Fair and Square: Issues of Fairness and Computation in Partition Problems

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Selbstständigkeitserklärung

Ich versichere an Eides Statt, dass die Dissertation von mir selbstständig und ohne unzulässige fremde Hilfe unter Beachtung der „Grundsätze zur Sicherung guter wissenschaftlicher Praxis an der Heinrich-Heine-Universität Düsseldorf“ erstellt worden ist. Desweiteren erkläre ich, dass ich eine Dissertation in der vorliegenden oder in ähnlicher Form noch bei keiner anderen Institution eingereicht habe.

Teile dieser Arbeit wurden bereits in den folgenden Schriften veröffentlicht oder zur Begutachtung eingereicht: Baumeister et al. [2014], Baumeister et al. [2017], Nguyen et al. [2015], Nguyen et al. [2018], Heinen et al. [2015], Nguyen and Rothe [2016], Nguyen et al. [2016].

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Zusammenfassung

Diese Arbeit befasst sich mit der gerechten Aufteilung nicht teilbarer Güter und mit hedonischen Spielen. Beides sind dynamische Gebiete in der Forschung zu Multiagentensystemen und zur künstlichen Intelligenz. Die gerechte Aufteilung nicht teilbarer Güter betrachtet die Verteilung von Gütern unter potentiell gegensätzlichen Interessen von Agenten, während hedonische Spiele Probleme der Koalitionsbildung modellieren. Bei der Aufteilung nicht teilbarer Güter führen wir scoring-basierte Allokationsregeln und -korrespondenzen ein. Agenten übermitteln ordinale Präferenzen über einzelne Güter an den sozialen Planer, der dann einen fixierten Scoring-Vektor verwendet, um eine additive Darstellung dieser ordinalen Präferenzen zu generieren. Diese Darstellung wird danach für die Optimierung der sozialen Wohlfahrt zunutze gemacht, wobei wir uns auf die utilitaristische und egalitaristische soziale Wohlfahrt und auf die Leximinpräordnung konzentrieren. Wir untersuchen diese Regeln und Korrespondenzen bezüglich axiomatischer Eigenschaften und ihrer Berechnungskomplexität.

Dann erweitern wir das scoring-basierte Modell, um Strategiesicherheit mit Mengenerweiterungen aus der Sozialwahltheorie zu untersuchen. Für die utilitaristische soziale Wohlfahrt charakterisieren wir Strategiesicherheit mit der Gärdenfors-Erweiterung bezüglich der Struktur des Scoring-Vektors.

Für den Fall, dass Agenten stattdessen kardinale Präferenzen übermitteln, untersuchen wir eine Fairnesshierarchie, die von Bouveret and Lemaître [2016] vorgeschlagen wurde, bezüglich k-additiven Nutzenfunktionen und beantworten eine Frage von denselben Autoren über die Beziehung von Effizienz und einem starken Fairnessbegriff. In derselben Arbeit untersuchen wir ranggewichteten Utilitarismus, der Ungleichheit reduziert, und zeigen, dass die Approximierbarkeit von Rangdiktatorfunktionen eng mit der Approximierbarkeit der egalitaristischen sozialen Wohlfahrt verbunden ist. Explorativ untersuchen wir die Verbindung zwischen beiden Fairnessansätzen für den Max-Min-Anteil und den ranggewichteten Utilitarismus mit Gewichtsvektoren, die Ungleichheit reduzieren.

Bei hedonischen Spielen führen wir lokale Fairnessbegriffe ein. Diese Fairnessbegriffe hängen nur von den einzelnen Präferenzen der Agenten ab und sind erfüllt, wenn der Nutzen eines jeden Agenten über einer individuellen Schwelle liegt. Inspiriert von Fairnessbegriffen aus der gerechten Aufteilung, untersuchen wir Max-Min-, Große-Koalitions-, und Min-Max-Fairness im Modell der lokalen Fairness. Wir zeigen, dass jede nashstabile Koalitionsstruktur auch min-max-fair ist und dass die drei Fairnessbegriffe auch unter allgemeinen Präferenzen eine Hierarchie bilden. Für additiv-separable hedonische Spiele kollabiert die Hierarchie. Bezüglich Große-Koalitions- und Max-Min-Fairness ist das Berechnen der individuellen Schwellwerte in Polynomialzeit möglich, aber für Min-Max-Fairness coNP-vollständig. Die Existenzprobleme sind NP-schwer in allen drei Fällen.

Der letzte Beitrag dieser Arbeit ist eine neue Darstellung von Präferenzen bei hedonischen Spielen. Im Gegensatz zu vorherigen Arbeiten, die annehmen, dass Präferenzen unter den Agenten unabhängig voneinander sind, führen wir soziale Präferenzen im Kontext von freund-orientierten hedonischen Spielen ein. Wir unterscheiden drei Grade der Selbstlosigkeit. Abhängig vom Grad benutzt ein Agent entweder die klassischen Präferenzen zuerst und in Fällen von Gleichständen zusätzlich die Durchschnittsmeinung der Freunde oder umgekehrt. Zwischen diesen beiden Extremen betrachten wir auch einen Grad, bei dem der Präferenz eines Agenten dasselbe Gewicht wie den Präferenzen der Freunde zugewiesen wird. Für diese Präferenztypen zeigen wir, dass sie sich von den klassischen Darstellungen unterscheiden. Wir benutzen Eigenschaften aus der Sozialwahltheorie und überprüfen, dass diese Präferenzen sich natürlich verhalten. Auch gibt es in allen drei Fällen immer nashstabile Koalitionsstrukturen und wir beginnen die Komplexitätsuntersuchung für andere Stabilitätskonzepte.

Abstract

This thesis deals with fair division of indivisible goods and with hedonic games. Both are vibrant fields in the multiagent systems and artificial intelligence community. Fair division of indivisible goods is concerned with the distribution of goods under possibly conflicting interests of agents, whereas hedonic games model issues of coalition formation. For fair division of indivisible goods, we introduce scoring-based allocation rules and correspondences. Agents submit ordinal preferences over single goods to the social planner, who then uses a fixed scoring vector to generate additive representations of these ordinal preferences. Next, this representation is leveraged for social welfare optimization, focusing on utilitarian and egalitarian social welfare, and on the leximin preorder. We study these rules and correspondences with respect to axiomatic properties and computational complexity.

