for any

$$l \geq \frac{594\delta-228\delta^2+24\delta^3-486}{216\delta-72\delta^2+8\delta^3-243}. \tag{1.67}$$

Since the right-hand side of (1.67) is for any δ not lager than 2, then (1.67) holds for any δ and any $l \geq 2$. Using $p_B^{x,dev}\left(p_A^{x*}\right)$ in (1.66), we can calculate the market share of firm B in the first period:

$$\alpha\left(p_A^{x*}, p_B^{x,dev}\left(p_A^{x*}\right)\right) = \frac{81l+108\delta-54l\delta-26\delta^2+12l\delta^2-162}{(l-1)(10\delta^2-117\delta+243)}.$$

The comparison shows that

$$\alpha\left(p_A^{x*}, p_B^{x,dev}\left(p_A^{x*}\right)\right) - \frac{1}{l-1} = \frac{3\left(27l+75\delta-18l\delta-12\delta^2+4l\delta^2-135\right)}{(l-1)(10\delta^2-117\delta+243)}. \tag{1.68}$$

The right-hand side of (1.68) is non-negative if

$$l \geq \frac{-75\delta+12\delta^2+135}{-18\delta+4\delta^2+27}. \tag{1.69}$$

Since the right-hand side of (1.69) is not smaller than 5 for any δ, (1.69) does not hold for any δ and any $\max\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\} \leq l \leq 4$. Hence, the optimal deviation price of firm B follows from

$$\alpha\left(p_A^{x*}, p_B^{x,dev}\left(p_A^{x*}\right)\right) = \tfrac{1}{l-1}$$

and is given by

$$p_B^{x,dev}\left(p_A^{x*}\right) = \tfrac{54l+120\delta-36l\delta-24\delta^2+8l\delta^2-189}{81-30\delta}, \tag{1.70}$$

which is non-negative if

$$l \geq \tfrac{-120\delta+24\delta^2+189}{-36\delta+8\delta^2+54}. \tag{1.71}$$

The right-hand side of (1.71) is for any δ smaller than $\max\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\}$, such that (1.71) holds for any δ and any $\max\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\} \leq l \leq 4$. Hence, $p_B^{x,dev}\left(p_A^{x*}\right)$ in (1.70) is the optimal deviation price of firm B. Using the latter we calculate the difference between the equilibrium and the deviation profits of firm B:

$$-\tfrac{l^2\left(52\delta^3-204\delta^2+297\delta-243\right)-l\left(352\delta^3-1608\delta^2+2862\delta-2430\right)+588\delta^3-3123\delta^2+6642\delta-6075}{3(10\delta-27)^2}. \tag{1.72}$$

The numerator of (1.72) is a quadratic function of l, which opens downwards and has a non-positive discriminant (for any δ). It follows that for any δ and any l the expression in (1.72) is non-negative, such that firm B does not have an incentive to deviate.

Deviation on $(l-2)/\left[2(l-1)\right] \leq \alpha(\cdot) \leq 1/(l-1)$.

i) The incentives of firm A. If firm A deviates, then its profit is given by (1.13). Keeping p_B^x at p_B^{x*} in (1.24) and taking the derivative of (1.13) with respect to p_A^x yields the deviation price of firm A:

$$p_A^{x,dev}\left(p_B^{x*}\right) = \tfrac{108l+6\delta+93l\delta+12\delta^2-48\delta^3-46l\delta^2+16l\delta^3-54}{-60\delta^2+102\delta+162}, \tag{1.73}$$

which is non-negative if

$$l \geq -\tfrac{6\delta+12\delta^2-48\delta^3-54}{93\delta-46\delta^2+16\delta^3+108}. \tag{1.74}$$

As the right-hand side of (1.74) is for any δ smaller than 1, (1.74) holds for any δ and any l. Using (1.73), we calculate the market share of firm B in the first period:

$$\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right), p_B^{x*}\right) = -\tfrac{54\delta-54l-63l\delta-54\delta^2+38l\delta^2+108}{(l-1)(162-60\delta^2+102\delta)}.$$

The comparison shows that

$$\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right),p_B^{x*}\right)-\tfrac{1}{l-1}=-\tfrac{\left(156\delta-54l-63l\delta-114\delta^2+38l\delta^2+270\right)}{6(l-1)(-10\delta^2+17\delta+27)}. \tag{1.75}$$

The right-hand side of (1.75) is non-positive if

$$l\leq l_7\left(\delta\right):=\tfrac{156\delta-114\delta^2+270}{63\delta-38\delta^2+54}. \tag{1.76}$$

The other comparison shows that

$$\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right),p_B^{x*}\right)-\tfrac{l-2}{2(l-1)}=\tfrac{48\delta-27l+12l\delta-6\delta^2-8l\delta^2+54}{6(l-1)(-10\delta^2+17\delta+27)},$$

which is non-negative if

$$l\leq l_8\left(\delta\right):=\tfrac{48\delta-6\delta^2+54}{-12\delta+8\delta^2+27}. \tag{1.77}$$

Note that if $\delta=\left(24-3\sqrt{7}\right)/19\approx0.85$, then $l_7\left(\delta\right)=l_8\left(\delta\right)=4$. Consider $\max\left\{l_8\left(\delta\right),\bar{l}_4\left(\delta\right)\right\}\leq l\leq4$, in which case (1.77) does not hold and $p_A^{x,dev}\left(p_B^{x*}\right)$ follows from $\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right),p_B^{x*}\right)=\left(l-2\right)/\left[2(l-1)\right]$. As firm A's profit is continuous, we conclude that firm A does not have an incentive to deviate. Consider now $\max\left\{l_7\left(\delta\right),\bar{l}_3\left(\delta\right)\right\}\leq l\leq4$, in which case (1.76) does not hold and $p_A^{x,dev}\left(p_B^{x*}\right)$ follows from $\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right),p_B^{x*}\right)=1/(l-1)$. As firm A's profit is continuous, we conclude that firm A does not have an incentive to deviate. Consider finally $\max\left\{\bar{l}_3\left(\delta\right),\bar{l}_4\left(\delta\right)\right\}\leq l\leq\min\left\{l_7\left(\delta\right),l_8\left(\delta\right)\right\}$, in which case both (1.76) and (1.77) are fulfilled and the optimal deviation price of firm A is given by (1.73). Using the latter price we calculate the difference between the equilibrium and the deviation profits of firm A:

$$\tfrac{\delta\left[l^2\left(-100\delta^3+372\delta^2+531\delta+2268\right)-l\left(-120\delta^3+276\delta^2+6552\delta+6156\right)-360\delta^3+2520\delta^2+6120\delta+3240\right]}{36(\delta+1)(10\delta-27)^2}. \tag{1.78}$$

For any δ, the function in the brackets in (1.78) is quadratic in l, looks upwards and has two roots both of which are smaller than $\max\left\{\bar{l}_3\left(\delta\right),\bar{l}_4\left(\delta\right)\right\}$. Hence, the expression in (1.78) is non-negative for any δ and any $\max\left\{\bar{l}_3\left(\delta\right),\bar{l}_4\left(\delta\right)\right\}\leq l\leq\min\left\{l_7\left(\delta\right),l_8\left(\delta\right)\right\}$, such that firm A does not have an incentive to deviate.

i) The incentives of firm B. If firm B deviates, then its profits are given by (1.14). Keeping p_A^x at p_A^{x*} in (1.24) and taking the derivative of (1.14) with respect to p_B^x yields the deviation price of firm B:

$$p_B^{x,dev}\left(p_A^{x*}\right) = \frac{27l+45\delta-18l\delta-12\delta^2+4l\delta^2-54}{81-30\delta}, \tag{1.79}$$

which is non-negative if

$$l \geq \frac{-45\delta+12\delta^2+54}{-18\delta+4\delta^2+27}. \tag{1.80}$$

Since the right-hand side of (1.80) is for any δ not larger than 2, then (1.80) holds for any δ and any $l \geq 2$. Using (1.79) we can calculate the equilibrium share of firm B in the first period:

$$\alpha\left(p_A^{x*}, p_B^{x,dev}\left(p_A^{x*}\right)\right) = \frac{27l+45\delta-18l\delta-12\delta^2+4l\delta^2-54}{(l-1)(81-30\delta)}.$$

The comparison shows that

$$\alpha\left(p_A^{x*}, p_B^{x,dev}\left(p_A^{x*}\right)\right) - \frac{l-2}{2(l-1)} = \frac{(9-4\delta)(6\delta-3l-2l\delta+6)}{6(27-10\delta)(l-1)}. \tag{1.81}$$

The right-hand side of (1.81) is non-negative if

$$l \leq \bar{l}_2\left(\delta\right) = \frac{6(1+\delta)}{2\delta+3}. \tag{1.82}$$

Note that for any δ and any $\max\left\{\bar{l}_3\left(\delta\right), \bar{l}_4\left(\delta\right)\right\} \leq l \leq 4$, (1.82) does not hold, such that the optimal deviation price of firm B follows from $\alpha\left(p_A^{x*}, p_B^{x,dev}\left(p_A^{x*}\right)\right) = (l-2)/[2(l-1)]$. As firm B's profits are continuous, we conclude that it does not have an incentive to deviate.

Conclusion from 4.a) We conclude that for any δ and any $\max\left\{\bar{l}_3\left(\delta\right), \bar{l}_4\left(\delta\right)\right\} \leq l \leq 4$ there exists the equilibrium (1.24).

4.b) Consider now the candidate equilibrium (1.28).

Deviation on $\alpha\left(\cdot\right) \leq (l-2)/[2(l-1)]$.

i) The incentives of firm A. If firm A deviates, its profit is given by (1.20). Keeping p_B^x at p_B^{x*} in (1.28) and taking the derivative of (1.20) with respect to p_A^x yields the deviation price of firm A:

$$p_A^{x,dev}\left(p_B^{x*}\right) = \frac{l\left(12\delta^3-192\delta^2+27\delta+972\right)-32\delta^3+396\delta^2+675\delta-486}{-48\delta^2+270\delta+1458}. \tag{1.83}$$

The price $p_A^{x,dev}\left(p_B^{x*}\right)$ is non-negative if

$$l \geq -\frac{675\delta+396\delta^2-32\delta^3-486}{27\delta-192\delta^2+12\delta^3+972}. \tag{1.84}$$

The right-hand side of (1.84) is for any δ smaller than 1, such that (1.84) holds for any δ and any l and $p_A^{x,dev}\left(p_B^{x*}\right) \geq 0$. Using $p_A^{x,dev}\left(p_B^{x*}\right)$ in (1.83) we can calculate the market share of firm B in the first period:

$$\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right), p_B^{x*}\right) = -\frac{-l\left(14\delta^2+99\delta-162\right)+40\delta^2+207\delta-324}{-(l-1)(-16\delta^2+90\delta+486)}. \tag{1.85}$$

The numerator of (1.85) is non-negative if

$$l \geq \frac{207\delta+40\delta^2-324}{99\delta+14\delta^2-162}. \tag{1.86}$$

The right-hand side of (1.86) is for any δ smaller or equal to 2, such that (1.86) is fulfilled for any δ and any $l \geq 2$. The denominator of (1.86) is negative for any δ and any l. It then follows that for any δ and any $l \geq 2$, $\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right), p_B^{x*}\right) \geq 0$ holds.

The comparison shows that

$$\alpha\left(p_A^{x,dev}\left(p_B^{x*}\right), p_B^{x*}\right) - \frac{l-2}{2(l-1)} = \frac{3\left[-l\left(2\delta^2+48\delta+27\right)+8\delta^2+99\delta+54\right]}{2(l-1)(-8\delta^2+45\delta+243)}. \tag{1.87}$$

The right-hand side of (1.87) is non-positive if

$$l \geq \frac{99\delta+8\delta^2+54}{48\delta+2\delta^2+27}. \tag{1.88}$$

The right-hand side of (1.88) is for any δ smaller than $\max\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\}$. Hence, for any δ and any $\max\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\} \leq l \leq 4$, (1.88) holds. We conclude that $p_A^{x,dev}\left(p_B^{x*}\right)$ in (1.83) is the optimal deviation price of firm A. Using the latter price we can calculate the difference between the equilibrium and the deviation profits of firm A:

$$\frac{\delta\left[-264l^2\delta^3+1116l^2\delta^2-2619l^2\delta-2916l^2+1184l\delta^3-2676l\delta^2+2754l\delta-11\,664l-1472\delta^3+1200\delta^2+9585\delta+43\,740\right]}{12(9-\delta)(8\delta+27)^2}. \tag{1.89}$$

The expression in the brackets in the numerator of (1.89) is a quadratic function with respect to l, which opens downwards (for any δ) and has two roots, one of which is negative for any δ and the other one is smaller than $\max\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\}$ for any δ:

$$\frac{-\left(1184\delta^3-2676\delta^2+2754\delta-11\,664\right)+4(8\delta+27)\sqrt{-149\delta^4+2472\delta^3-12\,033\delta^2+10\,530\delta+55\,404}}{-2(264\delta^3-1116\delta^2+2619\delta+2916)}, \quad (1.90)$$
$$\frac{-\left(1184\delta^3-2676\delta^2+2754\delta-11\,664\right)-4(8\delta+27)\sqrt{-149\delta^4+2472\delta^3-12\,033\delta^2+10\,530\delta+55\,404}}{-2(264\delta^3-1116\delta^2+2619\delta+2916)}.$$

It then follows that for any δ and any $\max\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\} \leq l \leq 4$ the expression in the brackets in (1.89) is negative, which implies that firm A has an incentive to deviate.

Conclusion from 4.b) We conclude that for any δ and any $\max\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\} \leq l \leq 4$ there does not exist the equilibrium (1.28).

Conclusion from 4). We conclude that for any δ and any $\max\left\{\bar{l}_3(\delta), \bar{l}_4(\delta)\right\} \leq l \leq 4$ there exists the unique equilibrium (1.24).

5) Consider finally $\bar{l}_4(\delta) \leq l \leq \bar{l}_3(\delta)$, which yields three candidate equilibria, (1.15), (1.24) and (1.28).

5.a) Consider first the candidate equilibrium (1.28).

Deviation on $\alpha(\cdot) \leq (l-2)/[2(l-1)]$.

i) The incentives of firm A. Note that the right-hand side of (1.88) is for any δ smaller than $\bar{l}_4(\delta)$. Note also that the second expression in (1.90) is for any δ smaller than $\bar{l}_4(\delta)$. Then similar to the analysis in *4.b)* we conclude that for any δ and any $\bar{l}_4(\delta) \leq l \leq \bar{l}_3(\delta)$, firm A has an incentive to deviate.

Conclusion from 5.a). We conclude that for any δ and any $\bar{l}_4(\delta) \leq l \leq \bar{l}_3(\delta)$, the equilibrium (1.28) does not exist.

5.b) We consider now the candidate equilibrium (1.15).

Deviation on $\alpha \leq (l-2)/[2(l-1)]$.

i) The incentives of firm A. From the analysis in *1.b)* it follows that for any δ and any $\bar{l}_4(\delta) \leq l \leq \bar{l}_3(\delta)$, firm A has an incentive to deviate.

Conclusion from 5.b). We conclude that for any δ and any $\bar{l}_4(\delta) \leq l \leq \bar{l}_3(\delta)$, the equilibrium (1.15) does not exist.

5.c) We consider finally the candidate equilibrium (1.24).

i) The incentives of firm B. We can use the results of the analysis in *1.a)* to conclude that firm B does not have an incentive to deviate on $(l-2)/[2(l-1)] \leq \alpha(\cdot) \leq 1/(l-1)$. Note also that for any δ it holds that $l_5(\delta) < \bar{l}_4(\delta)$, with $l_5(\delta)$

being defined in (1.44). Then we can use the results of the analysis in *1.a)* to conclude that firm B does not have an incentive to deviate on $\alpha(\cdot) \geq 1/(l-1)$ either.

ii) The incentives of firm A.

Deviation on $\alpha(\cdot) \geq 1/(l-1)$**.** Note that for any δ it holds that $\bar{l}_5(\delta) \leq \bar{l}_4(\delta)$, with $\bar{l}_5(\delta)$ being defined in (1.41). Based on this result we conclude from the analysis in *1.a)* that for any δ and any $\bar{l}_4(\delta) \leq l \leq \bar{l}_3(\delta)$, firm A does not have an incentive to deviate.

Deviation on $(l-2)/[2(l-1)] \leq \alpha(\cdot) \leq 1/(l-1)$**.** We first note that the right-hand side of (1.33) is for any δ not smaller than $\bar{l}_3(\delta)$. Consider first δ and l, for which (1.35) holds. We can then use the results of the analysis in *1.a)* to conclude that for any δ and any $\bar{l}_4(\delta) \leq l \leq \bar{l}_3(\delta)$, firm A does not have an incentive to deviate. Consider next δ and l, for which (1.35) does not hold, such that $p_A^{x,dev}(p_B^{x*})$ follows from $\alpha\left(p_A^{x,dev}(p_B^{x*}), p_B^{x*}\right) = 1/(l-1)$. We showed, however, above that firm A does not have an incentive to deviate on $\alpha(\cdot) \geq 1/(l-1)$ for any δ and any $\bar{l}_4(\delta) \leq l \leq \bar{l}_3(\delta)$.

Conclusion from 5.c). We conclude that for any δ and any $\bar{l}_4(\delta) \leq l \leq \bar{l}_3(\delta)$, the equilibrium (1.24) exists.

Conclusion from 5. We conclude that for any δ and any $\bar{l}_4(\delta) \leq l \leq \bar{l}_3(\delta)$, the unique equilibrium is (1.24).

Part 3. Consider finally $l \geq 4$, in which case second-period profits are given by different functions depending on α.

i) Consider first $\alpha \leq 1/(l-1)$. The profits of the second period are given by (1.8). Firm A chooses p_A^x to maximize the profit (1.20). Firm B chooses p_B^x to maximize the profit (1.21). Solving simultaneously firms' first-order conditions yields the equilibrium prices (1.24), both of which are positive for any δ and any $l \geq 4$. These prices yield the market share of firm B in the first period given by (1.25), which is positive for any δ and any $l \geq 4$. The comparison shows that

$$\alpha(p_A^{x*}, p_B^{x*}) - \tfrac{1}{l-1} = \tfrac{9l+29\delta-8l\delta-45}{(27-10\delta)(l-1)}. \tag{1.91}$$

The right-hand side of (1.91) is non-positive if

$$l \leq \bar{l}_6(\delta) := \tfrac{45-29\delta}{9-8\delta}. \tag{1.92}$$

We conclude that for any δ the equilibrium (1.24) can only exist if $l \leq \bar{l}_6(\delta)$.

ii) Consider now $1/(l-1) < \alpha < (l-2)/[2(l-1)]$. In this case second-period profits are given by (1.10). Then firm A chooses p_A^x to maximize the profits:

$$\begin{aligned} & \left(l - p_A^x + p_B^x\right) p_A^x + \tfrac{\delta}{9}\left[\left(2\alpha(l-1)+1\right)^2 + \left(2l-1-\alpha(l-1)\right)^2\right] \\ = \; & \left(l - p_A^x + p_B^x\right) p_A^x + \tfrac{\delta}{9}\left[\left(2p_A^x - 2p_B^x - 1\right)^2 + \left(2l - p_A^x + p_B^x\right)^2\right]. \end{aligned} \tag{1.93}$$

Firm B chooses p_B^x to maximize the profits:

$$\begin{aligned} & \left(p_A^x - p_B^x - 1\right) p_B^x + \tfrac{\delta}{9}\left[\left(\alpha(l-1)-1\right)^2 + \left(l-2-2\alpha(l-1)\right)^2\right] \\ = \; & \left(p_A^x - p_B^x - 1\right) p_B^x + \tfrac{\delta}{9}\left[\left(p_A^x - p_B^x - 2\right)^2 + \left(l - 2p_A^x + 2p_B^x\right)^2\right]. \end{aligned} \tag{1.94}$$

Solving simultaneously firms' first-order conditions yields the prices:

$$p_A^{x*} = \tfrac{l(18-14\delta)+6\delta-9}{27-20\delta} \text{ and } p_B^{x*} = \tfrac{l(9-6\delta)+14\delta-18}{27-20\delta}. \tag{1.95}$$

Note that $p_A^{x*} \geq 0$ if

$$l \geq \tfrac{9-6\delta}{18-14\delta}. \tag{1.96}$$

Since for any δ the right-hand side of (1.96) is smaller than 1, $p_A^{x*} \geq 0$ holds for any δ and any $l \geq 4$. Note also that $p_B^{x*} \geq 0$ if

$$l \geq \tfrac{18-14\delta}{9-6\delta}. \tag{1.97}$$

Since for any δ the right-hand side of (1.97) is not larger than 2, $p_B^{x*} \geq 0$ holds for any δ and any $l \geq 4$. Using the prices (1.95) we can calculate the market share of firm B in the first period:

$$\alpha\left(p_A^{x*}, p_B^{x*}\right) = \tfrac{9l+12\delta-8l\delta-18}{(l-1)(27-20\delta)}.$$

The comparison shows that

$$\alpha\left(p_A^{x*}, p_B^{x*}\right) - \tfrac{l-2}{2(l-1)} = -\tfrac{l(9-4\delta)+16\delta-18}{2(27-20\delta)(l-1)}. \tag{1.98}$$

The right-hand side of (1.98) is non-positive if

$$l \geq \tfrac{18-16\delta}{9-4\delta}. \tag{1.99}$$

Since the right-hand side of (1.99) is for any α not larger than 2, (1.99) holds for any δ and any $l \geq 4$. The other comparison shows that

$$\alpha\left(p_A^{x*}, p_B^{x*}\right) - \tfrac{1}{l-1} = \tfrac{l(9-8\delta)+32\delta-45}{(27-20\delta)(l-1)}. \tag{1.100}$$

The right-hand side of (1.100) is non-negative if

$$l \geq \bar{l}_8\left(\delta\right) := \tfrac{45-32\delta}{9-8\delta}.$$

We conclude that for any δ, equilibrium (1.95) can only exist if $l \geq \bar{l}_8\left(\delta\right)$.

iii) Consider finally $\alpha \geq (l-2)/[2(l-1)]$. The profits of the second period are given by (1.9). Firm A chooses p_A^x to maximize the profits (1.26). Firm B chooses p_B^x to maximize the profits (1.27). Solving simultaneously firms' first-order conditions yields the equilibrium prices (1.28). These prices yield the market share of firm B in the first period (1.29). As follows from (1.30), for any δ and any $l \geq 4$ it holds that $\alpha\left(p_A^{x*}, p_B^{x*}\right) \leq 1$. The other comparison shows that

$$\alpha\left(p_A^{x*}, p_B^{x*}\right) - \tfrac{l-2}{2(l-1)} = -\tfrac{-l(10\delta-9)+16\delta-18}{2(8\delta+27)(l-1)}. \tag{1.101}$$

If $\delta < 0.9$, then the right-hand side of (1.101) is non-negative if

$$l \leq \bar{l}_7\left(\delta\right) := \tfrac{18-16\delta}{9-10\delta}. \tag{1.102}$$

If $\delta > 0.9$, then the opposite to (1.102) inequality should hold. Finally, if $\delta = 0.9$, then the right-hand side of (1.101) is positive for any l. Note now that $\bar{l}_7\left(0.75\right) = 4$ and for any $\delta < 0.75$ it holds that $\bar{l}_7\left(\delta\right) < 4$, such that there is no $l \geq 4$, which satisfies (1.102) in that case. Note also that if $\delta > 0.9$, then $\bar{l}_7\left(\delta\right) < 0$. We conclude that the equilibrium with $\alpha \geq (l-2)/[2(l-1)]$ does not exist for any l if $\delta < 0.75$, can exist for $l \leq \bar{l}_7\left(\delta\right)$ if $0.75 \leq \delta < 0.9$ and can exist for any l if $\delta \geq 0.9$.

In the following we analyze in turn every candidate equilibrium.

6.a) Consider first the candidate equilibrium (1.28).

i) The incentives of firm A.

Deviation on $1/(l-1) \leq \alpha(\cdot) \leq (l-2)/[2(l-1)]$. If firm A deviates, it realizes the profit (1.93). Keeping p_B^x at p_B^{x*} in (1.28) and taking the derivative of (1.93) with respect to p_A^x yields the deviation price of firm A:

$$p_A^{xdev}\left(p_B^{x*}\right) = \frac{-l\left(-60\delta^3+360\delta^2+189\delta-972\right)-160\delta^3+228\delta^2+54\delta-486}{-240\delta^2-378\delta+1458}. \tag{1.103}$$

This price is non-negative if

$$l \geq \frac{54\delta+228\delta^2-160\delta^3-486}{189\delta+360\delta^2-60\delta^3-972}. \tag{1.104}$$

Since the right-hand side of (1.104) is for any δ smaller than 1, then (1.104) holds for any l. Using the price (1.103) we can calculate the market share of firm B in the first period:

$$\alpha\left(p_A^{xdev}\left(p_B^{x*}\right), p_B^{x*}\right) = -\frac{99l\delta-72\delta-162l+14l\delta^2+324}{(l-1)(-80\delta^2-126\delta+486)}.$$

The comparison shows that

$$\alpha\left(p_A^{xdev}\left(p_B^{x*}\right), p_B^{x*}\right) - \frac{l-2}{2(l-1)} = \frac{81l+54\delta+36l\delta+80\delta^2-26l\delta^2-162}{2(l-1)(40\delta^2+63\delta-243)}. \tag{1.105}$$

The right-hand side of (1.105) is non-positive if

$$l \geq -\frac{54\delta+80\delta^2-162}{36\delta-26\delta^2+81}. \tag{1.106}$$

Since the right-hand side of (1.106) is for any δ not larger than 2, then (1.106) holds for any δ and any $l \geq 2$. The other comparison shows that

$$\alpha\left(p_A^{xdev}\left(p_B^{x*}\right), p_B^{x*}\right) - \frac{1}{l-1} = \frac{162l+198\delta-99l\delta+80\delta^2-14l\delta^2-810}{2(l-1)(-40\delta^2-63\delta+243)}. \tag{1.107}$$

The right-hand side of (1.107) is non-negative if

$$l \geq \frac{198\delta+80\delta^2-810}{99\delta+14\delta^2-162}. \tag{1.108}$$

We first consider the case (1.108), such that the optimal deviation price of firm A is given by (1.103). Then the difference between the equilibrium and the deviation profits is equal to

$$\frac{\delta\left[l^2(-3176\delta^3+6372\delta^2+8991\delta-20412)-l(-12160\delta^3+19152\delta^2+43092\delta-55404)-12800\delta^3+5760\delta^2+35640\delta-29160\right]}{36(9-5\delta)(8\delta+27)^2}. \tag{1.109}$$

The function in the brackets of the numerator of (1.109) is quadratic in l, opens downwards for any δ and has two roots, both of which are not larger than 2. Hence, for any δ and any $l \geq 2$, it takes non-positive values. We conclude that for any δ and any l, which satisfy (1.108), firm A has an incentive to deviate. Consider now the other case with

$$l < \frac{198\delta+80\delta^2-810}{99\delta+14\delta^2-162}. \tag{1.110}$$

Then the deviation price of firm A follows from $\alpha\left(p_A^{xdev}\left(p_B^{x*}\right), p_B^{x*}\right) = 1/\left(l-1\right)$ and is given by

$$p_A^{xdev}\left(p_B^{x*}\right) = \frac{l\left(-6\delta^2+21\delta+27\right)+16\delta^2+30\delta+108}{24\delta+81}, \tag{1.111}$$

which is positive for any δ and any l. Using the price (1.111), we can calculate the difference between the equilibrium and the deviation profits of firm A:

$$\frac{-l^2\left(-149\delta^3+189\delta^2+1053\delta-729\right)+l\left(-496\delta^3+11134\delta^2+3726\delta-7290\right)+320\delta^3-1296\delta^2+405\delta+18225}{9\left(8\delta+27\right)^2}. \tag{1.112}$$

Remember that the equilibrium (1.93) can exist only if $\delta \geq 0.75$. For any $\delta \geq 0.75$, the quadratic function in the numerator of (1.112) opens downwards and has two roots:

$$\begin{aligned} l_9\left(\delta\right) &= \frac{-\left(-496\delta^3+11134\delta^2+3726\delta-7290\right)+6(8\delta+27)\sqrt{3\delta(8\delta^3-70\delta^2+51\delta+270)}}{-2(-149\delta^3+189\delta^2+1053\delta-729)}, \\ l_{10}\left(\delta\right) &= \frac{-\left(-496\delta^3+11134\delta^2+3726\delta-7290\right)-6(8\delta+27)\sqrt{3\delta(8\delta^3-70\delta^2+51\delta+270)}}{-2(-149\delta^3+189\delta^2+1053\delta-729)}. \end{aligned}$$

Note that $l_9\left(\delta\right) < 0$ for any $\delta \geq 0.75$. For any $\delta \geq 0.75$ it also holds that $l_{10}\left(\delta\right) \geq 4$ and $l_{10}\left(\delta\right)$ satisfies (1.110). Finally, for any $0.75 \leq \delta < 0.9$ it holds that $l_{10}\left(\delta\right) \leq \bar{l}_7\left(\delta\right)$. We conclude that for any $\delta \geq 0.75$, firm A does not have an incentive to deviate if $4 \leq l \leq l_{10}\left(\delta\right)$.

Deviation on $\alpha\left(\cdot\right) \leq 1/\left(l-1\right)$. If firm A deviates, then its profit is (1.20). Keeping p_B^x at p_B^{x*} in (1.28) and taking the derivative of (1.20) with respect to p_A^x yields the price:

$$p_A^{xdev}\left(p_B^{x*}\right) = \frac{972l+675\delta+27l\delta+396\delta^2-32\delta^3-192l\delta^2+12l\delta^3-486}{-48\delta^2+270\delta+1458}. \tag{1.113}$$

This price is non-negative if

$$l \geq -\frac{675\delta+396\delta^2-32\delta^3-486}{27\delta-192\delta^2+12\delta^3+972}. \tag{1.114}$$

Note that for any δ, the right-hand side of (1.114) is smaller than 1, such that (1.114) holds for any δ and any l. Using the price (1.113) we calculate the market share of firm B in the first period:

$$\alpha\left(p_A^{xdev}\left(p_B^{x*}\right), p_B^{x*}\right) = \frac{-l\left(14\delta^2+99\delta-162\right)+40\delta^2+207\delta-324}{(-16\delta^2+90\delta+486)(l-1)}. \tag{1.115}$$

The numerator in (1.115) is non-negative if

$$l \geq \frac{207\delta+40\delta^2-324}{99\delta+14\delta^2-162}. \tag{1.116}$$

The right-hand side of (1.116) is for any δ not larger than 2, such that (1.116) holds for any δ and any $l \geq 2$. The denominator of (1.115) is negative for any δ and any l. We conclude that $\alpha\left(p_A^{xdev}\left(p_B^{x*}\right), p_B^{x*}\right) \geq 0$ holds for any δ and any $l \geq 2$. The other comparison shows that

$$\alpha\left(p_A^{xdev}\left(p_B^{x*}\right), p_B^{x*}\right) - \frac{1}{l-1} = \frac{162l+117\delta-99l\delta+56\delta^2-14l\delta^2-810}{2(l-1)(-8\delta^2+45\delta+243)}. \tag{1.117}$$

The numerator in (1.117) is non-positive if

$$l \leq \frac{117\delta+56\delta^2-810}{99\delta+14\delta^2-162}. \tag{1.118}$$

In the following we will restrict attention to the case (1.118) only, because it includes $4 \leq l \leq l_{10}(\delta)$, where firm A does not have an incentive to deviate, as we showed above. Under (1.118), the optimal deviation price of firm A is given by (1.113). Using (1.113), we calculate the difference between the equilibrium and the deviation profits of firm A:

$$\frac{\delta\left[-l^2\left(264\delta^3-1116\delta^2+2619\delta+2916\right)+l\left(1184\delta^3-2676\delta^2+2754\delta-11\,664\right)-1472\delta^3+1200\delta^2+9585\delta+43740\right]}{12(9-\delta)(8\delta+27)^2}. \tag{1.119}$$

The function in the brackets in the numerator of (1.119) is quadratic in l, opens downwards and has two roots both of which are smaller than 4 for any δ. It then follows that for any δ and any $4 \leq l \leq l_{10}(\delta)$, the term in (1.119) is non-positive, such that firm A has an incentive to deviate.

Conclusion from 6.a) We conclude that for any $l \geq 4$ the equilibrium (1.28) does not exist.

6.b) Consider now the candidate equilibrium (1.24).

Deviation on $1/(l-1) \leq \alpha(\cdot) \leq (l-2)/[2(l-1)]$.

i) The incentives of firm A. If firm A deviates, then its profit is (1.93). Keeping p_B^x at p_B^{x*} in (*1.24*) and taking the derivative of (1.93) with respect to p_A^x yields the deviation price of firm A:

$$p_A^{xdev}\left(p_B^{x*}\right)=\frac{972l+513\delta-972l\delta-426\delta^2+240\delta^3+312l\delta^2-80l\delta^3-486}{300\delta^2-1350\delta+1458}. \tag{1.120}$$

The right-hand side of (1.120) is non-negative if

$$l\geq\frac{513\delta-426\delta^2+240\delta^3-486}{972\delta-312\delta^2+80\delta^3-972}. \tag{1.121}$$

The right-hand side of (1.121) is for any δ smaller than 1, such that (1.121) holds for any δ and any l. Using (1.120) we calculate the market share of firm B in the first period:

$$\alpha\left(p_A^{xdev}\left(p_B^{x*}\right),p_B^{x*}\right)=\frac{162l+243\delta-162l\delta+12\delta^2+16l\delta^2-324}{(l-1)(100\delta^2-450\delta+486)}.$$

The comparison shows that

$$\alpha\left(p_A^{xdev}\left(p_B^{x*}\right),p_B^{x*}\right)-\frac{l-2}{2(l-1)}=-\frac{81l+207\delta-63l\delta-112\delta^2+34l\delta^2-162}{2(l-1)(50\delta^2-225\delta+243)}. \tag{1.122}$$

The right-hand side of (1.122) is non-positive if

$$l\geq\frac{-207\delta+112\delta^2+162}{-63\delta+34\delta^2+81}. \tag{1.123}$$

The right-hand side of (1.123) is for any δ not larger than 2, such that (1.123) holds for any δ and any $l\geq 2$. The other comparison shows that

$$\alpha\left(p_A^{xdev}\left(p_B^{x*}\right),p_B^{x*}\right)-\frac{1}{l-1}=\frac{162l+693\delta-162l\delta-88\delta^2+16l\delta^2-810}{2(l-1)(50\delta^2-225\delta+243)}. \tag{1.124}$$

The right-hand side of (1.124) is non-negative if

$$l\geq\frac{-693\delta+88\delta^2+810}{-162\delta+16\delta^2+162}. \tag{1.125}$$

We showed above that the equilibrium (1.24) can only exist if $l\leq\bar{l}_6(\delta)$, with $\bar{l}_6(\delta)$ being defined in (1.92). The comparison shows that

$$\bar{l}_6(\delta)-\frac{-693\delta+88\delta^2+810}{-162\delta+16\delta^2+162}=\frac{3(27-10\delta)\delta}{2(8\delta^2-81\delta+81)}\geq 0 \text{ for any } \delta.$$

Consider first

$$l\leq\frac{-693\delta+88\delta^2+810}{-162\delta+16\delta^2+162}, \tag{1.126}$$

in which case the optimal deviation price of firm A follows from $\alpha\left(p_A^{xdev}\left(p_B^{x*}\right), p_B^{x*}\right) = 1/\left(l-1\right)$. As the profits of firm A are continuous, we conclude that firm A does not have an incentive to deviate when (1.126) holds.

We consider now the other case and assume that (1.125) holds, such that (1.120) is the optimal deviation price of firm A. Using this price we calculate the difference between the equilibrium and the deviation profits of firm A:

$$\frac{\delta\left[l^2\left(1024\delta^3-11520\delta^2+22032\delta-11664\right)-l\left(6464\delta^3-74376\delta^2+155844\delta-90396\right)+9076\delta^3-111888\delta^2+253935\delta-160380\right]}{36(9-5\delta)(10\delta-27)^2}. \quad (1.127)$$

The function in the brackets in the numerator of (1.127) is quadratic in l, opens downwards for any δ and has two roots, one of which does not fulfill (1.125). The other root does and is given by

$$\bar{l}_9\left(\delta\right) := \frac{\left(6464\delta^3-74376\delta^2+155844\delta-90396\right)-12\left(80\delta^2-306\delta+243\right)\sqrt{5\delta^2-54\delta+81}}{2(1024\delta^3-11520\delta^2+22032\delta-11664)}.$$

For any δ it holds that $\bar{l}_9\left(\delta\right) \leq \bar{l}_6\left(\delta\right)$. We conclude then that firm A deviates for any δ if $\bar{l}_9\left(\delta\right) < l \leq \bar{l}_6\left(\delta\right)$ and does not deviate if

$$\frac{-693\delta+88\delta^2+810}{-162\delta+16\delta^2+162} < l \leq \bar{l}_9\left(\delta\right).$$

Conclusion from i). We conclude that for any δ firm A deviates if $\bar{l}_9\left(\delta\right) < l \leq \bar{l}_6\left(\delta\right)$ and does not deviate if $4 \leq l \leq \bar{l}_9\left(\delta\right)$.

ii) The incentives of firm B. If firm B deviates, then its profit is (1.94). Keeping p_A^x at p_A^{x*} in (1.24) and taking the derivative of (1.94) with respect to p_B^x yields the deviation price of firm B:

$$p_B^{xdev}\left(p_A^{x*}\right) = \frac{-l\left(40\delta^3-156\delta^2+270\delta-243\right)+120\delta^3-468\delta^2+702\delta-486}{150\delta^2-675\delta+729}. \quad (1.128)$$

The right-hand side of (1.128) is non-negative if

$$l \geq \frac{702\delta-468\delta^2+120\delta^3-486}{270\delta-156\delta^2+40\delta^3-243}. \quad (1.129)$$

For any δ, the right-hand side of (1.129) is not larger than 2, such that (1.129) holds for any δ and any $l \geq 2$. Using (1.128) we calculate the share of firm B in the first period:

$$\alpha\left(p_A^{x*}, p_B^{xdev}\left(p_A^{x*}\right)\right) = \frac{81l+216\delta-108l\delta-66\delta^2+32l\delta^2-162}{(l-1)(50\delta^2-225\delta+243)}.$$

The comparison shows that

$$\alpha\left(p_A^{x*}, p_B^{xdev}\left(p_A^{x*}\right)\right) - \frac{l-2}{2(l-1)} = -\frac{-l\left(14\delta^2+9\delta-81\right)+32\delta^2+18\delta-162}{2(l-1)(50\delta^2-225\delta+243)}. \tag{1.130}$$

The right-hand side of (1.130) is non-positive if

$$l \geq \frac{18\delta+32\delta^2-162}{9\delta+14\delta^2-81}. \tag{1.131}$$

For any δ the right-hand side of (1.131) is not larger than 2, such that (1.131) holds for any δ and any $l \geq 2$. The other comparison shows that

$$\alpha\left(p_A^{x*}, p_B^{xdev}\left(p_A^{x*}\right)\right) - \frac{1}{l-1} = \frac{(9-4\delta)(9l+29\delta-8l\delta-45)}{(l-1)(50\delta^2-225\delta+243)}. \tag{1.132}$$

The right-hand side of (1.132) is non-negative if

$$l \geq \bar{l}_6\left(\delta\right) = \frac{45-29\delta}{9-8\delta}. \tag{1.133}$$

We showed above that the equilibrium (1.24) can exist only if $l \leq \bar{l}_6\left(\delta\right)$. It follows then from (1.133) that the optimal deviation price of firm B is given by $\alpha\left(p_A^{x*}, p_B^{xdev}\left(p_A^{x*}\right)\right) = 1/\left(l-1\right)$. From the fact that the profits of firm B are continuous, we conclude then that firm B does not have an incentive to deviate.

Conclusion from ii). We conclude that for any δ and $l \geq 4$ firm B does not have an incentive to deviate.

Deviation on $\alpha\left(\cdot\right) \geq \left(l-2\right)/\left[2\left(l-1\right)\right]$.

i) The incentives of firm A. If firm A deviates, then its profit is (1.26). Keeping p_B^x at p_B^{x*} in (1.24) and taking the derivative of (1.26) with respect to p_A^x yields the optimal deviation price of firm A:

$$p_A^{xdev}\left(p_B^{x*}\right) = -\frac{-l\left(80\delta^3-318\delta^2+621\delta+972\right)+240\delta^3-234\delta^2+567\delta+486}{-300\delta^2+270\delta+1458}. \tag{1.134}$$

The right-hand side of (1.134) is non-negative if

$$l \geq \frac{567\delta-234\delta^2+240\delta^3+486}{621\delta-318\delta^2+80\delta^3+972}. \tag{1.135}$$

The right-hand side of (1.135) is for any δ smaller than 1, such that (1.135) holds for any δ and any l. Using (1.134) we calculate the market share of firm B in the first period:

$$\alpha\left(p_A^{xdev}\left(p_B^{x*}\right), p_B^{x*}\right) = \frac{297\delta - 162l - 189l\delta - 212\delta^2 + 114l\delta^2 + 324}{(l-1)(100\delta^2 - 90\delta - 486)}.$$

The comparison shows that

$$\alpha\left(p_A^{xdev}\left(p_B^{x*}\right), p_B^{x*}\right) - 1 = -\frac{l\left(14\delta^2 - 99\delta + 324\right) - 112\delta^2 + 207\delta - 162}{2(l-1)(-50\delta^2 + 45\delta + 243)}. \tag{1.136}$$

The right-hand side of (1.136) is non-positive if

$$l \geq \frac{-207\delta + 112\delta^2 + 162}{-99\delta + 14\delta^2 + 324}. \tag{1.137}$$

The right-hand side of (1.137) is for any δ smaller than 1, such that (1.137) holds for any δ and any l. The other comparison shows that

$$\alpha\left(p_A^{xdev}\left(p_B^{x*}\right), p_B^{x*}\right) - \frac{l-2}{2(l-1)} = -\frac{l\left(64\delta^2 - 144\delta + 81\right) - 112\delta^2 + 207\delta - 162}{2(l-1)(-50\delta^2 + 45\delta + 243)}. \tag{1.138}$$

The right-hand of (1.138) is non-negative if

$$l \leq l_{11}(\delta) := \frac{-207\delta + 112\delta^2 + 162}{-144\delta + 64\delta^2 + 81}. \tag{1.139}$$

Consider first $\max\{4, l_{11}(\delta)\} \leq l \leq \bar{l}_9(\delta)$, in which case (1.139) does not hold and the optimal deviation price of firm A follows from $\alpha\left(p_A^{xdev}\left(p_B^{x*}\right), p_B^{x*}\right) = (l-2)/[2(l-1)]$ and is given by

$$p_A^{xdev}\left(p_B^{x*}\right) = \frac{l\left(16\delta^2 - 54\delta + 135\right) - 48\delta^2 + 66\delta - 108}{162 - 60\delta}. \tag{1.140}$$

The numerator of (1.140) is non-negative if

$$l \geq \frac{-66\delta + 48\delta^2 + 108}{-54\delta + 16\delta^2 + 135}. \tag{1.141}$$

The right-hand side of (1.141) is for any δ smaller than 1, such that (1.141) holds for any δ and any l. Using (1.140) we calculate the difference between the equilibrium and the deviation profits of firm A:

$$-\frac{l^2\left(436\delta^3 - 2376\delta^2 + 2025\delta - 729\right) - l\left(2816\delta^3 - 15\,444\delta^2 + 14\,418\delta - 2916\right) + 4324\delta^3 - 23\,868\delta^2 + 23\,652\delta - 2916}{36(10\delta - 27)^2}. \tag{1.142}$$

The numerator of (1.142) is a quadratic function of l, which opens downwards for any δ and has two roots:

$$\frac{\left(2816\delta^3-15\,444\delta^2+14\,418\delta-2916\right)+6(27-10\delta)\sqrt{3\delta(36\delta^3-200\delta^2+195\delta+108)}}{2(436\delta^3-2376\delta^2+2025\delta-729)},$$
$$\frac{\left(2816\delta^3-15\,444\delta^2+14\,418\delta-2916\right)-6(27-10\delta)\sqrt{3\delta(36\delta^3-200\delta^2+195\delta+108)}}{2(436\delta^3-2376\delta^2+2025\delta-729)}.$$

Since for any δ both roots are smaller than 4, the numerator of (1.142) is negative for any δ and any $\max\{4, l_{11}(\delta)\} \leq l \leq \bar{l}_9(\delta)$, such that firm A does not have an incentive to deviate.

Consider now $4 \leq l \leq \min\left\{l_{11}(\delta), \bar{l}_9(\delta)\right\}$, in which case (1.139) holds and (1.134) is the optimal deviation price of firm A. Then the difference between the equilibrium and the deviation profits of firm A is

$$\frac{\delta\left[-l^2\left(2092\delta^3-8796\delta^2+6615\delta-2916\right)+l\left(9472\delta^3-36876\delta^2+10530\delta+11664\right)-11\,388\delta^3+42\,264\delta^2+5805\delta-43\,740\right]}{12(5\delta+9)(10\delta-27)^2}. \tag{1.143}$$

The function in the numerator of (1.143) is quadratic in l, opens upwards for any δ and has two roots:

$$\frac{-\left(9472\delta^3-36\,876\delta^2+10\,530\delta+11\,664\right)+4(27-10\delta)\sqrt{-(3485\delta^4-16\,032\delta^3+801\delta^2+42\,930\delta-55\,404)}}{-2(2092\delta^3-8796\delta^2+6615\delta-2916)},$$
$$\frac{-\left(9472\delta^3-36\,876\delta^2+10\,530\delta+11\,664\right)-4(27-10\delta)\sqrt{-(3485\delta^4-16\,032\delta^3+801\delta^2+42\,930\delta-55\,404)}}{-2(2092\delta^3-8796\delta^2+6615\delta-2916)},$$

both of which are for any δ smaller than 4, such that the numerator of (1.143) is non-negative for any δ and any $4 \leq l \leq \min\left\{l_{11}(\delta), \bar{l}_9(\delta)\right\}$ and firm A does not have an incentive to deviate.

