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Dedicated to Anna

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Introduction

Understanding of the functioning of markets and, in particular, determinants of market outcomes is crucial for effective competition policy and regulation. The research presented in this thesis is concerned with distinct aspects of competition economics. Chapter 1 analyzes how imperfect information about consumer preferences relates to the stability of tacit collusion in a classical setup. Chapter 2 and 3 depart from classical assumptions about consumer behavior by incorporating psychologically well-founded behavioral assumptions. Chapter 2 studies how focused thinking (Kőszegi and Szeidl, 2013) affects the design of supply and service contracts in different competitive environments. Chapter 3 investigates how self-deception (Brunnermeier and Parker, 2005) about potentially negative consequences of purchasing behavior on others affects competition of firms, which are differentiated with respect to whether their production causes negative externalities. Thereby, this thesis contributes to the applied theoretical literature on competition economics by providing new insights.

Chapter 1 on "Private Information, Price Discrimination, and Collusion" is joint work with Alexander Rasch and Shiva Shekhar. We analyze firms' ability to sustain collusion in a setting in which horizontally differentiated firms can price-discriminate based on private information regarding consumers' preferences. In particular, firms receive private signals which can be noisy (e.g., big data predictions). We find that there is a non-monotone relationship between signal quality and sustainability of collusion. Starting from a low level, an increase in signal precision first facilitates collusion. There is a watershed, however, from which any further increase renders collusion less sustainable. Our analysis provides important insights for competition policy. In particular, a ban on price discrimination can help to prevent collusive behavior as long as signals are sufficiently noisy.

Chapter 2 on "Attention-Driven Demand for Bonus Contracts" is joint work with Markus Dertwinkel-Kalt and Mats Köster. It starts out with the observation that supply contracts (e.g., for electricity, telephony, or banking services) typically include a series of small, regular payments made by consumers and a single, large bonus (e.g., a monetary payment, or a premium such as a smartphone) that consumers receive at some point during the contractual period. Bonus payments, however, create inefficiencies such as sending out checks and redeeming them. Non-monetary bonus premiums may involve other inefficiencies, for instance, if the consumer values the bonus below its actual selling price. We offer a novel explanation for the frequent occurrence of such *bonus contracts*, which builds on a recent model of attentional focusing. Specifically, we show that a monopolist offers a bonus contract only for low-value goods, while independent of the consumers' valuation for the product, firms standing in competition always offer bonus contracts. Thus, competition does not eliminate, but exacerbates inefficiencies arising from contracting with focused agents. Common contract schemes on markets for electricity, telephony, and bank accounts mirror our findings.

In Chapter 3 entitled "Self-Deception and Social Responsibility in Markets" I study how the possibility to exploit *moral wiggle room* through *self-deception* can affect market outcomes. For this purpose, I consider a market in which consumers are concerned but uncertain about whether their purchasing decisions harm others. Instead of resolving uncertainty, however, they can willingly distort their beliefs. Firms, which differ in their production technology with respect to whether their production harms others or not, take this into account when competing in prices. I show that avoidance of information about the impact of purchasing decisions on others in order to maintain self-deception can arise endogenously in the market. Moreover, self-deception can distort market demand toward low-cost production that harms others and render costly mitigation of externalities less profitable. Thereby, I identify a new channel through which competition is affected, namely self-deception. Through this channel, consumers perceive firms as less differentiated in favor of the low-cost firm, as they tend to be overoptimistic about whether its production harms others in order to benefit from a relatively low price.

This yields policy implications. If low-cost production causes externalities, which cannot be prohibited, information provision and campaigns are not sufficient for their mitigation as long as consumers can deceive themselves about their presence. Then, taxing externalities, subsidizing its mitigation or introducing a binding price floor can reduce monetary incentives for remaining ignorant and lead to more informed decision making. Moreover, if using a high-cost, externality-mitigating technology becomes less profitable through self-deception, both innovation and entry can become less likely if access to this technology causes fixed costs.

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Chapter 1

Private Information, Price Discrimination, and Collusion

Co-authored with Alexander Rasch and Shiva Shekhar

1.1 Introduction

In this paper, we analyze sustainability of tacit collusion in a setting in which horizontally differentiated firms can price-discriminate based on private information about consumer preferences. In particular, prices for different consumer groups can be based on private and imperfect signals. For example, firms may price-discriminate on (possibly imprecise) big data predictions. These aspects are of high relevance in industries like traditional brick-and-mortar and online retailing.

In order to price-discriminate between different consumer segments, most firms in these industries collect data on their own customers through different channels (e.g., loyalty programs, cookies) or buy data from data-collection firms. In the US, for example, the second-largest discount store retailer Target uses a data-mining program to assign many different predictors to customers.[1] The quality of data and the precision of predictions, however, can crucially affect firms' pricing decisions.[2] In particular, data quality is rarely perfect. In the example of Target, their "pregnancy prediction" was flawed. Pregnancy-related mailers were sent out to women for months after a miscarriage.[3] While we abstract from both systematic informational advantages of a firm due to its past sales and correlation of predictions due to similar data sources or algorithms in the main part of the paper, we address these issues as robustness tests.

At the same time, antitrust policy is highly concerned with collusive behavior in these industries, especially with tacit collusion in online retailing. The acuteness of this issue can be seen by the recent stern warning of the Competition and Market Authority (CMA) in the UK. The warning was issued after the CMA had found signs of price coordination among retailers in different markets on platforms such as Amazon.[4]

[1] See `http://www.nytimes.com/2012/02/19/magazine/shopping-habits.html` (accessed on June 2, 2017). See also Esteves (2014) for these and other examples.

[2] For discussions of this issue in different contexts, see Liu and Serfes (2004), Esteves (2004, ch. 2), and Colombo (2016) among others.

[3] As Charles Duhigg, a journalist with the New York Times, puts it: "I can't tell you what one shopper is going to do, but I can tell you with 90 percent accuracy what one shopper is going to do if he or she looks exactly like one million other shoppers. You expect that there is some spillage there, and as a result that you will give the wrong message to a certain number of people." See, `https://6thfloor.blogs.nytimes.com/2012/02/21/behind-the-cover-story-how-much-does-target-know/` (accessed on June 2, 2017).

[4] For more details see, for instance, `https://www.theguardian.com/business/2016/nov/07/online-sellers-price-fixing-competition-and-markets-authority-amazon` (accessed on June 2, 2017).

We find that the critical discount factor necessary to sustain tacit collusion is non-monotone in signal precision. In particular, an increase in precision reduces the critical discount factor whenever the level of signal precision lies below a certain threshold. From levels above this threshold, an increase in precision leads to a higher critical discount factor. The intuition behind this finding is as follows. In our model, collusive profits are independent of signals, whereas both deviation profits and competitive profits, which serve as punishment for deviations, depend on how precise signals are. Deviation profits are weakly increasing in signal quality, as price discrimination allows to target consumers more effectively. At the same time, competitive profits are falling in signal quality. Competition gets fiercer, as both firms can price-discriminate more effectively. Hence, improvements in signal precision have opposing effects on the critical discount factor, as both the gains from deviation and the losses from punishment increase. We show that below the threshold, the gain from defecting outweighs the loss from punishment. Intuitively, potential misrecognition of consumers renders deviation from collusive prices relatively unprofitable. Above the threshold, the reverse turns out to be true. As consumers can be targeted effectively, deviation becomes relatively tempting.

This paper adds to the combination of two strands of theoretical industrial organization literature: third-degree price discrimination and collusion, both among horizontally differentiated firms. In the first strand, Bester and Petrakis (1996) show that third-degree price discrimination by using coupons intensifies competition in markets that are segmented exogenously by consumer preferences. In a similar setup, Shaffer and Zhang (1995) illustrate that the possibility of third-degree price discrimination leads to a prisoner's dilemma. Corts (1998) then generalizes these findings. Under best-response asymmetries, that is, firms find different groups of consumers most valuable, third-degree price discrimination leads to profits lower than under uniform pricing. Fudenberg and Tirole (2000) analyze the impact of third-degree price discrimination in a dynamic context in which learning about consumer preferences is endogenous from the purchasing history. After the first period, firms learn about the preferences of their own customers. In the second period, poaching can take place through price discrimination. They also find third-degree price discrimination, which they refer to as behavior-based price discrimination, results in more intense competition and hence lower profits. Villas-Boas (1999) extends their setup to long-lived firms and overlapping consumer generations and finds that competition is intensified if firms and consumers are patient.

While in the previous contributions, firms have or obtain perfect information about consumer preferences, Esteves (2009, 2014) analyzes the impact of imperfect information. She shows that improving the quality of information also results in lower competitive profits under third-degree price discrimination. If information is imperfect, potential misrecognition of consumers dampens competition. As information becomes more accurate, firms can better target different consumer groups, which results in more intense competition. She argues that imperfect information can also be understood as a reduced-form of imperfect learning. Colombo (2016) explicitly investigates the impact of imperfect information in the dynamic context of Fudenberg and Tirole (2000). First-period learning is noisy, as firms cannot recognize every first-period consumers and hence only learn the preference of a proportion. He finds that there is an inverse U-shaped relationship between quality of information and competitive profits, whereby the result of Fudenberg and Tirole (2000) is nested. Following Stole (2007), fiercer competition due to third-degree price discrimination creates incentives to commit to uniform pricing, that is, firms may seek to collude. In our paper, we focus on how potential misrecognition as in Esteves (2009, 2014) affects the scope for tacit collusion.

Combining the two strands, Liu and Serfes (2007) consider the impact of information on collusion. In their setup, however, information is publicly available and its quality is defined by the number of market segments. Then, an increase in information quality is equivalent to an increase of the number of perfectly distinguishable segments and hence the number of segment-specific prices. The authors analyze different collusive schemes. Their main finding is that collusion becomes harder to sustain as the number of market segments increases. Helfrich and Herweg (2016), which is closest to our work, consider two settings with perfect information in which price discrimination leads to either best-response symmetries or best-response asymmetries. Compared to the situation in which there is a ban on price discrimination, the authors show that third-degree price discrimination helps to fight collusion under both best-response symmetries and best-response asymmetries.

The findings from the theoretical literature on the relationship between collusion and third-degree price discrimination can thus be summarized as follows: When price discrimination is based on perfect information, theory predicts that third-degree price discrimination renders collusion less likely. Then, the implication for antitrust policy is that a legal ban on price third-degree price discrimination helps to fight tacit collusion. In most markets, however, information is not perfect. We contribute to this literature by relaxing the assumption of third-degree price discrimination under perfect information. By generalizing parts of the results in Helfrich and Herweg (2016), our analysis provides an important insight, namely that the outcomes can be fundamentally different when firms' information about consumer preferences is private and imperfect.

The rest of the paper is organized as follows. Section 1.2 introduces the model. In Section 1.3, we derive the relevant payoffs for the case that firms can price-discriminate as well as for the case of uniform pricing and determine the critical discount factors. Then, we compare sustainability of collusion in the two pricing regimes. In Section 1.4, we discuss the robustness of our results. Section 1.5 concludes.

1.2 Model

In this section, we first introduce the stage game, which is a static Bertrand pricing game of incomplete information. Thereafter, we describe the supergame, which is an infinite repetition of the stage game.

Stage Game

We consider a model of incomplete information developed in Armstrong (2006), which is a variant of Esteves (2014, 2004, chap. 2). Consider a linear city à la Hotelling (1929) with two symmetric firms, A and B, which are located at $\ell_A = 0$ and $\ell_B = 1$, respectively. Firms' marginal and fixed costs are normalized to zero. They compete in prices p_i with $i \in \{A, B\}$. We analyze two different pricing schemes: (i) third-degree price discrimination and (ii) uniform pricing.

Consumers of mass one are uniformly distributed along the line and derive a gross utility from buying the product, which is normalized to one. Additionally, they incur linear transport costs τ per unit of distance. Hence, when buying from firm i and paying price p_i, a consumer located at x derives net utility

$$U(x; p_i) = 1 - p_i - \tau|\ell_i - x|.$$

Consumers' outside option is normalized to zero.[5]

[5] Our results hold qualitatively if the outside option is located at each end node of the line as in Bénabou and Tirole (2016) (see the discussion below).

In our setup, there are two groups of consumers, L and R, consisting of the left and right half of the linear city, respectively. Given equal prices, consumers in group L (group R) prefer firm A (firm B). Synonymously, we can call consumers in group L (group R) loyal to firm A (firm B).[6]

Consumer types are private information. When facing a consumer, each firm i receives a noisy private signal $s_i \in \{s_L, s_R\}$ indicating the consumer's preference. Signal precision is measured by probability σ and drawn independently for each firm.[7] In other words, with probability σ, information about a consumer's preference is correctly passed on to a firm through the signal. With probability $1-\sigma$, the preference is misrecognized. We assume that the signal is weakly informative, that is, $\sigma \in [1/2, 1]$. Thereby, our setup nests the following two extreme cases: (i) the signal does not convey any information, that is, $\sigma = 1/2$, and (ii) market segments are perfectly distinguishable, that is, $\sigma = 1$.[8] The timing of the game is summarized below in detail.

1. Firms independently receive a private signal for any consumer along the linear city.[9] If a consumer is located at $x \in [0, 1/2]$, each firm receives signal s_L with probability σ and signal s_R with probability $1-\sigma$. If a consumer is located at $x \in (1/2, 1]$, each firm receives signal s_R with probability σ and signal s_L with probability $1-\sigma$.

2. Firms simultaneously set prices. Under price discrimination, firms can condition their prices on their private signal, whereas they set a single price under uniform pricing.

3. Consumers decide from which firm to buy, and payoffs are realized.

[6]As in Esteves (2014), the definition of brand loyalty is similar to Raju et al. (1990, p. 279), where "the degree of brand loyalty is defined to be the minimum difference between the prices of the two competing brands necessary to induce the loyal consumers of one brand to switch to the competing brand".

[7]In Section 1.4, we relax both the assumptions of independent and symmetric signals.

[8]The first case hence represents the classic model, whereas the second case corresponds to a segmented market with two distinguishable segments (see Liu and Serfes, 2007 and Helfrich and Herweg, 2016).

[9]In principle, an infinite family of random variables could lead to subtle measure theoretic issues. As the random variables are independent, the proposition of Andersen and Jessen guarantees the existence of such an infinite family (for the formal argument, see, e.g., Schmidt, 2009, Proposition 10.6.2 on p. 210). Another solution to these issues is an alternative interpretation of the model: A representative consumer could be randomly drawn from the linear city at the beginning of each stage game. Then, firms receive only one signal and compete for the single consumer.

As signals are private, firms do not know their competitor's payoff function. Hence, we consider a game of incomplete information. In order to solve the stage game, we use the notion of Bayesian Nash equilibrium. Our tie-breaking rule is the following: whenever a consumer values the outside option and a firm equally, she chooses the firm, and if she is indifferent between the firms, she chooses randomly.[10]

Dynamic Game

In order to study the scope for collusive behavior, we extend our setup to a game of infinite horizon. In the infinitely repeated game, the stage game described above is played in each period $t = 0, \ldots, \infty$. Firms are long-lived, that is, they play over the entire sequence of the infinitely repeated game. Expected payoff in period t is defined as the stage game payoff plus the discounted value of the stream of future payoffs determined by the continuation game strategy profile. Firms' common discount factor is $\delta \in (0, 1)$. Consumers are short-lived, that is, they only play for a single period and are replaced by a new cohort of consumers in the subsequent period.[11] As a consequence, intertemporal price discrimination is not possible. Their payoff is given by their net utility in the respective period. All players are payoff-maximizing. Consumers are perfectly informed. Hence, their payoff is deterministic.

As the stage game is Bayesian, we use the notion of perfect Bayesian equilibrium when analyzing the dynamic game. We refrain from explicitly stating the set of players' beliefs as part of the equilibrium description. In addition, as consumers are short-lived, firms cannot learn their preferences over time. The same holds true for beliefs regarding the signals of the competitor, as these are independent across periods.

Further, we assume that firms use grim-trigger strategies as defined in Friedman (1971) to support collusive outcomes. Thereby, we follow the related literature and can compare results. On the other hand, we want to focus on the impact of signal quality on the following trade-off for a firm: (i) long-term gains from collusive behavior compared to competitive outcomes against (ii) short-term gains by deviating unilaterally from collusive behavior. This seems plausible to us especially when thinking about tacit collusion without a certain punishment mechanism, where defection might lead to competition for an undetermined time horizon. Grim-trigger

[10] Ties are not outcome-relevant as the distribution of consumers is atomless.

[11] As argued in Section 1.4.1, asymmetric signal quality can be interpreted as relaxing the assumption of short-lived consumers.

strategies generate exactly this trade-off, as punishment coincides with competitive outcomes. If, instead, optimal penal codes as in Lambson (1987) and Abreu (1988) are employed in our setup, punishment payoffs become deterministic and finite. Then, firms still trade off gains from deviation against losses from punishment, but do not take into account competitive outcomes at all by construction.[12] The stationary strategy profile can be summarized as follows:

- In the starting period $t = 0$, each firm charges the collusive price. In any subsequent period $t = 1, \ldots, \infty$, each firm
  - charges the collusive price as long as it does not observe any other price in period $t - 1$ and
  - plays Bayesian Nash equilibrium strategies else.
- Consumers buy from the firm providing the highest net utility if it weakly exceeds the value of the outside option. If a consumer is indifferent between the two firms, she chooses randomly.

In order to verify whether the suggested strategy profile constitutes a perfect Bayesian equilibrium, we need to verify that the one-shot-deviation principle (OSDP) is satisfied (for a formal argument, see Hendon et al., 1996). Given firms' strategies and beliefs over consumers' preferences and the respective competitors private information, this is true if and only if the following inequality is satisfied in any period t:

$$\frac{\pi^c}{1-\delta} \geq \pi^d + \frac{\delta \pi^*}{1-\delta}, \tag{1.1}$$

where π^*, π^c, and π^d denote competitive (punishment) payoffs, collusive payoffs, and deviation payoffs, respectively. From this, it follows that the critical discount factor is defined by

$$\delta \geq \frac{\pi^d - \pi^c}{\pi^d - \pi^*} =: \bar{\delta}. \tag{1.2}$$

All things equal, a lower (higher) punishment or deviation payoffs facilitates collusion (makes collusion harder to sustain), that is, the set of discount factors which satisfy OSDP becomes larger (smaller). The opposite is true for the respective change in the collusive payoffs Put differently, lower (higher) gains from defecting (i.e., $\pi^d - \pi^c$) and higher (lower) losses from punishment (i.e., $\pi^c - \pi^*$) make collusion easier (harder) to sustain.

[12]See Appendix B for a characterization of an optimal penal code in our game.

Throughout the analysis, we focus on equilibria in which the market is covered, that is, all consumers along the line buy from one of the two firms. For this purpose, we impose the following assumption on consumers' transport costs:

Assumption 1.1. $\tau \in [0, 2/3]$.

Assumption 1.1, which is common in the related literature, guarantees that the market is fully served under uniform pricing.[13,14]

1.3 Analysis and Results

In this section, we derive the critical discount factors for the case of price discrimination and the uniform pricing case. From the comparison of these two cases, we provide policy implications for a ban on price discrimination.

1.3.1 Price Discrimination

In order to evaluate firms' ability to sustain collusion under price discrimination, we need to derive the payoffs under competition, deviation, and collusion. Firms want to condition their prices on the signal they receive as long as it is informative: After observing signal s_L, firm i charges $p_{i,L}$, and after observing s_R, it charges $p_{i,R}$. For demand under competition to be well-defined, suppose for now that given prices of firm B, it has to hold for firm A that $p_{B,L} \leq p_{A,R} \leq p_{A,L} \leq p_{B,R}$ and given prices of firm A, it has to hold for firm B that $p_{A,R} \leq p_{B,L} \leq p_{B,R} \leq p_{A,L}$. The intuition behind the restrictions is that, on the one hand, a firm does not find it profitable to charge lower prices from its loyal consumers than its rival. Neither it finds it profitable to charge lower prices from consumers that prefer the firm than from consumers that prefer its competitor. On the other hand, a firm cannot attract any consumer that is loyal to its competitor by charging a higher price. The remaining conditions are without loss of generality and can be specified differently. It will be shown later in this subsection that equilibrium prices indeed satisfy all conditions.

[13]For the case of price discrimination, the market is covered for larger values of the transport costs as prices tend to be lower. To ensure better comparability, we use the more restrictive upper bound on the transport-cost parameter.

[14]Instead, one could follow Bénabou and Tirole (2016) by assuming that the outside option is costly, that is, it is located at either end of the linear city. Then, Assumption 1.1 would not be needed. This, however, would not change our results qualitatively but make the comparison to the above mentioned literature less clean.

In order to derive expected demand of a firm conditional on its private signal, we need to distinguish all possible outcomes, where an outcome is characterized by a tuple (s_j, s_k) with $j, k \in \{L, R\}$, and where the first (second) element is the signal of firm A (firm B). Since signals are independently drawn for each consumer, firms can either receive identical signals ($j = k$) or different signals ($j \neq k$), that is, the set of possible signal realizations is given by $S := \{(s_L, s_L), (s_L, s_R), (s_R, s_L), (s_R, s_R)\}$. As firms condition their prices on their private signal, they have to take into account four different indifferent consumers, which determine the probability of winning a certain consumer or, equivalently, the market share in a segment, for any signal tuple. To this end, $\tilde{x}_1$ denotes the indifferent consumer for tuple (s_L, s_L), $\tilde{x}_2$ for (s_R, s_L), $\tilde{x}_3$ for (s_L, s_R), and $\tilde{x}_4$ for (s_R, s_R). Solving for each, we get

$$1 - p_{A,L} - \tau\tilde{x}_1 = 1 - p_{B,L} - \tau(1 - \tilde{x}_1) \Leftrightarrow \tilde{x}_1 = \frac{1}{2} - \frac{p_{A,L} - p_{B,L}}{2\tau},$$

$$1 - p_{A,L} - \tau\tilde{x}_2 = 1 - p_{B,R} - \tau(1 - \tilde{x}_2) \Leftrightarrow \tilde{x}_2 = \frac{1}{2} - \frac{p_{A,L} - p_{B,R}}{2\tau},$$

$$1 - p_{A,R} - \tau\tilde{x}_3 = 1 - p_{B,L} - \tau(1 - \tilde{x}_3) \Leftrightarrow \tilde{x}_3 = \frac{1}{2} - \frac{p_{A,R} - p_{B,L}}{2\tau},$$

and

$$1 - p_{A,R} - \tau\tilde{x}_4 = 1 - p_{B,R} - \tau(1 - \tilde{x}_4) \Leftrightarrow \tilde{x}_4 = \frac{1}{2} - \frac{p_{A,R} - p_{B,R}}{2\tau}.$$

Due to the restriction of the set of feasible prices above, it holds true that $\tilde{x}_1, \tilde{x}_3 \in [0, 1/2]$ and $\tilde{x}_2, \tilde{x}_4 \in [1/2, 1]$. For firm A, the probability of winning consumer $x \in L$ given (s_L, s_L) is equal to $2x_1$. In the same firm and segment, the probability of winning the consumer given (s_L, s_R) is equal to $2(x_2 - 1/2)$. In both cases, the winning probability is equivalent to the firm's expected market share in segment L. The remaining cases can be derived analogously.

The notion of Bayesian Nash equilibrium requires that firm i—after receiving a signal—updates its beliefs regarding the respective consumer's actual preference and regarding the signal of its competitor. As signal realizations are independent across firms and periods, the updating process is independent in each stage game. A firm's posterior belief that a consumer prefers firm A given signal s_L is

$$\Pr(L|s_L) = \frac{\Pr(s_L|L)\Pr(L)}{\Pr(s_L|L)\Pr(L) + \Pr(s_L|R)\Pr(R)} = \sigma,$$

which is equal to the precision of the signal due to symmetry. Conditional on this, firm i's posterior belief that firm j has received signal s_L is equal to the conditional probability of this event, namely σ. In the remaining cases, beliefs are updated analogously.

Then, firm A's expected demand conditional on receiving signal s_L can be derived as

$$\begin{aligned} D_A\left(p_{A,L}, p_{B,L}, p_{B,R}|s_L\right) &= \sigma\left(2\sigma\tilde{x}_1 + 1 - \sigma\right) + 2\sigma(1-\sigma)\left(\tilde{x}_2 - \frac{1}{2}\right) \\ &= \sigma\left(1 - \frac{p_{A,L} - \sigma p_{B,L} - (1-\sigma)p_{B,R}}{\tau}\right). \end{aligned} \tag{1.3}$$

Similarly, conditional on receiving signal signal s_R, firm A's expected demand can be derived as

$$\begin{aligned} D_A\left(p_{A,R}, p_{B,L}, p_{B,R}|s_R\right) &= (1-\sigma)\left(2\sigma\tilde{x}_3 + 1 - \sigma\right) + 2\sigma^2\left(\tilde{x}_4 - \frac{1}{2}\right) \\ &= 1 - \sigma - \frac{\sigma p_{A,R} - \sigma(1-\sigma)p_{B,L} - \sigma^2 p_{B,R}}{\tau}. \end{aligned} \tag{1.4}$$

Expected demand for firm B conditional on its signal realization can be derived analogously. In the following, we solve for the different

Competition

We start by analyzing the competitive payoffs, that is, the static Bayesian Nash equilibrium payoff of the stage game as defined in Section 1.2. These are used as punishment payoffs in the dynamic game.[15] The maximization problem of firm i is given as

$$\begin{aligned} \max_{p_{i,L}, p_{i,R}} \mathbb{E}\left[\pi_i\right] = {} & p_{i,L}\Pr(s_i = s_L)D_{i,L}\left(p_{A,L}, p_{B,L}, p_{B,R}|s_L\right) \\ & + p_{i,R}\Pr(s_i = s_R)D_{i,R}\left(p_{A,R}, p_{B,L}, p_{B,R}|s_R\right), \end{aligned} \tag{1.5}$$

where $\Pr(s_i = s_L) = \Pr(s_i = s_R) = 1/2$. Differentiating with respect to prices and solving the system of first-order conditions gives symmetric equilibrium prices of

$$p^*_{A,L} = p^*_{B,R} = \frac{2\tau}{1+2\sigma} \quad \text{and} \quad p^*_{A,R} = p^*_{B,L} = \frac{\tau}{\sigma(1+2\sigma)},$$

[15] The results from this part are equivalent to Armstrong (2006).

where $p^*_{A,R} < p^*_{A,L}$ and $p^*_{B,L} < p^*_{B,R}$ hold as long as the signal is informative. Then, price discrimination allows firms to set higher prices for those consumers who are signaled to be located more closely to their own location, that is, consumers with a higher willingness to pay for their product. Thereby, the market is segmented into four as in Fudenberg and Tirole (2000) under informative signals, although firms can only distinguish between two consumer groups. In each half, there are consumers served by their preferred firm, and consumers poached by the less preferred firm, i.e., $0 < \tilde{x}^*_1 < \tilde{x}^*_2 = \tilde{x}^*_3 = 1/2 < \tilde{x}^*_4 < 1 \ \forall \sigma \in (1/2, 1]$. The equilibrium payoff for each firm amounts to

$$\pi^* = \frac{\tau\left(1+4\sigma^2\right)}{2\sigma\left(1+2\sigma\right)^2}.$$

We observe that firms' payoffs are decreasing in the signal precision. As Esteves (2014) points out, an increase in signal precision has two opposing effects: On the one hand, misrecognition of consumers decreases, which means that a firm can charge more from its loyal consumers, while reducing the price to those consumers who are loyal to its rival. In other words, a firm can poach more effectively. On the other hand, since the rival behaves more aggressively as well when poaching loyal consumers, a firm optimally responds by reducing its prices. In this setup, it turns out that the latter effect (competition) outweighs the increase in prices due to reduction in misrecognition (information). Hence, competition is intensified with a rise in signal precision. As a result, for any $\sigma \in (1/2, 1]$, payoffs are strictly lower than static Bayesian Nash equilibrium payoffs under uniform pricing, as we have best-response asymmetries.

Collusion

Under full collusion, firms maximize industry profits by minimizing total transport costs, that is, firms divide the market in two and each firm serves its own turf. In our game, this allocation can only be induced by charging symmetric prices. As firms try to extract the maximal surplus from consumers net of transport costs, it is not optimal to attract consumers in the competitor's turf. Put differently, firms will not price-discriminate based on private information about consumers' preferences. Instead, they will set a single price for all consumers such that the marginal consumer located at 1/2 is indifferent between buying and not buying, that is, $1 - p^c - \tau|\ell_i - 1/2| = 0$. We summarize these considerations in the following lemma:

Lemma 1.1. *Collusive prices and payoffs are given by*

$$p^c = 1 - \frac{\tau}{2}$$

and

$$\pi^c = \frac{1}{2} - \frac{\tau}{4}.$$

We observe that price discrimination cannot lead to higher payoffs compared to uniform pricing, as firms can only distinguish two consumer groups.[16]

Deviation

In order to characterize the optimal deviation strategy, we need to define the following thresholds for τ:[17]

$$\tau_1 := \frac{2(1-\sigma)}{5-3\sigma}, \quad \tau_2 := \frac{2\sigma}{2+3\sigma}, \quad \text{and} \quad \tau_3 := \frac{2(1-\sigma)}{1+\sigma}.$$

It is easily checked that $\tau_1, \tau_2, \tau_3 \in [0, 2/3]$ for any $\sigma \in [1/2, 1]$ and that $\tau_2 \lesseqgtr \tau_3$ for $\sigma \lesseqgtr 1/\sqrt{2}$. The following lemma characterizes optimal deviation behavior:

Lemma 1.2. *The optimal deviation from collusive prices yields the following prices and payoffs, which are continuous and differentiable in both σ and τ:*

$$p^d_{A,L} = p^d_{B,R} = \begin{cases} 1 - \frac{3\tau}{2} & \textit{if } \tau \in [0, \tau_1] \\ \frac{1}{2} + \frac{\tau(3\sigma-1)}{4(1-\sigma)} & \textit{if } \tau \in (\tau_1, \tau_3], \\ 1 - \frac{\tau}{2} & \textit{if } \tau \in \left(\tau_3, \frac{2}{3}\right], \end{cases}$$

$$p^d_{A,R} = p^d_{B,L} = \begin{cases} 1 - \frac{3\tau}{2} & \textit{if } 0 \leq \tau \leq \tau_2, \\ \frac{1}{2} - \frac{\tau(3\sigma-2)}{4\sigma} & \textit{if } \tau_2 < \tau \leq \frac{2}{3}, \end{cases}$$

[16] In this setup, firms do not price-discriminate under collusion, which is also the case in Helfrich and Herweg (2016) and Liu and Serfes (2007) (with two segments). This is due to the fact that we only allow for a left and a right market, i.e., two signals. The present model could easily be extended to more signals, which would yield price discrimination also under collusion. At the same time, results would not change qualitatively (in particular, see the deviation incentives for low values of signal precision and transport costs below). For tractability reasons, we restrict our attention to two signals.

[17] The derivation of these thresholds is part of the proof of Lemma 1.2 in Appendix A.

and,

$$\pi^d = \begin{cases} 1 - \frac{3\tau}{2} & \text{if } \tau \in [0, \tau_1], \\ \frac{(3\tau(1+\sigma)+2(1-\sigma))^2 - 32\tau^2}{32\tau(1-\sigma)} & \text{if } \tau \in (\tau_1, \min\{\tau_2, \tau_3\}], \\ \frac{\tau}{8\sigma(1-\sigma)} + \frac{4(\tau+1)-15\tau^2}{32\tau} & \text{if } \sigma \in \left[\frac{1}{2}, \frac{1}{\sqrt{2}}\right] \wedge \tau \in \left(\tau_2, \tau_3\right], \\ \frac{2-3\tau+\sigma(2-\tau)}{4} & \text{if } \sigma \in (\frac{1}{\sqrt{2}}, 1] \wedge \tau \in (\tau_3, \tau_2], \\ \frac{\tau^2(2-\sigma)^2+4\sigma^2(\tau+1)+8\tau\sigma(1-\tau)}{32\tau\sigma} & \text{if } \tau \in \left(\max\{\tau_2, \tau_3\}, \frac{2}{3}\right]. \end{cases}$$

Proof. See Appendix A.

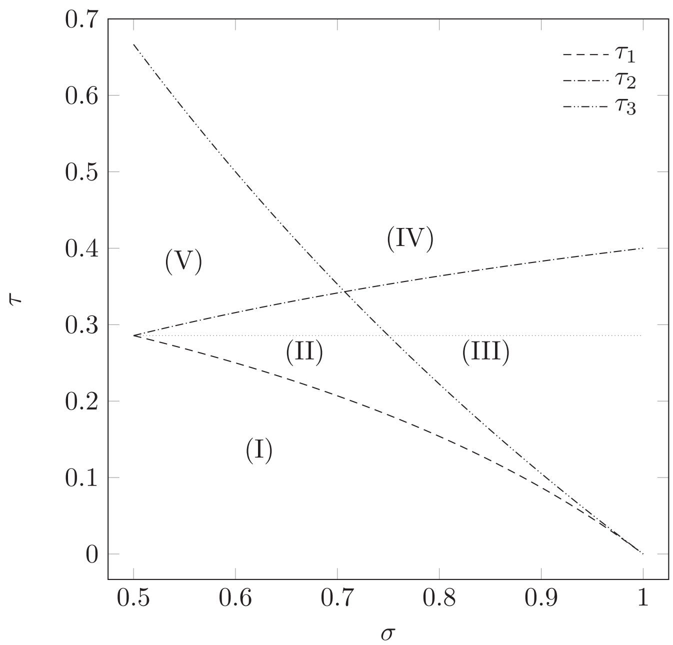

Figure 1.1: Characterization of deviation strategies.

Note: the dotted horizontal line at 2/7 gives the threshold below which a deviating firm wants to serve the whole market in the case of uniform pricing.

Corresponding to the cases in Lemma 1.2, Figure 1.1 divides all combinations of parameter values of σ and τ into regions I–V. Intuitively, the regions are divided corresponding to the following considerations: (i) does a deviating firm want to serve all consumers in its competitor's turf (I–III) or not (IV, V), and (ii) does it want to charge uniform prices (I), price-discriminate cautiously (II, V), or aggressively (III, IV). The first consideration is well known from the related literature on uniform pricing (see, e.g., Chang, 1991). The second consideration is unambiguous if information is perfect (see Helfrich and Herweg, 2016 and Liu and Serfes, 2007). Then, a deviator price-discriminates aggressively by charging the collusive price from its loyal consumers while poaching its competitor's loyal consumers with low prices. If information is imperfect, however, the quality of information plays an additional role. If signals are relatively noisy, the firm might misrecognize consumers' preferences, which can be costly. Then, it prefers to charge rather similar prices conditional on the signal received. Thereby, it avoids losing infra-margins by offering a relatively low price to a loyal customer as well as foregoing demand when offering a relatively high price to a disloyal consumer. If the signal is sufficiently reliable, however, misrecognition becomes less likely, and hence the firm prefers to act more aggressively by charging the collusive price to consumers it expects to be loyal and rather low poaching prices to consumers it expects to be loyal to its competitor. The behavior of a deviating firm in each region is explained in detail below.

In region I, transport costs are very low, and hence a deviating firm captures the entire market by setting a uniform price independent of σ. In region II, transport costs are still sufficiently low such that the deviator wants to capture all consumers, whereas it prefers to price-discriminate between the different groups depending on the signal it receives. To be precise, it still wants to charge a price independent of σ to its competitor's consumers, while the price it wants to charge to its own consumers rises in σ. This results in an increasing price difference between signals. In region III, the deviator still captures all consumers and wants to price its competitor's consumers as before. The relatively precise signal, however, makes it profitable for the deviator to charge the collusive price to consumers it expects to be loyal. In region IV, transport costs are high such that the firm finds it too costly to capture all of its competitor's consumers. As it can price-discriminate between signals, and signals are relatively precise, it still wants to charge the collusive price to its own consumers. The price difference, however, increases in σ, as the price it wants to charge to its competitor's consumers decreases in σ. In region V, transport costs are again high such that it is too costly for the deviating firm to serve all consumers whose signal indicates a preference for its competitor. It is too costly as well to charge the collusive price to consumers it expects to be loyal, as the signal is relatively noisy. This price, however, increases in σ and hence the price difference between the signals increases as well.

Critical Discount Factor

Using the payoffs derived in the three above scenarios, we can determine the critical discount factor $\bar{\delta} := \bar{\delta}(\sigma, \tau)$ as defined in Condition (1.2) as characterized in the following proposition:

Proposition 1.1. *When firms can price-discriminate, the critical discount factor $\bar{\delta}$ is a continuous and differentiable function of σ and τ with the following properties:*

(i) $\bar{\delta}$ is non-monotone in the signal quality such that $\partial\bar{\delta}/\partial\sigma < 0$ (> 0) holds for low (high) values of σ.

(ii) $\bar{\delta}$ is non-monotone in the transport costs such that $\partial\bar{\delta}/\partial\tau < 0$ (> 0) holds for low (high) values of τ.

Proof. See Appendix A.

Let us have a closer look at the intuition behind these findings. By construction, the collusive payoff is independent of signal quality, whereas the deviation payoff and the Bayesian Nash equilibrium payoff depend on it, as we can see from the analysis above. More precisely, for a given value of the transport-cost parameter, the deviation payoffs are weakly increasing in signal quality, as targeting consumers becomes easier. At the same time, Bayesian Nash equilibrium payoffs are falling in signal quality, as competition gets fiercer. Hence, increasing signal quality has opposing effects on the critical discount factor, as both the gains from deviation and the losses from punishment increase. For perfect signal quality, Helfrich and Herweg (2016) and Liu and Serfes (2007) find that the destabilizing effect dominates. As a consequence, collusion is harder to sustain under price discrimination than under uniform pricing, that is, $\bar{\delta}(1/2, \tau) < \bar{\delta}(1, \tau)$.

Now consider the case in which signal quality is imperfect. From a low level of signal precision, as precision increases the gain from defecting increases relatively slower than the loss from punishment. For the case with relatively low transport costs, this is intuitive. A deviating firm finds it profitable to capture the entire market irrespective of the signal precision (see region I). Meanwhile, competition is intensified as σ increases, and hence the loss from punishment increases.

The case in which transport costs are relatively high is more involved. On the one hand, punishment payoffs are decreasing as before. On the other hand, deviation payoffs increase in σ (see regions II, IV, V). As signal quality is relatively low, a deviating firm expects to misrecognize consumers often and hence price-discriminates cautiously, that is, the difference in prices conditional on signals is rather low. In our model, this misrecognition effect slows down the increase in deviation payoff relative to the increase in loss from punishment. As a result, collusion is facilitated.

From a high level of signal precision, as precision increases the misrecognition effect becomes less pronounced. The deviating firm price-discriminates more aggressively, that is, the difference in prices conditional on signals is rather high. Thereby, the increase in deviation payoffs outweighs the increase in loss from punishment impeding collusion. We thus shed light on the intermediate cases between uniform pricing ($\sigma = 1/2$) and price discrimination conditional on perfect information ($\sigma = 1$) and show that sustainability of collusion is non-monotonic in signal quality. Moreover, it turns out that there is a non-monotonic relationship between sustainability of collusion and transportation cost. The logic behind this result can be derived from Figure 1.1 similarly.