Then we extend the scoring-based model to analyze strategy-proofness using set extensions from social choice theory. For utilitarian social welfare, we characterize strategy-proofness using the Gärdenfors extension in terms of the structure of the scoring vector.

For the case when agents submit cardinal preferences, we study a fairness hierarchy proposed by Bouveret and Lemaître [2016] with respect to k-additive utility functions and answer a question raised by the same authors about the relationship between efficiency and a strong fairness notion. In the same work we study inequality-reducing rank-weighted utilitarianism and show that the approximability of rank dictator functions is intimately connected with the approximability of egalitarian social welfare. Then we look at the connection between both approaches to fairness for the max-min share and rank-weighted utilitarianism with inequality-reducing weight vectors in an exploratory manner.

We introduce local fairness notions for hedonic games. These are fairness notions that depend on an agent's individual preferences only and are satisfied if the utility of every agent is above a certain threshold. Inspired by fairness notions from fair division, we study max-min, grand-coalition, and min-max fairness in the framework of local fairness. We show that every Nash-stable coalition structure is min-max fair, and that the three fairness notions form a hierarchy, even under general preferences. For additively separable hedonic games, the hierarchy collapses. Computing individual thresholds is possible in polynomial time for grand-coalition and max-min fairness but is coNP-complete for min-max fairness. The existence problems are NP-hard in all three cases.

The last contribution of this thesis is a new representation of preferences for hedonic games. In contrast to prior work that assumes that preferences are independent among agents, we introduce social preferences in the context of friend-oriented hedonic games. We distinguish three degrees of altruism. Depending on the degree, an agent either uses the classical preferences first and the average opinion of this agent's friends as a tie-breaker, or vice versa. Between these two extremes we consider a degree where an agent's preference is given the same weight as the friends' preferences. For these preference types, we show that each is different from classical representations. Using properties from social choice, we check that these preferences behave naturally. Moreover, in all three cases Nash-stable coalition structures always exist and we initiate the study of the computational complexity for other stability notions.

Contents

1 Introduction

Around the time of World War II the field of game theory (von Neumann and Morgenstern [1944]) and the field of fair division (Steinhaus [1948, 1949]) gained increasing popularity. On the one hand, game theory analyzes interactions of rational agents. One can distinguish between noncooperative game theory, where the focus is on individual agents, and cooperative game theory that studies coalitions of agents. The latter is also characterized by the existence of binding agreements (Chalkiadakis et al. [2012]). In cooperative game theory one can make the assumption that members of a coalition can transfer utility to other members in the same coalition without restriction. This assumption makes sense when members have a common currency and it leads to the study of payoff divisions. However, in cooperative games with non-transferable utility it is more important which coalitions form, often using the assumption that agents care about their coalitions only. Such games are called hedonic games, and a key notion there is stability. Intuitively, agents should not have an incentive to leave their current group. Applications of hedonic games are, for instance, the modeling of stable marriage problems (Gale and Shapley [1962]). In this setting a matching is not stable if two unmarried partners would like to leave their current partners to form a new marriage. The research on matching led to its application for matching medical students to residency programs (Roth and Peranson [1999]). Other possible applications of hedonic games are the study of the composition of research teams (Alcalde and Revilla [2004]) and distributed task allocation (Saad et al. [2011]).

Fair division, on the other hand, deals with procedures and criteria to allocating goods fairly under conflicting interests. One can distinguish between divisible and indivisible goods. A heterogeneous divisible good is, for example, a cake with different toppings and a homogeneous divisible good could be water, whereas a house can be considered an indivisible good. A key notion in fair division of indivisible goods is envy-freeness. Intuitively, agents should not feel envy towards other agents' bundles. Applications of fair division are, for instance, fair allocation of resources of earth observation satellites (Lemaître et al. [1999]), dividing family silver (Pratt and Zeckhauser [1990]), assigning students to courses (Budish [2011]), dividing the rent (Gal et al. [2016]), or assigning computing resources (Ghodsi et al. [2011]). See the website by Goldman and Procaccia [2015] that provides access to fair division methods.

Recently, both topics have become more popular in multiagent systems and artificial intelligence research, in particular in a field called computational social choice. This field is concerned with computational questions in areas of collective decision making, including topics such as voting, fair division, coalition formation, and judgment aggregation. Research in this area has also become more relevant: With the increasing ubiquitous use of computation, human interaction with software agents is growing (e.g., Preist and van Tol [1998]). The interaction is not limited to humans receiving decision-support from software agents, but also spans scenarios where software agents take the lead. Jennings et al. [2014] coined the term "human-agent collectives" for that.

Because humans do not always behave completely rationally and selfishly (Fehr and Schmidt [2006]), software agents and decision-support systems have to incorporate aspects of fairness as well. These additional requirements may affect the computational complexity of those problems. Since, in the end, fair solutions have to be found computationally, the study of their computational complexity becomes important. Moreover, it may guide the search for favorable fairness notions.

Motivation and Outline

Chapter 2 provides the general context and the background of the succeeding chapters.

In fair division, agents need to express their preferences. A common solution is to use cardinal preferences, where agents use numbers to express their utility for single goods or bundles. However, this solution can be problematic because it requires agents to determine intensities. Another idea is to use ordinal preferences. In this case agents specify a preference relation that indicates, e.g., whether good a is preferred to good b. To determine cardinal preferences over single goods, an agent has to define (implicitly) a ranking of these goods. Therefore, it seems plausible to say that ordinal preferences are easier to elicit. The work in Chapter 3 models fair division problems with ordinal preferences over single goods. We study the performance of these allocation rules in terms of both axiomatic and computational properties. In Chapter 4 we analyze strategy-proofness of allocation correspondences based on utilitarian social welfare.

One approach to fair division of indivisible goods with cardinal preferences is to consider fairness criteria and to search for allocations that satisfy them. Another approach is to maximize a collective utility function. A priori these two approaches are orthogonal, for an allocation that is, e.g., envy-free does not have to maximize some collective utility function, and vice versa. In Chapter 5 we study the approximability of inequality-reducing collective utility functions and their relationship to fairness notions.