Conclusion from i). We conclude that for any δ and $4 \leq l \leq \bar{l}_9(\delta)$ firm A does not have an incentive to deviate.

ii) The incentives of firm B. If firm B deviates, then it realizes the profit (1.27). Keeping p_A^x at p_A^{x*} in (1.24) and taking the derivative of (1.27) with respect to p_B^x yields the deviation price of firm B:

$$p_B^{xdev}(p_A^{x*}) = \frac{-l\left(8\delta^3-72\delta^2+216\delta-243\right)+24\delta^3-228\delta^2+594\delta-486}{30\delta^2-351\delta+729}. \tag{1.144}$$

The right-hand side of (1.144) is non-negative if

$$l \geq \frac{594\delta-228\delta^2+24\delta^3-486}{216\delta-72\delta^2+8\delta^3-243}. \tag{1.145}$$

The right-hand side of (1.145) is for any δ not larger than 2, such that (1.145) holds for any δ and any $l \geq 2$. Using (1.144) we calculate the market share of firm B in the first period:

$$\alpha\left(p_A^{x*}, p_B^{xdev}\left(p_A^{x*}\right)\right) = \frac{81l+108\delta-54l\delta-26\delta^2+12l\delta^2-162}{(l-1)(10\delta^2-117\delta+243)}.$$

The comparison shows that

$$\alpha\left(p_A^{x*}, p_B^{xdev}\left(p_A^{x*}\right)\right) - 1 = -\frac{-l\left(2\delta^2+63\delta-162\right)+16\delta^2+9\delta-81}{(l-1)(10\delta^2-117\delta+243)}. \tag{1.146}$$

The right-hand side of (1.146) is non-positive if

$$l \geq \frac{9\delta+16\delta^2-81}{63\delta+2\delta^2-162}. \tag{1.147}$$

The right-hand side of (1.147) is for any δ smaller than 1, such that (1.147) holds for any δ and any l. The other comparison shows that

$$\alpha\left(p_A^{x*}, p_B^{xdev}\left(p_A^{x*}\right)\right) - \frac{l-2}{2(l-1)} = -\frac{-l\left(14\delta^2+9\delta-81\right)+32\delta^2+18\delta-162}{2(l-1)(10\delta^2-117\delta+243)}. \tag{1.148}$$

The right-hand side of (1.148) is non-negative if

$$l \leq \frac{18\delta+32\delta^2-162}{9\delta+14\delta^2-81}. \tag{1.149}$$

The right-hand side of (1.149) is for any δ not larger than 2, such that (1.149) does not hold for any δ and any $l > 2$. Hence, $p_B^{xdev}\left(p_A^{x*}\right)$ follows from $\alpha(\cdot) = (l-2)/[2(l-1)]$ and is given by

$$p_B^{xdev}\left(p_A^{x*}\right) = \frac{l\left(16\delta^2-42\delta+27\right)-48\delta^2+120\delta-54}{162-60\delta}. \tag{1.150}$$

The right-hand side of (1.150) is non-negative if

$$l \geq \frac{-120\delta+48\delta^2+54}{-42\delta+16\delta^2+27}. \tag{1.151}$$

The right-hand side of (1.151) is for any δ not larger than 2, such that (1.151) holds for any δ and any $l \geq 2$. Hence, (1.150) is the optimal deviation price of firm B. Using (1.150) we calculate the difference between the equilibrium and the deviation profits of firm B:

$$-\frac{l^2\left(244\delta^3-432\delta^2+81\delta-729\right)-l\left(1664\delta^3-4104\delta^2+2268\delta-2916\right)+2896\delta^3-8964\delta^2+7128\delta-2916}{36(10\delta-27)^2}. \quad (1.152)$$

The numerator of (1.152) is a quadratic function of l, which opens downwards for any δ and has two roots:

$$\frac{1664\delta^3-4104\delta^2+2268\delta-2916-12\left(81-10\delta^2-3\delta\right)\sqrt{\delta(9-4\delta)}}{2\left(244\delta^3-432\delta^2+81\delta-729\right)}, \frac{1664\delta^3-4104\delta^2+2268\delta-2916+12\left(81-10\delta^2-3\delta\right)\sqrt{\delta(9-4\delta)}}{2\left(244\delta^3-432\delta^2+81\delta-729\right)},$$

both of which are for any δ smaller than 4. Hence, for any δ and any $4 \leq l \leq \bar{l}_9(\delta)$ the numerator of (1.152) is non-positive, such that firm B does not have an incentive to deviate.

Conclusion from ii). We conclude that for any δ and any $4 \leq l \leq \bar{l}_9(\delta)$ firm B does not have an incentive to deviate.

Conclusion from 6.b). We conclude that equilibrium (1.24) exists for any δ and any $4 \leq l \leq \bar{l}_9(\delta)$.

6.c) Consider finally the candidate equilibrium (1.95).

Deviation on $\alpha(\cdot) \leq 1/(l-1)$.

i) The incentives of firm A. If firm A deviates, then its profit is (1.20). Keeping p_B^x at p_B^{x*} in (1.95) and taking the derivative of (1.20) with respect to p_A^x yields the deviation price of firm A:

$$p_A^{xdev}\left(p_B^{x*}\right) = \frac{l\left(92\delta^2-360\delta+324\right)-208\delta^2+405\delta-162}{40\delta^2-414\delta+486}. \quad (1.153)$$

The right-hand side of (1.153) is non-negative if

$$l \geq \frac{-405\delta+208\delta^2+162}{-360\delta+92\delta^2+324}. \quad (1.154)$$

Since the right-hand side of (1.154) is for any δ smaller than 1, (1.154) holds for any δ and l. Using (1.153) we calculate the market share of firm B in the first period:

$$\alpha\left(p_A^{xdev}\left(p_B^{x*}\right), p_B^{x*}\right) = \frac{l\left(80\delta^2-234\delta+162\right)-220\delta^2+531\delta-324}{(l-1)(40\delta^2-414\delta+486)}. \quad (1.155)$$

The right-hand side of (1.155) is non-negative if

$$l \geq \frac{-531\delta+220\delta^2+324}{-234\delta+80\delta^2+162}. \quad (1.156)$$

For any δ the right-hand side of (1.156) is not larger than 2, such that (1.156) holds for any δ and any $l \geq 2$. The other comparison shows that

$$\alpha\left(p_A^{xdev}\left(p_B^{x*}\right), p_B^{x*}\right) - \frac{1}{l-1} = \frac{l\left(80\delta^2-234\delta+162\right)-260\delta^2+945\delta-810}{2(l-1)(20\delta^2-207\delta+243)}. \tag{1.157}$$

The right-hand side of (1.157) is non-positive if

$$l \leq l_{12}(\delta) := \frac{-945\delta+260\delta^2+810}{-234\delta+80\delta^2+162}.$$

We showed above that the equilibrium (1.95) can only exist if $l \geq \bar{l}_8(\delta)$. The comparison shows that

$$l_{12}(\delta) - \bar{l}_8(\delta) = \frac{3\delta(27-20\delta)}{2(40\delta^2-117\delta+81)} \geq 0 \text{ for any } \delta.$$

We next analyze two cases. Consider first $l \geq l_{12}(\delta)$, in which case the optimal deviation price of firm A follows from $\alpha\left(p_A^{xdev}\left(p_B^{x*}\right), p_B^{x*}\right) = 1/(l-1)$. As firm A's profits are continuous, we conclude that firm A does not have an incentive to deviate.

Consider next $\bar{l}_8(\delta) \leq l \leq l_{12}(\delta)$, in which case the optimal deviation price of firm A is given by (1.153). Then the difference between the equilibrium and the deviation profits of firm A is

$$-\frac{\delta\left[l^2\left(5120\delta^3-20\,736\delta^2+27\,216\delta-11\,664\right)-l\left(31\,360\delta^3-136\,368\delta^2+194\,076\delta-90\,396\right)+a(\delta)\right]}{36(9-\delta)(20\delta-27)^2}, \text{ with} \tag{1.158}$$

$$a(\delta) := 47120\delta^3 - 212616\delta^2 + 319869\delta - 160380.$$

The function in the brackets of (1.158) is quadratic in l, opens downwards for any δ and has two roots:

$$\frac{\left(31\,360\delta^3-136\,368\delta^2+194\,076\delta-90\,396\right)+12\left(160\delta^2-396\delta+243\right)\sqrt{5\delta^2-54\delta+81}}{2(5120\delta^3-20\,736\delta^2+27\,216\delta-11\,664)},$$

$$\bar{l}_{10}(\delta) \;:\; = \frac{\left(31\,360\delta^3-136\,368\delta^2+194\,076\delta-90\,396\right)-12\left(160\delta^2-396\delta+243\right)\sqrt{5\delta^2-54\delta+81}}{2(5120\delta^3-20\,736\delta^2+27\,216\delta-11\,664)}.$$

The first root is for any δ smaller than $\bar{l}_8(\delta)$, while the other root for any δ fulfills $\bar{l}_8(\delta) \leq \bar{l}_{10}(\delta) < l_{12}(\delta)$. Hence, on the interval $\bar{l}_8(\delta) \leq l < l_{12}(\delta)$, firm A does not deviate if $\bar{l}_{10}(\delta) \leq l < l_{12}(\delta)$ and deviates otherwise.

Conclusion from i). Combining the results from both cases we conclude that firm A does not deviate if $l \geq \bar{l}_{10}(\delta)$ and deviates if $\bar{l}_8(\delta) \leq l < \bar{l}_{10}(\delta)$.

ii) The incentives of firm B. If firm B deviates, then its profit is given by (1.21). Keeping p_A^x at p_A^{x*} in (1.95) and taking the derivative of (1.21) with respect to p_B^x yields the deviation price of firm B:

$$p_B^{xdev}\left(p_A^{x*}\right)=\frac{l\left(16\delta^2-81\delta+81\right)-24\delta^2+153\delta-162}{80\delta^2-288\delta+243}. \tag{1.159}$$

The right-hand side of (1.159) is non-negative if

$$l\geq\frac{-153\delta+24\delta^2+162}{-81\delta+16\delta^2+81}. \tag{1.160}$$

The right-hand side of (1.160) is for any δ smaller than 4, such that (1.160) holds for any δ and any $l\geq 4$. Using (1.159) we calculate the market share of firm B in the first period:

$$\alpha\left(p_A^{x*},p_B^{xdev}\left(p_A^{x*}\right)\right)=\frac{l\left(40\delta^2-117\delta+81\right)-80\delta^2+225\delta-162}{(l-1)(80\delta^2-288\delta+243)}. \tag{1.161}$$

The right-hand side of (1.161) is non-negative if

$$l\geq\frac{-225\delta+80\delta^2+162}{-117\delta+40\delta^2+81}. \tag{1.162}$$

Note that for any δ and any $l\geq\bar{l}_8(\delta)$, the inequality (1.162) holds. The other comparison shows that

$$\alpha\left(p_A^{x*},p_B^{xdev}\left(p_A^{x*}\right)\right)-\frac{1}{l-1}=\frac{(9-5\delta)(9l+32\delta-8l\delta-45)}{(l-1)(80\delta^2-288\delta+243)}. \tag{1.163}$$

The right-hand side of (1.163) is non-positive if $l\leq\bar{l}_8(\delta)$ holds. Since the equilibrium (1.95) can only exist if $l\geq\bar{l}_8(\delta)$, the optimal deviation price of firm B follows from $\alpha(\cdot)=1/(l-1)$. As the profits of firm B are continuous, we conclude that firm B does not have an incentive to deviate.

Conclusion from ii). We conclude that for any δ and any $l\geq\bar{l}_8(\delta)$, firm B does not have an incentive to deviate.

Deviation on $\alpha(\cdot)\geq(l-2)/[2(l-1)]$.

i) The incentives of firm A. If firm A deviates, then its profit is (1.26). Keeping p_B^x at p_B^{x*} in (1.95) and taking the derivative of (1.26) with respect to p_A^x yields the deviation price of firm A:

$$p_A^{xdev}\left(p_B^{x*}\right)=\frac{-l\left(-240\delta^2+99\delta+324\right)-220\delta^2+162\delta+162}{200\delta^2+90\delta-486}, \tag{1.164}$$

which is non-negative if

$$l\geq\frac{162\delta-220\delta^2+162}{99\delta-240\delta^2+324}. \tag{1.165}$$

Since for any δ the right-hand side of (1.165) is smaller than 1, then (1.165) holds for any δ and any l. Using (1.164) we calculate the market share of firm B in the first period:

$$\alpha\left(p_A^{xdev}\left(p_B^{x*}\right),p_B^{x*}\right)=-\frac{144\delta-162l-117l\delta-280\delta^2+180l\delta^2+324}{2(l-1)(-100\delta^2-45\delta+243)}.$$

The comparison shows that

$$\alpha\left(p_A^{xdev}\left(p_B^{x*}\right),p_B^{x*}\right)-1=\frac{l\left(20\delta^2+207\delta-324\right)+80\delta^2-234\delta+162}{2(l-1)(-100\delta^2-45\delta+243)}. \tag{1.166}$$

The right-hand side of (1.166) is non-positive if

$$l\geq-\frac{-234\delta+80\delta^2+162}{207\delta+20\delta^2-324}. \tag{1.167}$$

Since for any δ the right-hand side of (1.167) is smaller than 1, (1.167) holds for any δ and any l. The other comparison shows that

$$\alpha\left(p_A^{xdev}\left(p_B^{x*}\right),p_B^{x*}\right)-\frac{l-2}{2(l-1)}=\frac{(9-8\delta)(9l+10\delta-10l\delta-18)}{2(l-1)(100\delta^2+45\delta-243)}. \tag{1.168}$$

The right-hand side of (1.168) is non-negative if

$$-l\left(10\delta-9\right)+10\delta-18\leq 0 \tag{1.169}$$

holds. We have to distinguish between the cases: *a)* if $\delta<0.9$, then (1.169) holds for $l\leq\left(18-10\delta\right)/\left(9-10\delta\right)$ and *b)* if $\delta\geq 0.9$, then (1.169) holds for any l. Remember that the equilibrium (1.95) exists if $l\geq\bar{l}_8\left(\delta\right)$. Note next that if $\delta=0.75$, then $\bar{l}_8\left(\delta\right)=\left(18-10\delta\right)/\left(9-10\delta\right)$. If $\delta<0.75$, then for any $l\geq\bar{l}_8\left(\delta\right)$ (1.169) does not hold. Similarly, if $0.75\leq\delta<0.9$ and $l>\left(18-10\delta\right)/\left(9-10\delta\right)$, then (1.169) does not hold either, in which case the optimal deviation price of firm A follows from $\alpha\left(\cdot\right)=\left(l-2\right)/\left[2\left(l-1\right)\right]$. Hence, in these cases firm A does not have an incentive to deviate because its profits are continuous.

Consider the remaining two cases:

c) $0.75 \leq \delta < 0.9$ and $\bar{l}_8(\delta) \leq l \leq (18-10\delta)/(9-10\delta)$, *d)* $\delta \geq 0.9$ and $l \geq \bar{l}_8(\delta)$. In these cases (1.169) is fulfilled and (1.164) is the optimal deviation price of firm A. Using (1.164) we calculate the difference between the equilibrium and the deviation profits of firm A:

$$-\frac{\delta\left[l^2\left(6800\delta^3-27720\delta^2+39933\delta-20412\right)-l\left(16000\delta^3-74160\delta^2+112428\delta-55404\right)+12800\delta^3-51840\delta^2+68040\delta-29160\right]}{36(5\delta+9)(20\delta-27)^2}. \quad (1.170)$$

The function in the brackets in (1.170) is quadratic in l, opens downwards and has two roots both of which are for any δ smaller than 4. Hence, the expression in (1.170) is non-negative for any δ and any $l \geq 4$, which implies that firm A does not have an incentive to deviate.

Conclusion from i). We conclude that firm A does not have an incentive to deviate.

ii) The incentives of firm B. If firm B deviates, then its profit is (1.27). Keeping p_A^x at p_A^{x*} in (1.95) and taking the derivative of (1.27) with respect to p_B^x yields the deviation price of firm B:

$$p_B^{xdev}\left(p_A^{x*}\right) = \frac{l\left(14\delta^2-81\delta+81\right)-46\delta^2+180\delta-162}{20\delta^2-207\delta+243}, \quad (1.171)$$

which is non-negative if

$$l \geq \frac{-180\delta+46\delta^2+162}{-81\delta+14\delta^2+81}. \quad (1.172)$$

The right-hand side of (1.172) is for any δ not larger than 2, such that (1.172) holds for any δ and any $l \geq 2$. Using (1.171) we calculate the market share of firm B in the first period:

$$\alpha\left(p_A^{x*}, p_B^{xdev}\left(p_A^{x*}\right)\right) = \frac{81l+90\delta-63l\delta+20\delta^2-162}{(l-1)(20\delta^2-207\delta+243)}.$$

The comparison shows that

$$\alpha\left(p_A^{x*}, p_B^{xdev}\left(p_A^{x*}\right)\right) - \frac{l-2}{2(l-1)} = -\frac{(9-5\delta)(9l+16\delta-4l\delta-18)}{2(l-1)(20\delta^2-207\delta+243)},$$

which is non-negative if

$$l \leq \frac{18-16\delta}{9-4\delta}. \quad (1.173)$$

The right-hand side of (1.173) is for any δ not larger than 2, such that (1.173) does not hold for any δ and any $l > 2$. Hence, the optimal deviation price of firm B

follows from $\alpha(\cdot) = (l-2)/[2(l-1)]$, which together with the fact that the profits of firm B are continuous, implies that firm B does not have an incentive to deviate.

Conclusion from ii). We conclude that firm B does not have an incentive to deviate**.**

Conclusion from 6.c) We conclude that the equilibrium (*1.95*) exists for any δ and any $l \geq \bar{l}_{10}(\delta)$.

In Proposition 1.2 we use the following new notation:

$$\begin{aligned}
h_1(\delta) &: = \bar{l}_1(\delta) = \tfrac{2+\delta}{1+\delta}, \\
h_2(\delta) &: = l_4(\delta) = \frac{24\delta^3+124\delta^2+176\delta+76+4(\delta+1)(2\delta+3)\sqrt{(\delta+1)(9-\delta)}}{2(4\delta^3+24\delta^2+49\delta+28)}, \\
h_3(\delta) &: = l_2(\delta) = \frac{-120\delta^3+276\delta^2+6552\delta+6156+36(27-10\delta)(1+\delta)\sqrt{(\delta+1)(9-\delta)}}{2(-100\delta^3+372\delta^2+531\delta+2268)}, \\
h_4(\delta) &: = \bar{l}_9(\delta) = \frac{\left(6464\delta^3-74376\delta^2+155844\delta-90396\right)-12\left(80\delta^2-306\delta+243\right)\sqrt{5\delta^2-54\delta+81}}{2(1024\delta^3-11520\delta^2+22032\delta-11664)}, \\
h_5(\delta) &: = \bar{l}_{10}(\delta) = \frac{\left(31\,360\delta^3-136\,368\delta^2+194\,076\delta-90\,396\right)-12\left(160\delta^2-396\delta+243\right)\sqrt{5\delta^2-54\delta+81}}{2(5120\delta^3-20\,736\delta^2+27\,216\delta-11\,664)}.
\end{aligned}$$

We can now summarize our results as follows. If $l \leq h_1(\delta)$, then in equilibrium in the first period firms charge prices (1.19) and realize profits (1.13) and (1.14). If $h_1(\delta) < l \leq h_2(\delta)$, then firms realize the same profits but charge the prices (1.15). If $h_3(\delta) \leq l \leq \min\{h_4(\delta), h_5(\delta)\}$, then in equilibrium firms charge prices (1.24) and realize profits (1.20) and (1.21). Finally, if $l \geq \max\{h_4(\delta), h_5(\delta)\}$, then in equilibrium firms charge prices (1.95) and realize profits (1.93) and (1.94).

Note finally that if $\delta \lesssim 0.98$, then $\min\{h_4(\delta), h_5(\delta)\} = h_4(\delta)$ and the other way around if $\delta > 0.98$. Our results show that if $h_2(\delta) < l < h_3(\delta)$ and $h_4(\delta) < l < h_5(\delta)$, then no equilibrium in pure strategies in the first period exists. If $h_5(\delta) < l < h_4(\delta)$, then two equilibria exist, where in the first period firms charge prices (1.24) or (1.95). *Q.E.D.*

Proof of Corollary 1.1. We use the results on the equilibrium market share of firm B derived in the proof of Proposition 1.2. If $l \leq h_1(\delta)$, then $\alpha^*(\delta, l) = 0$. If $h_1(\delta) < l \leq h_2(\delta)$, then $\alpha^*(\delta, l) = [l(1+\delta) - 2 - \delta] / [(2\delta+3)(l-1)]$. If $h_3(\delta) \leq l \leq \min\{h_4(\delta), h_5(\delta)\}$, then $\alpha^*(\delta, l) = [l(9-8\delta) + 19\delta - 18] / [(l-1)(27-10\delta)]$. Finally, if $l \geq \max\{h_4(\delta), h_5(\delta)\}$, then $\alpha^*(\delta, l) = [l(9-8\delta) + 6(2\delta-3)] / [(l-1)(27-20\delta)]$. Taking the derivatives of these market shares yields the results stated in the Corollary. *Q.E.D.*

Proof of Corollary 1.2. To prove the Corollary we will use the results stated in the proof of Proposition 1.2. In that proof we derived the equilibrium adjusted profits (divided by $\underline{t}(1-2x)$ and multiplied by $(l-1)$) of each firm on a given location on firm A's turf. Note that these profits do not depend on the location and firms are symmetric. Hence, to analyze how the ability to collect additional flexibility data influences profits it is sufficient to compare the sum of both firms' adjusted equilibrium profits on some location. Precisely, for the case without additional customer data we evaluate the respective profits at $\delta = 0$ and then multiply them with $1+\delta$ to get the discounted sum of profits over two periods. In the following we derive first the profits without additional customer data.

Consider $l \leq 2$. Evaluating the sum of (1.13) and (1.14) at (1.19) and $\delta = 0$ and then multiplying with $1+\delta$ yields

$$(1+\delta)(l-1). \tag{1.174}$$

Consider $2 < l \leq 5$. Evaluating the sum of (1.20) and (1.21) at (1.24) and $\delta = 0$ and then multiplying with $1+\delta$ yields

$$\frac{(1+\delta)\left(5l^2-8l+5\right)}{9}. \tag{1.175}$$

Consider finally $l > 5$. Evaluating the sum of (1.93) and (1.94) at (1.95) and $\delta = 0$ and then multiplying with $1+\delta$ yields again (1.175).

We now compare a firms' equilibrium discounted profits with and without the additional customer data. If $l \leq h_1(\delta)$, then $\alpha^*(\delta, l) = 0$, such that no additional data is revealed in equilibrium and profits do not depend on firms' ability to collect it. Consider $h_1(\delta) < l \leq 2$. The difference between the sum of (1.13) and (1.14) evaluated at (1.15) and (1.174) is equal to

$$\frac{l^2\left(\delta^3+6\delta^2+10\delta+5\right)-l\left(2\delta^3+14\delta^2+28\delta+17\right)+\delta^3+8\delta^2+19\delta+14}{(2\delta+3)^2}. \tag{1.176}$$

The nominator of (1.176) is a quadratic function with respect to l, which opens upwards (for any δ) and has two roots: $r_1(\delta) := (\delta^2+6\delta+7)/(\delta^2+5\delta+5)$ and $r_2(\delta) := h_1(\delta) = (2+\delta)/(1+\delta)$. Note that for any δ, $r_1(\delta) < h_1(\delta)$ holds, such that for any $l > h_1(\delta)$ the nominator of (1.176) is positive and firms are better off when they can obtain additional customer data.

Consider $2 < l \leq h_2(\delta)$. The difference between the sum of (1.13) and (1.14) evaluated at (1.15) and (1.175) is equal to

$$-\frac{\delta\left[l^2\left(11\delta^2+26\delta+15\right)-l\left(50\delta^2+146\delta+105\right)+47\delta^2+152\delta+123\right]}{9(2\delta+3)^2}. \tag{1.177}$$

The function in the brackets in (1.177) is quadratic in l, opens upwards (for any δ) and has two roots:

$$\begin{aligned} r_3(\delta) &= \frac{\left(50\delta^2+146\delta+105\right)-3(2\delta+3)\sqrt{3(2\delta+3)(2\delta+5)}}{2(11\delta^2+26\delta+15)}, \\ r_4(\delta) &= \frac{\left(50\delta^2+146\delta+105\right)+3(2\delta+3)\sqrt{3(2\delta+3)(2\delta+5)}}{2(11\delta^2+26\delta+15)}. \end{aligned}$$

For any δ it holds $r_3(\delta) < 2$ and $r_4(\delta) > h_2(\delta)$, such that for any $\delta > 0$ and $2 < l \leq h_2(\delta)$, the function in the brackets in (1.177) is negative and firms are better off when they can collect additional customer data.

Consider $h_3(\delta) \leq l \leq \min\{h_4(\delta), h_5(\delta)\}$. The difference between the sum of (1.20) and (1.21) evaluated at (1.24) and (1.175) is equal to

$$-\frac{\delta\left[l^2\left(560\delta^2-2380\delta+2079\right)-l\left(2480\delta^2-10864\delta+9855\right)+2525\delta^2-11452\delta+10665\right]}{9(10\delta-27)^2}. \tag{1.178}$$

The function in the brackets in (1.178) is quadratic in l, opens upwards (for any δ) and has two roots:

$$\begin{aligned} r_5(\delta) &= \frac{\left(2480\delta^2-10\,864\delta+9855\right)-(27-10\delta)\sqrt{3(1648\delta^2-5084\delta+3855)}}{2(560\delta^2-2380\delta+2079)}, \\ h_6(\delta) &:= r_6(\delta) = \frac{\left(2480\delta^2-10\,864\delta+9855\right)+(27-10\delta)\sqrt{3(1648\delta^2-5084\delta+3855)}}{2(560\delta^2-2380\delta+2079)}. \end{aligned}$$

For any δ it holds that $r_5(\delta) < h_3(\delta)$ and $h_3(\delta) < r_6(\delta) < \min\{h_4(\delta), h_5(\delta)\}$. Hence, for any $\delta > 0$ firms are (weakly) better off with the ability to collect additional customer data if $h_3(\delta) \leq l \leq h_6(\delta)$ and are worse off otherwise. Note that $\partial h_6(\delta)/\partial\delta > 0$ for any δ.

Consider finally $l \geq \max\{h_4(\delta), h_5(\delta)\}$. The difference between the sum of (1.93) and (1.94) evaluated at (1.95) and (1.175) is equal to

$$-\frac{8\delta\left[l^2\left(80\delta^2-209\delta+135\right)-l\left(240\delta^2-662\delta+459\right)+80\delta^2-209\delta+135\right]}{9(20\delta-27)^2}. \tag{1.179}$$

The function in the brackets of (1.179) is quadratic in l, opens upwards (for any δ) and has two roots:

$$\begin{aligned} r_7(\delta) &= \tfrac{\left(240\delta^2-662\delta+459\right)-(27-20\delta)\sqrt{80\delta^2-244\delta+189}}{2(80\delta^2-209\delta+135)} \text{ and} \\ r_8(\delta) &= \tfrac{\left(240\delta^2-662\delta+459\right)+(27-20\delta)\sqrt{80\delta^2-244\delta+189}}{2(80\delta^2-209\delta+135)}. \end{aligned}$$

Note that for any δ it holds that $r_n(\delta) < \min\{h_4(\delta), h_5(\delta)\}$, with $n = 7, 8$. Hence, for any $\delta > 0$ and any $l \geq \min\{h_4(\delta), h_5(\delta)\}$, the nominator of (1.179) is positive and firms are worse off with the ability to collect additional customer data.

We finally summarize our results (for $\delta > 0$). Note that $h_1(1) = 1.5$. Hence, for any $l \leq 1.5$, irrespective of δ, profits do not depend on whether firms can collect additional customer data. For any $1.5 < l < 2$ profits are larger when firms can collect data if $\delta > h_1^{-1}(l)$ and are same otherwise. Remember next that for any δ it holds that $\partial h_n(\delta)/\partial\delta > 0$, with $n = 2, ..., 6$. It also holds that $h_3(1) \approx 2.89 < h_6(0) \approx 3.07$. Hence, for any $2 \leq l \leq 3.07$ firms are better off with the ability to collect additional data irrespective of the discount factor. Note next that $h_6(1) \approx 4$. Hence, for $3.07 < l \leq 4$ firms are better of if $\delta > h_6^{-1}(l)$ and are (weakly) worse off otherwise. Finally, if $l > 4$, then firms are worse off with the ability to collect additional data irrespective of the discount factor. *Q.E.D.*

Proof of Corollary 1.3. For any l and δ we first calculate the equilibrium discounted social welfare over two periods and then subtract equilibrium profits to dirive consumer surplus. We then analyze how social welfare and consumer surplus change when firms are able to recognize consumers.

Part 1. i) If $l \leq h_1(\delta)$, then each firm serves all consumers on any location on its turf in both periods, such that social welfare is

$$SW_1^{1+2}(l,\delta) = v(1+\delta) - \tfrac{2(1+\delta)}{1-\underline{t}} \int_{\underline{t}}^{1} \int_{0}^{1/2} (tx)\, dxdt = v(1+\delta) - \tfrac{(1+\delta)(l+1)\underline{t}}{8}.$$

The discounted profits of a firm over two periods are given by the sum of (1.13) and (1.14) evaluated at (1.19), multiplied by $\underline{t}(1-2x)/(l-1))$ and integrated over $x \in [0, 1/2]$, which yields the discounted profits of both firms over two periods: $\Pi_1^{1+2}(l,\delta) = (1+\delta)\underline{t}/2$. The difference between $SW_1^{1+2}(l,\delta)$ and $\Pi_1^{1+2}(l,\delta)$ yields consumer surplus: $CS_1^{1+2}(l,\delta) = v(1+\delta) - (1+\delta)(l+5)\underline{t}/8$.

ii) If $h_1(\delta) < l \leq h_2(\delta)$, then the discounted profits of a firm over two periods are given by the sum of (1.13) and (1.14) evaluated at (1.15), multiplied by $\underline{t}(1-2x)/(l-1)$ and integrated over $x \in [0, 1/2]$, which yields the discounted profits of both firms over two periods:

$$\Pi_2^{1+2}(l, \delta) = \frac{\underline{t}\left[l^2\left(\delta^3+6\delta^2+10\delta+5\right)-l\left(-2\delta^3-2\delta^2+7\delta+8\right)-3\delta^3-8\delta^2-2\delta+5\right]}{2(2\delta+3)^2(l-1)}.$$

We calculate now social welfare. On each location on its turf a firm serves consumers with $t \geq t^{\alpha}(\cdot) = \underline{t}(1+\delta)(l+1)/(2\delta+3)$ (we used (1.11) to compute $t^{\alpha}(\cdot)$) in the first period and all consumers in the second period, which yields the discounted social welfare over two periods, $SW_2^{1+2}(l, \delta) =$

$$\begin{aligned} & v(1+\delta) - \frac{2}{1-\underline{t}} \int_{\frac{\underline{t}(1+\delta)(l+1)}{2\delta+3}}^{1} \int_0^{1/2} (tx)\, dxdt \\ & - \frac{2}{1-\underline{t}} \int_{\underline{t}}^{\frac{\underline{t}(1+\delta)(l+1)}{2\delta+3}} \int_0^{1/2} [t(1-x)]\, dxdt - \frac{2\delta}{1-\underline{t}} \int_{\underline{t}}^{1} \int_0^{1/2} (tx)\, dxdt \\ = {} & v(1+\delta) - \frac{\underline{t}\left[l^2\left(4\delta^3+18\delta^2+25\delta+11\right)+l\left(4\delta^2+8\delta+4\right)-4\delta^3-22\delta^2-41\delta-25\right]}{8(l-1)(2\delta+3)^2}. \end{aligned}$$

Subtracting $\Pi_2^{1+2}(l, \delta)$ from $SW_2^{1+2}(l, \delta)$ we get the discounted consumer surplus over two periods:

$$CS_2^{1+2}(l, \delta) = v(1+\delta) + \frac{\underline{t}\left[-l^2\left(8\delta^3+42\delta^2+65\delta+31\right)+l\left(-8\delta^3-12\delta^2+20\delta+28\right)+16\delta^3+54\delta^2+49\delta+5\right]}{8(2\delta+3)^2(l-1)}.$$

iii) If $h_3(\delta) \leq l \leq \min\{h_4(\delta), h_5(\delta)\}$, then the discounted profits of a firm over two periods are given by the sum of (1.20) and (1.21) evaluated at the prices (1.24) multiplied by $\underline{t}(1-2x)/(l-1)$ and integrated over $x \in [0, 1/2]$, which yields the discounted profits of both firms over two periods:

$$\Pi_3^{1+2}(l, \delta) = -\frac{\underline{t}\left[l^2\left(20\delta^3-60\delta^2+378\delta-1215\right)-l\left(560\delta^3-2448\delta^2+2781\delta-1944\right)+675\delta^3-3084\delta^2+3240\delta-1215\right]}{6(10\delta-27)^2(l-1)}.$$

We calculate now social welfare. On each location on its turf a firm serves in the first period consumers with $t \geq t^{\alpha}(\cdot) = \underline{t}[l(9-8\delta)+9(1+\delta)]/(27-10\delta)$ and in the second period all consumers on segment α and those with $t \geq \underline{t}[l+1+\alpha(l-1)]/3 = 3(4l+\delta-2l\delta+1)\underline{t}/(27-10\delta)$ on segment $1-\alpha$ (we used (1.12) to compute α), which yields the discounted social welfare over two periods, $SW_3^{1+2}(l,\delta) =$

$$\begin{aligned}
&= \; v(1+\delta) - \tfrac{2}{1-\underline{t}} \int\limits_{\frac{\underline{t}[l(9-8\delta)+9(1+\delta)]}{27-10\delta}}^{1} \int\limits_{0}^{1/2} (tx)\, dxdt - \tfrac{2}{1-\underline{t}} \int\limits_{\underline{t}}^{\frac{\underline{t}[l(9-8\delta)+9(1+\delta)]}{27-10\delta}} \int\limits_{0}^{1/2} [t(1-x)]\, dxdt \\
&\quad - \tfrac{2\delta}{1-\underline{t}} \int\limits_{\underline{t}}^{\frac{\underline{t}[l(9-8\delta)+9(1+\delta)]}{27-10\delta}} \int\limits_{0}^{1/2} (tx)\, dxdt - \tfrac{2\delta}{1-\underline{t}} \int\limits_{\frac{\underline{t}[l(9-8\delta)+9(1+\delta)]}{27-10\delta}}^{\frac{3(4l+\delta-2l\delta+1)\underline{t}}{27-10\delta}} \int\limits_{0}^{1/2} [t(1-x)]\, dxdt \\
&\quad - \tfrac{2\delta}{1-\underline{t}} \int\limits_{\frac{3(4l+\delta-2l\delta+1)\underline{t}}{27-10\delta}}^{1} \int\limits_{0}^{1/2} (tx)\, dxdt \\
&= \; v(1+\delta) - \frac{\underline{t}\left[l^2\left(44\delta^3-312\delta^2+27\delta+891\right)-l\left(-216\delta^3+252\delta^2+144\delta-324\right)-244\delta^3+114\delta^2+1071\delta-2025\right]}{8(l-1)(10\delta-27)^2}.
\end{aligned}$$

Subtracting $\Pi_3^{1+2}(l,\delta)$ from $SW_3^{1+2}(l,\delta)$ we get the discounted consumer surplus over two periods, $CS_3^{1+2}(l,\delta) =$

$$= v(1+\delta) + \frac{\underline{t}\left[l^2\left(-52\delta^3+696\delta^2+1431\delta-7533\right)-l\left(2888\delta^3-10\,548\delta^2+10\,692\delta-6804\right)+3432\delta^3-12\,678\delta^2+9747\delta+1215\right]}{24(10\delta-27)^2(l-1)}.$$

iv) If $l > \max\{h_4(\delta), h_5(\delta)\}$, then the discounted profits of a firm are given by the sum of (1.93) and (1.94) evaluated at (1.95), multiplied by $\underline{t}(1-2x)/(l-1)$ and integrated over $x \in [0, 1/2]$, which yields the discounted profits of both firms over two periods:

$$\Pi_4^{1+2}(l,\delta) = -\frac{\underline{t}\left[l^2\left(-1360\delta^3+1728\delta^2+2835\delta-3645\right)-l\left(-1280\delta^3+144\delta^2+6480\delta-5832\right)-1360\delta^3+1728\delta^2+2835\delta-3645\right]}{18(20\delta-27)^2(l-1)}.$$

We now compute social welfare. On each location on its turf a firm serves in the first period consumers with $t \geq t^{\alpha}(\cdot) = \underline{t}[(9-8\delta)(l+1)]/(27-20\delta)$. In the second period a firm serves consumers with $\underline{t}(36-28\delta+9l-8l\delta)/(81-60\delta) \leq t \leq t^{\alpha}(\cdot)$ on segment α and consumers with $\underline{t}(9-8\delta+36l-28l\delta)/(81-60\delta) \leq t \leq 1$ on segment $1-\alpha$, which yields the discounted social welfare over two periods,

$SW_4^{1+2}(l,\delta) =$

$$v(1+\delta) - \frac{2}{1-\underline{t}} \int\limits_{\frac{\underline{t}[(9-8\delta)(l+1)]}{27-20\delta}}^{1} \int\limits_{0}^{1/2} (tx)\,dxdt - \frac{2}{1-\underline{t}} \int\limits_{\underline{t}}^{\frac{\underline{t}[(9-8\delta)(l+1)]}{27-20\delta}} \int\limits_{0}^{1/2} [t(1-x)]\,dxdt$$

$$-\frac{2\delta}{1-\underline{t}} \int\limits_{\frac{\underline{t}[36-28\delta+9l-8l\delta]}{81-60\delta}}^{\frac{\underline{t}[(9-8\delta)(l+1)]}{27-20\delta}} \int\limits_{0}^{1/2} (tx)\,dxdt - \frac{2\delta}{1-\underline{t}} \int\limits_{\frac{\underline{t}[9-8\delta+36l-28l\delta]}{81-60\delta}}^{1} \int\limits_{0}^{1/2} (tx)\,dxdt$$

$$-\frac{2\delta}{1-\underline{t}} \int\limits_{\underline{t}}^{\frac{\underline{t}[36-28\delta+9l-8l\delta]}{81-60\delta}} \int\limits_{0}^{1/2} [t(1-x)]\,dxdt - \frac{2\delta}{1-\underline{t}} \int\limits_{\frac{\underline{t}[(9-8\delta)(l+1)]}{27-20\delta}}^{\frac{\underline{t}[9-8\delta+36l-28l\delta]}{81-60\delta}} \int\limits_{0}^{1/2} [t(1-x)]\,dxdt$$

$$= v(1+\delta) - \frac{\underline{t}\left[-l^2\left(-4144\delta^3+6696\delta^2+4455\delta-8019\right)-l\left(512\delta^3-3168\delta^2+5508\delta-2916\right)-10256\delta^3+17\,784\delta^2+8181\delta-18225\right]}{72(27-20\delta)^2(l-1)}.$$

Subtracting $\Pi_4^{1+2}(l,\delta)$ from $SW_4^{1+2}(l,\delta)$ we get the discounted consumer surplus over two periods, $CS_4^{1+2}(l,\delta) = v(1+\delta) +$

$$+\frac{\underline{t}\left[l^2\left(-9584\delta^3+13608\delta^2+157795\delta-22599\right)-l\left(-5632\delta^3+3744\delta^2+20412\delta-20412\right)+4816\delta^3-10872\delta^2+3159\delta+3645\right]}{72(20\delta-27)^2(l-1)}.$$

In the following we analyze how social welfare and consumer surplus change when firms become able to target consumers based on their behavior.

Part 2. i) If $l \leq h_1(\delta)$, then social welfare and consumer surplus do not change with behavior-based targeting.

ii) If $h_1(\delta) < l < 2$, the comparison of social welfare shows that

$$SW_2^{1+2}(l,\delta) - (1+\delta)\,SW_1^{1+2}(l,0) = -\frac{\underline{t}\left[l^2\left(\delta^2+2\delta+1\right)+l\left(2\delta^2+4\delta+2\right)-3\delta^2-10\delta-8\right]}{4(2\delta+3)^2(l-1)}. \quad (1.180)$$

The function in the brackets in the nominator of (1.180) is quadratic in l, opens upwards (for any δ) and has two roots, $h_1(\delta)$ and the negative one (for any δ), such that for any $l > h_1(\delta)$ and δ the right-hand side of (1.180) is negative. The other comparison shows that

$$CS_2^{1+2}(l,\delta) - (1+\delta)\,CS_1^{1+2}(l,0) = -\frac{\underline{t}\left[l^2\left(2\delta^3+13\delta^2+22\delta+11\right)-l\left(4\delta^3+26\delta^2+52\delta+32\right)+2\delta^3+13\delta^2+28\delta+20\right]}{4(2\delta+3)^2(l-1)}. \quad (1.181)$$

The function in the brackets in the nominator of (1.181) is quadratic in l, opens upwards (for any δ) and has two roots, $h_1(\delta)$ and the other smaller than 1 (for any

δ), such that for any $l > h_1(\delta)$ and δ the right-hand side of (1.181) is negative. We conclude that both social welfare and consumer surplus decrease when firms can recognize consumers.

iii) If $2 \le l \le h_2(\delta)$, the comparison of social welfare shows that

$$SW_2^{1+2}(l,\delta) - (1+\delta)\, SW_3^{1+2}(l,0) = \frac{t\delta\left[l^2\left(4\delta^2+7\delta+3\right)+l\left(8\delta^2+14\delta+6\right)-32\delta^2-101\delta-78\right]}{36(2\delta+3)^2(l-1)}. \tag{1.182}$$

The function in the brackets in (1.182) is quadratic in l, opens upwards (for any δ) and has two roots, one of which is negative an the other one is larger than $h_2(\delta)$ for any δ. We conclude that for any $2 \le l \le h_2(\delta)$ and any $\delta > 0$ social welfare is smaller with behavioral targeting. The other comparison shows that

$$CS_2^{1+2}(l,\delta) - (1+\delta)\, CS_3^{1+2}(l,0) = \frac{t\delta\left[l^2\left(26\delta^2+59\delta+33\right)-l\left(92\delta^2+278\delta+204\right)+62\delta^2+203\delta+168\right]}{36(2\delta+3)^2(l-1)}. \tag{1.183}$$

The function in the brackets of the nominator of (1.183) is quadratic in l, opens upwards (for any δ) and has two roots, one of which is smaller than 1 and the other one is larger than $h_2(\delta)$ for any δ. It follows that for any $2 \le l \le h_2(\delta)$ and any $\delta > 0$ consumers are worse off with behavioral targeting.

iv) If $h_3(\delta) \le l < 5$, the comparison of social welfare shows that

$$SW_3^{1+2}(l,\delta) - (1+\delta)\, SW_3^{1+2}(l,0) = \frac{\delta t\left[l^2\left(352\delta^2-1016\delta+918\right)+l\left(-772\delta^2+254\delta+1026\right)-152\delta^2+4987\delta-7182\right]}{36(10\delta-27)^2(l-1)}. \tag{1.184}$$

The function in the brackets of (1.184) is quadratic in l, opens upwards (for any δ) and has two roots, one of which is negative (for any δ) and the other one is

$$l_{13}(\delta) := \frac{-\left(-772\delta^2+254\delta+1026\right)+6(27-10\delta)\sqrt{225\delta^2-1016\delta+1045}}{2(352\delta^2-1016\delta+918)}.$$

Note that $\partial l_{13}(\delta)/\partial\delta < 0$. It also holds that $l_{13}(\delta) = h_3(\delta) \approx 2.28$ if $\delta \approx 0.19$, $l_{13}(\delta) > h_3(\delta)$ if $\delta < 0.19$, with an opposite sign otherwise. Hence, if $\delta > 0.19$, then social welfare increases for any δ and any $h_3(\delta) \le l < 5$. If $\delta \le 0.19$, then social welfare increases if $l_{13}(\delta) < l < 5$ and (weakly) decreases otherwise. The other comparison shows that $CS_3^{1+2}(l,\delta) - (1+\delta)\, CS_3^{1+2}(l,0) =$

$$= \frac{\delta t\left[l^2\left(1472\delta^2-5776\delta+5076\right)-l\left(5732\delta^2-21982\delta+18684\right)+4898\delta^2-17917\delta+14148\right]}{36(10\delta-27)^2(l-1)}. \tag{1.185}$$

The function in the brackets in (1.185) is quadratic in l, opens upwards (for any δ) and has two roots one of which is smaller than $h_3(\delta)$ (for any δ) and the other is

$$l_{14}(\delta) := \frac{\left(5732\delta^2-21982\delta+18684\right)+2(27-10\delta)\sqrt{3(3347\delta^2-9712\delta+7068)}}{2(1472\delta^2-5776\delta+5076)}.$$

Note that $\partial l_{14}(\delta)/\partial\delta < 0$. It also holds that $l_{14}(\delta) = h_3(\delta) \approx 2.61$ if $\delta \approx 0.48$, $l_{14}(\delta) > h_3(\delta)$ if $\delta < 0.48$, with an opposite sign otherwise. Hence, if $\delta > 0.48$, then consumer surplus increases for any δ and any $h_3(\delta) \leq l < 5$. If $\delta \leq 0.48$, then consumer surplus increases if $l_{14}(\delta) < l < 5$ and (weakly) decreases otherwise.

v) If $5 \leq l \leq \min\{h_4(\delta), h_5(\delta)\}$, the comparison of social welfare shows that

$$SW_3^{1+2}(l,\delta) - (1+\delta)\,SW_4^{1+2}(l,0) = \frac{\delta \underline{t}\left[l^2\left(352\delta^2-1016\delta+918\right)+l\left(-772\delta^2+254\delta+1026\right)-152\delta^2+4987\delta-7182\right]}{36(10\delta-27)^2(l-1)}. \quad (1.186)$$

The function in the brackets in the nominator of (1.186) is quadratic in l, opens upwards (for any δ) and has two roots both of which are smaller than 5 (for any δ). It follows that for any $5 \leq l \leq \min\{h_4(\delta), h_5(\delta)\}$ and any $\delta > 0$ social welfare is larger with behavioral targeting. The other comparison shows that $CS_3^{1+2}(l,\delta) - (1+\delta)\,CS_4^{1+2}(l,0) =$

$$= \frac{\underline{t}\delta\left[l^2\left(1472\delta^2-5776\delta+5076\right)-l\left(5732\delta^2-21982\delta+18684\right)+4898\delta^2-17917\delta+14148\right]}{36(10\delta-27)^2(l-1)}. \quad (1.187)$$

The function in the brackets in the nominator of (1.187) is quadratic in l, opens upwards (for any δ) and has two roots both of which are smaller than 5 (for any δ). It follows that for any $5 \leq l \leq \min\{h_4(\delta), h_5(\delta)\}$ and any $\delta > 0$ consumer surplus is larger with behavioral targeting.

vi) If $l \geq \max\{h_4(\delta), h_5(\delta)\}$, the comparison of social welfare shows that

$$SW_4^{1+2}(l,\delta) - (1+\delta)\,SW_4^{1+2}(l,0) = \frac{\underline{t}\delta\left[l^2\left(128\delta^2-392\delta+297\right)+l\left(1056\delta^2-2944\delta+2052\right)+128\delta^2-392\delta+297\right]}{36(20\delta-27)^2(l-1)}. \quad (1.188)$$

The function in the brackets in the nominator of (1.188) is quadratic in l, opens upwards (for any δ) and has two roots both of which are negative (for any δ), such that for any $l \geq \max\{h_4(\delta), h_5(\delta)\}$ and any $\delta > 0$ social welfare is larger with behavioral targeting. The other comparison shows that

$$CS_4^{1+2}(l,\delta) - (1+\delta)\,CS_4^{1+2}(l,0) = \frac{\underline{t}\delta\left[l^2\left(1408\delta^2-3736\delta+2457\right)-l\left(2784\delta^2-7648\delta+5292\right)+1408\delta^2-3736\delta+2457\right]}{36(20\delta-27)^2(l-1)}. \quad (1.189)$$

The function in the brackets in the nominator of (1.189) is quadratic in l, opens upwards (for any δ) and has two roots both of which are smaller than 5 (for any δ), such that for any $l \geq \max\{h_4(\delta), h_5(\delta)\}$ and any $\delta > 0$ consumer surplus is larger with behavioral targeting.