Proposition 1.1 provides new insights for competition policy. In our setup, an increase in signal precision leads to lower consumer prices under competition due to best-response asymmetries. If signals are perfect, both competitive prices and the likelihood of collusive behavior are lowest. Either effect benefits consumers. We know from the analysis above that an increase in signal quality from a relatively low level facilitates collusion. In this area, any policy that deregulates access to or usage of consumer data resulting in a gain in predictive power of firms' algorithms[18] can also support collusive behavior. In particular, regulators should be alarmed if such deregulation is demanded by the industry. While a single firm always gains from an increase in its predictive power, an increase of all firms' predictive power drives down competitive payoffs. Deregulation, however, might enable firms to coordinate their prices. From a relatively high level, an increase in predictive power impede collusive behavior. In this area, any policy concerned with consumer privacy that restricts predictive power of firms can come at the cost of collusive behavior.

[18] To a certain extent, it seems natural to assume a positive relation between the amount and variety of available data and predictive power. Yoganarasimhan (2017) provides evidence for this relation in the context of search queries. She finds that personalized search, especially long-term and across-session, helps to improve accuracy of suggested results significantly.

1.3.2 Uniform Prices

The above case nests the scenario in which firms are not allowed to price-discriminate, because outcomes are the same as in the situation in which signals are uninformative (i.e., $\sigma = 1/2$). Hence, punishment payoffs reduce to

$$\pi_u^* = \frac{\tau}{2},$$

collusive payoffs to

$$\pi_u^c = \pi^c = \frac{1}{2} - \frac{\tau}{4},$$

and deviation payoffs to

$$\pi_u^d = \begin{cases} 1 - \frac{3\tau}{2} & \text{if } 0 \leq \tau \leq \frac{2}{7}, \\ \frac{1}{8} + \frac{\tau}{32} + \frac{1}{8\tau} & \text{if } \frac{2}{7} < \tau \leq \frac{2}{3}. \end{cases}$$

Given these payoffs and Condition (1.2), it immediately follows that the critical discount factor is given as

$$\bar{\delta}_u = \begin{cases} \frac{2-5\tau}{4(1-2\tau)} & \text{if } 0 \leq \tau \leq \frac{2}{7}, \\ \frac{2-3\tau}{2+5\tau} & \text{if } \frac{2}{7} < \tau \leq \frac{2}{3}. \end{cases}$$

By construction, $\bar{\delta}_u$ is independent of σ. It decreases in the transport-cost parameter, that is, $\partial\bar{\delta}_u/\partial\tau < 0$, as established in Chang (1991).

1.3.3 Comparison

We can now compare the critical discount factors in the two scenarios, namely price discrimination and uniform prices. Profits are to a large extent affected differently by the possibility to price-discriminate. Figure 1.2 illustrates for all permissible parameter values of signal quality and transport costs when the two critical discount factors coincide.

When signal quality does not provide any information (i.e., $\sigma = 1/2$), price discrimination is not feasible. Hence, the critical discount factors are equal. When signal quality is perfect (i.e., $\sigma = 1$), we know from Liu and Serfes (2007) and Helfrich and Herweg (2016) that the linear city is divided into two distinguishable markets. Then, collusion is harder to sustain under price discrimination than uniform prices.

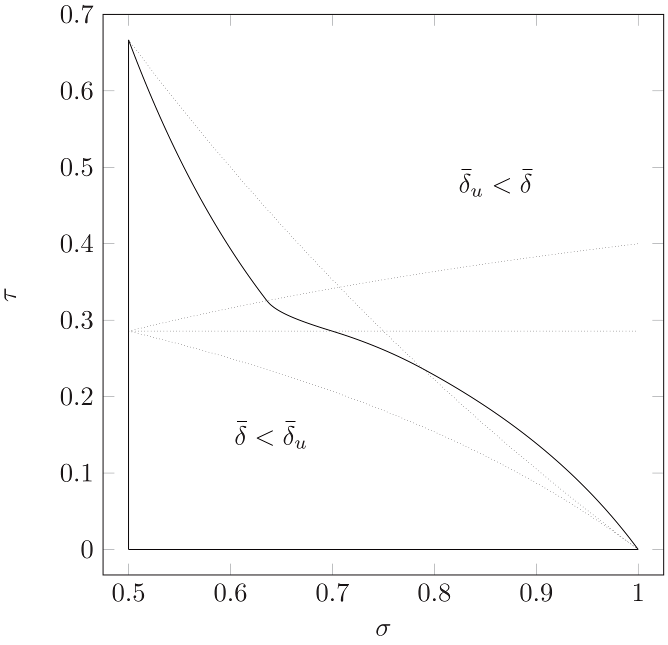

Figure 1.2: Comparison of the critical discount factors with and without price discrimination for all permissible parameter values.

Note: For those parameter combinations represented by the solid lines, the two critical discount factors coincide, that is, $\bar{\delta} = \bar{\delta}_u$. The dotted lines separate the different regions with respect to the deviation strategies for the cases with and without price discrimination.

For $\sigma \in (1/2, 1)$, there is a non-monotonic relationship of the critical discount factor in signal quality under price-discrimination as stated in Proposition 1.1. In particular, starting from $\sigma = 1/2$, the critical discount factor first decreases and then after a cut-off increases in signal quality. At the same time, the critical discount factor under uniform pricing remains unchanged. The corollary below immediately follows.

Corollary 1.1. *For any $\tau \in (0, 2/3)$, there exists a threshold $\tilde{\sigma}(\tau) \in (1/2, 1)$ such that for $\sigma = \tilde{\sigma}(\tau)$, we have $\bar{\delta} = \bar{\delta}_u$. Moreover, for any $\sigma \lesseqgtr \tilde{\sigma}(\tau)$, it holds true that $\bar{\delta} \lesseqgtr \bar{\delta}_u$.*

From the above corollary, we can derive the following policy implications. A ban on price discrimination facilitates collusion as in Liu and Serfes (2007) and Helfrich and Herweg (2016) as long as signal quality is relatively high. Else, we find that a ban on price discrimination hinders collusion.

1.4 Robustness

In this section, we test the robustness of our main results by relaxing some of the assumptions imposed on the signal structure. In particular, we consider the cases of asymmetric signal quality and correlated signals.

1.4.1 Asymmetric Signal Quality

In this subsection, we relax the assumption of symmetric information accuracy. Similar to Esteves (2014), we assume that the signal a firm receives is a function of the respective consumer's preference. We consider the following case: The signal a firm receives when facing a loyal consumer is weakly more precise than the signal it receives when facing a disloyal consumer. Let us denote the probability that the signal is correct if the consumer is loyal (disloyal) by σ_1 (σ_2) and assume that $1/2 \leq \sigma_2 \leq \sigma_1$. Thereby, we address the concern that a firm might know most about the characteristics of its loyal consumers and hence should be able to identify these with higher probability, which can also be interpreted as a short-cut approach to modeling consumers who live for more than a single period and firms which have access to an imperfect tracking technology similar to the one defined in Colombo (2016).

Consider the set S, which contains all possible signal tuples (s_j, s_k), and let $f(s_j, s_k|x \in l)$ denote the joint probability density function conditional on consumer x's preference $l \in \{L, R\}$. We impose the following assumption on the functional form of $f(\cdot)$:

Assumption 1.2.

$$f(s_j, s_k | x \in L) = \begin{cases} \sigma_1\sigma_2 & \textit{for} \quad (s_L, s_L), \\ \sigma_1(1-\sigma_2) & \textit{for} \quad (s_L, s_R), \\ (1-\sigma_1)\sigma_2 & \textit{for} \quad (s_R, s_L), \\ (1-\sigma_1)(1-\sigma_2) & \textit{for} \quad (s_R, s_R), \end{cases}$$

and

$$f(s_j, s_k | x \in R) = \begin{cases} (1-\sigma_2)(1-\sigma_1) & \textit{for} \quad (s_L, s_L), \\ (1-\sigma_2)\sigma_1 & \textit{for} \quad (s_L, s_R), \\ \sigma_2(1-\sigma_1) & \textit{for} \quad (s_R, s_L), \\ \sigma_2\sigma_1 & \textit{for} \quad (s_R, s_R). \end{cases}$$

The density function under Assumption 1.2 is well-defined and nests the extreme case of symmetric signals (for $\sigma_1 = \sigma_2 = \sigma$). As before, after observing signal s_i, firm i has to infer on the consumer's actual preference and on the signal s_j received by its competitor. Suppose firm i receives signal s_L. Applying Bayes' rule, its updated belief that a consumer prefers firm A, and its competitor has received the same signal is

$$\Pr(s_L, L | s_L) = \frac{f(s_L, s_L | L) \Pr(L)}{f_{s_i}(s_L | L) \Pr(L) + f_{s_i}(s_L | R) \Pr(R)} = \frac{\sigma_1\sigma_2}{1 + \sigma_1 - \sigma_2},$$

where f_{s_i} denotes the marginal distribution of s_i. In the remaining cases, beliefs are updated analogously. Given beliefs, we can specify each firm's maximization problem and determine mutual best responses similarly to the main analysis (see the Appendix). Firms optimally set prices equal to

$$p^*_{A,L} = p^*_{B,R} = \frac{2\tau\sigma_1}{\sigma_2 + 2\sigma_1\sigma_2} \quad \text{and} \quad p^*_{A,R} = p^*_{B,L} = \frac{\tau}{\sigma_2 + 2\sigma_1\sigma_2},$$

where $p^*_{A,R} < p^*_{A,L}$ and $p^*_{B,L} < p^*_{B,R}$ as long as the signal is informative. The resulting equilibrium payoff for each firm amounts to

$$\pi^* = \frac{\tau(1 + 4\sigma_1^2)}{2\sigma_2(1 + 2\sigma_1)^2}.$$

These payoffs serve as punishment payoffs in the dynamic game as defined in Section

1.2 and equal those derived in Section 1.3 for $\sigma_1 = \sigma_2 = \sigma$ by construction. The intuition from the symmetric case can be misleading here by suggesting a similar relation between punishment payoffs and average signal quality. In fact, we observe that the more asymmetric the signal quality is, the higher the punishment payoffs are—namely, they rise in σ_1 and fall in σ_2. When σ_1 increases, firms can better identify their loyal consumers allowing for an increase of their price. On the other hand, when σ_2 decreases, firms more often misrecognize their disloyal consumers leading to less aggressive poaching, as costly mistakes become more likely. Overall, signal asymmetry softens competition. As deviation payoffs are affected in the same way (see the proof of Proposition 1.2), it is not clear from an ex-ante perspective how signal asymmetry translates into the critical discount factor $\bar{\delta}_{\text{asy}}$. The following proposition summarizes our result:

Proposition 1.2. *For any $\sigma_2 < \sigma_1$, the critical discount factor $\bar{\delta}_{\text{asy}}$ is strictly larger compared to both cases of symmetric signal quality $\sigma = \sigma_1$ and $\sigma = \sigma_2$. In addition, $\bar{\delta}_{\text{asy}}$ is non-monotone in σ_1 and σ_2.*

Proof. See Appendix A.

By construction, the critical discount factors in the symmetric and asymmetric case are equivalent for $\sigma_1 = \sigma_2 = \sigma \in [1/2, 1]$. Starting from $\sigma_1 = \sigma_2 = 1/2$, we can see from the proof of Proposition 1.2 that a marginal increase in both dimensions leads to a marginal reduction of $\bar{\delta}_{\text{asy}}$. From continuity and Proposition 1.1, it immediately follows that we can always find $1/2 \leq \sigma_2 < \sigma_1$, such that $\bar{\delta}_{\text{asy}} < \bar{\delta}$. Then, collusion is more likely in terms of set inclusion if price discrimination is permitted compared to the case of no price discrimination. The corollary below summarizes this argument:

Corollary 1.2. *For $\sigma_2 < \sigma_1$, a ban on price discrimination helps to fight collusion if signals are sufficiently noisy.*

1.4.2 Correlated Signals

In this subsection, we relax the assumption of independent signal realizations by allowing for positive correlation of the private signals received by the firms.[19] This

[19] As random variables are correlated, the proposition of Andersen and Jessen does not apply. In order to avoid subtle measure theoretic issues, consider the representative consumer as alternative interpretation of the model, which is outlined in Footnote 9.

is natural, as firms might, for instance, use similar algorithms in order to infer on consumer types from available data or obtain consumer data from similar sources.

Consider the set of all signal tuples S and let $g(s_j, s_k|x \in l)$ denote the joint probability density function conditional on consumer x's preference $l \in \{L, R\}$. We assume the following functional form of $g(\cdot)$:

Assumption 1.3.

$$g(s_j, s_k|x \in L) = \begin{cases} \sigma^2 + \gamma & \textit{for} \quad (s_L, s_L), \\ \sigma(1-\sigma) - \gamma & \textit{for} \quad (s_L, s_R), (s_R, s_L), \\ (1-\sigma)^2 + \gamma & \textit{for} \quad (s_R, s_R), \end{cases}$$

and

$$g(s_j, s_k|x \in R) = \begin{cases} (1-\sigma)^2 + \gamma & \textit{for} \quad (s_L, s_L), \\ \sigma(1-\sigma) - \gamma & \textit{for} \quad (s_L, s_R), (s_R, s_L), \\ \sigma^2 + \gamma & \textit{for} \quad (s_R, s_R), \end{cases}$$

where $\gamma \in [0, \sigma(1-\sigma)]$ *measures the degree of correlation.*

The density function under Assumption 1.3 is well-defined and nests the two extreme cases: (i) independent signals (for $\gamma = 0$) and (ii) perfectly correlated signals (for $\gamma = \sigma(1-\sigma)$). The second case is equivalent to a model with imperfect public information about consumer preferences. It is easily checked that the interval $[0, \sigma(1-\sigma)]$ is non-empty for $\sigma \in [1/2, 1)$. In the following, we solve for the Bayesian Nash equilibrium of the stage game. As before, after observing signal s_i, firm i has to infer on the consumer's actual preference and on the signal of its competitor. For illustration, suppose that firm i receives signal s_L. Applying Bayes' rule, its posterior belief that a consumer prefers firm A, and firm j receives the same signal is

$$\Pr(s_L, L|s_L) = \frac{g(s_L, s_L|L)\Pr(L)}{g_{s_i}(s_L|L)\Pr(L) + g_{s_i}(s_L|R)\Pr(R)} = \sigma^2 + \gamma,$$

where g_{s_i} denotes the marginal distribution of s_i. In the remaining cases, beliefs are updated similarly. Given beliefs, we can specify each firm's maximization problem and determine mutual best responses analogously to the main analysis (see the Appendix). Firms optimally set prices equal to

$$p^*_{A,L} = p^*_{B,R} = \frac{\tau(\gamma + 2\sigma^2)}{\sigma(2\gamma + \sigma + 2\sigma^2)} \quad \text{and} \quad p^*_{A,R} = p^*_{B,L} = \frac{\tau(\gamma + \sigma)}{\sigma(2\gamma + \sigma + 2\sigma^2)},$$

where $p^*_{A,L} < p^*_{A,R}$ when the signal is informative. Resulting equilibrium payoffs for each firm are

$$\pi^* = \frac{\tau\left(2\gamma^2 + 2\gamma\sigma(2\sigma+1) + 4\sigma^4 + \sigma^2\right)}{4\sigma\left(2\gamma + 2\sigma^2 + \sigma\right)^2}.$$

These payoffs are the punishment payoffs in the dynamic game as defined in Section 1.2. By construction, punishment payoffs are equal to those derived in Section 1.3 for $\gamma = 0$. Furthermore, we observe that these payoffs fall as γ is rising, that is, gains from collusion are higher. As collusive prices are set uniformly and hence optimal deviation only depends on a firm's private signal, collusive and deviating payoffs remain unchanged compared to the symmetric-signal case. We therefore arrive at the following proposition:

Proposition 1.3. *For any $\gamma > 0$, the critical discount factor $\bar{\delta}_{\text{cor}}$ is strictly lower compared to the case of independent signal quality σ. In addition, $\bar{\delta}_{\text{cor}}$ is non-monotone in σ.*

Proof. See Appendix A.

At the lower and upper bound of σ, the cases of correlated and independent signals are equivalent by construction and hence the critical discount factors are equal. The following corollary directly results from Propositions 1.1 and 1.3:

Corollary 1.3. *For any $\gamma > 0$, the probability that a ban on price discrimination facilitates collusion is strictly lower compared to the case of independent signal quality σ. Furthermore, the difference strictly increases in γ.*

1.5 Conclusion

The use of big data—especially consumer data—for pricing strategies has substantially increased in recent times. Big data predictions of consumer preferences have been improving tremendously. Imprecision, however, is still an important factor when firms make their pricing decisions.

In this paper, we focus on the impact of data-driven price-discrimination strategies on the scope for tacit collusion. We find enhanced prediction of consumer preferences results in a U-shaped effect on firms' ability to sustain collusion. Compared to uniform pricing, we find that for low levels of predictive capabilities, collusion is easier to sustain under price discrimination. For sufficiently high levels, we find that collusion is harder to sustain under a discriminatory pricing than under uniform pricing. Thereby, potential misrecognition of consumers plays a crucial role.

Thereby, we provide the following policy implications. Although not central to designing data policy, data regulation should also take into account adverse effects on competition. In particular, deregulation of access to or usage of consumer data facilitates coordinated behavior of firms as long as initial predictions of consumer preferences are weak. In contrast, for relatively strong predictions, policies intending to restrict access to and usage of consumer data facilitate coordinated behavior among firms. Moreover, the effect of a legal ban on price discrimination on firms' ability to collude crucially depends on the quality of predictions. On a more general note and related to the above-mentioned aspect, one may argue that when the exchange of consumer data leads to higher signal precision towards perfect information, competition authorities should be less concerned with regard to collusive activity than in the case in which firms exchange data on prices, demands, etc. At the same time, the model we employ does not allow to draw conclusions with regard to welfare, as we do not take into account consumer preferences for privacy or other adverse effects due to discrimination of consumers.

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Appendix A: Proofs

Proof of Lemma 1.2. Without loss of generality, suppose that firm B sets the collusive price and firm A deviates unilaterally. As firm B charges p^c regardless of its signal, we have both $\tilde{x}_1 = \tilde{x}_2$ and $\tilde{x}_3 = \tilde{x}_4$. Substituting this into Equations (1.3) and (1.4), firm A expects its demand conditional on receiving signal s_L to be

$$D_{A,L} = \sigma + 2\left(1-\sigma\right)\left(\tilde{x}_1 - \frac{1}{2}\right), \tag{1.6}$$

and its demand conditional on receiving signal s_R to be

$$D_{A,R} = (1-\sigma) + 2\sigma\left(\tilde{x}_3 - \frac{1}{2}\right). \tag{1.7}$$

Then, the maximization problem of firm A is given as

$$\max_{p^d_{A,L}, p^d_{A,R}} \mathbb{E}\left[\pi^d_A\right] = \frac{1}{2}\left(p^d_{A,L} D_{A,L} + p^d_{A,R} D_{A,R}\right),$$

with $p_{B,L} = p_{B,R} = p^c$. Taking first order conditions with respect to firm A's deviation prices, we get inner solutions

$$p^{d^*}_{A,L} = \frac{1}{2} + \frac{\tau(3\sigma-1)}{4(1-\sigma)} \quad \text{and} \quad p^{d^*}_{A,R} = \frac{1}{2} - \frac{\tau(3\sigma-2)}{4\sigma}.$$

Using these, we make the following observations:

- $\tau > \dfrac{2(1-\sigma)}{5-3\sigma} =: \tau_1 \implies \tilde{x}_1 < 1,$
- $\tau > \dfrac{2\sigma}{2+3\sigma} =: \tau_2 \implies \tilde{x}_3 < 1,$
- $\tau < \dfrac{2(1-\sigma)}{1+\sigma} =: \tau_3 \implies p^{d^*}_{A,L} < p^c,$

where $p^c = 1 - \tau/2$. Thereby, it holds that $\tau_3 > \tau_2$ if and only if $\sigma < 1/\sqrt{2}$. Consequently, for $\sigma < 1/\sqrt{2}$, we obtain the order of parameters $0 < \tau_1 < \tau_2 < \tau_3 < 2/3$. On the other hand, for $\sigma > 1/\sqrt{2}$, we obtain the order of parameters $\tau_1 < \tau_3 < \tau_2 < 2/3$. In the following, we determine the optimal deviation behavior of firm A conditional on τ by distinguishing the following five cases:

Case (i): For $\tau \leq \tau_1$, we infer from our observations above that firm A optimally sets prices such that $\tilde{x}_1 = \tilde{x}_3 = 1$ in order to take over the whole market, that is,

$p^d_{A,L} = p^d_{A,R} = 1 - 3\tau/2$. Thereby, its conditional expected demand as defined in Equations (1.6) and (1.7) is equal to 1/2 regardless of the signal. Then, the expected payoff from deviating is given by

$$\pi^d_A = 1 - \frac{3\tau}{2}.$$

Case (ii): For $\tau_1 < \tau \leq \min\{\tau_2, \tau_3\}$, we infer from our observations above that firm A optimally sets prices such that $1/2 < \tilde{x}_1 < \tilde{x}_3 = 1$, that is, $p^d_{A,L} = p^{d^*}_{A,L}$ and $p^d_{A,R} = 1 - 3\tau/2$. Substituting this into Equations (1.6) and (1.7), we get an expected deviation payoff of

$$\pi^d_A = \frac{(3\tau(1+\sigma) + 2(1-\sigma))^2 - 32\tau^2}{32\tau(1-\sigma)}.$$

Case (iii): Suppose $\sigma \leq 1/\sqrt{2}$. For $\tau_2 < \tau \leq \tau_3$, we infer from our observations above that firm A optimally sets prices such that $\tilde{x}_1, \tilde{x}_3 < 1$, that is, $p^d_{A,L} = p^{d^*}_{A,L}$ and $p^d_{A,R} = p^{d^*}_{A,L}$. Substituting this into Equations (1.6) and (1.7), we get an expected deviation payoff of

$$\pi^d_A = \frac{\tau}{8\sigma(1-\sigma)} + \frac{4(\tau+1) - 15\tau^2}{32\tau}.$$

Case (iv): For now suppose $\sigma > 1/\sqrt{2}$. For $\tau_3 < \tau \leq \tau_2$, we infer from our observations above that firm A optimally sets prices such that $\tilde{x}_1 < \tilde{x}_3 = 1$, that is, $p_{A,L} = p^c$ and $p^d_{A,R} = 1 - 3\tau/2$. By Assumption 1.1, firm A does not find it profitable to charge more than p^c from its loyal consumers as long as firm B uniformly charges p^c. Substituting this into Equations (1.6) and (1.7), we get an expected deviation payoff of

$$\pi^d_A = \frac{2 - 3\tau + \sigma(2-\tau)}{4}.$$

Case (v): For $\tau > \max\{\tau_2, \tau_3\}$, we infer from our observations above that firm A optimally sets prices such that $\tilde{x}_1, \tilde{x}_3 < 1$, that is, $p_{A,L} = p^c$ and $p^d_{A,R} = p^{d^*}_{A,L}$. For the same reason as before, by Assumption 1.1, firm A charges p^c from its loyal consumers as long as firm B uniformly charges p^c. Substituting this into Equations (1.6) and (1.7), we get an expected deviation payoff of

$$\pi^d_A = \frac{\tau^2(2-\sigma)^2 + 4\sigma^2(\tau+1) + 8\tau\sigma(1-\tau)}{32\tau\sigma}.$$

Now, it is straightforward to check that for both prices and deviation payoffs their respective left-hand and right-hand limits for σ and τ approaching the bounds of Case (i)–(iv) from above are equal. Hence, they are continuous.

Further, it is straightforward to check that there are no kinks in both prices and deviation payoffs since the respective left-hand and right-hand limits of their derivatives for σ and τ approaching the bounds of Cases (i)–(iv) are equal. Hence, they are differentiable. □

Proof of Proposition 1.1. Taking the collusive payoffs from Lemma 1.1, the deviation payoffs from Lemma 1.2 and the punishment payoffs as given in Section 1.3.1, we can solve for the critical discount factor as defined in Condition (1.2). As only the functional form of the deviation payoff is changing with τ, we distinguish the five cases as defined in the proof of Lemma 1.2, that is:

Case (i): For $\tau \leq \tau_1$, we get

$$\bar{\delta} = \frac{\sigma(2\sigma+1)^2(2-5\tau)}{(2\sigma+1)^2(4\sigma-6\sigma\tau)-2\left(4\sigma^2+1\right)\tau}.$$

Case (ii): For $\tau_1 < \tau \leq \min\{\tau_2, \tau_3\}$, we get

$$\bar{\delta} = \frac{2\sigma(2\sigma+1)^2\left((\sigma(3\tau-2)+3\tau+2)^2+(8\tau-16)(\tau-\sigma\tau)-32\tau^2\right)}{2\sigma(2\sigma+1)^2\left((\sigma(3\tau-2)+3\tau+2)^2-32\tau^2\right)-32\left(4\sigma^2+1\right)\tau(\tau-\sigma\tau)}.$$

Case (iii): Suppose $\sigma \leq 1/\sqrt{2}$. For $\tau_2 < \tau \leq \tau_3$, we get

$$\bar{\delta} = \frac{\left((\sigma(3\tau-2)+3\tau+2)^2+(8\tau-16)(\tau-\sigma\tau)-32\tau^2\right)}{\left((\sigma(3\tau-2)+3\tau+2)^2-32\tau^2\right)-32\left(4\sigma^2+1\right)\tau(\tau-\sigma\tau)}.$$

Case (iv): Now suppose $\sigma > 1/\sqrt{2}$. For $\tau_3 < \tau \leq \tau_2$, we get

$$\bar{\delta} = \frac{\sigma(2\sigma+1)^2(\sigma(\tau-2)+2\tau)}{(\sigma(\sigma(4\sigma(\sigma+4)+21)+3)+2)\tau-2\sigma(\sigma+1)(2\sigma+1)^2}.$$

Case (v): For $\tau > \max\{\tau_2, \tau_3\}$, we get

$$\bar{\delta} = \frac{(2\sigma+1)^2(\sigma(\tau+2)-2\tau)^2}{C},$$

where $C := 4\tau\left(\sigma^2(2\sigma+3)^2+\sigma\tau\right)+\left(\sigma^2(4(\sigma-11)\sigma-95)-12\right)\tau^2+4\sigma^2(2\sigma+1)^2+8\sigma\tau$. From continuity and differentiability of all payoff functions entering Condition (1.2)—namely π^*, π^c, π^d—continuity and differentiability of $\bar{\delta}$ with respect to σ and

τ immediately follows.

In order to do comparative statics, we take the derivative of $\bar{\delta}$ as defined in Condition (1.2) with respect to σ, that is

$$\frac{\partial \bar{\delta}}{\partial \sigma} = \frac{\frac{\partial \pi^d}{\partial \sigma}\left(\pi^c - \pi^*\right) + \frac{\partial \pi^*}{\partial \sigma}\left(\pi^d - \pi^c\right)}{\left(\pi^d - \pi^*\right)^2}.$$

We observe that

$$\frac{\partial \bar{\delta}}{\partial \sigma} \gtrless 0 \Leftrightarrow \frac{\partial \pi^d}{\partial \sigma}\left(\pi^c - \pi^*\right) + \frac{\partial \pi^*}{\partial \sigma}\left(\pi^d - \pi^c\right) \gtrless 0.$$

Exploiting this, we show that $\partial\bar{\delta}/\partial\sigma|_{\sigma=1/2} < 0$ and $\partial\bar{\delta}/\partial\sigma|_{\sigma=1} > 0$ in all relevant cases. Figure 1.1 nicely illustrates which parameter ranges of τ have to be considered for the respective extreme value of σ. For $\sigma = 1/2$, we have $\tau_1 = \tau_2 = 2/7 < \tau_3 = 2/3$. For $\sigma = 1$, we have $\tau_1 = \tau_3 = 0 < \tau_2 = 2/5$. We obtain the following:

- If $\sigma = 1/2$, we observe that
  - for $\tau \in \left(0, \frac{2}{7}\right]$, $\frac{\partial \bar{\delta}}{\partial \sigma}|_{\sigma=\frac{1}{2}} = \frac{4\tau(5\tau-2)}{(4-8\tau)^2} < 0$,
  - for $\tau \in \left(\frac{2}{7}, \frac{2}{3}\right]$, $\frac{\partial \bar{\delta}}{\partial \sigma}|_{\sigma=\frac{1}{2}} = -\frac{32\tau^2}{(5\tau+2)^2} < 0$.
- If $\sigma = 1$, we observe that
  - for $\tau \in \left(0, \frac{2}{5}\right]$, $\frac{\partial \bar{\delta}}{\partial \sigma}|_{\sigma=1} = \frac{9(3\tau-2)}{46\tau-36} > 0$,
  - for $\tau \in \left(\frac{2}{5}, \frac{2}{3}\right]$, $\frac{\partial \bar{\delta}}{\partial \sigma}|_{\sigma=1} = \frac{24(\tau-2)\tau(\tau(149\tau-4)-108)}{(\tau(143\tau-108)-36)^2} > 0$.

Hence, $\bar{\delta}$ is non-monotonic with respect to σ.

In order to do comparative statics of $\bar{\delta}$ with respect to τ, we apply the implicit function theorem to the binding case of Inequality (1.1). We get

$$\frac{\partial \bar{\delta}}{\partial \tau} = \frac{\frac{\partial}{\partial \tau}\left(\pi^d - \pi^c\right) + \frac{\partial}{\partial \tau}\left(\pi^* - \pi^d\right)\bar{\delta}}{\pi^d - \pi^*}.$$

Exploiting that $\pi^d > \pi^*$, the sign of the above expression only depends on the sign of the numerator. It is straightforward to verify that the numerator is strictly negative in Case (i)–(iv) as defined in the proof of Lemma 1.2. Only in Case (v) the sign of the numerator can change. Solving for τ, we get

$$\left(\frac{\partial \pi^d}{\partial \tau} - \frac{\partial \pi^c}{\partial \tau}\right)\left(1 - \bar{\delta}\right) + \bar{\delta}\frac{\partial\left(\pi^* - \pi^c\right)}{\partial \tau} < 0 \Leftrightarrow \tau < \frac{2\sigma(2\sigma+1)^2}{\sigma(4\sigma(3\sigma+5)-5)+2} =: \tilde{\tau}.$$

We observe that $\tilde{\tau} \in (\max\{\tau_2, \tau_3\}, 2/3)$. Given this, we conclude that for $\tau \in (\max\{\tau_2, \tau_3\}, \tilde{\tau})$, the numerator is negative and hence it holds true that $\partial\bar{\delta}/\partial\tau < 0$. For $\tau \in (\tilde{\tau}, 2/3]$, the numerator is positive and hence it holds true that $\partial\bar{\delta}/\partial\tau > 0$. Finally, the numerator is zero at $\tau = \tilde{\tau}$ and hence it holds true that $\partial\bar{\delta}/\partial\tau = 0$. □

Proof of Proposition 1.2. As payoffs under collusion remain unchanged, we are left with determining punishment and deviation payoffs. Then, we compute the critical discount factor $\bar{\delta}_{\text{asy}}$. Finally, we show that the critical discount factor is always increasing in signal asymmetry compared to the symmetric case.

Lets first determine punishment payoffs. Given beliefs as derived in Section 1.4.1, we obtain expected demand of firm A conditional on receiving signals s_L and s_R, respectively, of

$$D_A(p_{A,L}, p_{B,L}, p_{B,R}|s_L) = \frac{1}{\sigma_1 + 1 - \sigma_2} \times \left(2\sigma_1\sigma_2\tilde{x}_1 + \sigma_1(1-\sigma_2) + 2(1-\sigma_1)\sigma_2\left(\tilde{x}_2 - \frac{1}{2}\right)\right) \quad (1.8)$$

and

$$D_A(p_{A,L}, p_{B,L}, p_{B,R}|s_R) = \frac{1}{\sigma_2 + 1 - \sigma_1} \times \left(2(1-\sigma_1)\sigma_2\tilde{x}_3 + (1-\sigma_1)(1-\sigma_2) + 2\sigma_2\sigma_1\left(\tilde{x}_4 - \frac{1}{2}\right)\right), \quad (1.9)$$

with $\tilde{x}_1$–$\tilde{x}_4$ referring to the indifferent consumers as defined in the main analysis. Firm A's maximization problem is then defined as in Equation (1.5). Firm B's maximization problem is determined analogously. Solving first-order conditions with respect to prices simultaneously, we get optimal prices

$$p^*_{A,L} = p^*_{B,R} = \frac{2\tau\sigma_1}{\sigma_2 + 2\sigma_1\sigma_2} \quad \text{and} \quad p^*_{A,R} = p^*_{B,L} = \frac{\tau}{\sigma_2 + 2\sigma_1\sigma_2},$$

where $p^*_{A,R} < p^*_{A,L}$ and $p^*_{B,L} < p^*_{B,R}$ as long as the signal is informative. The resulting equilibrium payoff for each firm amounts to

$$\pi^* = \frac{\tau(1 + 4\sigma_1^2)}{2\sigma_2(1 + 2\sigma_1)^2}.$$

Next, lets determine deviation payoffs. Without loss of generality, suppose that firm B sets the collusive price and firm A deviates unilaterally. As firm B charges p^c

regardless of its signal, we have both $\tilde{x}_1 = \tilde{x}_2$ and $\tilde{x}_3 = \tilde{x}_4$. Substituting this into Equations (1.8) and (1.9), firm A expects its demand conditional on receiving signal s_L to be

$$D_{A,L} = \frac{1}{\sigma_1 + 1 - \sigma_2}\left(\sigma_1 + 2\left(1 - \sigma_1\right)\left(\tilde{x}_1 - \frac{1}{2}\right)\right), \tag{1.10}$$

and its demand conditional on receiving signal s_R to be

$$D_{A,R} = \frac{1}{\sigma_2 + 1 - \sigma_1}\left((1 - \sigma_2) + 2\sigma_2\left(\tilde{x}_3 - \frac{1}{2}\right)\right). \tag{1.11}$$

Then, the maximization problem of firm A is given as

$$\max_{p^d_{A,L}, p^d_{A,R}} \mathbb{E}\left[\pi^d_A\right] = \frac{1}{2}\left(\frac{p^d_{A,L}D_{A,L}}{\sigma_1 + 1 - \sigma_2} + \frac{p^d_{A,R}D_{A,R}}{\sigma_2 + 1 - \sigma_1}\right),$$

with $p_{B,L} = p_{B,R} = p^c$. Taking first order conditions with respect to firm A's deviation prices, we get inner solutions

$$p^{d^*}_{A,L} = \frac{1}{2} + \frac{\tau(3\sigma_1 - 1)}{4(1 - \sigma_1)} \quad \text{and} \quad p^{d^*}_{A,R} = \frac{1}{2} - \frac{\tau(3\sigma_2 - 2)}{4\sigma_2}.$$

Using these, we make the following observations:

- $\tau > \dfrac{2(1-\sigma_1)}{5 - 3\sigma_1} =: \tau_1 \implies \tilde{x}_1 < 1,$
- $\tau > \dfrac{2\sigma_2}{2 + 3\sigma_2} =: \tau_2 \implies \tilde{x}_3 < 1,$
- $\tau < \dfrac{2(1-\sigma_1)}{1 + \sigma_1} =: \tau_3 \implies p^{d^*}_{A,L} < p^c,$

where $p^c = 1 - \tau/2$. The thresholds are ordered as $\tau_1 < \tau_2 < \tau_3$ if $\sigma_1 < (1 + \sigma_2)/(1 + 2\sigma_2)$ and $\sigma_2 < 1/\sqrt{2}$. Else, thresholds are ordered as $\tau_1 < \tau_3 < \tau_2$. In the following, we determine the optimal deviation behavior of firm A conditional on τ by distinguishing the following five cases:

Case (i): For $\tau \leq \tau_1$, we infer from our observations above that firm A optimally sets prices such that $\tilde{x}_1 = \tilde{x}_3 = 1$ in order to take over the whole market, that is, $p^d_{A,L} = p^d_{A,R} = 1 - 3\tau/2$. Thereby, its conditional expected demand as defined in Equations (1.10) and (1.11) is equal to 1/2 regardless of the signal. Then, the expected payoff from deviating is given by

$$\pi^d_A = 1 - \frac{3\tau}{2}.$$

Case (ii): For $\tau_1 < \tau \leq \min\{\tau_2, \tau_3\}$, we infer from our observations above that firm A optimally sets prices such that $\tilde{x}_1 < \tilde{x}_3 = 1$, that is, $p^d_{A,L} = p^{d*}_{A,L}$ and $p^d_{A,R} = 1 - 3\tau/2$. Substituting this into Equations (1.10) and (1.11), we get an expected deviation payoff of

$$\pi^d_A = \frac{(3\tau(1+\sigma_1) + 2(1-\sigma_1))^2 - 32\tau^2}{32\tau(1-\sigma_1)}.$$

Case (iii): Suppose $\sigma_1 < (1+\sigma_2)/(1+2\sigma_2)$ and $\sigma_2 \leq 1/\sqrt{2}$. For $\tau_2 < \tau \leq \tau_3$, we infer from our observations above that firm A optimally sets prices such that $\tilde{x}_1, \tilde{x}_3 < 1$, that is, $p^d_{A,L} = p^{d*}_{A,L}$ and $p^d_{A,R} = p^{d*}_{A,L}$. Substituting this into Equations (1.10) and (1.11), we get an expected deviation payoff of

$$\pi^d_A = \frac{D}{32(\sigma_1 - 1)\sigma_2\tau},$$

where $D := 4(\sigma_1 - 1)\left(\tau(3\sigma_1 - 3\sigma_2 + 1)\sigma_2(-\sigma_1 + \sigma_2 + 1)\right) + \tau^2(9(\sigma_1 - 1)\sigma_2^2 - 3\sigma_1(3\sigma_1 + 2)\sigma_2 + 4\sigma_1 + 11\sigma_2 - 4)\sigma_2$.

Case (iv): Suppose $\sigma_1 \geq (1+\sigma_2)/(1+2\sigma_2)$ and $\sigma_2 > 1/\sqrt{2}$. For $\tau_3 < \tau \leq \tau_2$, we infer from our observations above that firm A optimally sets prices such that $\tilde{x}_1 < \tilde{x}_3 = 1$, that is, $p_{A,L} = p^c$ and $p^d_{A,R} = 1 - 3\tau/2$. By Assumption 1.1, firm A does not find it profitable to charge more than p^c from its loyal consumers as long as firm B uniformly charges p^c. Substituting this into Equations (1.10) and (1.11), we get an expected deviation payoff of

$$\pi^d_A = \frac{2 - 3\tau + \sigma_1(2-\tau)}{4}.$$

Case (v): For $\tau > \max\{\tau_2, \tau_3\}$, we infer from our observations above that firm A optimally sets prices such that $\tilde{x}_1, \tilde{x}_3 < 1$, that is, $p_{A,L} = p^c$ and $p^d_{A,R} = p^{d*}_{A,L}$. For the same reason as before, by Assumption 1.1, firm A charges p^c from its loyal consumers as long as firm B uniformly charges p^c. Substituting this into Equations (1.10) and (1.11), we get an expected deviation payoff of

$$\pi^d_A = \frac{1}{32}(16\sigma_1 - 4(2\sigma_1 + 3)\tau + \frac{\sigma_2(2-3\tau)^2}{\tau} + \frac{4\tau}{\sigma_2} + 8).$$

Now, it is straightforward to check that for both prices and deviation payoffs their respective left-hand and right-hand limits for σ_1, σ_2 and τ approaching the bounds of Case (i)–(iv) from above are equal. Hence, they are continuous.

Further, it is straightforward to check that there are no kinks in both prices and deviation payoffs since the respective left-hand and right-hand limits of their derivatives for σ_1, σ_2 and τ approaching the bounds of Case (i)–(iv) are equal. Hence, they are differentiable.