Hedonic games and fair division of indivisible goods share similarities: A hedonic game consists of agents (or players, we will use both terms interchangeably) and their preferences. A solution (coalition structure) is a partition of the agents into disjoint coalitions. A key property of hedonic games is that agents have preferences over coalitions that contain them only. Thus, it does not matter to agent i whether another agent is in coalition A or in coalition B if i is in neither coalition. This property corresponds to the no externalities assumption, which is commonly made in fair division of indivisible goods. There, agents have preferences over bundles only, and a solution (allocation) is a partition of the goods into disjoint bundles. Since agents and goods are indivisible in our setting, it becomes clear that this connection opens up opportunities for cross-fertilization between hedonic games and fair division of indivisible goods. The work in Chapter 6 takes a first step in this direction by transferring fairness notions from fair division to the theory of hedonic games.

Since agents need to rank exponentially many coalitions, various compact representations have been proposed. A simple representation is that every agent partitions all the other agents into friends and enemies. This representation is then used to compare coalitions. However, it is not without problems. Counterintuitive preferences may arise, where, e.g., an agent a is indifferent between being in a coalition consisting of a friend and an enemy and being in a coalition with a friend and this friend's friend (who is also an enemy to agent a). We extend hedonic games to social preferences and study axiomatic and stability aspects in Chapter 7.

We conclude in Chapter 8, where we discuss the results and possible future work.

2 Background

Fair division of indivisible goods and the theory of hedonic games can be studied in the more general model of social choice. In the general model agents have preferences over alternatives and one studies procedures that map to, e.g., single alternatives, multiple alternatives, rankings, or lotteries. When alternatives correspond to partitions (allocations or coalition structures), it can be seen that fair division and hedonic games are subsets of social choice. Therefore, positive results from social choice can be applied to fair division and hedonic games, whereas impossibility results for special cases also hold for the general case of social choice. However, computational results do not necessarily translate to the more general setting because of the potential blow-up in the representation.

See the recent overview of computational social choice by Brandt et al. [2016], specifically the chapters by Bouveret et al. [2016] on fair division of indivisible goods and by Aziz and Savani [2016] on hedonic games. Other surveys on fair division are by Brams and Taylor [1996], Moulin [1988, 2004], Lang and Rothe [2015], Nguyen et al. [2013], and Chevaleyre et al. [2006]. For hedonic games, see the surveys by Woeginger [2013a] and Hajduková [2006]. A general survey on fairness in multiagent systems is by de Jong et al. [2008]. For algorithmic game theory and multiagent systems in general, see the books by Nisan et al. [2007] and Shoham and Leyton-Brown [2009]. The works by Peleg and Sudhölter [2007] and Chalkiadakis et al. [2012] give comprehensive overviews of cooperative game theory, while the textbooks by Papadimitriou [1994a], Rothe [2005], and Arora and Barak [2009] deal with computational complexity. The works by Vazirani [2003], Williamson and Shmoys [2011], and Schuurman and Woeginger [2011] present aspects of the theory of approximation.

For an overview of other-regarding preferences, see the book chapter by Fehr and Schmidt [2006]. Models with other-regarding preferences are studied in noncooperative game theory (see, e.g., the paper by Hoefer and Skopalik [2013]) and, most notably, in the context of experimental economics (see, e.g., the work by Fehr and Schmidt [1999]).

In Section 2.1 we briefly highlight the key notions from computational complexity and the theory of approximation that are used in the succeeding chapters. Some chapters borrow ideas from voting theory, which is discussed in Section 2.2. Section 2.3 and Section 2.4 present a short overview of relevant and related work in the theory of fair division of indivisible goods and hedonic games.

2.1 Computational Complexity & Theory of Approximation

The study of computational complexity was initiated by Hartmanis and Stearns [1965] based on the computational model introduced by Turing [1936]. NP is a complexity class that contains all decision problems that are decidable in deterministic polynomial time by a verifier in the following way: If the input is a yes-instance, then it is accepted for some polynomial-size witness. If the input is a no-instance, then for any polynomial-size witness the input and the witness are rejected. Similarly, PSPACE-problems can be decided in deterministic polynomial space. A polynomial-time many-one reduction is a polynomial-time computable function that maps instances from some problem to another problem in a membership-preserving way. We say a problem is $\mathcal{C}$-hard if every $\mathcal{C}$-problem can be many-one reduced to it in polynomial time. $\mathcal{C}$-complete problems are the hardest problems in $\mathcal{C}$ because they are in $\mathcal{C}$ and $\mathcal{C}$-hard. For instance, any deterministic polynomial-time algorithm for some NP-complete problem would imply such algorithms for any NP-complete problem. The complexity class coNP contains the complements of problems that are in NP. In a seminal work Cook [1971] found the first coNP-complete problem, disjunctive normal form (DNF) tautology, which asks whether a given formula in DNF is a tautology. This is equivalent to the UNSAT problem, which asks whether a given formula in conjunctive normal form is unsatisfiable. The complement of the UNSAT problem is the SAT problem. Therefore the SAT problem is NP-complete. Also see the works by Levin [1973] and Trakhtenbrot [1984] for the discovery of NP-complete problems. Karp [1972] showed NP-completeness for various natural combinatorial problems. One of these problems, which is used frequently in the succeeding chapters, is Partition. It is defined as given a list of numbers, is there an index set such that the list of numbers can be partitioned into two parts of equal sum total? Wechsung [1985] defined the boolean hierarchy over NP. The first level is NP, the second level (also known as DP) contains problems that can be written as the intersection of an NP and a coNP problem, the kth level consists of problems that are the union of a problem of the second level and of the $(k-2)$th level for $k \geq 3$ (Rothe [2005]). Meyer and Stockmeyer [1972] and Stockmeyer [1976] introduced the polynomial hierarchy, which contains problems that are presumably harder than NP-complete problems. Informally, whereas the quantifier definition of NP contains an existential quantifier for the yes-case, one can also consider complexity classes that can be defined via a bounded sequence of alternating existential and universal quantifiers. The sequence of quantifiers is then followed by a polynomial-time computation. One of these complexity classes is Σ_2^p which contains all decision problems for which there is a polynomial-time bounded Turing machine that accepts yes-instances if there is a polynomial size witness such that for all polynomial size witnesses it accepts.