We now summarize our results for $\delta > 0$. We first combine the results from cases *i)* and *ii)*. Note that $h_1(1) = 1.5$. If $l \leq 1.5$, both social welfare and consumer surplus do not change with targeted pricing. If $1.5 < l < 2$, they decrease for $\delta > h_1^{-1}(\delta)$ and do not change otherwise. We now combine the results from cases *iii)* and *iv)*. Note that $\partial h_2(\delta)/\partial\delta > 0$, $h_2(0) = 2$ and $h_2(1) \approx 2.67$. Hence, if $2 \leq l < 2.28$ ($2 \leq l < 2.61$), then social welfare (consumer surplus) decreases with behavioral targeting for any l and δ. If $2.28 \lessapprox l < 2.29$ ($2.61 \lessapprox l < 2.62$), then social welfare (consumer surplus) (weakly) decrease for $\delta \leq l_{13}^{-1}(\delta)$ ($\delta \leq l_{14}^{-1}(\delta)$), decrease for $\delta \geq h_2^{-1}(\delta)$ and increases otherwise. Finally, if $2.29 \lessapprox l \lessapprox 2.67$ ($2.62 \lessapprox l \lessapprox 2.67$), then social welfare and consumer surplus increase for $\delta \leq h_3^{-1}(\delta)$ and decrease for $\delta \geq h_2^{-1}(\delta)$. From cases *v)* and *vi)* it follows that for any $l > 2.67$ both social welfare and consumer surplus increase for any $\delta > 0$. *Q.E.D.*

1.7 References

1. Armstrong M. (2006) Recent Developments in the Economics of Price Discrimination. In: Blundell, R. and Newey, W.K. and Persson, T., (eds.) Advances in Economics and Econometrics: Theory and Applications: Ninth World Congress: volume II. (pp. 97-141). Cambridge University Press: Cambridge, UK.

2. Baye I., Sapi G. (2019) Should Mobile Marketers Collect Other Data Than Geo-Location? The Scandinavian Journal of Economics; 121; pp. 647- 675.

3. Bester H., Petrakis E. (1996) Coupons and Oligopolistic Price Discrimination. International Journal of Industrial Organization; 14; pp. 227-242.

4. BIA Kelsey (2015) Mobile Will Grab 11.5% of Total Local Media Revenues by 2019. http://www.biakelsey.com/mobile-will-grab-11-5-of-total-local-media-revenues-by-2019/.

5. Borenstein S. (1985) Price Discrimination in Free-Entry Markets. RAND Journal of Economics; 16; pp. 380-397.

6. Carroni, E. (2016) Competitive Customer Poaching with Asymmetric Firms. International Journal of Industrial Organization; 48; pp. 173-206.

7. Chen Y., Li X., Sun M. (2017) Competitive Mobile Geo Targeting. Marketing Science; 36; pp. 666-682.

8. Chen Y., Narasimhan C., Zhang Z. J. (2001) Individual Marketing with Imperfect Targetability. Marketing Science; 20; pp. 23-41.

9. Chen Y., Zhang Z. J. (2009) Dynamic Targeted Pricing with Strategic Consumers. International Journal of Industrial Organization; 27; pp. 43-50.

10. Colombo, S. (2016) Imperfect Behavior-Based Price Discrimination. Journal of Economics & Management Strategy; 25; pp. 563-583.

11. Corts K. S. (1998) Third-Degree Price Discrimination in Oligopoly: All-Out Competition and Strategic Commitment. RAND Journal of Economics; 29; pp. 306-323.

12. Danaher P. J., Smith M. S., Ranasinghe K., Danaher T. S. (2015) Where, When, and How Long: Factors that Influence the Redemption of Mobile Phone Coupons. Journal of Marketing Research; 52; pp. 710-725.

13. Dubé J.-P., Fang Z., Fong N., Luo X. (2017) Competitive Price Targeting with Smartphone Coupons. Marketing Science; 36; pp. 944-975.

14. Esteves R.-B. (2009) Price Discrimination with Partial Information: Does it Pay off? Economics Letters; 105; pp. 28-31.

15. Esteves R.-B. (2010) Pricing with Customer Recognition. International Journal of Industrial Organization; 28; pp. 669-681.

16. Fudenberg D., Tirole J. (2000) Customer Poaching and Brand Switching. RAND Journal of Economics; 31; pp. 634-657.

17. Fudenberg D., Villas-Boas J.M. (2006), Behavior-Based Price Discrimination and Customer Recognition, pp. 377-433, in: T. Hendershott (eds.), Handbook in Information Systems, Vol. 1, Emerald Group Publishing Limited, UK.

18. Jentzsch N., Sapi G., Suleymanova I. (2013) Targeted Pricing and Customer Data Sharing Among Rivals. International Journal of Industrial Organization; 31; pp. 131-144.

19. Liu Q., Serfes K. (2004) Quality of Information and Oligopolistic Price Discrimination. Journal of Economics & Management Strategy; 13; pp. 671-702.

20. Liu Q., Shuai J. (2013) Multi-dimensional Price Discrimination. International Journal of Industrial Organization; 31; pp. 417-428.

21. Liu Q., Shuai, J. (2016) Price Discrimination with Varying Qualities of Information. B.E. Journal of Economic Analysis and Policy; 16; pp. 1093-1121.

22. Luo X., Andrews M., Fang Z., Phang C. W. (2017) Mobile Targeting. Management Science; 60; pp. 1738-1756.

23. Shaffer G., Zhang Z. J. (1995) Competitive Coupon Targeting. Marketing Science; 14; pp. 395-416.

24. Shaffer G., Zhang Z. J. (2000) Pay to Switch or Pay to Stay: Preference-Based Price Discrimination in Markets with Switching Costs. Journal of Economics and Management Strategy; 9; pp. 397-424.

25. Shaffer G., Zhang Z. J. (2002) Competitive One-to-One Promotions. Management Science; 48; pp. 1143-1160.

26. Shin J., Sudhir K. (2010) A Customer Management Dilemma: When Is It Profitable to Reward One's Own Customers? Marketing Science; 29; pp. 671-689.

27. Thisse J.-F., Vives X. (1988) On the Strategic Choice of Spatial Price Policy. American Economic Review; 78; pp. 122-137.

28. Thumbvista (2015) Behavioral Targeting – Using Consumer Behavior For Advertising. https://thumbvista.com/2015/04/mobile-advertising-using-consumer-location- behavioral-targeting/.

29. Tode C. (2014) Dunkin' Donuts' Winning Mobile Triple Play: Geofencing, Behavioral Targeting and Coupons. Mobile Commerce Daily. http://www.retaildive.com/ex/mobilecom-mercedaily/dunkin-donuts-winning-mobile-triple-play-geofencing-behavioral-targeting-and-coupons.

30. Villas-Boas J. M. (1999) Dynamic Competition with Customer Recognition. RAND Journal of Economics; 30; pp. 604-631.

31. Won, S. B. (2017). Two-Dimensional Price Discrimination With Uncertainty About Willingness-To-Pay. Annals of Regional Science; 58; pp. 513-541.

Chapter 2

Imperfect Customer Recognition with Location Data

2.1 Introduction

Modern information technologies rapidly improve the opportunities of firms and interested parties to raise customer data of various types and precision to use it for targeted marketing. Within these datasets some information can be considered as almost perfect, for instance, data on consumer location. In our increasingly digital world, information on a potential consumer's name and home address can be easily gained and used to approach her with personalized offers. Additionally, the widespread use of smartphones provides a vast amount of GPS-based location data, enabling targeted pricing depending on a consumer's real-time location. However, location is not the only factor firms would like to consider when designing personal pricing. Other factors, such as health or income, also influence a consumer's response to targeted offers and can be summarized as her "flexibility". In contrast to location, this information is not directly observable and has to be inferred from consumers' behavior or purchase history, giving rise to (possibly imperfect) behavior-based price discrimination. Depending on a firm's technological ability to recognize its customers, the collected flexibility data will be more or less precise.

The literature on behavior-based price discrimination typically analyze its profit and welfare implications under the assumption that the first-period customer recognition is perfect in the sense that all first-period consumers are correctly recognized. But one can think about various reasons why the customer recognition might be imperfect. First, this can be due to simple imperfect process or imperfect technology related reasons in the sense that, e.g., the sales assistant in the store does not completely raise the data in stressful situations or the database to recall consumers is technically limited. Another reason could be the existence of an online next to an offline payment opportunity. Consumers who want to redeem the coupon might be able to pay the amount directly and the firms would say "conveniently" online before patronizing the store or driving to the specific activity. Other consumers, instead, pay after the arrival in cash and are thereby harder to recognize again afterwards. A third and very interesting reason why the customer recognition might be imperfect is due to privacy policy regulations. Firms might be forced to delete stored information after some time, such that there is only a fraction of past customers in the database. It is also a common practice to provide the opportunity to agree on the purchase of a newsletter and the use of customer data to "benefit" from more personal offers in connection with an online purchase. This option is typically ticked and can be deleted what only a fraction of consumers do.[1] This share would be reasonably different, when the regulation would require that this option is not initially ticked, such that the consumers have to tick it themselves. Therefore, privacy policy regulations directly impact the recognition quality and makes the following imperfect customer recognition analysis particularly interesting to consider welfare implications.

The aim of this paper is to analyze how imperfect customer recognition affects firms' profits, social welfare and consumer surplus in a competitive setting, where firms hold consumer location data of perfect quality and are able to derive additional consumer flexibility data from observing their purchase histories.[2]

We consider consumers who differ along two dimensions: their locations on a unit line and the strength of their transport cost parameters, to which we refer as their "flexibility". There are two firms in the market, which sell homogeneous products at the two ends of a unit line over two periods. In each period firms hold perfect information on consumer locations. At the same time, firms are able to recognize a share of their first-period customers in the second period, which allows to estimate (with certain precision) their flexibility and price discriminate respectively in the second period. We consider two scenarios, where consumers are either myopic or sophisticated. Sophisticated consumers maximize the discounted surplus over both periods and take into account how their first-period purchases will influence the charged prices in the second period. In contrast, myopic consumers maximize their one-period surplus and ignore the dependence of the future targeted prices on their initial choices. We also vary the level of consumer heterogeneity in flexibility and consider two versions of our model with "relatively homogeneous" and "relatively heterogeneous" consumers. Finally, we allow for different accuracy levels of customer recognition to analyze how the profit and welfare effects of combining perfect location data with the flexibility information depend on the quality of the latter.

We get the following results. When consumers are relatively homogeneous, for any

[1]It is reasonable to assume that the probability that a consumer deletes the cross is unrelated to her flexibility.

[2]Note that our analysis and results are equivalent to the case when location would be interpreted as preference. In general can our results easily be transferred to other (more general) settings due to the flexible shape of our model. E.g., it also incorporates and analyzes the case that firms ex ante have no information and consumers only differ in one dimension, usually considered in the literature.

precision of customer recognition and consumer sophistication, the profit effect of combining behavioral targeting with perfect price discrimination based on customer locations is (weakly) positive.[3] Moreover, when consumer recognition becomes more accurate, firms' discounted profits over two periods (weakly) increase. We also find that with relatively homogeneous consumers, the interests of firms on the one hand and those of consumers and social welfare on the other hand are always opposed.[4] When consumers are relatively heterogeneous, firms are always worse off when they combine behavioral flexibility data with perfect location data.[5] However, firms' discounted profits may increase when customer recognition becomes more precise. This happens when recognition is already sufficiently precise and consumers sophisticated or myopic with a sufficiently high discount factor.[6] Interestingly, firms' and consumer interests are not necessarily opposed and all can benefit from the improved accuracy of customer recognition.

As it is standard in the price discrimination literature, we can explain these results by distinguishing between the rent-extraction and competition effects of additional (more precise) customer data. With more (precise) customer data firms are potentially able to extract more rents from consumers, to which we refer as the "rent-extraction effect". At the same time additional customer data impacts the strategic behavior of the firms to which we refer as the "competition effect". With relatively homogeneous consumers and in the absence of flexibility data, every firm serves all

[3]When consumers are relatively homogeneous, firms prefer to monopolize their turf even without the additional flexibility data in the short term. Accordingly, they can fully exploit their advantage on their turf and decide whether or not to invest in flexibility information by sharing some customers with the rival in the first period. Logically, they only make use of this strategy when it leads to higher profits which is, however, only the case when recognition is sufficiently precise to recover the first-period investments and consumers myopic. Sophisticated consumers anticipate the higher second-period targeting price of the home firm and react with a more elastic first-period demand, which makes higher prices to share customers inefficient.

[4]The effects on welfare are always opposed to the effects on profits when consumers are relatively homogeneous, since firms benefit by starting to share customers, leading to higher transport costs and prices.

[5]Firms initially share customers on each location when consumers are relatively heterogeneous, such that the additional flexibility data leads to an increase in competition, since each firm can now compete for the different customer groups on each location separately, making both firms worse off.

[6]However, firms more likely benefit from a higher accuracy when consumers are sophisticated, since their strategic purchase decision in this case leads to an inverse u-shaped price sensitivity, such that competition increases firstly further, but is soften when the customer recognition is sufficiently precise. Accordingly, prices and thereby profits are an u-shaped function of the recognition precision, while profits interestingly exceed the ones with myopic consumers when recognition is sufficiently precise, such that firms in fact want to educate consumer about the use of their data in this case.

consumers on any location on its turf. As a result, the rival prices initially most aggressively. Thus, with additional behavioral (flexibility) data firms enjoy only the (positive) rent-extraction effect, which increases their profits, because there is no scope for the rival to become more aggressive in pricing. Moreover, with more accurate customer recognition the positive effect on profits become stronger. When consumers are relatively heterogeneous, every firm serves only a share consumers on any location on its turf (those with relatively large transport cost parameters). As a result, the rival is moderate in pricing and additional customer data intensifies competition. The final impact on firms' profits is driven by a complex interplay of rent-extraction and competition effect depending on data quality and consumer sophistication, discussed in the analysis section of this paper.

Finally, there are several further findings in this paper, which differ from most other previous findings on behavior-based price discrimination and are worth to highlight. First, we find that firms actually manage to influence the market outcome in their favor by strategically choosing their first-period price under whole game considerations. Second, profit and welfare effects may depend on the discount factor. And third, first-period prices are in all cases not necessarily higher than under uniform pricing models and depend fundamentally on the level of recognition precision.

The rest of the paper is organized as follows. In Section 2.2, we discuss the related literature. Section 2.3 introduces the model, which we analyze in Section 2.4. Section 2.5 provides the welfare analysis. In Section 2.6, we compare the results with myopic and sophisticated consumers. Finally, Section 2.7 concludes.

2.2 Related Literature

This paper contributes to two strands of literature. First, to the literature on competitive and behavior-based price discrimination and second, to the works which analyze how the profits and welfare effects of customer data depends on its quality.

The literature on competitive and behavior-based price discrimination was initiated by the seminal articles of Thisse and Vives (1988) and Fudenberg and Tirole (2000).[7] Thisse and Vives were the first to demonstrate the negative effect of first-degree price discrimination based on perfect location in a static Hotelling-model.

[7] A survey of literature on behavior-based price discrimination is provided in Fudenberg and Villas-Boas (2006).

Fudenberg and Tirole consider a dynamic Hotelling-type model, where in the second period firms use behavioral data on consumer purchases in the initial period for third-degree price discrimination. When consumers are uniformly distributed on the Hotelling line, similar to the result of Thisse and Vives, they also find that firms are worse off with the ability to discriminate on consumer locations (framed as preferences). The reason behind this result is that a consumer's first-period purchase reveals her (relative) preference for one firm, inducing the rival to target this consumer more aggressively. In response to this aggressive pricing a firm has to reduce its prices as well, leading to lower profits over two periods for both firms. Following these seminal contributions, many subsequent articles confirmed the detrimental effect of customer data on profits in different settings.[8] The common feature of these works is that according to Corts' (1998) terminology, they consider settings characterized by best-response asymmetry, where a "strong market" of one firm is at the same time the "weak market" of the other. Corts shows that in such markets, access to customer data gives rise to an unambiguous profit effect which, however, can be either positive or negative. The above mentioned articles demonstrate the negative profit effect of customer data. Nevertheless, this result is not general and, for instance, if firms are sufficiently asymmetric, customer data may boost profits in a model with best-response asymmetry (see Shaffer and Zhang, 2000 and 2002 or Carroni, 2016).[9] Similarly, a positive effect of customer data is identified in Jentzsch *et al.* (2013), Baye and Sapi (2019) and Baye *et al.* (2018), when consumers are sufficiently homogeneous in their preferences.

The works most related to ours are Baye *et al.* (2018) and Colombo (2016). Baye *et al.* consider a two-period model, where consumers differ along two dimensions: their locations and transport cost parameters.[10] They assume firms hold perfect data on the former dimension and are able to collect information on the latter dimension through observing consumers' purchases in the first period. Within this set-up, they vary consumers' heterogeneity in the dimension to reveal and find that this set-up nests the results of the already mentioned papers as special cases. Therefore, they

[8]See, for instance, Shaffer and Zhang (1995), Bester and Petrakis (1996), Villas-Boas (1999) and Esteves (2010).

[9]Alternatively, Chen and Zhang (2009) consider a market with three consumer groups. Two of them strictly purchase from the preferred firm, while the third group buy from the firm with the lower price. They find that firms are better off in this setting, since firms charge higher first-period prices to identify its loyal consumers.

[10]Other models considering two-dimensional heterogeneous consumers can be found, e.g., in Borenstein (1985), Armstrong (2006) and Won (2017).

demonstrate the importance of the level of consumer differentiation in preferences when considering the profit effects of consumer behavior-based pricing. Importantly, within their analysis the authors assume that every firm can identify all its former customers with perfect precision. We contribute by allowing for imperfect customer recognition, where every firm can identify only a share of its customers, and sophisticated consumers. Accordingly, we fill the gap between the two extreme cases of perfect customer recognition (meaning usual behavior-based price discrimination) and no customer recognition (meaning uniform pricing) within their results and are able to analyze the impact of an increasing recognition accuracy while also enlarging their analysis to sophisticated customers. Colombo also considers a two-period behavior-based price discrimination model and assumes imperfect customer recognition. However, he considers consumers who differ only in their locations and he doesn't allow for varying levels of consumer heterogeneity. He finds that customer recognition is always detrimental for firms, while the profits are u-shaped in the level of information accuracy when consumers are sophisticated. We find a similar result when consumers are relatively heterogeneous. However, the result is reversed with relatively homogeneous consumers, where firms (weakly) gain from the ability to recognize customers and profits increase in the recognition accuracy. In this case customer data doesn't intensify competition and firms gain from the improved ability to discriminate among customers. Thus, our more flexible model provide a much broader picture of the issue.

The other works which analyze the profit effect of customer data depending on its quality are Chen *et al.* (2001) and Esteves (2014). However, these contributions model information accuracy in a different way. They consider a one-shot game, where firms receive a noisy (so eventually erroneous) signal about consumers' preference and price discriminate accordingly.[11] Therefore, in their setting firms have information on all consumers, however, these information are potentially wrong, such that inaccuracy in their setting mean that the probability of the signal to be wrong is higher. In contrast, we consider a two-period game where firms generate the information endogenously and the inaccuracy arises from the fact that information on consumers in the second period is incomplete, meaning that there is no (additional) information about certain consumers. However, despite these differences,

[11]Further related works include Liu and Serfes (2004), Liu and Shuai (2016) or Baye and Sapi (2019), where customer data allows the firms to segment consumers. With data of better quality, customer segments become finer.

our model also incorporates their results to a large extend. Due to the one-period competition with given data, we have that their setting corresponds to our analysis in the second stage. Additionally, Chen *et al.* (2001) consider the same consumer structure as Chen and Zhang (2009). Since the only price-sensitive consumer group strictly buys from the firm with the lower price, firms compete for (relatively) homogeneous consumers in the sense of our model. In this case, we find a strictly positive impact of a higher data accuracy. Also in Chen *et al.* (2001) profits firstly increase with a higher data accuracy, however, when accuracy is very high loyal and switching consumers can be very clearly distinguished and profits decrease due to the fierce competition for switchers. We do not find this effect in our model, since each firm prefers to serve all consumers on her turf even without the additional information, such that an increasing information accuracy can never harm firms. In contrast and in line with our model, Esteves (2014) considers the case of uniformly distributed consumers, each of them being price-sensitive. Since consumers overall do not strictly prefer one specific firm in her setting, we have that firms compete for relatively heterogeneous consumers in the sense of our model. In this case, provided firms in our model share customers equally as in her model, the findings of both models coincide and the impact of a higher data accuracy on firms' (second-period) profits is monotonically negative.

2.3 The Model

We consider two firms, A and B, located at the two endpoints of the unit interval (with $x_A = 0$ and $x_B = 1$), producing two brands of the same product at zero marginal costs. The firms compete over two periods and set prices simultaneously in each period. There is a unit mass of consumers buying at most one unit of the product. Each consumer has an address $x \in [0, 1]$ on the line, which corresponds to her (average) physical location (e.g., home address or average GPS-location).[12] If a consumer does not buy at her location, she incurs linear transport costs proportional to the distance to the firm. We follow Jentzsch *et al.* (2013), Baye and Sapi (2019) and Baye *et al.* (2018) and assume that consumers differ not only in their locations, but also in their transport costs per unit distance t, to which we refer as consumer

[12] We use the average physical location such that the targeted location of the consumer does not change between the two periods.

flexibility, with $t \in [\underline{t}, \overline{t}]$.[13] Higher levels of t correspond to lower flexibility, because a consumer needs a higher discount to make her choose the firm she likes less. Without loss of generality, we normalize $\overline{t}$ (so the lowest flexibility) to 1 such that: $t \in [\underline{t}, 1]$, with $0 \leq \underline{t} < 1$ and measure the level of consumer heterogeneity l by the ratio of the lowest to the highest flexibility: $l := 1/\underline{t}$, with $l \in (1, \infty)$. Each consumer is uniquely characterized by a pair (x, t), with x and t being uniformly and independently distributed according to the density functions: $f_t = 1/(1 - \underline{t})$, $f_x = 1$ and $f_{t,x} = 1/(1 - \underline{t})$.

The utility of a consumer (x, t) from buying at firm $i = \{A, B\}$ is

$$U_i(p_i(x), t, x) = \upsilon - t\,|x - x_i| - p_i(x), \tag{2.1}$$

where $\upsilon > 0$ denotes the basic utility and $p_i(x)$ is the price of firm i at location x. We assume that the basic utility is high enough such that all consumers purchase in equilibrium and every consumer buys from the firm, which provides her a higher utility.[14]

Customer data. Table 2.1 presents the types and quality of customer data firms hold within the model as well as the time firms obtain the information.

Type of customer data	Time obtained	Quality
Location	Given in each period	Perfect
Flexibility	Inferred from 1-st period purchase decision	Imperfect
Own past customer	Inferred from 1-st period purchase decision	Imperfect
Rival's past customer	Never	Zero

Table 2.1: Customer data available to the firms.

Motivated by the example of mobile advertising where location data delivered by GPS-signals is almost perfect and consumers individually addressable, we assume that firms hold perfect information on consumers' physical location and can dis-

[13]Different from Jentzsch *et al.* (2013) and Baye and Sapi (2019), in Baye *et al.* (2018) and in this model flexibility data is endogenously generated.

[14]As usual, we assume that a consumer buys from the closer firm in case of indifference. If $x = 1/2$, then she buys from firm A.

criminate between consumers on different locations in both periods. Consumers' flexibility is instead completely unknown in the first period, but firms can infer at least consumers' relative flexibility by observing their first-period purchase decisions and use this information in the second period. However, given that the knowledge about past purchases is typically private to the firm and even the information about own past customers is often imperfect, we assume that firms do not observe the purchase history (and thereby do not obtain relative flexibility information) from all consumers. Only a share $\alpha \in [0, 1]$ of own past customers are correctly recognized, while following Colombo (2016) we assume that α is the same for both firms.[15] Given this imperfect recognition, we have that each firm can separate consumers in two groups after the first stage. An identified past customers group and a foreign customers group (consisting of unidentified own past customers and former customers of the rival). The purchase history and thereby the relative flexibility is only revealed in the first group, such that firms can target only these customers with a price that considers consumers' location and relative flexibility in the second period.[16] The second-period price for the second group depend once again solely on consumers' location, since the firm can not distinguish whether the specific consumer bought at the rival or simply got not recognized as an own previous customer, such that the relative flexibility can not be inferred and exploited.[17]

To fix ideas, consider Figure 2.1 and 2.2 presenting the distribution of consumers in period 1 and 2. Firms can target each x separately in both periods, such that the first-period purchase decisions generate a cut-off point "k" in the consumer flexibility distribution on every location.[18] All consumers with a lower flexibility (meaning higher t) bought at their more preferred firm, while the consumers with a higher flexibility bought at the more distant rival, provided its price was sufficiently low.[19]

[15]In Baye *et al.* (2018), the technology to recognize previous customers is perfect, i.e. $\alpha = 1$, as usual in the literature of behavior-based price discrimination. Thereby, own and rival's past customer are perfectly distinguishable which is a special case in our setting.

[16]There is practical evidence e.g., via a field experiment by Danaher *et al.* (2015) that distance as well as the previous redemption history determine whether a coupon is used. Firms should therefore use both informations to design the coupon.

[17]Also following Colombo (2016), we exclude the possibility of arbitrage between consumers.

[18]It turns out that k is the same on every x in equilibrium.

[19]A standard revealed-preference argument implies that if a consumer on $x < 1/2$ with $t = k$ bought from firm A (B) in the first period, then all consumers with $t > k$ ($t < k$) made the same choice.

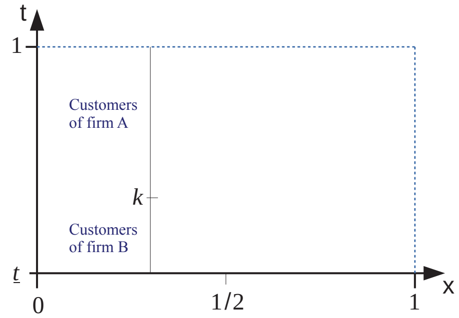


Figure 2.1: First-period market outcome at some x on firm A's turf.

Both firms recognize the share α of their previous customers, such that each x consists of customers belonging to one of three groups after the first period.

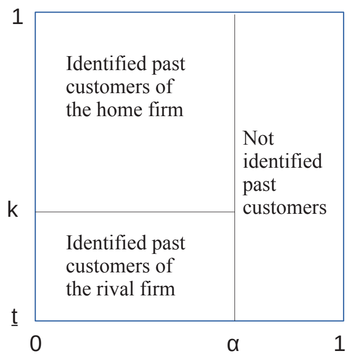


Figure 2.2: Customer groups in the second period on a certain location.

In the second period, both firms choose one (home) price to maximize the profits on the group of identified own past customers and another (foreign) price designed to maximize the profits on the other two groups jointly. Consequently, we have that firms compete on their identified customer group with their home price against the foreign price of the other firm, while firms compete for generally not identified consumers with their foreign price against each other.

We assume that firms are forward-looking and maximize the discounted sum of profits over both periods. With respect to consumers, we consider two cases with all consumers being either myopic or sophisticated. Sophisticated consumers maximize the discounted surplus over both periods and take the impact of their purchasing decision in the first period on the prices they are charged with in the future period into account. In contrast, myopic consumers maximize their one-period surplus and ignore the dependence of the future targeted prices on their choices in the first period.

We follow Jentzsch *et al.* (2013) and consider two extreme versions of our model with respect to the level of consumer heterogeneity in flexibility. In the first version, we assume that $\underline{t} \geq 1/2$ $(l \leq 2)$ and refer to the consumers in this case as "relatively homogeneous", since the difference between the lowest and the highest flexibility parameters is relatively low. In the second version, we assume $\underline{t} = 0$ and refer to the consumers in this case as "relatively heterogeneous", since $\lim_{\underline{t} \to 0} 1/\underline{t} \to \infty$ and the difference between the lowest and the highest transport cost parameter is very large.
We look for a subgame-perfect Nash equilibrium in pure strategies and solve the game backwards starting from the second period.

2.4 Equilibrium Analysis

We introduce now some useful notations and discuss the results subsequently. Since the game is symmetric, it is sufficient to restrict the analysis to a single location x on firm A's turf $(x < 1/2)$ and then extend the results to all the other locations. Given that we analyze a location on firm A's turf representatively, we often refer to it as the home firm.

Second period. In the second period, each firm can discriminate consumers on different locations and on each location it can distinguish at most two groups of consumers: the own identified customers and the foreign consumers (which include the own unidentified customers and the rival's customers of the first period). Therefore, both firms can specify (up to) two different prices on each location in the second period and we denote the prices with p_{iH} for the identified previous (home) consumers of firm $i = A, B$ and p_{iF} for the unidentified (foreign) consumers of firm $i = A, B$. On any location and for given prices, the home firm attracts the less flexible consumers in each customer group of the second period, i.e., those with sufficiently high transport cost parameters: $t \geq (p_{Ai} - p_{Bj})/(1 - 2x)$ with $i, j = H, F$. Given that the less flexible consumers who bought at firm A in the first period are partially recognized as previous customers by firm A and analogously, firm B also recognize the share α of its previous customers on $x < 1/2$ (with relatively low transport cost parameters), the home firm's expected second-period profit on x (provided prices are not too different and denoted by the superscript "2" to indicate the second period) is given by:

$$\Pi_A^2(x) = \alpha\left(\frac{1-\frac{p_{AH}-p_{BF}}{1-2x}}{1-\underline{t}}\right)p_{AH} + \alpha\left(\frac{k-\frac{p_{AF}-p_{BH}}{1-2x}}{1-\underline{t}}\right)p_{AF} + (1-\alpha)\left(\frac{1-\frac{p_{AF}-p_{BF}}{1-2x}}{1-\underline{t}}\right)p_{AF} \quad (2.2)$$

The first term in equation (2.2) is the profit of firm A from its previous identified customers, while the second and third terms are the profits of firm A from the foreign consumers, which includes the consumers identified by firm B and those who are not recognized by any of the firms.

Figure 2.3 depicts demand regions when poaching actually takes place on all customer groups in the second period (i.e., firms' prices are not too different).

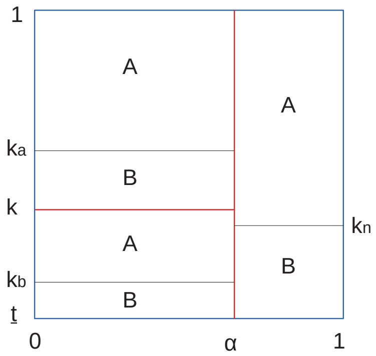


Figure 2.3: Second-period market outcome in case of poaching.

Similarly, the expected profit of firm B in the second period on some $x < 1/2$ is given by:

$$\Pi_B^2(x) = \alpha\left(\frac{\frac{p_{AF}-p_{BH}}{1-2x}-\underline{t}}{1-\underline{t}}\right)p_{BH} + \alpha\left(\frac{\frac{p_{AH}-p_{BF}}{1-2x}-k}{1-\underline{t}}\right)p_{BF} + (1-\alpha)\left(\frac{\frac{p_{AF}-p_{BF}}{1-2x}-\underline{t}}{1-\underline{t}}\right)p_{BF} \quad (2.3)$$

Each firm choose the price p_{iH} in order to maximize the profits on their group of identified own customers and p_{iF} in order to maximize the profits on the unidentified customer groups (which can not be distinguished) jointly.

First period. In the first period, each firm can discriminate consumers only with respect to their location, such that firms can (only) charge one price on each location, we denote as p_A for the home firm and p_B for the rival. Given the expected second-period equilibrium outcome, each firm charge its first-period price so as to maximize its discounted profits over both periods, respectively, while firms value future profits by $\delta \in (0, 1]$.[20] Therefore, in the first period, home and rival firm maximize on each x the following profit functions:

$$\Pi_A(x) = \left(\frac{1-k}{1-\underline{t}}\right) p_A + \delta \Pi_A^2(x) \tag{2.4}$$

$$\Pi_B(x) = \left(\frac{k-\underline{t}}{1-\underline{t}}\right) p_B + \delta \Pi_B^2(x), \tag{2.5}$$

with $k = (p_A - p_B)/(1-2x)$ in the myopic consumers case and a more complex (and within the relatively homogeneous and heterogeneous consumers case varying) expression when consumers are sophisticated, derived within the following analysis.[21]

2.4.1 Relatively Homogeneous Consumers

The following Lemma states the equilibrium of the second period when consumers are relatively homogeneous and analyzes how profits change when customer data of the first period becomes more accurate.

Lemma 2.1. (*Second period. Relatively Homogeneous Consumers.*)
Consider an arbitrary x on the turf of firm $i = A, B$ and assume that consumers are relatively homogeneous.

In equilibrium, firm $i = A, B$ serves all consumers on any x on its turf independently of the accuracy of customer recognition and its first-period market share by charging the prices $p_{iH}(x) = k|1-2x|$ and $p_{iF}(x) = \underline{t}|1-2x|$ to the identified and foreign consumers, respectively. The rival cannot do better than charging $p_{jH}(x) = p_{jF}(x) = 0$, such that firms realize profits $\Pi_i^2(x) = [\alpha(k-1)(k-\underline{t}) + (\underline{t}-1)\underline{t}](1-2x)/(\underline{t}-1)$ and $\Pi_j^2(x) = 0$.

Profits increase monotonically in the recognition accuracy and are given by the inverse u-shaped function of k, which gets a maximum at $k = (1+\underline{t})/2$.

In the second period, firms choose their prices in order to maximize their profits on

[20] In the case of sophisticated consumer, we assume that firms' and consumers' discount factors coincide.
[21] All the omitted proofs are contained in the Appendix.

each consumer group. Solving for the home firm's optimal price for its recognized past customers, we get the best-response function:

$$p_{AH}(x;p_{BF}) = \begin{cases} p_{BF} + k\,(1-2x) & \text{if} \quad p_{BF} \geq (1-2k)\,(1-2x) \\ \frac{p_{BF}+(1-2x)}{2} & \text{if} \quad p_{BF} < (1-2k)\,(1-2x)\,. \end{cases} \tag{2.6}$$

Inspecting (2.6), we can identify two possible strategies: either monopolization or sharing. If the price of the rival is sufficiently high, the home firm's best response is to follow a monopolization strategy by choosing a relatively low price, which allows to serve all recognized past customers. Otherwise, the home firm's optimal response is to share some of the recognized past customers with the rival because serving all of them would require a too substantial price reduction. For relatively homogeneous consumers, i.e. $\underline{t} \geq 1/2$, the condition $p_{BF} \geq (1-2k)\,(1-2x)$ is fulfilled for any p_{BF}, because $k \geq 1/2$ must hold. Therefore, the home firm clearly monopolizes all identified customers in this case, since intuitively (when consumers do not differ a lot among each other) even a small reduction in the price is sufficient to capture all customers, which makes the monopolization strategy efficient irrespective of the price charged by the rival. For the same reason, we arrive at the analog clear monopolization result when considering the home firm's best response on the foreign consumer group under relatively homogeneous consumers, yielding to a zero price of the rival for both consumer groups in equilibrium and the home firm serves all consumers on its turf for any α and k.

Since the home firm monopolizes both consumer groups on its turf either way, meaning for all recognition levels and all given first-period market outcomes including this one with $k = \underline{t}$, implying no revealed flexibility information through the first-period market interaction, we get the straightforward second-period profit implications of α and k, depicted in Figure 2.4.

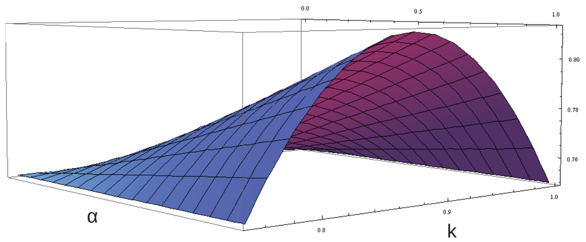


Figure 2.4: Second-period profit pattern as a function of α and k for l=3/2.

The rival charges a price of zero anyhow, such that the home firm can exploit the additional data uncontested. Accordingly, whenever $k \in (\underline{t}, 1)$ such that firms shared consumers in the first period and some flexibility data can be inferred, the second-period profits monotonically increase with the recognition precision, since then more customers (who have shown to be relatively inflexible) are correctly recognized and the home firm can charge the higher home price more often. For a given α, profits are intuitively maximized when firms shared consumers equally in the first period, i.e. $k = (1 + \underline{t})/2$, since consumers are then divided into two equally small groups and the data quality is maximized.

Lemma 2.2. (*First period. Relatively Homogeneous Consumers.*)

Consider some x on the turf of firm $i = A, B$ and assume that consumers are relatively homogeneous. The equilibrium of the first period depend on consumer sophistication and the level of consumer heterogeneity.

*1) Consider **myopic** consumers:*

i) If consumer heterogeneity is relatively small, i.e. $l \leq 1 + [1/(1 + \delta)]$, in equilibrium firm i monopolizes location x with the price $p_i(x) = \underline{t}|1 - 2x|$, while the rival charges the price $p_j(x) = 0$.

ii) Otherwise, there exists $\hat{\alpha}(\delta, l) := [1/(l - 1) - 1]/\delta$, such that in equilibrium firm i lets the rival attract the more flexible consumers and serves only those with $t \geq k =: [(1 + \alpha\delta)(1 + \underline{t})]/(3 + 2\alpha\delta)$, if the recognition precision is high enough, i.e., $\alpha > \hat{\alpha}(\delta, l)$. If $\alpha \leq \hat{\alpha}(\delta, l)$, in equilibrium firm i monopolizes location x. It holds that $\partial\hat{\alpha}(\delta, l)/\partial\delta < 0$ and $\partial\hat{\alpha}(\delta, l)/\partial l < 0$.

*2) Consider **sophisticated** consumers:*

In equilibrium firm i monopolizes location x independently of the level of consumer heterogeneity with the price $p_i(x) = \underline{t}|1 - 2x|$. The rival charges the price $p_j(x) = 0$.

We know from Lemma 2.1 that with relatively homogeneous consumers in the second period any firm monopolizes any location on its turf for any market shares of the first period. This strategy maximizes the home firm's one-period profits. However, given the positive impact of data on the second-period profits, a firm may find it optimal to deviate from this strategy in the first period and let the rival attract some of the more flexible consumers, in order to gain flexibility data about consumers on x and use it for targeting in the following period. It is quite straightforward that this strategy can, first of all, only be profitable if consumers are sufficiently different among each other, i.e. $l > 1 + [1/(1+\delta)]$. Otherwise the variation in consumer flexibility is so low that even a perfect recognition of the first-period purchase decision does not validate possible investments. Given that consumers are sufficiently different, it depends on firm's ability to exploit consumers first-period purchase decision, i.e. α, whether the investment in sharing some consumers in the first period (and thereby the gained customer data) is actually valuable enough. Only when α is (for given δ and l) high enough, we have that the home firm can charge the higher (home) price in the second period to sufficiently many consumers, such that the sharing strategy is overall profitable. Otherwise, the home firm has to stay with its short-run optimal monopolization strategy. Since the actual value of the gained customer data is determined by the interplay of α, δ and l jointly, with all factors favoring the data value, we have that the critical α for the revealing strategy is smaller for higher δ and l. Finally, when consumers are myopic, the home firm shares intuitively more customers with the rival when revealing is already profitable and either α, δ or l increases further, since then (once again) the gained customer data becomes more valuable and the home firm prefers to reveal more of it.

The equilibrium of the first period is different when consumers are sophisticated. Precisely, in this case every firm monopolizes any location on its turf, independently of the level of consumer heterogeneity. This is due to the behavior of the sophisticated consumers, who correctly anticipate that the home firm will serve all consumers on a given location in the second period, regardless of its first-period market share. However, the price a consumer has to pay in the second period depends on whether she bought at the home firm or its rival previously. Precisely, in the former case the price is $p_{AH} = k(1-2x)$ and higher than in the latter case with $p_{AF} = \underline{t}(1-2x)$. While making their purchasing decisions in the first period, sophisticated consumers take this difference into account and the cut-off consumer must be indifferent between buying in both periods from the home firm and therefore paying with the recognition probability α the higher price and buying at the rival in the first period to ensure the low price in the second period, leading to the following condition:[22]

$$p_A + kx + \alpha\delta(p_{AH} + kx) = p_B + k(1-x) + \alpha\delta(p_{AF} + kx) \tag{2.7}$$

[22] In case of being unidentified by the firms, meaning the absence of BBPD, the second period does as usual not impact the first-period purchase decision, since the second-period action is not restricted by the first-period action.

Inserting the second-period prices and solving (2.7) for k, we get:

$$k = \frac{p_A - p_B}{(1-\alpha\delta)(1-2x)} - \frac{\alpha\delta\underline{t}}{1-\alpha\delta} \tag{2.8}$$

Comparing this function with the first-period demand when consumers are myopic, $k = (p_A - p_B)/(1-2x)$, we observe that in the former case demand is more elastic. To get an intuition for this result, it is useful to note that (2.7) can also be rewritten as:

$$k = \frac{p_A - p_B}{1-2x} + \frac{\delta\alpha}{1-2x}(p_{AH} - p_{AF}) \tag{2.9}$$

The first term states the well-known direct effect, indicating a higher market share of the rival in the first period after an increase in the home firm's price p_A. The second term, instead, states the (additional) indirect effect of a home firm's first-period price increase and thereby an increase in k, transferred in the second period. When k increases, the home firm's second-period price for the identified consumers p_{AH} also increases while p_{AF} is unaffected, leading to an additional incentive for the consumers to buy from the rival in the first period after an increase in p_A.

Given the more elastic demand, the home firm can not increase its price that much when targeting a certain k. Therefore, we have that compared to the case with myopic consumers, the home firm faces lower first-period profits when revealing the same amount of data, such that revealing is less attractive with sophisticated consumers. Finally, to understand why the revealing strategy is in fact completely prevented with sophisticated consumers, note that demand becomes more and more elastic for higher values of α and δ, since then the probability of being recognized and getting charged with the higher price in the second period as well as the importance of future prices overall is higher. Therefore, the increasingly positive effects of α and δ on the second-period profits are countervailed by their increasingly negative impact on the first-period profits, driving the clear monopolization result.

Using Lemma 2.1 and 2.2 we are able to calculate total discounted profits over two periods when firms (have the opportunity to) combine behavioral data with location information and can compare these with the profits when firms target only based on location as well as analyze the impact of an improved recognition precision. Proposition 2.1 summarizes our results.

Proposition 2.1. (*Profit effect of customer recognition and impact of an improved recognition precision. Relatively Homogeneous Consumers.*)

The profit effect of combining perfect location information with behavioral flexibility data as well as the impact of an improved recognition precision depends on the level of consumer heterogeneity, recognition precision and the discount factor.

1) Consider ***myopic*** *consumers:*

i) If consumers are very similar in their preference, i.e. $l \leq 1 + [1/(1+\delta)]$*, then profits do not respond to firms' ability to recognize consumers.*

ii) Otherwise, firms benefit from the ability to recognize past customers only if the recognition precision is high enough, i.e. $\alpha > \hat{\alpha}(\delta, l)$*, and total discounted profits increase monotonically in* α*. If* $\alpha \leq \hat{\alpha}(\delta, l)$*, then profits do not respond to firms' ability to collect behavioral data.*

2) Consider ***sophisticated*** *consumers:*

Profits do not change when firms have the opportunity to combine location targeting with behavioral data and the impact of an improved recognition precision is neutral.