Taking the collusive payoffs from Lemma 1.1, the deviation payoffs from the above analysis and the punishment payoffs as given in Section 1.4.1, we can solve for the critical discount factor as defined in Equation (1.2). As only the functional form of the deviation payoff is changing with τ, we distinguish the five cases as defined in the proof of Lemma 1.2, that is:

Case (i): For $\tau \leq \tau_1$, we get

$$\bar{\delta}_{asy} = \frac{(2\sigma_1+1)^2\sigma_2(5\tau-2)}{2\left(4\sigma_1^2+1\right)\tau+2(2\sigma_1+1)^2\sigma_2(3\tau-2)}.$$

Case (ii): For $\tau_1 < \tau \leq \min\{\tau_2, \tau_3\}$, we get

$$\bar{\delta}_{asy} = \frac{(2\sigma_1+1)^2\sigma_2\left(-15\tau^2+\sigma_1^2(2-3\tau)^2+2\sigma_1(\tau+2)(5\tau-2)-4\tau+4\right)}{E},$$

where $E := (2\sigma_1{+}1)^2\sigma_2\left(4(\sigma_1-1)^2+(9\sigma_1(\sigma_1+2)-23)\tau^2-12\left(\sigma_1^2-1\right)\tau\right){+}16(\sigma_1{-}1)\left(4\sigma_1^2+1\right)\tau^2$.

Case (iii): Suppose $\sigma_1 < (1+\sigma_2)/(1+2\sigma_2)$ and $\sigma_2 \leq 1/\sqrt{2}$. For $\tau_2 < \tau \leq \tau_3$, we get

$$\bar{\delta}_{\text{asy}} = \frac{F}{G}$$

where $F := (2\sigma_1+1)^2(\sigma_2\left(4(\sigma_1-1)^2+(\sigma_1(9\sigma_1-2)-3)t^2-12(\sigma_1-1)^2\tau\right)-4(\sigma_1-1)\tau^2-(\sigma_1-1)\sigma_2^2(2-3\tau)^2)$, and $G := (2\sigma_1+1)^2\sigma_2(-11\tau^2+\sigma_1^2(2-3\tau)^2+2\sigma_1(\tau+2)(3\tau-2)+4\tau+4-(\sigma_1-1)\sigma_2(2-3\tau)^2)+4(\sigma_1-1)(4\sigma_1(3\sigma_1-1)+3)\tau^2$.

Case (iv): Suppose $\sigma_1 \geq (1+\sigma_2)/(1+2\sigma_2)$ and $\sigma_2 > 1/\sqrt{2}$. For $\tau_3 < \tau \leq \tau_2$, we get

$$\bar{\delta}_{\text{asy}} = \frac{(2\sigma_1+1)^2\sigma_2((\sigma_1+2)\tau-2\sigma_1)}{2\left(4\sigma_1^2+1\right)\tau+(2\sigma_1+1)^2\sigma_2((\sigma_1+3)\tau-2(\sigma_1+1))}.$$

Case (v): For $\tau > \max\{\tau_2, \tau_3\}$, we get

$$\bar{\delta}_{\text{asy}} = \frac{(2\sigma_1+1)^2\left(4\tau^2-4\sigma_2\tau(2\sigma_1(\tau-2)+\tau+2)+\sigma_2^2(2-3t)^2\right)}{H},$$

where $H := (2\sigma_1+1)^2\left(\sigma_2^2(2-3\tau)^2+\sigma_2\tau(16\sigma_1-4(2\sigma_1+3)\tau+8)\right)-16\sigma_1(3\sigma_1-1)+3)\tau^2$. From continuity and differentiability of all payoff functions entering Condition (1.2)—namely π^*, π^c, π^d—continuity of $\bar{\delta}$ with respect to σ_1, σ_2 and τ immediately follows.

Using this, we show that $\bar{\delta}_{\text{asy}} > \max\{\bar{\delta}(\sigma=\sigma_1), \bar{\delta}(\sigma=\sigma_2)\}$. For this to hold, it is sufficient that $\partial\bar{\delta}_{\text{asy}}/\partial\sigma_1|_{\sigma_1=\sigma_2=\sigma} > 0$ and $\partial\bar{\delta}_{\text{asy}}/\partial\sigma_2|_{\sigma_1=\sigma_2=\sigma} < 0$. Why is this? Starting from the symmetric case, asymmetry can be created by either $\sigma_1 > \sigma$ or $\sigma_2 < \sigma$. Straightforward calculations immediately verify that the stepwise derivatives of $\bar{\delta}_{\text{asy}}$ actually satisfy the sufficient conditions.

Finally, we show that $\bar{\delta}_{\text{asy}}$ is non-monotonic in σ_1 and σ_2. At $\sigma_1 = \sigma_2 = \frac{1}{2}$ and $\sigma_1 = \sigma_2 = 1$, we have $\bar{\delta}_{\text{asy}} = \bar{\delta}$ by construction. Hence, by exploiting Proposition 1.1, it is sufficient to show that $\bar{\delta}_{\text{asy}}$ is decreasing around the lower bound of its support. By evaluating the relevant cases, we obtain the following:

- for $\tau \in \left[0, \frac{2}{7}\right]$, $\frac{\partial\bar{\delta}}{\partial\sigma_1}|_{\sigma_1=\sigma_2=\frac{1}{2}} + \frac{\partial\bar{\delta}}{\partial\sigma_2}|_{\sigma_1=\sigma_2=\frac{1}{2}} = \frac{\tau(5\tau-2)}{4(1-2\tau)^2} < 0$,
- for $\tau \in \left(\frac{2}{7}, \frac{2}{3}\right]$, $\frac{\partial\bar{\delta}}{\partial\sigma_1}|_{\sigma_1=\sigma_2=\frac{1}{2}} + \frac{\partial\bar{\delta}}{\partial\sigma_2}|_{\sigma_1=\sigma_2=\frac{1}{2}} = -\frac{32t^2}{(5t+2)^2} < 0$,

where $\tau_1 = \tau_2 = 2/7$ and $\tau_3 = 2/3$ for $\sigma = 1/2$. By continuity of $\bar{\delta}_{\text{asy}}$, there exist $\sigma_1 > \sigma_2 \geq 1/2$, such that the above signs of the derivative continue to hold. □

Proof of Proposition 1.3. As collusion and deviation payoffs remain unchanged, we are left with determining punishment payoffs. Then, we argue how $\bar{\delta}_{asy}$ is affected.

Lets determine punishment payoffs. Given beliefs as derived in Section 1.4.2, we obtain expected demand of firm A conditional on receiving signals s_L and s_R, respectively, of

$$D_A(p_{A,L}, p_{B,L}, p_{B,R}|s_L) = 2(\sigma^2+\gamma)\tilde{x}_1 + (\sigma(1-\sigma)-\gamma) + 2\left((1-\sigma)\sigma-\gamma\right)\left(\tilde{x}_2 - \frac{1}{2}\right)$$

and

$$D_A(p_{A,L}, p_{B,L}, p_{B,R}|s_R) = 2(\sigma^2+\gamma)\left(\tilde{x}_4 - \frac{1}{2}\right) + 2\left((1-\sigma)\sigma-\gamma\right)\tilde{x}_3 + \left((1-\sigma)^2+\gamma\right),$$

with $\tilde{x}_1$–$\tilde{x}_4$ referring to the indifferent consumers from above. Expected payoffs of firm A are then defined as in (1.5), and the decision problem of firm B is derived analogously. Solving first-order conditions with respect to prices simultaneously, we get optimal prices

$$\frac{\partial \pi^*}{\partial \gamma} = -\frac{(1-2\sigma)^2 \sigma \tau}{2\left(2\gamma + 2\sigma^2 + \sigma\right)^3} < 0 \quad \forall \gamma \in [0, \sigma(1-\sigma)], \sigma \in \left(0, \frac{1}{2}\right), \tau \in \left(0, \frac{2}{3}\right].$$

We further observe, that collusion and deviation payoffs only depend on a firm's private signal and hence are defined as in Section 1.3. It follows immediately from the definition of the critical discount factor in 1.2 that the lower is the punishment payoff, the less patient players have to be in order to sustain collusion. Hence, we conclude that for any σ and $\gamma > 0$, we have $\bar{\delta}_{\text{cor}} < \bar{\delta}$. In addition, $\bar{\delta}_{\text{cor}}$ is continuous in σ.

Finally, we show that $\bar{\delta}_{\text{cor}}$ is non-monotonic in σ. At $\sigma = 1/2$ and $\sigma = 1$, we have $\bar{\delta}_{\text{cor}} = \bar{\delta}$ by construction. Hence, from the above observations and Proposition 1.1, the non-monotonicity immediately follows. □

Appendix B: Optimal Punishment

In this section, we characterize an optimal penal code. As defined in Section 1.2, consumers are short-lived and signals about their preferences are on the path of play and get redrawn every period. As a result, the structure of the supergame is recursive as well as perfect Bayesian equilibria are recursive as if we employed the notion of subgame perfect equilibrium.[20] We assume that firms cannot price below marginal costs, that is, prices must be non-negative.[21] The game and strategy profile is as described above except for punishment.

In order to derive optimal penal codes, we first need to determine the *minmax* payoff of firm $i = 1, 2$—the stick. Due to positive transport costs and strategic complementarity, the worst firm $j \neq i$ can do to i is charging $p^o := 0$ irrespectively of its private signal s_j. Given this, we can specify beliefs over consumers' preferences and the relevant indifferent consumers analogously to Section 1.3. Firm i faces the

[20] The reason why we do not use subgame perfect equilibrium directly is purely technical. Our supergame does not have a proper subgame. Information and beliefs, however, are not outcome-relevant across periods as outlined above.

[21] In general, one could allow for below-marginal-cost pricing during the punishment period in order to increase its effectiveness. To which extent such a costlier punishment can be incentivized needs to be examined.

following optimization problem:

$$\max_{p_{i,L},p_{i,R}} \mathbb{E}\left[\pi_i\right] = \frac{1}{2}\left(\sigma p_{i,L}\left(1-\frac{p_{A,L}}{\tau}\right)+(1-\sigma)p_{i,R}\left(1-\frac{p_{A,R}}{\tau}\right)\right).$$

As the objective function is concave, the optimal solution is $p_{i,L} = p_{i,R} = \tau/2 =: p^{mx}$. Then, firm i's *minmax* payoff is given by

$$\pi^{mx} = \frac{\tau}{8}.$$

Next, we have to make sure that it is incentive compatible for firm j to punish firm i after observing a deviation from charging the collusive price—the carrot. As punishment is costly for firm j, it has to be compensated after charging a zero price for T periods. In our game, the most efficient compensation is reversion to collusive behavior as defined in Lemma 1.1, which provides each firm with payoff π^c. First, we need to find the minimum amount of punishment periods T^* such that punishment is incentive compatible for any discount factor δ. Observing that punishment is most efficient if the deviator charges p^o as well throughout the respective T periods,[22] we define the following punishment strategy profile:

- If firm j observes an unexpected deviation of firm i from p^c in any period t, both firms charge p^o in periods $t+1$ to $t+T^*$. Then,
  - if a firm deviates from p^o in any period $t' \in \{t+1,\ldots,t+T^*\}$, both firms charge p^o in periods $t'+1$ to $t'+T^*$, and
  - if there is no deviation from p^o throughout T^* periods, both firms charge p^c again.

To see why this is optimal, lets define T^* such that a firm is indifferent between the following scenarios: (i) receiving zero payoffs for T periods and afterwards receiving π^c for the rest of the game; and (ii) deviating to p^{mx} in period t, receiving zero payoffs for T periods and afterwards receiving π^c for the rest of the game. Hence, T^* solves

$$V^p = \pi^{mx} + \delta V^p,$$

[22] One can easily verify, that the critical discount factor is strictly larger when allowing the deviator to receive *minmax* payoffs during punishment phase.

where $V^p := 0 + \delta 0 + \dots \delta^{T-1} 0 + \delta^T \pi^c$. The implicit solution is given by

$$\delta^{T^*} = \frac{\pi^{mx}}{\pi^c}.$$

As $\delta \in (0,1)$, π^{mx} is bounded from above, and π^c is bounded away from zero, T^* is finite for any $\tau > 0$. We observe that the larger δ, the larger T^*. The intuition behind this trade-off is that the more patient firms are, the more tempted they are to trade π^{mx} in period t against delaying the future stream of π^c by a single period.

Finally, we substitute for the payoff stream from optimal penal codes V^p in Inequality (1.1) to get the following condition for OSDP to hold:

$$\frac{\pi^c}{1-\delta} \geq \pi^d + \frac{\delta \left(\delta^{T^*} \pi^c \right)}{1-\delta}.$$

Substituting for the implicit characterization of T^*, we obtain

$$\delta \geq \frac{\pi^d - \pi^c}{\pi^d - \pi^{mx}} =: \bar{\delta}_{mx}.$$

It is easily verified that $\bar{\delta}_{mx} < \bar{\delta}$ as $\pi^{mx} < \pi^*$ for all σ and τ.[23] Since V^p is independent of signal quality, $\bar{\delta}_{mx}$ always rises in σ.

[23] Moreover, $\bar{\delta}_{mx}$ is lower compared to the critical discount factors in the case of asymmetric signal quality and correlated signals.

Chapter 2

Attention-Driven Demand for Bonus Contracts

Co-authored with Markus Dertwinkel-Kalt and Mats Köster

2.1 Introduction

Supply contracts (e.g., for electricity, telephony, or banking services) typically include many payments, one of which often represents a bonus payment to consumers. More specifically, such *bonus contracts* involve a series of small, regular payments to be made by subscribers, and a single, large bonus (e.g., a monetary payment, or a premium such as a smartphone) that is paid to consumers at some point during the contractual period. As transfers are to be put and tracked, each of these payments generates transaction costs. Bonus payments, in addition, involve checks that need to be send out and redeemed. Non-monetary bonus premiums may involve other inefficiencies, for instance, if the consumer values the bonus below its actual selling price. Thus, abandoning bonuses and reducing the number of transfers to be made by consumers may in general increase efficiency.[1] In this sense, the predominant use of bonus contracts appears puzzling through the lens of the classical model.

We offer a novel explanation for the frequent occurrence of bonus contracts that builds on a recent model of attentional focusing by Kőszegi and Szeidl (2013). Accordingly, consumers select an option which performs particularly well in those choice dimensions where the available alternatives differ a lot, while dimensions along which the available options are rather similar tend to be neglected in the decision-making process. In our setup, the choice dimensions correspond to the different payments specified in a contract. For illustrative reasons, suppose a consumer decides whether to sign some bonus contract for a certain good. Here, the large bonus payment attracts a great deal of attention as the difference between obtaining the bonus if the contract is signed and not getting the bonus otherwise is large. In contrast, regular fees (at least if sufficiently small) play only a minor role. The difference between paying one rate if the contract is signed and paying zero otherwise is relatively small, so that none of the regular payments attracts much attention. Thus, the inclusion of a large bonus at the cost of slightly higher monthly payments can persuade a consumer to sign a contract which she might abandon otherwise.

In this paper, we derive a firm's optimal contract choice if consumers are focused thinkers. Irrespective of the market structure, this contract exhibits two general features. On the one hand, payments to be made by consumers are equally dispersed

[1]In particular situations, spreading payments over time can serve other purposes such as relaxing budget or credit constraints, so that a contract with several regular payments is not inefficient per se.

over the contractual period in order to minimize consumers' focus on costs. On the other hand, the contract involves at most one bonus payment, and if this bonus is non-zero, it will be always maximal.[2] These features create a decision situation that is highly imbalanced with respect to the dispersion of the costs and benefits of the contract. In general, the more imbalanced a decision situation is, the stronger is the distortion of a focused thinker's valuation for a good, so that a consumer's willingness to pay for a subscription can be maximized by concentrating its benefits and equally dispersing its costs over all feasible payments. This is achieved with a contract that includes a single, maximal bonus payment as well as dispersed and rather small regular payments.

In a first step, we analyze a monopolistic market and show that a monopolist offers a bonus contract only for low-value goods. If consumers already have a high valuation for the product, the payments to be made by consumers are relatively high. In this case, setting a bonus at the cost of increased consumer payments cannot shift the consumer's attention solely toward the bonus, but draws attention also to the increased consumer payments. Thus, setting a bonus does not pay off for high-value products.

In a second step, we consider a perfectly competitive market and show that competition forces firms to offer bonus contracts (at least in a symmetric equilibrium), independent of the consumers' valuation for the product. If none of the firms pays a bonus, competition drives down regular payments to cost. Relative to these low regular payments the maximal bonus would attract much attention and each firm could obtain a competitive advantage by offering it. Thus, in any (symmetric) competitive equilibrium, consumers sign a bonus contract.

Thereby, our results mirror a practice that is common, among others, on markets for electricity, telephony, and bank accounts. For illustration, consider the electricity retail market. Competition authorities in the European Union regard this market as split into two separate markets, one of which consists of *loyal consumers* who stay with their default provider, and the other one consists of *switching consumers* (see, for instance, Haucap *et al.*, 2013, pp. 282). This view is supported by recent empirical studies suggesting that a substantial share of consumers do not even consider *switching the provider* as a viable option, so that their default provider de facto serves as a monopolist for this group (e.g., Handel, 2013; Hortaçsu *et al.*,

[2] As we discuss in more detail in Section 2, it seems reasonable to assume that bonus payments are bounded.

2017). Since electricity is essential for running most devices, consumers' valuation can be assumed to be high. Our model predicts—as it is observed in practice—that electricity providers will not offer a bonus contract to their loyal consumers, but a contract that involves only (relatively high) monthly fees. In contrast, firms fiercely compete for switching consumers, who search for the best deal in the market. As electricity is a homogeneous good, we predict that firms fiercely compete for consumers' attention by offering bonus contracts. That is indeed ongoing practice: on German comparison websites, for instance, virtually every power provider offers a large bonus payment instead of a reduction in regular fees in order to attract new customers.

Under standard assumptions, the common design of bonus contracts—that is, small regular payments uniformly dispersed over the contractual period and a single bonus paid at *some* point in time—is hard to reconcile with the classical model or established behavioral approaches such as hyperbolic discounting. According to the classical model, consumers should be indifferent between a bonus payment and a reduction of regular payments as long as the contract's net present value stays the same. As a consequence inefficient bonuses should not occur in equilibrium. If consumers are hyperbolic discounters and therefore present-biased, it is suboptimal for a firm to pay a bonus at *some* point during the contractual period, since a present-biased agent prefers to obtain the bonus payment as soon as possible. Also, hyperbolic discounters would prefer a back-loaded instead of a uniform payment stream. In practice, however, the bonus is often paid at *some* point during the contractual period, and regular payments are small and constant (for a more thorough discussion of this issue see Section 2.4.1).[3]

Our study adds to a growing body of theoretical and empirical research that has investigated and supported the importance of attentional focusing for economic choice. Accordingly, a decision maker automatically focuses on eye-catching choice features. These salient aspects of an option obtain an over-proportionate weight in the decision making process, while less prominent attributes tend to be neglected. A key implication of attentional focusing is *a bias toward concentration* (Kőszegi and Szeidl, 2013) whereby a decision maker puts disproportionately more attention toward concentrated than dispersed outcomes. Dertwinkel-Kalt *et al.* (2017a) demonstrate concentration bias in a laboratory experiment.[4] Applied to industrial

[3]See, for instance, `https://www.marktwaechter-energie.de/aerger-mit-energieversorgern/boni/`, accessed on August 1, 2017.

[4]In a series of papers, Bordalo *et al.* (2012, 2013, 2016) have developed *salience theory*, which

organization, attentional focusing can explain, for instance, why drastic (minor) innovations yield decommoditized (commoditized) markets (Bordalo *et al.*, 2016). We apply attentional focusing in order to delineate how firms design contracts to attract focused thinkers.

We proceed as follows. In Section 2.2 we present the model. In Section 2.3 we derive the equilibrium in a monopolistic and a perfectly competitive market, respectively. In Section 2.4 we discuss the related literature, before Section 2.5 concludes.

2.2 Model

There are L firms offering a homogeneous product at zero production costs, and a unit mass of homogeneous consumers who value the good at $v \geq 0$ and purchase at most one unit.

Contract Space. Each firm k can offer an $M + N$-part tariff that consists of

(i) $M \geq 1$ bonus payments $b_1^k, \ldots, b_M^k \geq 0$ to be paid to consumers, and

(ii) $N \geq 2$ regular payments $p_1^k, \ldots, p_N^k \geq 0$ to be made by consumers.

While we interpret the regular payments by consumers, p_i^k, as payments to be made at different points in time, we stay agnostic on when different bonus payments are made. As we will show in the next section, the assumption of a fixed number of bonus payments is without loss of generality. In contrast, without imposing further restrictions, a fixed number of payments to be made by consumers entails a loss. But, on the one hand, it seems plausible to assume that consumers aggregate payments they have to make for a specific good within a short time period, so that firms may not be able to increase the perceived number of payments beyond a certain threshold.[5] And, on the other hand, if each additional payment to be made by

shares Kőszegi and Szeidl's central assumption that dimensions where alternatives differ much attract much attention. Experimental support for salience theory has been provided by Dertwinkel-Kalt *et al.* (2017b).

[5]In an experimental study, Dertwinkel-Kalt *et al.* (2017a) find that subjects regard payments as separate that are dispersed over several weeks, but aggregate payments that are split within a day. Given this evidence, it seems plausible to assume that consumers aggregate all payments they have to cover with one salary, for instance. Then, there is no reason for firms to disperse payments between two paydays as this raises transaction costs, but does not impact on consumers' valuation for the contract. Supportive of this, in Europe where salaries are typically paid monthly also most

consumers comes along with an increasing transaction cost (i.e., transaction costs incurred by consumers are a convex function of the number of regular payments), an "optimal" number of regular payments exists and N could be understood as to be chosen optimally by the firms. We discuss implications of this interpretation in the next section when analyzing the robustness of our results.

We also limit the maximum bonus that firms can pay. In other words, we impose a floor on the total price a firm could charge (see Heidhues and Kőszegi, 2018, for a broader discussion).

Assumption 2.1. *The sum of bonus payments is bounded from above by some $\bar{b} > 0$.*

Since even large firms face financial constraints, in practice firms cannot afford very large bonus payments. More importantly, a very large bonus may create incentives for the consumers to betray the firm and to not fulfill the contract. Finally, offering too large bonus payments might make consumers suspicious in that they believe something fishy to be going on. In this sense, setting a bonus beyond some level $\bar{b}$ may never pay off for a firm.

Timing of the Game. In a first stage, each firm $k \in \{1, \dots, L\}$ chooses a contract

$$\boldsymbol{c}^k := (v, b_1^k, \dots, b_M^k, p_1^k, \dots, p_N^k).$$

In a second stage, consumers decide whether and from which firm to buy the product. Formally, each consumer chooses a contract from the set

$$\mathcal{C} := \{\boldsymbol{c}^k \mid 0 \leq k \leq L\},$$

where $\boldsymbol{c}^0 := (0, \dots, 0) \in \mathbb{R}^{M+N+1}$ refers to the outside option of not buying the product.

For simplicity, firms and consumers adopt the same discount factor which may be determined by the market interest rate. Throughout our analysis we assume that all payments refer to present values (i.e., real instead of nominal sums). While this assumption is not crucial for our qualitative insights, it allows us to abstract from discounting.

supply contracts (i.e., mobile or electricity contracts) involve monthly payments. In the US where salaries are often paid weekly also supply contracts often involve weekly payments.

A Firm's Problem. Each firm k designs a contract $\boldsymbol{c}^k$ in order to maximize her profits,

$$\pi_k(\boldsymbol{c}^k, \boldsymbol{c}^{-k}) := D_k \cdot \left(\sum_{i=1}^{N} p_i^k - \sum_{j=1}^{M} \left[\, b_j^k \; + \; \mathbb{1}_{\mathbb{R}_{>0}}(b_j^k) \cdot c \, \right] \right),$$

where $D_k = D_k\Big(\boldsymbol{c}^k, \boldsymbol{c}^{-k}\Big)$ corresponds to the share of consumers choosing the contract offered by firm k from the set $\mathcal{C}$, where $\mathbb{1}_{\mathbb{R}_{>0}}$ is the indicator function on the interval of positive, real numbers, and where $c > 0$ are per-customer transaction costs for each additional bonus payment. We argue below why we regard it as a plausible assumption that $c > 0$.

A Consumer's Problem. We assume that consumers are *focused thinkers* (Kőszegi and Szeidl, 2013, henceforth: KS). Focused thinkers put an excessive weight on the salient choice dimension(s) of a contract, while they partly neglect less prominent attributes. Following KS, we assume that payments at different points in time as well as a good's quality (or its value to consumers) correspond to different choice dimensions.[6] Moreover, we assume that consumers also perceive the different bonus payments as distinct attributes.[7] Altogether, we assume that the N regular payments to be made by consumers, the M bonus payments offered by the firms, and the consumption value of the product all represent distinct choice dimensions.

Given these assumptions, a focused thinker chooses a contract from the choice set $\mathcal{C}$ in order to maximize her *focus-weighted utility* given by

$$\tilde{U}\Big(\boldsymbol{c}^k \Big| \, \mathcal{C}\Big) := \begin{cases} g(\Delta^v)v - \sum_{i=1}^{N} g(\Delta_i^p)p_i^k + \sum_{j=1}^{M} g(\Delta_j^b)b_j^k & \text{if } k > 0, \\ 0 & \text{if } k = 0, \end{cases}$$

whereby the weights on the different choice dimensions are determined by a *focusing function* $g : \mathbb{R}_+ \to \mathbb{R}_+$. According to KS, the weight on price component i, $g(\Delta_i^p)$,

[6] While KS do not analyze a model of industrial organization, they point out that it is plausible to assume that in such models quality represents a choice dimension that is distinct from the price dimension(s); also the related model by Bordalo *et al.* (2013) adapts the assumption that quality constitutes a separate choice dimension.

[7] This assumption is particularly plausible if some of the bonus payments refer to non-monetary premia such as smartphones or other gadgets while others refer to monetary payments. In addition, it is straightforward to show that our results would not change if consumers do not perceive the different bonus payments as distinct attributes, but aggregate them into a single attribute.

depends on the range of attainable utility along this choice dimension denoted as

$$\Delta_i^p := \max_{0\leq k\leq L} p_i^k - \min_{0\leq k\leq L} p_i^k = \max_{0\leq k\leq L} p_i^k,$$

where the equality follows from the fact that the outside option does not involve any regular payments. Analogously, the weight on bonus payment j depends on the range of attainable utility along this bonus attribute, which we denote as Δ_j^b, and the weight on the product's consumption value depends on the utility range in this choice dimension, Δ^v, which is spanned by v in the case that the consumer buys and 0 in the case that she does not buy.

Following KS, we assume that the weight assigned to a certain attribute increases in the utility range along this choice dimension that is attainable given $\mathcal{C}$. This captures the intuition that large contrasts are particularly salient (see, e.g., Schkade and Kahneman, 1998), so that choice dimensions along which the available options differ a lot attract a great deal of attention.

Assumption 2.2 (Contrast Effect)**.** *The focusing function g is strictly increasing with $g' > 0$.*

In addition, we assume that the contrast effect is sufficiently strong.

Assumption 2.3 (Convexity)**.** *The function $h : \mathbb{R}_+ \to \mathbb{R}_+$ with $h(x) := g(x)x$ is convex.*

Notice that Assumption 2.3 is not very restrictive as it admits for convex, linear, and mildly concave focusing functions. In fact, it is violated only for strongly concave focusing functions.[8]

Contractual Inefficiencies. We assume that making bonus payments is inefficient, as it involves costs for issuing, sending, and tracking checks, for instance. Formally, for each non-zero bonus payment, a firm has to bear per-customer transaction costs $c > 0$. We neglect transaction costs related to regular payments made by consumers for the moment, but discuss the relevance of such costs in the next section when analyzing the robustness of our results.

[8]More formally, Assumption 2.3 holds if and only if $-\frac{g''(x)x}{g'(x)} < 2$ for any $x \in \mathbb{R}_+$, which is satisfied for any convex or linear focusing function and for concave focusing functions with a first derivative that is not too elastic. In particular, a power function $g(x) = x^\alpha$, $\alpha > 0$, satisfies Assumption 2.3.

In order to allow firms to increase a consumer's focus-weighted utility using a bonus payment and to break even at the same time, we assume that the costs of paying a bonus are not too large relative to the maximum bonus itself, that is,

$$\frac{c}{\bar{b}} < \frac{g(\bar{b})}{g(\bar{b}/N + c/N)} - 1. \tag{2.1}$$

While this assumption allows firms to benefit from using bonus contracts, it is not very restrictive and becomes weaker for larger values of N or $\bar{b}$, respectively. Suppose, for instance, that the number of regular payments is $N = 24$, that the maximum bonus is $\bar{b} = 120$, and that the focusing function is the identity function. In this case, the costs of making a bonus payment could be much larger than the maximum bonus itself without violating (2.1).

2.3 Equilibrium Analysis

In this section, we first analyze under which conditions a monopolist offers a bonus contract. Second, we derive equilibrium contracts in a perfectly competitive market. Finally, we discuss the robustness of our results and provide applications to markets for electricity supply, telephony services, and bank accounts. All missing proofs can be found in Appendix A.

2.3.1 Monopolistic Market

Suppose that a single firm monopolizes the market (i.e., $L = 1$). For brevity, we drop the index k in this subsection. Then, the monopolist's maximization problem is given by

$$\max_{\boldsymbol{c}} \pi(\boldsymbol{c}) \quad \text{subject to} \quad \sum_{i=1}^{N} g(p_i)p_i \leq g(v)v + \sum_{j=1}^{M} g(b_j)b_j, \text{ and } \sum_{j=1}^{M} b_j \leq \bar{b},$$

and the optimal contract offer is characterized in the following lemma.

Lemma 2.1. *A contract* $\boldsymbol{c} = (v, b_1, \ldots, b_M, p_1, \ldots, p_N)$ *maximizes the monopolist's profit only if*

(i) the payments made by consumers are equally spread across periods, that is, $p_1 = \ldots = p_N$*,*

(ii) if bonus payment(s) are made, the bonus is maximal, that is, $\sum_{j=1}^{M} b_j = \bar{b}$, *and*

(iii) the contract involves at most a single bonus payment, that is, if a bonus payment is made, then $b_j = \bar{b}$ *for some* $j \in \{1, \ldots, M\}$ *and* $b_i = 0$ *for any* $i \neq j$.

Since the monopolist can fully extract the consumers' willingness to pay, he offers a contract that maximizes focus-weighted utility conditional on extracting it. According to the contrast effect a focused thinker's attention is directed to particularly large payments, so that the monopolist can minimize the consumers' perceived costs by dispersing the regular payments uniformly over the entire contractual period. More formally, suppose that one of the regular payments was larger than the others, and, without loss of generality, let $p_1 > p_i$ for all $i \in \{2, \ldots, N\}$. Then, since $g(p_i)p_i$ is convex (Assumption 2.3), decreasing p_1 by ϵ and increasing each of the other payments by $\epsilon/(N-1)$ lowers the perceived costs of the contract, while keeping revenue constant. As a result, a necessary condition for maximizing the consumers' willingness to pay (and therefore the monopolist's profit) is that all payments to be made by consumers are of equal size. In contrast, if the monopolist chooses to pay a bonus, it should attract as much attention as possible, which is achieved by setting the maximal bonus, $\bar{b}$, and concentrating it into a single payment.

Yet, the monopolist will not always choose a bonus contract. A bonus will be offered if and only if (i) the consumers' valuation for the good is sufficiently low and (ii) the inefficiency that arises from a bonus payment is sufficiently small.

Proposition 2.1. *There exists a threshold value* $\hat{c} > 0$ *and, for any* $c < \hat{c}$, *a threshold value* $\hat{v}(c) > 0$ *such that the monopolist offers a bonus contract if and only if* $c < \hat{c}$ *as well as* $v < \hat{v}(c)$. *In addition, the threshold value* $\hat{v}$ *monotonically decreases in* c *on* $[0, \hat{c})$.

Even if transaction costs are low, the monopolist offers a bonus only if the consumers' valuation for the product is sufficiently low. Only if the consumers' valuation and therefore the regular payments are low, the monopolist can increase its relatively small margin by setting a bonus that grabs attention. If the valuation is high, consumers are already willing to accept relatively high regular payments. Then, the focus on the bonus—even if it is maximal—cannot outweigh the consumers' focus on the even higher regular payments that are necessary to make a bonus contract profitable. Consequently, the monopolist cannot benefit from offering a bonus payment.

In order to put the preceding result into perspective, we consider an example.

Example 1. *Suppose that the focusing function is linear with $g(x) = x$. Then, we obtain a threshold value $\hat{c} = (\sqrt{N} - 1)\bar{b}$ and, for any $c < \hat{c}$, a threshold value $\hat{v}(c) = \frac{(N-1)\bar{b}^2 - (2\bar{b}+c)c}{2\sqrt{N}(\bar{b}+c)}$.*

Since $\hat{v}(c)$ strictly increases with the inefficiency arising from bonus payments, Example 1 further suggests that the monopolist will offer a bonus contract only if the regular payments he would charge when not paying a bonus lie strictly below $\frac{\bar{b}}{2}\frac{(N-1)}{N}$. Suppose, for instance, that the number of regular payments is $N = 24$, and that the maximum bonus is $\bar{b} = 120$. If the consumers' valuation is high enough, so that the monopolist would already charge regular payments $p > 57.5$ when not paying a bonus, then offering a bonus contract would not increase his profits. And with a concave focusing function—such as $g(x) = \sqrt{x}$, which is consistent with experimental evidence by Dertwinkel-Kalt *et al.* (2017a)—regular payments when not paying a bonus had to be even lower to make a bonus contract profitable.

2.3.2 Competitive Market

Suppose that at least two firms compete for customers. As the product is homogeneous, firms fiercely compete for consumers' attention, and, as we will see below, bonus contracts play an even larger role than in a monopolistic market, despite the inefficiencies they produce. The (symmetric) equilibria of the game are characterized in the following proposition.

Proposition 2.2. *For $L = 2$, an equilibrium exists and any equilibrium has following properties:*

(i) the market is covered and firms earn zero profits,

(ii) payments to be made by consumers are equally spread across periods (i.e., $p_1^k = \ldots = p_N^k$), and both firms charge the exact same regular payments (i.e., $p_i^1 = p_i^2$), and

(iii) both firms offer the maximum bonus (i.e., $\sum_{j=1}^{M} b_j = \bar{b}$) using a single bonus payment.

For $L \geq 3$, a symmetric equilibrium exists and any symmetric equilibrium satisfies (i) – (iii).[9]

[9]Notice that for $L \geq 3$ also asymmetric equilibria exist, where at least two firms offer

As in the monopoly case, the payments to be made by consumers are equally spread over the contractual period. Thereby, firms minimize the consumers' perceived costs of the contract for a fixed revenue. Given that the remaining firms offer the equilibrium contract, no firm can benefit from unilaterally decreasing some payment and increasing some others. In doing so, the firm would induce consumers to focus more on increased payments, that is, exactly on those choice dimensions along which it offers a worse deal compared to the other firms. At the same time, the focus-weight attached to the decreased payment does not change as it is determined by the other firms' higher regular payments. To sum up, a price hike attracts more attention than the corresponding price cuts, so that the firm cannot benefit from such a contract adjustment.

In contrast to the monopoly case, competing firms always offer a bonus contract (at least in the symmetric equilibrium), irrespective of the consumers' valuation for the product or service. For the sake of a contradiction, suppose that firms do not offer a bonus in the symmetric equilibrium. Since firms must earn zero profits in any equilibrium, the payments to be made by consumers have to be zero. Then, any firm could benefit from offering a single bonus payment of size $\bar{b}$ and increasing each regular payment to $\frac{c+\bar{b}}{N} + \epsilon$ for some sufficiently small $\epsilon > 0$, since Assumption 2.2 together with Eq. (2.1) ensure that—given zero regular payments offered by the other firms—consumers focus more on the bonus payment than on the increase in regular payments. In addition, in the monopoly case we have already discussed that a bonus attracts as much attention as possible if it is concentrated into a single payment. Therefore, firms offer at most one bonus payment, and a similar argument as above implies that this bonus must be maximal. Although the other firms' choices impact on consumers' attention allocation, raising the bonus to the maximal level increases the focus on the own contract's advantage—the large bonus—by more than it increases the focus on the higher regular payments, since in equilibrium regular payments are lower than the maximal bonus payment.

Given that all firms charge a single bonus payment of size $\bar{b}$ and regular payments $\frac{c+\bar{b}}{N}$, firms earn zero profits and no firm has an incentive to deviate. In fact, the only way to attract consumers would involve a decrease in some regular payment(s),

$(v, 0, \ldots, 0)$ and serve the market, while at least one firm offers a contract with regular payments exceeding the maximum bonus payment. Importantly, these asymmetric equilibria are neither robust to assuming that clearly dominated options do not affect a consumer's attention allocation nor to assuming that firms want to maximize demand for a given profit level. In this sense, we argue that the symmetric equilibria delineated above are the only plausible equilibria.

which is profitable for a firm only if it either increases some other regular payment(s) or decreases the bonus by an even larger amount. Since the focusing function is increasing by Assumption 2.2 and the maximal bonus exceeds the regular payments in equilibrium, a firm cannot find a deviation that is better than decreasing one regular payment and uniformly increasing the remaining regular payments. But still, given the other firms' equilibrium offers, the price increases would attract more attention than the price decrease, so that consumers would not choose this new contract.[10]

Importantly, even though paying a bonus creates an inefficiency, bonus contracts are more prevalent in competitive rather than monopolistic markets. Since firms standing in competition only care about beating the best offer of their competitors and not necessarily about maximizing the consumers' willingness to pay, they use bonus payments more often. More precisely, by increasing the regular payments in order to cover a bonus payment, a firm in a competitive market makes not only her own offer worse but also makes the consumers' outside option—the best competitor's offer—look worse. Hence, only the incremental change over and above the competitors matters, so that paying a bonus is indeed a good idea, since in equilibrium the regular payments are lower than the maximum bonus. In contrast, for a monopolist even then an increase in the regular payments might not pay off since it lowers the consumers' willingness to pay due to the fact that also the inframarginal payments are weighted more. Altogether, under competition firms cannot attract consumers without offering an attention-grabbing, though inefficient bonus while a monopolist will not offer a bonus if the consumers' valuation and therefore the regular payments are high. This leads us to the following statement.

Corollary 2.1. *If the consumers' valuation for the product is sufficiently high, the contractual inefficiencies are strictly lower in a monopolistic than in a competitive market.*

2.3.3 Robustness

Here, we will argue in how far our findings take over to more general setups.

[10]Notably, the idea of avoiding high and therefore attention-grabbing prices is also relevant in the model by de Clippel *et al.* (2014) where firms compete for consumers' inattention (to the own price) by making price components non-salient. Here, each firm avoids to charge a sum that exceeds the other firms' payments as this would attract a great deal of attention, thereby deterring consumers from signing the respective contract.

Heterogeneous Consumers. So far, we have assumed that consumers are homogeneous, both with respect to their valuation for the product, v, and their focusing function, $g(\cdot)$. First, suppose that consumers have the same valuation for the product, but are heterogeneous with respect to the curvature of their focusing function, and that firms can only offer a single contract. Then, a monopolist offers a contract that sets the consumer type that has, among those that should be attracted, the flattest focusing function indifferent between buying and not buying. Depending on v and the focusing function of the indifferent type, either no bonus or the maximal bonus will be set. Consumers with a stronger focusing bias (i.e., a steeper focusing function) will also be attracted by that contract as they appreciate the bonus even more. If there are at least two firms competing for consumers, the bonus will be set maximal (at least in the symmetric equilibrium) and the periodical payments are so that firms earn zero profits. Intuitively, suppose that some consumers are not susceptible to focusing, and are therefore indifferent between a contract with a maximal bonus and zero bonus, respectively, as long as the net-payment (and hence firms' profits) are held constant. This implies that, if a small fraction of consumers have focused-weighted utility, competing firms want to exploit this by offering a bonus contract.