An alternative characterization of NP was provided by Arora and Safra [1998] and Arora et al. [1998]. Instead of requiring a deterministic verifier, it is enough to consider probabilistic verifiers that only use a logarithmic amount of randomness and read a constant number of witness bits (instead of all the bits). This characterization has implications for the approximability of NP-complete problems (see, e.g., the paper by Håstad [2001]). The study of approximability started with Johnson [1974], but there were also earlier works by Graham [1966, 1969] and Garey et al. [1972]. A c-approximation algorithm for optimization problems is a polynomial-time algorithm that returns a solution that is within a factor of c from the optimal solution. APX is the approximation class that contains all NP-optimization problems that can be approximated within a constant factor (Ausiello et al. [2003]). Garey and Johnson [1978] defined the terms polynomial-time approximation scheme (PTAS) and fully polynomial-time approximation scheme (FPTAS). A PTAS for maximization problems is a $(1-\varepsilon)$-approximation algorithm for any $\varepsilon > 0$ that has a running time that is polynomial in the input size. An FPTAS is a PTAS where the running time is polynomial both in the input size and in $1/\varepsilon$. Garey and Johnson [1978] introduced strong NP-completeness as well. A number problem is strongly NP-complete if it is NP-complete even if numbers are given in unary (equivalently, if numbers are bounded by a polynomial in the input size). In Chapter 5 we use the connection that a strongly NP-complete problem cannot have an FPTAS, unless P = NP.

Downey and Fellows [1995a] introduced the class FPT of all fixed-parameter tractable problems. A problem is fixed-parameter tractable with respect to some parameter, if there is an algorithm whose running time can be bounded by a polynomial and a computable function that depends on the parameter. W[1] contains all problems for which there is a parameterized reduction to some parameterized circuit satisfiability problem with circuits having weft at most one (Downey and Fellows [1995b]).

Regarding function problems, the complexity class #P contains all functions that output the number of accepting paths of some nondeterministic polynomial-time Turing machine (Valiant [1979]).

FNP is the function analogue of NP where the output is a witness, if it exists, and "no" otherwise.

The class PPAD (polynomial parity argument directed), defined by Papadimitriou [1994b], contains function problems, for which a solution is always guaranteed to exists by a parity argument on a directed graph, and is defined in terms of the problem EndOfTheLine.

Johnson et al. [1988] considered the class PLS (polynomial local search) that contains problems, for which a better solution in the neighborhood can be found in polynomial time.

2.2 Social Choice

Social choice dates back at least to the time of Pliny around 50 AD (see the book by McLean and Urken [1995]), but the "Golden Age" of social choice was around the time of the French revolution. The most relevant model for this work is the following: There is a set of alternatives and a set of voters. Voters have preferences over the alternatives as linear orders. A collection of linear orders is a profile. By a social choice function we mean a mapping from the set of profiles to a (nonempty) set of alternatives. Borda [1781] reinvented (McLean and Urken [1995]) the following rule: Each alternative receives from each voter points equal to the number of alternatives that are worse than this alternative. Thus, the first ranked alternative of some voter receives from this particular voter $m-1$ points, and the last ranked alternative receives from this voter 0 points, where m is the number of alternatives. Alternatives with the maximum point total win. Equivalently, one can consider a scoring vector $s = (m-1, m-2, \dots, 0)$ and the point score of alternative a is $\sum_{v \in V} s_{rank(a,v)}$, where $rank(a, v)$ denotes the position of alternative a in the linear order of voter v. This procedure can be generalized to arbitrary scoring vectors $(s_1, s_2, \dots, s_m)$ with $s_i \geq s_{i+1}$, $i < m$, and $s_1 > s_m$. Chapters 3 and 4 deal with scoring vectors as well. Another influential work is by Condorcet [1785] who noticed that the majority relation can cycle, proposed a rule based on pairwise-comparison, and found the so-called Condorcet jury theorem.

A normative way to compare voting rules is the axiomatic method. This method plays a crucial role in this work and is used throughout the following chapters. In the context of voting rules, some of the key axioms are anonymity (permuting the voters does not change the outcome), neutrality (permuting the alternatives does not change the outcome), and monotonicity (a winning alternative whose ranking is improved ceteris paribus does not lose suddenly). However, not every combination of axioms is possible. Arrow [1950] concluded that an independence axiom (independence of irrelevant alternatives) is incompatible with efficiency in the context of social welfare functions, which return a ranking of alternatives, in the sense that any such voting rule is a dictatorship. Similarly, Gibbard [1973] and Satterthwaite [1975] showed that any voting rule that returns exactly one alternative, is onto, and is strategy-proof (misrepresentation of preferences is never profitable) is a dictatorship.

Given this last result, some works in computational social choice study the computational complexity of manipulating elections (e.g., Bartholdi et al. [1989b], Conitzer et al. [2007], and Conitzer and Walsh [2016] provide an overview). Related problems are the winner determination problem, control, and bribery, studied by, e.g., Bartholdi et al. [1989a], Hemaspaandra et al. [1997], Bartholdi et al. [1992], Hemaspaandra et al. [2007], and Faliszewski et al. [2009]. Also see the overviews by Faliszewski and Rothe [2016] and Baumeister and Rothe [2015].

An alternative approach is to consider correspondences. Set extensions are used to consider sets of alternatives (Barberà et al. [2004]). A generalization of the result by Gibbard [1973] and Satterthwaite [1975] is by Duggan and Schwartz [2000]. In Chapter 4 we consider the set extensions by Kelly [1977], Gärdenfors [1976], and Fishburn [1972]. More recent work on strategy-proofness of correspondences is by Brandt [2015, 2011], Brandt and Brill [2011], and Brandt and Geist [2016].

2.3 Fair Division of Indivisible Goods

A fair division setting consists of a set of agents, a set of indivisible goods, and the agents' preferences. A common assumption is that agents care only about the goods that they receive. Therefore, agents' preferences are usually over single goods or over bundles of goods. One can distinguish between ordinal preferences and cardinal preferences. For ordinal preferences, one usually considers linear orders, although there are some works that also deal with ties.

Representation

For cardinal preferences, the common representation forms are the bundle form (an explicit listing of all bundles), the k-additive form, and representations via weighted goals using propositional formulas. In Chapter 5 we consider the k-additive form. This representation form was proposed originally in the context of fuzzy measures by Grabisch [1997]. Later Conitzer et al. [2005] introduced them in the context of combinatorial auctions and Chevaleyre et al. [2004, 2008] in fair division. Chevaleyre et al. [2004, 2008] compared the k-additive form to the bundle form and found that there are utility functions which are representable in polynomial space in the k-additive form but not in the bundle form, and vice versa. Uckelman et al. [2009] found that the utility functions that can be generated by goal bases that consist of propositional formulas restricted to positive k-cubes is exactly the class of k-additive utility functions, whereas the restriction to positive k-clauses leads to normalized k-additive utility functions. They showed other equivalences for similar restrictions and considered succinctness and complexity questions as well. Uckelman and Endriss [2010] performed a classification when the max operator is used for weights instead of the sum operator. Also see the work by Bouveret et al. [2005] for complexity results with logic-based languages.