Figure 2.5 illustrates Proposition 2.1 by depicting how the total discounted profits over two periods, Π, change with the recognition precision, α, depending on consumer sophistication (we set $l = 10/6$ and $\delta = 1$).

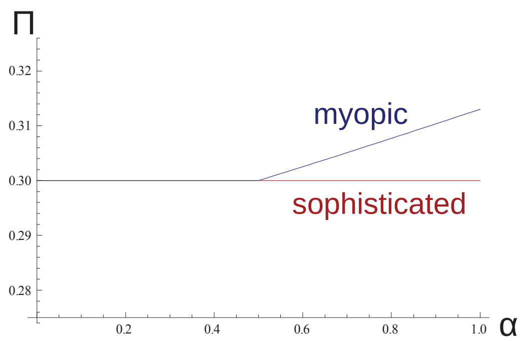


Figure 2.5: Total discounted profits as a function of α depending on consumer sophistication for $l = 10/6$ and $\delta = 1$.

In case of sophisticated consumers, firms strictly monopolize their turf in both periods, since customer recognition is (given the more elastic demand) too costly in the first period. Accordingly, firms never use the ability to reveal flexibility data and profits never react to α, as in the figure. In contrast, when consumers are myopic, firms (at least potentially) invest in some information, if the gained customer data is sufficiently valuable. Given $\underline{t} = 0.6$ and $\delta = 1$, we have that $l > 1 + [1/(1+\delta)]$, such that consumers are sufficiently heterogeneous to make the sharing strategy profitable when the recognition technology is sufficiently precise, i.e., $\alpha > 1/2$ in this example. Accordingly, we see that profits increase above the level when no behavioral data would be available to firms, when consumers are myopic and $\alpha > 1/2$, since then investing in customer recognition becomes beneficial, while profits consequently further increase with an improved recognition precision in this case due to more and more efficient first-period customer sharing. Otherwise, profits are also with myopic consumers constant and unaffected by the recognition precision, since firms then (once again) deny to invest in the data and nothing changes compared to the case when firms would not be able to collect additional data.[23]

2.4.2 Relatively Heterogeneous Consumers

Previous literature on behavioral targeting (see e.g., Fudenberg and Tirole, 2000 or Colombo, 2016) established that in the second period poaching takes place, such that some of the identified customers of each firm buy in the second stage from the other firm. For our analysis with relatively heterogeneous consumers, i.e. $t \in [0, 1]$, we will for now also assume that poaching takes place in equilibrium and show afterwards that this is indeed the case.

Lemma 2.3. (*Second period. Relatively Heterogeneous Consumers.*)

Consider an arbitrary x on the turf of firm $i = A, B$ and assume that consumers are relatively heterogeneous. In the poaching equilibrium where each firm loses some of its identified customers of the first period, firms charge the prices:

$$\begin{aligned}
p_{iH}(x) &= \frac{(8 - \alpha(2 + (2+\alpha)k))|1 - 2x|}{3\,(4 - \alpha^2)} \\
p_{jH}(x) &= \frac{(4 + 2\alpha(-2 + k) - \alpha^2 k)\,|1 - 2x|}{3\,(4 - \alpha^2)} \\
p_{iF}(x) &= \frac{(8 + \alpha(-8 + 2(2+\alpha)k))|1 - 2x|}{3\,(4 - \alpha^2)} \\
p_{jF}(x) &= \frac{(4 - \alpha(4 - 3\alpha + 4k + 2\alpha k))|1 - 2x|}{3\,(4 - \alpha^2)}
\end{aligned}$$

[23] Given the positive impact of δ and l on the value of customer data, the upward-sloping part of the myopic profit pattern would move to the left (right) and become steeper (flatter) with higher (lower) values of δ and l.

Profits of firm i (j) decrease monotonically in α for $k > 0.47$ ($k > 0.24$) and are u-shaped in α otherwise.

Profits of firm i (j) increase (decrease) monotonically in k for $\alpha \leq 0.76$ and are u-shaped in k otherwise with the minimum at $k = (-16+24\alpha-2\alpha^2)/(8\alpha+6\alpha^2+\alpha^3)$ $\big(k = (8-12\alpha+10\alpha^2)/(8\alpha+6\alpha^2+\alpha^3)\big)$.

Since firms do not monopolize their turf in this second-period equilibrium, we have that the impact of more accurate customer data on firms' profits in the second period is more complex than with relatively homogeneous consumers. Considering the impact of α, we first have to note that, intuitively, p_{AF} strictly decreases in α, since it targets relatively more flexible consumers when the recognition precision increases. This clearly induces firm B to also reduce p_{BH}, however, the reaction of p_{BF} is more differentiated. The foreign price of firm B is given by, $p_{BF} = [\alpha\ p_{AH} + (1-\alpha)\ p_{AF} - \alpha k(1-2x)]/2$, and a higher α has thereby two effects. First, it decreases p_{BF} due to the decrease of p_{AF}. However, for an increasing α, firm B puts more emphasis on p_{AH} when setting p_{BF}, since it then competes with this price against p_{AH} on a relatively larger interval. Accordingly, the negative impact of the decreasing p_{AF} diminishes in α, while p_{AH} is clearly higher than p_{AF}, such that the level of p_{BF} increasingly benefits from putting more emphasis on the former price. Whether the positive effect actually dominates at some point, such that p_{BF} and thereby p_{AH} increase when the recognition precision improves further, depends intuitively on k. Next to α, this is the second parameter determining the size of the pricing areas and when k is high, p_{BF} still competes very often against p_{AF} even for high α, holding the negative and positive effect on a high and respectively low level. Instead, when k is low, every increase in α enlarges the area where p_{BF} competes against p_{AH} a lot, such that the negative and positive effect diminishes and respectively increases relatively fast. Accordingly, we have that k has to be sufficiently low to find that the positive effect outweighs the negative one at some point, such that p_{BF} and thereby p_{AH} increase when the recognition precision improves further (while p_{AF} and thereby p_{BH} still decrease in α). Given that for high k all prices decrease in α, while for sufficiently small k some of them are u-shaped and additionally the importance of these prices on firms' profits increase with smaller k, since the area where these prices are active enlarges, we have that firms' profits first monotonically decrease in α for high k, while they are u-shaped for sufficiently small

k and become increasingly more convex when k decreases. The profits of firm A start to recover earlier, since a lower k (once again) puts more emphasis on p_{AH} and p_{BF}. While p_{AH} is clearly higher than p_{AF}, we have that p_{BF} can be higher than p_{BH} due to above, however, this is only the case when α is high and k sufficiently low, leading to the fact that the profits of firm B require a lower k to follow an u-shaped pattern in α.

Considering the impact of k, the fact that p_{BF} is higher than p_{BH} when α is high and k sufficiently low becomes important. Firms intuitively prefer to play against the higher price of the rival more often. For relatively small α, we have that p_{BF} is higher than p_{BH} for any k, such that firm A strictly prefers a higher k to play against the higher p_{BH} more often. This is additionally supported by the fact that when α is small, firm A's profits depend to a large extend on p_{AF}, which intuitively increases in k, since it then targets more inflexible consumers. With the same intuition, we have that firm B prefers a lower k when α is relatively small, such that firms' interests are opposed in this case. However, this is not necessarily the case for high α. When α is high, there is a certain k up to which p_{BF} is higher than p_{BH}. Accordingly, firm A prefers a further decreasing k when k is initially low and a further increasing k otherwise. Similarly, also firm B prefers a further decreasing k when α is high and k sufficiently low in order to play against the higher p_{AH} with the in this case higher p_{BF} more often. Further in line with firm A, firm B prefers an increasing k when k is initially very high, since then p_{BH} is much higher than p_{BF} and firm B prefers to use this price more often, while the interest to play against the higher p_{AH} is mitigated due to the fact that p_{AF} is with high k also very high. Consequently, we have that both firms' second-period profits are u-shaped in k and firms' interests with respect to a change in k often aligned, when α is high. Note finally, that when α is high, the minima in firms' profit pattern in k are both around $k = 0.4$, such that both firms prefer an decreasing k when it is below this threshold. Interestingly, this threshold is clearly above the one-period profits maximizing first-period cut-off $k = 1/3$.

Lemma 2.4. (*First period. Relatively Heterogeneous Consumers.*)

Consider an arbitrary x on the turf of firm $i = A, B$ and assume that consumers are relatively heterogeneous. In equilibrium, firms share the consumers on x in the first stage. The equilibrium market share of firm i depend on consumer sophistication.

1) If consumers are ***myopic****, firm i serves those consumers with:*

$$t \geq k =: \frac{-36 + \alpha(36 - 8\delta) + \alpha^2(-9 + 16\delta)}{-108 + 108\alpha + 4\alpha^3\delta + \alpha^2(-27 + 16\delta)}$$

2) If consumers are ***sophisticated****, firm i serves those consumers with:*

$$t \geq k =: \frac{-36 + 30\alpha^3\delta + 4\alpha(9 + 10\delta) - \alpha^2(9 + 68\delta)}{-108 + 67\alpha^3\delta + 108\alpha(1 + \delta) - \alpha^2(27 + 164\delta)}$$

In contrast to the case when consumers are relatively homogeneous, firms always charge prices at which some information on consumer flexibility is revealed when consumers are relatively heterogeneous, independently of consumer sophistication. This is due to the high level of consumer heterogeneity ($\underline{t} = 0$), such that the home firm had to charge a price of zero to serve all consumers, which makes the monopolization strategies inefficient and led firms opt for a sharing strategy. By inspecting the first-period cut-off k in both cases, we recognize that they fulfill the necessary conditions for poaching to occur in the second period. However, to ensure a non-negative market share of firm B on the general unidentified customers in the second period when consumers are myopic, we have to ensure that $\alpha < 0.967$, such that we restrict the following analysis on $\alpha \in [0, 0.966]$ when consumers are myopic.[24]

Figure 2.6 depicts firms' first-period market shares (given by k) in α. Intuitively, we find that both are starting in $k = 1/3$ for $\alpha = 0$, given that this maximizes the one-period profits.

Considering the further pattern of k for higher α first for the case when consumers are myopic, we perfectly recognize the revealed impact of k on the second-period profits. We know from Lemma 2.3 that firms' interests with respect to k in the second period are opposed for relatively small α. Accordingly, we find that the equilibrium k in the first period does not vary much from the one-period maximum.

[24]Note that it would be sufficient to ensure that $\delta \leq [3\left(12 - 16\alpha + 5\alpha^2\right)]/4a^3$, such that one could support the full (or a higher) range of α when lowering the range of δ accordingly.

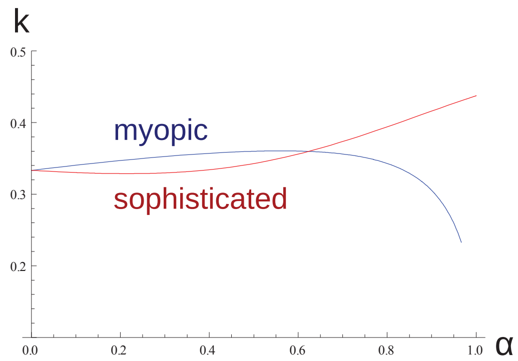


Figure 2.6: First-period market cut-off as a function of α depending on consumer sophistication for $\delta = 1$.

Firm A raises its price to increase k which also induces firm B to raise its price, however, firm B ensures that k does not increase too much. We also know from the previous second-period analysis that for high α, both firms prefer a lower k in the second period, provided that $k < 0.4$ which is fulfilled. Accordingly, firms' interests are aligned and both firms set their first-period price to strategically decrease k in this area, leading to the massive drop of k at the end of the α-distribution depicted in Figure 2.6 and the overall inverse u-shaped pattern. Finally, since the importance of these second-period profit considerations clearly increases with firms' valuation of future profits, we have that the pattern of k in α becomes more concave in δ. For $\delta = 0$, we would intuitively observe a constant k on the one-period profit maximum.

The pattern of the equilibrium k in α is very different when consumers are sophisticated. In this case, the effect of k on the second-period profits is complemented by consumers' strategic purchase decision in the first period, driving this difference. Sophisticated consumers correctly anticipate that the cut-off consumer gets poached in the second period, if she gets recognized. Therefore, the cut-off consumer must be indifferent between buying at firm A in the first period and then for the poaching price from firm B in the second period if she gets recognized, or the other way around, leading to the following condition:[25]

$$p_A + kx + \delta\alpha(p_{BF} + k(1-x)) = p_B + k(1-x) + \delta\alpha(p_{AF} + kx) \tag{2.10}$$

[25] Once again, in case of being unidentified by the firms, meaning the absence of BBPD, the second period does not impact the first-period purchase decision, since the second-period action is not restricted by the first-period action.

Inserting the second-period prices and solving (2.10) for k, we get:

$$k = \frac{(6-3\alpha)(p_A - p_B)}{(6+\alpha(-3+(-6+7\alpha)\delta))(1-2x)} + \frac{\alpha(3\alpha-2)\delta}{6+\alpha(-3+(-6+7\alpha)\delta)} \tag{2.11}$$

Analog to the case when consumers are relatively homogeneous, we recognize that this function looks very different compared to the first-period demand when consumers are myopic, $k = (p_A - p_B)/(1-2x)$. The additional factor, independent from the first-period prices, captures the effect of the second-period poaching prices on the sophisticated consumer's first-period choice and determines the u-shaped pattern of k in α. Precisely, we know from Lemma 2.3 that p_{AF} strictly decreases in α. Also p_{BF} decreases (firstly) in α, while the reaction is at first even higher than the reaction of p_{AF}. Accordingly, consumers at first increasingly prefer to buy from firm A, to pay the increasingly cheaper poaching price of firm B in the second period, lowering the first-period market share of firm B. However, due to the increasingly positive reaction of p_{BF} to higher values of α discussed above, we find that the reactions of the poaching prices increasingly turn in favor of p_{AF} in consumers' considerations. Accordingly, it is increasingly attractive for consumers to buy from firm B in the first period, to pay the poaching price of the home firm in the second period. This induces k to increase for higher values of α, countervailing and in fact dominating the second-period profit effect of k, such that the equilibrium k raises in α for sufficiently high α. Once again, since the importance of these second-period price considerations clearly increases with consumers' valuation of future surplus, we have that the pattern of k in α becomes more convex in δ. For $\delta = 0$, we would intuitively observe that the pattern in case of sophisticated and myopic consumers coincide on the one-period profit maximum.

Considering (2.11) further and also in line with the case when consumers are relatively homogeneous, we recognize that the first-period demand elasticity is very different compared to the constant first-period demand elasticity when consumers are myopic. When consumers are sophisticated, we find that $|\partial k/\partial p_i|$ is inverse u-shaped in α, such that for an increasing α starting at $\alpha = 0$, first-period demand becomes first more sensitive to price variations, while this effect is reversed in sufficiently high ranges of α. To get an intuition for this result, note that (2.10) can also be rewritten as:

$$k = \frac{p_A - p_B}{(1-\alpha\delta)(1-2x)} + \frac{\alpha\delta}{(1-\alpha\delta)(1-2x)}(p_{BF} - p_{AF}) \tag{2.12}$$

Once again, we recognize the direct effect in the first term, indicating a higher k after a price increase in p_A. Turning to the indirect effect of a higher k transferred in the second period, we first have to recall from the previous analysis that p_{BF} is decreasing in k while p_{AF} increases.[26] Since the marginal consumer switches firms in the second period if she gets recognized, these price reactions reduce her incentives to switch to firm B in the first period after an increase in p_A. Consequently, we have a positive direct and a negative indirect effect, while we can note by considering (2.12) that both effects raise in α. Due to the fact that the direct effect firstly dominates for sufficient low values of α before the indirect effect starts to dominate for a sufficiently precise recognition, we observe an inverse u-shaped price sensitivity pattern in α.

Due to this inverse u-shaped price sensitivity, the first-period prices follow an u-shaped pattern in α when consumers are sophisticated. Interestingly, we also find that they exceed their initial level at some point. Therefore, we have that the famous finding in the literature (e.g., in Fudenberg and Tirole, 2000), that first-period prices are higher under behavior-based price discrimination than under uniform pricing when consumers are sophisticated, relies on their assumption of perfect recognition. In general, first-period prices are only higher than these under uniform pricing when the recognition is sufficiently precise. Additionally, when consumers are myopic, Fudenberg and Tirole (2000) and many other contributions find that prices in these two schemes coincide on the one-period maximum. We find instead, that they are inverse u-shaped in α and the home firm's price drops below the initial value for sufficiently high α, i.e., below the one-period maximum, while the rival's price stays above this threshold.[27] Accordingly, we get a more complex pricing result compared to the usual findings also when consumers are myopic, arising from the fact that in our setting firms manage to change the prices strategically.

Using Lemma 2.3 and 2.4 we are once again able to calculate total discounted profits over two periods when firms combine behavioral data with location information and can compare these with the profits when firms target only based on location as well as analyze the impact of an improved recognition precision. Proposition 2.2 summarizes our results.

[26]Intuitively, p_{AF} increases in k since it targets more inflexible consumers, while firm B reduces its emphasis on the higher p_{AH} when setting p_{BF}, such that p_{BF} decreases in k.

[27]Note interestingly that for sufficiently large δ, the rival's first-period price raises again in α when α is sufficiently high. This is driven by the analyzed incentive to decrease k in this case.

Proposition 2.2. (*Profit effect of customer recognition and impact of an improved recognition precision. Relatively Heterogeneous Consumers.*)

The ability to recognize past customers and combining perfect location information with behavioral flexibility data is always detrimental for firms. The impact of an improved recognition precision depend on consumer sophistication and firm discount factor.

*1) Consider **myopic** consumers:*

i) If $\delta < 0.82$*, then profits decrease monotonically in the recognition precision, such that firms are always worse off with an improved recognition precision.*

ii) Otherwise, profits are (slightly) u-shaped in the recognition precision, such that firms only benefit from an improved recognition precision when α *is very high and are worse off otherwise.*

*2) Consider **sophisticated** consumers:*

Profits are, independently of δ*, u-shaped in the recognition precision, such that firms only benefit from an improved recognition precision when* α *is sufficiently high and are worse off otherwise.*

The intuition behind the result in case consumers are sophisticated is quite intuitive. We know from Lemma 2.3 that each firm's second-period profits in α follow an u-shaped distribution for $k \leq 0.47$ on a location of its own turf and for $k \leq 0.24$ on a location of the rival's turf. Considering firms' total second-period profits over all locations, we find that they follow an u-shaped distribution for $k \leq 0.41$, which is always fulfilled in equilibrium, such that second-period profits are generally u-shaped in α. In combination with the inverse u-shaped demand elasticity to first-period price variations when consumers are sophisticated, we get a strengthening of this u-shaped relationship in α, leading to the clear result. This strengthening effect is missing when consumers are myopic. Instead, first-period profits are inverse u-shaped in this case due to the revealed inverse u-shaped first-period pricing pattern. Therefore, we only get an overall u-shaped pattern in α for sufficiently high values of δ, such that more emphasis is put on the second-period profits and the u-shaped second-period relationship is thereby not too moderated. Additionally, we know from the first-period analysis that the u-shaped second-period relationship is strengthened when k is smaller. However, when δ is small, firms do not manipulate k that much downwards from the one-period maximum. This additionally mitigates the u-shaped effect for small δ, such that firms' profits are monotonically decreasing in α when consumers are myopic and δ low. Since the magnitude of the inverse u-shaped first-period relationship also decrease with lower δ, we have that this effect can only offset the u-shaped second-period relationship and never translates into the two-period discounted profit pattern, even for small δ.

Finally, since profits in the u-shaped cases never recover to their initial level, we get the result that the ability to recognize past customers and combining location targeting with behavioral customer data is always detrimental for firms, which can also be seen in Figure 2.7.

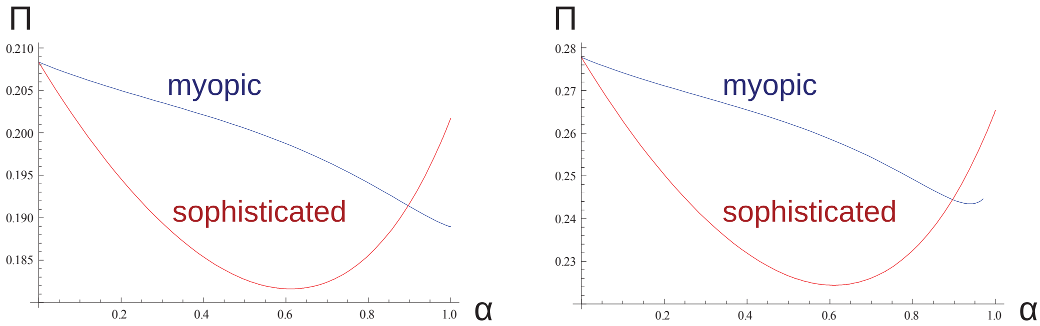


Figure 2.7: Total discounted profits as a function of α depending on consumer sophistication for $\delta = 1/2$ and $\delta = 1$.

Given the equilibrium outcomes when consumers are relatively homogeneous and heterogeneous, we can now turn to their welfare implications, while we consider the interesting intersection of the profit pattern in Figure 2.7 in Section 2.6.

2.5 Welfare

In the following section, we analyze the social welfare and consumer surplus implications of different levels of recognition accuracy. Given the market coverage in our Hotelling-setup, total social welfare is determined by the transport costs, while consumer surplus also depends on firms' profits.

2.5.1 Relatively Homogeneous Consumers

It follows from Proposition 2.1 that we have to distinguish between two cases. First, if either consumers are sophisticated or myopic but $\alpha \leq \hat{\alpha}(\delta, l)$, then every firm serves all consumers on any location on its turf in both periods, which yields the discounted social welfare over two periods:

$$W = v(1+\delta) - \frac{2(1+\delta)}{1-\underline{t}} \int_{\underline{t}}^{1} \int_{0}^{1/2} (tx)\, dxdt. \tag{2.13}$$

Otherwise, every firm shares some consumers on any location on its turf in the first period, such that welfare over two periods is given by:

$$\begin{aligned} W = v(1+\delta) \quad - \quad & \left[\frac{2}{1-\underline{t}} \left(\int_{\underline{t}}^{k} \int_{0}^{1/2} (t(1-x))\, dxdt + \int_{k}^{1} \int_{0}^{1/2} (tx)\, dxdt \right) \right. \\ & \left. + \frac{2\delta}{1-\underline{t}} \int_{\underline{t}}^{1} \int_{0}^{1/2} (tx)\, dxdt \right], \end{aligned} \tag{2.14}$$

with $k = (p_A - p_B)/(1-2x)$. Subtracting the equilibrium profits from social welfare yields consumer surplus.

The following Proposition summarizes how social welfare and consumer surplus change when firms (have the opportunity to) combine location targeting with behavioral data and states the impact of an improved recognition precision.

Proposition 2.3. (*Welfare effect of customer recognition and impact of an improved recognition precision. Relatively Homogeneous Consumers.*)

The social welfare and consumer surplus effect of firms' ability to combine perfect location information with behavioral flexibility data as well as the impact of an improved recognition precision depend on the level of consumer heterogeneity, the level of recognition precision and the discount factor.

1) Consider ***myopic*** *consumers:*

i) If consumers are very similar in their preference, i.e. $l \leq 1 + [1/(1+\delta)]$*, social welfare and consumers surplus do not respond to firms' ability to recognize past customers and the impact of an improved recognition precision is neutral, irrespective of the level of* α*.*

ii) Otherwise, firms' ability to recognize past customers is only detrimental for social welfare and consumer surplus if the recognition precision is high enough, i.e. $\alpha > \hat{\alpha}(\delta, l)$*, in which case the impact of an improved recognition precision is strictly negative. If* $\alpha \leq \hat{\alpha}(\delta, l)$*, then social welfare and consumer surplus do not respond to firms' ability to collect behavioral data and the impact of an improved recognition precision is neutral.*

*2) Consider **sophisticated** consumers:*

Social welfare and consumer surplus do not change when firms have the opportunity to combine location targeting with behavioral data and the impact of an improved recognition precision is neutral.

The intuition of Proposition 2.3 is straightforward. According to Proposition 2.1, the home firm serves for all values of α all consumers on her turf in both periods, whenever consumers are sophisticated or myopic but $\alpha \leq \hat{\alpha}(\delta, l)$. Therefore, transport costs and prices do not change with α and social welfare as well as consumer surplus stay constant. Instead, if consumers are myopic and $\alpha > \hat{\alpha}(\delta, l)$, the home firm shares some consumers in the first period and the market share of the rival raises with α. Therefore, welfare decreases in α due to a higher fraction of consumers incurring higher transport costs through buying at the more distant firm in the first period. Additionally, prices in the first period raise in α and more identified customers of the home firm have to pay a higher price in the second period. This leads to an additional negative effect of α on consumer surplus and strengthens the welfare drop, as depicted in Figure 2.8.

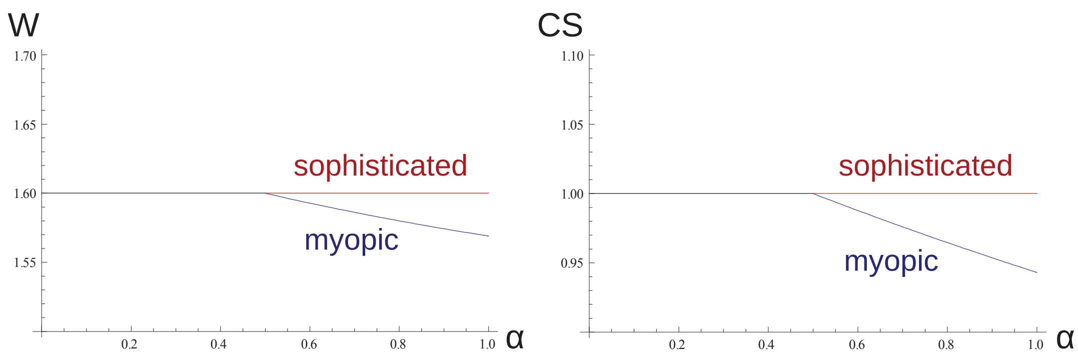


Figure 2.8: Total discounted social welfare and consumer surplus as a function of α for $v = 1$, $l = 10/6$ and $\delta = 1$.

2.5.2 Relatively Heterogeneous Consumers

When consumers are relatively heterogeneous, then firms share consumers on any location in equilibrium in both periods, which yields the discounted social welfare over two periods:

$$
\begin{aligned}
W = v(1+\delta) \quad - \quad & \Bigg[2\left(\int_0^k \int_0^{1/2} (t(1-x))\,dxdt + \int_k^1 \int_0^{1/2} (tx)\,dxdt\right) \\
& + 2\delta\Bigg(\alpha \int_{ka}^1 \int_0^{1/2} (tx)\,dxdt + \alpha \int_k^{ka} \int_0^{1/2} (t(1-x))\,dxdt \\
& + \alpha \int_{kb}^k \int_0^{1/2} (tx)\,dxdt + \alpha \int_0^{kb} \int_0^{1/2} (t(1-x))\,dxdt \\
& + (1-\alpha) \int_{kn}^1 \int_0^{1/2} (tx)\,dxdt + (1-\alpha) \int_0^{kn} \int_0^{1/2} (t(1-x))\,dxdt\Bigg)\Bigg],
\end{aligned} \tag{2.15}
$$

where ka, kb and kn are the second-period cut-offs in the different customer groups, i.e., identified customers of firm A, firm B and the general non-identified customers. These cut-offs differ in the myopic and sophisticated consumers case. Subtracting the equilibrium profits from social welfare yields once again consumer surplus.

The following Proposition summarizes how social welfare and consumer surplus change when firms combine location targeting with behavioral data and states the impact of an improved recognition precision when consumers are relatively heterogeneous.

Proposition 2.4. (*Welfare effect of customer recognition and impact of an improved recognition precision. Relatively Heterogeneous Consumers.*)

The social welfare and consumer surplus effect of firms' ability to combine perfect location information with behavioral customer data as well as the impact of an improved recognition precision depend on consumer sophistication and the discount factor.

1) Consider ***myopic*** *consumers:*

i) If $\delta \leq 0.99$, social welfare and consumer surplus increase monotonically in the recognition precision, such that firms' ability to recognize past customers is strictly beneficial and the impact of an improved recognition precision strictly positive.

ii) Otherwise, social welfare still monotonically increases while consumer surplus is (slightly) inverse u-shaped in the recognition precision, such that an improved recognition precision is only beneficial for consumer surplus when α is not too high

and detrimental otherwise. Firms' ability to recognize consumers overall is strictly beneficial for social welfare and consumer surplus.
2) Consider ***sophisticated*** *consumers:*

Social welfare and consumer surplus are, independently of δ*, inverse u-shaped in the recognition precision, such that an improved recognition precision is only beneficial when* α *is sufficiently low and detrimental otherwise. Firms' ability to recognize consumers overall is strictly beneficial for consumer surplus, while it is from a social welfare perspective beneficial for low values of* α *and detrimental otherwise.*

The effects are once again directly related to the previous analyzed equilibrium outcome. The equilibrium outcomes for the different levels of α when consumers are myopic lead to a monotonically decrease in transport costs and thereby an monotonically increase in social welfare.[28] Instead, consumer surplus drops at the end of the α-distribution for very high values of δ since, according to Proposition 2.2, prices and thereby profits recover in this area and consumers have to pay more. Due to the combination of this price effect with the monotonically increase in social welfare, we get that the (slight) drop only occurs for $\delta > 0.99$ despite the fact that profits start to recover at the end of the α-distribution already at $\delta \geq 0.82$. Within these two values, the positive welfare effect strictly dominates the negative price effect in consumer surplus. Finally, since the drop is only very small, we have that the overall effect of firms' ability to include behavioral data is strictly positive from a social welfare and consumer surplus perspective.

When consumers are sophisticated, the equilibrium outcomes for the different levels of α lead to an u-shaped pattern in transport costs and thereby an inverse u-shaped pattern in social welfare.[29] Due to the combination with the u-shaped profit function in α, according to Proposition 2.2, this inverse u-shaped relationship clearly also holds for the effect of α on consumer surplus. However, since u-shaped profits at the end of the α-distribution do not fully recover their initial losses, we find that the consumer surplus pattern is lifted at the end of the distribution.[30] Actually, this

[28]Note interestingly that although k is firstly increasing in α, this effect is completely compensated by the decreasing sum of transport costs in the second period due to this new first-period outcome.

[29]In this case, k is firstly decreasing in α while it is rapidly increasing afterwards. Therefore we find that both, the initially positive first-period effect as well as the negative first-period effect for medium or high values of α can not be vanished by the second-period results due to this new first-period outcome and thereby translate into the respective transport cost changes.

[30]This is once again in line with the finding of Colombo (2016) as his model refers to this special case of relatively heterogeneous and sophisticated consumers in our model. Note however, that we

leads to the result that the inverse u-shaped consumer surplus pattern never crosses their initially level at $\alpha = 0$, while the inverse u-shaped social welfare function always does, such that the effect of including behavioral data is only strictly positive from a consumer surplus perspective, as depicted in Figure 2.9.[31]

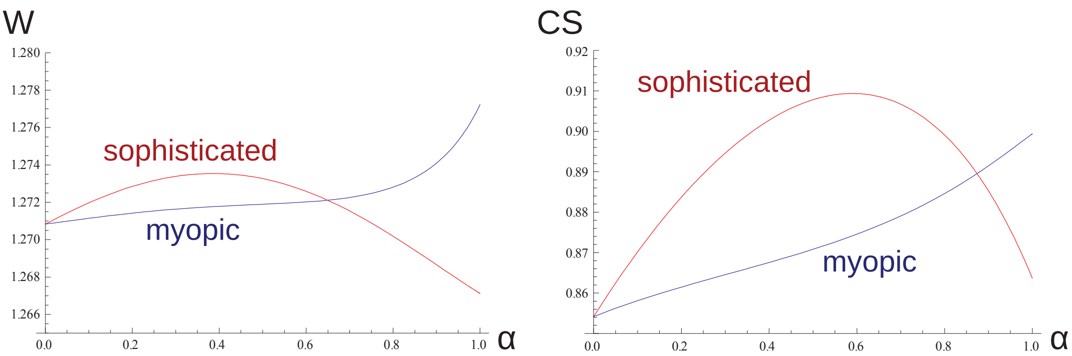


Figure 2.9: Total discounted social welfare and consumer surplus as a function of α for $v = 1$ and $\delta = 1/2$.

Very interesting to note is that the effects of an increasing recognition precision from the firms' and welfare perspective are not necessarily opposed. When consumers are relatively heterogeneous and myopic, firms prefer a higher recognition precision at the end of the α-distribution for $\delta \geq 0.82$, while a higher recognition is strictly beneficial for social welfare. Also consumer surplus increase monotonically in α for $\delta \leq 0.99$ (and even for $\delta > 0.99$ we find that the minimum of the profit function is below the consumer surplus maximum), such that we once again find possible analogies with the effect of α on firms' profits. Similarly, there are some analogies when recognition precision decreases and consumers are relatively heterogeneous and sophisticated.

Analog to the profit pattern in Figure 2.7, we observe in Figure 2.9 that the social welfare and consumer surplus pattern in case of myopic and sophisticated consumers intersect at some α. The following section considers this observation intensively.

get a different result with respect to social welfare, where he finds a monotonically negative pattern. This is due to the fact that the firms in our model achieve to manipulate the prices strategically, which changes the equilibrium outcome and thereby transport cost, determining social welfare.

[31] That the overall effect of firms' ability to recognize consumer on social welfare depends on the specific recognition precision is in fact unique to this benchmark in our analysis.

2.6 Comparison of the Equilibria with Myopic and Sophisticated Consumers

In this section, we analyze how firms' profits and welfare depend on consumer sophistication. Comparing the discounted sum of profits and welfare for given parameters α, δ and l for the cases of myopic and sophisticated consumers, we are able to answer the question whether firms or a social agent have an incentive to educate consumers and inform them about the use of behavioral data collected in the first period for targeting in the subsequent period. Precisely, we introduce the stage "0" in the first period, where each firm or a social agent can inform consumers about the usage of their behavioral data, in which case all of them become sophisticated.
The following Corollary summarizes our results for firms' education incentives.

Corollary 2.1. (*Firms' incentives to educate consumers.*)

Firms' incentives to educate consumers depend on the level of consumer heterogeneity and the recognition precision.
1) If consumers are ***relatively homogeneous****, then the discounted sum of profits over two periods is (weakly) higher when consumers are myopic, such that firms have no incentive to educate them.*
2) If consumers are ***relatively heterogeneous****, then there exits a cut-off $\alpha^P(\delta)$, such that profits are higher under myopic consumers for $\alpha \leq \alpha^P(\delta)$ and lower otherwise. Therefore, firms want to educate consumers when α is sufficiently large.*

The intuition for the result when consumers are relatively homogeneous follows straightforward from the clear profit effects in Proposition 2.1. We showed that with relatively homogeneous consumers, firms' profits are the same under myopic and sophisticated consumers except for the case $\alpha > \hat{\alpha}(\delta, l)$, in which firms realize higher profits when consumers are myopic. Therefore, we find that firms have no incentive to educate consumers in this case. More interesting is the result that when consumers are relatively heterogeneous, firms in fact are better off with sophisticated consumers for sufficiently large α and therefore can potentially be interested in educating them, as depicted in Figure 2.7. This result stems from the different first-period price sensitivity pattern in these cases. When consumers are myopic, first-period price sensitivity is constant in α, while it is inverse u-shaped in α when consumers are sophisticated, depicted in Figure 2.10.

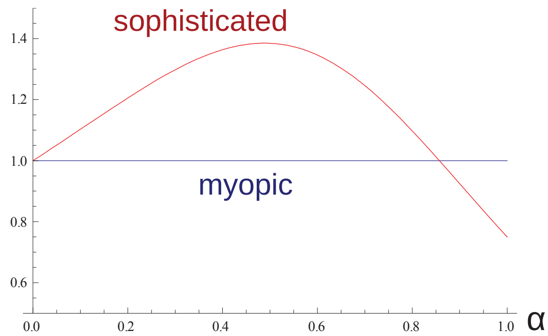


Figure 2.10: First-period price sensitivity as a function of α on $x = 0$ for $\delta = 1$.

It can be clearly seen that for high α the strength of the negative indirect effect is high enough to push the first-period price sensitivity when consumers are sophisticated significantly below the level when consumers are myopic, leading to higher first-period prices when consumers are sophisticated and the result in Corollary 2.1.

The following Corollary summarizes our results from the social perspective.

Corollary 2.2. (*Social incentives to educate consumers.*)

A social agent's incentives to educate consumers depend on the level of consumer heterogeneity and the recognition precision.
*1) If consumers are **relatively homogeneous**, then discounted social welfare and consumer surplus over two periods are (weakly) higher when consumers are sophisticated, such that a social agent has a clear incentive to educate them.*
*2) If consumers are **relatively heterogeneous**, then there exits a cut-off $\alpha^W(\delta)$, such that social welfare is higher under sophisticated consumers for $\alpha < \alpha^W(\delta)$ and lower otherwise. Additionally, there exits a cut-off $\alpha^{CS}(\delta)$, such that consumer surplus is higher under sophisticated consumers for $\alpha < \alpha^{CS}(\delta)$ and lower otherwise. Therefore, a social agent want to educate consumers only when α is sufficiently low.*

The intuition for the result when consumers are relatively homogeneous are, once again, straightforward and directly related to the previous Corollary. Only when consumers are myopic, the home firm possibly follows a profitable sharing strategy, leading to more transport cost and higher prices for the consumers, such that there is a clear social interest to educate consumers to prevent this scenario when consumers are relatively homogeneous. More interesting are once again the results when consumers are relatively heterogeneous, arising from the interplay of welfare and profit effects in α. When consumers are myopic, welfare increases monotonically in α, while it is inverse u-shaped for sophisticated consumers. Additionally, we have that the initial increase for low α is higher when consumers are sophisticated, leading to a preferred education in this situation. However, due to the massive increase of k for high α when consumers are sophisticated, we have that the inverse u-shaped pattern in this case crosses the initial value for $\alpha = 0$, such that this education incentive is clearly reversed at some point, leading to $\alpha^W(\delta)$. The same pattern holds intuitively also for the consumer surplus analysis due to the interplay of this welfare effect with the profit comparison in Corollary 2.1, indicating higher prices for sophisticated consumers for high values of α. Since for high α social welfare is higher and prices lower with myopic consumers, we find that consumer education is once again only preferred for sufficiently low α, leading to $\alpha^{CS}(\delta)$.

Very interesting to note is that the education incentives in both Corollaries are not necessarily opposed. The thresholds $\alpha^P(\delta)$, $\alpha^W(\delta)$ and $\alpha^{CS}(\delta)$ vary only slightly in δ, such that $\alpha^P \approx 0.89$, $\alpha^W \approx 0.65$ and $\alpha^{CS} \approx 0.87$. Therefore, there is an interval where firms and a social agent, both from a welfare and a consumer surplus perspective, do not want to educate consumers.

Instead, with respect to the general effect of combining behavioral data with location targeting, we find that firms' and consumers' incentives are always opposed. Firms can generally benefit from this opportunity only when consumers are relatively homogeneous, while this opportunity is always detrimental for consumer surplus in this case and vice versa.

2.7 Conclusion

In this paper, we analyzed the impact of imperfect customer recognition on firms' profits, social welfare and consumer surplus within a behavior-based price discrimination setting, in which consumers vary along two dimensions while firms hold perfect information on one of these dimensions (consumer's location). Within our flexible set-up, we varied consumer heterogeneity (in flexibility) and sophistication and have thereby identified various different effects of firms' ability to recognize past customer and combine behavioral data with location targeting as well as an increasing recognition precision.

For *relatively homogeneous* consumers: We find that profits strictly increase in the recognition precision, while social welfare and consumer surplus strictly decrease provided that consumers are myopic and recognition sufficiently precise. Otherwise, profits and welfare do not respond to firms' ability to recognize consumers and the impact of an improved recognition precision is neutral.

For *relatively heterogeneous* consumers: We find that the ability to recognize customers is overall always detrimental for firms. When consumers are myopic, profits are strictly decreasing in the recognition precision for $\delta < 0.82$ while they are (slightly) u-shaped otherwise without exceeding their initial level. Additionally, social welfare strictly increase in the recognition precision while consumer surplus strictly increase for $\delta \leq 0.99$ and follows an (slight) inverse u-shaped pattern otherwise without dropping below its initial level. When consumers are sophisticated, profits are generally u-shaped in the recognition precision without exceeding their initial level, while social welfare is inverse u-shaped and drops below its initial value when the recognition is sufficiently precise. Finally, consumer surplus is inverse u-shaped without dropping below its initial level.

The various effects demonstrate that firms and policy makers need to consider the market environment very carefully when predicting possible effects or aiming to take actions. Depending on the specific situation, an improved recognition precision or an education of consumers can have very different effects. Therefore, our results are giving scope for a careful firm and consumer policy, while firms' and social interests are not necessarily opposed.

2.8 Appendix

Proof of Lemma 2.1. As firms are symmetric, we will restrict attention to the turf of firm A. Consider some $x < 1/2$. The expected profit of firm A on x is given by,

$$\Pi_A^2(x) = \alpha \left(\frac{1 - \frac{p_{AH} - p_{BF}}{1-2x}}{1-\underline{t}} \right) p_{AH} + \alpha \left(\frac{k - \frac{p_{AF} - p_{BH}}{1-2x}}{1-\underline{t}} \right) p_{AF} + (1-\alpha) \left(\frac{1 - \frac{p_{AF} - p_{BF}}{1-2x}}{1-\underline{t}} \right) p_{AF} \quad (2.16)$$

Maximizing the expected profit of firm A yields the best-response function,

$$p_{AH}(x; p_{BF}) = \begin{cases} p_{BF} + k(1-2x) & \text{if} \quad p_{BF} \geq (1-2k)(1-2x) \\ \frac{p_{BF}+(1-2x)}{2} & \text{if} \quad p_{BF} < (1-2k)(1-2x), \end{cases} \quad (2.17)$$

which depends on the ratio $1/k$. If $1/k \leq 2$, then $1 - 2k \leq 0$ and $p_{AH}(x; p_{BF}) = p_{BF} + k(1-2x)$ irrespective of p_{BF}, such that firm A optimally serves all recognized previous consumers for any p_{BF}. When consumers are relatively homogeneous, it holds that $\underline{t} \geq 1/2$, which implies that $1/k \leq 2$, because $\underline{t} \leq k$ holds. It follows that firm A serves all of its recognized consumers on x. In the proposed equilibrium firm A serves also all of the unidentified customers, such that in equilibrium firm B charges $p_{BH}(x) = 0$ and $p_{BF}(x) = 0$, because it would have an incentive to deviate from any positive price. Hence, $p_{AH}(x) = k(1-2x)$ and $p_{AF}(x) = \underline{t}(1-2x)$.

For the above prices to constitute the equilibrium, none of the firms should have an incentive to deviate. Since firm B clearly can not gain from a deviation, we are left to check that firm A has no incentive to raise the price $p_{AF}(x)$. The derivative of (2.16) evaluated at $p_{AH}(x) = k(1-2x)$, $p_{AF}(x) = \underline{t}(1-2x)$, $p_{BH}(x) = 0$ and $p_{BF}(x) = 0$ must be non-positive, yielding the condition $\underline{t} \geq (1 - \alpha + \alpha k)/2$, which is always fulfilled when consumers are relatively homogeneous.

Inserting the equilibrium prices yields the second-period profits on x:

$$\Pi_A^2(x) = \frac{(\alpha(-1+k)(k-\underline{t}) + (-1+\underline{t})\underline{t})}{\underline{t}-1}(1-2x) \ , \ \Pi_B^2(x) = 0. \quad (2.18)$$

Taking the derivative with respect to α, we get,

$$\frac{\partial \Pi_A^2(x)}{\partial \alpha} = \frac{(k-1)(k-\underline{t})(1-2x)}{\underline{t}-1} > 0 \ , \qquad \frac{\partial \Pi_B^2(x)}{\partial \alpha} = 0. \quad (2.19)$$

Taking the derivative with respect to k, we get,

$$\frac{\partial \Pi_A^2(x)}{\partial k} = \frac{(\alpha(k-1)+\alpha(k-\underline{t}))(1-2x)}{\underline{t}-1} = 0 \Leftrightarrow k = \frac{1+\underline{t}}{2}\,, \quad \frac{\partial \Pi_B^2(x)}{\partial k} = 0. \tag{2.20}$$

The Lemma follows. *Q.E.D.*

Proof of Lemma 2.2. In the first period firms maximize their discounted sum of profits over both periods on each location. Consider first myopic consumers, such that firm A chooses $p_A(x)$ to maximize the profits:

$$\Pi_A(x) = \left(\frac{1-k}{1-\underline{t}}\right) p_A(x) + \delta\left[\frac{(\alpha(-1+k)(k-\underline{t}) + (\underline{t}-1)\underline{t})(1-2x)}{\underline{t}-1}\right], \tag{2.21}$$

given the transport cost parameter of the indifferent consumer:

$$k = \frac{p_A - p_B}{1-2x}. \tag{2.22}$$

Accordingly, firm B chooses $p_B(x)$ to maximize the profits:

$$\Pi_B(x) = \left(\frac{\frac{p_A-p_B}{1-2x} - \underline{t}}{1-\underline{t}}\right) p_B(x). \tag{2.23}$$

Solving firms' first-order conditions yields the prices:

$$p_A(x) = \frac{(2+2\alpha\delta-\underline{t})(1-2x)}{3+2\alpha\delta} \text{ and } p_B(x) = \frac{(1-\alpha\delta(\underline{t}-1)-2\underline{t}))(1-2x)}{3+2\alpha\delta}. \tag{2.24}$$

Note that second-order conditions are also fulfilled. For the prices (2.24) to constitute the equilibrium, it must hold that $\underline{t} < k \leq 1$, which yields the condition:

$$\underline{t} < \frac{(1+\alpha\delta)(1+\underline{t})}{3+2\alpha\delta} \leq 1. \tag{2.25}$$

Note that the right-hand side of (2.25) is fulfilled for any α, δ, $\underline{t}$ and $\partial k/\partial\alpha > 0$. The left-hand side of (2.25) is fulfilled, if

$$\underline{t} < \frac{1+\alpha\delta}{2+\alpha\delta}. \tag{2.26}$$

Or equivalently, using $l = 1/\underline{t}$ and solving for α,

$$\alpha > \hat{\alpha}(\delta, l) := \frac{1}{\delta}\left(\frac{1}{l-1} - 1\right), \tag{2.27}$$

with $\partial\hat{\alpha}(\delta, l)/\partial\delta < 0$ and $\partial\hat{\alpha}(\delta, l)/\partial l < 0$. Given that $\alpha \in [0, 1]$, the condition can not be fulfilled when consumer heterogeneity is relatively small, i.e. $l \leq 1 + [1/(1+\delta)]$. Note finally that if (2.27) holds, then $p_A(x) > 0$ and $p_B(x) > 0$.