Second, suppose that consumers differ only in their valuation for the product, and each firm can only offer a single contract. Also in this case, the main insights of Propositions 1 and 2 still hold. A monopolist makes the consumer type that has, among those that should be attracted, the lowest valuation indifferent between buying and not buying, and maximizes profits along the lines of Proposition 1. Competitive firms set the maximal bonus in any case, charging regular payments that allow them to break even.

Third, suppose that consumers are heterogeneous and that firms can perfectly discriminate between them, that is, each firm can offer a different contract to each consumer type. In addition, assume that each consumer type can only see the contracts(s) tailored to it. Then, all of our preceding results apply separately to each consumer group. In the next section we discuss applications where we regard the distinction between two consumer types—loyals and switchers—as plausible. In these examples it also seems reasonable to assume that each consumer type is not aware of the contracts tailored to the other type.

Maximal Bonus Payment. We also have assumed that the maximal bonus a firm can pay is bounded and that this upper bound is independent of the consumers' valuation v. In practice, it may be plausible to assume that the maximal bonus increases in v. Our result on the monopolistic outcome carries over to this case, as long as the maximal bonus increases only mildly in the consumers' valuation v. As an illustration, we extend Example 1 as follows.

Example 2. *Suppose that the focusing function is linear with $g(x) = x$ and that the maximal bonus is given by $\overline{b}(v) := \beta v + \delta$ with $\beta, \delta > 0$. This implies a threshold value $\hat{c} = (\sqrt{N} - 1)\delta$. In addition, there exists some $\hat{\beta} > 0$ such that, for any $\beta < \hat{\beta}$, the monopolist offers a bonus contract if and only if $c < \hat{c}$ as well as*

$$v < \hat{v}(c, \beta) = \frac{\beta\Big[(N-1)\delta - c\Big] + \sqrt{N}\left[\sqrt{(c+\delta)^2 + \beta c\Big(\beta c - 2\sqrt{N}\delta\Big)} - (c+\delta)\right]}{\beta\Big[2\sqrt{N} - \beta(N-1)\Big]},$$

whereby $\hat{v}(c, \beta) > 0$ if both $\beta < \hat{\beta}$ and $c < \hat{c}$.

While in general it depends not only on the valuation v, but also on the curvature of the focusing function g whether a monopolist offers a bonus contract if the maximal bonus increases with v, we find that the competitive outcome does not change qualitatively. More specifically, even if the maximal bonus is an increasing function of v, then in any (symmetric) competitive equilibrium all firms offer a bonus contract (as in Proposition 2), so that our finding that a bonus payment is always made under competition, but not necessarily in a monopolistic market, is robust such an extension of our baseline model.

Endogenous Attribute Space. Moreover, we have considered the case with a fixed number of bonus and regular payments, respectively. Obviously, as firms want to pay at most a single bonus, such a restriction on the number of bonus payments is without loss of generality. Just assuming a fixed number of regular payments without imposing further restrictions (e.g. transaction costs for regular payments) entails a loss of generality, however, as then firms would always want to increase the number of regular payments. Instead, we could assume that firms can freely choose the number of bonus and regular payments, but that consumers incur transaction costs that are increasing and convex in the number of non-zero regular payments.

As we prove in Appendix B, given these assumptions, the qualitative insights from Propositions 2.1 and 2.2 remain valid. If in addition the transaction costs are sufficiently convex, then also our result on the comparison of the monopolistic and the competitive outcome remains to hold; that is, if transaction costs are sufficiently convex and the consumers' valuation for the product is sufficiently high, the contractual inefficiencies are strictly lower in a monopolistic than in a competitive market. In order to illustrate this result, imagine a cost function that is almost flat until some point and then becomes pretty steep pretty fast. In this case, it is easy to see that a monopolist would choose the same number of regular payments as competitive firms would do, so that the only welfare-relevant difference between the monopolistic and the competitive outcome refers to the question whether the monopolist pays a bonus or not.

2.3.4 Applications

Next, we combine our preceding results to discuss three applications. Consider a market with $L \geq 2$ firms, where each firm $k \in \{1, \ldots, L\}$ has a share of *loyal consumers* $\alpha_k > 0$ with $\sum_{k=1}^{L} \alpha_k < 1$. A consumer who is loyal to firm k only considers her tailor-made contract by firm k, and buys as long as this contract gives her a non-negative focus-weighted utility. The remaining consumers, a share $1-\sum_{k=1}^{L} \alpha_k$, we call *switching consumers*. They observe all contracts except for those tailored to the loyal consumers. Finally, suppose that the consumers' valuation for the good is so high that a monopolist would not offer a bonus contract.

We then predict that firms offer different contracts for loyal and for switching consumers. Since each firm k acts as a monopolist for its loyal consumers, it offers a contract without a bonus payment as defined in Lemma 2.1. In contrast, firms fiercely compete for switching consumers and offer them the bonus contract defined in Proposition 2.2.

Application I: Electricity Supply Contracts. In practice, power consumption is not binary, but continuous. Consumers do not only decide whether or not to consume, but also how much to consume. In many countries such as Germany, however, electricity suppliers charge fixed monthly pre-payments that are based on a consumer's estimated power consumption. Arguably, even though the actual monthly fees are not fixed, this contract design (involving pre-payments) might make consumers ex-ante reason as if the regular payments were fixed. Thus, as long as a consumer does not expect an additional payment at the end of the year (or expects it to be small), we regard our assumption on the agent's choice as being binary as reasonable. Furthermore, in many countries (e.g., in Germany, the UK, or the US), the electricity market consists of local default providers and several smaller entrants. Empirical studies suggest that a substantial share of consumers do not consider switching from their default provider to a cheaper alternative as a feasible option (Hortaçsu *et al.*, 2017). As a consequence, our formal setup that distinguishes between loyal and switching consumers matches the market for electricity supply.

As predicted by our model, loyal consumers are typically charged rather high regular payments and do not receive a bonus. In contrast, but in line with our model, electricity suppliers fiercely compete for the remaining consumers who are willing to switch (i.e., who compare offers across providers) by offering large bonus payments that are paid after the subscription.[11] As mentioned above, it is a common feature of electricity supply contracts that the (often monthly) payments to be made by consumers are constant over the contractual period, even though actual usage is measured and therefore also billed only once a year.[12] Such a contract design involving regular (pre-)payments that are equally dispersed over the contractual period is also optimal according to our model as—unlike contracts conditioning payments in each period on the actual per-period usage—it minimizes the consumers' focus on costs. To sum up, our model helps to understand the contract design of electricity suppliers.

Application II: Telephony and Internet Contracts. Our model also fits the market for mobile phone contracts. Although in most OECD countries there are several providers of telecommunication services, a substantial share of consumers have never switched their provider.[13] While contracts designed for new customers typically include valuable features (such as smartphones, tablets, special discounts, or bonus payments), customers who extend an already existing contract usually obtain worse offers that do not involve such a bonus.[14] This observation is suggestive for our predictions that firms offer contracts without bonus payments to their loyal consumers,

[11]See, for instance, `http://www.handelsblatt.com/politik/konjunktur/oekonomie/nachrichten/anbieterwechsel-die-teure-traegheit-der-verbraucher/3560414-all.html`, accessed on August 6, 2017.

[12]See, for instance, `https://www.verbraucherzentrale.de/energieversorger-rechnungen`, or `https://www.gov.uk/guidance/gas-meter-readings-and-bill-calculation`, both accessed on August 1, 2017.

[13]See, for instance, `https://www.oecd.org/sti/consumer/40679279.pdf`, pp. 32, accessed on August 4, 2017.

[14]See, for instance, `http://money.cnn.com/2015/03/18/smallbusiness/tmobile-uncarrier/index.html`, accessed on August 4, 2017.

and compete for switching consumers with bonus contracts. In line with our model, firms often advertise flat-rate contracts, that is, contracts involving payments to be made by consumers that are equally spread over the contractual period and that do not depend on actual usage frequency. Analogous tariff structures are common on the market for Internet contracts.[15]

Application III: Bank Accounts. The retail banking industry serves as another example that our setup applies to. In most EU countries, a considerable share of consumers do not even consider the option to switch their bank as a viable alternative, although there are several competitors in the market.[16] While account management fees are usually dispersed over the contractual period, banks try to attract new customers by offering a large switching bonus that is typically paid after the contract is signed and certain conditions (e.g., minimal monthly deposits) are satisfied.[17] As predicted by our model, banks offer bonus payments only to those consumers who are willing to switch and open a new account, but not to their existing customers.

2.4 Related Literature

In this section, we discuss how our paper relates to the existing literature on the common design of contracts in markets for consumption goods. Notably, none of the previous approaches can explain the frequent use of bonus contracts—including a single, large bonus to be paid at some point during the contractual period and regular payments that are equally dispersed over time.

2.4.1 Exponential and Hyperbolic Discounting

According to the classical model, as proposed by Samuelson (1937), an agent maximizes her expected intertemporal utility, which (i) is additively separable across payoffs received at different points in time and (ii) satisfies exponential discounting (i.e., payoffs t periods ahead are discounted by δ^t for some discount factor $\delta < 1$). A classical agent should be indifferent between any allocation of payments across time that has the same present value. Therefore, firms will avoid inefficient bonus

[15] Notably, for telephony and Internet not only flat-rate contracts exist, but also contracts that depend on actual usage, in which case the periodical payments differ to some degree.

[16] See, for instance, `http://www.ec.europa.eu/competition/sectors/financial_services/inquiries/sec_2007_106.pdf`, p. 66, accessed on August 2, 2017.

[17] See, for instance, `https://www.welt.de/finanzen/geldanlage/article126159643/Hohe-Praemien-fuer-Girokonten-bringen-Nachteile.html`, accessed on August 2, 2017.

payments. If in addition we impose the common assumption that the marginal utility from money decreases (i.e., preferences can be represented by a concave utility function over monetary outcomes), the use of large bonus payments becomes even less attractive.

In order to match evidence on present-biased behavior, more recent approaches to intertemporal decision making have assumed that discounting is hyperbolic (for seminal contributions, see, Chung and Herrnstein, 1967, and Loewenstein and Prelec, 1992) or quasi-hyperbolic (Laibson, 1997). Hyperbolic discounters strictly prefer to receive the bonus payment as soon as possible and to shift the regular payments into the future. Thus, if consumers discount hyperbolically, we would expect back-loaded rather than uniform payment streams, which does not fit the common practice for many types of contracts. The practice of uniform regular payment streams might be in line with models of quasi-hyperbolic discounting, however, which predict that agents are simply indifferent between all allocations of future payments that have the same present value. But still, a model of quasi-hyperbolic discounting cannot explain why the use of contracts including a single, large bonus is predominant on many markets. In addition, it cannot explain why bonuses are often paid with a substantial delay. Notably, both features may be driven by practical reasons: regular payments might be uniform either to keep contracts simple or to address consumers' preferences for consumption smoothing, and bonuses may be paid with a delay as firms do not want to deal with deadbeat customers who quickly receive a bonus and then fail to make the regular payments. Nevertheless, the existence of the bonus payment and the high frequency of the regular payments cannot be explained hereby.

2.4.2 Switching Costs and Automatic-Renewal Contracts

Models on switching costs (e.g., Klemperer, 1995) can explain why consumers may abstain from switching providers. Accordingly, consumers need to be compensated for switching costs, which may be achieved by paying a bonus to consumers (see, Farrell and Klemperer, 2007, for a discussion). These models, however, are agnostic with respect to the timing and the dispersion of payments specified in a contract. In addition, these models cannot explain why consumers are compensated via a single, large bonus payment and not via lower regular payments.

Relatedly, Johnen (2018) studies a market in which firms offer automatic-renewal contracts to consumers who are inert in the sense that they forgo benefits from switching to another firm. If a consumer underestimates the probability of failing to cancel a contract (e.g., due to limited attention or a naive present-bias), firms can exploit this consumer by offering an attractive teaser rate that increases after the automatic renewal of the contract. Although his approach provides a plausible explanation for offering attractive teaser rates, it does not make specific predictions on whether firms should use a bonus payment to attract consumers or whether they should simply lower the regular payments, as in his model only the predicted net present value matters. Interestingly, also in Johnen (2018) competitive firms focus more on exploiting consumer mistakes than a monopolist does, so that similar to our findings also in his model the monopolistic outcome can be more efficient than the competitive one.[18]

2.4.3 Partitioned Pricing, Shrouding, and Socially Wasteful Products

We have shown that attentional focusing can explain why firms frequently partition a product's total price into several price components in order to increase a consumer's willingness to pay (for empirical evidence see Morwitz *et al.*, 1998). Indeed, a number of older studies observe that consumers systematically underestimate a product's overall price if it is partitioned into several price components that the consumer is simultaneously (*partitioned pricing*) or sequentially (*drip pricing*) informed about (e.g., Carlson and Weathers, 2008; Ahmetoglu *et al.*, 2014). More recent experimental evidence from the lab (Dertwinkel-Kalt *et al.*, 2017a) and from the field (Dertwinkel-Kalt *et al.*, 2018) suggests, however, that simply splitting up the total price without dispersing the price components over time does not affect a consumer's willingness to pay.

While also the literature on shrouded price components (e.g., Gabaix and Laibson, 2006; Heidhues *et al.*, 2016) can explain the success of partitioned pricing, it necessarily assumes that a share of consumers are not aware of some price components when making the purchase decision or the importance thereof when mispredicting their future behavior. In contrast, a model of focusing can also explain these effects if all information is readily available. Because several smaller prices attract less attention than a single, but large one, they are underweighted. As a

[18] There is a small, but recently growing literature on the distorting effects of competition (e.g., Carlin, 2009; Gabaix *et al.*, 2016; Friedrichsen, 2018). See Johnen (2018) for a discussion of the mechanisms in these papers.

consequence, the focusing model can account for the fact that a uniform dispersion of the total price over time increases a consumer's willingness to pay even if the consumer is fully informed about all price components.

Finally, our study connects to the literature demonstrating that even socially wasteful products can survive competition and may be sold in a competitive equilibrium. Heidhues *et al.* (2016) argue that, if a part of the product's price is shrouded, some consumers may not anticipate the product's total price at the moment they make the purchase decision, so that these consumers may purchase a good at a price that strictly exceeds their valuation. Thus, even a socially wasteful product—that is, a product for which the production costs lie above the consumers' valuation—may generate positive demand. Since firms typically have no incentive to unshroud the additional price, selling a socially wasteful product can be the equilibrium outcome in a perfectly competitive market. Also in our model socially wasteful products might be sold in a competitive equilibrium even if consumers are aware of the entire price.[19] By focusing on the contract's outstanding feature (i.e., the large bonus payment), consumers may overestimate the value of a deal and sign contracts for socially wasteful products.

2.5 Conclusion

Bonus contracts create two distinct inefficiencies. On the one hand, bonus payments are typically sent out as checks that need to be issued and tracked, while non-monetary bonuses such as included premiums may give an imperfect fit to the consumer's actual preferences. On the other hand, bonus contracts yield an imbalanced decision situation— benefits are concentrated in the form of a single, large bonus payment while costs are dispersed over many small payments—in which focused thinkers tend to make suboptimal decisions.

We have shown that these inefficiencies are not eliminated by competition, but can only be overcome by regulation. Indeed, firms *have to* exploit attentional focusing under competitive pressure, so that bonus contracts are even more frequent on competitive than on monopolistic markets. By enhancing the use of bonus payments, competition even exacerbates the inefficiencies arising from contracting with

[19] Since our model assumes zero production costs, a product that consumers value at $v < 0$ is socially wasteful. While we assume $v \geq 0$, in fact our analysis also holds if v is negative, but sufficiently close to zero.

focused agents.[20]

From a policy perspective, our study suggests that a legal ban on bonus payments could have favorable consequences. On the one hand, a legal ban on bonus payments eliminates the inherent inefficiency of paying bonuses. On the other hand, it creates choice environments that are balanced, that is, where in equilibrium all payments receive the same amount of attention. Notably, making bonus payments is not necessary to encourage consumers to switch providers, as firms could instead lower the regular payments to attract consumers (see, Farrell and Klemperer, 2007, for a discussion of different modeling approaches). Hence, even if consumers incur costs for switching to another provider, a ban on bonus payments does not impair competitive forces. Altogether, we argue that prohibiting the use of bonus contracts does not only reduce the direct inefficiencies arising from bonus payments, but could also induce better decisions by consumers.

[20]Interestingly, firms standing in competition cannot benefit from exploiting the focusing bias. As in other behavioral models (see, for instance, DellaVigna and Malmendier, 2004; Gabaix and Laibson, 2006), competition drives down firms' profits to zero, *even though* consumers' decision biases are fully exploited.

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Appendix A: Proofs

For brevity, we denote $\tilde{v} := g(v)v$ the focus-weighted consumption value of the product. In addition, we suppress the consumption value dimension of a contract throughout the Appendix.

Proof of Lemma 2.1. The proof proceeds in two steps. First, we rewrite the monopolist's maximization problem and characterize the optimal payments to be made by consumers. Second, we argue that the monopolist offers either the maximal bonus (i.e., $\sum_{j=1}^{M} b_j = \bar{b}$) or no bonus. Third, we show that the monopolist pays at most one bonus.

1. STEP: In order to solve the monopolist's maximization problem, we set up the Lagrangian

$$\mathcal{L}(\boldsymbol{c}, \boldsymbol{\mu}, \eta, \boldsymbol{\gamma}, \lambda) := \sum_{i=1}^{N} p_i - \sum_{j=1}^{M} b_j - \lambda \left(\sum_{i=1}^{N} g(p_i)p_i - \tilde{v} - \sum_{j=1}^{M} g(b_j)b_j \right) - \eta \left(\sum_{j=1}^{M} b_j - \bar{b} \right) - \sum_{j=1}^{M} \gamma_j(-b_j) - \sum_{i=1}^{N} \mu_i(-p_i),$$

where $\lambda, \eta, \gamma_j, \mu_i \geq 0$, which yields the following Karush-Kuhn-Tucker Conditions:

$$\frac{\partial \mathcal{L}}{\partial p_i} = 1 - \lambda \left[g(p_i) + g'(p_i)p_i \right] + \mu_i \leq 0, \tag{KKT$_i^p$-1}$$

holding with equality if $p_i > 0$, and

$$\frac{\partial \mathcal{L}}{\partial b_j} = -1 + \lambda \left[g(b_j) + g'(b_j)b_j \right] - \eta + \gamma_j \begin{cases} \leq 0 & \text{if } b_j = 0, \\ = 0 & \text{if } 0 < b_j < \bar{b}, \\ \geq 0 & \text{if } b_j = \bar{b}, \end{cases} \tag{KKT$_j^b$-1}$$

as well as the condition on the participation constraint

$$\lambda \cdot \left(\sum_{i=0}^{N} g(p_i)p_i - \tilde{v} - \sum_{j=1}^{M} g(b_j)b_j \right) = 0, \tag{KKT-PC}$$

and conditions on price constraints, that is,

$$\mu_i \cdot (-p_i) = 0 \tag{KKT$_i^p$-2}$$

for any $i \in \{1, \ldots, N\}$, and conditions on bonus constraints, that is,

$$\gamma_j \cdot (-b_j) = 0 \tag{KKT$_j^b$-2}$$

for any $j \in \{1, \ldots, M\}$ as well as

$$\eta \left(\sum_{j=1}^{M} b_j - \bar{b} \right) = 0. \tag{KKT-BC}$$

First, we characterize the optimal payments to be made by consumers (i.e., part (i) of our lemma). We observe that at least one p_i has to be larger than zero, as otherwise $\lambda = 0$ by (KKT-PC) and therefore $\frac{\partial \mathcal{L}}{\partial p_i} > 0$ by (KKT$_i^p$-1); a contradiction. Hence, from now on suppose $p_i > 0$ for some $i \in \{1, \ldots, N\}$. Then, since $1 + \mu_i > 0$, Condition (KKT$_i^p$-1) gives $\lambda > 0$. Together with (KKT-PC), this yields

$$\sum_{i=1}^{N} g(p_i) p_i = \tilde{v} + \sum_{j=1}^{M} g(b_j) b_j. \tag{2.2}$$

Next, we show that this implies $p_i > 0$ for any $i \in \{1, \ldots, N\}$. For the sake of contradiction, suppose $p_j = 0$ for some $j \in \{1, \ldots, N\}$. Then, Condition (KKT$_j^p$-1) yields $1 + \mu_j \leq \lambda g(0)$. As there is at least one $p_i > 0$, Conditions (KKT$_i^p$-1) and (KKT$_i^p$-2) yield $1 = \lambda \Big[g(p_i) + g'(p_i) p_i \Big]$. Together, these considerations give

$$1 \leq 1 + \mu_j \leq \lambda \underbrace{\Big[g(0) + g'(0) 0 \Big]}_{= g(0)} \overset{\text{A.3}}{<} \lambda \left[g(p_i) + g'(p_i) p_i \right] = 1, \tag{2.3}$$

a contradiction. Thus, we have $p_i > 0$ for any $i \in \{1, \ldots, N\}$. In addition, Conditions (KKT$_i^p$-1) and (KKT$_j^p$-1) require $p_i = p_j$ for $i, j \in \{1, \ldots, N\}$, which completes the proof of part (i).

2. STEP: Given the results derived above, we show that *the monopolist either offers the maximal bonus (i.e., $\sum_{j=1}^{M} b_j = \bar{b}$) or no bonus at all.* For the sake of a contradiction, suppose that the monopolist offers a contract with $0 < \sum_{j=1}^{M} b_j < \bar{b}$. Thus, we have $\eta = 0$ by (KKT-BC), and, for any $b_j > 0$, also $\gamma_j = 0$ by (KKT$_j^b$-2). Using the same arguments as in the first step, we conclude from Conditions (KKT$_i^p$-1) and (KKT$_j^b$-1) that either $b_j = p_i = p'$ or $b_j = 0$. Let $m \in \{1, \ldots, M\}$ bonus payments be non-zero and notice that $m < N$, as otherwise profits would be zero.

Then, (KKT-PC) implies that $g(p')p' = \tilde{v}/(N-m)$, so that the monopolist earns

$$\pi' = \frac{\tilde{v}}{g(p')} - m \cdot c. \tag{2.4}$$

Suppose that the monopolist instead does not offer any bonus payments; that is, $b_j = 0$ for any $j \in \{1, \dots, M\}$. By the first step, we have $p_i = p''$ for any $i \in \{1, \dots, N\}$, and Condition (KKT-PC) yields $g(p'')p'' = \tilde{v}/N$, so that the monopolist earns

$$\pi'' = \frac{\tilde{v}}{g(p'')}. \tag{2.5}$$

Since $g(x)x$ is a strictly increasing function, by Assumption 2, we conclude $p'' < p'$ from

$$g(p'')p'' = \frac{\tilde{v}}{N} < \frac{\tilde{v}}{N-m} = g(p')p'.$$

Then, for any $c \geq 0$, we obtain $\pi'' > \pi'$ by Assumption 2; a contradiction. As a consequence, the monopolist will never offer a contract with $0 < \sum_{j=1}^{M} b_j < \bar{b}$.

3. STEP: Given the results from the preceding steps, we next show that *the monopolist offers at most one bonus payment.* Suppose that $\sum_{j=1}^{M} b_j = \bar{b}$ and that $m \geq 1$ bonus payments are non-zero. By the first step, we have $p_i = p'''(m)$, $i \in \{1, \dots, N\}$, and (KKT-PC) yields

$$g(p'''(m))p'''(m) = \frac{1}{N} \cdot \left[\tilde{v} + \sum_{j=1}^{m} g(b_j)b_j\right]. \tag{2.6}$$

For any $m > 1$, Assumption 2 immediately implies that

$$g(\bar{b})\bar{b} = \sum_{j=1}^{m} g(\bar{b})b_j \overset{\text{A.2}}{>} \sum_{j=1}^{m} g(b_j)b_j$$

holds. Thus, by Assumption 2 and Eq. (2.6), we have $p'''(1) > p'''(m)$ for any $m > 1$. As the bonus payment is fixed and as less bonus payments imply lower costs, the monopolist will choose at most one bonus payment, which was to be proven. □

Proof of Proposition 2.1. By Lemma 2.1, the monopolist offers either a bonus contract with a single bonus payment, $\boldsymbol{c}^{bon} := (\bar{b}, 0, \dots, 0, p^{bon}, \dots, p^{bon})$, or a contract without any bonus payments, $\boldsymbol{c}^{no} := (0, \dots, 0, p^{no}, \dots, p^{no})$. We have also seen in the proof of Lemma 1 that $p^{bon} = p^{bon}(v, \bar{b})$ is implicitly defined by $g(p^{bon})p^{bon} =$

$\frac{1}{N}\Big[\tilde{v}+g(\bar{b})\bar{b}\Big]$, and that $p^{no} = p^{no}(v)$ is implicitly given by $g(p^{no})p^{no} = \frac{\tilde{v}}{N}$. We proceed in two steps. First, we neglect the cost of paying a bonus, that is, we set $c = 0$. Second, we allow for positive costs of paying a bonus, that is, $c > 0$.

1. STEP: Let $c = 0$. Then, the monopolist offers a bonus contract $\boldsymbol{c}^{bon}$ if and only if

$$\frac{\tilde{v} + \Big(g(\bar{b}) - g(p^{bon})\Big)\,\bar{b}}{g(p^{bon})} > \frac{\tilde{v}}{g(p^{no})}$$

or, equivalently,

$$\bar{b}\,\Big(g(\bar{b}) - g(p^{bon})\Big) > \tilde{v}\,\left(\frac{g(p^{bon}) - g(p^{no})}{g(p^{no})}\right). \tag{2.7}$$

We proceed as follows: first, we verify that $\pi(\boldsymbol{c}^{bon}) - \pi(\boldsymbol{c}^{no})$ monotonically decreases in v, which implies that (2.7) is more likely to hold for small values of v. Second, we argue that (2.7) is violated as v approaches infinity while it is fulfilled as v approaches zero.

Recall that $p^{bon} > p^{no}$. Then, by the Implicit Function Theorem, we obtain

$$\begin{aligned}\frac{\partial}{\partial v}\Big(\pi(\boldsymbol{c}^{bon}) - \pi(\boldsymbol{c}^{no})\Big) &= N \cdot \left(\frac{\partial}{\partial v}p^{bon}(v,\bar{b}) - \frac{\partial}{\partial v}p^{no}(v)\right) \\ &= \frac{\partial \tilde{v}}{\partial v} \cdot \left(\frac{1}{g(p^{bon}) + g'(p^{bon})p^{bon}} - \frac{1}{g(p^{no}) + g'(p^{no})p^{no}}\right),\end{aligned}$$

which is strictly negative by Assumption 2.3 and $p^{bon} > p^{no}$. Thus, $\pi(\boldsymbol{c}^{bon}) - \pi(\boldsymbol{c}^{no})$ monotonically decreases in v, which was to be proven.

Next, suppose that v approaches infinity, and notice that the left-hand side of (2.7) is negative for sufficiently large values of v, while the right-hand side of (2.7) is non-negative for any $v \geq 0$. Hence, (2.7) is violated in the limit of v approaching infinity. Finally, we consider the limit for v approaching zero. By Assumption 2.2, this implies that also $\tilde{v} := g(v)v$ approaches zero. First, it is easy to see that in this limit the left-hand side of Inequality (2.7) is strictly larger than zero, as

$$\lim_{v\to 0} g\Big(p^{bon}(v,\bar{b})\Big)p^{bon}(v,\bar{b}) = \frac{g(\bar{b})\bar{b}}{N} < g(\bar{b})\bar{b},$$

and therefore $\lim_{v\to 0} p^{bon}(v,\bar{b}) < \bar{b}$. Second, the right-hand side of Inequality (2.7) is zero in the limit of v approaching zero since $g(0) > 0$.

Combining the above results and using the fact that $\pi(\boldsymbol{c}^{bon}) - \pi(\boldsymbol{c}^{no})$ is continuous in v, we conclude by the Intermediate Value Theorem that there exists some threshold value $v' > 0$ such that the monopolist offers a bonus contract if and only if $v < v'$.

2. STEP: Let $c > 0$. Then, the monopolist offers a bonus contract $\boldsymbol{c}^{bon}$ if and only if

$$\frac{\tilde{v} + \Big(g(\bar{b}) - g(p^{bon})\Big)\,\bar{b}}{g(p^{bon})} - c > \frac{\tilde{v}}{g(p^{no})}$$

or, equivalently,

$$c < \underbrace{\frac{\bar{b}\,\Big(g(\bar{b}) - g(p^{bon})\Big)}{g(p^{bon})} - \tilde{v}\left(\frac{g(p^{bon}) - g(p^{no})}{g(p^{no})g(p^{bon})}\right)}_{=\pi(\boldsymbol{c}^{bon})\big|_{c=0} - \pi(\boldsymbol{c}^{no})}. \tag{2.8}$$

We have already seen in the first step that the right-hand side of Inequality (2.8) monotonically decreases in v. Hence, the monopolist offers a bonus contract for some $v > 0$ only if

$$\begin{aligned} c &< \lim_{v\to 0}\left[\frac{\bar{b}\left[g(\bar{b}) - g\Big(p^{bon}(v,\bar{b})\Big)\right]}{g\Big(p^{bon}(v,\bar{b})\Big)} - \tilde{v}\left(\frac{g\Big(p^{bon}(v,\bar{b})\Big) - g\Big(p^{no}(v)\Big)}{g\Big(p^{no}(v)\Big)g\Big(p^{bon}(v,\bar{b})\Big)}\right)\right] \\ &= \lim_{v\to 0}\frac{\bar{b}\left[g(\bar{b}) - g\Big(p^{bon}(v,\bar{b})\Big)\right]}{g\Big(p^{bon}(v,\bar{b})\Big)} \\ &=: \hat{c}. \end{aligned}$$

By the same arguments as in the first step, for any $c < \hat{c}$, there exists some $\hat{v}(c)$ such that the monopolist offers a bonus contract if and only if $v < \hat{v}(c)$, which was to be proven.

Specifically, the function $\hat{v} : [0, \hat{c}) \to \mathbb{R}_+$ is implicitly given by

$$\underbrace{\frac{\tilde{v}(\hat{v}) + \left[g(\bar{b}) - g\Big(p^{bon}(\hat{v},\bar{b})\Big)\right]\bar{b}}{g\Big(p^{bon}(\hat{v},\bar{b})\Big)} - \frac{\tilde{v}(\hat{v})}{g\Big(p^{no}(\hat{v})\Big)} - c}_{=:F(\hat{v},c)} = 0.$$

By construction, we have $\hat{v}(\hat{c}) = 0$. In addition, the Implicit Function Theorem yields

$$\frac{\partial \hat{v}}{\partial c} = -\frac{\frac{\partial}{\partial c} F(\hat{v}, c)}{\frac{\partial}{\partial \hat{v}} F(\hat{v}, c)} = -\frac{(-1)}{\dfrac{\partial}{\partial \hat{v}} \Big[\pi(\boldsymbol{c}^{bon}) \Big|_{c=0} - \pi(\boldsymbol{c}^{no}) \Big]} < 0,$$

since we have seen above that the denominator is strictly negative. This completes the proof. □

Proof of Proposition 2.2. For illustrative purposes, we only consider the case of $L = 2$, but we solve for the essentially unique equilibrium without restricting ourselves to symmetric equilibria. The generalization of the arguments to the case of $L > 2$ is straightforward when restricting the analysis to symmetric equilibria.

The proof proceeds in seven steps. First, we show that the standard Bertrand logic applies, so that in equilibrium firms earn zero profits, and consumers are indifferent between both offers. Second, we show that equilibrium payments made by consumers are equally spread across periods. Third, we argue that both firms charge the same regular payments, which also implies that both offer the same overall bonus. Fourth, we show that firms offer at most one bonus payment, which in turn implies that both firms offer essentially the same contract. Fifth, we argue that firms either offer the maximal bonus or no bonus. Sixth, we show that firms offer a bonus in equilibrium. Seventh, we prove that a unique equilibrium exists.

1. STEP: We show that *firms earn zero profits in any equilibrium.* For the sake of a contradiction, suppose firm $k \in \{1, 2\}$ earns strictly positive profits in equilibrium, which implies

$$\tilde{v} + \sum_{i=1}^{M} g(\Delta_j^b) b_j^k \geq \sum_{i=1}^{N} g(\Delta_i^p) p_i^k, \text{ and } \sum_{i=1}^{M} g(\Delta_j^b)(b_j^k - b_j^{-k}) \geq \sum_{i=1}^{N} g(\Delta_i^p)(p_i^k - p_i^{-k}),$$
$$\text{and } \sum_{i=1}^{N} p_i^k > \sum_{j=1}^{M} b_j^k.$$

Hence, we have $p_i^k > 0$ for at least one $i \in \{1, \ldots, N\}$. Without loss of generality, let $p_1^k > 0$ and $\sum_{i=1}^{N} p_i^k - \sum_{j=1}^{M} \Big[b_j^k + \mathbb{1}[b_j^k > 0] \cdot c \Big] \geq \sum_{i=1}^{N} p_i^{-k} - \sum_{j=1}^{M} \Big[b_j^{-k} + \mathbb{1}_{\mathbb{R}_{>0}}(b_j^k) \cdot c \Big]$. This immediately implies that firm $-k$ earns at most

$$(1 - D_k) \cdot \left(\sum_{i=1}^{N} p_i^k - \sum_{j=1}^{M} \Big[b_j^k + \mathbb{1}_{\mathbb{R}_{>0}}(b_j^k) \cdot c \Big] \right) \tag{2.9}$$

for some $D_k \leq 1$. By deviating to another contract $\boldsymbol{c}^{-k} = (b_1^k, \ldots, b_M^k, p_1^k - \epsilon, \ldots, p_N^k)$ for some $\epsilon > 0$, firm $-k$ can earn $\sum_{i=1}^N p_i^k - \sum_{j=1}^M \left[b_j^k + \mathbb{1}_{\mathbb{R}_{>0}}(b_j^k) \cdot c \right] - \epsilon$, which exceeds (2.9) for ϵ sufficiently small. Hence, firm $-k$ has an incentive to deviate; a contradiction. As a consequence, firms earn zero profits in equilibrium. Finally, it is straightforward to see that, in equilibrium, consumers are indifferent between both firms' offers. Otherwise, the firm that serves the market could slightly adjust its contract and earn strictly positive profits.

2. STEP: We show that *all payments to be made by consumers are of the same size, that is,* $p_i^k = p^k$ *for any* $i \in \{1, \ldots, N\}$. For the sake of a contradiction, suppose that there exist $i, j \in \{1, \ldots, N\}$ such that firm k offers a contract with $p_i^k \neq p_j^k$ in equilibrium. In this case, maximal payment $p_{\max} := \min\{p_1^k, \ldots, p_N^k\}$ strictly exceeds minimal payment $p_{\min} := \min\{p_1^k, \ldots, p_N^k\}$. Without loss of generality, let $p_{\max} = p_1^k$ and $p_{\min} = p_2^k$. As firms earn zero profits in equilibrium, firm $-k$ could profitably deviate to a contract $\tilde{\boldsymbol{c}}^{-k} = (b_1^k, \ldots, b_M^k, p_1^k - \epsilon, p_2^k + \epsilon + \epsilon', p_3^k, \ldots, p_N^k)$ for some $\epsilon, \epsilon' > 0$ such that $p_1^k > p_2^k + \epsilon + \epsilon'$. Obviously, all consumers choose contract $\tilde{\boldsymbol{p}}^{-k}$ if

$$g(p_1^k)[p_1^k - \epsilon] + g(p_2^k + \epsilon + \epsilon')[p_2^k + \epsilon + \epsilon'] + \sum_{l=3}^{N} g(\Delta_l^p) p_l^k < \sum_{l=1}^{N} g(\Delta_l^p) p_l^k,$$

or, equivalently,

$$g(p_1^k)p_1^k - g(p_1^k)[p_1^k - \epsilon] > g(p_2^k + \epsilon + \epsilon')[p_2^k + \epsilon + \epsilon'] - g(p_2^k + \epsilon + \epsilon')p_2^k.$$

Rearranging this inequality yields

$$g(p_1^k)\epsilon > g(p_2^k + \epsilon + \epsilon')[\epsilon + \epsilon'],$$

which is satisfied for ϵ' sufficiently small. Hence, firm $-k$ indeed has a profitable deviation; a contradiction. As a consequence, in equilibrium, we must have $p_i^k = p_j^k$ for any two payments $i, j \in \{1, \ldots, N\}$, and any firm $k \in \{1, 2\}$.

3. STEP: We show that *both firms offer the same regular payments, that is,* $p_i^k = p$ *for any* $k \in \{1, 2\}$ *and any* $i \in \{1, \ldots, N\}$. For the sake of a contradiction, let $p_i^k = p^k > p^{-k} = p_i^{-k}$. Since consumers are indifferent between both contracts, we conclude that $\sum_{j=1}^M b_j^k > \sum_{j=1}^M b_j^{-k}$. Hence, at least one bonus payment of firm k exceeds the corresponding bonus payment of firm $-k$. Without loss of generality,

let $b_1^k > b_1^{-k}$.