Collective Utility Functions

Solutions are partitions (allocations) of the set of goods into bundles, one for each agent. Depending on the setting, the assumption of completeness can be made, that is, that every good is assigned to some agent. A fundamental property for allocations is Pareto optimality. It is an efficiency requirement that says that there is no allocation where every agent is weakly better off and some agent is strictly better off. Allocations can be assessed based on binary criteria such as envy-freeness (no agent envies another agent's bundle) or proportionality (every agent receives a proportional share) or based on collective utility functions. The approach of collective utility functions requires cardinal preferences (or at least a cardinalization). Common collective utility functions (see, e.g., the book chapter by d'Aspremont and Gevers [2002]) are utilitarian social welfare, where agents' utility values are summed up; egalitarian social welfare, where only the utility value of the worst-off agent is relevant; and Nash social welfare, where agents' utility values are multiplied. The latter was introduced by Nash [1950] in the context of bargaining games. Another approach that is frequently used is the leximin preorder. Intuitively, an allocation that maximizes the leximin preorder maximizes the utility of the worst-off agent, then the utility of the second poorest agent, and so on.

Chevaleyre et al. [2004, 2008] established that the corresponding decision problem of maximizing utilitarian social welfare with respect to the bundle form is an NP-complete problem, and this is also the case for the k-additive form, if $k \geq 2$. Roos and Rothe [2010] and Nguyen et al. [2014] examined the computational complexity of social welfare optimization under different representation forms (bundle and k-additive form and straight-line program representation) and for different notions of social welfare (utilitarian, egalitarian, and Nash social welfare). Almost all of these problems are NP-hard. In addition, they gave approximability and inapproximability results. Ramezani and Endriss [2010] demonstrated that the maximization problem for Nash social welfare is NP-hard.

In terms of approximability results, in the context of scheduling Woeginger [1997] gave a polynomial-time approximation scheme for maximizing egalitarian social welfare when agents have identical additive utility functions. This algorithm can also be used as a PTAS for computing the max-min share value. Bezáková and Dani [2005, 2004] studied the approximability of egalitarian social welfare for arbitrary additive utility functions. Their algorithm returns allocations with egalitarian social welfare at least $\mathrm{OPT} - \alpha$, where α denotes the largest value in any utility function for a single good. Moreover, they provided a $(1/(m-n+1))$-approximation and showed that it is NP-hard to approximate within $1/2 + \varepsilon$, which is the best known lower bound so far, where n denotes the number of agents and m the number of goods. Golovin [2005] considered the "Big Goods/Small Goods" case and gave an $\mathcal{O}(1/\sqrt{n})$-approximation. Bansal and Sviridenko [2006] looked at the case where every good has an intrinsic value, that is, agents either value a good at zero or some fixed value, which is the same for all agents. Using ideas from the scheduling literature they found an $\mathcal{O}(\log\log\log n/\log\log n)$-approximation for this special case. Asadpour and Saberi [2010] improved on the general problem with an $\Omega(1/\sqrt{n}\log^3 n)$ guarantee. The best bound so far is by Chakrabarty et al. [2009] with an $\widetilde{\mathcal{O}}(1/m^{\varepsilon})$-approximation for every $\varepsilon > 0$. Bateni et al. [2009] studied further restrictions by bounding the number of agents that can have positive value for a good. They showed that a degree bound of 3 is as hard as the general case and gave an approximation algorithm for a degree bound of 2. Ferraioli et al. [2014] considered another restriction, the k-division problem. In this problem every agent receives exactly k goods. They established a connection to matroid theory and gave approximation algorithms.

Approximability in fair division was also studied by Aziz et al. [2016d] in the context of chore division. Under the assumption that a max-min share allocation exists, they showed that it is strongly NP-hard to compute one. For the chore setting, a $(1/2)$-approximation exists. Furthermore, they considered the optimal max-min share. This is the largest fraction, depending on the instance, of the max-min share that can be guaranteed to every agent. They designed both for the chores and goods setting polynomial-time approximation schemes when the number of agents is constant (also see the paper by Nguyen et al. [2017]).

Cole and Gkatzelis [2015] examined the approximability of the average Nash social welfare, which is an APX-hard problem (Lee [2017]). They established a $(1/2e^{1/e})$-approximation, which was improved to a $(1/2)$-approximation through a tight analysis by Cole et al. [2017]. Also see the paper by Brânzei et al. [2017] on strategic aspects under this objective.

Other kinds of collective utility functions stem from the study of inequality measurement. Some key works are by Lorenz [1905] who proposed the Lorenz curve as a way to make changes in income distributions visible, by Dalton [1920] who introduced the so-called Pigou-Dalton property that should be satisfied by inequality indices (if a higher-income agent transfers income to a lower-income agent, then inequality should "diminish"), and by Atkinson [1970] who used tools from decision-making under uncertainty to rank income distributions. Prior works are also by, e.g., Blackorby and Donaldson [1978], who studied the collective utility function induced by the Gini index, and by Weymark [1981], who generalized the Gini index to decreasing weights. Yager [1988] introduced ordered weighted averages in the context of multicriteria decision-making. This idea can be used to capture the Gini index as well.

Contemporary research on inequality was done by Golden and Perny [2010]. They investigated Lorenz domination, a refinement of Pareto domination, and proposed to iterate Lorenz domination, if two vectors are incomparable. Based on this, they defined infinite Lorenz domination, showed that it has a representation as an ordered weighted average with decreasing weights, and gave a mixed-integer programming (MIP) formulation for the problem. Lesca and Perny [2010] studied fair division with additive utility functions and volume constraints on bundle sizes. They provided MIP formulations for maximizing the social welfare function related to the Gini index. In addition, they considered nonanonymous settings where agents have different weights that are also nonadditive as well. The model that is closest to the model in Chapter 5 is by Endriss [2013]. He studied fair division with the objective of reducing inequality. In his work inequality is measured by the Gini, Robin Hood, and Atkinson index (for some social welfare orderings). For these measures, he presents a modular way to construct integer programs to minimize them.