If $\alpha \leq [1/(l-1) - 1]/\delta$, firm A serves all consumers at x in equilibrium. In this equilibrium firm A charges the highest possible price, which leads to all consumers buying her product:

$$p_A(x) = p_B + \underline{t}\,(1 - 2x)\,. \tag{2.28}$$

Given firm A's action, $p_B\,(x) = 0$ must hold, because firm B would have an incentive to deviate downwards from any positive price. Hence,

$$p_A(x) = \underline{t}(1 - 2x) \text{ and } p_B(x) = 0. \tag{2.29}$$

For the prices (2.29) to constitute the equilibrium, none of the firms should have an incentive to deviate. Precisely, firm A should not have an incentive to increase its price. The derivative of (2.21) evaluated at $p_A(x) = \underline{t}(1 - 2x)$ and $p_B(x) = 0$ must be non-positive, yielding the condition $\alpha \leq [1/(l-1) - 1]/\delta$, which is the opposite of (2.27) and therefore always holds in the monopolization strategy area.

Consider now sophisticated consumers.

Consumers buy at firm A in the first period, whenever,

$$p_A + tx + \alpha\delta(p_{AH} + tx) \leq p_B + t(1 - x) + \alpha\delta(p_{AF} + tx). \tag{2.30}$$

Inserting the second-period prices and solving (2.30) for k yields the threshold for the indifferent consumer:

$$t \geq k := \frac{p_A - p_B}{(1 - \alpha\delta)(1 - 2x)} - \frac{\alpha\delta\underline{t}}{1 - \alpha\delta}. \tag{2.31}$$

Solving firms' first-order conditions given this formula yields the prices:

$$p_A(x) = \frac{(2 - \underline{t} + \alpha\delta(3\underline{t} - 2)(1 - 2x)}{3 - \alpha\delta}\,,\, p_B(x) = \frac{(\alpha\delta - 1)(2\underline{t} - 1)(1 - 2x)}{3 - \alpha\delta}. \tag{2.32}$$

For the prices (2.32) to constitute the equilibrium, it must hold that $\underline{t} < k \leq 1$, which yields the condition:

$$\underline{t} < \frac{1 + \underline{t} - \alpha\delta\underline{t}}{3 - \alpha\delta} \leq 1. \tag{2.33}$$

Note that the right-hand side of (2.33) is fulfilled for any α, δ and $\underline{t}$. However, the left-hand side of (2.33) is never fulfilled, because this requires:

$$\underline{t} < \frac{1}{2}, \tag{2.34}$$

which is never fulfilled when consumers are relatively homogeneous. Therefore, the condition can not be fulfilled and firm A serves all consumers at x in equilibrium. In this equilibrium firm A charges the highest possible price, which leads to all consumers buying her product:

$$p_A(x) = p_B + \underline{t}\,(1 - 2x)\,. \tag{2.35}$$

Given firm A's action, $p_B\,(x) = 0$ must hold, because firm B would have an incentive to deviate downwards from any positive price. Hence,

$$p_A(x) = \underline{t}(1 - 2x) \text{ and } p_B(x) = 0. \tag{2.36}$$

For the prices (2.36) to constitute the equilibrium, none of the firms should have an incentive to deviate. Precisely, firm A should not have an incentive to increase its price. The derivative of the expected profit evaluated at $p_A(x) = \underline{t}(1 - 2x)$ and $p_B(x) = 0$ must be non-positive, yielding the condition $\underline{t} \geq 1/2$, which is the opposite of (2.34) and always fulfilled when consumers are relatively homogeneous. *Q.E.D.*

Proof of Proposition 2.1. Using Lemma 2.1 and 2.2, we can calculate each firm's discounted profits over two periods on any location on firm A's turf. Note that they consist of a common factor multiplied by $(1 - 2x)$ and $\int_0^{1/2}(1 - 2x)\,dx = \int_{1/2}^{1}(2x - 1)\,dx = 1/4$.

Therefore, each firm's total discounted profits over two periods (over all x) are due to symmetry given by the sum of both firms' profits on some $x < 1/2$ divided by

$4(1-2x)$, i.e.:

$$\Pi_i = \frac{\Pi_A(x)}{4(1-2x)} + \frac{\Pi_B(x)}{4(1-2x)}. \tag{2.37}$$

In case consumers are myopic and $\alpha > [1/(l-1)-1]/\delta$, this results in the expression:

$$\Pi_i = \frac{-5-\alpha^3\delta^3(-1+\underline{t})^2+(8-9\delta)\underline{t}+(-5+9\delta)\underline{t}^2+2\alpha^2\delta^2(-1+\underline{t})(3+2(-1+\delta)\underline{t})}{4(3+2\alpha\delta)^2(-1+\underline{t})} + \frac{\alpha\delta(-10-4(-4+3\delta)\underline{t}+(-7+12\delta)\underline{t}^2)}{4(3+2\alpha d)^2(-1+\underline{t})}. \tag{2.38}$$

Note that taking the derivative with respect to α yields,

$$\frac{\partial\Pi_i}{\partial\alpha} = \frac{\delta\left(10+9\alpha^2\delta^2(-1+\underline{t})^2+2\alpha^3\delta^3(-1+\underline{t})^2-16\underline{t}+\underline{t}^2+2\alpha\delta\left(8-14\underline{t}+5\underline{t}^2\right)\right)}{4(3+2\alpha\delta)^3(1-\underline{t})} > 0, \tag{2.39}$$

for any α, δ and $\underline{t}$.

In all other cases, the resulting expression for each firm's total discounted profits over two periods (over all x) is given by:

$$\Pi_i = \frac{1}{4}\left(\underline{t}+\delta\underline{t}\right), \tag{2.40}$$

such that,

$$\frac{\partial\Pi_i}{\partial\alpha} = 0. \tag{2.41}$$

Note finally that if $\alpha > [1/(l-1)-1]/\delta$, then (2.38) > (2.40).

The Proposition follows. *Q.E.D.*

Proof of Lemma 2.3. Consider some $x < 1/2$. Given that poaching occurs and $\underline{t} = 0$ when consumers are relatively heterogeneous, the expected second-period profits of firm A on x are given by,

$$\Pi_A^2(x) = \alpha\left(1-\frac{p_{AH}-p_{BF}}{1-2x}\right)p_{AH} + \alpha\left(k-\frac{p_{AF}-p_{BH}}{1-2x}\right)p_{AF} + (1-\alpha)\left(1-\frac{p_{AF}-p_{BF}}{1-2x}\right)p_{AF}. \tag{2.42}$$

Similarly, the rival's expected second-period profits on x are given by,

$$\Pi_B^2(x) = \alpha\left(\frac{p_{AF}-p_{BH}}{1-2x}\right)p_{BH} + \alpha\left(\frac{p_{AH}-p_{BF}}{1-2x}-k\right)p_{BF} + (1-\alpha)\left(\frac{p_{AF}-p_{BF}}{1-2x}\right)p_{BF}. \tag{2.43}$$

Maximizing these profit functions with respect to p_{AH}, p_{AF}, p_{BH} and p_{BF} and solving the system of equations simultaneously, we find the equilibrium prices:

$$p_{AH}(x) = \frac{(8-\alpha(2+(2+\alpha)k))(1-2x)}{3\left(4-\alpha^2\right)} \tag{2.44}$$

$$p_{BH}(x) = \frac{\left(4+2\alpha(-2+k)-\alpha^2 k\right)(1-2x)}{3\left(4-\alpha^2\right)} \tag{2.45}$$

$$p_{AF}(x) = \frac{(8+\alpha(-8+2(2+\alpha)k))(1-2x)}{3\left(4-\alpha^2\right)} \tag{2.46}$$

$$p_{BF}(x) = \frac{(4-\alpha(4-3\alpha+4k+2\alpha k))(1-2x)}{3\left(4-\alpha^2\right)} \tag{2.47}$$

Note that given these prices and the best response functions, poaching occurs whenever the first-period cut-off fulfills jointly the conditions $k < (4+2\alpha-3\alpha^2)/(12-2\alpha-4\alpha^2)$ and $k > (2\alpha-2)/(2\alpha^2+\alpha-6)$. In this case, the prices are also positive and second-order conditions fulfilled. Finally, note that we have additionally to ensure that $k \leq (3\alpha-2)/4\alpha$, such that $0 \leq (p_{AF}-p_{BF})/(1-2x) \leq 1$.
Whenever poaching in pure strategies does not occur, the second-period equilibrium is in mixed strategies, which following Colombo (2016) we do not explicitly consider.

Inserting the equilibrium prices in (2.42) and (2.43) yields the second-period profits on x:

$$\Pi_A^2(x) = \frac{k^2(1-2x)\left(16\alpha^2+20\alpha^3+8\alpha^4+\alpha^5\right)}{9\left(-4+\alpha^2\right)^2} + \frac{k(1-2x)\left(64\alpha-64\alpha^2-40\alpha^3+4\alpha^4\right)}{9\left(-4+\alpha^2\right)^2} + \frac{(1-2x)\left(64-64\alpha+32\alpha^2+4\alpha^3\right)}{9\left(-4+\alpha^2\right)^2} \tag{2.48}$$

and

$$\Pi_B^2(x) = \frac{k^2(1-2x)\left(16\alpha^2+20\alpha^3+8\alpha^4+\alpha^5\right)}{9\left(-4+\alpha^2\right)^2} + \frac{k(1-2x)\left(-32\alpha+32\alpha^2-16\alpha^3-20\alpha^4\right)}{9\left(-4+\alpha^2\right)^2} + \frac{(1-2x)\left(16-16\alpha+8\alpha^2-8\alpha^3+9\alpha^4\right)}{9\left(-4+\alpha^2\right)^2}. \tag{2.49}$$

Taking the derivatives with respect to α, we get,

$$\frac{\partial \Pi_A^2(x)}{\partial \alpha} = \frac{\left(-256(-1+k)-128(-2+k)^2\alpha-48\left(-3-6k+5k^2\right)\alpha^2-32\left(2-2k+5k^2\right)\alpha^3+\left(-4+40k-40k^2\right)\alpha^4+k^2\alpha^6\right)(1-2x)}{9(-4+\alpha^2)^3}$$

and

$$\frac{\partial \Pi_B^2(x)}{\partial \alpha} = \frac{\left(64(1+2k)-128(1+k)^2\alpha-48\left(-3-6k+5k^2\right)\alpha^2-32\left(5-8k+5k^2\right)\alpha^3+\left(8+16k-40k^2\right)\alpha^4+k^2\alpha^6\right)(1-2x)}{9(-4+\alpha^2)^3}.$$

Within the poaching boundaries, $\partial \Pi_A^2(x)/\partial \alpha < 0$ for $k > 0.47$ and $\partial \Pi_B^2(x)/\partial \alpha < 0$ for $k > 0.24$.

By setting $\partial \Pi_A^2(x)/\partial \alpha = 0$ and $\partial \Pi_B^2(x)/\partial \alpha = 0$, we find unique solutions representing two minima according to the second derivatives, which are given by,

$$\begin{aligned} \frac{\partial^2 \Pi_A^2(x)}{\partial \alpha^2} = {} & 8\Big[64(-2+k)^2 + 48\left(-7-2k+5k^2\right)\alpha + 32\left(13-13k+10k^2\right)\alpha^2 \\ & +8\left(-8-28k+25k^2\right)\alpha^3 + 12\left(2-2k+5k^2\right)\alpha^4 \\ & +\left(1-10k+7k^2\right)\alpha^5\Big](1-2x) \Big/ 9\left(-4+\alpha^2\right)^4 \end{aligned}$$

and

$$\begin{aligned} \frac{\partial^2 \Pi_B^2(x)}{\partial \alpha^2} = {} & 8\Big[64(1+k)^2 + 48\left(-4-8k+5k^2\right)\alpha + 32\left(10-7k+10k^2\right)\alpha^2 \\ & +8\left(-11-22k+25k^2\right)\alpha^3 + 12\left(5-8k+5k^2\right)\alpha^4 \\ & +\left(-2-4k+7k^2\right)\alpha^5\Big](1-2x) \Big/ 9\left(-4+\alpha^2\right)^4 \end{aligned}$$

and always positive for any α and k within the poaching boundaries.

By inspecting (2.48) and (2.49), we note that profits are u-shaped in k and by taking the derivatives with respect to k, we get,

$$\begin{aligned} \frac{\partial \Pi_A^2(x)}{\partial k} = {} & \frac{2\alpha\left(16+8(-3+k)\alpha+(2+6k)\alpha^2+k\alpha^3\right)(1-2x)}{9(-2+\alpha)^2(2+\alpha)} = 0 \\ & \Leftrightarrow k = \frac{-16+24\alpha-2\alpha^2}{8\alpha+6\alpha^2+\alpha^3} \end{aligned}$$

and

$$\begin{aligned} \frac{\partial \Pi_B^2(x)}{\partial k} = {} & \frac{2\alpha\left(-8+4(3+2k)\alpha+2(-5+3k)\alpha^2+k\alpha^3\right)(1-2x)}{9(-2+\alpha)^2(2+\alpha)} = 0 \\ & \Leftrightarrow k = \frac{8-12\alpha+10\alpha^2}{8\alpha+6\alpha^2+\alpha^3}. \end{aligned}$$

Note finally that when $\alpha \leq 0.76$, these minima are to the left and respectively to right of the poaching boundaries.

The Lemma follows. *Q.E.D.*

Proof of Lemma 2.4. The discounted sum of profits over both periods on each location are given by,

$$\Pi_A(x) = ((1-k))\, p_A + \delta \Pi_A^2(x), \tag{2.50}$$

with $\Pi_A^2(x)$ given by (2.48) and

$$\Pi_B(x) = k * p_B + \delta \Pi_B^2(x), \tag{2.51}$$

with $\Pi_B^2(x)$ given by (2.49).
When consumers are myopic, firms maximize (2.50) and (2.51) given,

$$k = \frac{p_A - p_B}{1-2x}. \tag{2.52}$$

When consumers are sophisticated, consumers buy at firm A, whenever,

$$p_A + tx + \delta\alpha(p_{BF} + t(1-x)) \leq p_B + t(1-x) + \delta\alpha(p_{AF} + tx). \tag{2.53}$$

Inserting the second-period prices and solving (2.53) for k yields that those consumers buy at firm A with transport costs:

$$t \geq k := \frac{(6-3\alpha)(p_A - p_B)}{(6+\alpha(-3+(-6+7\alpha)\delta))(1-2x)} + \frac{\alpha(3\alpha-2)\delta}{6+\alpha(-3+(-6+7\alpha)\delta)}. \tag{2.54}$$

Maximizing (2.50) and (2.51) given (2.54) yields the equilibrium prices when consumers are sophisticated.

When consumers are myopic, we find:

$$\begin{aligned} p_A(x) =& \Big[2\Big(432 - 528\alpha^2\delta + 48\alpha(-9+5\delta) + \alpha^5\delta(3+8\delta) - 4\alpha^3\left(-27-63\delta+8\delta^2\right) \\ &+3\alpha^4\left(-9-10\delta+8\delta^2\right)\Big)(1-2x)\Big] \\ &\Big/\Big(3\left(-4+\alpha^2\right)\left(-108+108\alpha+4\alpha^3\delta+\alpha^2(-27+16\delta)\right)\Big) \end{aligned} \tag{2.55}$$

$$\begin{aligned} p_B(x) =& \Big[\Big(432 - 864\alpha^2\delta + 48\alpha(-9+8\delta) + 2\alpha^5\delta(3+8\delta) + \alpha^3\left(108+528\delta-64\delta^2\right) \\ &+3\alpha^4\left(-9-36\delta+16\delta^2\right)\Big)(1-2x)\Big] \\ &\Big/\Big(3\left(-4+\alpha^2\right)\left(-108+108\alpha+4\alpha^3\delta+\alpha^2(-27+16\delta)\right)\Big) \end{aligned} \tag{2.56}$$

Note that prices are positive for any α and δ and second-order conditions fulfilled. Inserting these prices in (2.52) yields:

$$k = \frac{-36 + \alpha(36 - 8\delta) + \alpha^2(-9 + 16\delta)}{-108 + 108\alpha + 4\alpha^3\delta + \alpha^2(-27 + 16\delta)}. \tag{2.57}$$

When consumers are sophisticated, we find:

$$\begin{aligned} p_A(x) =& \Big[\Big(864 - 239\alpha^6\delta^2 + 48\alpha^2\delta(32 + 5\delta) - 48\alpha(18 + 23\delta) + \alpha^5\delta(231 + 580\delta) \\ & -8\alpha^3\left(-27 + 44\delta^2\right) - \alpha^4\left(54 + 708\delta + 232\delta^2\right)\Big)(1 - 2x)\Big] \\ & \Big/ \Big(3\left(-4 + \alpha^2\right)\left(-108 + 67\alpha^3\delta + 108\alpha(1 + \delta) - \alpha^2(27 + 164\delta)\right)\Big) \end{aligned} \tag{2.58}$$

$$\begin{aligned} p_B(x) =& \Big[\Big(432 - 230\alpha^6\delta^2 + 96\alpha^2\delta(9 + 2\delta) - 48\alpha(9 + 13\delta) + \alpha^5\delta(159 + 568\delta) \\ & -4\alpha^3\left(-27 - 42\delta + 76\delta^2\right) - \alpha^4\left(27 + 540\delta + 256\delta^2\right)\Big)(1 - 2x)\Big] \\ & \Big/ \Big(3\left(-4 + \alpha^2\right)\left(-108 + 67\alpha^3\delta + 108\alpha(1 + \delta) - \alpha^2(27 + 164\delta)\right)\Big) \end{aligned} \tag{2.59}$$

Note once again that prices are positive for any α and δ and second-order conditions fulfilled. Inserting these prices in (2.54) yields:

$$k = \frac{-36 + 30\alpha^3\delta + 4\alpha(9 + 10\delta) - \alpha^2(9 + 68\delta)}{-108 + 67\alpha^3\delta + 108\alpha(1 + \delta) - \alpha^2(27 + 164\delta)} \tag{2.60}$$

Note importantly that these cut-offs fulfill the conditions for poaching to occur in the second period. To ensure $k \leq (3\alpha - 2)/4\alpha$ when consumers are myopic, we have to restrict: $\delta \leq [3\left(12 - 16\alpha + 5\alpha^2\right)]/4a^3$. For simplicity, we use the restriction $\alpha \leq 0.966$ when consumers are myopic in the following. *Q.E.D.*

Proof of Proposition 2.2. Using Lemma 2.3 and 2.4, we can once again calculate each firm's discounted profits over two periods on any location on firm A's turf. Again they consist of a common factor multiplied by $(1 - 2x)$, while $\int_0^{1/2}(1 - 2x)\,dx = \int_{1/2}^{1}(2x - 1)\,dx = 1/4$, such that each firm's total discounted profits over two periods (over all x) are due to symmetry given by the sum of both firms' profits on some $x < 1/2$ divided by $4(1 - 2x)$.

When consumers are myopic, we get:

$$
\begin{aligned}
\Pi_i =\ & \frac{-155520(1+\delta) + 8\alpha^9\delta^2(3+14\delta) + 1728\alpha(135+161\delta) + 12\alpha^8\delta\left(-27-98\delta+72\delta^2\right)}{4(3(-2+\alpha)(2+\alpha)^2(-108+108\alpha+4\alpha^3\delta+\alpha^2(-27+16\delta))^2)} \\
&+ \frac{-48\alpha^3\left(2025-1143\delta+368\delta^2\right) + 96\alpha^2\left(-405-1143\delta+448\delta^2\right)}{4(3(-2+\alpha)(2+a)^2(-108+108\alpha+4\alpha^3\delta+\alpha^2(-27+16\delta))^2)} \\
&+ \frac{-6\alpha^6\left(1215+4437\delta-2736\delta^2+128\delta^3\right) - 24\alpha^4\left(-2025+5463\delta+672\delta^2+128\delta^3\right)}{4(3(-2+\alpha)(2+\alpha)^2(-108+108\alpha+4\alpha^3\delta+\alpha^2(-27+16\delta))^2)} \\
&+ \frac{-4\alpha^5\left(-1215-23841\delta+5040\delta^2+704\delta^3\right) + \alpha^7\left(1215+3375\delta+864\delta^2+1600\delta^3\right)}{4(3(-2+\alpha)(2+\alpha)^2(-108+108\alpha+4\alpha^3\delta+\alpha^2(-27+16\delta))^2)}
\end{aligned} \tag{2.61}
$$

Taking the derivatives with respect to α, we get,

$$
\begin{aligned}
\frac{\partial \Pi_i}{\partial \alpha} =\ & \delta\Big[4852224 + 32\alpha^{12}\delta^2(-3+8\delta) + 20736\alpha^2(2919+16\delta) - 20736\alpha(1101+32\delta) \\
&+ 384\alpha^3\left(-261225+14004\delta+128\delta^2\right) + 384\alpha^4\left(270297-45702\delta+152\delta^2\right) \\
&- 192\alpha^5\left(350811-111006\delta+3352\delta^2\right) + 6\alpha^{10}\left(405+1314\delta+1984\delta^2-576\delta^3\right) \\
&- 16\alpha^6\left(-1699299+707292\delta-102624\delta^2+256\delta^3\right) + 3\alpha^{11}\left(-81+504\delta-1376\delta^2+512\delta^3\right) \\
&+ 24\alpha^7\left(-273213+76572\delta-31200\delta^2+896\delta^3\right) \\
&- 18\alpha^8\left(-47655-29448\delta+11744\delta^2+2816\delta^3\right) \\
&- 3\alpha^9\left(17253+66456\delta-39584\delta^2+12032\delta^3\right)\Big] \\
&\Big/\Big(3(-2+\alpha)^2(2+\alpha)^3(-108+108\alpha+4\alpha^3\delta+\alpha^2(-27+16\delta))^3\Big)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 \Pi_i}{\partial \alpha^2} =\ & \Big[-46223032320\delta + 38026506240\alpha\delta - 123848165376\alpha^2\delta + 227493909504\alpha^3\delta - 272235520512\alpha^4\delta \\
&+ 228720029184\alpha^5\delta - 140323518720\alpha^6\delta + 63725470848\alpha^7\delta - 21200113152\alpha^8\delta + 4981216176\alpha^9\delta \\
&- 769106664\alpha^{10}\delta + 67840740\alpha^{11}\delta - 2427570\alpha^{12}\delta - 2866544464\delta^2 + 1719926784\alpha\delta^2 + 517570560\alpha^2\delta^2 \\
&- 19886653440\alpha^3\delta^2 + 50232213504\alpha^4\delta^2 - 63958781952\alpha^5\delta^2 + 51106609152\alpha^6\delta^2 - 27183154176\alpha^7\delta^2 \\
&+ 9042430464\alpha^8\delta^2 - 1236570624\alpha^9\delta^2 - 321770880\alpha^{10}\delta^2 + 174477888\alpha^{11}\delta^2 - 28553472\alpha^{12}\delta^2 \\
&+ 1636848\alpha^{13}\delta^2 - 40824\alpha^{14}\delta^2 + 2916\alpha^{15}\delta^2 - 148635648\alpha^2\delta^3 + 69009408\alpha^3\delta^3 - 241532928\alpha^4\delta^3 \\
&+ 3359563776\alpha^5\delta^3 - 6097047552\alpha^6\delta^3 + 4791343104\alpha^7\delta^3 - 1861014528\alpha^8\delta^3 + 135406080\alpha^9\delta^3 \\
&+ 208127232\alpha^{10}\delta^3 - 75821184\alpha^{11}\delta^3 - 5184\alpha^{12}\delta^3 + 3120768\alpha^{13}\delta^3 - 150336\alpha^{14}\delta^3 - 15552\alpha^{15}\delta^3 \\
&+ 9437184\alpha^4\delta^4 + 3145728\alpha^5\delta^4 + 30670848\alpha^6\delta^4 - 287440896\alpha^7\delta^4 + 217104384\alpha^8\delta^4 - 39002112\alpha^9\delta^4 \\
&- 23900160\alpha^{10}\delta^4 + 19433472\alpha^{11}\delta^4 + 340992\alpha^{12}\delta^4 - 2084352\alpha^{13}\delta^4 - 125184\alpha^{14}\delta^4 + 40320\alpha^{15}\delta^4 \\
&+ 768\alpha^{16}\delta^4 - 1703936\alpha^8\delta^5 + 8159232\alpha^9\delta^5 + 3424256\alpha^{10}\delta^5 + 1155072\alpha^{11}\delta^5 + 1536000\alpha^{12}\delta^5 \\
&+ 651264\alpha^{13}\delta^5 + 63488\alpha^{14}\delta^5 - 12288\alpha^{15}\delta^5 - 2048\alpha^{16}\delta^5\Big] \\
&\Big/\Big(3(-2+\alpha)^3(2+\alpha)^4(-108+108\alpha+4\alpha^3\delta+\alpha^2(-27+16\delta))^4\Big)
\end{aligned}
$$

By inspecting $\partial\Pi_i/\partial\alpha$, we find that when $\alpha \in [0, 0.966]$: $\partial\Pi_i/\partial\alpha < 0$ for $\delta < 0.82$. When $\delta \geq 0.82$, $\partial\Pi_i/\partial\alpha = 0$ has an unique solution $\alpha^{myo}(\delta)$ decreasing in δ, with $\alpha^{myo}(1) \approx 0.94$.

As $\partial^2\Pi_i/\partial\alpha^2|_{\alpha^{myo}(\delta)} > 0$, we have a minimum and the profits are u-shaped in α. Note finally, that $\Pi_i|_{\alpha=0.966} < \Pi_i|_{\alpha=0}$ for any δ.

When consumers are sophisticated, we get:

$$
\begin{aligned}
\Pi_i = \; & \frac{15143\alpha^{10}\delta^3 + 155520(1+\delta) - \alpha^9\delta^2(19329 + 51625\delta) - 1728\alpha(135 + 419\delta + 180\delta^2)}{4(3(2-\alpha)(2+\alpha)^2(108 - 67\alpha^3\delta - 108\alpha(1+\delta) + \alpha^2(27+164\delta))^2} \\
& + \frac{\alpha^8\delta(8343 + 81708\delta + 33770\delta^2) + 96\alpha^2(405 + 11133\delta + 11912\delta^2 + 1620\delta^3)}{4(3(2-\alpha)(2+\alpha)^2(108 - 67\alpha^3\delta - 108\alpha(1+\delta) + \alpha^2(27+164\delta))^2} \\
& + \frac{-48\alpha^3(-2025 + 9513\delta + 34648\delta^2 + 11980\delta^3) + \alpha^7(-1215 - 42417\delta - 42840\delta^2 + 29492\delta^3)}{4(3(2-\alpha)(2+\alpha)^2(108 - 67\alpha^3\delta - 108\alpha(1+\delta) + \alpha^2(27+164\delta))^2} \\
& + \frac{24\alpha^4(-2025 - 9171\delta + 41304\delta^2 + 36244\delta^3) + 2\alpha^6(3645 + 11205\delta - 94968\delta^2 + 72892\delta^3)}{4(3(2-\alpha)(2+\alpha)^2(108 - 67\alpha^3\delta - 108\alpha(1+\delta) + \alpha^2(27+164\delta))^2} \\
& + \frac{-4\alpha^5(1215 - 47277\delta - 864\delta^2 + 155308\delta^3)}{4(3(2-\alpha)(2+\alpha)^2(108 - 67\alpha^3\delta - 108\alpha(1+\delta) + \alpha^2(27+164\delta))^2}
\end{aligned}
\tag{2.62}
$$

Taking the derivatives with respect to α, we get,

$$
\begin{aligned}
\frac{\partial\Pi_i}{\partial\alpha} = \; & \Big[776355584\delta - 308551680\alpha\delta + 469421568\alpha^2\delta - 391723776\alpha^3\delta + 263450880\alpha^4\delta - 199158912\alpha^5\delta \\
& + 128210688\alpha^6\delta - 48553344\alpha^7\delta + 6807888\alpha^8\delta + 1428840\alpha^9\delta - 624510\alpha^{10}\delta + 62451\alpha^{11}\delta \\
& - 233570304\alpha\delta^2 + 1052393472\alpha^2\delta^2 - 1870649856\alpha^3\delta^2 + 1788680448\alpha^4\delta^2 - 1209275136\alpha^5\delta^2 \\
& + 857962368\alpha^6\delta^2 - 604060416\alpha^7\delta^2 + 277375104\alpha^8\delta^2 - 55191888\alpha^9\delta^2 - 5280120\alpha^{10}\delta^2 + 4078890\alpha^{11}\delta^2 \\
& - 484785\alpha^{12}\delta^2 + 233902080\alpha^2\delta^3 - 1177257984\alpha^3\delta^3 + 2394361344\alpha^4\delta^3 - 2622275328\alpha^5\delta^3 \\
& + 1884159744\alpha^6\delta^3 - 1257609600\alpha^7\delta^3 + 912084480\alpha^8\delta^3 - 494297472\alpha^9\delta^3 + 131322624\alpha^{10}\delta^3 \\
& + 1970976\alpha^{11}\delta^3 - 8422638\alpha^{12}\delta^3 + 1226583\alpha^{13}\delta^3 - 77967360\alpha^3\delta^4 + 433219584\alpha^4\delta^4 - 991861248\alpha^5\delta^4 \\
& + 1233863936\alpha^6\delta^4 - 972025088\alpha^7\delta^4 + 633604736\alpha^8\delta^4 - 450747648\alpha^9\delta^4 + 276401664\alpha^{10}\delta^4 \\
& - 94046208\alpha^{11}\delta^4 + 5270320\alpha^{12}\delta^4 + 5421194\alpha^{13}\delta^4 - 1014581\alpha^{14}\delta^4\Big] \\
& \Big/ \Big(12(-2+\alpha)^2(2+\alpha)^3(-108 + 67\alpha^3\delta + 108\alpha(1+\delta) - \alpha^2(27+164\delta))^3\Big)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2\Pi_i}{\partial\alpha^2} = \; & \Big[-4120657920\delta + 17835282432\alpha\delta - 42572666880\alpha^2\delta + 71961467904\alpha^3\delta - 88004040192\alpha^4\delta + 76884982272\alpha^5\delta \\
& - 47903208192\alpha^6\delta + 21403066752\alpha^7\delta - 6895383552\alpha^8\delta + 1599939216\alpha^9\delta - 261407736\alpha^{10}\delta + 27652428\alpha^{11}\delta \\
& - 1439046\alpha^{12}\delta - 23887872\delta^2 + 16876781568\alpha\delta^2 - 81613578240\alpha^2\delta^2 + 207564705792\alpha^3\delta^2 - 366683148288\alpha^4\delta^2 \\
& + 475658625024\alpha^5\delta^2 - 449787976704\alpha^6\delta^2 + 306259978752\alpha^7\delta^2 - 149108397696\alpha^8\delta^2 + 51671442240\alpha^9\delta^2 \\
& - 12680372448\alpha^{10}\delta^2 + 2179619280\alpha^{11}\delta^2 - 252222336\alpha^{12}\delta^2 + 17324280\alpha^{13}\delta^2 - 625968\alpha^{14}\delta^2 + 44712\alpha^{15}\delta^2 \\
& + 23887872\alpha\delta^3 - 25409175552\alpha^2\delta^3 + 136271462400\alpha^3\delta^3 - 372461248512\alpha^4\delta^3 + 694037680128\alpha^5\delta^3 \\
& - 953875132416\alpha^6\delta^3 + 969766553088\alpha^7\delta^3 - 717623421696\alpha^8\delta^3 + 380505831552\alpha^9\delta^3 - 142738984512\alpha^{10}\delta^3 \\
& + 37538368704\alpha^{11}\delta^3 - 6883524288\alpha^{12}\delta^3 + 858735648\alpha^{13}\delta^3 - 62524440\alpha^{14}\delta^3 + 1518102\alpha^{15}\delta^3 - 106191\alpha^{16}\delta^3 \\
& + 16850681856\alpha^3\delta^4 - 99405275136\alpha^4\delta^4 + 292443590656\alpha^5\delta^4 - 577435453440\alpha^6\delta^4 + 840449727488\alpha^7\delta^4 \\
& - 914128136192\alpha^8\delta^4 + 731672265216\alpha^9\delta^4 - 422094554368\alpha^{10}\delta^4 + 172097149568\alpha^{11}\delta^4 - 48849375264\alpha^{12}\delta^4 \\
& + 9608556496\alpha^{13}\delta^4 - 1293248872\alpha^{14}\delta^4 + 98706540\alpha^{15}\delta^4 - 394100\alpha^{16}\delta^4 - 4212670464\alpha^4\delta^5 + 27038633984\alpha^5\delta^5 \\
& - 85345179648\alpha^6\delta^5 + 178536807424\alpha^7\delta^5 - 274545456640\alpha^8\delta^5 + 317873496576\alpha^9\delta^5 - 273961555712\alpha^{10}\delta^5 \\
& + 171807102848\alpha^{11}\delta^5 - 76438641408\alpha^{12}\delta^5 + 23551326736\alpha^{13}\delta^5 - 4956722680\alpha^{14}\delta^5 \\
& + 711606732\alpha^{15}\delta^5 - 60255158\alpha^{16}\delta^5\Big] \Big/ \Big((-2+\alpha)^3(2+\alpha)^4\left(108 - 67\alpha^3\delta - 108\alpha(1+\delta) + \alpha^2(27+164\delta)\right)^4\Big)
\end{aligned}
$$

When $\alpha \in [0,1]$, $\partial\Pi_i/\partial\alpha = 0$ has an unique solution $\alpha^{soph}(\delta) \approx 0.61$.
As $\partial^2\Pi_i/\partial\alpha^2 > 0$ for all values of α and δ, we have a minimum and the profits are

u-shaped in α.

Note finally, that $\Pi_i|_{\alpha=1} < \Pi_i|_{\alpha=0}$ for any δ. The Proposition follows. *Q.E.D.*

Proof of Proposition 2.3. When consumers are sophisticated or myopic but $\alpha \leq \hat{\alpha}(\delta, l)$, the discounted social welfare over two periods is given by,

$$W = v(1+\delta) - \frac{2(1+\delta)}{1-\underline{t}} \int_{\underline{t}}^{1} \int_{0}^{1/2} (tx)\, dxdt, \tag{2.63}$$

which results in:

$$W = v(1+\delta) - \frac{(1+\delta)\left(1-\underline{t}^2\right)}{8(1-\underline{t})}. \tag{2.64}$$

Subtracting the equilibrium profits from social welfare yields consumer surplus, i.e.,

$$CS = W - 2 \cdot \Pi_i, \tag{2.65}$$

with Π_i given by (2.40).

Taking the derivatives with respect to α, we get,

$$\frac{\partial W}{\partial a} = 0 \; , \; \frac{\partial CS}{\partial \alpha} = 0 \tag{2.66}$$

for any α, δ and $\underline{t}$.

When consumers are myopic and $\alpha > \hat{\alpha}(\delta, l)$, the discounted social welfare over two periods is given by,

$$\begin{aligned} W = v(1+\delta) \; - & \left[\frac{2}{1-\underline{t}}\left(\int_{\underline{t}}^{\frac{(1+\alpha\delta)(1+\underline{t})}{3+2\alpha\delta}} \int_{0}^{1/2} (t(1-x))dxdt + \int_{\frac{(1+\alpha\delta)(1+\underline{t})}{3+2\alpha\delta}}^{1} \int_{0}^{1/2} (tx)dxdt\right)\right. \\ & \left. + \frac{2\delta}{1-\underline{t}} \int_{\underline{t}}^{1} \int_{0}^{1/2} (tx)\, dxdt\right], \end{aligned}$$

which results in:

$$W = v(1+\delta) - \frac{\delta\left(1-\underline{t}^2\right)}{8(1-\underline{t})} - \frac{2\left(\frac{1}{8}\left(\frac{1}{2} - \frac{(1+\alpha\delta)^2(1+\underline{t})^2}{2(3+2\alpha\delta)^2}\right) + \frac{3}{16}\left(-\underline{t}^2 + \frac{(1+\alpha\delta)^2(1+\underline{t})^2}{(3+2\alpha\delta)^2}\right)\right)}{1-\underline{t}}. \tag{2.67}$$

Subtracting the equilibrium profits from social welfare yields once again consumer surplus, i.e.,

$$CS = W - 2 \cdot \Pi_i, \tag{2.68}$$

with Π_i given by (2.38).

Taking the derivatives with respect to α, we get,

$$
\begin{aligned}
\frac{\partial W}{\partial \alpha} &= \frac{\delta(1+\alpha\delta)(1+\underline{t})^2}{2(3+2\alpha\delta)^3(-1+\underline{t})} < 0 \\
\frac{\partial CS}{\partial \alpha} &= \frac{\delta\left(11+9\alpha^2\delta^2(-1+\underline{t})^2+2\alpha^3\delta^3(-1+\underline{t})^2-14\underline{t}+2\underline{t}^2+\alpha\delta\left(17-26\underline{t}+11\underline{t}^2\right)\right)}{2(3+2\alpha\delta)^3(-1+\underline{t})} < 0
\end{aligned}
$$

for any α, δ and $\underline{t}$.
The Proposition follows. *Q.E.D.*

Proof of Proposition 2.4. Calculating the cut-off consumers in both periods when consumers are myopic, we get:

$$
\begin{aligned}
k = \frac{p_A - p_B}{1-2x} &=: \frac{-36+\alpha(36-8\delta)+\alpha^2(-9+16\delta)}{-108+108\alpha+4\alpha^3\delta+\alpha^2(-27+16\delta)} \\
ka = \frac{p_{AH} - p_{BF}}{1-2x} &=: \frac{4\left(-18-3\alpha+\alpha^4\delta+2\alpha^2(9+\delta)+\alpha^3(-6+4\delta)\right)}{(2+\alpha)\left(-108+108\alpha+4\alpha^3\delta+\alpha^2(-27+16\delta)\right)} \\
kb = \frac{p_{AF} - p_{BH}}{1-2x} &=: \frac{-72+96\alpha+3\alpha^3+4\alpha^2(-9+2\delta)}{(2+\alpha)\left(-108+108\alpha+4\alpha^3\delta+\alpha^2(-27+16\delta)\right)} \\
kn = \frac{p_{AF} - p_{BF}}{1-2x} &=: \frac{-36+48\alpha-15\alpha^2+4\alpha^3\delta}{-108+108\alpha+4\alpha^3\delta+\alpha^2(-27+16\delta)}
\end{aligned}
$$

Plugging these values in (2.15) yields the discounted social welfare over two periods when consumers are myopic.
Taking the derivative with respect to α, we get,

$$
\begin{aligned}
\frac{\partial W}{\partial \alpha} = & -\delta\Big[248832+13824\alpha(-21+16\delta)+3456\alpha^2\left(-963-112\delta+48\delta^2\right)+28\alpha^{10}\delta\left(99-288\delta+128\delta^2\right) \\
& -64\alpha^3\left(-169047+8928\delta+8752\delta^2\right)+32\alpha^4\left(-397305+59904\delta+9216\delta^2+256\delta^3\right) \\
& +48\alpha^5\left(148149-50472\delta+12544\delta^2+384\delta^3\right)+6\alpha^8\left(8181-19440\delta+14592\delta^2+896\delta^3\right) \\
& -8\alpha^6\left(231579-135864\delta+48528\delta^2+3712\delta^3\right)-4\alpha^7\left(-25677-7344\delta+17616\delta^2+10624\delta^3\right) \\
& +\alpha^9\left(-8181+12168\delta-896\delta^2+15360\delta^3\right)\Big]\Big/\Big(4(2+\alpha)^3\left(-108+108\alpha+4\alpha^3\delta+\alpha^2(-27+16\delta)\right)^3\Big),
\end{aligned}
$$

which is positive for all values of α and δ.

Subtracting the equilibrium profits given by (2.62) from social welfare yields consumer surplus.
Taking the derivatives with respect to α, we get,

$$\begin{aligned}
\frac{\partial CS}{\partial \alpha} = & -\delta\Big[41803776 - 663552\alpha(285+4\delta) + 41472\alpha^2\left(10815 - 112\delta + 48\delta^2\right) + 4\alpha^{12}\delta\left(2079 - 6240\delta + 3200\delta^2\right) \\
& - 768\alpha^3\left(824985 - 54000\delta + 10832\delta^2\right) + 6144\alpha^4\left(87561 - 18180\delta + 1827\delta^2 + 16\delta^3\right) \\
& + 1536\alpha^5\left(-174798 + 75987\delta - 2046\delta^2 + 80\delta^3\right) + 288\alpha^7\left(-26595 - 18252\delta - 1288\delta^2 + 256\delta^3\right) \\
& + 96\alpha^8\left(6885 + 59886\delta - 9996\delta^2 + 832\delta^3\right) - 96\alpha^9\left(8262 - 405\delta + 3362\delta^2 + 3088\delta^3\right) \\
& + 3\alpha^{11}\left(-8829 + 5112\delta + 20352\delta^2 + 5120\delta^3\right) - 64\alpha^6\left(-1121931 + 666684\delta - 33384\delta^2 + 9152\delta^3\right) \\
& - 6\alpha^{10}\left(-44145 + 66600\delta - 45312\delta^2 + 25472\delta^3\right)\Big] \\
& \Big/\Big(12(-2+\alpha)^2(2+\alpha)^3\left(-108 + 108\alpha + 4\alpha^3\delta + \alpha^2(-27+16\delta)\right)^3\Big)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 CS}{\partial \alpha^2} = & \Big[9137111040\delta - 67372756992\alpha\delta + 195762604032\alpha^2\delta - 320128091136\alpha^3\delta + 340723542528\alpha^4\delta \\
& - 254881728000\alpha^5\delta + 139759167744\alpha^6\delta - 57023803008\alpha^7\delta + 17090932608\alpha^8\delta - 3594273264\alpha^9\delta \\
& + 483152040\alpha^{10}\delta - 34143444\alpha^{11}\delta + 695466\alpha^{12}\delta + 2866544464\delta^2 - 2579890176\alpha\delta^2 + 175177728\alpha^2\delta^2 \\
& + 30695915520\alpha^3\delta^2 - 75212955648\alpha^4\delta^2 + 84469506048\alpha^5\delta^2 - 55509442560\alpha^6\delta^2 + 23399165952\alpha^7\delta^2 \\
& - 6173646336\alpha^8\delta^2 + 550540800\alpha^9\delta^2 + 297789696\alpha^{10}\delta^2 - 126562176\alpha^{11}\delta^2 + 19805472\alpha^{12}\delta^2 \\
& - 1998432\alpha^{13}\delta^2 + 326592\alpha^{14}\delta^2 - 23328\alpha^{15}\delta^2 - 429981696\alpha\delta^3 + 2800189440\alpha^2\delta^3 - 4948770816\alpha^3\delta^3 \\
& + 2165170176\alpha^4\delta^3 - 1837596672\alpha^5\delta^3 + 5434490880\alpha^6\delta^3 - 5485363200\alpha^7\delta^3 + 2245238784\alpha^8\delta^3 \\
& - 30917376\alpha^9\delta^3 - 328599936\alpha^{10}\delta^3 + 99655488\alpha^{11}\delta^3 + 3641760\alpha^{12}\delta^3 - 5088960\alpha^{13}\delta^3 + 762912\alpha^{14}\delta^3 \\
& - 172368\alpha^{15}\delta^3 + 16632\alpha^{16}\delta^3 - 169869312\alpha^3\delta^4 + 277217280\alpha^4\delta^4 + 384761856\alpha^5\delta^4 - 758218752\alpha^6\delta^4 \\
& + 265371648\alpha^7\delta^4 + 283533312\alpha^8\delta^4 - 37048320\alpha^9\delta^4 - 127027200\alpha^{10}\delta^4 + 11811840\alpha^{11}\delta^4 \\
& + 19012608\alpha^{12}\delta^4 - 3816960\alpha^{13}\delta^4 - 344064\alpha^{14}\delta^4 + 419328\alpha^{15}\delta^4 - 49920\alpha^{16}\delta^4 - 3145728\alpha^5\delta^5 \\
& - 4718592\alpha^6\delta^5 - 393216\alpha^7\delta^5 + 16580608\alpha^8\delta^5 - 8060928\alpha^9\delta^5 - 18841600\alpha^{10}\delta^5 - 6094848\alpha^{11}\delta^5 \\
& + 540672\alpha^{12}\delta^5 - 559104\alpha^{13}\delta^5 - 590848\alpha^{14}\delta^5 - 30720\alpha^{15}\delta^5 + 25600\alpha^{16}\delta^5\Big] \\
& \Big/\Big(3(-2+\alpha)^3(2+\alpha)^4\left(-108 + 108\alpha + 4\alpha^3\delta + \alpha^2(-27+16\delta)\right)^4\Big)
\end{aligned}$$

By inspecting $\partial CS/\partial\alpha$, we find that when $\alpha \in [0, 0.966]$: $\partial CS/\partial\alpha > 0$ for $\delta \leq 0.99$. When $\delta > 0.99$, $\partial CS/\partial\alpha = 0$ has an unique solution $\alpha^{myoCS}(\delta)$ decreasing in δ, with $\alpha^{myoCS}(1) \approx 0.964$.
As $\partial^2 CS/\partial\alpha^2|_{\alpha^{myoCS}(\delta)} < 0$, we have a maximum and consumer surplus is inverse u-shaped in α.
Note finally, that $CS|_{\alpha=0.966} > CS|_{\alpha=0}$ for any δ.