In a first step, we argue that firm k offers a single bonus payment. For the sake of a contradiction, suppose further that firm k offers at least two bonus payments in equilibrium. Then, firm k could profitably deviate to a contract $\tilde{\boldsymbol{c}}^k = (\sum_{j=1}^M b_j^k, 0, \ldots, 0, p^k + \epsilon, p^k, \ldots, p^k)$ for some $\epsilon > 0$ since all consumers choose the contract $\tilde{\boldsymbol{c}}^k$ if

$$\begin{aligned}
&-g(p^k)(N-1)p^k - g(p^k+\epsilon)[p^k+\epsilon] + g\left(\sum_{j=1}^M b_j^k\right)\left(\sum_{j=1}^M b_j^k\right) \\
&\quad= -g(p^k)Np^k + \sum_{j=1}^M g(\max\{b_j^k, b_j^{-k}\})b_j^k \\
&\qquad - p^k[g(p^k+\epsilon) - g(p^k)] - g(p^k+\epsilon)\epsilon + \sum_{j=1}^M \left[g\left(\sum_{j=1}^M b_j^k\right) - g(\max\{b_j^k, b_j^{-k}\})\right] b_j^k \\
&\quad> -g(p^k)Np^{-k} + \sum_{j=1}^M g(\max\{b_j^k, b_j^{-k}\})b_j^{-k} \\
&\qquad - p^{-k}[g(p^k+\epsilon) - g(p^k)] + \left[g\left(\sum_{j=1}^M b_j^k\right) - g(\max\{b_1^k, b_1^{-k}\})\right] b_1^{-k} \\
&\qquad + \sum_{j=2}^M \left[g(b_j^{-k}) - g(\max\{b_j^k, b_j^{-k}\})\right] b_j^{-k} \\
&\quad= -g(p^k)(N-1)p^{-k} - g(p^k+\epsilon)p^{-k} + g\left(\sum_{j=1}^M b_j^k\right) b_1^{-k} + \sum_{j=2}^M g(b_j^{-k})b_j^{-k}.
\end{aligned}$$

As consumers must be indifferent between both contracts in equilibrium, we have

$$-g(p^k)Np^k + \sum_{j=1}^M g(\max\{b_j^k, b_j^{-k}\})b_j^k = -g(p^k)Np^{-k} + \sum_{j=1}^M g(\max\{b_j^k, b_j^{-k}\})b_j^{-k},$$

so that the above inequality holds if and only if

$$\begin{aligned}
&\overbrace{g(p^k+\epsilon)\epsilon}^{\to 0 \text{ as } \epsilon\to 0} + [p^k - p^{-k}]\overbrace{[g(p^k+\epsilon) - g(p^k)]}^{\to 0 \text{ as } \epsilon\to 0} < \overbrace{\left[g\left(\sum_{j=1}^M b_j^k\right) - g(\max\{b_1^k, b_1^{-k}\})\right]}^{>0 \text{ by A.2}} \overbrace{[b_1^k - b_1^{-k}]}^{>0} \\
&\qquad + \underbrace{\sum_{j=2}^M \left[g\left(\sum_{j=1}^M b_j^k\right) - g(\max\{b_j^k, b_j^{-k}\})\right] b_j^k}_{>0 \text{ by our assumption towards a contradiction}} - \sum_{j=2}^M \underbrace{\left[g(b_j^{-k}) - g(\max\{b_j^k, b_j^{-k}\})\right]}_{\leq 0 \text{ by A.2}} b_j^{-k};
\end{aligned}$$

that is, if and only if ϵ is sufficiently small; a contradiction. Thus, given our initial assumption that k charges higher regular payments than $-k$, firm k must offer a single bonus payment. Thus, from now on, let $b_j^k = 0$ for any $j \neq 1$, and notice that $b_1^k > p^k$, as otherwise firm k would earn positive profits

In a second step, we argue that—given that firm k offers a single bonus payment and higher regular payments—firm $-k$ could profitably deviate to a contract

$$\tilde{\boldsymbol{c}}^{-k} = (b_1^{-k} + \epsilon, b_2^{-k}, \ldots, b_M^{-k}, p^{-k} + \epsilon + \epsilon', p^{-k}, \ldots, p^{-k})$$

for some $\epsilon, \epsilon' > 0$ such that $b_1^k > b_1^{-k} + \epsilon$ and $p_1^{-k} + \epsilon + \epsilon' < p_1^k$ since all consumers choose $\tilde{\boldsymbol{c}}^{-k}$ if

$$-g(p^k)[Np^{-k} + \epsilon + \epsilon'] + g(b_1^k)[b_1^{-k} + \epsilon] + \sum_{j=2}^{M} g(b_j^{-k})b_j^{-k} > -g(p^k)Np^k + g(b_1^k)b_1^k$$

or, equivalently,

$$-g(p^k)[\epsilon + \epsilon'] + g(b_1^k)\epsilon > \underbrace{g(p^k)N[p^{-k} - p^k] + g(b_1^k)[b_1^k - b_1^{-k}] - \sum_{j=2}^{M} g(b_j^{-k})b_j^{-k}}_{=0 \text{ as consumers must be indifferent between contracts}}.$$

This inequality is satisfied for ϵ' sufficiently small since $g(b_1^k) > g(p^k)$; a contradiction. As a consequence, in equilibrium, both firms offer the same regular payments. This further implies that $\sum_{j=1}^M b_j^k = \sum_{j=1}^M b_j^{-k}$, as otherwise at least one firm would earn positive profits; that is, either both firms offer a bonus contract or none does so.

4. STEP: We show that *firms offer at most one bonus payment in equilibrium*. For the sake of a contradiction, suppose that firm k offers at least two bonus payments. By STEP 3, we have $\sum_{j=1}^M b_j^k = \sum_{j=1}^M b_j^{-k}$, and therefore $\sum_{j=1}^M b_j^{-k} > b_j^k$ for any $j \in \{1, \ldots, N\}$. Denote the payment to be made by consumers in each period by p, which is the same across periods by STEP 2 and the same across firms by STEP 3. Then, firm $-k$ could profitably deviate to $\tilde{\boldsymbol{c}}^{-k} = (\sum_{j=1}^M b_j^{-k}, 0, \ldots, 0, p + \epsilon, p, \ldots, p)$

for some $\epsilon > 0$ since all consumers choose $\tilde{\boldsymbol{c}}^{-k}$ if

$$-g(p)(N-1)p - g(p+\epsilon)[p+\epsilon] + g\left(\sum_{j=1}^{M} b_j^{-k}\right)\left(\sum_{j=1}^{M} b_j^{-k}\right) > -g(p)(N-1)p - g(p+\epsilon)p + g\left(\sum_{j=1}^{M} b_j^{-k}\right) b_1^k + \sum_{j=2}^{M} g(b_j^k)b_j^k,$$

or, equivalently,

$$g(p+\epsilon)\epsilon < \overbrace{g\left(\sum_{j=1}^{M} b_j^{-k}\right)\left(\sum_{j=1}^{M} b_j^{-k}\right) - \underbrace{\left[g\left(\sum_{j=1}^{M} b_j^{-k}\right) b_1^k + \sum_{j=2}^{M} g(b_j^k)b_j^k\right]}_{<\sum_{j=1}^{M} g\left(\sum_{j=1}^{M} b_j^{-k}\right) b_j^k \text{ by A.2}}}^{>0},$$

which holds if and only if ϵ is sufficiently small; a contradiction. Thus, firms offer at most one bonus payment in equilibrium.

5. STEP: We show that *firms either offer the maximal bonus or no bonus at all.* We already know that each firm offers at most one bonus and that bonus firms offer the same overall bonus payment. Without loss of generality, we can assume that both firms use the same bonus attribute; that is, we can solve the game as if there is only one bonus attribute, say, $b_1^k = b_1$. Again, denote the payment to be made by consumers in each period by p.

For the sake of a contradiction, suppose that $0 < b_1 < \bar{b}$ in equilibrium. Then, firm k could profitably deviate to a contract $\tilde{\boldsymbol{c}}^k = (b_1+\epsilon, 0, \ldots, 0, p+\frac{\epsilon+\epsilon'}{N}, \ldots, p+\frac{\epsilon+\epsilon'}{N})$ for some $\epsilon, \epsilon' > 0$ such that $\bar{b} \geq b_1 + \epsilon > p + \frac{\epsilon+\epsilon'}{N}$ since all consumers choose the contract $\tilde{\boldsymbol{c}}^k$ if

$$-g\left(p+\frac{\epsilon+\epsilon'}{N}\right) N \left[p+\frac{\epsilon+\epsilon'}{N}\right] + g(b_1+\epsilon)[b_1+\epsilon] > -g\left(p+\frac{\epsilon+\epsilon'}{N}\right) Np + g(b_1+\epsilon)b_1,$$

or, equivalently,

$$g(b_1+\epsilon)\epsilon > g\left(p+\frac{\epsilon+\epsilon'}{N}\right)[\epsilon+\epsilon'].$$

This inequality is satisfied for ϵ' sufficiently small since $g(b_1+\epsilon) > g\left(p+\frac{\epsilon+\epsilon'}{N}\right)$. As a consequence, firms either pay the maximal bonus or no bonus at all.

6. STEP: Notice that there are only two equilibrium candidates left (again we assume that both firms use the same bonus attribute, which is in fact without loss): ei-

ther both firms offer $\boldsymbol{c}^{no} = (0, \dots, 0)$ or both firms offer $\boldsymbol{c}^{bon} = \left(\bar{b}, 0, \dots, 0, \frac{c+\bar{b}}{N}, \dots, \frac{c+\bar{b}}{N}\right)$. We show that *both firms offering* $\boldsymbol{c}^{no}$ *cannot be an equilibrium.*

For the sake of contradiction, suppose that both firms offer the contract $\boldsymbol{c}^{no}$ in equilibrium. Now, firm k could profitably deviate to a contract

$$\tilde{\boldsymbol{c}}^k = \left(\bar{b}, 0, \dots, 0, \frac{c+\bar{b}}{N} + \epsilon, \frac{c+\bar{b}}{N}, \dots, \frac{c+\bar{b}}{N}\right)$$

for some $\epsilon > 0$ since all consumers choose $\tilde{\boldsymbol{c}}^k$ if

$$\underbrace{-g\left(\frac{c+\bar{b}}{N}\right)(N-1)\left(\frac{c+\bar{b}}{N}\right) - g\left(\frac{c+\bar{b}}{N}+\epsilon\right)\left(\frac{c+\bar{b}}{N}+\epsilon\right)}_{\to -g\left(\frac{c+\bar{b}}{N}\right)[c+\bar{b}] \text{ as } \epsilon \to 0} + g(\bar{b})\bar{b} > 0,$$

which holds for sufficiently small ϵ by Eq. (2.1). Hence, both firms offering $\boldsymbol{c}^{no}$ is not an equilibrium, which was to be proven.

7. STEP: It remains to be proven that *both firms offering contract* $\boldsymbol{c}^{bon}$ *is indeed an equilibrium.* We show that firm k has no incentive to deviate. In order to attract consumers, firm k has to reduce some payment p_i^k for $i \in \{1, \dots, N\}$ by an amount $\epsilon > 0$, as increasing the bonus payment is not feasible. In order to benefit from this deviation, it has to increase some other payments p_j^k, $j \neq i$, to be made by consumers, or decrease the bonus payment b_1^k by an overall amount $\epsilon' > \epsilon$. As $g(\cdot)$ is increasing by Assumption 2.2, the most effective way of increasing payments is to equally spread ϵ' over all payments to be made by consumers, namely p_j^k for $j \neq i$. Then, the price cut ϵ is weighted by $g\left(\frac{c+\bar{b}}{N}\right)$, while each price increase $\frac{\epsilon'}{N-1}$ is weighted by $g\left(\frac{c+\bar{b}}{N} + \frac{\epsilon'}{N-1}\right)$. Thus, this deviation attracts consumers if and only if

$$g\left(\frac{c+\bar{b}}{N}\right)\epsilon > g\left(\frac{c+\bar{b}}{N} + \frac{\epsilon'}{N-1}\right)\epsilon',$$

which can only be satisfied for $\epsilon > \epsilon'$; a contradiction. Hence, firm k has no incentive to deviate, so that both firms offering the contract $\boldsymbol{c}^{bon}$ is an equilibrium. Since this was the last remaining equilibrium candidate, the equilibrium is unique.

Finally, consider the case of $L > 2$ firms. It is straightforward to show that in the essentially unique symmetric equilibrium all firms offer the contract $\boldsymbol{c}^{bon}$. This completes the proof. □

Appendix B: Endogenous Attribute Space

B.1: Model

We extend our baseline model from Section 2 in the following two ways: suppose first that the number of bonus payments and the number of regular payments are unbounded and second that consumers buying at firm k incur transaction costs, $\tau(N^k)$, depending on the number of non-zero regular payments specified in firm k's contract, which we denote as N^k.

We assume that the transaction costs are strictly increasing and convex in the number of non-zero regular payments. For technical reasons and without loss of generality, we treat τ as a twice continuously differentiable function from $\mathbb{R}_+$ to $\mathbb{R}_+$ and we further assume that $\tau', \tau'' > 0$. In addition, we impose certain conditions on the cost function: (i) $\tau(2) + 2g\left(\frac{c+\bar{b}}{2}\right)\left(\frac{c+\bar{b}}{2}\right) < g(\bar{b})\bar{b}$, (ii) $\tau'(2) < g'\left(\frac{c+\bar{b}}{2}\right)\left(\frac{c+\bar{b}}{2}\right)^2$, and (iii) $\lim_{N\to\infty}\tau'(N) = \infty$. Notice that Assumptions (i) and (ii) are the natural extensions of Condition (2.1), which we imposed on the inefficiency arising from paying a bonus in order to allow firms to increase a consumer's focus-weighted utility using a bonus payment and to break even at the same time. Assumption (iii) is a typical Inada-Condition to ensure that a profit-maximizing number of regular payments exists.

The remainder of Appendix B is organized as follows. In Section B.2 we derive a monopolist's optimal contract offer. In Section B.3 we characterize (symmetric) competitive equilibria and compare the competitive outcome to the monopolistic one. Importantly, as long as transaction costs are sufficiently convex, our main result (i.e., Corollary 2.1) is robust to this extension.

B.2: Monopolistic Market

The monopolist's optimal contract offer is characterized in the following lemma.

Lemma 2.2. *A contract maximizes the monopolist's profit only if*

(i) the regular payments made by consumers are equally spread across N^{mon} periods, that is, $p_i(N^{mon}) = p(N^{mon})$ for any $i \in \{1, \dots, N^{mon}\}$, whereby $N^{mon} \in \{\lfloor N^* \rfloor, \lceil N^* \rceil\}$ and N^* is the unique solution to

$$g'(p(N))p(N)^2 = \tau'(N),$$

(ii) and, if bonus payment(s) are made, the maximal bonus is paid using a single payment.

Proof. In order to prove the statement, we can make use of the insights derived in Lemma 2.1, where we have characterized the optimal contract offer for a fixed number of regular and bonus payments, respectively. Indeed, the second part immediately follows from Lemma 2.1 since, even if the number of potential bonus payments is fixed, the monopolist will not want to pay more than one bonus. Thus, it remains to be shown that also (i) holds.

Remember that we have seen in the proof of Lemma 2.1 that regular payments have to be of equal size and that there is either a single bonus payment that is maximal or none bonus at all. In addition, we know that for a given number of regular payments, $N \in \mathbb{N}$, it has to hold that

$$Ng(p(N))p(N) = \tilde{V} - \tau(N), \tag{2.10}$$

where $\tilde{V} = \tilde{v} + g(\bar{b})\bar{b}$ if a bonus is paid and $\tilde{V} = \tilde{v}$ otherwise. Notice that consumers are willing to buy at a price of zero only if $N \leq \tau^{-1}(\tilde{V}) =: \overline{N}^{mon}$, where τ^{-1} is the inverse of the transaction cost function, which indeed exists as τ is strictly increasing. This in turn implies that the optimal regular payments are characterized by (2.10) as long as N lies weakly below $\overline{N}^{mon}$.

Now ignore the integer constraint for a moment and suppose that $N \in (0, \overline{N}^{mon})$ holds. Then, when applying the Implicit Function Theorem to (2.10), we obtain

$$p'(N) = -\frac{1}{N}\frac{g(p(N))p(N) + \tau'(N)}{g(p(N)) + g'(p(N))p(N)} < 0. \tag{2.11}$$

Since the size of the bonus payment is independent of N and as both $N = 0$ and $N = \overline{N}^{mon}$ imply zero profit, the monopolist chooses N as to maximize $Np(N)$ subject to $N \in (0, \overline{N}^{mon})$. In addition, as the function $Np(N)$ is continuous in N on the interval $(0, \overline{N}^{mon})$ and also strictly positive by (2.10), it has at least one local maximum in this interval, so that—ignoring integer constraints—the optimal number of regular payments solves

$$
\begin{aligned}
0 &= p(N) + Np'(N), \\
&= p(N) - \frac{g(p(N))p(N) + \tau'(N)}{g(p(N)) + g'(p(N))p(N)} \\
&= \frac{1}{g(p(N)) + g'(p(N))p(N)} \cdot \Big[g'(p(N))p(N)^2 - \tau'(N) \Big].
\end{aligned} \tag{2.12}
$$

Here, the second equality follows from (2.11) and the last equality is a simple rearrangement. Hence, we conclude that the optimal number of payments has to solve

$$
g'(p(N))p(N)^2 = \tau'(N). \tag{2.13}
$$

Since Assumption 2.3 and Eq. (2.11) imply that the left-hand side of (2.13) strictly decreases in N and since $\tau'' > 0$ implies that the right-hand side of (2.13) strictly increases in N, there exists a unique solution to (2.13), which further implies that $Np(N)$ has a unique local maximum, N^*, on the interval $(0, \overline{N}^{mon})$. Finally, as $Np(N)$ strictly increases (decreases) for any $N < N^*$ ($N > N^*$), the statement follows immediately when taking the integer constraint into account. □

Before we can prove the analogue to Proposition 2.1, the next lemma derives further properties of the monopolist's optimal contract that will be useful in the proof later on.

Lemma 2.3. *The monopolist's contract offer delineated in Lemma 2.2 satisfies:*

(i) $\frac{\partial N^*}{\partial v} > 0$ *and* $\lim_{v \to \infty} \frac{\partial N^*}{\partial v} = 0$.

(ii) There exists some $v' \in \mathbb{R}_+$ *such that for any* $v > v'$ *we have* $\frac{\partial N^{mon}}{\partial v} = 0$.

(iii) There exists some $v'' \in \mathbb{R}_+$ *such that for any* $v > v''$ *we have* $p(N^{mon}) > \bar{b}$.

(iv) There is some $\overline{\tau} \in \mathbb{R}_+$ *so that for any cost function with* $\tau''(\cdot) > \overline{\tau}$ *the monopolist chooses the same number of regular payments irrespective of whether she pays a bonus or not.*

Proof. First, we derive some preliminary results. Subsequently, we directly prove the statements.

PRELIMINARIES: First, when applying the Implicit Function Theorem to (2.10), we obtain

$$
\frac{\partial}{\partial \tilde{V}} p(N, \tilde{V}) = \frac{1}{N} \frac{1}{g'(p(N, \tilde{V}))p(N, \tilde{V}) + g(p(N, \tilde{V}))}. \tag{2.14}
$$

Second, when applying the Implicit Function Theorem to (2.13), we obtain

$$\begin{aligned}
\frac{dN^*}{d\tilde{V}} &= -\frac{g''(p(N^*,\tilde{V}))p(N^*,\tilde{V})^2\frac{\partial p}{\partial \tilde{V}} + 2g'(p(N^*,\tilde{V}))p(N^*,\tilde{V})\frac{\partial p}{\partial \tilde{V}}}{g''(p(N^*,\tilde{V}))p(N^*,\tilde{V})^2\frac{\partial p}{\partial N} + 2g'(p(N^*,\tilde{V}))p(N^*,\tilde{V})\frac{\partial p}{\partial N} - \tau''(N^*)} \\
&= -\left(\frac{\partial p/\partial \tilde{V}}{\partial p/\partial N}\right)\cdot\left(\frac{1}{1-\frac{\tau''(N^*)}{g''(p(N^*,\tilde{V}))p(N^*,\tilde{V})^2\frac{\partial p}{\partial N}+2g'(p(N^*,\tilde{V}))p(N^*,\tilde{V})\frac{\partial p}{\partial N}}}\right) \\
&= \left(\frac{1}{g(p(N^*,\tilde{V})p(N^*,\tilde{V})+\tau'(N^*)}\right)\cdot\left(\frac{1}{1-\frac{1}{\frac{\partial p}{\partial N}}\frac{\tau''(N^*)}{g''(p(N^*,\tilde{V}))p(N^*,\tilde{V})^2+2g'(p(N^*,\tilde{V}))p(N^*,\tilde{V})}}\right) \\
&= \frac{1}{g(p(N^*,\tilde{V})p(N^*,\tilde{V})+\tau'(N^*)+N^*\tau''(N^*)\cdot\left(\frac{g(p(N^*,\tilde{V}))+g'(p(N^*,\tilde{V}))p(N^*,\tilde{V})}{g''(p(N^*,\tilde{V}))p(N^*,\tilde{V})^2+2g'(p(N^*,\tilde{V}))p(N^*,\tilde{V})}\right)} \\
&> 0,
\end{aligned}$$

where the second equality is a simple re-arrangement, the third equality follows from inserting (2.11) and (2.14), and the last equality follows from inserting (2.11) once more.

PART (i): Since $\frac{dN^*}{d\tilde{V}} > 0$ and since $\frac{dN^*}{d\tilde{V}} < \frac{1}{\tau'(N^*)}$, we obtain (i) simply from the fact that $\tilde{V}$ increases with v and goes to infinity as v approaches infinity and that $\lim_{N\to\infty}\tau'(N) = \infty$.

PART (ii): Follows immediately from (i).

PART (iii): Follows immediately from (2.10), when taking the limit of v to infinity and keeping in mind that N^{mon} is constant for sufficiently large values of v by (ii).

PART (iv): Follows immediately from the fact that $N^* \geq 1$ and that $\lim_{\overline{\tau}\to\infty}\frac{dN^*}{d\tilde{V}} = 0$. □

Using the above lemmata, we can fully characterize the monopolist's contract offer. In particular, the following proposition shows that our previous result on the monopolistic outcome still holds if transaction costs are sufficiently convex.

Proposition 2.3. *The following statements hold true:*

(i) There exist a threshold value $\check{c} > 0$ and, for any $c < \check{c}$, a threshold value $\check{v}_1(c) > 0$ such that the monopolist offers a bonus contract if $c < \check{c}$ and $v < \check{v}_1(c)$.

(ii) *For any* $c > 0$*, there exists a threshold value* $\check{v}_2(c) \geq 0$ *such that the monopolist does not offer a bonus contract if* $v > \check{v}_2(c)$.

(iii) *If transaction costs are sufficiently convex (i.e., if* $\tau''(N)$ *is sufficiently large for any* N*), then* $\check{v}_1(c) = \check{v}_2(c) = \check{v}(c)$ *for any* $c < \check{c}$ *and* $\check{v}$ *monotonically decreases in* c *on* $[0, \check{c})$.

Proof. PART (i): Obviously, if $v = 0$, the monopolist can earn positive profits only when offering a bonus contract. As $\tau(2) + g(c/2)(c/2) < g(\bar{b})\bar{b}$ by assumption, the monopolist can indeed earn strictly positive profits using a bonus contract even if $v = 0$. The statement then follows from the fact that the monopolist's profit is continuous in v conditional on offering a certain type of contract (i.e., a bonus contract or a contract without a bonus payment).

PART (ii): Follows immediately from Lemma 2.3 Part (iii) using basically the same arguments as in the proof of Proposition 2.1.

PART (iii): By Lemma 2.3 Part (iv), the monopolist chooses the same number of regular payments irrespective of whether she pays a bonus or not. Given this fact, the proof is analogous to that of Proposition 2.1. □

B.3: Competitive Market

Next, we analyze the competitive outcome in our extended model with transaction costs.

Proposition 2.4. *For* $L = 2$*, an equilibrium exists and any equilibrium has following properties:*

(i) *the market is covered and firms earn zero profits,*

(ii) *the regular payments made by consumers are equally spread across* N_k^{com} *periods, that is,* $p_i^k(N_k^{com}) = p^k(N_k^{com})$ *for* $i \in \{1, \ldots, N_k^{com}\}$ *and* $k \in \{1, 2\}$*, whereby* $\underline{N} \leq N_k^{com} \leq \lceil N^{**} \rceil$ *and* N^{**} *is the unique solution to*

$$g'\left(\frac{c+\bar{b}}{N}\right)\left(\frac{c+\bar{b}}{N}\right)^2 = \tau'(N)$$

while $\underline{N}$ *is the maximum of two and the smallest natural number* N *that sat-*

isfies

$$\frac{c+\bar{b}}{N+1} \leq \frac{\tau(N+1)-\tau(N)}{g\left(\frac{c+\bar{b}}{N}\right)-g\left(\frac{c+\bar{b}}{N+1}\right)}, \tag{2.15}$$

(iii) both firms offer the maximum bonus using a single bonus payment, and

(iv) both firms provide the exact same focus-weighted utility to consumers.

For $L \geq 3$, a symmetric equilibrium exists and any such equilibrium satisfies properties (i) – (iv). In addition, for any $L \geq 2$, there exists a symmetric equilibrium with $N_k^{com} \in \{\lfloor N^{**} \rfloor, \lceil N^{**} \rceil\}$.

Proof. We prove the statement for $L = 2$, while the proof for $L \geq 3$ is a straightforward extension. Again, we can make use from the insights derived in the main text, namely, Proposition 2.2. For instance, we already know that firms earn zero profits in equilibrium and that consumers are indifferent between both offers, that is, Part (iv) immediately follows from Proposition 2.2. In addition, it follows directly from Proposition 2.2 that firms offer at most one bonus payment. Hence, without loss of generality, let $M = 1$ in the following.

The remainder of the proof proceeds in four steps. In a first step, we show that in any equilibrium $N_k^{com} \geq 2$, which in turn implies that firms offer bonus contracts. In a second step, we prove that in any equilibrium $N_k^{com} \leq \lceil N^{**} \rceil$. In a third step, we show that a symmetric equilibrium with $N^{com} \in \{\lfloor N^{**} \rfloor, \lceil N^{**} \rceil\}$ exists. In a fourth step, we show that an equilibrium with $N^k \in \{2, \ldots, \lfloor N^{**} \rfloor\}$ exists if and only if (2.15) holds at $N = N^k$ and that (2.15) is more likely to be fulfilled for larger values of N.

1. STEP: By Proposition 2.2, we know that for $M = 1$ and a fixed number of regular payments $N \geq 2$, there exists a unique equilibrium in which both firms offer the contract

$$\boldsymbol{c}^{bon}(M=1,N) = \left(\bar{b}, \frac{c+\bar{b}}{N}, \ldots, \frac{c+\bar{b}}{N}\right).$$

Moreover, if firms choose at most one non-zero regular payment, they cannot profitably offer a bonus. But then the only other equilibrium candidate is setting all regular payments to zero.

For the sake of a contradiction, suppose that firms do not offer a bonus payment in equilibrium, but set all regular payments to zero. Then, firm k could profitably deviate to a contract

$$\tilde{\boldsymbol{c}}^k(M=1, N^k=2) = \left(\bar{b}, \frac{c+\bar{b}+\epsilon}{2}, \frac{c+\bar{b}+\epsilon}{2}\right)$$

for some $\epsilon > 0$ since all consumers choose $\tilde{\boldsymbol{c}}^k(M=1, N^k=2)$ if

$$g(\bar{b})\bar{b} - 2g\left(\frac{c+\bar{b}+\epsilon}{2}\right)\left(\frac{c+\bar{b}+\epsilon}{2}\right) - \tau(2) > 0,$$

which holds for sufficiently small values ϵ by the assumption that $\tau(2)+2g\left(\frac{c+\bar{b}}{2}\right)\left(\frac{c+\bar{b}}{2}\right) < g(\bar{b})\bar{b}$; a contradiction. Hence, we have $N_k^{com} \geq 2$ in any equilibrium.

2. STEP: For the sake of a contradiction, suppose that $N_k^{com} > \lceil N^{**} \rceil$ holds in equilibrium. Then, firm k could profitably deviate to a contract

$$\tilde{\boldsymbol{c}}^k(M=1, N^k) = \left(\bar{b}, \frac{c+\bar{b}+\epsilon}{N^k}, \dots, \frac{c+\bar{b}+\epsilon}{N^k}\right)$$

for $N^k \in \{\lfloor N^{**} \rfloor, \lceil N^{**} \rceil\}$ and some $\epsilon > 0$ since all consumers choose $\tilde{\boldsymbol{c}}^k(M=1, N^k)$ if

$$\begin{aligned}
&-N^k g\left(\frac{c+\bar{b}+\epsilon}{N^k}\right)\left(\frac{c+\bar{b}+\epsilon}{N^k}\right) - \tau(N^k) \\
&\qquad > -N^k g\left(\frac{c+\bar{b}+\epsilon}{N^k}\right)\left(\frac{c+\bar{b}}{N_{-k}^{com}}\right) - [N_{-k}^{com} - N^k] g\left(\frac{c+\bar{b}}{N_{-k}^{com}}\right)\left(\frac{c+\bar{b}}{N_{-k}^{com}}\right) - \tau(N_{-k}^{com}).
\end{aligned} \tag{2.16}$$

Notice that the right-hand side of the above inequality is smaller than

$$-N_{-k}^{com} g\left(\frac{c+\bar{b}}{N_{-k}^{com}}\right)\left(\frac{c+\bar{b}}{N_{-k}^{com}}\right) - \tau(N_{-k}^{com})$$

by Assumption 2.2 and that

$$-N^k g\left(\frac{c+\bar{b}+\epsilon}{N^k}\right)\left(\frac{c+\bar{b}+\epsilon}{N^k}\right) - \tau(N^k) > -N_{-k}^{com} g\left(\frac{c+\bar{b}}{N_{-k}^{com}}\right)\left(\frac{c+\bar{b}}{N_{-k}^{com}}\right) - \tau(N_{-k}^{com})$$

by our assumption toward a contradiction and the definition of N^{**} as the unique minimizer of $Ng\left(\frac{c+\bar{b}}{N}\right)\left(\frac{c+\bar{b}}{N}\right) + \tau(N)$. Consequently, Inequality (2.16) holds for sufficiently small values of ϵ; a contradiction. Hence, we conclude that $N_k^{com} \leq \lceil N^{**} \rceil$

in any equilibrium.

3. STEP: Suppose that both firms offer the contract

$$\boldsymbol{c}^{bon}(M=1,N^{com}) = \left(\bar{b}, \frac{c+\bar{b}}{N^{com}}, \ldots, \frac{c+\bar{b}}{N^{com}}\right),$$

where N^{com} is chosen as to minimize $Ng\left(\frac{c+\bar{b}}{N}\right)\left(\frac{c+\bar{b}}{N}\right) + \tau(N)$; that is, $N^{com} \in \{\lfloor N^{**} \rfloor, \lceil N^{**} \rceil\}$. By STEP 3, no firm has an incentive to increase the number of regular payments, which by the way implies that $\underline{N} \leq \lceil N^{**} \rceil$. In addition, notice that the regular payments of firm k would determine the focus-weights if it decides to decrease the number of regular payments in a way that allows for non-negative profits. But then, by the definition of N^{com}, decreasing the number of regular payments cannot increase focus-weighted utility and yield non-negative profits at the same time. Hence, no firm has an incentive to decrease the number of regular payments and therefore no incentive to deviate, which was to be proven.

4. STEP: Suppose that both firms offer the contract

$$\boldsymbol{c}_k^{bon}(M=1,N_k^{com}) = \left(\bar{b}, \frac{c+\bar{b}}{N_k^{com}}, \ldots, \frac{c+\bar{b}}{N_k^{com}}\right),$$

where $N_k^{com} \in \{2, \ldots, \lfloor N^{**} \rfloor\}$. First, suppose that both firms choose the same number of regular payments, that is, $N_1^{com} = N_2^{com} = N^{com}$. Since $N^{com} \leq \lfloor N^{**} \rfloor$, by same argument as in STEP 3, firms do not have an incentive to decrease the number of regular payments. In addition, firms do not have an incentive to increase the number of payments if and only if

$$\begin{aligned} &N^{com} g\left(\frac{c+\bar{b}}{N^{com}}\right)\left(\frac{c+\bar{b}}{N^{com}}\right) - \tau(N^{com}) \\ &\quad > N^{com} g\left(\frac{c+\bar{b}}{N^{com}}\right)\left(\frac{c+\bar{b}}{N^{com}+1}\right) + g\left(\frac{c+\bar{b}}{N^{com}+1}\right)\left(\frac{c+\bar{b}}{N^{com}+1}\right) - \tau(N^{com}+1), \end{aligned}$$

which holds if and only if (2.15) holds at $N = N^{com}$.

Second, notice that

$$\underbrace{\frac{\partial}{\partial N} \frac{c+\bar{b}}{N+1}\left[g\left(\frac{c+\bar{b}}{N}\right) - g\left(\frac{c+\bar{b}}{N+1}\right)\right]}_{<0 \text{ by A.3}} - \underbrace{\frac{\partial}{\partial N}\Big[\tau(N+1) - \tau(N)\Big]}_{<0 \text{ as } \tau''>0} < 0,$$

which in turn implies that (2.15) is more likely to hold for larger values of N.

Third, let $N_1^{com} \neq N_2^{com}$. If $N_k^{com} \leq N_{-k}^{com} - 2$, firm k could profitably deviate to the contract

$$\tilde{\boldsymbol{c}}_k^{bon}(M = 1, N_k^{com} + 1) = \left(\bar{b}, \frac{c + \bar{b} + \epsilon}{N_k^{com} + 1}, \ldots, \frac{c + \bar{b} + \epsilon}{N_k^{com} + 1}\right)$$

for some sufficiently small $\epsilon > 0$, as $N_k^{com} \leq \lfloor N^{**} \rfloor - 1$ and as firm k's regular payments would fully determine the focus-weights. If $N_k^{com} = N_{-k}^{com} - 1$, then

$$\begin{aligned}
&N_k^{com} g\left(\frac{c + \bar{b}}{N_k^{com}}\right)\left(\frac{c + \bar{b}}{N_k^{com}}\right) - \tau(N_k^{com}) \\
&\quad = N_k^{com} g\left(\frac{c + \bar{b}}{N_k^{com}}\right)\left(\frac{c + \bar{b}}{N_k^{com} + 1}\right) + g\left(\frac{c + \bar{b}}{N_k^{com} + 1}\right)\left(\frac{c + \bar{b}}{N_k^{com} + 1}\right) - \tau(N_k^{com} + 1),
\end{aligned}$$

has to hold, as consumers have to be indifferent between both contracts in equilibrium. But then (2.15) holds at $N = N_k^{com}$ and as it is more likely to hold for larger values of N it also holds at $N = N_{-k}^{com}$. This completes the proof. □

The preceding proposition shows that the competitive equilibrium has the same qualitative properties as before, namely, firms offer a single, maximum bonus payment and the regular payments are of equal size. The only difference compared to our baseline model is that there can exist multiple equilibria that differ in the number of non-zero regular payments. It is easy to see, however, that this multiplicity vanishes for sufficiently convex transaction costs. Consequently, as long as the transaction cost function is sufficiently convex, also our result on the comparison of monopolistic and competitive outcomes remains qualitatively the same.

Corollary 2.2. *If transaction costs are sufficiently convex (i.e., if $\tau''(N)$ is sufficiently large for any N), there is a unique (symmetric) competitive equilibrium. In this equilibrium all firms choose the same number of regular payments as a monopolist would do. If in addition the consumers' valuation for the product is sufficiently high, the contractual inefficiencies are strictly lower in a monopolistic than in a competitive market.*

Proof. As τ becomes more convex, the right-hand side of (2.15) becomes smaller for small values of N and larger for large values of N. Hence, $\underline{N}$ becomes larger as τ becomes more convex and eventually only one equilibrium candidate survives. In addition, as τ becomes more convex, both N^* and N^{**} become less sensitive to the level of the regular payments, so that for sufficiently convex transaction costs $N^{mon} = N^{com}$. □

Chapter 3

Self-Deception and Social Responsibility in Markets

3.1 Introduction

The production of various consumption goods causes negative externalities such as violations of human and animal rights or environmental pollution, which are not regulated, not legally prohibited or not prosecuted.[1] A combination of growing awareness of such potential harm to others and the idea that purchasing behavior (individually or as a group) can incentivize better forms of production influences many consumers' purchasing decisions (see, e.g., Bartling *et al.*, 2015; Pigors and Rockenbach, 2016b; Sutter *et al.*, 2016). The intention to alter purchasing behavior may arise from altruism or warm glow (see, e.g., Baron, 2009) as well as distributional or moral concerns (see, e.g., Pigors and Rockenbach, 2016a; see Bénabou and Tirole, 2010 for social- or self-image concerns arising from moral concerns). At the same time, there is an increasing number of firms investing in production technologies or activities, which mitigate negative externalities (see Bénabou and Tirole, 2010 for a comprehensive discussion), whereby motives of owners or managers can range from expected profits to intrinsic motivation. Consequently, socially responsible behavior in markets is of increasing importance.

Market shares, however, of such *socially responsible* products, which are typically more expensive, are still extremely small relative to consumers' stated preferences for such products.[2] Of course, this gap might be partially driven by cheap talk in order to signal social responsibility to the interviewer or oneself, or could simply arise because the price differences between externality-free and externality-causing products exceeds the difference in willingness to pay.[3] Moreover, in many markets there is uncertainty about whether production actually harms others and whether measures for mitigation of such harm are actually effective or, for instance, purely serving marketing purposes (see, e.g., Bartling *et al.*, 2015; Pigors and Rockenbach, 2016a). Even established labels like the *EU certified organic food label* may not fully assure consumers.[4]

[1] See, for instance, https://www.deutschlandfunk.de/migranten-in-italien-die-neuen-sklaven-europas and https://www.deutschlandfunk.de/textilfabriken-in-suedasien-mehr-oeffentliches-bewusstsein , all accessed on August 12, 2018. I may refer to negative externalities as externalities in the remainder of the paper.

[2] See, for instance, https://www.gfk-verein.org/compact/fokusthemen/nachhaltig-konsumieren-nur-ein-lippenbekenntnis, accessed on August 12, 2018. See also Pigors and Rockenbach (2016b) for a brief review of related marketing literature.

[3] See, for instance, https://www.handelsblatt.com/unternehmen/handel-konsumgueter/textilbranche-warum-hat-es-nachhaltige-kleidung-so-schwer, accessed on August 12, 2018.

[4] See, for instance, https://www.daserste.de/information/ratgeber-service/vorsicht-

Given the wealth of information nowadays, in particular on the Internet, consumers should be able to resolve uncertainty in many cases at relatively low cost. Uncertainty, however, may give rise to *moral wiggle room*. To be precise, consumers may avoid information in order to deceive themselves about the consequences of their purchasing behavior, while benefiting from relatively low prices.[5] A large experimental literature demonstrates that especially social decision contexts are prone to exploitation of moral wiggle room in order to justify more selfish behavior if there is uncertainty about harm to others (see, e.g., the seminal paper by Dana *et al.*, 2007). More specifically, Ehrich and Irwin (2005) let experimental subjects in the role of consumers decide whether to acquire information about different product attributes and find that especially those who indicate a preference for sustainability avoid information on this dimension.

This paper studies how the possibility to exploit moral wiggle room through *self-deception* can affect market outcomes. For this purpose, I consider a market in which consumers are concerned but uncertain about whether their purchasing decisions harm others. Instead of resolving uncertainty, however, they can willingly distort their beliefs. Firms, which differ in their production technology with respect to whether their production harms others or not, take this into account when competing in prices. I show that avoidance of information about the impact of purchasing decisions on others in order to maintain self-deception can arise endogenously in the market. Moreover, self-deception can distort market demand toward low-cost production that harms others and render costly mitigation of externalities less profitable. Thereby, I identify a new channel through which competition is affected, namely self-deception. Through this channel, consumers perceive firms as less differentiated in favor of the low-cost firm, as they tend to be overoptimistic about whether its production harms others in order to benefit from a relatively low price.

I consider the following market setup. Two competing firms offer a homogeneous product, whereas their production technologies differ. One is of low costs and potentially causes externalities harming others, while the other is more costly as it prevents such harm. Consumers suffer from being aware of harming others to various degrees. Through this channel, firms are vertically differentiated as long as

verbraucherfalle/sendung/schwindel-mit-dem-eu-bio-siegel, accessed on August 12, 2018.

[5]See, for instance, https://www.tagesspiegel.de/wirtschaft/weniger-fleisch-viel-obst-die-grosse-heuchelei-bei-der-ernaehrung accessed on August 12, 2018.

harm cannot be ruled out. Initially, consumers are uncertain about whether low-cost production is at the expense of others. After observing prices, they can resolve uncertainty in order to make an informed purchasing decision. In order to capture the idea that typically consumers cannot fully infer on harm from firms' prices, I assume that consumers are χ-*cursed* as in Eyster and Rabin (2005). This guarantees an element of uncertainty about whether low-cost production harms others in this stylized market environment. Thereby, an arbitrarily small level of cursedness is sufficient for the main results to hold.