Ordered weighted averages are used in related fields as well. For instance, Skowron et al. [2016] studied them in the context of multiwinner elections to model agents' preferences. Goldsmith et al. [2014] used them in the context of voting to extend scoring rules. Instead of taking the sum of a candidate's score, individual scores are weighted using this approach.

Fairness Criteria

Another approach to assess allocations is to use binary criteria as in Chapter 5. However, some criteria are not always met, e.g., envy-free allocations may not exist. De Keijzer et al. [2009] showed that deciding whether there is an allocation that is envy-free and Pareto-optimal with additive utility functions is Σ_2^p-complete. With respect to the parameterized complexity of existence problems for Pareto-optimal envy-free allocations, Bliem et al. [2016] studied the parameters "number of agents," "number of goods," and "number of different utility values." They provided FPT and both classical and parameterized hardness results. For example, with respect to the parameter "number of agents," they proved hardness for levels of the boolean hierarchy under dichotomous preferences. For additive preferences in unary, they showed W[1]-hardness and FPT membership when utilities are from $\{0, 1\}$.

Dickerson et al. [2014] gave a probabilistic analysis for the existence of envy-free allocations under additive utility functions. They concluded that if the number of goods is in $\Omega(n \ln n)$, then the probability of existence goes to 1 as the number of goods goes to infinity. The proof shows in fact that the allocation that maximizes utilitarian social welfare is envy-free. If the number of goods is too small, then an envy-free allocation may not exist. They supported the theoretical study with experiments. There they found a phase-transition, i.e., the probability of existence of an envy-free allocation rises steeply when the number of goods becomes large enough. As the number of goods rises, the problem becomes easier and the MIP solvers need less time for the problem.

An idea to cope with the nonexistence of envy-free allocations is to introduce degrees to measure the amount of envy-freeness (Lipton et al. [2004]). However, this problem is NP-hard, and there is a ($1/1+\alpha$)-approximation algorithm, where α is the largest utility value. For the minimum envy-ratio problem, they showed that there is a PTAS for identical utility functions and that there is an FPTAS when the number of agents is constant. Likewise, Nguyen and Rothe [2014] studied degrees of envy and average Nash social welfare. They demonstrated that there is an FPTAS for minimizing certain degrees of envy if the number of agents is constant and gave an inapproximability result. For matching settings (equal number of agents and goods), the problems are solvable in polynomial time. Average Nash social welfare can be approximated within $1/(m-n+1)$, whereas for identical utility functions there is a PTAS.

Somewhat similarly, proportional allocations may not always exist. Therefore, it makes sense to find a lower bound on the utility that every agent can receive (Hill [1987]). Markakis and Psomas [2011] showed that the lower bound proposed by Hill [1987] can be attained and found in polynomial time. Gourvès et al. [2013], within a more general model of matroids, found a family of functions that improves on the previous family of lower bound functions.

With regards to boolean fairness criteria, the work by Budish [2011] studies the combinatorial assignment problem in a course allocation setting. He proposed the notion of approximate competitive equilibrium from equal incomes, a relaxed version of competitive equilibrium from equal incomes (CEEI). A competitive equilibrium from equal incomes is a pair of an allocation and a price vector, every agent maximizes their utility subject to the budget constraint. It is fair because every agent has equal income. However, it may not always exist in discrete settings. The relaxation proposed by Budish [2011] guarantees the existence in the studied setting and works by giving each agent only approximately the same budget and by requiring that the market clears only in an approximate sense. In addition, he considered two new fairness notions: max-min share and envy-freeness up to one good, adapting proportionality and envy-freeness from the cake-cutting setting. The max-min share is the bundle that agents can guarantee themselves when playing the cut-and-choose game for bundles. An allocation satisfies the max-min share property if every agent receives at least their max-min share. Interestingly, approximate CEEI satisfies these fairness notions in an approximate sense. The key property here is that budget inequality can become arbitrarily small. In addition, approximate CEEI satisfies a weaker form of strategy-proofness. Unfortunately, it is PPAD-complete to find such an equilibrium (Othman et al. [2016]), but Othman et al. [2010] showed how to compute it efficiently in practice.

The existence problem for ordinary CEEI allocations is strongly NP-hard (Aziz [2015a]) under additive utility functions. The verification problem is coNP-complete. Hardness also holds for the case of deciding whether a suitable allocation exists for a given price vector, and vice versa (Brânzei et al. [2015]). Brânzei et al. [2016] considered a restriction to single-minded agents and characterized a relaxation of competitive equilibrium from equal incomes.

Gourvès et al. [2014] adapted the notions of equitability, proportionality, and envy-freeness to the matroid setting and studied (inspired by Budish [2011]) near fairness notions. For near proportionality, they demonstrated an algorithm to find such a base allocation in polynomial time. For two agents, they gave a decentralized algorithm to find a nearly envy-free base allocation in polynomial time.

Every allocation that satisfies competitive equilibrium from equal incomes is envy-free. This follows from the fact that all agents have the same income. If they envied another agent, they could buy their bundle. Bouveret and Lemaître [2016] considered a hierarchy of fairness notions for additive utility functions. They showed that the five fairness notions of max-min share, proportionality, min-max share, envy-freeness, and competitive equilibrium from equal incomes form a hierarchy with max-min share being the least demanding fairness property. This is similar to a result in Chapter 6. On the computational side, they established that computing the bounds for max-min share and min-max share is intractable (NP-complete and coNP-complete, respectively). Moreover, they found that when agents' preferences are strict in the sense that distinct bundles are assigned distinct utility values, then any CEEI allocation is Pareto-optimal, leaving open whether the converse is also true. Next, they considered restrictions and investigated when max-min share allocations exist. For example, if there are exactly three more objects than agents, then there is always a max-min share allocation. They concluded their study with experimental results; most notably they found that all their generated instances admitted a max-min share allocation. For k-additive utility functions, they were able to show that the existence problem for max-min share allocations is NP-hard. This research direction is continued in Chapter 5.