Calculating the cut-off consumers in both periods when consumers are sophisticated, we get:

$$
\begin{aligned}
k = \frac{p_A - p_B}{1-2x} &=: \frac{-36 + 30\alpha^3\delta + 4\alpha(9+10\delta) - \alpha^2(9+68\delta)}{-108 + 67\alpha^3\delta + 108\alpha(1+\delta) - \alpha^2(27+164\delta)} \\
ka = \frac{p_{AH} - p_{BF}}{1-2x} &=: \frac{-72 - 24\alpha^2(-3+\delta) + 57\alpha^4\delta + 12\alpha(-1+6\delta) - 4\alpha^3(6+23\delta)}{(2+\alpha)\left(-108 + 67\alpha^3\delta + 108\alpha(1+\delta) - \alpha^2(27+164\delta)\right)} \\
kb = \frac{p_{AF} - p_{BH}}{1-2x} &=: \frac{-72 - 10\alpha^4\delta + 24\alpha(4+3\delta) - 12\alpha^2(3+11\delta) + \alpha^3(3+72\delta)}{(2+\alpha)\left(-108 + 67\alpha^3\delta + 108\alpha(1+\delta) - \alpha^2(27+164\delta)\right)} \\
kn = \frac{p_{AF} - p_{BF}}{1-2x} &=: \frac{-36 + 27\alpha^3\delta + 12\alpha(4+3\delta) - \alpha^2(15+64\delta)}{-108 + 67\alpha^3\delta + 108\alpha(1+\delta) - \alpha^2(27+164\delta)}
\end{aligned}
$$

Plugging these values in (2.15) yields the discounted social welfare over two periods when consumers are sophisticated. Taking the derivatives with respect to α, we get,

$$
\begin{aligned}
\frac{\partial W}{\partial \alpha} = \Big[& -9953328\delta + 2903040\alpha\delta + 2208384\alpha^2\delta - 12498624\alpha^3\delta + 13833504\alpha^4\delta - 6831216\alpha^5\delta + 1549368\alpha^6\delta \\
& - 91044\alpha^7\delta - 22842\alpha^8\delta + 3807\alpha^9\delta + 2681856\alpha\delta^2 - 8280576\alpha^2\delta^2 5708160\alpha^3\delta^2 + 42311232\alpha^4\delta^2 \\
& - 57128544\alpha^5\delta^2 + 35663760\alpha^6\delta^2 - 10828296\alpha^7\delta^2 + 1074060\alpha^8\delta^2 + 197730\alpha^9\delta^2 - 46971\alpha^{10}\delta^2 \\
& - 2529792\alpha^2\delta^3 + 8425216\alpha^3\delta^3 + 4298112\alpha^4\delta^3 - 47260992\alpha^5\delta^3 + 76146336\alpha^6\delta^3 - 57873840\alpha^7\delta^3 \\
& + 22389192\alpha^8\delta^3 - 3339100\alpha^9\delta^3 - 377496\alpha^{10}\delta^3 + 139320\alpha^{11}\delta^3 + 843264\alpha^3\delta^4 - 3128576\alpha^4\delta^4 \\
& - 429696\alpha^5\delta^4 + 17049024\alpha^6\delta^4 - 33002336\alpha^7\delta^4 + 29935632\alpha^8\delta^4 - 14168184\alpha^9\delta^4 + 2860852\alpha^{10}\delta^4 \\
& + 154800\alpha^{11}\delta^4 - 115240\alpha^{12}\delta^4 \Big] \Big/ \left(4(2+\alpha)^3 \left(-108 + 67\alpha^3\delta + 108\alpha(1+\delta) - \alpha^2(27+164\delta)\right)^3 \right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 W}{\partial \alpha^2} = \Big[& -1522851184\delta - 629296128\alpha\delta + 3417458688\alpha^2\delta - 5901050880\alpha^3\delta + 5656946688\alpha^4\delta - 3501159552\alpha^5\delta \\
& + 1438031232\alpha^6\delta - 368815680\alpha^7\delta + 50423472\alpha^8\delta - 2458188\alpha^9\delta + 32845824\delta^2 + 214990848\alpha\delta^2 \\
& + 3158341632\alpha^2\delta^2 - 15374333952\alpha^3\delta^2 + 29345649408\alpha^4\delta^2 - 32324023296\alpha^5\delta^2 + 22918609152\alpha^6\delta^2 \\
& - 10508424192\alpha^7\delta^2 + 2919142800\alpha^8\delta^2 - 426233664\alpha^9\delta^2 + 26411832\alpha^{10}\delta^2 - 2012040\alpha^{11}\delta^2 \\
& + 251505\alpha^{12}\delta^2 - 32845824\alpha\delta^3 - 223838208\alpha^2\delta^3 - 4905197568\alpha^3\delta^3 + 25659721728\alpha^4\delta^3 \\
& - 54767704320\alpha^5\delta^3 + 67819769856\alpha^6\delta^3 - 53842051584\alpha^7\delta^3 + 27584637696\alpha^8\delta^3 - 8629178832\alpha^9\delta^3 \\
& + 1452670848\alpha^{10}\delta^3 - 103840128\alpha^{11}\delta^3 + 4737096\alpha^{12}\delta^3 - 614583\alpha^{13}\delta^3 + 238436352\alpha^3\delta^4 \\
& + 2973677568\alpha^4\delta^4 - 18513967104\alpha^5\delta^4 + 44766990336\alpha^6\delta^4 - 62005146624\alpha^7\delta^4 + 54619091712\alpha^8\delta^4 \\
& - 31085738496\alpha^9\delta^4 + 11000229888\alpha^{10}\delta^4 - 2188484352\alpha^{11}\delta^4 + 190885848\alpha^{12}\delta^4 - 1735848\alpha^{13}\delta^4 \\
& - 59609088\alpha^4\delta^5 - 705207296\alpha^5\delta^5 + 4987729920\alpha^6\delta^5 - 13444332544\alpha^7\delta^5 + 20663468032\alpha^8\delta^5 \\
& - 20183746176\alpha^9\delta^5 + 12815230592\alpha^{10}\delta^5 - 5136402368\alpha^{11}\delta^5 + 1183836432\alpha^{12}\delta^5 - 120262684\alpha^{13}\delta^5 \Big] \\
& \Big/ \left(2(2+\alpha)^4 \left(108 - 67\alpha^3\delta - 108\alpha(1+\delta) + \alpha^2(27+164\delta)\right)^4 \right)
\end{aligned}
$$

When $\alpha \in [0,1]$, $\partial W/\partial\alpha = 0$ has an unique solution $\alpha^{sophW}(\delta) \approx 0.39$.
As $\partial^2 W/\partial\alpha^2|_{\alpha^{sophW}} < 0$, we have a maximum and social welfare is inverse u-shaped in α.
Note finally, that $W|_{\alpha=1} < W|_{\alpha=0}$ for any δ.

Subtracting the equilibrium profits given by (2.63) from social welfare yields consumer surplus.

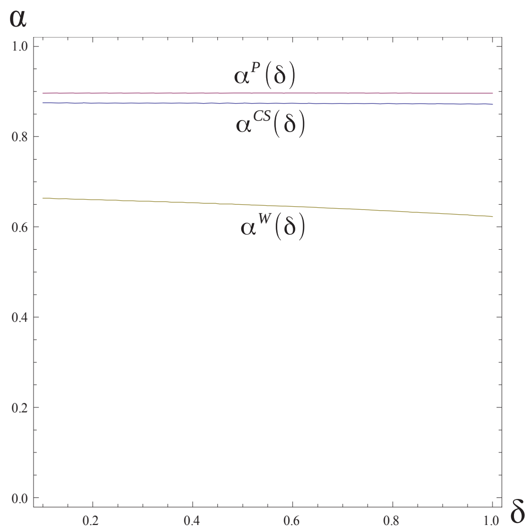
Taking the derivatives with respect to α, we get,

$$
\begin{aligned}
\frac{\partial CS}{\partial \alpha} = \Big[& -1672151048\delta + 663883776\alpha\delta - 950164992\alpha^2\delta + 615672576\alpha^3\delta - 204291072\alpha^4\delta + 112845312\alpha^5\delta \\
& - 114353856\alpha^6\delta + 56928096\alpha^7\delta - 8149248\alpha^8\delta - 2811024\alpha^9\delta + 1134810\alpha^{10}\delta - 113481\alpha^{11}\delta \\
& + 499322880\alpha\delta^2 - 2236336128\alpha^2\delta^2 + 3780214272\alpha^3\delta^2 - 3025969920\alpha^4\delta^2 + 1208148480\alpha^5\delta^2 \\
& - 475483392\alpha^6\delta^2 + 478830528\alpha^7\delta^2 - 304930656\alpha^8\delta^2 + 67382928\alpha^9\delta^2 + 10846008\alpha^{10}\delta^2 - 7000938\alpha^{11}\delta^2 \\
& + 828657\alpha^{12}\delta^2 - 498161664\alpha^2\delta^3 + 2485976064\alpha^3\delta^3 - 4845837312\alpha^4\delta^3 + 4651117056\alpha^5\delta^3 \\
& - 2274537216\alpha^6\delta^3 + 765194112\alpha^7\delta^3 - 632573568\alpha^8\delta^3 + 506233920\alpha^9\delta^3 - 159938424\alpha^{10}\delta^3 \\
& - 7757460\alpha^{11}\delta^3 + 14040948\alpha^{12}\delta^3 - 2035206\alpha^{13}\delta^3 + 166053888\alpha^3\delta^4 - 914101248\alpha^4\delta^4 + 2018638848\alpha^5\delta^4 \\
& - 2267368960\alpha^6\delta^4 + 1342144768\alpha^7\delta^4 - 460806784\alpha^8\delta^4 + 273242496\alpha^9\delta^4 - 258648000\alpha^{10}\delta^4 \\
& + 113115240\alpha^{11}\delta^4 - 5198564\alpha^{12}\delta^4 - 8995108\alpha^{13}\delta^4 + 1683442\alpha^{14}\delta^4 \Big] \\
& \Big/ \Big(12(-2+\alpha)^2(2+\alpha)^3 \left(-108 + 67\alpha^3\delta + 108\alpha(1+\delta) - \alpha^2(27+164\delta)\right)^3 \Big)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 CS}{\partial \alpha^2} = \Big[& 177009131520\delta - 68134182912\alpha\delta + 136313155584\alpha^2\delta - 196004468736\alpha^3\delta + 214813928448\alpha^4\delta \\
& - 172823528448\alpha^5\delta + 98251937280\alpha^6\delta - 38741462784\alpha^7\delta + 10623011328\alpha^8\delta - 2124084384\alpha^9\delta \\
& + 344776176\alpha^{10}\delta - 45437112\alpha^{11}\delta + 3297996\alpha^{12}\delta - 1672151048\delta^2 - 68832903168\alpha\delta^2 + 303570395136\alpha^2\delta^2 \\
& - 670621151232\alpha^3\delta^2 + 1028740331520\alpha^4\delta^2 - 1196490175488\alpha^5\delta^2 + 1036466523648\alpha^6\delta^2 \\
& - 642659422464\alpha^7\delta^2 + 277143679872\alpha^8\delta^2 - 82277031744\alpha^9\delta^2 + 17372110176\alpha^{10}\delta^2 - 2908894032\alpha^{11}\delta^2 \\
& + 398028168\alpha^{12}\delta^2 - 27794988\alpha^{13}\delta^2 - 1017198\alpha^{14}\delta^2 + 72657\alpha^{15}\delta^2 + 167215104\alpha\delta^3 + 10303325798\alpha^2\delta^3 \\
& - 508333252608\alpha^3\delta^3 + 1227015032832\alpha^4\delta^3 - 2000885078016\alpha^5\delta^3 + 2456866391040\alpha^6\delta^3 \\
& - 2280226613760\alpha^7\delta^3 + 1542025642752\alpha^8\delta^3 - 731102163840\alpha^9\delta^3 + 236434547520\alpha^{10}\delta^3 \\
& - 52530992928\alpha^{11}\delta^3 + 8904914928\alpha^{12}\delta^3 - 1297469160\alpha^{13}\delta^3 + 110460060\alpha^{14}\delta^3 + 2352186\alpha^{15}\delta^3 \\
& - 189819\alpha^{16}\delta^3 - 69310218240\alpha^3\delta^4 + 376692916224\alpha^4\delta^4 - 987409113088\alpha^5\delta^4 + 1711834656768\alpha^6\delta^4 \\
& - 2214496372736\alpha^7\delta^4 + 2188382142464\alpha^8\delta^4 - 1605776182272\alpha^9\delta^4 + 837627819520\alpha^{10}\delta^4 \\
& - 297744442112\alpha^{11}\delta^4 + 70521484224\alpha^{12}\delta^4 - 11998573024\alpha^{13}\delta^4 + 1818365872\alpha^{14}\delta^4 - 193525224\alpha^{15}\delta^4 \\
& - 159448\alpha^{16}\delta^4 + 17327554560\alpha^4\delta^5 - 103228186624\alpha^5\delta^5 + 293374046208\alpha^6\delta^5 - 542568175616\alpha^7\delta^5 \\
& + 740910504960\alpha^8\delta^5 - 776408675328\alpha^9\delta^5 + 613694283264\alpha^{10}\delta^5 - 350588480256\alpha^{11}\delta^5 \\
& + 137571916032\alpha^{12}\delta^5 - 35403523488\alpha^{13}\delta^5 + 6144317552\alpha^{14}\delta^5 - 941014392\alpha^{15}\delta^5 + 120757948\alpha^{16}\delta^5 \Big] \\
& \Big/ \Big(2(-2+\alpha)^3(2+\alpha)^4 \left(108 - 67\alpha^3\delta - 108\alpha(1+\delta) + \alpha^2(27+164\delta)\right)^4 \Big)
\end{aligned}
$$

When $\alpha \in [0,1]$, $\partial CS/\partial\alpha = 0$ has an unique solution $\alpha^{sophCS}(\delta) \approx 0.59$.
As $\partial^2 CS/\partial\alpha^2|_{\alpha^{sophCS}(\delta)} < 0$, we have a maximum and consumer surplus is inverse u-shaped in α.
Note finally, that $CS|_{\alpha=1} > CS|_{\alpha=0}$ for any δ.
The Proposition follows. *Q.E.D.*

Proof of Corollary 2.1 and 2.2. Corollary 2.1 and 2.2 directly follow from equating the above profit, social welfare and consumer surplus functions when consumers are myopic with those when consumers are sophisticated and solving for the (possible) root, yielding the clear results in the homogeneous case and the unique cut-offs $\alpha^P(\delta)$, $\alpha^W(\delta)$ and $\alpha^{CS}(\delta)$ in the heterogeneous case, depicted in the following picture. *Q.E.D.*



2.9 References

1. Armstrong M. (2006) Recent Developments in the Economics of Price Discrimination. In: Blundell, R. and Newey, W.K. and Persson, T., (eds.) Advances in Economics and Econometrics: Theory and Applications: Ninth World Congress: volume II. (pp. 97-141). Cambridge University Press: Cambridge, UK.

2. Baye I., Reiz T., Sapi G. (2018) Customer Recognition and Mobile Geo-Targeting, DICE Discussion Paper.

3. Baye I., Sapi G. (2019) Should Mobile Marketers Collect Other Data Than Geo-Location? The Scandinavian Journal of Economics; 121; pp. 647- 675.

4. Bester H., Petrakis E. (1996) Coupons and Oligopolistic Price Discrimination. International Journal of Industrial Organization; 14; pp. 227-242.

5. Borenstein S. (1985) Price Discrimination in Free-Entry Markets. RAND Journal of Economics; 16; pp. 380-397.

6. Carroni, E. (2016) Competitive Customer Poaching with Symmetric Firms. International Journal of Industrial Organization; 48; pp. 173-206.

7. Chen Y., Narasimhan C., Zhang Z. J. (2001) Individual Marketing with Imperfect Targetability. Marketing Science; 20; pp. 23-41.

8. Chen Y., Zhang Z. J. (2009) Dynamic Targeted Pricing with Strategic Consumers. International Journal of Industrial Organization; 27; pp. 43-50.

9. Colombo, S. (2016) Imperfect Behavior-Based Price Discrimination. Journal of Economics & Management Strategy; 25; pp. 563-583.

10. Corts K. S. (1998) Third-Degree Price Discrimination in Oligopoly: All-Out Competition and Strategic Commitment. RAND Journal of Economics; 29; pp. 306-323.

11. Danaher P. J., Smith M. S., Ranasinghe K., Danaher T. S. (2015) Where, When, and How Long: Factors Than Influence the Redemption of Mobile Phone Coupons. Journal of Marketing Research; 52; pp. 710-725.

12. Esteves R.-B. (2009) Price Discrimination with Partial Information: Does it Pay off? Economics Letters; 105; pp. 28-31.

13. Esteves R.-B. (2010) Pricing with Customer Recognition. International Journal of Industrial Organization; 28; pp. 669-681.

14. Esteves, R.-B. (2014) Price discrimination with private and imperfect information. Scandinavian Journal of Economics; 116; pp. 766–796.

15. Fudenberg D., Tirole J. (2000) Customer Poaching and Brand Switching. RAND Journal of Economics; 31; pp. 634-657.

16. Fudenberg D., Villas-Boas J.M. (2006), Behavior-Based Price Discrimination and Customer Recognition, pp. 377-433, in: T. Hendershott (eds.), Handbook in Information Systems, Vol. 1, Emerald Group Publishing Limited, UK.

17. Jentzsch N., Sapi G., Suleymanova I. (2013) Targeted Pricing and Customer Data Sharing Among Rivals. International Journal of Industrial Organization; 31; pp. 131-144.

18. Liu Q., Serfes K. (2004) Quality of Information and Oligopolistic Price Discrimination. Journal of Economics & Management Strategy; 13; pp. 671-702.

19. Liu Q., Shuai J. (2013) Multi-dimensional Price Discrimination. International Journal of Industrial Organization; 31; pp. 417-428.

20. Liu Q., Shuai, J. (2016) Price Discrimination with Varying Qualities of Information. B.E. Journal of Economic Analysis and Policy; 16; pp. 1093-1121.

21. Shaffer G., Zhang Z. J. (1995) Competitive Coupon Targeting. Marketing Science; 14; pp. 395-416.

22. Shaffer G., Zhang Z. J. (2000) Pay to Switch or Pay to Stay: Preference-Based Price Discrimination in Markets with Switching Costs. Journal of Economics and Management Strategy; 9; pp. 397-424.

23. Shaffer G., Zhang Z. J. (2002) Competitive One-to-One Promotions. Management Science; 48; pp. 1143-1160.

24. Thisse J.-F., Vives X. (1988) On the Strategic Choice of Spatial Price Policy. American Economic Review; 78; pp. 122-137.

25. Villas-Boas J. M. (1999) Dynamic Competition with Customer Recognition. RAND Journal of Economics; 30; pp. 604-631.

26. Won, S. B. (2017). Two-Dimensional Price Discrimination With Uncertainty About Willingness-To-Pay. Annals of Regional Science; 58; pp. 513-541.

Chapter 3

Behavioral Targeting and Incentives to Invest in Screening Technology

Co-authored with Astrid Kiekert

3.1 Introduction

In the past decades, the ability of firms in different industries to collect data and to use it for targeted marketing and pricing has improved rapidly. The increasing importance of online marketplaces and the emergence of sophisticated techniques to collect and store data on customers' past purchases are nowadays common for many industries. In retailing, for instance, many pieces of information are collected on purchases within loyalty programs (e.g., type and quantity of the product bought as well as its effective price) and are linked with the respective customers.[1] Especially in online marketplaces, plenty of customer data are collected, which can be used for personalized advertising and pricing.[2] Setting prices in subsequent periods based on a customer's purchasing history, a strategy called behavior-based price discrimination (BBPD), has therefore become increasingly prevalent. For instance, the price of a contract with an electricity provider or a telecommunication company depends on whether a consumer is an old or a new customer.[3]

While a large amount of customer data is a by-product of firms' operations (such as in the case of a telecommunication or an electricity firm), in many other cases firms can collect and use data for targeted activities only if they make the necessary strategic investment into respective customer data technologies. This explains the asymmetric outcomes in some industries, where firms are characterized by different abilities to perform targeted marketing and pricing.[4] In the German retailing sector, for instance, most full-range grocery stores and their subsidiaries (such as Rewe, Edeka, real, and Penny) participate in joint loyalty programs like "Payback" or "DeutschlandCard", while leading discounters, such as Aldi and Lidl, do not. The organic supermarket chain "denn's" chose an alternative strategy and set up its own

[1]See Ross (2013) for a discussion of the importance of price discrimination in the retailing sector.

[2]An anonymous computer scientist in online retailing stated that "... [i]ts common for big retail web sites to direct different users to different deals, offers, or items based on their purchase histories or cookies... And companies frequently offer special deals for customers with a few items in their shopping bags - from discounts on additional items, to free shipping, to coupons for future purchases" (Klosowski, 2013). This indicates that these firms do, at least, perform third-degree price discrimination on a regular basis.

[3]A study by the German price comparison webpage Check24 has shown that old customers pay up to 80 per cent more for a combination of Internet and telephone flatrate compared to new customers (Check24, 2016).

[4]For example, after Tesco introduced its "Clubcard" program in the UK in 1995, it took its main competitor Sainsbury until 2002 to set up its own loyalty card program called "Nectar" on a comparable scale (The Guardian, 2003).

customer retention program.[5] These examples clearly indicate that it is a strategic decision of a firm to invest in respective technologies and gain the ability to collect customer data and to analyze it for targeted pricing (see also Schenker, 2015).

In this project, we analyze incentives to invest into a screening technology which allows a firm to recognize its customers and to price discriminate respectively. We consider a Hotelling model with two periods, such that in the first period every firm makes its investment decisions and records its customers' purchases provided it holds the screening technology, while in the second period firms are able to price discriminate based on the collected customer data. We consider two scenarios with all consumers being either myopic or sophisticated. The latter anticipate that a firm with a screening technology will use the collected customer data for targeted pricing in the second period and therefore adjust their behavior in the first period accordingly. Myopic consumers, in contrast, do not realize this and base their purchase decisions only on the prices in the current period. The analysis of myopic consumers is of special relevance for policy as consumer naiveté expressed by their limited foresight is considered to be the main source of consumer harm by the European Commission (cf. Europe Economics, 2007).

Our main result is that the prisoner's dilemma problem, known since Thisse and Vives (1988) can be resolved in our model. It states that price discrimination along consumer brand preferences is unilaterally attractive, however, under competition it results in lower overall profits. Precisely, we show that when consumers are myopic, only one firm invests into the screening technology in equilibrium, which results in higher profits for both firms compared to the outcome where both of them are able to discriminate based on consumer behavior. When consumers are sophisticated, two symmetric equilibria exist, where either both firms invest into the screening technology or do not. Respectively, in the latter equilibrium firms avoid the prisoner's dilemma problem. We also show that both consumer surplus and social welfare are higher when consumers are sophisticated and the discount factor is large enough. We conclude that educating consumers about firms' ability to use customer data for price discrimination can improve welfare.

This article is organized as follows. In the next section we discuss the relevant literature. In Section 3.3 we present the model. In the three subsequent sections we

[5] While consumers with a "Payback"-card or "DeutschlandCard" can collect bonus points in a large variety of stores on- and offline, the owners of the "Meindenn's"-loyalty card can only collect bonus points when shopping in a denn's-store. Between 2012 and 2017, denn's took part in the "Payback"-program but then decided to set up its own loyalty program (see Schader, 2017, a, b).

provide the equilibrium analysis. Finally, Sections 3.7 concludes.

3.2 Related Literature

Our article contributes to the literature on competitive price discrimination with demand-side asymmetries, where consumers are classified into different groups depending on their preferences for firms. In this literature, Thisse and Vives (1988) were the first to demonstrate the famous prisoner's dilemma result in a standard Hotelling model: Price discrimination, despite being individually profitable, makes both firms worse off, because it intensifies competition. Precisely, every firm targets aggressively consumers with a preference for the rival's product, which in turn puts a downward pressure on that firm's own pricing.

An important question in this literature strand is whether firms are able to avoid the prisoner's dilemma result. Several studies indeed give an affirmative answer to this question and show that when firms are able to decide on the intensity of price discrimination, they choose to target less in order to soften competition. For instance, Chen and Iyer (2002) assume that firms have access to an exogenously provided database which allows to target a certain share of consumers in the market. Moreover, the more firms invest into the database, the larger is the share of consumers they are able to address with targeted prices. The authors show that even if the costs of investment into database are zero, in equilibrium, perfect addressability never prevails such that firms optimally restrict the own targeting ability. In a related study Liu and Serfes (2004) analyze firms' incentives to acquire customer data depending on its quality. They find that firms do not invest when data is of low quality and in this way avoid the prisoner's dilemma problem. The main difference of these articles from ours is that they assume that customer data is available exogenously and has been collected by third parties. In our analysis, in contrast, firms have to collect data about consumer preferences themselves through observing customer behavior. We show that in this case the ability of firms to avoid the prisoner's dilemma result depends strongly on consumer sophistication. Precisely, when consumers are sophisticated, i.e., they foresee that a firm with a screening technology will use the collected customer data for targeted pricing in the future, an equilibrium exists where none of the firms acquires the technology and respectively none of them performs targeted pricing in equilibrium.

Our study is closely related to the literature on behavior-based price discrimination (BBPD), where firms target consumers depending on their behavior in the previous periods.[6] The seminal paper in this strand is Fudenberg and Tirole (2000), who were the first to demonstrate the detrimental effect of BBPD on profits: The positive rent-extraction effect of more detailed information on consumer preferences is dominated by a negative competition effect, when each firm prices the loyal customers of its rival more aggressively. The same effect is found in Esteves (2010) who studies a setting with a discrete distribution of consumer preferences. Colombo (2016) assumes that firms have incomplete information about consumers' purchasing histories. He analyzes how the accuracy of behavioral customer data impacts on profits. The main difference of our study compared to these articles is that we consider firms that are initially not endowed with a screening technology which allows to record customer behavior and target customers respectively. In turn, it is a strategic decision of a firm whether to invest in such a technology. We show that both with myopic and sophisticated consumers firms are able to avoid the detrimental effect of behavioral targeting on profits through not investing into the screening technology.

Closely related to our work is Baye and Sapi (2020) who analyze investment decisions into technology which allows perfect targeting among a firm's past customers. In contrast, in our analysis a firm with a screening technology is only able to identify its own past customers and to price discriminate among them and the rival's customers respectively. While we get similar results in the case of myopic consumers, where only one of the firms invests into the respective technology, different from Baye and Sapi (2020), we show that with sophisticated consumers the equilibrium exists where both firms hold screening technology. Hence, in contrast to this study, we identify scope for a prisoner's dilemma result even when firms decide strategically on their ability to target consumers based on behavior.

3.3 The Model

We consider a model where two firms, A and B, compete in prices within two periods. In each period they sell horizontally differentiated products à la Hotelling. Firms are located at the respective ends of an interval with a unit length ($x_A = 0$, $x_B = 1$) and have zero production costs. There is a unit mass of consumers, each with an address

[6] Esteves (2009) and Fudenberg and Villas-Boas (2006) provide surveys of the articles in this field.

$x \in [0, 1]$ corresponding to her preference for the ideal product. Each consumer buys at most one unit of a product. If a consumer cannot buy her ideal product, she incurs linear transport costs proportional to the distance to the firm. The utility of a consumer with address x from buying the product of firm $i = A, B$ in each period is

$$U_i(p_i, x) = v - t|x\text{-}x_i| - p_i, \tag{3.1}$$

with $v > 0$ denoting the base utility and p_i the price of firm i in the respective period. The base utility is assumed to be high enough such that each consumer purchases in equilibrium while she buys the product from the firm providing her a higher utility.[7] Firms initially hold no customer data, but can invest in a screening technology in the first period while this decision is publicly observable. With this technology a firm can record consumers' purchasing decisions in the first period and price discriminate among them in the following period respectively. The timing of the game is as follows:

Period 1:

Stage 1 (Investment). Firms decide independently and simultaneously whether to invest into a screening technology.

Stage 2 (Competition with regular prices). First, firms publish independently and simultaneously their regular prices. Consumers observe these prices and make their purchasing decisions.

Period 2:

(Competition with regular and targeted prices). In case that a firm invested into the screening technology, it observes consumers' first-period choices and discriminates respectively by charging different prices to the own and the rival's past customers. Otherwise, a firm sets a uniform price. Targeted prices are set after uniform prices, because firms are typically more flexible to change prices, which are targeted at smaller customer groups.[8]

The timing of the price setting behavior in the second period is in line with a large body of literature on competitive price discrimination (see e.g., Thisse and Vives, 1988; Shaffer and Zhang, 1995; Liu and Serfes, 2004, 2006; Choudhary et al.,

[7] As in Liu and Serfes (2006), in case of a tie in utilities, we assume that a consumer chooses the firm nearby, the consumer with $x = 0.5$ visits firm A.

[8] Furthermore, if firms decide on overall and customized prices simultaneously a Nash equilibrium does not always exist. For instance, Silbye (2010) shows in a model where firms learn the exact preferences of their customers in the first period that no Nash equilibrium in pure strategies exists in the second period. Therefore, he solves for an equilibrium in mixed strategies.

2005; Baye and Sapi, 2020).

Given the investment stage's outcome in the first period, three subgames can emerge in the following period where i) only one firm invested, ii) both firms invested, and iii) no firm invested. To subgames i) and ii) we will refer to as the asymmetric and symmetric subgame, respectively, while subgame iii) is also symmetric, however, constitute a standard Hotelling model in both periods. We assume that firms maximize the discounted sum of profits over both periods and discount future profits by $\delta \in [0, 1]$. On the consumer side, we consider two cases: All consumers being either myopic or sophisticated. Myopic consumers do not anticipate that customer data will be used for price discrimination in the second period. In contrast, sophisticated consumers maximize their expected surplus over both periods, expecting to get targeted offers in the second period from a firm (firms) which invested into the screening technology.[9]

We solve for a subgame-perfect Nash equilibrium in pure strategies and start the analysis from the second period.

3.4 Equilibrium Analysis of the Second Period

In subgame iii) where none of the firms invested into a screening technology, firms' profits in equilibrium are $\Pi_A = \Pi_B = t/2$, because in each period we get the standard result of a Hotelling model. To analyze the second period in subgames i) and ii), we denote with α the market share of firm A in the first period, constituting the segment of firm A.[10] To consumers with $x \leq \alpha$ ($x > \alpha$) we also refer as "segment α" ("segment $1 - \alpha$"). The respective variables are marked with a superscript "α" or "$1 - \alpha$" depending on whether they refer to segment α or $1 - \alpha$.[11] For instance, with $p^{\alpha}_{A,2}$ we denote the price of firm A on segment α in the second period.

[9] Sophisticated consumers discount future surplus by the same discount factor as firms do (cf. e.g., Klemperer 1995; Chen, 1997; Fudenberg and Tirole, 2000; Liu and Serfes, 2006).

[10] A standard revealed preference argument yields that if a consumer with $x = \alpha$ bought at firm A in the first period, then all consumers with address $x < \alpha$ bought at that firm as well.

[11] We use subscripts to indicate the respective firm and period.

3.4.1 Asymmetric Subgame: Only one firm invests into the screening technology

We assume without loss of generality that it is firm A, which holds the technology allowing to practice third-degree price discrimination while firm B has to set a uniform price to all consumers irrespective of their first-period decisions.

Given that firm B sets a uniform price to all consumers, firm A's best-reply function on segment α is

$$p_{A,2}^{\alpha}(p_{B,2}) = \begin{cases} \frac{t+p_{B,2}}{2}, & \text{if } p_{B,2} < t(4\alpha-1) \\ p_{B,2}+t(1-2\alpha), & \text{if } p_{B,2} \geq t(4\alpha-1), \end{cases} \tag{3.2}$$

while on segment $1-\alpha$ it is

$$p_{A,2}^{1-\alpha}(p_{B,2}) = \begin{cases} 0, & \text{if } p_{B,2} \leq t(2\alpha-1) \\ \frac{p_{B,2}+t(1-2\alpha)}{2}, & \text{if } t(2\alpha-1) < p_{B,2} < t(3-2\alpha) \\ p_{B,2}-t, & \text{if } p_{B,2} \geq t(3-2\alpha). \end{cases} \tag{3.3}$$

According to the best-response function (3.2), if the price of firm B is relatively small, then firm A charges a price, such that both firms serve consumers on segment α in equilibrium. However, if the price of firm B is large enough, then firm A charges a price targeted at the least loyal consumer on segment α and serves all the consumers there. Consider now the best-response function (3.3). If the price of firm B is sufficiently small, firm A is not able to serve any consumer on segment $1-\alpha$, in which case it cannot do better than setting a price of zero. If, in contrast, the price of firm B is sufficiently high, firm A charges the price targeted at the most loyal consumer of firm B and serves the whole segment $1-\alpha$. In the intermediate case, both firms serve consumers on segment $1-\alpha$.

Using the best-response functions (3.2) and (3.3), we arrive at the equilibria of the second period, as stated in Lemma 3.1.[12]

Lemma 3.1. *(Second period. Asymmetric subgame.) Assume that only firm A invested in screening technology in the first period. The equilibrium of the second period depends on the share of firm A in the first period (α) as follows:*

i) If α is relatively small, $\alpha \in [0; 0.46]$, firm A serves all consumers on its

[12] All the omitted proofs are contained in the Appendix.

segment while firm B loses consumers on its segment. Firm B serves only those with $x > (5+2\alpha)/8$ *and charges a price* $p_{B,2}(x,\alpha) = [t(3-2\alpha)]/2$. *The price of firm A on its segment is* $p^{\alpha}_{A,2}(x,\alpha) = [t(5-6\alpha)]/2$, *on the segment of firm B it charges the price* $p^{1-\alpha}_{A,2} = [t(5-6\alpha)]/4$. *Firms realize profits:* $\Pi_{A,2} = [5t(5+4\alpha-12\alpha^2)]/32$ *and* $\Pi_{B,2} = [t(3-2\alpha)^2]/16$, *respectively.*

ii) If α *is intermediate,* $\alpha \in (0.46; 0.93]$, *both firms lose consumers on their segments. Firm A serves only those consumers with* $x \leq (3+\alpha)/8$ *on its segment and those with* $x \leq (3+5\alpha)/8$ *on the segment of firm B. Firm B charges the price* $p_{B,2} = [t(1+\alpha)]/2$. *The price of firm A on its segment is* $p^{\alpha}_{A,2} = [t(3+\alpha)]/4$, *on the segment of firm B it charges the price* $p^{1-\alpha}_{A,2} = [3t(1-\alpha)]/4$. *Firms realize profits:* $\Pi_{A,2} = [t(9-6\alpha+5\alpha^2)]/16$ *and* $\Pi_{B,2} = [t(1+\alpha)^2]/8$, *respectively.*

iii) If α *is relatively large,* $\alpha \in (0.93; 1]$, *firm A loses consumers on its own segment while it serves all consumers on the segment of firm B. More precisely, firm A serves consumers with* $x \leq (1+4\alpha)/8$. *Firm B charges the price* $p_{B,2} = [t(4\alpha-1)]/2$. *The price of firm A on its segment is* $p^{\alpha}_{A,2} = [t(4\alpha+1)]/4$, *on the segment of firm B it charges the price* $p^{1-\alpha}_{A,2} = [t(4\alpha-3)]/2$. *Firms realize profits:* $\Pi_{A,2} = [t(120\alpha-47-48\alpha^2)]/32$ *and* $\Pi_{B,2} = [t(1-4\alpha)^2]/16$, *respectively.*

Despite the fact that firm A has a competitive advantage over firm B in the asymmetric subgame, we find that the second-period equilibrium crucially depends on the optimal strategy of firm B. Precisely, being bound to set a uniform price to all customers, firm B has to decide whether to target both segments or implicitly focus on only one customer group. Firm A in turn reacts optimally to firm B's price in line with the best-response functions (3.2) and (3.3).

Consider case i) of Lemma 3.1. Given a relatively small market share of firm A in the first period, firm A can only differentiate between a rather small interval of very loyal customers and a rather large interval consisting of own loyal customers and loyal customers of firm B.[13] In this case, firm B would have to set a very low uniform price in order to attract some of the very loyal customers of firm A in the former group, which makes this strategy unprofitable. Instead, firm B optimally charges a relatively high price and loses some of the consumers it served in the previous period. This strategy can be referred to as a rent-extraction strategy on the larger customer group including its own loyal customers. Consequently, firm A monopolizes its former segment in this case and clearly extends its first-period market share in

[13] We call a customer loyal if she always chooses a certain firm when prices are the same.

the second period, with an intuitively increasing second-period market share in α.

At $\alpha = 0.46$, firm B switches its strategy for the following intermediate values of α, since its former segment becomes increasingly too small for the previous rent-extraction strategy. Instead, firm B opts for a market-sharing strategy on both segments in the second period and sets a relatively low uniform price in order to attract also customers on firm A's segment. However, firm A clearly utilizes its competitive advantage to price each segment separately such that for lower values of $\alpha \in [0.46, 0.93]$, firm A intuitively extends its first-period market share in the second period. Only for $\alpha > 0.6$ leads firm B's new market-sharing strategy also to a higher second-period market share of firm B compared to the first period. With such relatively high values of α contains firm B's segment only rather loyal customers of this firm such that it can more easily hold many of them even against a rather low special price of firm A for these customers. Additionally, firm A's segment contains relatively many loyal customers of firm B such that it is also relatively easy for firm B to attract some of them, leading to an overall higher second-period market share of firm B compared to the first period.

At $\alpha = 0.93$, firm B switches its strategy again for the remaining values of α. With such high values of α, we have that the segment of firm B is very small and contains also only the most loyal customers of firm B. This leads firm A to price this customer group extremely aggressively. Given that the segment of firm A contains not the most but also very loyal customers of firm B, it refuses to defend its segment against the very low price of firm A and instead sets its uniform price to attract its rather loyal customers on the segment of firm A solely. Thereby, despite losing its whole segment to firm A, firm B extends its first-period market share in the second period, since firm A can intuitively not sustain this very asymmetric first-period market outcome even with its competitive advantage. This effect is further strengthened for increasing values of α, since firm A's advantage additionally clearly vanishes with α.

Lemma 3.1 also presents the equilibrium targeting strategy of firm A. Quite intuitively, firm A charges a higher price to its own past customers than to those of the rival, because the former (the latter) revealed their preference for firm A's (B's) product through the respective purchases in the first period. Also intuitively, we have that for small and intermediate values of α, the price of firm B takes values between the equilibrium prices of firm A on the two segments, such that the prices in the second period can be ordered as: $p^{\alpha}_{A,2} > p_{B,2} > p^{1-\alpha}_{A,2}$ for $\alpha \in [0, 0.933]$. While firm A is able to use its pricing flexibility to extract rents from its past customers and at the same time price aggressively the former customers of the rival to gain market shares among them, firm B has to trade-off these incentives with its uniform price, resulting in a price level in the intermediate range. However, when α is very large ($\alpha > 0.93$), firm B charges a price which is higher than both discriminatory prices of firm A, such that the order of prices is: $p_{B,2} > p^{\alpha}_{A,2} > p^{1-\alpha}_{A,2}$. In this case contains segment α almost the whole interval of consumer preferences, while segment $1-\alpha$ contains only the most loyal customers of firm B. Hence, the aggressive pricing strategy of firm A on segment $1-\alpha$ to gain market shares prevails. However, given the large variety of consumer tastes on segment α, firm A now also trades-off its rent-extraction purposes on its own loyal customers with its incentives to gain market shares on the loyal customers of firm B with its uniform price on segment α, just as firm B does, which focuses on this segment and let firm A monopolize segment $1-\alpha$ for such high values of α. Due to the special situation that firm B, in this case, competes only for its rather and not for its most loyal customers, we have that firms B is less willing to fight for market shares than firm A and puts more emphasis on extracting rents from its rather loyal customers on segment α, leading to this ordering of prices.

Firms' resulting profits for the second period depending on α are displayed in Figures 3.1 and 3.2.

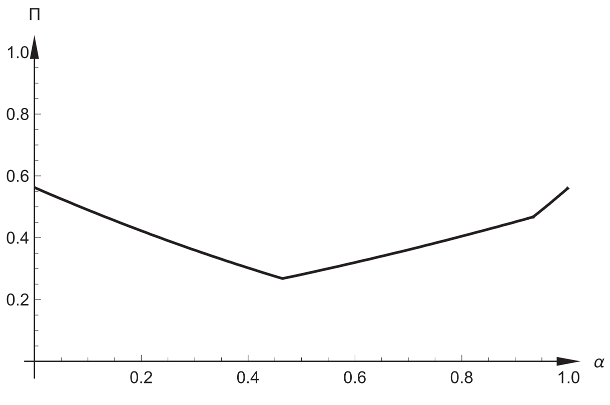


Figure 3.1: Second-period profits of firm B depending on α in the asymmetric subgame with $t = 1$.

First, it is intuitive that the profit of firm B has a continuous course, since firm B is the price leader in our setting. The kinks arise due to the abrupt changes in firm B's pricing strategy depending on α in the second period. To fully understand the pattern of both firms' second-period profit functions, we have to enhance our previous market-sharing and pricing analysis further.

Starting with the profit function of firm B, it starts intuitively with a downward sloping trend in α, since, according to Lemma 3.1, firms compete over a decreasing segment $1-\alpha$ when α is relatively small, leading to fiercer competition and therefore lower prices and profits with increasing α. The same, but simply reversed, reasoning explains the upward sloping trend at the end of the profit pattern, resulting in the overall u-shaped pattern of firm B's profits. Competition over segment α decreases when the segment enlarges, i.e. α increases, allowing firms to raise prices and profits increase. However, for initially intermediate values of α, we have a slightly more complex situation. In this case, firms compete on both intervals with opposing competition effects due to an increase in α. While segment α becomes larger and competition softens, segments $1-\alpha$ shrinks and competition intensifies. Being able to price each segment separately, firm A can individually account these different effects, leading to a price increase on segment α and a price decrease on segment $1-\alpha$. Being bound to set only one uniform price for both segments, firm B has to react and balance both effects together with the price reactions of firm A. Since competition softens on the rather disloyal customers of firm B, while competition intensifies on the more and more loyal customers of firm B, we find that overall the positive price effect dominates for firm B such that firm B's price increases with α and profits recover in the middle of firm B's profit pattern.

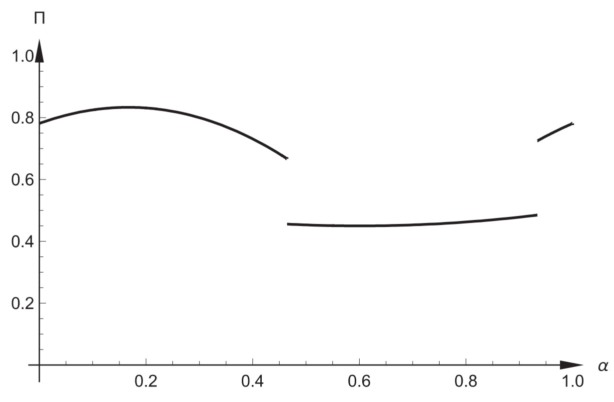


Figure 3.2: Second-period profits of firm A depending on α in the asymmetric subgame with $t = 1$.

Turning to the profit function depending on α of firm A, we first observe an inverse u-shaped pattern. This is due to the opposing effects of better customer data to extract rents on segment α and the higher competition on segment $1-\alpha$ with increasing but initially smaller values of α. At first, the positive effect of recognizing more loyal customers and being able to extract rents from them dominates the negative effect of higher competition on segment $1-\alpha$ for firm A. However, with increasingly better data of firm A, firm B fights increasingly harder on segment $1-\alpha$, reducing the profits of firm A on this and also implicitly on segment α, since the lower price of firm B puts also a downward pressure on firm A's price on segment α. At some point, the increasing competition effect starts to dominate and accordingly, we observe the inverted u-shaped patter for initially smaller values of α. Due to the abrupt change of firm B's pricing strategy at $\alpha = 0.46$, we observe in the following an abrupt decrease in firm A's profit pattern depending on α in contrast to the continuous profit function of firm B. Firm B at once follows a market-sharing strategy on both segments and introduce an considerable price decrease for intermediate values of α, hurting firm A's profits abrupt and significantly. Following this drop, profits of firm A in α exhibit an almost constant but slightly u-shaped pattern.[14] As discussed for the profit pattern of firm B, we have that firm A can address the opposing competition effects of an increase in α on the different segments in this interval individually, leading each others competition effect to almost offset. Finally, at the end of firm A's profit function depending on α, profits of firm A abruptly ascend once again due to the second, but this time for firm A positive, price strategy change of firm B and increase further with α just as the profits of firm B. Firm A monopolizes the segment of firm B in this interval, however, due to the high customer loyalty to firm B of these customers and the relatively small length of this interval, we have that firm A cannot make considerable profits on this segment anyway. Therefore is a shrinking and hence even less profitable segment $1-\alpha$ rather irrelevant for firm A, while the softer competition on segment α boosts firm A's profits. In fact, firm A's price on segment $1-\alpha$ even raises with α on this interval due to the softer competition of firm B's price, rendering additionally the negative effect of a shrinking segment $1-\alpha$.

Finally comparing both profit functions, it is straightforward that firm A's second-period profits are higher than those of firm B for any α due to its competitive advantage.

[14]Interestingly, the minimum of firm A's profit function is at $\alpha = 0.6$, exactly at the point where firm A starts to loose market shares in the second period compared to the first period.