Self-deception is incorporated into the model through Spiegler (2008), which is a simplified and extremely tractable single-period version of Brunnermeier and Parker (2005), where beliefs can be distorted. To be precise, consumers can form motivated beliefs about whether low-cost production harms others as long as there is uncertainty. Thereby, affective gains from distorting beliefs in a self-serving way are traded off against material losses from distorting the purchasing decision. An uninformed consumer distorts her decision whenever self-deception tempts her to choose the low-cost product, while she had chosen the high-cost product under rational expectations. Empirical evidence from controlled experiments for the relevance of the basic economic trade-off is provided by Zimmermann (2018). The possibility to hide behind uncertainty gives moral wiggle room to consumers. In particular, consumers can willingly avoid information in order to benefit from self-deception at the cost of an informed purchasing decision.

The mechanism behind the first result is the following. If an uninformed consumer intends to purchase from the low-cost firm, she finds it optimal to underestimate the likelihood of harming others by her purchasing decision. In contrast, distorting beliefs is not beneficial when purchasing the high-cost product, as its production prevents harm. Suppose that low-cost production harms others and that the low-cost product is offered at a lower price than the high-cost product such that both products attract consumers. As consumers are cursed, they cannot fully infer on harm from observed prices, while uncertainty can be resolved at no cost.

Then, three groups of consumers can be distinguished regarding how much they dislike if their purchasing decision harms others: (i) consumers who prefer the low-cost product regardless of information about harm, (ii) consumers who prefer the low-cost product if they are uninformed or learn that its production is harmless, while they prefer the high-cost product if they learn that low-cost production harms others, and (iii) consumers who prefer the high-cost product unless they learn that low-cost production is harmless.

Consumers of the first and third group do not distort their decision given prices. A consumer of the first group remains ignorant in order to secure benefits from self-deception. To be more precise, being overoptimistic reduces her disutility from anticipation of harm. As her decision is not distorted given prices, she cannot benefit from information. In contrast, a consumer of the third group fully appreciates the instrumental value of information. As she does not rule out that low-cost production is harmless, she expects the bargain of paying a lower price without causing harm.

A consumer of the second group, however, can benefit from distorting her decision toward the low-cost product given prices. Her trade-off is the following. Avoiding information insures her against paying a high price in the case of learning that low-cost production harms others. At the same time, it allows her to benefit from self-deception while paying a low price. Then, she forgoes the bargain of paying the low price without causing harm in the case of learning that low-cost production is harmless. It turns out that, given prices as defined above, there is always a threshold level of how much a consumer suffers from harm such that the consumer is indifferent. Consumers who suffer less choose to remain ignorant in order to benefit from self-deception, while consumers who suffer more prefer to make an informed purchasing decision.

Thereby, firms' demand is defined, which is affected by self-deception if low-cost production harms others. Then, firms indeed behave as in a model of vertical differentiation, whereby they are differentiated through how uninformed consumers perceive the likelihood of harm. As a result, prices are as defined above and the market is covered.

If there are no externalities, in contrast, the low-cost firm induces all consumers to become informed by setting a price equal to its competitor. To be precise, all consumers belong to the first group if prices are equal, as harm cannot be ruled out before deciding about information. Then, all consumers strictly prefer the high-cost product unless they learn that low-cost production is harmless. Due to the expected bargain, information is beneficial. This leads to a market outcome in which the low-cost firm serves the market at the high cost firm's production cost, as products are homogeneous.

The driving force behind the second result is that, as long as low-cost production harms others, self-deceiving consumers perceive firms as less differentiated compared to not deceiving themselves. Given prices, this renders the low-cost product relatively more attractive, which leads to a competitive advantage. At the same time, however, firms are perceived as less vertically differentiated, which intensifies competition. It turns out that the high-cost firm's reaction to self-deception is a reduction in its price. While the low-cost firm's reaction is ambiguous, self-deception distorts demand toward low-cost production as long as demand does not react too sensitive to the high-cost firm's price cut. Then, self-deception harms the high-cost firm as it loses profits compared to the case in which consumers cannot deceive themselves. If self-deception leads to higher demand for the high-cost firm, it loses profits as long as its inframarginal losses are not outweighed.

These findings provide important policy implications. Suppose policymakers pursue mitigating externalities caused by low-cost production, which cannot be prohibited by law or contracts, while they care relatively less about anticipated utility. Then, information provision and campaigns are not sufficient as long as consumers can avoid information and deceive themselves about the presence of externalities. In contrast, taxing externalities, subsidizing its mitigation or introducing a binding price floor can reduce monetary incentives for remaining ignorant and lead to more informed decision making.

Moreover, if using the high-cost, externality-mitigating technology becomes less profitable through self-deception, both innovation and entry can become less likely if access to this technology causes fixed costs.

The remainder of the paper is organized in the following way. In Section 3.2, I relate the paper to the existing literature. In Section 3.3, I first introduce the model formally and then discuss important model assumptions more in detail. In Section 3.4, I establish the main results. Finally, Section 3.5 provides a discussion of the results and a concludes.

3.2 Related Literature

In this section, I review the related literature and argue how this paper contributes to the different strands.

3.2.1 Self-Deception and Information Avoidance

This paper builds on Spiegler (2008), which is a version of Brunnermeier and Parker's 2005 model of self-deception in which beliefs can be distorted. Optimal choice of such motivated beliefs trades off affective gains against material losses from distorted decisions. Self-deception can be maintained through information avoidance (see, e.g., Loewenstein, 2006; Bénabou, 2015). Then, benefits from self-deception are traded off against making an informed decision.

This paper adds to the literature by providing a theoretical framework in which information avoidance in favor of self-deception arises endogenously. In this paper, avoiding information about the impact of purchasing decisions on others in order to maintain self-deception is an endogenous market outcome if low-cost production harms others. Given firms' prices, consumers trade off benefits from self-deception against the cost of an informed purchasing decision, which is anticipated by firms. Oster *et al.* (2013) provide field evidence for information avoidance in favor of motivated beliefs in a medical context of Huntington's disease (HD) testing. HD limits life expectancy significantly and hence crucially affects economic decisions like retirement. Tests are perfectly predictive and of little economic cost. They find, however, that a large proportion of individuals at risk avoid testing, express optimistic beliefs about life-expectancy and behave as if they do not have HD. Huck *et al.* (2017) argue that this result may be driven by self-selection of optimistic individuals into avoiding the test. In order to control for self-selection, they conduct experiments with real-effort tasks and find strong evidence for information avoidance in favor of motivated beliefs and according performance. In both papers, formal models are provided in which there is a similar trade-off between remaining ignorant in order to maintain self-deception as in Brunnermeier and Parker (2005) and appreciating the instrumental value of information. In this paper, I show that information avoidance arises endogenously in a market environment in which moral wiggle room can be exploited through self-deception.

In an experimental setup concerned with pro-social behavior, Dana *et al.* (2007) introduce moral wiggle room into a binary version of the dictator game. Subjects in the role of a dictator can either choose a high or intermediate amount of money for themselves. Initially, it is uncertain whether the receiver's corresponding payoff is either low or intermediate as in the standard case. Despite the possibility to resolve uncertainty at no cost, they find information avoidance in favor of significantly less generous behavior relative to a baseline without initial uncertainty. In other words, they demonstrate that some subjects hide behind uncertainty in order to exploit moral wiggle room in order to behave selfishly. Bénabou and Tirole (2011) interpret this and similar findings in subsequent experiments as clear indicator of self-deception about potentially negative impact of decisions on others. Moreover, Bénabou (2015) argues that social decision contexts are particularly prone to self-serving beliefs.

Grossman and van der Weele (2017) follow the idea that information may be avoided in such contexts in order to maintain a social self-image despite acting selfishly, which is also suggested in Dana *et al.* (2007). They derive theoretical predictions from a Bayesian signaling game in which agents care about both self-image and payoffs of others as in Bénabou and Tirole (2006) and Bodner and Prelec (2002). In their model, contribution to a public good is costly but does not necessarily have positive impact. Although uncertainty can be resolved, information avoidance may serve as an excuse for acting selfishly, which gives rise to moral wiggle room. While information avoidance itself negatively affects self-image, agents can derive positive self-image from the belief that they would contribute to the public good if they knew it had an impact without bearing the cost for contributing. Then, agents also trade off benefits from self-deception against the value of information.[6] In an experimental setup similar to Dana *et al.* (2007), they test their predictions which confirm previous experimental results on information avoidance and allow to test their model against outcome-based preferences and social-image concerns. Their results suggest self-signaling as a driver for information avoidance.

This paper adds to the literature by demonstrating complementarity of self-image concerns and motivated beliefs in providing incentives for information avoidance in social decision contexts. Although Grossman and van der Weele's prediction of information avoidance in favor of selfish behavior coincides qualitatively with consumer behavior in this paper, the mechanism crucially differs. In particular, beliefs are undistorted under self-signaling. Under motivated beliefs, selfish behavior is accompanied by a distorted, overoptimistic belief. This affects, for instance, quantitative predictions of information avoidance and the willingness to pay to remain ignorant. Hence, self-image concerns and motivated beliefs can complement each other in providing incentives for information avoidance in social decision contexts. Policy implications, however, will depend on the exact interaction of the two mechanisms.

[6] In a self-signaling model, the agent signals its social type, from which she derives utility, to an observer-self through her actions, as she cannot observe her true type anymore after taking actions. Hence, the value of information is not only instrumental. In addition, its choice affects the agent's utility through self-signaling.

3.2.2 Social Responsibility in Markets

Market outcomes with social preferences have been extensively studied over time (see, e.g., Dufwenberg *et al.*, 2011 and references therein). Sutter *et al.* (2016), however, point out that despite its popularity among some of the most important founders of modern economics, the topic of how markets relate to socially responsible behavior has been rediscovered only recently. Shleifer (2004), for instance, argues that socially responsible behavior in markets is challenged severely by competitive pressure driven by cost reductions. Since Falk and Szech (2013) have presented controlled experimental evidence on how markets erode socially responsible behavior, a mainly experimental literature has emerged trying to identify explanations.

This paper adds to the literature by providing a theoretical framework in which socially responsible behavior in markets can be eroded through self-deception, from which testable predictions can be derived. Bartling *et al.* (2015) study socially responsible behavior in a laboratory market that is very similar to the market setup analyzed in this paper. They show that there is a persistent preference among market participants for avoiding externalities caused by low-cost production. In particular, their result is robust to limiting initial information of subjects in the role of consumers about firms' production technologies. Thereby, consumers who do not acquire information typically buy the cheapest product that is most likely harming others. Consumers who acquire information tend to prefer the externality-free product that is typically more expensive. In this respect, our findings are consistent. As Bartling *et al.* (2015) do not find a significant drop in socially responsible behavior due to initial uncertainty, information acquisition seems to be instrumental rather than strategical. Their experimental design, however, provides limited scope for self-deception. I explicitly model the possibility to exploit moral wiggle room in a market environment. Thereby, this paper provides a theoretical framework in which self-deception can indeed erode socially responsible behavior in markets in terms of both the amount of socially responsible purchasing decisions and investment or entry incentives for firms. Testable predictions can be derived for future research.

3.2.3 Corporate Social Responsibility

Following Bénabou and Tirole (2010), corporate social responsibility (CSR) refers to voluntary social activities of firms that go beyond legal and contractual obligations. In case of market failures as a result of government failures, CSR can serve as decentralized correction. Moreover, economic agents can promote values that are not sufficiently addressed by policy. Three notions of CSR are discussed: taking a long-term perspective in contrast to short-term bias to profit maximization, socially responsible behavior on behalf of stakeholders, and insider-initiated corporate philanthropy. While in practice the motivation for CSR is usually a mix of these three notions, most of the implications in this paper are derived with respect to the first and the second. Despite the economic relevance and increasing prominence of the topic, Heidhues and Kőszegi (2018) point out that the economics literature on CSR is relatively small.

This paper contributes to the literature by analyzing how uncertainty about CSR activities affects firms' incentives to invest in such activities if consumers with heterogeneous preferences for CSR can exploit moral wiggle room. Most closely related to this paper is Baron (2009), where a morally managed firm bears additional costs in order to mitigate externalities caused by its production beyond legal and regulatory requirements. A self-interested firm can choose to do so. Consumers heterogeneously value such activities, that is, products are vertically differentiated through CSR. In particular, fraction δ of consumers does not value CRS, while the valuation of fraction $1 - \delta$ is uniformly distributed. In equilibrium, the self-interested firm chooses not to invest in CSR, as maximal differentiation softens price competition. This is beneficial for both firms benefit. Baron assumes that consumers can perfectly identify CSR and its motives and focuses on how citizens can increase CSR activities through social pressure. In this paper, I introduce uncertainty about the effectiveness of costly CSR activities relative to low-cost production and allow consumers to exploit moral wiggle room by hiding behind uncertainty. Thereby, I identify a new channel through which incentives to invest in CSR can be affected negatively, namely self-deception.

3.3 Model

First, I introduce the formal setup. Second, I provide a discussion of important model assumptions.

3.3.1 Formal Setup

Consider a market with two single-product firms labeled h and l. Firm h produces at constant marginal costs $c > 0$ without causing negative externalities. Firm l produces at constant marginal costs of zero, but depending on its type $\omega \in \Omega := \{bad, good\}$ may cause negative externalities of $e > 0$—namely, if $\omega = bad$. The realization of ω is known by the firms but not by the consumers. Let the common prior probability distribution over types be $q(\omega)$ with full support. As Ω is binary, $q(\omega)$ can simply be describe by $q := q(\omega = bad)$. Each firm chooses its price p_i, whereby it is common knowledge that firms cannot price below marginal costs. Firm i's strategy prescribes a probability distribution $\sigma_i(p_i|\omega)$ over feasible prices for each type ω. In the following, I only allow for pure firm strategies, that is, $\sigma_i(p_i|\omega)$ is degenerate.[7] Each firm's objective is to maximize its expected profits.

There is a unit mass of consumers with unit demand. Consumer type θ's material payoff from consuming product $i \in \{h, l\}$ is given by $v - p_i - \theta E[e]$. The private value of consumption for either firm's product $v > 0$ is equal to all consumers. The consumer's type is denoted by $\theta \in (0, \infty)$ and determines her negative valuation for externalities. Consumer types are distributed according to density function $f(\theta)$ with full support and cumulative distribution function $F(\theta)$. Let $f(\theta)$ be continuously differentiable and log-concave. Further, I assume that the type of a consumer is private information. The distribution of types, however, shall be public information. I will explain in detail how expectations over externalities are formed below.

I depart from the classical rationality paradigm by assuming that consumers are χ-cursed as in Eyster and Rabin (2005), whereby I rule out that consumers are fully rational at the beginning of the game, that is, $\chi \in (0, 1]$. Then, consumers play best response to the following objective belief: firms play type-dependent strategy profile $\sigma_{h,l}(p_h, p_l|\omega) := (\sigma_h(p_h|\omega), \sigma_l(p_l|\omega))$ with probability $1 - \chi$ and type-independent average-strategy profile $\bar{\sigma}_{h,l}(p_h, p_l) := q\sigma_{h,l}(p_h, p_l|\omega = bad) + (1 - q)\sigma_{h,l}(p_h, p_l|\omega = good)$ with probability χ. As a result, objective beliefs of a χ-cursed consumer about firm l's type along the path of play are defined by

$$b(\omega|p_h, p_l, \sigma_{h,l}, \chi) := \left((1 - \chi)\frac{\sigma_{h,l}(p_h, p_l|\omega)}{\bar{\sigma}_{h,l}(p_h, p_l)} + \chi \right) q(\omega). \tag{3.1}$$

[7] Except for the Bertrand outcome in the case of no externalities, the proofs hold for mixed strategies as can be seen in Appendix 3.5.

Hence, a belief along the path of play is bounded away from zero or one as it is a convex combination of a belief that is updated through Bayes' rule and the common prior, where the weight is determined by χ.

Consumers can decide whether to receive a perfectly informing signal about firm l's type ω or not denoted by $m = 1$ and $m = 0$, respectively. Technically, this requires that perfect information renders consumers fully rational, that is, $\chi = 0$. Subsequently, consumers choose whether to purchase one of the products or not. Denote the product choice by $a \in A := \{a_h, a_l, a_0\}$, where subscripts h and l refer to the respective firm's product and subscript 0 refers to a costless zero-value outside option.

At the same time, consumers form motivated beliefs about ω denoted by the probability distribution $r(\omega)$. In the spirit of Spiegler (2008), I require that the support of $r(\omega)$ is weakly contained in the support of $b(\omega|p_h, p_l, \sigma_{h,l}, \chi)$. In this model, the choice of motivated beliefs trades off anticipatory gains from overoptimistic beliefs about externalities against material losses from a distorted product choice. I formalize this idea similar to Spiegler (2008), which builds on Brunnermeier and Parker (2005).

Let $u(a, p_h, p_l|\theta, \omega)$ denote a consumer's material payoff from product choice a and prices depending on her own and firm l's type. Expected overall utility is a convex combination of anticipated and expected material utility, where $\alpha \in (0, 1)$ denotes consumers' weight of anticipatory utility. Similar to $q(\omega)$, $r(\omega)$ can simply be described by $r := r(\omega = bad)$. Given prices and information choice, a consumer chooses product-belief pair (a, r) in order to maximize the following expression:

$$\max_{a,r} \alpha \sum_{\omega \in \Omega} r(\omega) u(a, p_h, p_l|\theta, \omega) + (1 - \alpha) \sum_{\omega \in \Omega} b(\omega|p_h, p_l, \sigma_{h,l}, \chi) u(a, p_h, p_l|\theta, \omega)$$
$$\text{s.t.} \quad a \in \arg\max_{a'} \sum_{\omega \in \Omega} r(\omega) u(a', p_h, p_l|\theta, \omega). \tag{3.2}$$

Similarly to $q(\omega)$ and $r(\omega)$, objective beliefs $b(\omega|p_h, p_l, \sigma_{h,l}, \chi)$ simply can be described by $b(p_h, p_l, \sigma_{h,l}, \chi) := b(\omega = bad|p_h, p_l, \sigma_{h,l}, \chi)$. If a consumer is certain about ω, the problem above boils down to choosing a in order to maximize material utility. Hence, a consumer can form motivated beliefs only if she stays uninformed about ω, that is, $m = 0$. If she chooses to become informed, that is, $m = 1$, anticipated and expected material payoffs are equal as $r = b(\omega = bad|p_h, p_l, \sigma_{h,l}, \chi)$. For simplicity, I assume that $\alpha = 0$ if $m = 1$. Consumers anticipate this, as they shall not be naive about the consequences from their information choice.

As the choice of (a, r) depends on a consumer's information, both a and r are functions of m as well as ω if $m = 1$. Consequently, her strategy must prescribe the choice for each information set. For notational convenience, I suppress the functions' arguments and refer to actions as (m, a, r). A consumer's strategy is a probability distribution $\sigma_c(m, a, r|p_h, p_l, \theta)$ over feasible actions (m, a, r) conditional on observed prices and her type. Consumers' objective is to maximize expected overall utility. The timing of the game can be summarized as follows:

t=0: Nature draws firm l's type of externality ω.

t=1: Firms learn ω. Then, they simultaneously choose $\sigma_i(p_i|\omega)$.

t=2: Consumers observe prices and update objective beliefs. Then, each consumer chooses m first and subsequently (a, r) conditionally on m, as prescribed by $\sigma_c(m, a, r|p_h, p_l, \theta)$.

In order to solve the game of incomplete information described above, I introduce the notion of χ perfectly-cursed equilibrium (χ-PCE) (see Eyster and Rabin, 2002 and 2005), which is defined in the following. Let G denote the game as described above. Further, let $\bar{G}^\chi$ be G's χ-virtual game, which is equivalent to G except for the following two assumptions. First, consumers act as if their expected material utility from product choice a and prices is derived from material payoffs defined by

$$\bar{u}^\chi(a, p_h, p_l|\theta, \omega) := (1 - \chi)u(a, p_h, p_l|\theta, \omega) + \chi \sum_{\omega' \in \Omega} q(\omega')u(a, p_h, p_l|\theta, \omega'). \quad (3.3)$$

Second, r can always be chosen from $[0, 1]$ as long as consumers are cursed. Then, strategy profile σ is a χ-PCE of G if it is a perfect Bayesian equilibrium (PBE) of $\bar{G}^\chi$.

I impose the following tie-breaking rules: (i) whenever a consumer is indifferent between becoming informed and staying uninformed, she chooses information, (ii) whenever a consumer is indifferent between firms' products, she chooses the firm that provides larger value to the market, and (iii) whenever a consumer is indifferent between a firm's product and the outside option, she chooses the firm. Further, I assume that $0 < c < qe$. Thereby, an increase of production costs by c in order to avoid externality e with certainty is socially desirable. In addition, I assume that $c < v$ such that firm h can enter the market. Concluding, I impose regularity conditions on the distribution of consumer types stated in the two assumptions below.

Assumption 3.1.

$$\frac{|f'(\theta)|}{f(\theta)} < \frac{(1-\alpha)e}{v}, \ \forall \theta \in (0, \infty).$$

This assumption guarantees that demand is well behaved implying that second-order conditions are satisfied and the slope of each firm's best-response function lies between zero and one. Intuitively, it requires that demand is not too sensitive with respect to marginal price changes relative to the importance of the maximal cognitive distortion that may result from self-deception.

Assumption 3.2.

$$\lim_{(\theta \to 0)^+} \frac{F(\theta)}{f(\theta)} < \frac{c}{(1-\alpha)e}.$$

This assumption guarantees that firm l wants to be active in the market even if firm h charges its minimal price $p_h = c$. Technically, the assumption is relevant if and only if $\lim_{(\theta \to 0)^+} f(\theta) = 0$, as $\lim_{(\theta \to 0)^+} F(\theta) = 0$ by definition. Then, $f(\theta)$ shall not converge too fast relative to $F(\theta)$.

3.3.2 Discussion of Model Assumptions

The model described above is set up in order to study a consumer's trade-off between exploiting moral wiggle room and making an informed decision in a market context. In this section, I discuss some of the model assumptions more in detail.

Consumers' Valuation of Externalities. Moral wiggle room requires moral to matter. I assume that consumers care about the consequences of their purchasing decisions on third parties. Empirical evidence for such preferences in similar laboratory market contexts is provided by Bartling *et al.* (2015) amongst others. The density of consumer preferences is assumed to be log-concave for tractability. As put in Anderson *et al.* (1997), this requires that it should not be too convex. In particular, it should neither rise faster than exponentially nor decline slower than a negative exponential. Although more restrictive than quasi-concavity, this assumption should not be too restrictive. It is satisfied by several densities of probability distributions that are commonly used in economics like uniform, exponential, logistic or extreme value (for a comprehensive list, see, e.g., Bagnoli and Bergstrom, 2005).

While additive separability of disutility from externalities is crucial for feasibility, linear impact is not.

Production Technologies. For moral to matter, consumers must have the choice. As a benchmark, I assume that firm h offers an externality-free option using a costly technology. Production of firm l is less costly but potentially harms others. Its costs are normalized to zero without loss of generality. The externality-free benchmark is certainly extreme. An increasing number of firms, however, put serious effort into corporate social responsibility (for a critical discussion, see Bénabou and Tirole, 2010). The benchmark can also be interpreted as a production technology that causes an amount of externalities tolerable for consumers or sufficiently low for bearing a credible label. Assuming a binary externality space serves expositional clearness. Allowing for some finite number is a straightforward relaxation.

Cursed Consumers and Information. What gives rise to moral wiggle room in my model is uncertainty about the consequences of purchasing decisions. Thereby, I assume that consumers do not learn whether firm l's production causes externalities or not unless they actively decide to become informed. In particular, I rule out that consumers can draw full inference from firms' pricing behavior by exploiting the extremely intuitive notion of cursedness as introduced by Eyster and Rabin (2005). It will turn out that an arbitrarily small level of cursedness is sufficient for the main result to hold. The nice feature is that consumers' objective beliefs off the path of play are bounded away from zero and one in the same way as on the path (for a discussion, see Eyster and Rabin, 2002). This allows to define firms' demand as a function of their prices. Without further restrictions, this would not be possible if, for instance, firms applied type-dependent strategies with ϵ-trembling hands.

Perfect information as a benchmark can also be interpreted as some amount of indications sufficient to convince a consumer, that is, information cannot be denied anymore or be interpreted in a self-serving manner (see, e.g., Bénabou, 2015 for illustration and discussion of reality denial). It is crucial, however, that information is fully understood in contrast to firms' prices. While the assumption of costless information is not crucial, it allows to study the pure trade-off between exploiting moral wiggle room and benefiting from the instrumental value of information. Moreover, it is not important for one-shot interactions whether uninformed consumers suffer from externalities after their purchase as long as there remains the chance to suffer at some point in time. They might experience existence of externalities through the news or information campaigns at some later point in time. With such an interpretation in mind, the model could still provide insights for repeated interactions. Finally, I rule out that firms can actively educate consumers, as credible information is available at no cost. In addition, firms have incentives for misreporting.

Motivated Beliefs. In my model, consumers can exploit moral wiggle by deceiving themselves about the presence of externalities. In particular, I exploit a simplified and extremely tractable single-period version of Brunnermeier and Parker (2005) presented in Spiegler (2008), where consumers can deceive themselves by choosing beliefs. I refer to this choice variable as motivated beliefs. Intuitively, anticipatory gains from overoptimistic beliefs are traded off against material losses from a distorted decision. Empirical evidence for this economic trade-off from controlled experiments is provided by Zimmermann (2018).

As stated in Brunnermeier and Parker (2005), the concept of anticipatory utility has regained attention in economics through Loewenstein (1987) amongst others and its formal incorporation into economic models can be traced back to Geanakoplos *et al.* (1989) and Caplin and Leahy (2001). Here, consumers derive utility from both anticipation and expectation of externalities. The former can be distorted by motivated beliefs. The latter remains a function of objective beliefs and can materialize as consequence of a distorted decision. Alternatively, it can be interpreted as cognitive dissonance (see, e.g., Akerlof and Dickens, 1982 for a discussion of this concept). While distorting beliefs, one could argue that consumers interpret signals or information in a self-serving way over time (see Zimmermann, 2018). I explicitly allow consumers, however, to learn whether externalities occur or not at no cost. Self-deception can be maintained through information avoidance (see, e.g., Loewenstein, 2006; Bénabou, 2015). Then, benefits from self-deception are traded off against making an informed decision. Oster *et al.* (2013) and Huck *et al.* (2017) provide empirical evidence from the field and the laboratory, respectively, for the relevance of this trade-off in different economic contexts.

Following Spiegler (2008), I allow a consumer to deceive herself about the presence of externalities as long as she is not fully convinced or informed. Thereby, I am agnostic about the question when self-deception is not possible anymore. If there is a threshold level below certainty, the result holds if consumers are sufficiently cursed. In the original game G's χ-virtual game $\bar{G}^{\chi}$, I need to relax this assumption by allowing self-deception unless the consumer is informed. The reason is of technical nature. In game $\bar{G}^{\chi}$, consumers form rational beliefs but their utility is cursed and hence there remains noise. The assumption guarantees the required behavioral equivalence of the two games.

Finally, cursedness does not apply to anticipatory utility. Else, the choice of motivated beliefs would be restricted by the bounds of objective beliefs. This asymmetry, however, requires additional structure for information decision $m = 1$ in game $\bar{G}^{\chi}$. Considering $m = 1$, consumers anticipate that they cannot choose r anymore in either game. In G, it is sufficient that a consumer anticipates $r = b$. In $\bar{G}^{\chi}$, anticipated and expected utility are determined by different utility functions due to the required behavioral equivalence of the two games. As a consequence, $r = b$ is not sufficient. For simplicity, I assume that $\alpha = 0$ if $m = 1$.

Tie-Breaking Rules. It will become clear during the analysis that the tie-breaking rules are not outcome relevant in the case of externalities as the distribution of consumer preferences is atomless. In the case of no externalities, firms' payoffs will turn out to be discontinuous. Then, tie-breaking rules should be part of the equilibrium concept as suggested by Jackson *et al.* (2002). Applying the logic from Blume (2003), it follows from the analysis below that the tie-breaking rules I have imposed above neither affect firms' equilibrium outcomes nor consumers' optimal product choice. Unless information acquisition in the case of no externalities or welfare are under consideration, they can be replaced by any random split.

3.4 Analysis and Results

In this section, I analyze the model described above. Arguments of functions are suppressed where it is notationally convenient and not needed for understanding. All missing proofs are presented in Appendix 3.5. First, I illustrate optimal consumer behavior. Observe that the material payoff from product choice a_h is independent of firm l's type ω for any consumer. Consequently, overall utility is simply $v - p_h$. Material payoff from product choice a_l is $v - p_l - \theta e$ if $\omega = bad$ and $v - p_l$ if $\omega = good$. An informed consumer ($m = 1$) is neither cursed ($\chi = 0$) nor can she motivate her beliefs ($\alpha = 0$). Her overall utility from product choice a_l is also deterministic and hence comparison of the alternatives is straightforward. For an uninformed consumer ($m = 0$), overall utility from product choice a_l depends on both her objective and motivated beliefs. In particular, the choice of motivated beliefs is intertwined with her product choice. For illustration, consider a consumer of type θ and take $m = 0$ as well as some prices $p_h, p_l \leq v$ as given. Then, it follows from the problem in (3.2) that the consumer finds product-belief pair (a_h, r) optimal only if

$$v - p_h \geq r(v - p_l - \theta e) + (1 - r)(v - p_l) \Leftrightarrow r \geq \frac{p_h - p_l}{\theta e}. \tag{3.4}$$

Define the set of motivated beliefs consistent with product-belief pair (a_h, r) being optimal as $R_h(\theta) := [(p_h - p_l)/\theta e, 1] \cap [0, 1]$. Choice (a_l, r) is optimal only if the inequality sign in (3.4) is reversed. Analogously, define the set of motivated beliefs consistent with with product-belief pair (a_l, r) being optimal as $R_l(\theta) := [0, (p_h - p_l)/\theta e] \cap [0, 1]$. The lemma below characterizes product-belief choice of an uninformed consumer.

Lemma 3.1. *If a consumer of type θ chooses $m = 0$, the following holds true:*

(i) If (a_h, r) is chosen, then the motivated belief is some $r \in R_h(\theta)$.

(ii) If (a_l, r) is chosen, then the motivated belief is $r = 0$.

The intuition behind this result is straightforward. Optimal choice of motivated beliefs trades off anticipated gains against expected material losses from a distorted decision. As overall utility from product choice a_h is independent of externalities, the program in (3.2) only requires motivated beliefs to rationalize a_h. Suppose the consumer is uninformed. Then, her expected overall utility from product choice a_l, however, decreases in both anticipated and expected externalities. Underestimating the perceived likelihood of externalities and thereby anticipating higher gains is the best she can do when considering firm l's product. As in Spiegler (2008), the result is indeed extreme. The reason is that only product choice a_l bears the risk of causing material losses, whose extend are independent of motivated beliefs. As a consumer does not incur any other costs for deceiving herself, she optimally chooses not to anticipate any potential loss.

Which product-belief pair does an uninformed χ-cursed consumer optimally choose? Take some prices $p_h, p_l \leq v$, and the consumer's corresponding objective belief about the presence of externalities, b, as given. For any $b \in (0, 1)$, the objective belief has full support and hence motivated beliefs can indeed be chosen.[8] It follows from Lemma 3.1 and the problem in (3.2) that an uninformed consumer

[8]Full support of the χ-cursed consumer's objective beliefs along the path of play immediately follows from the definition in (3.1). In the proof of Lemma 3.1, I formally argue that the exact same bounds apply to her objective beliefs off the path of play.

of type θ finds product-belief pair (a_h, r) with $r \in R_h(\theta)$ optimal if and only if

$$v - p_h \geq v - p_l - (1-\alpha)b\theta e \Leftrightarrow \theta \geq \frac{p_h - p_l}{(1-\alpha)be} =: \hat{\theta}.$$

At the same time, product-belief pair $(a_l, 0)$ is optimal for any uninformed consumer of type $\theta \leq \hat{\theta}$. Given prices, a consumer distorts her decision if she chooses $(a_l, 0)$, while she had chosen a_h if motivated beliefs cannot be formed, that is, $\alpha = 0$.

The main economic insight of Lemma 3.1 is that self-deception reduces perceived product differentiation in favor of firm l's product. Given both objective and motivated beliefs, the product is chosen as in a classical model of vertical differentiation. For a consumer, the perceived quality differs by how much she dislikes both anticipated and expected externalities.[9] Hence, underestimating the perceived likelihood of externalities through self-deception increases perceived quality of firm l's product. As perceived quality of firm h's externality-free product remains unchanged, self-deception leads to less perceived product differentiation by rendering firm l's product more attractive. Implications for competitive outcomes will be discussed below.

In this model, consumers anticipate that self-deception is possible only if they forgo the instrumental value of information. What is the optimal information choice? Take some prices $p_l < p_h \leq v$ as given and consider three groups of consumers depending on their type θ: (i) consumers who prefer firm l's product regardless of their information about externalities, that is,

$$v - p_l - \theta e > v - p_h \Leftrightarrow \theta < \frac{p_h - p_l}{e} =: \check{\theta},$$

(ii) consumers who prefer firm l's product if they are uninformed or learn that externalities do not occur and firm h's product if they learn that externalities occur, that is, $\theta \in (\check{\theta}, \hat{\theta})$, and (iii) consumers who prefer firm h's product unless they learn that externalities do not occur, that is, $\theta > \hat{\theta}$. The first group of consumers never wants to become perfectly informed, as

$$v - p_l - (1-\alpha)b\theta e > b\,(v - p_l - \theta e) + (1-b)\,(v - p_l) \Leftrightarrow \alpha > 0.$$

Why is that? Learning about the presence of externalities does not affect their

[9]See Baron (2009) for a model of complete information in which firms are vertically differentiated through certain production externalities.

product choice. It merely reduces overall utility as they cannot deceive themselves anymore. The second group of consumers is faced with the following trade-off: Avoiding information allows to self-deceive about the presence of externalities. At the same time, it insures against paying a relatively high price in the case of learning that externalities actually occur. On the other hand, they certainly forgo the expected bargain of obtaining an externality-free product at a lower price in the case of learning that externalities do not occur. How is this resolved? A consumer of this group chooses to avoid information if and only if

$$v - p_l - (1-\alpha)b\theta e > b\left(v - p_h\right) + (1-b)\left(v - p_l\right) \Leftrightarrow \theta < \frac{p_h - p_l}{(1-\alpha)e} =: \tilde{\theta},$$

where $\check{\theta} < \tilde{\theta} < \hat{\theta}$. Interestingly, $\tilde{\theta}$ does not depend on objective beliefs. To see why, rearrange the inequality above in order to obtain a consumer's benefit from ignorance net her forgone bargain from learning that no externalities occur, that is,

$$b(v - p_l - (1-\alpha)\theta e) > b\left(v - p_h\right).$$

Intuitively, the consumer prefers to hide behind uncertainty in order to exploit moral wiggle room by deceiving herself. She does not benefit from paying the higher price for the safe option sufficiently unless she learns that externalities do occur. But even under certainty, she would prefer to remain self-deceiving in order to choose the low-cost product. A consumer of type $\theta \geq \tilde{\theta}$ sufficiently values the safe option and hence prefers to resolve uncertainty. The instrumental value of information insures her against suffering from externalities.

In Grossman and van der Weele (2017), a similar logic can be found. There, agents care about both self-image and payoffs of others. Acting socially responsible is costly but not necessarily avoiding negative consequences on others' payoffs. This gives rise to moral wiggle room, as selfish behavior is not necessarily worse. Although uncertainty can be resolved, information avoidance may serve as an excuse for acting selfishly. Agents can still derive self-image from the belief that they would act socially responsible if they knew it had an impact without bearing the costs. The third group of consumers always wants to become perfectly informed, as

$$v - p_h \leq b\left(v - p_h\right) + (1-b)\left(v - p_l\right) \Leftrightarrow p_l \leq p_h.$$

Even if deceiving themselves under uncertainty, these consumers suffer so much from expected externalities that they intend to buy firm h's safe option. Hence, they can only benefit from learning about the presence of externalities as they can bet on a lower price without stakes. They decide as if they are classical agents which cannot form motivated beliefs.

How do competitive firms react to the outlined consumer behavior? As $\tilde{\theta}$ is determined independently of objective beliefs, firms' demand is well defined for each of firm l's externality types ω. If externalities occur, that is, $\omega = bad$, firms compete in products that are perceived to be vertically differentiated. For illustration purposes, consider only prices such that both firms cover the market, that is, $p_l < p_h \leq v$.[10] Given prices, any consumer of type $\theta < \tilde{\theta}$ chooses to avoid information, does not anticipate externalities and strictly prefers firm l's product. Consequently, the firm's price maximizes against demand $F(\tilde{\theta})$ given firm h's price. Any consumer of type $\theta \geq \tilde{\theta}$ chooses to become informed, learns that externalities occur and hence strictly prefers firm h's product. Firm h, however, cannot cash this in. By increasing its price, it induces consumers of types slightly above $\tilde{\theta}$ to avoid this information. Hence, the firm chooses a price that maximizes against demand $1 - F(\tilde{\theta})$ given firm l's price. If externalities do not occur, that is, $\omega = good$, again $\tilde{\theta}$ determines which consumers avoid information and which become informed. Then, however, any informed consumer learns that externalities do not occur and prefers firm l's product as long as its price does not exceed that of firm h. This gives rise to a Bertrand outcome. The proposition below characterizes the unique equilibrium.

Proposition 3.1. *Under Assumptions 3.1 and 3.2, there exists a unique equilibrium in the original game G. This equilibrium has the following properties:*

(i) Firms' equilibrium strategies, $\sigma_h^*(p_h|\omega)$ and $\sigma_l^*(p_l|\omega)$, are pure and type-dependent.

(ii) If firm l's type is $\omega = bad$, $p_l^* < p_h^* \leq v$, that is, both firms cover the market. Then,

- *all consumers of types $\theta < \tilde{\theta}^*$ avoid information, deceive themselves about the presence of externalities and choose firm l's product, and*
- *all consumers of types $\theta \geq \tilde{\theta}^*$ become perfectly informed and choose firm h's product.*

[10] In equilibrium, the market is always covered and both firms must be active if externalities occur, as shown in the proof of Proposition 3.1.

(iii) If firm l's type is $\omega = good$, $p_h^* = p_l^* = c$. Then, all consumers become perfectly informed and choose firm l's product.

Why are firms' equilibrium strategies type-dependent? Suppose that firm l's type is $\omega = bad$. Then, firms are perceived to be vertically differentiated and compete for the marginal consumer determined by type $\tilde{\theta}$. As the distribution of consumer types $f(\theta)$ is quasi-concave and not too extreme by Assumptions 3.1 and 3.2, there always exists a unique price pair that is mutual best response given optimal consumer behavior and objective beliefs. In particular, firm h's optimal price always exceeds the one of firm l, that is, both firms are active on the market. Neither firm can monopolize the market as preference type θ is distributed on $(0, \infty)$. If firm l's type is $\omega = good$, however, a Bertrand logic drives down prices to firm h's marginal costs. Consequently, all consumers choose to become informed and learn that externalities do not occur. By the tie-breaking rule, firm l wins all consumers as it adds larger value to market.