Despite the experimental results that suggest that there is always an allocation that gives every agent their max-min share, Procaccia and Wang [2014] showed that max-min share allocations do not always exist. The construction is complicated (a simpler counterexample, which only needs a linear number of goods, was given by Kurokawa et al. [2016]) and eluded experiments. If one settles for a ($^2/_3$)-approximation, then such an approximate max-min share allocation always exists (Procaccia and Wang [2014]), and it can be found in polynomial time (Amanatidis et al. [2015], Barman and Murthy [2017]). This was improved by Ghodsi et al. [2017]. They showed that a ($^3/_4$)-approximation always exists and demonstrated a PTAS for finding such an allocation. For three agents, Amanatidis et al. [2015] provided a ($^7/_8$)-approximation algorithm, improving on the ($^3/_4$)-approximation algorithm by Procaccia and Wang [2014]. For the case where item values are from $\{0, 1, 2\}$, they gave an exact algorithm. Kurokawa et al. [2016] performed a probabilistic analysis and showed that max-min share allocations exist with high probability when utility functions are sampled independently from distributions of constant variance. For the uniform distribution, Amanatidis et al. [2015] provided better bounds on the number of agents.

Nash social welfare can be seen as a compromise between utilitarian social welfare and egalitarian social welfare. A balanced utility vector maximizes Nash social welfare which has an egalitarian flavor, but it does not suffer from the drowning effect (lack of Pareto optimality). Surprisingly, under additive utility functions Caragiannis et al. [2016] showed that allocations that maximize Nash social welfare are Pareto-optimal, satisfy envy-freeness up to one good, and constitute a $\Theta(^1/\sqrt{n})$-approximation of the max-min share.

The question of how much social welfare is lost when requiring fairness was studied by Caragiannis et al. [2012]. They considered the three fairness notions of envy-freeness, proportionality, and equitability. In the case of indivisible goods, they showed that the price of proportionality is $n - 1 + {^1/_n}$ and that the price of envy-freeness is $\Theta(n)$, that is, a factor of n is lost in terms of social welfare, when envy-freeness is demanded. For a small number of goods, Kurz [2016] showed that the price of fairness improves. A related idea is to consider the price of envy-freeness when goods can be sold. Under the assumption of additive utility functions and the existence of a constant that determines the loss when selling goods, Karp et al. [2014] confirmed that the price of envy-freeness can decrease. Furthermore, they showed that maximizing social welfare with sellable goods is NP-complete and gave a fully polynomial time approximation scheme for a special case.

Experiments

Although the envy-freeness condition is intriguing, it is not a priori clear that it is a criterion nonexpert humans use to solve fair division problems. Herreiner and Puppe [2007] performed an exploratory questionnaire study on allocating indivisible goods where strategic considerations played no role. The focus was on the background of the participants. Participants were given ten allocation problems. Then they were asked which allocation they found the fairest. They found that the background (economics or law students) does not matter. Moreover, the participants' written explanations for their choices are based on either procedures or equity. Two of these procedures were assigning the good to the agent who values it most or assigning to each agent their most valued good. Herreiner and Puppe [2009] reported on lab experiments on bargaining games. They found that the participants relied more upon inequality aversion than envy-freeness. Envy-freeness was only a secondary criterion to further discriminate between Pareto-optimal allocations. In another experiment they stated that agents behaved seemingly according to a "Conditional Pareto Improvement from Equal Split" condition (Herreiner and Puppe [2010]). The condition states that agents start from an allocation that is as equitable as possible. If it is not Pareto-optimal, they will perform Pareto improvements as long as inequality does not become subjectively too large. According to Herreiner and Puppe [2010] interpersonal comparisons were done using cardinal preferences or by using the rank in case of ordinal preferences.

Whether more sophisticated procedures are preferable is not clear. On the one hand, Schneider and Krämer [2004] performed an experiment where participants had to choose between fair division procedures. More participants preferred a sophisticated procedure to divide-and-choose than vice versa. They also found that solutions became worse when the protocol no longer had to be followed and that there is a connection between the choice of the fair division procedure and the psychological profile of participants. On the other hand, Dupuis-Roy and Gosselin [2011] performed a study where participants' satisfaction and perceived fairness was measured before and after using fair division procedures. They found that solutions of sophisticated procedures are perceived as less fair than simple procedures. In a similar vein, Holcombe [1997] argued in favor of better distinguishing between procedural and outcome fairness.

Ordinal-Preference Models

Most works discussed above assume cardinal preferences. Since elicitation is easier for ordinal preferences, Chapters 3 and 4 consider an ordinal-preference model of fair division. Gärdenfors [1973] studied such a model in the context of assignment problems. He assumed that agents are only assigned one good and showed that certain combinations of axioms lead to impossibility results. Then he proposed assignment functions that do satisfy some of the criteria. Most notably, some of the assignment functions assign scores to the goods, which is similar to the approach in Chapter 3. Wilson [1977] continued the work by Gärdenfors [1973] on assignment functions by trying to adapt methods from the matching literature to the assignment setting.

The works that are closest to Chapter 3 are by Edelman and Fishburn [2001], which is based on works by Brams and Fishburn [2000] and by Brams and King [2005]. Edelman and Fishburn [2001] supposed that all agents agree on the ranking of single goods (but may disagree over the ranking of bundles). Brams and King [2005] assumed ordinal preferences over single goods and separability instead. They proved that an allocation is Pareto-optimal if and only if there is a policy for a sequential allocation procedure such that the allocation results from that policy. They studied maximin and Borda maximin allocations. Maximin allocations maximize the minimum rank that an agent assigns to a received good, and Borda maximin allocations maximize egalitarian social welfare when preferences are cardinalized according to the Borda scoring vector. Brams et al. [2003] extended and complemented the study by Brams and King

[2005] and gave characterizations of Pareto-optimal allocations and envy-free allocations. Bouveret et al. [2010] continued that research direction and analyzed the existence of envy-free allocations when preferences are lifted using monotonicity and separability. They called the representation of these preferences SCI-nets, which are precondition-free, singleton-comparing conditional importance networks (Bouveret et al. [2009]). Since the lifting process leads to incomparabilities (i.e., the induced preferences are not complete), they defined possible and necessary variants of envy-freeness and Pareto optimality: An allocation is possibly envy-free if there is a completion of the preferences such that the allocation is envy-free with respect to the completion. It is necessarily envy-free if it holds for every completion. They showed that if k distinct goods are at the top position, then there is a complete and possibly envy-free allocation if and only if $m \geq 2n - k$, where m is the number of goods and n the number of agents. This characterization also holds if possible Pareto optimality is required instead of completeness of allocations. For necessary envy-freeness, they proved that the problem of deciding whether such an allocation exists is NP-hard in general. This is also true when possible or necessary Pareto optimality is required. However, for two agents, this problem becomes easy again via network-flow techniques. Likewise, in Chapter 3 we distinguish between properties holding in a possible and in a necessary sense.