We next recall the second-period equilibrium of the well-known symmetric subgame in order to compare them and being able to better asses our results of the asymmetric subgame.

3.4.2 Symmetric Subgame: Both firms hold the screening technology

The second period of the symmetric subgame where both firms hold the screening technology is analogous to the model considered in Fudenberg and Tirole (2000) and the special case of the uniform distribution of consumer preferences. Lemma 3.2 basically repeats their findings.

Lemma 3.2. *(Second period. Symmetric subgame, cf. Fudenberg and Tirole (2000).) Assume that both firms invested in the screening technology in the first period. The equilibrium of the second period depends on the share of firm A in the first period (α) as follows:*

i) If α is relatively small, $\alpha \in [0, 0.25]$, firm A serves all consumers on its segment while firm B loses consumers on its segment and serves only those with $x > (3+2\alpha)/6$. The price of firm A on its segment is $p^{\alpha}_{A,2} = t(1-2\alpha)$, on the segment of firm B it charges the price $p^{1-\alpha}_{A,2} = [t(3-4\alpha)]/3$. Firm B charges the price $p^{\alpha}_{B,2} = 0$ on the segment of firm A, on its own segment its price is $p^{1-\alpha}_{B,2} = [t(3-2\alpha)]/3$. Firms realize profits: $\Pi_{A,2} = \alpha t(1-2\alpha) + [t(3-4\alpha)^2]/18$ and $\Pi_{B,2} = [t(3-2\alpha)^2]/18$, respectively.

ii) If α is intermediate, $\alpha \in [0.25, 0.75]$, both firms lose consumers on their segments. Firm A serves only consumers with $x \leq (2\alpha+1)/6$ on its segment and those with $x \leq (3+2\alpha)/6$ on the segment of firm B. The price of firm A on its segment is $p^{\alpha}_{A,2}(x, \alpha) = [t(1+2\alpha)]/3$, on the segment of firm B it charges the price $p^{1-\alpha}_{A,2} = [t(3-4\alpha)]/3$. Firm B charges the price $p^{\alpha}_{B,2} = [t(4\alpha-1)]/3$ on the segment of firm A, on its own segment its price is $p^{1-\alpha}_{B,2} = [t(3-2\alpha)]/3$. Firms realize profits: $\Pi_{A,2} = [t(1+2\alpha)^2]/18 + [t(3-4\alpha)^2]/18$ and $\Pi_{B,2} = [t(4\alpha-1)^2]/18 + [t(3-2\alpha)^2]/18$, respectively.

iii) If α is relatively large, $\alpha \in [0.75, 1]$, firm A loses consumers on its segment while firm B serves all consumers on its segment. More precisely, firm A serves those consumers with $x \leq (1+2\alpha)/6$. The price of firm A on its segment is $p^{\alpha}_{A,2}(x, \alpha) = [t(1+2\alpha)]/3$, on the segment of firm B it charges the price $p^{1-\alpha}_{A,2} = 0$. Firm B charges *the price* $p^{\alpha}_{B,2}(x,\alpha) = [t(4\alpha - 1)]/3$ *on the segment of firm A, on its own segment it charges the price* $p^{1-\alpha}_{B,2} = t(2\alpha - 1)$*. Firms realize profits:* $\Pi_{A,2} = [t(1 + 2\alpha)^2]/18$ *and* $\Pi_{B,2} = t(1 - \alpha)(2\alpha - 1) + [t(4\alpha - 1)^2]/18$*, respectively.*

We keep the discussion of the symmetric subgame short, as Fudenberg and Tirole (2000) provide the detailed intuitions in a similar setup. Following the results of Fudenberg and Tirole (2000), we find that if one firm's market share in first period was smaller than 1/4, it is willing to monopolize its segment in the second period even against an opponent's price of zero, due to the high customer loyalty on this segment. This strategy, however, leads interestingly to the somehow counter-intuitive situation that the price on the own segment will be lower than on the rival's one, due to the much fiercer competition on the relative small segment. The larger segment gets shared by both firms in this case, while firms share intuitively both segments in the second period for intermediate values of α. Turning to firms' second-period profits depending on α, Figure 3.3 illustrates the well known u-shaped pattern, indicating that in the second period firms suffer the most when firms followed their individual short-term optimal action and shared the market equally in the first period.

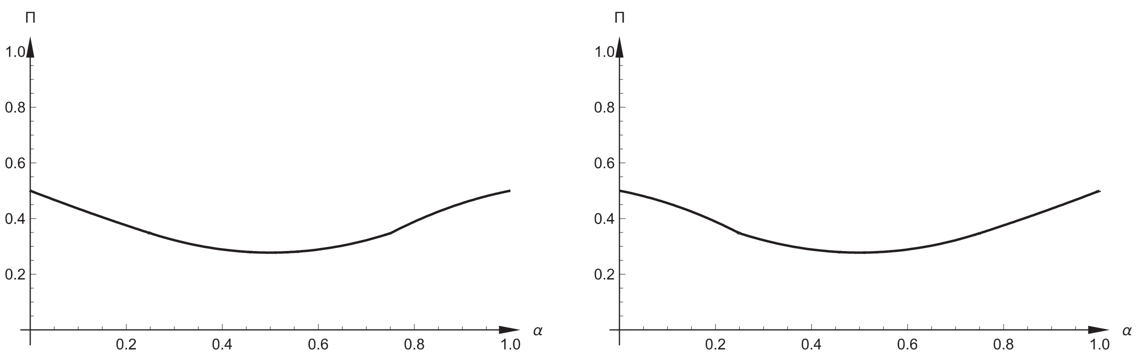


Figure 3.3: Second-period profits of firm B (left) and firm A (right) depending on α in the symmetric subgame with $t = 1$.

From a second-period point of view, firms would be best-off if one firm had monopolized the market in the first period, i.e. $\alpha = 0$ or 1. Thinking about the first-period market share as the quality of data, this means that data always hurts firms in the second period in the symmetric sugbame. The worst situation arises for the best data quality available, i.e. $\alpha = 0.5$, since customers can then be divided into two equally sized consumer groups, which additionally exhibit a clear preference for one of the firms (at equal prices) respectively. This enables firms to clearly differentiate between their own loyal customers and the rival ones, leading to the maximal competitive situation due to the mutual competitive pressure of each firm's poaching price.

The situation is somewhat different in the previous asymmetric subgame. While in this case the profit pattern of the firm without the technology also exhibits an u-shaped pattern, with only an slightly shifted minimum, we observe a fairly different picture for the firm holding the technology. Here, profits also follow an (erratic) u-shaped pattern, but only after the data quality exceeded a certain level. At first, starting without customer data, the firm having the data in fact benefits from them in the second period indicated by the inverse u-shaped pattern discussed above. Therefore, we face in both considered subgames the rather typical situation that less data tend to be better for firms. However, in the asymmetric subgame, at least some data could be in fact optimal for both firms. This arises due to the fact that the firm having the data is directly favored in the second period, while it would in turn have to favor the other firm without the data with a higher market share in the first period in order to create the desired amount and quality of data.[15] This profit sharing over the two periods could overall benefit both firms. Further, by comparing the profit levels of both firms in the symmetric and the asymmetric subgame, we first recognize that the profits of a firm having the data are clearly higher when it alone invested into the screening technology, since competition becomes fiercer when both firms have the technology and aim to poach customers of the rival respectively. More interestingly, also the profits of the firm without the data in the asymmetric subgame are, except for a small interval around each kink of the respective profit pattern, higher than those of a firm with data in the symmetric subgame due to this competition effect. This additionally indicates that both firms in fact could benefit from the opportunity to collect data overall, given that only one firm invests into the technology and firms are able to achieve the necessary (small) amount of customer data in the first period. We next analyze the first period and this presumption further.

[15]This strategy would also implicitly favor the firm without data further in the second period, since second-period profits of that firm drop less with less precise data.

3.5 Equilibrium Incentives to Acquire Screening Technology

We distinguish between the cases with all consumers being either myopic or sophisticated. In contrast to sophisticated consumers do myopic consumers not anticipate that collected customer data will be used by firms for targeting in the future. We will show that firms' incentives to invest into a screening technology crucially depend on consumers' ability to anticipate the future effect of their first-period purchase decisions.

3.5.1 Myopic Consumers

The following two Lemmas summarize our results on the equilibrium of the first period when consumers are myopic. We first consider the asymmetric subgame followed by the symmetric subgame.[16]

Asymmetric Subgame

Lemma 3.3. *(First period. Asymmetric subgame. Myopic consumers.) Assume that only firm A invested in the screening technology and consumers are myopic. The equilibrium of the first period depends on the discount factor.*

i) If the discount factor is small, $\delta \leq 0.01$, in equilibrium firm A charges a (weakly) lower price than the rival in the first period, with $p_{A,1} = [t(2\delta^2 - \delta - 48)]/(7\delta - 48)$ and $p_{B,1} = [2t(\delta^2 - 3\delta - 24)]/(7\delta - 48)$. At the same time, it serves (weakly) more consumers than firm B, precisely, those with $x \leq (\delta - 24)/(7\delta - 48)$. Over two periods, firm A realizes higher profits than the rival. Moreover, the profits of firm A (B) over two periods are higher (lower) compared to the subgame where neither firm acquires the technology.

ii) If the discount factor is relatively large, $\delta \geq 0.25$, in equilibrium firm A charges a higher price than the rival in the first period, with $p_{A,1} = [t(24 + 29\delta - 10\delta^2)]/(24 + 13\delta)$ and $p_{B,1} = [t(24 + 15\delta - 10\delta^2)]/(24 + 13\delta)$. At the same time, it serves less consumers than firm B, precisely, those with $x \leq (24-\delta)/(48+26\delta)$. Over two periods, firm A realizes higher profits than the rival and higher profits compared to the subgame where neither firm acquires the technology. Firm B realizes higher

[16] Since there are no first-period pure strategy equilibria for some values of the discount factor in the asymmetric subgame, we only consider the respective subset of δ in Lemma 3.3.

profits compared to the subgame where neither firm acquires the technology only for $\delta < 0.4$.

While making their first-period pricing decision, firms also consider the resulting effects on the second period. However, when the discount factor is small, firms are pretty impatient and care very little about the profits of the second period. This is the case in the first part of the Lemma. Instead, they essentially aim to maximize their profits in the first period, which results in the typical result of an (almost) equal market split between them. There are only small shifts to a slightly higher market share of firm A, depending on δ, due to the second-period effects. Given a very small discount factor in the asymmetric subgame, it is never optimal for firms to distort the short-term optimal market share in a way that firms end up in a different scenario than sharing both segments in the second period. Recalling the profit effects of α on second-period profits, we have that firm A's profits are (almost) constant, while firm B's profit slightly increase. This explains why firm A is more willing to fight for (slightly) higher market shares in the first period than firm B with higher discount factors. Comparing firms total profits over both periods, we get the typical result that data intensifies competition and harms firms. Firm A is only better off than firm B and compared to the situation without the opportunity to acquire the technology due to its competitive advantage.

Firms behave very differently when the discount factor is intermediate or large. Profits of the second period become relatively more important and especially firm A is interested to distort the short-term optimal market share in a way that it can monopolize its segment in the second period, granting a much higher second-period profit as displayed in Figure 3.2. Accordingly, firm A charges a relatively high price in order to lower its first-period market share. Since firm A moves with this investment on the right hand side of its inverse u-shaped second-period profit pattern, it is willing to invest more first-period market share with a higher discount factor. Most interestingly, all incentives of firm A are in line with those of firm B. Firm B obviously benefits in the first period from firm A being less competitive and additionally, when firms share only the segment of firm B in the second period, firm B also prefer a lower α from a second-period point of view. This lead to the interesting fact that for intermediate values of δ firm B indeed also benefits from the presence of data even though only the opponent is able to collect them. For higher discount factors, this relation is again reversed because even though firm B is also better off the higher the discount factor, since firm A then acts decreasingly aggressive in the first period and α shrinks, it is important to note that the marginal increase in firm B's profits decrease with δ. In contrast, in case neither firm invested in the technology, profits increase constantly with δ and dominate the profits of the firm not having the data in the asymmetric subgame for high discount factors.

Symmetric Subgame

Lemma 3.4. *(First period. Symmetric subgame. Myopic consumers.) Assume that both firms invested in the screening technology and consumers are myopic.*
In equilibrium, in the first period, both firms charge the price $p_1 = t$ *and firm A serves consumers with* $x \leq 0.5$. *Over two periods, both firms realize the profits of* $\Pi_{A,1+2} = \Pi_{B,1+2} = [t(9 + 5\delta)]/18$ *and are worse off compared to the subgame where none of them holds the technology.*

As firms are symmetric, they share the market equally in the first period. Moreover, they set the same prices in each period as in a standard Hotelling model due to consumer myopia. Accordingly, the second part of the Lemma directly follows from the dominating competition effect in the second period as described by Fudenberg and Tirole (2000) and Lemma 3.2.

Based on Lemma 3.3 and 3.4, we can conclude on firms' incentives to obtain screening technology, which are summarized in the following Proposition.[17] In order to clearly differentiate the total industry profit, social welfare, and consumer surplus in the different subgames, we use the superscripts "NN", "II", and "AS" respectively for the case that neither firm, both firms, or only one firm invests.

Proposition 3.1. *(Myopic consumers. Investment incentives and welfare.) If consumers are myopic, two asymmetric equilibria exist, where only one of the firms invests into the screening technology. Depending on the discount factor* δ*, we obtain the comparisons for total industry profits* (Π)*, social welfare* (SW)*, and consumer surplus* (CS) *as stated in Table 3.1.*

[17] Due to the restricted set of δ in Lemma 3.3, we restrict the set of δ in Proposition 3.1 accordingly.

	$\delta \leq 0.01$	$\delta \geq 0.25$
Π	$\Pi^{NN} \succ \Pi^{AS} \succ \Pi^{II}$	$\Pi^{AS} \succ \Pi^{NN} \succ \Pi^{II}$
SW	$SW^{NN} \succ SW^{AS} \succ SW^{II}$	$SW^{NN} \succ SW^{II} \succ SW^{AS}$
CS	$CS^{II} \succ CS^{AS} \succ CS^{NN}$	$CS^{II} \succ CS^{NN} \succ CS^{AS}$

Table 3.1: Comparison of total industry profits, social welfare, and consumer surplus over two periods in the different subgames when consumers are myopic.

The literature on competitive third-degree price discrimination has identified the famous prisoner's dilemma result, which states that firms are worse off when they are able to price discriminate. For instance, Fudenberg and Tirole (2000) derive this result in the context of behavior-based price discrimination. Our results show that this problem can be resolved when firms endogenously decide on their ability to target consumers, precisely, if the discount factor is large enough and consumers myopic. Then only one firm acquires the screening technology in equilibrium and both firms together realize higher joint profits than when none of them holds the technology.[18] Moreover, when $0.25 \leq \delta < 0.4$, not only the dominant profit increase of the firm holding the technology drives this result, but both firms individually gain from the asymmetric ability to target consumers in the second period, as explained in Lemma 3.3.[19]

We also find an asymmetric investment equilibrium for very small discount factors, however, in this case the loss of the firm without the technology is always greater than the gain of the firm holding the technology, such that the prisoner's dilemma is not resolved. Firms are nevertheless better off compared to the outcome where both of them initial have a targeting technology as is the case, for instance, in Fudenberg and Tirole (2000), such that the overall situation of firms is at least improved. This result directly follows from the less intense competition when only one of the firms can target consumers in the second period.

[18]Note that the asymmetric investment result seems a bit odd, due to the initial symmetry of the game. However, it matches many current market observations such that our results reasonably speak for still myopic consumers with respect to their data in the current state.

[19]Chen and Iyer (2002) and Baye and Sapi (2019) also find possibly asymmetric investments in customer data, however, their set-ups analyze one-shot games with exogenously provided data. For multi-period settings with the opportunity to invest into a technology to gain customer data endogenously, Baye and Sapi (2020) also find the asymmetric investment result with myopic consumers. However, in contrast to our result, firms are not able to resolve the prisoner's dilemma in their setting.

As the market is always covered in our model, social welfare is determined by consumer transportation costs. Social welfare is largest when none of the firms holds the technology, since every consumer buys at the closer firm in this case and transport costs are minimized. Accordingly, this set-up is clearly top ranked in Table 3.1. In the second benchmark set-up, when both firms initially hold the technology, the efficient equal market split occurs in the first period. However, in the second period each firm attracts 1/6 of the consumers on the rival's turf. In our set-up with myopic consumers, we find asymmetric investments for both intervals of the discount factor. In both cases, firms (weakly) distort the welfare efficient first-period outcome. However, compared to the case with both firms holding the technology, the resulting first-period welfare loss can be recouped in the second period when the discount factor is very low. In this case, the first-period equilibrium differs only slightly from the efficient one and its competitive advantage in the second period enables firm A mainly to prevent switching on its own turf than to attract considerable more consumers on firm B's turf, leading to a favorable situation for social welfare compared to the scenario with both firms holding the technology. In contrast, when the discount factor is higher, first-period market shares differ considerably from the efficient ones and even the fact that firm A defend its entire turf in the second period cannot recoup this initial welfare loss.[20]

Considering consumer surplus, we have to combine the above profit and transport cost effects. From the consumers' perspective, both effects are perfectly opposed when the discount factor is very low. While prices are the highest with no firm holding the technology, transport costs are minimized. Since the price effect dominates, we find that consumers favor the situation with both firms holding the technology, followed by the asymmetric subgame, due to the increased competition. With higher discount factors, firms make the highest profits while also transport costs are the highest, when firms ability to target consumers is asymmetric. Accordingly, consumer surplus is lowest in this situation. Comparing both firms holding the technology to none, once again, the former leads to higher consumer surplus due to the dominating competition effect. Finally, it is worth to note that firms' opportunity to decide endogenously on their ability to target consumers always harms consumers and favors firms with myopic consumers compared to the typical analyzed set-up with firms initially holding the technology.

[20]This is also due to firm A attracting consumers with a higher loyalty to firm B in the second period, compared to the symmetric subgame.

3.5.2 Sophisticated Consumers

In this chapter, we conduct a similar analysis as in the previous one under the assumption that all consumers are sophisticated. Sophisticated consumers correctly anticipate that their first-period purchase data will be used for targeted pricing in the following period. Accordingly, they consider the effect of their initial decision on future prices in the first period. As the game has only two periods, the equilibrium of the last period is the same with myopic and sophisticated consumers, since firms cannot make use of additional customer data gained during this period. Therefore, we only consider the first period here.

The following two Lemmas summarize our results on the first-period equilibrium when consumers are sophisticated for the asymmetric and symmetric subgame, respectively.[21]

Asymmetric Subgame

Lemma 3.5. *(First period. Asymmetric subgame. Sophisticated consumers.) Assume that only firm A invested in the screening technology and consumers are sophisticated. The equilibrium of the first period depends on the discount factor* δ*.*

i) If δ *is very small,* $\delta \in [0; 0.02)$*, in equilibrium, in the first period, firm A charges the price* $p_{A,1} = [t(82\delta - 96 - 17\delta^2)]/(50\delta - 96)$ *and serves (weakly) less consumers than firm B, namely those with* $x \leq (24 - 13\delta)/(48 - 25\delta)$*. Firm B charges the price* $p_{B,1} = [t(48\delta - 96 + \delta^2)]/(50\delta - 96)$*. Over two periods, firm A realizes (weakly) lower profits than the rival irrespective of the discount factor. Each firm realizes over two periods lower profits than in the case where neither of them obtains the screening technology.*

ii) For most values of the discount factor, $\delta \in (0.09; 0.81]$*, in equilibrium, in the first period, firm A charges the price* $p_{A,1} = [t(44\delta - 96 + 91\delta^2)]/(20\delta - 96)$ *and serves less consumers than the rival, namely those with* $x \leq (24 - 23\delta)/(48 - 10\delta)$ *. Firm B charges the price* $p_{B,1} = [t(68\delta - 96 + 47\delta^2)]/(20\delta - 96)$*. Over two periods, firm A realizes higher profits than the rival if* $\delta > 0.56$ *and lower profits otherwise. Each firm realizes over two periods lower profits than in the case where neither of them obtains the screening technology.*

[21] As with myopic consumers, there are no first-period pure strategy equilibria for some values of the discount factor in the asymmetric subgame with sophisticated consumers, such that we only consider the respective subset of δ in Lemma 3.5.

The striking finding of Lemma 3.5 is that despite its technological advantage and (typically) charging the lower price, firm A serves less consumers than its rival in the first period. This result depends on consumer sophistication, which makes consumers anticipate that they will be charged with the smaller poaching price by firm A in the second period when they buy at firm B in the initial period. As a result, those consumers on firm A's turf with a weaker preference for this firm actually prefer to buy at its rival in the first period, even when firm B charges the higher price. Hence, they are willing to invest into not being identified as a loyal customer of firm A and somehow "punish" it for investing into the technology. The resulting negative impact on the profits of firm A is so strong that, for a wide range of discount factors, it realizes lower overall profits than the rival without the screening technology. Only when δ is high enough, firm A makes more profits than firm B over both periods, since in this case a lower market share in the first period can be compensated with the resulting higher profits in the second period, especially when α is low enough such that firm A can monopolize its segment. However, compared to myopic consumers, asymmetrically investing into the technology is clearly less attractive with sophisticated consumers and as we will show, never constitute an equilibrium.

Symmetric Subgame

Lemma 3.6. *(First period. Symmetric subgame. Sophisticated consumers, cf. Fudenberg and Tirole (2000).)*

Assume that both firms hold the screening technology and consumers are sophisticated.

In equilibrium, in the first period, both firms charge the price $p_1 = [t(3+\delta)]/3$ *such that firm A serves those consumers with* $x \leq 0.5$*. Over two periods, both firms obtain profits of* $\Pi_{A,1+2} = \Pi_{B,1+2} = [t(9+8\delta)]/18$ *and are worse off compared to the subgame where none of them holds the technology.*

In contrast to the asymmetric subgame, consumers cannot punish firms for investing into the screening technology when both firms hold it. Even further, as Fudenberg and Tirole (2000) analyzed, first-period demand becomes less elastic when both firms hold the technology and consumers are sophisticated, such that first-period prices are in equilibrium even higher than with none of the firms holding the technology. However, since firms intuitively share the market equally in the first period, due to the initial symmetry in this subgame, they end up in the most competitive situation in the second period and the first-period gains cannot recoup the large profit decrease there.[22]

Using Lemmas 3.5 and 3.6, we derive the equilibrium incentives to obtain screening technology when consumers are sophisticated. Our results are summarized in Proposition 3.2.[23] In order to clearly differentiate the total industry profit, social welfare, and consumer surplus in the different subgames, we use the superscripts "NN", "II", and "AS" respectively for the case that neither firm, both firms, or only one firm invests.

Proposition 3.2. *(Sophisticated consumers. Investment incentives.) If consumers are sophisticated and the discount factor is small,* $\delta \in [0; 0.02)$*, or intermediate,* $\delta \in (0.09; 0.81]$*, there exist two symmetric equilibria in pure strategies, where both or neither firm invests in screening technology while the no-investing equilibrium is pareto-dominant. Depending on the discount factor* δ*, we obtain the relations for total industry profits (*Π*), social welfare (*SW*), and consumer surplus (*CS*) as stated in Table 3.2.*

	$\delta < 0.02$	$\delta \in (0.09, 0.81]$
Π	$\Pi^{NN} \succ \Pi^{II} \succ \Pi^{AS}$	$\Pi^{NN} \succ \Pi^{II} \succ \Pi^{AS}$
SW	$SW^{NN} \succ SW^{AS} \succ SW^{II}$	$SW^{NN} \succ SW^{II} \succ SW^{AS}$
CS	$CS^{AS} \succ CS^{II} \succ CS^{NN}$	$CS^{AS} \succ CS^{II} \succ CS^{NN}$

Table 3.2: Comparison of total industry profits, social welfare, and consumer surplus over two periods in the different subgames when consumers are sophisticated.

With myopic consumers, we showed that the famous prisoner's dilemma result in the context of third-degree price discrimination can be resolved when firms are able to decide endogenously about their ability to target consumers. Here, we find that firms are also better off with this endogenous investment opportunity when consumers are sophisticated. Precisely, we find two symmetric equilibria with either both or none of the firms investing in the technology, such that the prisoner's dilemma is (at least)

[22]Since first-period prices with myopic consumers in the symmetric subgame are identical to those with no firm holding the technology, sophisticated consumers are always better for firms when both firms initially hold the technology, as it provides higher first-period profits while second-period profits are identical.

[23]Due to the restricted set of δ in Lemma 3.5, we restrict the set of δ in Proposition 3.2 accordingly.

avoidable if the equilibrium with no investments can be sustained.[24] The intuition that no firm investing in the technology constitute an equilibrium follows in this context directly from Lemma 3.5, where we found no initial investment incentives if the rival doesn't have the technology due the punishment by sophisticated consumers in the first period. However, the other firm also prefers to invest if the rival already has the technology, since the less elastic first-period demand in the symmetric subgame is more profitable than being the favored non-investing firm in the asymmetric subgame, while second-period profits are similar in both cases. Hence, both firms investing constitute the second equilibrium when consumers are sophisticated and the asymmetric subgame is clearly worst in the profit ranking followed by both and then no firm investing, as found in Lemma 3.6.

Considering welfare and especially transport cost for social welfare, the equilibrium with no investments is obviously, once again, best ranked due to the implementation of the equal market split in each period. While having a different ordering of the asymmetric and the both firm investing subgame under transport cost considerations depending on the discount factor, we once again have a clear ordering when combining profits and transport costs for consumer surplus due to the dominating price effect. Consumer surplus is always highest in the asymmetric subgame followed by both and then no firm investing, since competition is in the latter always lower.

3.6 Comparison of the Equilibria with Myopic and Sophisticated Consumers

In this section, we compare total industry profits, consumer surplus, and social welfare in equilibrium depending on consumers sophistication. Our results are summarized in Corollary 3.1.[25]

Corollary 3.1. *(Comparison: Myopic vs. sophisticated consumers.)*
The comparison of equilibria with myopic and sophisticated consumers depends on the discount factor.

[24]Note that in contrast to the asymmetric investment equilibria with myopic consumers, we find the more standard symmetric equilibria in such an initial symmetric game with sophisticated consumers, as Colombo (2016).

[25]Due to the restricted set of δ in Proposition 3.1 and 3.2, we restrict the set of δ in Corollary 3.1 on the resulting common subset.

i) If $\delta \leq 0.01$, then depending on the equilibrium with sophisticated consumers two cases can emerge:
a) If in equilibrium firms do not invest into screening technology, total profits and social welfare are higher with sophisticated consumers than with myopic consumers, while consumer surplus is lower.
b) If both firms invested into screening technology in equilibrium, total profits and social welfare are lower with sophisticated consumers than with myopic consumers, while consumer surplus is higher.

ii) If $\delta \in [0.25, 0.81]$, then firms realize lower profits, while consumer surplus and social welfare are higher with sophisticated consumers than with myopic consumers, regardless of the equilibrium in the former case.

Corollary 3.1 shows that firms' and consumers' interests with respect to consumer education are always opposed. For instance, when firms prefer consumers to be myopic, the latter would be better off if they could anticipate price discrimination by the firm(s) holding the screening technology. Additionally, for most values of the discount factor, consumer surplus and social welfare are higher with sophisticated consumers than with myopic ones. However, firms would not educate consumers by informing them about the usage of their data in this case, because as we mentioned above, their interests are opposed to those of consumers. Only for a very small range of the discount factor, firms would prefer to educate consumers, which, however, would happen at the expense of the latter.[26]

Overall, we observe that for most values of the discount factor, firms' incentives to educate consumers are insufficient, which speaks for public policies aimed at informing consumers about firms' use of customer data for targeted pricing. Our conclusion provides a theoretical justification for policies considering consumer naiveté, expressed by their limited foresight, as a main source of consumer harm (see Europe Economics, 2007).

[26] As we start in a situation with myopic consumers, for which our model predicts asymmetric investments, it is much more reasonable to expect that after education, where we found the two symmetric equilibria, the firm without the technology catches up than to expect that the firm having the technology removes it. Accordingly, it makes sense to focus on the both firm investing equilibrium for our education analysis, which generally speak for public policies aimed at consumer education due to the clear favoring effect of consumer and typically also social surplus.

3.7 Conclusion

With increasing opportunities to collect and analyze customer data, behavior-based price discrimination became an affordable pricing strategy for firms in many industries. The seminal work by Fudenberg and Tirole (2000) has shown, however, that it has a detrimental effect on firm profits because of intensified competition. Precisely, firms end up in the prisoner's dilemma: While it is unilaterally optimal for a firm to use behavioral customer data for targeted pricing, both firms are overall worse off. In this paper we endogenize firms' ability to collect customer data through introducing the investment stage where firms decide whether or not to invest into screening technology, which allows to recognize customers. Our set-up reflects the observation that a firm's ability to practice behavioral targeting is a result of its strategic decision. We show that in our model firms are able to avoid the prisoner's dilemma problem. Precisely, when consumers are myopic, only one of the firms invests into screening technology. Provided the discount factor is sufficiently large, over both periods firms realize larger profits than in the subgame, where none of them can target based on behavior. When consumers are sophisticated, then two symmetric equilibria exist, where both firms either invest or not. While the former equilibrium corresponds to the situation considered in Fudenberg and Tirole (2000), in the latter equilibrium none of the firms practices behavioral targeting, which protects their profits. We also show that in most cases consumers are better off when they are able to foresee that the firm with a screening technology will use behavioral data for targeted pricing. This result supports the intuition that consumer naiveté may lead to consumer exploitation and justifies the introduction of consumer educating policies.

3.8 Appendix

Supplementary derivations for Section 3.4.1. We show first how we arrive at the best-response functions of firm A on segments α and $1-\alpha$ in the second period, (3.2) and (3.3), respectively. We start with the derivation of the demand for firm A's product among its own and the rival's former customers, which are given by (3.4) and (3.5), respectively.
On segment α, demand for firm A is

$$D^{\alpha}_{A,2}(p^{\alpha}_{A,2}, p_{B,2}) = \begin{cases} 0, & \text{if } p^{\alpha}_{A,2} \geq p_{B,2} + t \\ \frac{1}{2} + \frac{p_{B,2} - p^{\alpha}_{A,2}}{2t}, & \text{if } p_{B,2} + t(1-2\alpha) < p^{\alpha}_{A,2} < p_{B,2} + t \\ \alpha, & \text{if } p^{\alpha}_{A,2} \leq p_{B,2} + t(1-2\alpha) \end{cases} \tag{3.4}$$

while demand for firm B is $D^{\alpha}_{B,2}(p^{\alpha}_{A,2}, p_{B,2}) = \alpha - D^{\alpha}_{A,2}(p^{\alpha}_{A,2}, p_{B,2})$.
On segment $1-\alpha$, the demand for firm A's product is

$$D^{1-\alpha}_{A,2}(p^{1-\alpha}_{A,2}, p_{B,2}) = \begin{cases} 0, & \text{if } p^{1-\alpha}_{A,2} \geq p_{B,2} - t(2\alpha - 1) \\ \frac{1}{2} + \frac{p_{B,2} - p^{1-\alpha}_{A,2}}{2t} - \alpha, & \text{if } p_{B,2} - t < p^{1-\alpha}_{A,2} < p_{B,2} - t(2\alpha - 1) \\ 1-\alpha, & \text{if } p^{1-\alpha}_{A,2} \leq p_{B,2} - t \end{cases} \tag{3.5}$$

while demand for firm B is $D^{1-\alpha}_{B,2}(p^{1-\alpha}_{A,2}, p_{B,2}) = 1 - \alpha - D^{1-\alpha}_{A,2}(p^{1-\alpha}_{A,2}, p_{B,2})$.

Each demand function specifies three possible outcomes, where either one of the firms gains all consumers on a given segment or both of them serve consumers. Using these demand functions, we can directly derive the best-response functions (3.2) and (3.3), yielding five possible market outcomes (depending on $p_{B,2}$) as displayed in Table 3.3. The respective demands in each case are determined by the indifferent consumer given by firm A's optimal prices in the respective market-sharing situation.

Case	segment α	segment $1-\alpha$	$D_{A,2}$	$D_{B,2}$
(i)	mon. by A	shared	$\frac{p_{B,2}}{4t} + \frac{1+2\alpha}{4}$	$\frac{3-2\alpha}{4} - \frac{p_{B,2}}{4t}$
(ii)	shared	shared	$\frac{1-\alpha}{2} + \frac{p_{B,2}}{2t}$	$\frac{1+\alpha}{2} - \frac{p_{B,2}}{2t}$
(iii)	shared	mon. by A	$\frac{5-4\alpha}{4} + \frac{p_{B,2}}{4t}$	$\frac{4\alpha-1}{4} - \frac{p_{B,2}}{4t}$
(iv)	shared	mon. by B	$\frac{1}{4} + \frac{p_{B,2}}{4t}$	$\frac{3}{4} - \frac{p_{B,2}}{4t}$
(v)	mon. by A	mon. by A	1	0

Table 3.3: Notation of market-sharing scenarios.

We now turn to the proofs of Lemmata 3.1 to 3.6, Propositions 3.1 and 3.2, as well as Corollary 3.1.

Proof of Lemma 3.1. From combining the best-reply functions given in equations (3.2) and (3.3) it follows that how the market will be shared among both firms depends on α and $p_{B,2}$ as follows:

1.) Consider first the case where $\alpha \leq 2/3$.

1. a) If $p_{B,2} < t(2\alpha - 1)$, segment α will be shared while segment $1 - \alpha$ will be served only by firm B which corresponds to case (iv) in Table 3.3. In this case, firm B would want to set a price of

$$p^*_{B,2} = \frac{3t}{2} \tag{3.6}$$

which follows from maximizing $\Pi_{B,2} = \Big(3/4 - p_{B,2}/4t\Big)p_{B,2}$. As this price, however, does not fulfill the condition that $p_{B,2} < t(2\alpha - 1)$, this case cannot emerge in equilibrium.

1. b) If $t(2\alpha - 1) < p_{B,2} < t(4\alpha - 1)$, both segments will be shared which corresponds to case (ii) in Table 3.3. Maximizing its profit $\Pi_{B,2} = \Big((1+\alpha)/2 - p_{B,2}/2t\Big)p_{B,2}$ leads firm B to set an optimal price of

$$p^*_{B,2} = \frac{t(1+\alpha)}{2}. \tag{3.7}$$

This price fulfills the condition $t(2\alpha - 1) < p_{B,2} < t(4\alpha - 1)$ only if $3/7 < \alpha$.

1. c) If $t(4\alpha - 1) < p_{B,2} < t(3 - 2\alpha)$, segment α will be served by firm A, while segment $1 - \alpha$ will be shared among both firms. This corresponds to case (i) in Table 3.3 and in this case firm B would want to set a price of

$$p^*_{B,2} = \frac{t(3-2\alpha)}{2}. \tag{3.8}$$

If $\alpha < 1/2$, that price indeed fulfills the condition $t(4\alpha - 1) < p_{B,2} < t(3 - 2\alpha)$. Following the previous analysis, we can conclude that this price (and the corresponding market-sharing situation) constitute an equilibrium for $\alpha < 3/7$. For the range of parameters where $3/7 < \alpha < 1/2$, we have to compare the profits under the optimal prices from 1. b) and 1. c)

$$\Pi_{B,2}\left(p^*_{B,2}=\frac{t(1+\alpha)}{2}\right)=\frac{t(1+\alpha)^2}{8} \tag{3.9}$$

and

$$\Pi_{B,2}\left(p^*_{B,2}=\frac{t(3-2\alpha)}{2}\right)=\frac{t(3-2\alpha)^2}{16}, \tag{3.10}$$

which yields that the price $p^*_{B,2}=t(3-2\alpha)/2$ (cf. case (i)) will apply if $\alpha \leq 0.46$ since

$$\frac{(3-2\alpha)^2}{16} \geq \frac{(1+\alpha)^2}{8} \Leftrightarrow \alpha \leq \frac{8-5\sqrt{2}}{2} \approx 0.46. \tag{3.11}$$

If $0.46 < \alpha \leq 2/3$, firm B will set an optimal price of $p^*_{B,2}=t(1+\alpha)/2$.

1. d) If $t(3-2\alpha) < p_{B,2}$, both segments will be served by firm A. As firm B always has an incentive to lower its price, this is not an equilibrium.

2.) Consider now the case with $\alpha > 2/3$.

2. a) If $t(3-2\alpha) < p_{B,2} < t(4\alpha-1)$, segment α will be shared while segment $1-\alpha$ will be served only by firm A which corresponds to case (iii) in Table 3.3. In this scenario, firm B would want to maximize $\Pi_{B,2}=((4\alpha-1)/4-p_{B,2}/4t)p_{B,2}$ by setting

$$p^*_{B,2}=\frac{t(4\alpha-1)}{2}. \tag{3.12}$$

This price fulfills the condition $t(3-2\alpha) < p_{B,2} < t(4\alpha-1)$ only if $7/8 < \alpha$ (while for $\alpha < 7/8$, the optimal price from 1. b) applies). For the range of parameters where $7/8 < \alpha < 1$, we have to compare the profits under the optimal prices from 2. a) and 1. b)

$$\Pi_{B,2}\left(p^*_{B,2}=\frac{t(1+\alpha)}{2}\right)=\frac{t(1+\alpha)^2}{8} \tag{3.13}$$

and

$$\Pi_{B,2}\left(p^*_{B,2}=\frac{t(4\alpha-1)}{2}, \alpha > \frac{7}{8}\right)=\frac{t(1-4\alpha)^2}{16} \tag{3.14}$$

which yields that the price $p^*_{B,2}=t(4\alpha-1)/2$ (cf. case (iii)) will apply if $\alpha > 0.93$ since

$$\alpha > \frac{6+5\sqrt{2}}{14} \approx 0.93. \tag{3.15}$$

2. b) If $t(4\alpha-1) < p_{B,2}$, both segments will be served by firm A. As firm B always has an incentive to lower its price, this is no equilibrium.

Bringing together the results for all intervals, we summarize that for $\alpha \in [0; 0.46]$, case (i) will emerge, for $\alpha \in (0.46; 0.93]$ case (ii) and, finally, for $\alpha \in (0.93; 1]$ case (iii) will emerge, which is stated in Lemma 3.1. *Q.E.D.*

Proof of Lemma 3.2. We will only analyze segment α and then, using symmetry, make conclusions for segment $1-\alpha$. The address of the indifferent consumer is given by

$$x = \frac{1}{2} + \frac{p_{B,2} - p_{A,2}}{2t} \tag{3.16}$$

from which we derive firms' best-response functions. Maximizing the profit of firm A leads to

$$p_{A,2}(p_{B,2}) = \begin{cases} \frac{t+p_{B,2}}{2}, & \text{if } p_{B,2} < t(4\alpha - 1) \\ p_{B,2} - t(2\alpha - 1), & \text{if } p_{B,2} \geq t(4\alpha - 1). \end{cases} \tag{3.17}$$

The first part of the best-response function (3.17) results in the outcome, where both firms serve consumers on segment α and, hence, can be referred to as the market-sharing outcome. The second part of the best response (3.17) results in the monopoly outcome on segment α, because firm A serves all consumers there. The respective best-response function of firm B is given by

$$p_{B,2}(p_{A,2}) = \begin{cases} 0, & \text{if } p_{A,2} \leq t(1 - 2\alpha) \\ \frac{p_{A,2}+t(2\alpha-1)}{2}, & \text{if } t(1 - 2\alpha) < p_{A,2} < t(1 + 2\alpha) \\ p_{A,2} - t, & \text{if } p_{A,2} \geq t(1 + 2\alpha). \end{cases} \tag{3.18}$$

The first part of the best response (3.18) refers to the case where firm B does not gain any of the past customers of firm A even at a zero price. The second part refers to the market-sharing outcome and the last part gives rise to the outcome where firm A serves none of its past customers. Combining both reaction functions (equations (3.17) and (3.18)), we find that two types of equilibria might emerge: Either both firms share the segment of firm A, or the segment is monopolized by firm A. We obtain the following result, which under symmetry leads to Lemma 3.2: If $\alpha < 0.25$ firm A will serve all consumers on its segment. The respective prices are $p_{A,2} = t(1 - 2\alpha)$ and $p_{B,2} = 0$. If $\alpha > 0.25$, firms will share the market of segment α. More precisely, firm A will serve all consumers with an address $x < (1 + 2\alpha)/6$. The respective prices are $p_{A,2} = t(1 + 2\alpha)/3$ and $p_{B,2} = t(4\alpha - 1)/3$. *Q.E.D.*

Proof of Lemma 3.3. To obtain firms' profits in the first period, we have to distinguish between the three different cases how the market can be shared in the second period depending on α as derived in Lemma 3.1.

1.) Assume that firms' prices in the first period are such that $\alpha \leq (8-5\sqrt{2})/2 \approx 0.46$, to which we refer as the first interval. Then the discounted sum of firm B's profits is

$$\Pi_{B,1+2} = (1-\alpha)p_{B,1} + \frac{\delta t(3-2\alpha)^2}{16}, \tag{3.19}$$

where $\alpha = 1/2 + (p_{B,1} - p_{A,1})/2t$ is the address of the indifferent consumer. Firm A's discounted sum of profits is

$$\Pi_{A,1+2} = \alpha p_{A,1} + \frac{5\delta t(5+4\alpha-12\alpha^2)}{32}. \tag{3.20}$$

Taking the derivatives w.r.t. the respective first-period prices and solving the FOCs simultaneously gives the optimal first-period prices

$$p^*_{A,1} = \frac{t(24+29\delta-10\delta^2)}{24+13\delta} \tag{3.21}$$

and

$$p^*_{B,1} = \frac{t(24+15\delta-10\delta^2)}{24+13\delta}, \tag{3.22}$$

which are positive for all values of δ. The indifferent consumer indeed lies on the first interval under those prices if

$$\delta \geq \frac{24(7-5\sqrt{2})}{5(13\sqrt{2}-21)} \approx 0.13. \tag{3.23}$$

Both prices lead to total profits of firm A and firm B depending on the discount factor δ of

$$\Pi_{B,1+2} = \frac{t(576+1296\delta+645\delta^2-70\delta^3)}{2(24+13\delta)^2} \tag{3.24}$$

and

$$\Pi_{A,1+2} = \frac{t(576+1392\delta+931\delta^2+265\delta^3)}{2(24+13\delta)^2}. \tag{3.25}$$

We have to check whether there are any incentives for a deviation to a different interval. Note that in order to stay on the first interval, it must hold that $p_{A,1} > p_{B,1} - t(7-5\sqrt{2})$ and $p_{A,1} > p_{B,1} + t(1/7 - (5\sqrt{2})/7)$ (i.e. no deviation to a case where $\alpha > 0.46$).

1. a) Consider the deviation of firm A on the second interval. The price of firm A, which follows from the FOC is

$$p_{A,1}^{d} = \frac{2t(111\delta^2 - 25\delta^3 - 176\delta - 384)}{(24 + 13\delta)(5\delta - 32)} \tag{3.26}$$

where the index d stands for deviation. This follows from maximizing the deviation profit which firm A obtains if deviating to the second interval, $\Pi_{A,1+2} = \alpha p_{A,1}^{d} + \delta t(9 - 6\alpha + 5\alpha^2)/16$. Using the derived condition above, we find that firm A would deviate to the second interval with this price only if $\delta \leq 0.64$ and otherwise use the price of the corner solution given by

$$p_{A,1}^{d} = \frac{t(24 + 15\delta - 10\delta^2)}{24 + 13\delta} - t(7 - 5\sqrt{2}). \tag{3.27}$$

However, both prices on the second interval lead to lower profits compared to the profits that are obtained when staying on the first interval. Thus, a deviation to the second interval is not profitable.

1. b) Consider a deviation of firm A on the third interval. The optimal price (inner solution) that firm A would set when deviating to the third interval is

$$p_{A,1}^{d} = \frac{t(192 + 40\delta - 67\delta^2 - 60\delta^3)}{192 + 248\delta + 78\delta^2}. \tag{3.28}$$

This deviation price follows from maximizing the deviation profit $\Pi_{A,1+2} = \delta t(47 - 120\alpha + 48\alpha^2)/32 + \alpha p_{A,1}^{d}$.

However, it does not satisfy the condition that α indeed lies in the third interval ($p_{A,1} < p_{B,1} + t(1/7 - (5\sqrt{2})/7)$). Therefore, firm A would set the price of the corner solution if deviating to the third interval, which is

$$p_{A,1}^{d} = \frac{t(24 + 15\delta - 10\delta^2)}{24 + 13\delta} + t\left(\frac{1}{7} - \frac{5\sqrt{2}}{7}\right). \tag{3.29}$$

This price does not lead to higher profits than under $p_{A,1} = p_{A,1}^{*}$. We conclude that firm A doesn't have an incentive to deviate from $p_{A,1}^{*}$ for any $\delta \geq 0.13$.

1. c) Consider the deviation of firm B on the second interval. The FOC price is

$$p_{B,1}^{d} = \frac{2t(192 + 192\delta - 35\delta^2 + 5\delta^3)}{384 + 184\delta - 13\delta^2}. \tag{3.30}$$

This deviation price follows from maximizing the deviation profit $\Pi_{B,1+2} = (1 - \alpha)p_{B,1} + \delta t(1+\alpha)^2/8$. Under this price it holds that $0.46 < \alpha(p^*_{A,1}, p^d_{B,1}) \leq 0.93$. The SOC is also fulfilled, such that it is an optimal deviation price of firm B on this interval. This price yields a deviation profit of

$$\Pi^d_{B,1+2} = \frac{4t(1152 + 2592\delta + 834\delta^2 - 235\delta^3 + 115\delta^4)}{(16-\delta)(24+13\delta)^2}, \tag{3.31}$$

which is larger than the profit which firm B gets at $p_{B,1} = p^*_{B,1}$ if

$$\delta < \frac{3(\sqrt{12329} - 83)}{340} \approx 0.25. \tag{3.32}$$

However, firm B doesn't have an incentive to deviate on the second interval if $\delta \geq 0.25$.