What role does cursedness of consumers play here? Being χ-cursed implies that consumers cannot fully infer firm l's type from observed prices in the proposed separating equilibrium. They play against the belief that firms follow type-dependent strategy profile $\sigma_{h,l}^*(p_h, p_l|\omega)$ as proposed with probability $1-\chi$ and type-independent average-strategy profile $\bar{\sigma}_{h,l}^*$ as defined above with probability χ. After observing prices that are supported by $\sigma_{h,l}^*(p_h, p_l|\omega = bad)$, they do not fully update their belief about the presence of externalities. To be precise, $b = 1 - \chi(1-q)$. If they observe prices that are supported by $\sigma_{h,l}^*(p_h, p_l|\omega = good)$, they do not fully rule out that externalities occur. Then, $b = \chi q$. By assumption, motivated beliefs can be chosen only if consumers do not fully update. It follows that an arbitrarily small level of cursedness χ is sufficient for the proposition to hold. Objective beliefs off the path of play are bounded in the exact same way as on the path of play. As $\tilde{\theta}$ is independent of both objective and motivated beliefs, the specification of off-path beliefs is not crucial here. The proposition holds true for any $b \in [\chi q, 1 - \chi(1-q)]$ after observing prices off the equilibrium path.[11] Moreover, this implies that optimal consumer behavior is not additionally distorted by objective beliefs.

Why does no pooling equilibrium exist in which firms employ type-independent strategies? If firm l's type is $\omega = good$, it can always take over the entire market

[11]This is true because any $\theta < \tilde{\theta}$ distorts her beliefs maximally to $r = 0$. If beliefs are not distorted maximally, for instance, due to convex psychological costs, $\tilde{\theta}$ may indeed depend on objective and motivated beliefs. Then, beliefs off the path of play need to be restricted to $b = 1$.

for a price equal to firm h's price, independent of objective beliefs. Observing equal prices, all consumers choose to become informed, learn that externalities do not occur and prefer firm l by the tie-breaking rule. If $\omega = bad$, however, such a price leads to zero demand as all consumers learn that externalities do occur and hence strictly prefer firm h. As consumers are χ-cursed, beliefs off the equilibrium path always have full support. Then, firm l can benefit from slightly undercutting firm h independently of objective beliefs as it attracts all consumers of type $\theta < \tilde{\theta}$ at some positive margin.

The main implication of the proposed result is that information avoidance is an endogenous market outcome if the low-cost firm's production causes negative externalities. This is purely driven by the possibility to exploit moral wiggle room through self-deception. Consumers which benefit from such conduct, avoid information in order to remain ignorant. In particular, consumers which intend to buy the safe but more expensive option only if externalities are certain, hide behind uncertainty in order to choose the cheaper option. Given prices, they distort their decision in order to benefit from anticipated gains. Competition cannot prevent such behavior. Moreover, ignorant consumers are willing to pay to remain ignorant in order to maintain distorted beliefs. In contrast, consumers which make informed decisions are willing to pay for the instrumental value of information. Consumers of type $\theta < \tilde{\theta}^*$ avoid information if and only if externalities occur, that is, $\omega = bad$. Take equilibrium prices $p_h^* > p_l^*$ as given. Then, the willingness to pay to remain ignorant is given by the expected difference in overall utility from either information choice. For a consumer of group (i), this is equivalent to the net benefit of self-deception,

$$v - p_l^* - (1-\alpha)b\theta e - b\left(v - p_l^* - \theta e\right) - (1-b)\left(v - p_l^*\right) = \alpha b\theta e > 0.$$

Intuitively, it increases in the weight of anticipated utility α and a consumer's expected material loss from externalities, $b\theta e$. For a consumer of group (ii), the willingness to pay to remain ignorant is

$$v - p_l^* - (1-\alpha)b\theta e - b\left(v - p_h^*\right) - (1-b)\left(v - p_l^*\right) = b(p_h^* - p_l^* - (1-\alpha)\theta e) > 0 \Leftrightarrow \theta < \tilde{\theta}^*.$$

It increases in the price difference, in α and b, whereas it decreases in θ and e. The larger the price difference, the more she can safe by hiding behind uncertainty. Caring more about externalities , however, renders ignorance more costly as there remains the risk of suffering from realized externalities. At $\tilde{\theta}^*$, it is zero by construction. Hence, any consumer of type $\theta > \tilde{\theta}^*$ is willing to pay for information. The reverse logic applies. Also consumers of group (iii) are willing to pay for information, as

$$b(v - p_h) + (1 - b)(v - p_l) - v + p_h = (1 - b)(p_h - p_l).$$

Their expected difference in overall utility increases in the price difference as the potential bargain becomes more attractive. At the same time, it decreases in b as the bargain is believed to be less likely. As a benchmark, consider the case in which consumers cannot deceive themselves. To be precise, the benchmark corresponds to the original game G with $\alpha = 0$, which is ruled out above. As consumers are still χ-cursed, their objective beliefs are distorted. The perceived the likelihood of externalities, however, they cannot willingly distort how anymore. Taking this into account, it follows from the arguments above that no consumer is willing to pay to remain ignorant. Forgoing the instrumental value of information comes at no benefit. In particular, $\check{\theta}$ and $\tilde{\theta}$ coincide and all consumers of type $\theta \leq \tilde{\theta}$ are indifferent between becoming informed and not as they correctly assess the likelihood of externalities. Due to the tie-breaking rule, they choose to become informed as long as information is costless. All consumers of type $\theta > \tilde{\theta}$ are willing to pay for information as in the original game G with $\alpha > 0$.

As discussed above, self-deception distorts the perceived likelihood of externalities downwards. As a consequence, firm l's product becomes more attractive for given prices as its perceived quality increases. At the same time, consumers perceive firms as less differentiated, which intensifies competition (see, e.g., Shaked and Sutton, 1982). The proposition below summarizes the result.

Proposition 3.2. *Given Assumptions 3.1 and 3.2 and that firm l is of type $\omega = bad$, firm h's equilibrium price is lower in the original game G compared to a benchmark in which consumers cannot deceive themselves. The comparison of firm l's equilibrium price is ambiguous. Self-deception shifts demand toward firm l if one of the following holds true:*

(i) Demand is relatively sensitive to self-deception such that competitive pressure from firm h does not drive down firm l's price.

(ii) Demand is relatively sensitive to prices such that competitive pressure from firm h drives down firm l's price, whereby the effect is not too strong.

Intuitively, firm l can increase its price as uninformed consumers perceive its quality as higher through self-deception. As this renders firm h's product less attractive, firm h decreases its price in order to compensate consumers. Due to strategic complementarity, this creates downward pressure on firm l's price, which in turn allows firm h to slightly increase its price. Overall, firm h's price always falls and hence the firm loses infra-margins. The overall effect on firm l's price depends on both by how much firm h's price falls and how sensitive demand reacts to self-deception relative to price changes. The following cases can occur: (i) firm h reduces its price, while firm l increases its prices and still gains demand, (ii) both firms reduce prices, while demand shifts toward firm l, and (iii) both firms reduce prices, while demand shifts toward firm h. In the first case, demand is affected so strongly by the perceived increase of firm l's quality such that firm h's price reduction does not fully compensate for the net gain after firm l's price increase. Firm l can gain both infra- and extra-margins. In the second case, consumers react more sensitive to firm h's price reduction such that competitive pressure overweights and firm l reduces its price in turn. As long as demand is not too sensitive to prices relative to self-deception, firm l gains extra-margins. This holds true, for instance, if consumer preferences are distributed uniformly on a bounded support or exponentially. In the third case, however, consumers react so sensitive to firm h's price reduction that firm l cannot do better than reducing its price drastically in order to regain some of the consumers. Then, firm l loses both extra- and infra-margins, while firm h gains extra-margins. Interestingly, there is a discrete jump in prices and demand in comparison to the benchmark even for an arbitrarily small α.

An important economic implication is that self-deception can harm firm h, while firm l can benefit. Moreover, if demand is distorted toward firm l, some consumers distort their decision, or, in terms of Brunnermeier and Parker (2005), take excessive risk. Why is this? Interpret firm h's offer as safe with respect to externalities and firm l's offer as risky. The benchmark in which consumers cannot take excessive risk by construction determines which consumers should choose the risky option. If demand is higher in the original game, however, these additional, ignorant consumers take excessive risk as self-deception distorts their decision toward the risky option. With this interpretation in mind, demand distortion toward firm l as an endogenous market outcome is in line with the main finding in Brunnermeier and Parker (2005).

3.5 Discussion and Conclusion

This paper investigated how the possibility to exploit moral wiggle room through self-deception can affect market outcomes. It turned out that self-deception can lead to avoidance of information about the impact of purchasing decisions as an endogenous market outcome. Moreover, self-deception can distort market demand toward low-cost, externality-causing production and render costly mitigation of externalities less profitable.

This yields policy implications. Suppose policymakers pursue mitigating externalities that are caused by the production of consumer goods and cannot be prohibited by law or contracts, while they care relatively less about anticipated utility. Then, information provision and campaigns are not sufficient as long as consumers can deceive themselves about the presence of externalities. In contrast, taxing externalities, subsidizing its mitigation or introducing a binding price floor can reduce monetary incentives for remaining ignorant and lead to more informed decision making.

Moreover, if using the more costly, externality-mitigating technology becomes less profitable through self-deception, both innovation and entry can become less likely if access to this technology causes fixed costs.[12]

Finally, externalities can be interpreted differently. Consumption of the low-cost product may cause negative externalities on future-selfs, for instance, health or well-being. It might also indicate a drawback in terms of quality in the classical sense, which can be experienced only by usage. The question whether self-deception is relevant in these contexts, however, is open to future research.

[12]Recouping fixed costs requires that externalities occur. Else, there are zero profits. Cursed consumers, however, do not fully infer on the presence of externalities from observing entry or innovation.

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Appendix A: Proofs

Before providing proofs of the results from the main text, I derive the preliminary result stated in the lemma below as it proofs useful later.

Lemma 3.2. *Suppose that a consumer holds objective belief* $\bar{b}^{\chi}(\omega|p_h', p_l', \sigma_{h,l}) := \bar{b}^{\chi}$ *after observing some price pair* (p_h', p_l') *in game* $\bar{G}^{\chi}$*. Then, she behaves in* $\bar{G}^{\chi}$ *as if she holds objective belief* $b(\omega|p_h', p_l', \sigma_{h,l}, \chi) = (1-\chi)\bar{b}^{\chi} + \chi q(\omega)$ *in the original game* G*.*

Proof. Take firms' strategy profile $\sigma_{h,l}$ as given. In order to proof the statement of the lemma, I distinguish the following two cases: (i) a consumer observes prices along the path of play and chooses m and (a, r) conditionally on m, and (ii) a consumer observes prices off the path of play and chooses m and (a, r) conditionally on m.

Case (i): Consider game $\bar{G}^{\chi}$. After observing any price pair $(p_h', p_l') \in \text{supp}(\sigma_{h,l})$, a consumer updates her objective belief by applying Bayes' rule. Then, for any $\omega \in \Omega$,

$$\bar{b}^{\chi}(\omega|p_h', p_l', \sigma_{h,l}) := \frac{q(\omega)\sigma_{h,l}(p_h', p_l'|\omega)}{\bar{\sigma}_{h,l}(p_h', p_l')}.$$

Take sequential rationality as given, that is, (a, r) is chosen optimally conditional on m as well as ω if $m = 1$. Substitute $\bar{u}^{\chi}(a, p_h', p_l'|\theta, \omega)$ for $u(a, p_h', p_l'|\theta, \omega)$ in the non-anticipatory part of the objective in (3.2). Then, a consumer's expected overall utility from $m = 0$ is given by

$$\alpha\left[ru(a, p_h', p_l'|\theta, \omega = bad) + (1-r)u(a, p_h', p_l'|\theta, \omega = good)\right] + (1-\alpha) \times$$

$$\left[\bar{b}^{\chi}(\omega = bad|p_h', p_l', \sigma_{h,l}, \chi)\left((1-\chi)u(a, p_h', p_l'|\theta, \omega = bad) + \chi\sum_{\omega'\in\Omega} q(\omega')u(a, p_h', p_l'|\theta, \omega')\right) + \right.$$

$$\left.\bar{b}^{\chi}(\omega = good|p_h', p_l', \sigma_{h,l}, \chi)\left((1-\chi)u(a, p_h', p_l'|\theta, \omega = good) + \chi\sum_{\omega'\in\Omega} q(\omega')u(a, p_h', p_l'|\theta, \omega')\right)\right].$$

Rearranging yields

$$\alpha\left[ru(a,p_h',p_l'|\theta,\omega=bad)+(1-r)u(a,p_h',p_l'|\theta,\omega=good)\right]+(1-\alpha)\times$$
$$\Big[\Big(\underbrace{(1-\chi)\bar{b}^{\chi}(\omega=bad|p_h',p_l',\sigma_{h,l},\chi)+\chi q(\omega=bad)}_{=b(\omega=bad|p_h',p_l',\sigma_{h,l},\chi)}\Big)u\left(a,p_h',p_l'|\theta,\omega=bad\right)+$$
$$\Big(\underbrace{(1-\chi)\bar{b}^{\chi}(\omega=good|p_h',p_l',\sigma_{h,l},\chi)+\chi q(\omega=good)}_{=b(\omega=good|p_h',p_l',\sigma_{h,l},\chi)}\Big)u(a,p_h',p_l'|\theta,\omega=good)\Big]. \quad (3.5)$$

Given $m=0$, a consumer's expected overall utility from (a,r) is as in (3.5) after adjusting that (a,r) is not conditional on m. Substituting for $\alpha=0$ in (3.5) yields a consumer's expected overall utility from $m=1$ as she anticipates that r cannot be chosen anymore. For these cases, the argument immediately follows from the definition of objective beliefs along the path of play in the original game G as given in (3.1). Given $m=1$, $\chi=0$ by assumption and hence $\bar{b}^{\chi=0}(\omega|p_h',p_l',\sigma_{h,l})=b(\omega|p_h',p_l',\sigma_{h,l},\chi=0)$ and $\bar{u}^{\chi=0}(a,p_h',p_l'|\theta,\omega)=u(a,p_h',p_l'|\theta,\omega)$.

Case (ii): Consider again game $\bar{G}^{\chi}$. Suppose that after observing price pair $(p_h'',p_l'')\notin \text{supp}(\sigma_{h,l})$, a consumer updates her objective beliefs to some $\bar{b}^{\chi}(\omega|p_h'',p_l'',\sigma_{h,l})$. Substituting for (p_h'',p_l'') in (3.5) yields a consumer's expected overall utility from $m=0$. Given $m=0$, a consumer's expected overall utility from (a,r) is as in (3.5) after adjusting prices and that (a,r) is not conditional on m. Substituting for both prices and $\alpha=0$ in (3.5) yields a consumer's expected overall utility from $m=1$ as she anticipates that r cannot be chosen anymore. For these cases, the argument immediately follows for any $\bar{b}^{\chi}(\omega|p_h'',p_l'',\sigma_{h,l})$ off the path of play. Given $m=1$, $\chi=0$ by assumption and hence $\bar{b}^{\chi=0}(\omega|p_h'',p_l'',\sigma_{h,l})=b(\omega|p_h'',p_l'',\sigma_{h,l},\chi=0)$ and $\bar{u}^{\chi=0}(a,p_h'',p_l''|\theta,\omega)=u(a,p_h'',p_l''|\theta,\omega)$. □

Proof of Lemma 3.1. Consider a consumer of type θ and fix $m=0$. First, I argue that the consumer can always choose motivated belief r from $[0,1]$. In game $\bar{G}^{\chi}$, the choice of r is independent of the support of objective beliefs $\bar{b}^{\chi}$ by assumption. Consider the original game G. It immediately follows from the definition in (3.1) that objective beliefs along the path of play are bounded away from zero and one for any $\chi\in(0,1]$. Then, objective beliefs have full support and hence r can be chosen $[0,1]$. Consumers act in game G as if their utility is given by $\bar{u}^{\chi}(a,p_h',p_l'|\theta,\omega)$ as defined in (3.3) in game $\bar{G}^{\chi}$. It follows from Lemma 3.2 that objective beliefs off the path of play are bounded in the exact same way as on the path of play,[13] which completes the argument.

In order to proof the two statements of the lemma, I distinguish the following four cases: (i) the outside option is dominated by both products, (ii) only firm l's product is dominated by the outside option, (iii) only firm h's product is dominated by the outside option, and (iv) both firms' products are dominated by the outside option.

Case (i): Fix some prices $p_h, p_l \leq v$ and let motivated belief $r' \geq 0$ be such that the condition in (3.4) holds with equality. This implies $r \leq r' \in R_l(\theta)$ and $r \geq r' \in R_h(\theta)$. If a consumer of type θ chooses some (a_h, r), the optimization problem in (3.2) reduces to the constrained problem

$$\max_{r \in R_h} v - p_h,$$

which is independent of r. Hence, any motivated belief $r \in R_h(\theta)$ is optimal. If the consumer chooses some (a_l, r), the optimization problem in (3.2) reduces to the constrained problem

$$\max_{r \in R_l} v - p_l - (\alpha r + (1-\alpha)b)\,\theta e.$$

Here, I exploit Lemma 3.2 for notational convenience, that is, I substitute b for $(1-\chi)\bar{b}^{\chi} + \chi q$ in game $\bar{G}^{\chi}$. As the objective function is continuous and strictly decreasing in r, a motivated belief equal to $r = \inf R_l(\theta) = 0$ is optimal.

Case (ii): Fix some prices $p_h \leq v < p_l$. Then, (a_l, r) can never be optimal for any r. It follows from the tie-breaking rule and the problem in (3.2) that any consumer chooses some (a_h, r) if and only if

$$v - p_h \geq 0,$$

which is independent of r. As $p_h < p_l$, observe that $R_h(\theta) = [0,1]$ for any θ. Hence, any motivated belief $r \in [0,1]$ is optimal.

Case (iii): Fix some prices $p_l \leq v < p_h$. Then, (a_h, r) can never be optimal for any r. It follows from the tie-breaking rule and the problem in (3.2) that a consumer of type θ chooses some (a_l, r) only if

$$r \leq \frac{v - p_l}{\theta e}.$$

[13] See also Eyster and Rabin, 2002, p. 10 f. and fn. 9 for a discussion.

Define the set of motivated beliefs that is consistent with action a_l being optimal as $R_l'(\theta) := [0, (v - p_l)/\theta e] \cap [0, 1]$. If the consumer chooses some (a_l, r), the optimization problem in (3.2) reduces to the constrained problem

$$\max_{r \in R_l'} v - p_l - (\alpha r + (1 - \alpha) b)\, \theta e.$$

Again, I exploit Lemma 3.2 for notational convenience, that is, I substitute b for $(1 - \chi)\bar{b}^{\chi} + \chi q$ in game $\bar{G}^{\chi}$. As the objective function is continuous and strictly decreasing in r, a motivated belief of $r = \inf R_l'(\theta) = 0$ is optimal.

Case (iv): Fix some prices $p_h, p_l > v$. Then, both (a_h, r) and (a_l, r) are strictly dominated by (a_0, r) for any r. □

Proof of Proposition 3.1. In order to proof the proposition, I proceed by the following steps. First, I show existence of an $\chi - PCE$ under Assumptions 3.1 and 3.2 for any χ. Therefore, I need to show that there exists some PBE in original game G's χ-virtual game $\bar{G}^{\chi}$. Following Definition 3 from Eyster and Rabin (2002, p. 10), this is equivalent to showing existence of some strategy profile σ^* that satisfies the One-Shot-Deviation Principle (OSDP) for every player given objective beliefs and strategies of the others game $\bar{G}^{\chi}$ (for a formal argument, see Hendon *et al.*, 1996). Second, I show that the equilibrium is unique. Third, I verify that consumers, which choose product l and avoid information, deceive themselves in a sense that their motivated belief about the presence of externalities is equal to zero.

Step (i): First, I derive a consumer's equilibrium strategy conditional on observed prices and her type, $\sigma_c^*(m, a, r | p_h, p_l, \theta)$. Therefore, I pin down optimal behavior of consumers in game $\bar{G}^{\chi}$. To be precise, I derive the optimal choice of (a, r) for any consumer type θ given prices, her information choice m and objective beliefs $\bar{b}^{\chi}$. Then, I determine her optimal choice of m given optimal continuation play. Notationally, it is convenient to exploit Lemma 3.2 by substituting $b(\omega | p_h, p_l, \sigma_{h,l}, \chi)$ for $(1 - \chi)\bar{b}^{\chi}(\omega | p_h, p_l, \sigma_{h,l}, \chi) + \chi q(\omega)$ in $\bar{u}^{\chi}$. I distinguish the following four cases: (i) the outside option is dominated by both products, (ii) only firm l's product is dominated by the outside option, (iii) only firm h's product is dominated by the outside option, and (iv) both firms' products are dominated by the outside option. As $\chi = 0$ after choosing $m = 1$, a consumer updates her objective belief to $b = 1$ if $\omega = bad$ and $b = 0$ if $\omega = good$ regardless of observed prices. Then, she cannot motivate her belief r in game G as b is a point belief, that is, $r = b$. Additionally, $m = 1$ implies $\alpha = 0$. This rules out the choice of r in either game. For simplicity, I set $r = b$ in game $\bar{G}^{\chi}$ as well. After choosing $m = 0$, however, r can always be chosen from $[0, 1]$ as argued in the proof of Lemma 3.1.

Case (i): Fix some prices $p_h, p_l \leq v$. Consider a consumer of type θ and suppose for the moment that $m = 0$. Then, b lies in $[q, 1)$ for any $\chi \in (0, 1]$ regardless of whether prices on or off the path of play are observed as argued in the proof of Lemma 3.1. The lemma and the problem in (3.2) imply that (a_h, r) with $r \in R_h(\theta)$ is optimal if and only if

$$v - p_h \geq v - p_l - (1 - \alpha)b\theta e \Leftrightarrow \theta \geq \frac{p_h - p_l}{(1 - \alpha)be} =: \hat{\theta}(p_h, p_l|b). \tag{3.6}$$

On the other hand, θ finds $(a_l, 0)$ optimal if and only if $\theta \leq \hat{\theta}$. Observe that $\hat{\theta} \in (0, \infty)$ for any $p_h > p_l$ and that $(a_l, 0)$ cannot be optimal for any $p_h \leq p_l$.

Now, suppose that $m = 1$ and $\omega = bad$. Then, $\chi = 0$ and $\alpha = 0$ imply $b = 1$ and that r cannot be chosen. For simplicity, I set $r = b$ as mentioned above. It follows that θ finds $(a_h, 1)$ optimal if and only if

$$v - p_h \geq v - p_l - \theta e \Leftrightarrow \theta \geq \frac{p_h - p_l}{e} =: \check{\theta}(p_h, p_l). \tag{3.7}$$

On the other hand, θ finds $(a_l, 1)$ optimal if and only if $\theta \leq \check{\theta}$ and $\check{\theta} \in (0, \infty)$ for $p_h > p_l$. Observe that $(a_l, 0)$ cannot be optimal for any θ if $p_h \leq p_l$.

Finally, suppose that $m = 1$ and $\omega = good$. Then, $b = 0$ and hence the material payoff from either firm's product is independent of θ. Moreover, goods are homogeneous and firm l adds larger value to the market as it produces at strictly lower constant marginal costs than firm h. It follows from the tie-breaking rule that all consumers find $(a_h, 0)$ optimal if and only if

$$v - p_h > v - p_l \Leftrightarrow p_l > p_h.$$

On the other hand, all consumers find $(a_l, 0)$ optimal if and only if $p_h \geq p_l$.

Given these observations, what is the optimal information choice? Suppose for the moment that $p_h > p_l$ and observe that $\hat{\theta} > \check{\theta}$ for any $b \in (0, 1)$. A consumer of type $\theta \leq \check{\theta}$ finds information choice $m = 0$ optimal if and only if it expects larger overall utility given objective beliefs. By sequential rationality and anticipation of

$\alpha = 0$ for $m = 1$, this is equivalent to

$$v - p_l - (1-\alpha)b\theta e \geq b\,(v - p_l - \theta e) + (1-b)\,(v - p_l) \Leftrightarrow \alpha \geq 0,$$

which always holds true by assumption. Analogously, a consumer of type $\theta \in [\check{\theta}, \hat{\theta}]$ finds information choice $m = 0$ optimal if and only if

$$\begin{aligned} v - p_l - (1-\alpha)b\theta e &\geq b\,(v - p_h) + (1-b)\,(v - p_l) \\ \Leftrightarrow \theta &\leq \frac{p_h - p_l}{(1-\alpha)e} =: \tilde{\theta}(p_h, p_l). \end{aligned} \tag{3.8}$$

Observe that $\check{\theta} < \tilde{\theta} < \hat{\theta}$ for any $b \in (0,1)$. For $p_h > p_l$, it follows that any $\theta \in [\check{\theta}, \tilde{\theta}]$ finds $m = 0$ optimal and any $\theta \in [\tilde{\theta}, \hat{\theta}]$ finds $m = 1$ optimal. Finally, $\theta \geq \hat{\theta}$ finds information choice $m = 0$ optimal if and only if

$$v - p_h \geq b\,(v - p_h) + (1-b)\,(v - p_l) \Leftrightarrow p_l \geq p_h,$$

a contradiction. Hence, any $\theta \geq \hat{\theta}$ finds $m = 1$ optimal if $p_h > p_l$.

Now, suppose that $p_h \leq p_l$. By sequential rationality and anticipation of $\alpha = 0$ if $m = 1$, a consumer of type θ finds information choice $m = 0$ optimal if and only if

$$v - p_h \geq b\,(v - p_h) + (1-b)\,(v - p_h),$$

which always holds with equality. Hence, any θ is indifferent between choosing $m = 0$ and $m = 1$ and chooses $m = 1$ by the tie-breaking rule if $p_h \leq p_l$.

Case (ii): Fix some prices $p_h \leq v < p_l$. First, suppose that $m = 0$. Then, it follows from Lemma 3.1, the tie-breaking rule and the problem in (3.2) that (a_h, r) with some $r \in R_h(\theta)$ is optimal for any θ.

Second, suppose that $m = 1$. It follows from the tie-breaking rule that (a_h, b) is optimal for any θ independent of ω.

Given the above observations, what is the optimal information choice? By sequential rationality and anticipation of $\alpha = 0$ if $m = 1$, a consumer of type θ finds information choice $m = 0$ optimal if and only if

$$v - p_h \geq q\,(v - p_h) + (1-q)\,(v - p_h),$$

which always holds with equality. Hence, any θ is indifferent between $m = 0$ and

$m = 1$. By the tie-breaking rule, she chooses $m = 1$.

Case (iii): Fix some prices $p_l \leq v < p_h$. First, suppose that $m = 0$. It follows from Lemma 3.1, the tie-breaking rule and the problem in (3.2) that (a_0, r) with some $r \in ((v - p_l)/\theta e, 1]$ is optimal if and only if $\theta \geq \hat{\theta}(v, p_l|b)$ and $(a_l, 0)$ is optimal if and only if $\theta \leq \hat{\theta}(v, p_l|b)$. Observe that $\hat{\theta}(v, p_l|b) \in (0, \infty)$ for $p_l < v$ and that $(a_l, 0)$ cannot be optimal for any θ if $p_l = v$.

Second, suppose that $m = 1$ and $\omega = \mathit{bad}$. It follows from the tie-breaking rule that $(a_l, 1)$ is optimal if and only if $\theta \leq \check{\theta}(v, p_l)$ and $(a_0, 1)$ is optimal if and only if $\theta \geq \check{\theta}(v, p_l)$. Observe that $\check{\theta}(v, p_l) \in (0, \infty)$ for $p_l < v$. Hence, $(a_l, 1)$ cannot be optimal for any θ if $p_l = v$.

Finally, suppose that $m = 1$ and $\omega = \mathit{good}$. Then, a consumer's material payoff from firm l's product is independent of her type θ. It follows from the tie-breaking rule that any θ finds $(a_l, 0)$ optimal.

Given the above observations, what is the optimal information choice? For any $b \in (0, 1)$, any $\hat{\theta}(v, p_l|b) > \check{\theta}(v, p_l)$. By sequential rationality and anticipation of $\alpha = 0$ if $m = 1$, any $\theta \leq \check{\theta}(v, p_l)$ finds information choice $m = 0$ optimal if and only if

$$v - p_l - (1 - \alpha)b\theta e \geq b\,(v - p_l - \theta e) + (1 - b)\,(v - p_l) \Leftrightarrow \alpha \geq 0,$$

which always holds true by assumption. Analogously, $\theta \in [\check{\theta}(v, p_l), \hat{\theta}(v, p_l)]$ finds information choice $m = 0$ optimal if and only if

$$\begin{aligned} v - p_l - (1 - \alpha)b\theta e &\geq b\,(0) + (1 - b)\,(v - p_l) \\ \Leftrightarrow \theta &\leq \frac{v - p_l}{(1 - \alpha)e} = \tilde{\theta}(v, p_l). \end{aligned}$$

Observe that $\check{\theta}(v, p_l) < \tilde{\theta}(v, p_l) < \hat{\theta}(v, p_l|b)$ for any $b \in (0, 1)$. Further, any $\theta \in [\check{\theta}(v, p_l), \tilde{\theta}(v, p_l)]$ finds $m = 0$ optimal and any $\theta \in [\tilde{\theta}(v, p_l), \hat{\theta}(v, p_l|b)]$ finds $m = 1$ optimal. Finally, any $\theta \geq \hat{\theta}(v, p_l|b)$ finds information choice $m = 0$ optimal if and only if

$$0 \geq b\,(0) + (1 - b)\,(v - p_l)\,,$$

which can only hold true with equality for $p_l = v$. For $p_l < v$, the inequality above is violated. It follows from the tie-breaking rule that any $\theta \geq \hat{\theta}(v, p_l)$ finds $m = 1$

optimal for $p_l \leq v$.

Case (iv): Fix some prices $p_h, p_l > v$. First, suppose that $m = 0$. It follows from Lemma 3.1 and that neither (a_h, r) nor (a_l, r) can be optimal for θ for any r. Hence, $(a_0, 1)$ is optimal for any θ.

Second, suppose that $m = 1$. It follows that neither (a_h, b) nor (a_l, b) can be optimal for any θ independent of ω. Hence, (a_0, b) is optimal for any θ.

Given the above observations, what is the optimal information choice? By sequential rationality and anticipation of $\alpha = 0$ if $m = 1$, any consumer of type θ θ finds information choice $m = 0$ optimal if and only if

$$0 \geq b\,(0) + (1 - b)\,(0),$$

which always holds with equality. Hence, any θ is indifferent between $m = 0$ and $m = 1$. By the tie-breaking rule, she chooses $m = 1$.

Now, let $\sigma_c^*(m, a, r|p_h, p_l, \theta)$ denote the equilibrium strategy of a consumer conditional on observed prices and her type and define it in the following way: (i) $\sigma_c^*(m, a, r|p_h, p_l, \theta)$ assigns positive probability mass to information choice $m \in \{0, 1\}$ and conditional product-belief pairs $(a, r) \in A \times [0, 1]$ if and only if it is optimal for θ given prices and objective beliefs as derived above for game $\bar{G}^\chi$, and (ii) for each possible information choice $m \in \{0, 1\}$, it assigns positive probability mass to product-belief pairs $(a, r) \in A \times [0, 1]$ if and only if it is optimal for θ given the information choice, prices and objective beliefs as derived above for game $\bar{G}^\chi$.

Next, I derive firms' type-dependent and separating equilibrium strategies $\sigma_h^*(p_h|\omega)$ and $\sigma_l^*(p_l|\omega)$, which constitute profile $\sigma_{h,l}^*$. Hence, I consider a separating equilibrium in game $\bar{G}^\chi$ and assume that after observing price pair $(p_h, p_l) \notin \text{supp}(\sigma_{h,l}^*)$, all consumers hold some off-path belief $\bar{b}^\chi(\omega|p_h', p_l', \sigma_{h,l}^*) \in [0, 1]$. By Lemma 3.2, consumers behave in game $\bar{G}^\chi$ as if they hold off-path belief $b(\omega|p_h', p_l', \sigma_{h,l}^*, \chi) := (1 - \chi)\bar{b}^\chi(\omega|p_h', p_l', \sigma_{h,l}^*, \chi) + \chi q(\omega)$ in game G. Again, it is convenient to exploit Lemma 3.2 by substituting $b(\omega|p_h, p_l, \sigma_{h,l}^*, \chi)$ for $(1 - \chi)\bar{b}^\chi(\omega|p_h, p_l, \sigma_{h,l}^*, \chi) + \chi q(\omega)$ in $\bar{u}^\chi$ in game $\bar{G}^\chi$.

In order to determine $\sigma_h^*(p_h|\omega = \textit{good})$ and $\sigma_l^*(p_l|\omega = \textit{good})$, I claim that conditional on $\omega = \textit{good}$ and given $\sigma_c^*(m, a, r|p_h, p_l, \theta)$ as well as objective beliefs as defined above, it is optimal for firm $i = h, l$ to charge $p_i = c$. The following arguments shall proof the claim. Fix some pure firm equilibrium strategy profile $\sigma_{h,l}^*$. As I consider a separating equilibrium in game $\bar{G}^\chi$, the support of firms' pure equilibrium strategies must not overlap across types. To be precise, $\text{supp}(\sigma^*_{h,l}(p_h, p_l|\omega = \textit{bad})) \cap \text{supp}(\sigma^*_{h,l}(p_h, p_l|\omega = \textit{good})) = \varnothing$. Consider some $p'_h \in \text{supp}(\sigma^*_h(p_h|\omega = \textit{good})$. Then, firm l's best response is to set $p_l = p'_h$, whether $(p'_h, p'_h) \in \text{supp}(\sigma^*_{h,l}(p_h, p_l|\omega = \textit{bad}))$ or not. Why is that? In case of $(p'_h, p'_h) \notin \text{supp}(\sigma^*_{h,l}(p_h, p_l|\omega = \textit{good}))$, consumers update their objective belief about $\omega = \textit{bad}$ to some $\bar{b}^\chi \in [0, 1]$. By Lemma 3.2, they behave in game $\bar{G}^\chi$ as if they hold objective belief $b = (1 - \chi)\bar{b}^\chi + \chi q \in [q, 1)$ in the original game G. In case of $(p'_h, p'_h) \in \text{supp}(\sigma^*_{h,l}(p_h, p_l|\omega = \textit{good}))$, consumers update their objective belief to $\bar{b}^\chi = 0$. By Lemma 3.2, they behave in game $\bar{G}^\chi$ as if they hold objective belief $b = \chi q$ in the original game G. As $p_l = p'_h$, all consumers choose $m = 1$ in either case, learn that $\omega = \textit{good}$ and choose firm l's product due to the tie-breaking rule. As any $p_l < p'_h$ leads to a lower payoff for similar arguments and any $p_l > p'_h$ leads to zero demand, setting the exact same price as firm h is the best firm l can respond if $\omega = \textit{good}$.

Now, consider some $p'_l \in \text{supp}(\sigma^*_l(p_h|\omega = \textit{good})$ such that $p'_l > c$. Then, firm h's best response is to set $p_h = p'_l - \epsilon$ with ϵ arbitrarily small, whether $(p'_l - \epsilon, p'_l) \in \text{supp}(\sigma^*_{h,l}(p_h, p_l|\omega = \textit{bad}))$ or not. To see why this holds true, observe that in case of $(p'_l - \epsilon, p'_l) \notin \text{supp}(\sigma^*_{h,l}(p_h, p_l|\omega = \textit{good}))$, consumers update their objective belief to some $\bar{b}^\chi \in [0, 1]$. By Lemma 3.2, they behave in game $\bar{G}^\chi$ as if they hold objective belief $b = (1 - \chi)\bar{b}^\chi + \chi q \in [q, 1)$ in the original game G. In case of $(p'_h, p'_h) \in \text{supp}(\sigma^*_{h,l}(p_h, p_l|\omega = \textit{good}))$, consumers update their objective belief to $\bar{b}^\chi = 0$. By Lemma 3.2, they behave in game $\bar{G}^\chi$ as if they hold objective belief $b = \chi(1 - q)$ in the original game G. As $p_h = p'_l - \epsilon$, all consumers choose $m = 1$ in either case, learn that $\omega = \textit{good}$ and choose firm h's product as it is cheaper. Any $p_h \geq p'_l$ leads to zero demand as all consumers interested in firm h become informed. Hence, undercutting firm l slightly is the best firm h can respond if $\omega = \textit{good}$. Consequently, the only price pair that is mutual best response is given by (c, c) and hence firms strategies are pure and unique, that is, $\sigma^*_{h,l}(c, c|\omega = \textit{good}) = 1$.

Take $\sigma^*_{h,l}(p_h, p_l|\omega = \textit{good})$, $\sigma^*_c(m, a, r|p_h, p_l, \theta)$, some off-path beliefs, the tie-breaking rules and Lemma 3.2 as given. Then, I can derive each firm's demand conditional on type ω by

$$D_h\left(p_h, p_l | \omega = bad\right) := \begin{cases} 1 & \text{if} \quad p_h \leq \min\{v, p_l\}, \\ 1 - F(\tilde{\theta}(p_h, p_l)) & \text{if} \quad p_l < p_h \leq v, \\ 0 & \text{if} \quad \min\{v, p_l\} < p_h, \end{cases}$$

and

$$D_h\left(p_h, p_l | \omega = good\right) := \begin{cases} 1 & \text{if} \quad p_h < p_l \leq v \vee p_h \leq v < p_l, \\ 0 & \text{if} \quad \min\{v, p_l\} < p_h. \end{cases}$$

Similarly, demand for firm l conditional on objective beliefs b and type ω can be defined by

$$D_l\left(p_l, p_h | \omega = bad\right) := \begin{cases} F(\tilde{\theta}(v, p_l)) & \text{if} \quad p_l < v < p_h, \\ F(\tilde{\theta}(p_h, p_l)) & \text{if} \quad p_l < p_h \leq v, \\ 0 & \text{if} \quad \min\{v, p_h\} \leq p_l, \end{cases}$$

and

$$D_h\left(p_h, p_l | \omega = good\right) := \begin{cases} 1 & \text{if} \quad p_l \leq \min\{v, p_l\}, \\ 0 & \text{if} \quad \min\{v, p_h\} < p_l. \end{cases}$$

Why is demand well-defined? For any price pair $(p'_h, p'_l) \neq (c, c)$, consumers hold some objective belief $\bar{b}^\chi(p'_h, p'_l, \sigma^*_{h,l}) \in [0, 1]$ in game $\bar{G}^\chi$. Then, they behave as if they hold objective belief $b(p'_h, p'_l, \sigma^*_{h,l}, \chi) = (1 - \chi)\bar{b}^\chi(p'_h, p'_l, \sigma^*_{h,l}) + \chi q \in [q, 1)$ in game G. Observe that $\tilde{\theta}(p'_h, p'_l)$ as defined in (3.8) is independent of objective beliefs. Consumers of types $\theta > \tilde{\theta}$ decide for firm h's safe, externality-free option. Consumers of types $\theta < \tilde{\theta}$ decide to remain uninformed and choose firm l's product while motivating their belief about the presence of externalities to zero. If $(p_h, p_l) = (c, c)$, consumers hold objective belief $\bar{b}^\chi(c, c, \sigma^*_{h,l}) = 0$ in game $\bar{G}^\chi$, that is, they behave as if they hold objective belief $b(c, c, \sigma^*_{h,l}, \chi) = \chi q$ in game G. Given equal prices, all consumers decide to become perfectly informed, update their beliefs to $b = 0$ and choose firm l's product due to the tie-breaking rule. The lemma below summarizes useful properties of firms' equilibrium strategy profile $\sigma^*_{h,l}(p_h, p_l | \omega = bad)$.