1. d) Consider the deviation of firm B on the third interval. The FOC price is

$$p^d_{B,1} = \frac{t(24\delta + 45\delta^2 - 20\delta^3 - 96t - 84\delta t + 20\delta^2 t)}{2(24+13\delta)(\delta - 2t)}, \tag{3.33}$$

which follows from maximizing the deviation profit $\Pi_{B,1+2} = (1-\alpha)p_{B,1} + \delta t(1-4\alpha)^2/16$. That price, however, does not satisfy the condition $p_{B,1} > p_{A,1} - t(1/7 - (5\sqrt{2})/7)$, so that the optimal deviation price of firm B on the third interval is

$$p^d_{B,1} = \frac{t(24(6+5\sqrt{2}) + 5\delta(38+13\sqrt{2}) - 70\delta^2)}{168+91\delta}. \tag{3.34}$$

As the resulting deviation profit is lower than the profit at $p_{B,1} = p^*_{B,1}$, firm B doesn't have an incentive to deviate to the third interval.

Combining the results on the deviations of firm B on the two intervals, we conclude that it doesn't have an incentive to deviate if $\delta \geq 0.25$. Summarizing our results on the deviations of firms A and B, we conclude that the prices $p^*_{A,1}$ and $p^*_{B,1}$ stated in (3.21) and (3.22), respectively, constitute the equilibrium in the first period if $\delta \geq 0.25$.

2.) Assume that firms' prices in the first period are such that $0.46 \approx (8-5\sqrt{2})/2 < \alpha \leq (6+5\sqrt{2})/14 \approx 0.93$, to which we refer as the second interval. Then the discounted sum of firm B's profits is

$$\Pi_{B,1+2} = (1-\alpha)p_{B,1} + \frac{\delta t(1+\alpha)^2}{8}, \tag{3.35}$$

while for firm A it is

$$\Pi_{A,1+2} = \alpha p_{A,1} + \frac{\delta t(9-6\alpha+5\alpha^2)}{16}. \tag{3.36}$$

Taking the derivatives w.r.t. the respective first-period prices and solving them simultaneously gives the optimal first-period prices

$$p^*_{A,1} = \frac{t(2\delta^2-\delta-48)}{7\delta-48} \tag{3.37}$$

and

$$p^*_{B,1} = \frac{2t(\delta^2-3\delta-24)}{7\delta-48}, \tag{3.38}$$

which are positive for all values of δ. Plugging the derived prices into the address of the indifferent first-period consumer α, we get

$$\alpha = \frac{\delta-24}{7\delta-48} \tag{3.39}$$

which is increasing in δ and always strictly within the interval $\forall\ \delta \in [0,1]$.

The prices lead to total profits for firm A and firm B depending on the discount factor δ of

$$\Pi_{B,1+2} = \frac{4t(288+126\delta-57\delta^2+5\delta^3)}{(48-7\delta)} \tag{3.40}$$

and

$$\Pi_{A,1+2} = \frac{t(4608+4080\delta-1444\delta^2+109\delta^3)}{4(48-7\delta)^2}. \tag{3.41}$$

Note that firm A's profit is larger $\forall\ \delta \in [0,1]$.

2. a) Consider the deviation of firm A to the first interval. In that case it must hold that $p_{A,1} > p_{B,1} - t(7-5\sqrt{2})$. The optimal price in case of a deviation (inner solution) would be

$$p^d_{A,1} = \frac{2t(-384-596\delta-2\delta^2+15\delta^3)}{105d^2-608\delta-768} \tag{3.42}$$

which holds whenever $\delta > 0.17$. This deviation price follows from maximizing the profit which firm A would obtain if deviating to the first interval $\Pi_{A,1+2} = \alpha p_{A,1} + \delta t(4\alpha + 5 - 12\alpha^2)/32$. This price leads to a deviation profit of $\Pi^d_{A,1+2} = t(36864 + 79872\delta + 35668\delta^2 - 16004\delta^3 + 1311\delta^4)/(2(48 - 7\delta)^2(16 + 15\delta))$ which is always higher than $\Pi_{A,1+2}$ (3.41). Thus, firm A will deviate (with the inner solution) if $\delta > 0.17$. For $\delta \leq 0.17$, firm A would deviate with the corner solution given by

$$p^d_{A,1} = t(5\sqrt{2} - 7 + \frac{48 + 6\delta - 2\delta^2}{48 - 7\delta}). \tag{3.43}$$

The profit $\Pi^d_{A,1+2} = t[-10752(5\sqrt{2} - 7) - 80(-805 + 579\sqrt{2})\delta + (-10979 + 7890\sqrt{2})$ $\delta^2]/(32(7\delta - 48))$, that firm A obtains from deviating with this price, is larger than $\Pi_{A,1+2}$ (3.41) if $\delta > 0.01$.

2. b) Consider the deviation of firm A to the third interval. We find that it would deviate with a corner solution price of

$$p^d_{A,1} = \frac{1}{7}\Big(t - 5\sqrt{2}t + \frac{14t(\delta^2 - 24 - 3\delta)}{7\delta - 48}\Big) \tag{3.44}$$

that leads to a deviation profit of

$$\Pi^d_{A,1+2} = \frac{t(1536(1 - 5\sqrt{2}) - 80(-377 + 905\sqrt{2})\delta + 49(-89 + 220\sqrt{2})\delta^2)}{1568(7\delta - 48)}. \tag{3.45}$$

This profit is higher than $\Pi_{A,1+2}$ (3.41) if $\delta > 0.98$, such that firm A in this case will deviate to the third interval with the price $p^d_{A,1}$ (3.44).

2. c) Consider a deviation of firm B to the first interval. The FOC price in this case is

$$p^d_{B,1} = \frac{t(384 - 168\delta + 5\delta^2 + 2\delta^3)}{384 - 104\delta + 7\delta^2} \tag{3.46}$$

if $\delta > 0.21$.

This deviation price follows from maximizing the profit which firm B would obtain when deviating to the first interval $\Pi_{B,1+2} = (1 - \alpha)p_{B,1} + \delta t(3 - 2\alpha)^2/16$.

If $\delta \leq 0.21$, firm B would deviate with the price of the corner solution of

$$p^d_{B,1} = 7t - 5t\sqrt{2} + \frac{t(2\delta^2 - \delta - 48)}{7\delta - 48}. \tag{3.47}$$

Comparing the respective profits with $\Pi_{B,1+2}$ (3.40), we find that for $\delta > 0.6$ firm B would deviate with the inner solution to the first interval. Since equilibrium profits are always more profitable than deviating with the corner solution, firm B will only deviate with the inner solution as stated above. Moreover, firm B would not want to deviate to the third interval, because the deviation profit resulting from the price of the corner solution is always lower than $\Pi_{B,1+2}$ (3.40).

To sum up the results, we find that only for $\delta \leq 0.01$, there is indeed an equilibrium on the second interval with prices and profits as stated in equations (3.37), (3.38), (3.40), and (3.41).

3.) Assume that firms' first-period prices are such that $0.93 \approx (6+5\sqrt{2})/14 < \alpha \leq 1$, to which we refer as the third interval. Then the discounted sum of firm B's profits is

$$\Pi_{B,1+2} = (1-\alpha)p_{B,1} + \frac{\delta t(1-4\alpha)^2}{16}, \tag{3.48}$$

while for firm A it is

$$\Pi_{A,1+2} = \alpha p_{A,1} + \frac{\delta t(47-120\alpha+48\alpha^2)}{32}. \tag{3.49}$$

Taking the derivatives w.r.t. the respective first-period prices and solving them simultaneously gives the optimal first-period prices as

$$p^*_{A,1} = \frac{t(6-7\delta+6\delta^2)}{6+\delta} \tag{3.50}$$

and

$$p^*_{B,1} = \frac{3t(4-\delta+4\delta^2)}{2(6+\delta)}, \tag{3.51}$$

which are positive for all values of δ. Plugging in the derived prices into the address of the indifferent first-period consumer α, we get

$$\alpha = \frac{12+13\delta}{24+4\delta} \tag{3.52}$$

which is increasing in δ. This value, however, never lies in the interval $(0.93, 1]$, such that there is no equilibrium on the third interval. *Q.E.D.*

Proof of Lemma 3.4. Following Lemma 3.2, we have to distinguish between three different cases concerning the size of firm A's segment. We analyze each case separately.

1.) If firms' first-period prices are such that $\alpha < 0.25$, the discounted sum of profits takes the form

$$\Pi_{A,1+2}(p_{A,1}, p_{B,1}) = p_{A,1}\alpha + \delta t \left[\alpha(1-2\alpha) + \frac{(3-4\alpha)^2}{18} \right] \tag{3.53}$$

and

$$\Pi_{B,1+2}(p_{A,1}, p_{B,1}) = p_{B,1}(1-\alpha) + \frac{\delta t(3-2\alpha)^2}{18}. \tag{3.54}$$

Maximizing firms' profits, we obtain first period prices

$$p_{A,1} = \frac{t(81 + 90\delta - 22\delta^2)}{81 + 24\delta} \tag{3.55}$$

and

$$p_{B,1} = \frac{t(81 + 39\delta - 22\delta^2)}{81 + 24\delta} \tag{3.56}$$

which lead to $\alpha(p_{A,1}, p_{B,1}) = 9(3-\delta)/(2(27+8\delta))$. For any δ it holds that $\alpha \geq 0.25$, such that no equilibrium can exist in this range of α.

2.) If $0.25 < \alpha < 0.75$, the discounted sum of profits is

$$\Pi_{A,1+2}(p_{A,1}, p_{B,1}) = p_{A,1}\alpha + \delta t \left[\frac{(1+2\alpha)^2}{18} + \frac{(3-4\alpha)^2}{18} \right] \tag{3.57}$$

and

$$\Pi_{B,1+2}(p_{A,1}, p_{B,1}) = p_{B,1}(1-\alpha) + \delta t \left[\frac{(4\alpha-1)^2}{18} + \frac{(3-2\alpha)^2}{18} \right]. \tag{3.58}$$

Maximization of profits leads to $p_{A,1} = p_{B,1} = t$ and $\alpha = 0.5$, such that the discounted sum of profits simplifies to $\Pi_{A,1+2} = \Pi_{B,1+2} = t(9+5\delta)/18$. This constitutes an equilibrium since when the rival's price is fixed, there is no incentive to deviate for a firm such that α would lie outside the interval $[0.25, 0.75]$ irrespective of δ, because the function $\Pi_{i,1+2}(p_{i,1}, p_{j,1})$ is always concave.

3.) It follows from symmetry that no equilibrium exists if $\alpha > 0.75$, either. *Q.E.D.*

Proof of Proposition 3.1. We start to prove the first part of the Proposition, namely, with firms' unilateral incentives to invest in screening technology.

1.) Consider the case where $\delta \geq 0.25$. If firm B does not invest, firm A's profit if investing is

$$\Pi^{AS}_{A,1+2,}(\delta) = \frac{t(576 + 1392\delta + 931\delta^2 + 265\delta^3)}{2(24 + 13\delta)^2}, \tag{3.59}$$

while it is

$$\Pi^{NN}_{A,1+2}(\delta) = \frac{t(1 + \delta)}{2} \tag{3.60}$$

if neither firm invests. For any δ, the discounted sum of profits $\Pi^{AS}_{A,1+2,}(\delta)$ (3.59) is larger than $\Pi^{NN}_{A,1+2}(\delta)$ (3.60), such that firm A always invests. If firm A invests and firm B does not invest, firm B will obtain a discounted sum of profits of

$$\Pi^{AS}_{B,1+2}(\delta) = \frac{t(576 + 1296\delta + 645\delta^2 - 70\delta^3)}{2(24 + 13\delta)^2}. \tag{3.61}$$

If firm B invests and firm A also obtains screening technology, firm A's profits will be

$$\Pi^{II}_{A,1+2}(\delta) = \frac{t(9 + 5\delta)}{18} \tag{3.62}$$

while firm A's profits when it refuses to invest equal the profits $\Pi^{AS}_{B,1+2}(\delta)$ (3.61).

Since profits $\Pi^{AS}_{B,1+2}(\delta)$ (3.61) are larger than profits $\Pi^{II}_{A,1+2}(\delta)$ (3.62) for any δ, it follows that when firm B invests, firm A will not want to do so. Taking the analysis together and given symmetry, we can conclude that there are two asymmetric equilibria in pure strategies, namely those where respectively only one of the firms invests into screening technology.

2.) Consider the case where $\delta \leq 0.01$. The analysis is similar, with the same asymmetric equilibria emerging. The respective profits in the asymmetric subgames are

$$\Pi^{AS}_{A,1+2}(\delta) = \frac{t(4608 + 4080\delta - 1444\delta^2 + 109\delta^3)}{4(48 - 7\delta)^2} \tag{3.63}$$

and

$$\Pi^{AS}_{B,1+2}(\delta) = \frac{4t(288 + 126\delta - 57\delta^2 + 5\delta^3)}{(48 - 7\delta)^2}. \tag{3.64}$$

We now proof the analysis about industry profits, social welfare, and consumer surplus.

1.) We first summarize industry profits over two periods. When neither of the firms invests, total industry-profits are

$$\Pi^{NN}_{A,1+2}(\delta) + \Pi^{NN}_{B,1+2}(\delta) = \frac{t(1+\delta)}{2} + \frac{t(1+\delta)}{2} = t(1+\delta). \tag{3.65}$$

If both firms invest into screening technology, total industry-profits are

$$\Pi^{II}_{A,1+2}(\delta) + \Pi^{II}_{B,1+2}(\delta) = \frac{t(9+5\delta)}{18} + \frac{t(9+5\delta)}{18} = \frac{t(9+5\delta)}{9}. \tag{3.66}$$

If $\delta \leq 0.01$, in equilibrium, total industry-profits are

$$\begin{aligned}
&\Pi^{AS}_{A,1+2}(\delta) + \Pi^{AS}_{B,1+2}(\delta) = \\
&\frac{t(4608 + 4080\delta - 1444\delta^2 + 109\delta^3)}{4(48-7\delta)^2} + \frac{4t(288 + 126\delta - 57\delta^2 + 5\delta^3)}{(48-7\delta)^2} \\
&= \frac{t(9216 + 6096\delta - 2356\delta^2 + 189\delta^3)}{4(48-7\delta)^2}.
\end{aligned} \tag{3.67}$$

If $\delta \geq 0.25$, in equilibrium, total industry-profits are

$$\begin{aligned}
&\Pi^{AS}_{A,1+2}(\delta) + \Pi^{AS}_{B,1+2}(\delta) = \\
&\frac{t(576 + 1392\delta + 931\delta^2 + 265\delta^3)}{2(24+13\delta)^2} + \frac{(576 + 1296\delta + 645\delta^2 - 70\delta^3)}{2(24+13\delta)^2} \\
&= \frac{t(1152 + 2688\delta + 1576\delta^2 + 195\delta^3)}{2(24+13\delta)^2}.
\end{aligned} \tag{3.68}$$

2.) We now turn to the analysis of social welfare. When none of the firms holds screening technology, social welfare over two periods is

$$SW^{NN}_{1+2}(\delta) = (1+\delta)\Big(v - 2t\int_0^{\frac{1}{2}} x\,dx\Big) = (1+\delta)\Big(v - \frac{t}{4}\Big). \tag{3.69}$$

If both firms hold screening technology, social welfare is

$$SW^{II}_{1+2}(\delta) =$$

$$v(1+\delta)-t\Big(\int_0^{\frac{1}{2}} x\,dx+\int_{\frac{1}{2}}^{1}(1-x)\,dx\Big)-\delta t\Big(\int_0^{\frac{1}{3}} x\,dx+\int_{\frac{1}{3}}^{\frac{1}{2}}(1-x)\,dx+\int_{\frac{1}{2}}^{\frac{2}{3}} x\,dx+\int_{\frac{2}{3}}^{1}(1-x)\,dx\Big)$$

$$= v(1+\delta)-\frac{t(9+11\delta)}{36}. \tag{3.70}$$

In the asymmetric equilibrium, if $\delta \leq 0.01$, social welfare is

$$SW^{AS}_{1+2}(\delta) = v(1+\delta)-t\Big(\int_0^{\frac{\delta-24}{7\delta-48}} x\,dx+\int_{\frac{\delta-24}{7\delta-48}}^{1}(1-x)\,dx\Big)$$

$$-\delta t\Big(\int_0^{\frac{84-11\delta}{192-28\delta}} x\,dx+\int_{\frac{84-11\delta}{192-28\delta}}^{\frac{\delta-24}{7\delta-48}}(1-x)\,dx+\int_{\frac{\delta-24}{7\delta-48}}^{\frac{132-13\delta}{192-28\delta}} x\,dx+\int_{\frac{132-13\delta}{192-28\delta}}^{1}(1-x)\,dx\Big)$$

$$= v(1+\delta)-\frac{t(4608+3984\delta-1196\delta^2+53\delta^3)}{8(48-7\delta)^2}. \tag{3.71}$$

while social welfare if $\delta \geq 0.25$ is

$$SW^{AS}_{1+2}(\delta) = v(1+\delta)-t\Big(\int_0^{\frac{24-\delta}{48+26\delta}} x\,dx+\int_{\frac{24-\delta}{48+26\delta}}^{1}(1-x)\,dx\Big)-\delta t\Big(\int_0^{\frac{18+8\delta}{24+13\delta}} x\,dx+\int_{\frac{18+8\delta}{24+13\delta}}^{1}(1-x)\,dx\Big)$$

$$= v(1+\delta)-\frac{t(576+1344\delta+1061\delta^2+178\delta^3)}{4(24+13\delta)^2}. \tag{3.72}$$

3.) We now turn to the analysis of consumer surplus. Consumer surplus when none of the firms invests into screening technology is

$$SW^{NN}_{1+2}(\delta)-(\Pi^{NN}_{A,1+2}(\delta)+\Pi^{NN}_{B,1+2}(\delta)) = (1+\delta)\frac{4v-5t}{4}. \tag{3.73}$$

Consumer surplus when both firms invest into screening technology is

$$SW^{II}_{1+2}(\delta) - (\Pi^{II}_{A,1+2}(\delta) + \Pi^{II}_{B,1+2}(\delta)) = v(1+\delta) - \frac{t(45+31\delta)}{36}. \tag{3.74}$$

In the asymmetric equilibrium, if $\delta \leq 0.01$, consumer surplus is

$$SW^{AS}_{1+2}(\delta) - (\Pi^{AS}_{A,1+2}(\delta) + \Pi^{AS}_{B,1+2}(\delta)) = v(1+\delta) - \frac{t(23040+16176\delta-5908\delta^2+431\delta^3)}{8(48-7\delta)^2}, \tag{3.75}$$

while consumer surplus if $\delta \geq 0.25$ is

$$SW^{AS}_{1+2}(\delta) - (\Pi^{AS}_{A,1+2}(\delta) + \Pi^{AS}_{B,1+2}(\delta)) = v(1+\delta) - \frac{t(2880+6720\delta+4213\delta^2+568\delta^3)}{4(24+13\delta)^2}. \tag{3.76}$$

Comparing total industry-profits, social welfare, and consumer surplus leads to the order displayed in Table 3.1 of Proposition 3.1. *Q.E.D.*

Proof of Lemma 3.5. To calculate firms' profits in the first period we have to distinguish between the different cases depending on α as derived in Lemma 3.1.

1.) Assume that firms' prices in the first period are such that $0 < \alpha \leq (8-5\sqrt{2})/2 \approx 0.46$. The discounted sum of firm B's profits is then

$$\Pi_{B,1+2} = (1-\alpha)p_{B,1} + \frac{\delta t(3-2\alpha)^2}{16}, \tag{3.77}$$

where $\alpha = (4p_{A,1} - 4p_{B,1} - 4t + 5\delta t)/(2t(3\delta - 4))$ is the address of the indifferent consumer. The discounted sum of firm A's total profits is

$$\Pi_{A,1+2} = \alpha p_{A,1} + \frac{5\delta t(5+4\alpha-12\alpha^2)}{32}. \tag{3.78}$$

Taking the derivatives w.r.t. the respective first-period prices and solving the FOCs simultaneously gives the optimal first-period prices as

$$p^*_{A,1} = \frac{t(-96+44\delta+91\delta^2)}{-96+20\delta} \tag{3.79}$$

and

$$p^*_{B,1} = \frac{t(-96+68\delta+47\delta^2)}{-96+20\delta}, \tag{3.80}$$

which are both positive for $\delta \leq 0.81$.

The indifferent consumer indeed lies in the first interval under the derived first-period prices if $\delta > 0.09$. Both prices lead to the following total profits of firm A and firm B respectively depending on the discount factor δ

$$\Pi_{A,1+2} = \frac{t(2304 - 384\delta - 212\delta^2 + 553\delta^3)}{8(24 - 5\delta)^2} \tag{3.81}$$

and

$$\Pi_{B,1+2} = \frac{t(2304 + 768\delta - 1628\delta^2 - 579\delta^3)}{8(24 - 5\delta)^2}. \tag{3.82}$$

In a next step we have to check firms' incentives to deviate.

1. a) Consider the deviation of firm A to the second interval. The optimal price (inner solution) in this case is

$$p^d_{A,1} = \frac{t(3072 - 4096\delta + 912\delta^2 + 397\delta^3)}{4(5\delta - 24)(17\delta - 32)}. \tag{3.83}$$

This deviation price follows from maximizing the profit $\Pi_{A,1+2} = \alpha p^d_{A,1} + \delta t(9 - 6\alpha + 5\alpha^2)/16$. Controlling that firm A indeed deviates to the second interval, we find that this price holds only for $\delta \leq 0.19$. Otherwise, firm A will deviate by using the corner solution.

Both prices, however, lead to lower profits compared to those firm A gets at $p_{A,1} = p^*_{A,1}$. Thus, a deviation to the second interval is not profitable for firm A.

1. b) Consider the deviation of firm A to the third interval. The optimal price (inner solution) that firm A then would set is

$$p^d_{A,1} = \frac{t(-384 + 1088\delta - 576\delta^2 + 157\delta^3)}{4(\delta - 4)(5\delta - 24)}. \tag{3.84}$$

This deviation price follows from maximizing the profit $\Pi_{A,1+2} = \alpha p^d_{A,1} + \delta t(47 - 120\alpha + 48\alpha^2)/32$. This price, however, does not satisfy the condition that α lies indeed in the third interval. Therefore, firm A would set the price of the corner solution, but this does not lead to higher profits than staying on the first interval. As a result, firm A will not deviate from the first interval.

1. c) Consider the deviation of firm B to the second interval. The FOC price is then

$$p^d_{B,1} = \frac{t(768 - 448\delta - 348\delta^2 + 197\delta^3)}{2(5\delta - 24)(7\delta - 16)}. \tag{3.85}$$

This deviation price follows from maximizing the deviation profit $\Pi_{B,1+2} = (1 -$

$\alpha)p_{B,1} + \delta t(1+\alpha)^2/8$. With this price, the indifferent consumer indeed lies on the second interval if $\delta < 0.95$. Otherwise, the corner solution applies. The deviation profit with the corner solution, however, never leads to higher profits than with $p_{B,1} = p^*_{B,1}$. A deviation with the inner solution is only profitable for $0.83 < \delta < 0.95$, for which there is no equilibrium in pure strategies.

1. d) Consider the deviation of firm B to the third interval. The FOC price is then

$$p^d_{B,1} = \frac{t(192 - 304\delta - 137\delta^2 + 212\delta^3)}{4(3\delta - 2)(5\delta - 24)}. \tag{3.86}$$

This deviation price follows from maximizing the profit $\Pi_{B,1+2} = (1-\alpha)p_{B,1} + \delta t(1-4\alpha)^2/16$. This price only lies on the third interval for a small range of discount factors, namely if $\delta \in [0.76; 0.77]$. For other discount factors firm B will use the corner solution. Both deviation prices, however, lead to lower profits than under $p_{B,1} = p^*_{B,1}$.

To sum up, we find that the prices $p^*_{A,1}$ (3.79) and $p^*_{B,1}$ (3.80) and the respective profits $\Pi_{A,1+2}$ (3.81) and $\Pi_{B,1+2}$ (3.82) constitute an equilibrium on the first interval if $0.09 < \delta \leq 0.81$.

2.) Assume that firms' first-period prices are such that $0.46 \approx (8 - 5\sqrt{2})/2 < \alpha \leq (6 + 5\sqrt{2})/14 \approx 0.93$. The discounted sum of firm B's total profits is then

$$\Pi_{B,1+2} = (1-\alpha)p_{B,1} + \frac{\delta t(1+\alpha)^2}{8}, \tag{3.87}$$

where $\alpha = (4p_{A,1} - 4p_{B,1} - 4t + 3\delta t)/(t(3\delta - 8))$ is the address of the indifferent consumer. For firm A the sum of discounted total profits is

$$\Pi_{A,1+2} = \alpha p_{A,1} + \frac{\delta t(9 - 6\alpha + 5\alpha^2)}{16}. \tag{3.88}$$

Taking the derivatives w.r.t. the respective first-period prices and solving the FOCs simultaneously gives the optimal first-period prices as

$$p^*_{A,1} = \frac{t(-96 + 82\delta - 17\delta^2)}{-96 + 50\delta} \tag{3.89}$$

and

$$p^*_{B,1} = \frac{t(-96 + 48\delta + \delta^2)}{-96 + 50\delta}, \tag{3.90}$$

which are both positive for any $\delta \in [0, 1]$. The indifferent consumer indeed lies in the second interval for any $\delta \in [0, 1]$. Both prices lead to the following total profits of firm A and firm B respectively depending on the discount factor δ

$$\Pi_{A,1+2} = \frac{t(1152 - 564\delta - 349\delta^2 + 172\delta^3)}{(48 - 25\delta)^2} \tag{3.91}$$

and

$$\Pi_{B,1+2} = \frac{t(2304 - 1008\delta - 816\delta^2 + 373\delta^3)}{2(48 - 25\delta)^2}. \tag{3.92}$$

2. a) Consider a deviation of firm B to the first interval. If $0.09 \leq \delta < 0.63$, firm B will find it optimal to set a price (inner solution) of

$$p^d_{B,1} = \frac{t(-1536 + 2976\delta - 1728\delta^2 + 311\delta^3)}{4(8 - 7\delta)(25\delta - 48)} \tag{3.93}$$

and if $0.03 < \delta < 0.09$ and $0.63 \leq \delta \leq 1$, firm B will find it profitable to deviate with the corner solution given by

$$p^d_{B,1} = \frac{t(192(5\sqrt{2} - 8) - 4\delta(305\sqrt{2} - 444) + \delta^2(375\sqrt{2} - 509))}{4(25\delta - 48)}.$$

2. b) Consider a deviation of firm A to the first interval. If $\delta > 0.15$, it will deviate with a price (inner solution) of

$$p^d_{A,1} = \frac{t(768 - 248\delta - 430\delta^2 + 183\delta^3)}{(16 + 3\delta)(48 - 25\delta)}$$

and if $0.02 \leq \delta \leq 0.15$, firm A will deviate with a price (corner solution) of

$$p^d_{A,1} = \frac{t(192(5\sqrt{2} - 6) - 4\delta(305\sqrt{2} - 379) + \delta^2(375\sqrt{2} - 477))}{4(48 - 25\delta)}.$$

To sum up, we find that the prices $p^*_{A,1}$ (3.89) and $p^*_{B,1}$ (3.90) and the respective profits $\Pi_{A,1+2}$ (3.91) and $\Pi_{B,1+2}$ (3.92) constitute an equilibrium on the second interval if $\delta < 0.02$.

Moreover, we find that neither of the firms wants to deviate to the third interval. 3.) Assume that firms' first-period prices are such that $0.93 \approx (6 + 5\sqrt{2})/14 < \alpha \leq 1$. In this case, there is no equilibrium because firm A would always want to deviate to the first interval. *Q.E.D.*

Proof of Lemma 3.6. To calculate firms' profits in the first period we have to distinguish between the different cases, following Lemma 3.2. If $\alpha \in [0.25, 0.75]$, firms' total profits are, respectively

$$\Pi_{A,1+2} = \alpha p_{A,1} + \delta\Big(\frac{t(1+2\alpha)^2}{18} + \frac{t(3-4\alpha)^2}{18}\Big) \tag{3.94}$$

and

$$\Pi_{B,1+2} = (1-\alpha)p_{B,1} + \delta\Big(\frac{t(4\alpha-1)^2}{18} + \frac{t(3-2\alpha)^2}{18}\Big) \tag{3.95}$$

where

$$\alpha = \frac{3t + \delta t - 3p_{A,1} + 3p_{B,1}}{2t(3+\delta)} \tag{3.96}$$

is the address of the consumer who is indifferent between buying from firm A or from firm B in the first period. Maximizing firms' profits w.r.t. their respective price yields an equilibrium price of

$$p_{A,1} = p_{B,1} = \frac{t(3+\delta)}{3}. \tag{3.97}$$

The indifferent consumer α (3.96) with these price lies at $x = 1/2$. This leads to discounted profits over two periods for each firm of

$$\Pi_{A,1+2} = \Pi_{B,1+2} = \frac{t(9+8\delta)}{18}. \tag{3.98}$$

Both firms do not have an incentive to deviate, such that the prices $p_{A,1}$ and $p_{B,1}$ (3.97) constitute an equilibrium. If $\alpha \notin [0.25, 0.75]$, there is no further equilibrium. *Q.E.D.*

Proof of Proposition 3.2. We first show that two symmetric equilibria exist. In the first equilibrium, both firms do not to invest, while in the second one both firms obtain the screening technology.

1.) Consider first the case of a very low discount factor, $\delta < 0.02$ (cf. case i) of Lemma 3.5). If firm B does not invest, firm A's profit in case of investment is

$$\Pi^{AS}_{A,1+2,}(\delta) = \frac{t(1152 - 564\delta - 349\delta^2 + 172\delta^3)}{(48-25\delta)^2}, \tag{3.99}$$

while it is

$$\Pi^{NN}_{A,1+2}(\delta) = \frac{t(1+\delta)}{2} \tag{3.100}$$

if neither firm invests. For any δ, the discounted sum of profits $\Pi^{AS}_{A,1+2,}(\delta)$ (3.99) is smaller than $\Pi^{NN}_{A,1+2}(\delta)$ (3.100), such that firm A always decides not to invest. If firm B invests and firm A does so too, its profits will be

$$\Pi^{II}_{A,1+2}(\delta) = \frac{t(9+8\delta)}{18} \tag{3.101}$$

while profits when it refuses to invest will equal firm B's profits in the case that only firm A invests

$$\Pi^{AS}_{B,1+2}(\delta) = \frac{t(2304 - 1008\delta - 816\delta^2 + 373\delta^3)}{2(48-25\delta)^2}. \tag{3.102}$$

Since $\Pi^{AS}_{B,1+2}(\delta)$ (3.102) is smaller than the profits $\Pi^{II}_{A,1+2}(\delta)$ (3.101) for any δ, it follows that if firm B invests, firm A does so too. Taking the analysis together and given symmetry, we can conclude that there are two symmetric equilibria in pure strategies, namely those where both firms either decide to invest or not to invest into screening technology.

2.) Consider the case of $\delta \in (0.09, 0.81]$. Due to symmetry the analysis is similar with the same symmetric equilibria emerging. The respective profits in the asymmetric subgames are

$$\Pi^{AS}_{A,1+2}(\delta) = \frac{t(2304 - 384\delta - 212\delta^2 + 553\delta^3)}{8(24-5\delta)^2} \tag{3.103}$$

and

$$\Pi^{AS}_{B,1+2}(\delta) = \frac{t(2304 + 768\delta - 1628\delta^2 - 579\delta^3)}{8(24-5\delta)^2}. \tag{3.104}$$

We now proof the analysis about industry profits, social welfare, and consumer surplus.

1.) We first summarize industry profits over the two periods. When neither of the firms invests, total industry-profits are

$$\Pi^{NN}_{A,1+2}(\delta) + \Pi^{NN}_{B,1+2}(\delta) = \frac{t(1+\delta)}{2} + \frac{t(1+\delta)}{2} = t(1+\delta). \tag{3.105}$$

If both firms invest into screening technology, total industry-profits are

$$\Pi^{II}_{A,1+2}(\delta) + \Pi^{II}_{B,1+2}(\delta) = \frac{t(9+8\delta)}{18} + \frac{t(9+8\delta)}{18} = \frac{t(9+8\delta)}{9}. \tag{3.106}$$

If $\delta < 0.02$, total industry-profits in the asymmetric subgame are

$$\Pi^{AS}_{A,1+2}(\delta) + \Pi^{AS}_{B,1+2}(\delta) =$$

$$\frac{t(1152-564\delta-349\delta^2+172\delta^3)}{(48-25\delta)^2} + \frac{t(2304-1008\delta-816\delta^2+373\delta^3)}{2(48-25\delta)^2}$$

$$= \frac{t(4608-2136\delta-1514\delta^2+717\delta^3)}{2(48-25\delta)^2}. \tag{3.107}$$

If $\delta \geq 0.25$, total industry-profits in the asymmetric subgame are

$$\Pi^{AS}_{A,1+2}(\delta) + \Pi^{AS}_{B,1+2}(\delta) =$$

$$\frac{t(2304-384\delta-212\delta^2+553\delta^3)}{8(24-5\delta)^2} + \frac{t(2304+768\delta-1628\delta^2-579\delta^3)}{8(24-5\delta)^2}$$

$$= \frac{t(2304+192\delta-920\delta^2-13\delta^3)}{4(24-5\delta)^2}. \tag{3.108}$$

2.) We now turn to the analysis of social welfare. If none of the firms holds screening technology, social welfare over two periods is (cf. Proof of Proposition 3.1)

$$SW^{NN}_{1+2}(\delta) = (1+\delta)\Big(v - 2t\int_0^{\frac{1}{2}} x\ dx\Big) = (1+\delta)(v - \frac{t}{4}). \tag{3.109}$$

If both firms hold screening technology, social welfare is

$$SW^{II}_{1+2}(\delta) =$$

$$v(1+\delta) - t\Big(\int_0^{\frac{1}{2}} x\ dx + \int_{\frac{1}{2}}^{1} (1-x)\ dx\Big) - \delta t\Big(\int_0^{\frac{1}{3}} x\ dx + \int_{\frac{1}{3}}^{\frac{1}{2}} (1-x)\ dx + \int_{\frac{1}{2}}^{\frac{2}{3}} x\ dx + \int_{\frac{2}{3}}^{1} (1-x)\ dx\Big)$$

$$= v(1+\delta) - \frac{t(9+11\delta)}{36}. \tag{3.110}$$

In the asymmetric subgame, if $\delta < 0.02$, social welfare is

$$SW^{AS}_{1+2}(\delta) = v(1+\delta) - t\Big(\int\limits_0^{\frac{24-13\delta}{48-25\delta}} x\ dx + \int\limits_{\frac{24-13\delta}{48-25\delta}}^{1} (1-x)\ dx\Big)$$

$$-\delta t\Big(\int\limits_0^{\frac{21-11\delta}{48-25\delta}} x\ dx + \int\limits_{\frac{21-11\delta}{48-25\delta}}^{\frac{24-13\delta}{48-25\delta}} (1-x)\ dx + \int\limits_{\frac{24-13\delta}{48-25\delta}}^{\frac{66-35\delta}{96-50\delta}} x\ dx + \int\limits_{\frac{66-35\delta}{96-50\delta}}^{1} (1-x)\ dx\Big)$$

$$= v(1+\delta) - \frac{t(2304 + 264\delta - 2170\delta^2 + 733\delta^3)}{4(48-25\delta)^2}. \tag{3.111}$$

In the asymmetric subgame, if $\delta \in (0.09, 0.81]$, social welfare is

$$SW^{AS}_{1+2}(\delta) = v(1+\delta) - t\Big(\int\limits_0^{\frac{24-23\delta}{48-10\delta}} x\ dx + \int\limits_{\frac{24-23\delta}{48-10\delta}}^{1} (1-x)\ dx\Big) - \delta t\Big(\int\limits_0^{\frac{6\delta-18}{5\delta-24}} x\ dx + \int\limits_{\frac{6\delta-18}{5\delta-24}}^{1} (1-x)\ dx\Big)$$

$$= v(1+\delta) - \frac{t(576 + 480\delta - 59\delta^2 + 74\delta^3)}{4(24-5\delta)^2}. \tag{3.112}$$

3.) We now turn to the analysis of consumer surplus. If none of the firms invests into screening technology consumer surplus is (cf. Proof of Proposition 3.1)

$$SW^{NN}_{1+2}(\delta) - (\Pi^{NN}_{A,1+2}(\delta) + \Pi^{NN}_{B,1+2}(\delta)) = (1+\delta)\frac{4v-5t}{4}. \tag{3.113}$$

Consumer surplus when both firms invest into screening technology is

$$SW^{II}_{1+2}(\delta) - (\Pi^{II}_{A,1+2}(\delta) + \Pi^{II}_{B,1+2}(\delta)) = v(1+\delta) - \frac{t(45+43\delta)}{36}. \tag{3.114}$$

In the asymmetric subgame, if $\delta < 0.02$, consumer surplus is

$$SW^{AS}_{1+2}(\delta) - (\Pi^{AS}_{A,1+2}(\delta) + \Pi^{AS}_{B,1+2}(\delta)) = v(1+\delta) - \frac{t(11520 + 4008\delta + 5198\delta^2 - 2167\delta^3)}{4(48-25\delta)^2}. \tag{3.115}$$

In the asymmetric subgame, if $\delta \in (0.09, 0.81]$, consumer surplus is

$$SW^{AS}_{1+2}(\delta) - (\Pi^{AS}_{A,1+2}(\delta) + \Pi^{AS}_{B,1+2}(\delta)) = v(1+\delta) - \frac{t(2880 + 672\delta - 979\delta^2 + 61\delta^3)}{4(24-5\delta)^2}. \tag{3.116}$$

Comparing total industry-profits, social welfare, and consumer surplus leads to the order displayed in Table 3.2. *Q.E.D.*

Proof of Corollary 3.1. We start with the comparison between myopic and sophisticated consumers when the discount factor is low.

1.) If $\delta \leq 0.01$, in equilibrium one firm will invest if consumers are myopic and either both firms invest or both firms refuse to invest when consumers are sophisticated. Concerning profits, a comparison of the asymmetric equilibrium $\Pi^{AS}_{A,1+2}(\delta) + \Pi^{AS}_{B,1+2}(\delta)$(3.67) and the non-investing equilibrium $\Pi^{NN}_{A,1+2}(\delta) + \Pi^{NN}_{B,1+2}(\delta)$ (3.105) shows that the latter yields higher industry profits. The asymmetric equilibrium, however, leads to higher total industry profits than the equilibrium when consumers are sophisticated and both firms invest $\Pi^{II}_{A,1+2}(\delta) + \Pi^{II}_{B,1+2}(\delta)$ (3.106). Concerning social welfare, a comparison between the asymmetric equilibrium when consumers are myopic $SW^{AS}_{1+2}(\delta)$ (3.71), the non-investing-equilibrium $SW^{NN}_{1+2}(\delta)$ (3.109), and the investing-equilibrium $SW^{II}_{1+2}(\delta)$ (3.110) when they are sophisticated, shows that social welfare in the asymmetric equilibrium is lower than when both firms refuse to invest but higher than in the case where both firms obtain screening technology. Consumer surplus in the asymmetric equilibrium $SW^{AS}_{1+2}(\delta) - (\Pi^{AS}_{A,1+2}(\delta) + \Pi^{AS}_{B,1+2}(\delta))$ (3.75) is higher than in the non-investing equilibrium $SW^{NN}_{1+2}(\delta) - (\Pi^{NN}_{A,1+2}(\delta) + \Pi^{NN}_{B,1+2}(\delta))$ (3.113), but lower than in the equilibrium when both firms invest $SW^{II}_{1+2}(\delta) - (\Pi^{II}_{A,1+2}(\delta) + \Pi^{II}_{B,1+2}(\delta))$ (3.114).

2.) If $0.25 \leq \delta \leq 0.81$, such that there is an equilibrium in pure strategies irrespective of whether consumers are myopic or sophisticated, total industry profits when consumers are myopic $\Pi^{AS}_{A,1+2}(\delta) + \Pi^{AS}_{B,1+2}(\delta)$ (3.68) are higher compared to the equilibrium when they are sophisticated and no firm invests $\Pi^{NN}_{A,1+2}(\delta) + \Pi^{NN}_{B,1+2}(\delta)$ (3.105) and both firms invest $\Pi^{II}_{A,1+2}(\delta) + \Pi^{II}_{B,1+2}(\delta)$ (3.106). In contrast, social welfare in the asymmetric equilibrium when consumers are myopic $SW^{AS}_{1+2}(\delta)$ (3.72) is lower, irrespective of whether both firms invest $SW^{II}_{1+2}(\delta)$ (3.110) or do not invest $SW^{NN}_{1+2}(\delta)$ (3.109) when consumers are sophisticated. Finally, consumer surplus in the asymmetric equilibrium $SW^{AS}_{1+2}(\delta) - (\Pi^{AS}_{A,1+2}(\delta) + \Pi^{AS}_{B,1+2}(\delta))$ (3.76) is also lower than in both symmetric equilibria $SW^{NN}_{1+2}(\delta) - (\Pi^{NN}_{A,1+2}(\delta) + \Pi^{NN}_{B,1+2}(\delta))$ (3.113) and $SW^{II}_{1+2}(\delta) - (\Pi^{II}_{A,1+2}(\delta) + \Pi^{II}_{B,1+2}(\delta))$ (3.114). *Q.E.D.*

3.9 References

1. Baye I., Sapi G. (2019) Should Mobile Marketers Collect Other Data Than Geo-Location? The Scandinavian Journal of Economics; 121; pp. 647- 675.

2. Baye I., Sapi G. (2020) Consumer Foresight, Customer Data and Investment in Targeting Technology. Scottish Journal of Political Economy; 67(4); pp. 363-386.

3. Check24 (2016) Internetanbieter: Bestandskunden zahlen bis zu 80 Prozent mehr als Neukunden. Press release.

4. Chen Y. (1997) Paying Customers to Switch. Journal of Economics and Management Strategy; 6(4); pp. 877-897.

5. Chen Y., Iyer G. (2002) Research Note. Consumer Addressability and Customized Pricing. Marketing Science; 21(2); pp. 197-208.

6. Choudhary V., Ghose A., Mukhopadhyay T., Rajan U. (2005) Personalized Pricing and Quality Differentiation. Management Science; 51(7); pp. 1120-1130.

7. Colombo S. (2016) Imperfect Behavior-Based Price Discrimination. Journal of Economics and Management Strategy; 25(3); pp. 563-583.

8. Esteves R. B. (2009) A Survey on the Economics of Behaviour-Based Price Discrimination. Technical Report; NIPE-Universidade do Minho.

9. Esteves R. B. (2010) Pricing with Customer Recognition. International Journal of Industrial Organization; 28(6); pp. 669-681.

10. Europe Economics (2007) An Analysis of the Issue of Consumer Detriment and the Most Appropriate Methodologies to Estimate It. Final Report for DG SANCO. http://www.europe-economics.com/publications

11. Fudenberg D., Villas-Boas J.M. (2006) Behavior-Based Price Discrimination and Customer Recognition. Handbook on Economics and Information Systems; 1; pp. 377-436.

12. Fudenberg D., Tirole J. (2000) Customer Poaching and Brand Switching. RAND Journal of Economics; 31; pp. 634-657.

13. Klemperer P. (1995) Competition When Consumers Have Switching Costs: An Overview with Applications to Industrial Organization, Macroeconomics and International Trade; The Review of Economic Studies; 62(4); pp. 515-539.

14. Klosowski T. (2013) How Web Sites Vary Prices Based on Your Information (and What You Can Do About It). https://lifehacker.com/5973689/how-web-sites-vary-prices-based-on-your-information-and-what-you-can-do-about-it

15. Liu Q., Serfes K. (2004) Quality of Information and Oligopolistic Price Discrimination. Journal of Economics and Management Strategy; 13(4); pp. 671-702.

16. Liu Q., Serfes K. (2006) Customer Information Sharing among Rival Firms. European Economic Review; 50(6); pp. 1571-1600.

17. Neff J. (2014) Once the "Future", Are Print-at-Home Coupons Already in Decline? Catalina Bets On Fully Digital Coupons That Load to Loyalty Cards; AdAge

18. Ross B. (2013) Canned Goods - not Canned Prices with Personalized Pricing. How Grocers can Implement a Personalized Pricing Strategy in Four Steps. http://www.canadiangrocer.com/blog/canned-goods

19. Schader P. (2017) Denn's Biomarkt verabschiedet sich für "Mein denn's" von Payback.
https://www.supermarktblog.com/2017/05/31/denns-biomarktverabschiedet-sich-fur-mein-denns-von-payback

20. Schader P. (2017) "Werte passen nicht zusammen": Denn's Biomarkt erklärt seinen Payback-Abschied.
https://www.supermarktblog.com/2017/07/12/werte-passen-nicht-zusammen-denns-biomarkt-erklart-seinen-payback-abschied

21. Schenker R. (2015) Enterprise Customer Management. Can You Afford not to Take the Leap? Bond Brand Loyalty White Paper.
https://www.loyalty360.org/Loyalty360/media//ResearchAndReportDocs

22. Shaffer G., Zhang Z. J. (1995) Competitive Coupon Targeting. Marketing Science; 14(4); pp. 395-416.

23. Shaffer G., Zhang Z. J. (2002) Competitive One-to-One Promotions. Management Science; 48(9); pp. 1143-1160.

24. Silbye F. (2010) Topics in Competition Policy: Cartels, Leniency, and Price Discrimination. Mimeo.

25. The Guardian (2003) The Card up Their Sleeve. https://www.theguardian.com/lifeandstyle/2003/jul/19/shopping.features

26. Thisse J.-F., Vives X. (1988) On the Strategic Choice of Spatial Price Policy. The American Economic Review; 78; pp. 122-137.