Lemma 3.3. *Given $\sigma^*_{h,l}(p_h, p_l|\omega = good)$, $\sigma^*_c(m, a, r|p_h, p_l, \theta)$, off-path beliefs and the tie-breaking rules, any $\sigma^*_{h,l}(p_h, p_l|\omega = bad)$ supports prices such that both firms are active on the market.*

Proof. Can any $\sigma^*_{h,l}(p_h, p_l|\omega = bad)$ that supports prices such that no firm is active on the market be optimal and hence satisfy OSDP? For the sake of contradiction, fix some possibly mixed $\sigma^*_{h,l}(p_h, p_l|\omega = bad)$, where $\check{p}_h, \check{p}_l > v$ denote the lowest supported prices, that is, $\check{p}_h \leq \inf \operatorname{supp}(\sigma^*_h(p_h|\omega = bad))$ and $\check{p}_l \leq \inf \operatorname{supp}(\sigma^*_l(p_l|\omega = bad))$. By construction, either firm expects a payoff of zero. Hence, any $p'_h \in \operatorname{supp}(\sigma^*_h(p_h))$ is strictly dominated by any $p''_h \in (c, v]$ as it leads to a strictly positive market share with a strictly positive margin, a contradiction.

Can any $\sigma^*_{h,l}(p_h, p_l|\omega = bad)$ that supports prices such that only firm h is active on the market be optimal and hence satisfy OSDP? For the sake of contradiction, fix some possibly mixed $\sigma^*_{h,l}(p_h, p_l|\omega = bad)$ where $\check{p}_l$ denotes the lowest supported price of firm l, that is, $\check{p}_l \leq \inf \operatorname{supp}(\sigma^*_l(p_l|\omega = bad))$. If and only if $p_h \leq \min\{v, \check{p}_l\}$, only firm h is active on the market. If $\check{p}_l > c$, firm h can expect a positive payoff and hence any $p_h \in \operatorname{supp}(\sigma^*_h(p_h|\omega = bad))$ must lie in the non-empty set $(c, \min\{v, \check{p}_l\}]$. Observe that any $p'_h < \min\{v, \check{p}_l\}$ is strictly dominated by any $p''_h \in (p'_h, \min\{v, \check{p}_l\}]$ as firm h does not lose customers by increasing its price over this range. Thus, firm h's unique best-response to $\sigma^*_l(p_l|\omega = bad)$ is $\sigma^*_h(\min\{v, \check{p}_l\}|\omega = bad) = 1$. By construction, $\sigma^*_l(p_l|\omega = bad)$ yields an expected payoff of zero. Hence, any $p'_l \in \operatorname{supp}(\sigma^*_l(p_l|\omega = bad))$ is strictly dominated by any $p''_l \in (0, \check{p}_l)$ as it leads to a strictly positive market share with a strictly positive margin, a contradiction. If $\check{p}_l = c$, only firm h is active on the market if and only if $p_h = c$, a contradiction to separating equilibrium. If $\check{p}_l < c$, it is not possible that only firm h is active on the market.

Can any $\sigma^*_{h,l}(p_h, p_l|\omega = bad)$ that supports prices such that only firm l is active on the market be optimal and hence satisfy OSDP? For the sake of contradiction, fix some possibly mixed $\sigma^*_{h,l}(p_h, p_l|\omega = bad)$ where $\check{p}_h > v$ denotes the lowest supported price of firm h, that is, $\check{p}_h \leq \inf \operatorname{supp}(\sigma^*_h(p_h|\omega = bad))$. As firm l can expect a positive payoff, any $p_l \in \sigma^*_l(p_l|\omega = bad)$ must lie in the non-empty set $(0, v)$. By construction, $\sigma^*_h(p_h|\omega = bad)$ yields an expected payoff of zero. Hence, any $p'_h \in \operatorname{supp}(\sigma^*_h(p_h|\omega = bad))$ is strictly dominated by any $p''_h \in (c, v]$ as it leads to a strictly positive market share with a strictly positive margin given any $p'_l \in \operatorname{supp}(\sigma^*_l(p_l|\omega = bad))$, a contradiction. □

Which prices should profile $\sigma^*_{h,l}(p_h, p_l|\omega = good)$ support such that both firms are active on the market while OSDP is satisfied? Define firms' constrained maximization problems for $\omega = bad$:

$$\max_{p_h > c} \pi_h = D_h \left(p_h, p_l|\omega = bad, b\right) \left(p_h - c\right) \tag{3.9}$$

and

$$\max_{p_l > 0} \pi_l = D_l \left(p_l, p_h|\omega = bad, b\right) p_l, \tag{3.10}$$

Add the constraints following from Lemma 3.3 to (3.9), and set up the Lagrangian

$$\mathcal{L}(p_h, \eta_h, \lambda_h|\omega = bad, b) = [1 - F(\tilde{\theta}(p_h, p_l|b))](p_h - c) - \eta_h \left(-p_h + \max\{c, p_l\}\right) - \lambda_h(p_h - v),$$

where $\eta_h, \lambda_h \geq 0$. For optimality, the following Karush-Kuhn-Tucker conditions are necessary:

$$\frac{\partial \mathcal{L}}{\partial p_h} = -f(\tilde{\theta})\frac{\partial \tilde{\theta}}{\partial p_h}(p_h - c) + 1 - F(\tilde{\theta}) + \eta_h - \lambda_h = 0, \tag{3.11}$$

$$\eta_h \left(-p_h + \max\{c, p_l\}\right) = 0,$$

and

$$\lambda_h(p_h - v) = 0.$$

Similarly, add the constraints following from Lemma 3.3 to (3.10), and set up the Lagrangian

$$\mathcal{L}(p_l, \eta_l, \lambda_l|\omega = bad, b) = F(\tilde{\theta}(p_h, p_l|b))p_l - \eta_l \left(-p_l\right) - \lambda_l(p_l - \min\{v, p_h\}),$$

where $\eta_l, \lambda_l \geq 0$. For optimality, the following Karush-Kuhn-Tucker conditions are necessary:

$$\frac{\partial \mathcal{L}}{\partial p_l} = f(\tilde{\theta})\frac{\partial \tilde{\theta}}{\partial p_l}p_l + F(\tilde{\theta}) + \eta_l - \lambda_l = 0, \tag{3.12}$$

$$\eta_l \left(-p_l\right) = 0,$$

and

$$\lambda_l(p_l - \min\{v, p_h\}) = 0.$$

If the constraints are non-binding, complementary slackness requires that $\eta_h = \eta_l = \lambda_l = \lambda_h = 0$. Substituting this into (3.11) and (3.12) yields

$$p_h = c + \frac{1 - F(\tilde{\theta})}{f(\tilde{\theta})}(1 - \alpha)e, \tag{3.13}$$

and

$$p_l = \frac{F(\tilde{\theta})}{f(\tilde{\theta})}(1 - \alpha)e. \tag{3.14}$$

Log-concavity of $f(\theta)$ implies that both $F(\theta)$ and $1 - F(\theta)$ are log-concave (for a formal argument, see, e.g., Anderson *et al.*, 1997; Bagnoli and Bergstrom, 2005). Further, log-concavity implies that the following hazard-rate properties are satisfied:

$$\frac{\partial}{\partial\theta}\left(\frac{F(\theta)}{f(\theta)}\right) \geq 0 \geq \frac{\partial}{\partial\theta}\left(\frac{1 - F(\theta)}{f(\theta)}\right).$$

It follows that firms' objective functions are log-concave for $p_l < p_h < v$, which is sufficient for quasi-concavity. Thus, first-order conditions are sufficient for optimality.[14] Let $p_i(p_j)$ denote firm i's best response for $i, j = h, l$ and $i \neq j$. Define the support of $p_h(p_l)$ by the set $S_h := \{p_l | p_l < p_h(p_l) < v\}$ and the support of $p_l(p_h)$ by the set $S_l := \{p_h | p_l(p_h) < p_h < v\}$. Observe that the restrictions imply that $\tilde{\theta} \in (0, \infty)$ and that all consumers prefer either of the firm's products and not the outside option. The lemma below summarizes useful properties of firms' best responses.

Lemma 3.4. *Under Assumption 3.1, both (3.13) and (3.14) implicitly define best-response functions with positive slope less than one on S_h and S_l, respectively.*

Proof. First, I argue that over the support, each firm's best response is defined by a function. Observe that $\tilde{\theta}$ is strictly increasing in p_h and strictly decreasing in p_l. This implies that the right-hand side of (3.13) is non-increasing in p_h and the right-hand side of (3.14) is non-increasing in p_l due to the hazard-rate properties

[14]Moreover, Assumption 3.1 implies strict concavity of the problem for prices such that both firms are active on the market.

from above. Hence, there exists a unique solution to both (3.13) and (3.14). As $\tilde{\theta}$ is continuous in p_h and p_l, so is each firm's one-to-one mapping from its opponent's price to its best response.

Next, I show that the slope of each firm's best-response function lies between zero and one. Apply the implicit function theorem to Equation (3.13) in order to obtain the derivative of $p_h(p_l)$ on S_h, that is,

$$\frac{\partial p_h(p_l)}{\partial p_l} = \frac{f(\tilde{\theta}) + f'(\tilde{\theta})\frac{p_h(p_l)-c}{(1-\alpha)e}}{2f(\tilde{\theta}) + f'(\tilde{\theta})\frac{p_h(p_l)-c}{(1-\alpha)e}}.$$

Observe that the numerator is strictly positive if

$$\frac{|f'(\tilde{\theta})|}{f(\tilde{\theta})} < \frac{(1-\alpha)e}{p_h(p_l)-c}.$$

As $p_h(p_l) < v$ by construction, $(1-\alpha)e/(p_h(p_l)-c) > (1-\alpha)e/v$ for any $b \in (0,1)$ and hence Assumption 3.1 is sufficient for a strictly positive numerator. It immediately follows that the denominator is strictly positive as well and strictly smaller than the numerator. Hence, the slope of $p_h(p_l)$ always lies in $(0,1)$ on S_h. Analogously, apply the implicit function theorem to Equation (3.14) in order to obtain the derivative of $p_l(p_h)$ on S_l, that is,

$$\frac{\partial p_l(p_h)}{\partial p_h} = \frac{f(\tilde{\theta}) - f'(\tilde{\theta})\frac{p_l(p_h)}{(1-\alpha)e}}{2f(\tilde{\theta}) - f'(\tilde{\theta})\frac{p_l(p_h)}{(1-\alpha)e}}.$$

Observe that the numerator is strictly positive if

$$\frac{|f'(\tilde{\theta})|}{f(\tilde{\theta})} < \frac{(1-\alpha)e}{p_l(p_h)}.$$

As $p_l(p_h) < p_h < v$ by construction, $(1-\alpha)e/p_l(p_h) > (1-\alpha)e/v$ for any $b \in (0,1)$ and hence Assumption 3.1 is sufficient for a strictly positive numerator. It immediately follows that the denominator is strictly positive as well and strictly smaller than the numerator. Hence, the slope of $p_l(p_h)$ always lies in $(0,1)$ on S_l. □

It follows from Lemma 3.4 that a unique solution to the system of equations (3.13) and (3.14) exists if v is sufficiently large. Let (p_h^*, p_l^*) denote the solution.

Then, $p_l^* < p_h^* < v$ by construction. To see why $p_l^* < p_h^*$ exists, plug the minimal price pair $(p_h, p_l) = (c, 0)$ into (3.14) in order to obtain

$$0 = \underbrace{\frac{F(\tilde{\theta}(c,0))}{f(\tilde{\theta}(c,0))}}_{>0}(1-\alpha)e,$$

a contradiction. Keeping $p_h = c$ constant while letting p_l approach c from below yields

$$c = \underbrace{\lim_{(p_l \to c)^-} \frac{F(\tilde{\theta}(c,p_l))}{f(\tilde{\theta}(c,p_l))}(1-\alpha)e}_{=0},$$

a contradiction. To see why this is true, observe that $\lim_{(p_l \to c)^-} \tilde{\theta}(c, p_l) = 0$ by construction, $\lim_{(\theta \to 0)^+} F(\theta) = 0$ by definition and both $f(\theta) > 0$ for all $\theta \in (0, \infty)$ and $\lim_{(\theta \to 0)^+} F(\theta)/f(\theta) < c/(1-\alpha)e$ by Assumption 3.2. Thus, the hazard-rate properties and the intermediate value theorem imply that there exists a unique $p_l(c) \in (0, c)$. By Lemma 3.4, the slope of both $p_h(p_l)$ and $p_l(p_h)$ lies in $(0, 1)$. Hence, any $p_l(p_h) < p_h$ and any intersection of firms' best-response functions leads to $p_l^* < p_h^*$ as there exists a unique fix point if v is sufficiently large, which completes the argument. Can a deviation to some $p_h' \leq p_l^*$ be profitable for firm h if $p_l^* > c$? Any such p_h' attracts all consumers. Then, $p_h' = p_l^*$ is most profitable as firm h's payoff is linearly increasing on (c, p_l^*). Further, it follows from the first-order condition that firm h's payoff is differentiable and strictly increasing on (p_l^*, p_h^*) due to quasi-concavity. As $\lim_{(p_h \to p_l^*)^+} \hat{\theta}(p_h, p_l^*) = 0$, it holds true that $\lim_{(\hat{\theta} \to 0)^+}(1 - F(\hat{\theta})) = 1$ and hence the payoff is continuous on (c, p_h^*). As a result, a deviation to $p_h' = p_l^*$ leads to a smaller payoff. It follows that firms' type-dependent and pure equilibrium strategies $\sigma_h^*(p_h^*|\omega = bad) = 1$ and $\sigma_l^*(p_l^*|\omega = bad) = 1$ satisfy OSDP if v is sufficiently large.

It remains to show that there exist equilibrium strategies $\sigma_h^*(p_h|\omega = bad)$ and $\sigma_l^*(p_l|\omega = bad)$ that satisfy OSDP if v is not sufficiently large and hence (p_h^*, p_l^*)—the solution to the system of equations (3.13) and (3.14)—does not exist. By Lemma 3.3, $p_i > v$ can never be a best response for firm $i = h, l$. Moreover, OSDP can be satisfied only if both firms are active on the market. Let firms' type-dependent and pure equilibrium strategies be $\sigma_h^*(v|\omega = bad) = 1$ and $\sigma_l^*(p_l(v)|\omega = bad) = 1$. By Lemma 3.4, $p_l(v) < v$. Hence, $\tilde{\theta}(v, p_l(v)) \in (0, \infty)$ generating a strictly positive payoff for each firm. Observe that $\sigma_l^*(p_l|\omega = bad)$ satisfies OSDP by construction. It remains to show that $\sigma_h^*(p_h|\omega = bad)$ satisfies OSDP, that is, there is no incentive for firm h to deviate to some $p_h' < v$ given $\sigma_l^*(p_l|\omega = bad)$, $\sigma_{h,l}^*(p_h, p_l|\omega = good)$, $\sigma_c^*(m, a, r|p_h, p_l, \theta)$ and beliefs as defined above. Consider an auxiliary game that differs from game $\bar{G}^\chi$ in the following way: the private value $v' > v$ of every consumer is sufficiently large such that (p_h^*, p_l^*) exist. Then, $p_h^* > v$ and $p_l^* > p_l(v)$ by construction and Lemma 3.4 implies that $p_h(p_l(v)) > v$. Due to continuity and quasi-concavity of firm h's payoff in the auxiliary game, it follows that firm h's payoff strictly increases in p_h on $(p_l(v), p_h^*)$, which contains v. In game $\bar{G}^\chi$, firm h's payoff function is left-continuous at v and differentiable on $(p_l(v), v)$. As the games coincide for $p_h \leq v$ by construction, a deviation to any $p_h' \in (p_l(v), v)$ must lead to a smaller payoff in game $\bar{G}^\chi$. Can a deviation to some $p_h' \leq p_l(v)$ be profitable if $p_l(v) > c$? Any such p_h' attracts all consumers. Then, $p_h' = p_l(v)$ is optimal as firm h's payoff is linearly increasing on $(c, p_l(v))$. Further, it follows from the first-order condition that firm h's payoff is differentiable and strictly increasing on $(p_l(v), v)$ due to quasi-concavity. As $\lim_{(p_h \to p_l(v))^+} \hat{\theta}(p_h, p_l(v)) = 0$, $\lim_{(\hat{\theta} \to 0)^+}(1 - F(\hat{\theta})) = 0$ and hence the payoff is continuous on (c, v). Then, a deviation to $p_h' = p_l(v)$ leads to a smaller payoff. It follows that firms' type-dependent and pure equilibrium strategies $\sigma_h^*(v|\omega = bad) = 1$ and $\sigma_l^*(p_l(v)|\omega = bad) = 1$ satisfy OSDP if v is not sufficiently large.

When is v sufficiently large relative to c and $(1-\alpha)e$? Plug the solution to the system of equations (3.13) and (3.14)—denoted by (p_h^*, p_l^*)—into (3.13). Observe that (p_h^*, p_l^*) exists and hence v is sufficiently large if and only if

$$v - p_h > 0 \Leftrightarrow v - c - \frac{1 - F(\tilde{\theta}(p_h^*, p_l^*))}{f(\tilde{\theta}(p_h^*, p_l^*))}(1-\alpha)e > 0$$

$$\Leftrightarrow \frac{v - c}{(1-\alpha)e} > \frac{1 - F(\tilde{\theta}(p_h^*, p_l^*))}{f(\tilde{\theta}(p_h^*, p_l^*))}. \tag{3.15}$$

As the right-hand side of the inequality is independent of v, there always exist private values such that the inequality is satisfied. I conclude that under Assumptions 3.1 and 3.2, there always exists a strategy profile σ^* that satisfies OSDP for all players given objective beliefs as defined above. Then, there always exists a PBE in game $\bar{G}^\chi$, which constitutes a χ-PCE in the original game G, for any χ.

Step (ii): Next, I show that under Assumptions 3.1 and 3.2, the $\chi - PCE$ is unique. It follows from the arguments above, that strategy profile σ^* describes the unique separating PBE in game $\bar{G}^\chi$ as well as the unique separating $\chi - PCE$ in game G. Why is that? The equilibria neither depend on off-path beliefs nor on χ and hence the sets of equilibria coincide. Then, it remains to show that there does not exist any pooling $\chi - PCE$ in game G. In any such pooling $\chi - PCE$, firms' possibly mixed strategies must be type-independent, that is, $\sigma'_{h,l}(p_h, p_l|\omega = bad) = \sigma_{h,l}(p_h, p_l|\omega = good) = \sigma_{h,l}(p_h, p_l)$. Moreover, both firms must be active on the market. As optimal consumer behavior as prescribed by $\sigma^*_c(m, a, r|p_h, p_l, \theta)$ does not depend on firms' strategies, the arguments from Lemma 3.3 apply to this claim in game G after replacing type-dependent by type-dependent strategies. Then, firm l's largest supported price must be strictly lower than firm h's lowest supported price. Suppose that $\sigma'_{h,l}$ satisfies both requirements. Take $\sigma^*_c(m, a, r|p_h, p_l, \theta)$ and the tie-breaking rules as given. For any price pair $(p'_h, p'_l) \in \text{supp}(\sigma'_{h,l})$, consider $\tilde{\theta}(p'_h, p'_l)$ as defined in (3.6). Observe that any consumer of type $\theta < \tilde{\theta}(p'_h, p'_l)$ chooses information-product-belief triple $(0, a_l, 0)$. Any consumer of type $\theta > \tilde{\theta}$ chooses information-product-belief triple $(1, a_h, r)$ with $r \in R_h(\theta)$. If $\omega = good$, however, firm l benefits from setting $p''_l = \inf \text{supp}(\sigma'_h(p_h))$. Why is that? Suppose that after observing price pair (p'_h, p''_l), consumers update their objective off-path belief about $\omega = bad$ to some $b' \in [q, 1)$, where the bounds follow from Lemma 3.2. All consumers of type $\theta < \tilde{\theta}(p'_h, p''_l)$ remain choosing $(0, a_l, 0)$. All consumers of type $\theta > \tilde{\theta}(p'_h, p''_l)$ choose to become informed, learn that $\omega = good$ and choose firm l's product as $p''_l \leq p_h$. At the same time, $p''_l \geq p'_l$ by construction, which completes the argument. Hence, there does not exist pooling $\chi - PCE$ in game G.

Step (iii): Next I argue that any consumer of type $\theta < \tilde{\theta}^*$, where $\tilde{\theta}^*$ denotes the unique indifferent consumer type in the unique separating $\chi - PCE$ in game G as derived above, deceives herself about the likelihood of externalities if $\omega = bad$. Suppose that $\omega = bad$. After observing price pair (p^*_h, p^*_l), $\sigma^*_c(m, a, r|p_h, p_l, \theta)$ prescribes that a consumer chooses information-product-belief triple $(0, a_l, 0)$ if and only if $\theta < \tilde{\theta}^*$. The objective belief of e, however, is $b = 1 - \chi(1 - q)$. Hence, $r = 0 < b$ in equilibrium for any $\theta < \tilde{\theta}^*$. These consumers perceive the likelihood of externalities as $\alpha r + (1 - \alpha)b < b$, which completes the argument. Moreover, any consumer of type $\theta < \tilde{\theta}^*$ deceives herself about the likelihood of e at the cost of expected material losses from distorted decisions, $(1 - \alpha)b\theta e$, as defined in the program in (3.2). As self-deception is optimal only if the gain in anticipated utility outweighs expected material losses from distorted decision making, the net effect of $r = 0$ is $b\theta e - (1 - \alpha)b\theta e = \alpha b\theta e > 0$, an increasing function of the anticipatory-utility weight α and the consumer's expected material loss from externalities, $b\theta e$. □

Proof of Proposition 3.2. In order to proof the proposition, I first characterize the equilibrium of the benchmark. Second, I compare firms' equilibrium prices and the corresponding indifferent consumer type from the benchmark and the original game G.

Step (i): Consider the benchmark in which consumers cannot choose motivated beliefs. This corresponds to the original game G with the exception that $\alpha = 0$. As consumers are still χ-cursed, their objective beliefs are distorted. They cannot willingly distort, however, how they perceive the likelihood of externalities anymore, that is, $r = b$. Taking this into account, the arguments from the proof of Proposition 3.1 remain valid. As a result, there exists a unique equilibrium in the benchmark in which firms' equilibrium strategies, $\sigma_h^*(p_h|\omega)$ and $\sigma_l^*(p_l|\omega)$, are pure and type-dependent and the market is always covered. In particular, if firm l's type is $\omega = bad$, $p_h^* > p_l^*$ and all consumers of types $\theta < \tilde{\theta}^*$ choose firm l's product, while all consumers of types $\theta \geq \tilde{\theta}^*$ choose firm h's product. If firm l's type is $\omega = good$, $p_h^* = p_l^* = c$ and all consumers choose firm l's product.

Step (ii): Observe that prices and consumer behavior in the benchmark and the original game G coincide if $\omega = good$, that is, prices do not differ. Suppose that $\omega = bad$. In order to proof the claim, I provide comparative statics of firms' equilibrium prices and the corresponding indifferent consumer type with respect to $x := (1-\alpha)e$. Fix $\alpha' > 0$ of the original game. Then, the comparison is between $x = (1-\alpha')e$ and $x = e$, whereby x decreases in α on $[0, \alpha']$.

First, suppose that v is sufficiently large such that the solution to the system of equations (3.13) and (3.14) exists for $\alpha \in [0, \alpha']$. Exploit the implicit characterization of equilibrium prices and the corresponding indifferent consumer type as defined in (3.6) in order to define

$$y\left(x, p_h(x), \tilde{\theta}\left(x, p_h(x), p_l(x)\right)\right) := \frac{1 - F(\tilde{\theta})}{f(\tilde{\theta})}x + c - p_h = 0,$$

$$z\left(x, p_l(x), \tilde{\theta}\left(x, p_h(x), p_l(x)\right)\right) := \frac{F(\tilde{\theta})}{f(\tilde{\theta})}x - p_l = 0,$$

and

$$g\left(x, \tilde{\theta}\left(x, p_h(x), p_l(x)\right)\right) := \tilde{\theta} - \frac{c}{x} - \frac{1 - 2F(\tilde{\theta})}{f(\tilde{\theta})} = 0.$$

Applying the implicit function theorem yields the following system of equations:

$$\frac{\partial p_h}{\partial x} = -\left(\frac{\partial y}{\partial x} + \frac{\partial y}{\partial \tilde{\theta}}\frac{\partial \tilde{\theta}}{\partial x} + \frac{\partial y}{\partial \tilde{\theta}}\frac{\partial \tilde{\theta}}{\partial p_l}\frac{\partial p_l}{\partial x}\right)\left(\frac{\partial y}{\partial p_h} + \frac{\partial y}{\partial \tilde{\theta}}\frac{\partial \tilde{\theta}}{\partial p_h}\right)^{-1},$$

$$\frac{\partial p_l}{\partial x} = -\left(\frac{\partial z}{\partial x} + \frac{\partial z}{\partial \tilde{\theta}}\frac{\partial \tilde{\theta}}{\partial x} + \frac{\partial z}{\partial \tilde{\theta}}\frac{\partial \tilde{\theta}}{\partial p_h}\frac{\partial p_h}{\partial x}\right)\left(\frac{\partial z}{\partial p_l} + \frac{\partial z}{\partial \tilde{\theta}}\frac{\partial \tilde{\theta}}{\partial p_l}\right)^{-1},$$

$$\frac{\partial \tilde{\theta}}{\partial x} = -\left(\frac{\partial g}{\partial x} + \frac{\partial g}{\partial \tilde{\theta}}\left(\frac{\partial \tilde{\theta}}{\partial p_h}\frac{\partial p_h}{\partial x} + \frac{\partial \tilde{\theta}}{\partial p_l}\frac{\partial p_l}{\partial x}\right)\right)\left(\frac{\partial g}{\partial \tilde{\theta}}\right)^{-1}. \tag{3.16}$$

Substitute for $\partial\tilde{\theta}/\partial p_h = -\partial\tilde{\theta}/\partial p_l$ and solve simultaneously in order to obtain the effect of x on firms' equilibrium prices,

$$\frac{\partial p_h}{\partial x} = \Bigg(\underbrace{\frac{\partial g}{\partial x}\frac{\partial y}{\partial \tilde{\theta}} - \frac{\partial g}{\partial \tilde{\theta}}\frac{\partial y}{\partial x}}_{\text{Direct effect}}\Bigg)\Bigg(\underbrace{\frac{\partial g}{\partial \tilde{\theta}}\frac{\partial y}{\partial p_h}}_{\text{Indirect effect}}\Bigg)^{-1} \tag{3.17}$$

and

$$\frac{\partial p_l}{\partial x} = \Bigg(\underbrace{\frac{\partial g}{\partial x}\frac{\partial z}{\partial \tilde{\theta}} - \frac{\partial g}{\partial \tilde{\theta}}\frac{\partial z}{\partial x}}_{\text{Direct effect}}\Bigg)\Bigg(\underbrace{\frac{\partial g}{\partial \tilde{\theta}}\frac{\partial z}{\partial p_l}}_{\text{Indirect effect}}\Bigg)^{-1}. \tag{3.18}$$

Can the signs be determined unambiguously? Observe that $\partial y/\partial p_h = \partial z/\partial p_l = -f(\tilde{\theta})/x < 0$, as $f(\tilde{\theta}) > 0$ for all $\theta \in (0,\infty)$ by assumption. Further, I claim that

$$\frac{\partial g}{\partial \tilde{\theta}} = 1 + \frac{2f(\tilde{\theta})^2 + (1-2F(\tilde{\theta}))f'(\tilde{\theta})}{f(\tilde{\theta})^2} > 0 \Leftrightarrow 3f(\tilde{\theta})^2 + (1-2F(\tilde{\theta}))f'(\tilde{\theta}) > 0.$$

Why is this true? $f(\theta) > 0$ for all $\theta \in (0,\infty)$ by assumption. As $F(\theta)$ is log-concave, $f(\theta)^2 - F(\theta)f'(\theta) > 0$ and as $1-F(\theta)$ is log-concave, $f(\theta)^2 + (1-F(\theta))f'(\theta) > 0$ for all $\theta \in (0,\infty)$ by definition. It follows that

$$3f(\tilde{\theta})^2 + (1-2F(\tilde{\theta}))f'(\tilde{\theta}) > f(\tilde{\theta})^2 - F(\tilde{\theta})f'(\tilde{\theta}) + f(\tilde{\theta})^2 + (1-F(\tilde{\theta}))f'(\tilde{\theta}) > 0,$$

which completes the argument. Then, the indirect effects in (3.17) and (3.18) are both negative. In order to evaluate the indirect effects, observe that $\partial g/\partial x = c/x^2 > 0$. Further, it holds true that $\partial y/\partial x = f(\tilde{\theta})(p_h - c)/x^2 > 0$ and $\partial z/\partial x = f(\tilde{\theta})p_l/x^2 >$

0 as $f(\theta) > 0$ for all $\theta \in (0, \infty)$ by assumption. What is the sign of $\partial y/\partial\tilde{\theta}$? I claim that

$$\frac{\partial y}{\partial \tilde{\theta}} = -f'(\tilde{\theta})\frac{p_h - c}{x} - f(\tilde{\theta}) < 0 \Leftrightarrow -\frac{f'(\tilde{\theta})}{f(\tilde{\theta})} < \frac{x}{p_h - c}.$$

This holds true as $p_h < v$ by construction and hence $x/(p_h - c) > x/v > |f'(\tilde{\theta})|/f(\tilde{\theta})$ by Assumption 3.1. What is the sign of $\partial z/\partial\tilde{\theta}$? I claim that

$$\frac{\partial z}{\partial \tilde{\theta}} = -f'(\tilde{\theta})\frac{p_l}{x} + f(\tilde{\theta}) > 0 \Leftrightarrow \frac{f'(\tilde{\theta})}{f(\tilde{\theta})} < \frac{x}{p_l}.$$

This holds true as $p_l < v$ by Lemma 3.4 and hence $x/p_l > x/v > |f'(\tilde{\theta})|/f(\tilde{\theta})$ by Assumption 3.1. Then, the direct effect in (3.17) is negative. As a result, firm h's equilibrium price rises in x. The direction of the direct effect in (3.18), however, is ambiguous. Consequently, firm l's equilibrium price can rise or fall in x. Consider the effect of x on the indifferent consumer type in equilibrium as a function of the effect of x on equilibrium prices in (3.16). It immediately follows from the arguments above, that a negative effect on p_l contradicts a positive effect on $\tilde{\theta}$. Moreover, a negative effect on p_l is sufficient for a negative effect on $\tilde{\theta}$. If the effect on p_l is positive, it must be sufficiently small relative to the effect on p_h.[15] If the effect is positive and not sufficiently small, the effect on $\tilde{\theta}$ is positive.

Next, suppose that v is not sufficiently large for the solution to the system of equations (3.13) and (3.14) to exist for $\alpha \in [0, \alpha']$. Subtract the equation in (3.14) from equation $v = v$ and plug the difference into (3.6). Exploit the implicit characterization of the resulting indifferent consumer type and firm l's equilibrium price at $p_h = v$ in order to define

$$z\left(x, p_l(x), \tilde{\theta}\left(x, p_h(x), p_l(x)\right)\right) := \frac{F(\tilde{\theta})}{f(\tilde{\theta})}x - p_l = 0,$$

and

$$h\left(x, \tilde{\theta}\left(x, p_l(x)\right)\right) := \tilde{\theta} - \frac{v}{x} + \frac{F(\tilde{\theta})}{f(\tilde{\theta})} = 0.$$

[15] This condition is satisfied, for instance, if consumer types are distributed uniformly on a bounded support or exponentially.

Applying the implicit function theorem yields the following system of equations:

$$\frac{\partial p_l}{\partial x} = -\left(\frac{\partial z}{\partial x} + \frac{\partial z}{\partial \tilde{\theta}}\frac{\partial \tilde{\theta}}{\partial x}\right)\left(\frac{\partial z}{\partial p_l} + \frac{\partial z}{\partial \tilde{\theta}}\frac{\partial \tilde{\theta}}{\partial p_l}\right)^{-1},$$

$$\frac{\partial \tilde{\theta}}{\partial x} = -\left(\frac{\partial h}{\partial x} + \frac{\partial h}{\partial \tilde{\theta}}\frac{\partial \tilde{\theta}}{\partial p_l}\frac{\partial p_l}{\partial x}\right)\left(\frac{\partial h}{\partial \tilde{\theta}}\right)^{-1}. \tag{3.19}$$

Solve simultaneously in order to obtain the effect of x on firm l's equilibrium price,

$$\frac{\partial p_l}{\partial x} = \Bigg(\underbrace{\frac{\partial h}{\partial x}\frac{\partial z}{\partial \tilde{\theta}} - \frac{\partial h}{\partial \tilde{\theta}}\frac{\partial z}{\partial x}}_{\text{Direct effect}}\Bigg)\Bigg(\underbrace{\frac{\partial h}{\partial \tilde{\theta}}\frac{\partial z}{\partial p_l}}_{\text{Indirect effect}}\Bigg)^{-1}. \tag{3.20}$$

Can the sign be determined unambiguously? As shown above, $\partial z/\partial p_l < 0$. Further, I claim that

$$\frac{\partial h}{\partial \tilde{\theta}} = 1 + \frac{f(\tilde{\theta})^2 - F(\tilde{\theta})f'(\tilde{\theta})}{f(\tilde{\theta})^2} > 0 \Leftrightarrow 2f(\tilde{\theta})^2 - F(\tilde{\theta})f'(\tilde{\theta}) > 0.$$

Why is this true? It holds true that $f(\theta) > 0$ for all $\theta \in (0, \infty)$ by assumption. As $F(\theta)$ is log-concave, $f(\theta)^2 - F(\theta)f'(\theta) > 0$ for all $\theta \in (0, \infty)$ by definition. It follows that

$$2f(\tilde{\theta})^2 - F(\tilde{\theta})f'(\tilde{\theta}) > f(\tilde{\theta})^2 - F(\tilde{\theta})f'(\tilde{\theta}) > 0,$$

which completes the argument. Then, the indirect effect in (3.20) is negative. In order to evaluate the indirect effect, observe that $\partial h/\partial x = v/x^2 > 0$. As shown above, it holds true that $\partial z/\partial x > 0$ and $\partial z/\partial \tilde{\theta} > 0$. Then, the direction of the direct effect in (3.20) is ambiguous. As a result, firm l's equilibrium price can rise or fall in x. Consider the effect of x on the indifferent consumer type in equilibrium as a function of the effect of x on equilibrium prices in (3.19). It immediately follows from the arguments above, that a negative effect on p_l contradicts a positive effect on $\tilde{\theta}$. Moreover, a negative effect on p_l is sufficient for a negative effect on $\tilde{\theta}$. If the effect on p_l is positive, it must be sufficiently small.[16] If the effect is positive and not sufficiently small, the effect on $\tilde{\theta}$ is positive.

Finally, observe that a change in α affects the threshold valuation defined in

[16] This condition is satisfied, for instance, for the same distributions as in the case that v is sufficiently large.

(3.15), which determines whether equilibrium prices are an interior or a corner solution, whereby changes in prices are continuous. As a result, both cases considered above can be relevant at the same time for given v. As all arguments in the two cases considered above are qualitatively equivalent, they remain valid. □

Conclusion

In this thesis, I presented three essays on competition economics. In Chapter 1, we argued that the use of big data—especially consumer data—for pricing strategies has substantially increased in recent times. Big data predictions of consumer preferences have been improving tremendously. Imprecision, however, is still an important factor when firms make their pricing decisions.

We focused on the impact of data-driven price-discrimination strategies on the scope for tacit collusion. We have shown that enhanced prediction of consumer preferences results in a U-shaped effect on firms' ability to sustain collusion. Compared to uniform pricing, we have found that for low levels of predictive capabilities, collusion is easier to sustain under price discrimination. For sufficiently high levels, we find that collusion is harder to sustain under a discriminatory pricing than under uniform pricing. Thereby, potential misrecognition of consumers plays a crucial role.

Thereby, our study provides the following policy implications. Although not central to designing data policy, data regulation should also take into account adverse effects on competition. In particular, deregulation of access to or usage of consumer data facilitates coordinated behavior of firms as long as initial predictions of consumer preferences are weak. In contrast, for relatively strong predictions, policies intending to restrict access to and usage of consumer data facilitate coordinated behavior among firms. Moreover, the effect of a legal ban on price discrimination on firms' ability to collude crucially depends on the quality of predictions. On a more general note and related to the above-mentioned aspect, one may argue that when the exchange of consumer data leads to higher signal precision towards perfect information, competition authorities should be less concerned with regard to collusive activity than in the case in which firms exchange data on prices, demands, etc. At the same time, the model we employ does not allow to draw conclusions with regard to welfare, as we do not take into account consumer preferences for privacy or other adverse effects due to discrimination of consumers.

In Chapter 2, we argued that bonus contracts create two distinct inefficiencies. On the one hand, bonus payments are typically sent out as checks that need to be issued and tracked, while non-monetary bonuses such as included premiums may give an imperfect fit to the consumer's actual preferences. On the other hand, bonus contracts yield an imbalanced decision situation—benefits are concentrated in the form of a single, large bonus payment while costs are dispersed over many small payments—in which focused thinkers tend to make suboptimal decisions.

We have shown that these inefficiencies are not eliminated by competition, but can only be overcome by regulation. Indeed, firms *have to* exploit attentional focusing under competitive pressure, so that bonus contracts are even more frequent on competitive than on monopolistic markets. By enhancing the use of bonus payments, competition even exacerbates the inefficiencies arising from contracting with focused agents.[17]

From a policy perspective, our study suggests that a legal ban on bonus payments could have favorable consequences. On the one hand, a legal ban on bonus payments eliminates the inherent inefficiency of paying bonuses. On the other hand, it creates choice environments that are balanced, that is, where in equilibrium all payments receive the same amount of attention. Notably, making bonus payments is not necessary to encourage consumers to switch providers, as firms could instead lower the regular payments to attract consumers (see, Farrell and Klemperer, 2007, for a discussion of different modelling approaches). Hence, even if consumers incur costs for switching to another provider, a ban on bonus payments does not impair competitive forces. Altogether, it was argued that prohibiting the use of bonus contracts does not only reduce the direct inefficiencies arising from bonus payments, but could also induce better decisions by consumers.

Chapter 3 investigated how the possibility to exploit moral wiggle room through self-deception can affect market outcomes. It turned out that self-deception can lead to avoidance of information about the impact of purchasing decisions as an endogenous market outcome. Moreover, self-deception can distort market demand toward low-cost, externality-causing production and render costly mitigation of externalities less profitable.

This yields policy implications. Suppose policymakers pursue mitigating externalities that are caused by the production of consumer goods and cannot be prohibited by law or contracts, while they care relatively less about anticipated utility. Then, information provision and campaigns are not sufficient as long as consumers can deceive themselves about the presence of externalities. In contrast, taxing externalities, subsidizing its mitigation or introducing a binding price floor can reduce monetary incentives for remaining ignorant and lead to more informed decision making.

[17]Interestingly, firms standing in competition cannot benefit from exploiting the focusing bias. As in other behavioral models (see, for instance, DellaVigna and Malmendier, 2004; Gabaix and Laibson, 2006), competition drives down firms' profits to zero, *even though* consumers' decision biases are fully exploited.

Moreover, if using the more costly, externality-mitigating technology becomes less profitable through self-deception, both innovation and entry can become less likely if access to this technology causes fixed costs.[18]

Finally, externalities can be interpreted differently. Consumption of the low-cost product may cause negative externalities on future-selfs, for instance, health or well-being. It might also indicate a drawback in terms of quality in the classical sense, which can be experienced only by usage. The question whether self-deception is relevant in these contexts, however, is open to future research.

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[18]Recouping fixed costs requires that externalities occur. Else, there are zero profits. Cursed consumers, however, do not fully infer on the presence of externalities from observing entry or innovation.