Aziz et al. [2015d] introduced new fairness properties for ordinal preferences over single goods. They used stochastic dominance (SD) to lift preferences over goods to preferences over bundles. Bundle A is preferred to bundle B according to stochastic dominance if every cardinal utility function consistent with the ordinal preferences gives at least as much utility for A as for B. This is equivalent to preferences according to the responsive set extension, which is also used in Chapter 4. Based on this extension, they considered variants of proportionality and envy-freeness and showed how these notions relate to each other. Then they studied complexity issues and gave a polynomial-time algorithm to check whether an SD proportional allocation exists, generalizing a result by Pruhs and Woeginger [2012]. For weak SD proportionality, there is a polynomial-time algorithm when the number of agents is constant. Moreover, they answered a question by Bouveret et al. [2010] by showing that checking the existence of an allocation that is envy-free (for all considered notions) is NP-complete.

For proportional allocations, Darmann and Klamler [2016] investigated the existence of such allocations assuming the Borda scoring vector and balanced allocations, where all agents receive the same number of goods. If the number of goods is of the form $l \cdot n$ for l even, then a proportional allocation always exists and there is a polynomial-time algorithm for it. If l is odd and the number of agents n is odd, the statement also holds. However, this is no longer true for odd l and an even number of agents, but a nearly proportional allocation can be guaranteed for any l.

The work by Aziz et al. [2016a] deals with Pareto-optimal allocations under cardinal and ordinal preferences when agents have initial endowments. If an allocation is Pareto-dominated, it is individually rational for every agent to change the allocation. Therefore, testing for Pareto optimality is related to searching for individually rational improvements. Assuming additive utility functions, the authors showed that the existence of a polynomial-time algorithm for computing a Pareto-optimal and individually rational allocation implies the existence of a polynomial-time algorithm for testing Pareto optimality. They found that it is strongly coNP-complete to check whether an allocation is Pareto-optimal if every agent receives exactly two goods. Then they provided a pseudo-polynomial-time algorithm for the computation of a Pareto-optimal individually rational allocation if the number of agents is constant. Under lexicographic utilities they characterized the existence of Pareto-optimal allocations based on a certain graph. For ordinal preferences, the responsive set extension is used. For this setting, they showed that an allocation is possibly Pareto-optimal if it is Pareto-optimal with respect to the responsive set extension or the graph from the characterization for lexicographic utilities does not admit a specific cycle. In addition, they gave a characterization of necessary Pareto optimality. An allocation is necessarily Pareto-optimal if it is possibly Pareto-optimal and there is no "one-for-two Pareto improvement swap."

Working in the model of Chapter 3, Darmann and Schauer [2015] investigated the complexity of maximizing Nash social welfare for k-approval, Borda, and lexicographic scoring. Under k-approval maximization is easy, but the corresponding decision problem is NP-complete for the two other cases. On a related note, Garg et al. [2010] considered a paper reviewing scenario. Reviewers are assigned to papers (subject to their ordinal preferences) so that every paper is reviewed an adequate number of times, no person has to review too many papers and the leximin objective is maximized. This is similar to Chapter 3 when we consider reviewers as agents and papers as goods. NP-hardness emerges when preferences allow for at least three different ranks. For the two-valued case, they showed that there is a polynomial-time algorithm.

Instead of assuming ordinal preferences over single goods, Herreiner and Puppe [2002] studied the so-called descending demand procedure with the assumption that agents have preferences over all bundles of goods (assuming monotonicity). The descending demand procedure then asks agents (according to a fixed order) one by one for their most preferred bundle and checks whether a feasible allocation is possible. If this is not the case, the procedure asks for the second-most preferred bundles, and so on. This results in a Pareto-optimal allocation that maximizes the rank of the bundle of the worst-off agent.

Procedures

Chapter 3 considers a sequential allocation approach as well. This was already studied by Kohler and Chandrasekaran [1971]. Instead of submitting these preferences to a central authority agents take turns (according to a given policy) and pick their most desired good among the remaining goods. They considered sequential allocation games for two players with additive utility functions with the focus on finding optimal strategies under various assumptions. Bouveret and Lang [2011] continued the work on sequential allocation. Agents have ordinal preferences over single goods that are cardinalized using Borda, lexicographic, or quasi-indifferent scoring vectors as in Chapter 3. The authors assumed two uncertainty models for profiles. Under full-independence, agents' preferences are independent and uniformly distributed. Under full correlation, agents' preferences are all the same (this can be justified if there is an objectively best ranking). They studied expected utilitarian and egalitarian social welfare. Under full correlation, they find that all policies have the same expected utilitarian social welfare, while for egalitarian social welfare the corresponding decision problem becomes NP-hard. For full independence, the authors conjectured that computing expected social welfare is hard. Moreover, they studied manipulability. They found that the procedure is strategy-proof if all agents have identical preferences and gave results if lexicographic scoring is used (also see the studies by Bouveret and Lang [2014] and Aziz et al. [2017a] for results on manipulability).

Kalinowski et al. [2013a] answered the above conjecture in the negative. They showed that expected utility and expected utilitarian social welfare can be computed in polynomial time for any policy. Then they proved that the alternating policy maximizes expected utilitarian social welfare for two agents. A game-theoretic study was then initiated by Kalinowski et al. [2013b]. They computed for two agents the subgame perfect Nash equilibrium. For an unbounded number of agents, they showed that computing the subgame perfect Nash equilibrium is PSPACE-hard. Aziz et al. [2017c] considered the one shot variant of sequential allocation.

Aziz et al. [2015e] characterized outcomes of balanced, recursively balanced, and balanced alternation policies. They studied the complexity of decision problems that ask, in the possible case, whether there is a policy from some given policy class such that some agent receives a given item, set or subset or whether a given allocation is reached. Decision problems for the necessary case deal with questions that are true for all policies from some given policy class. All assignment questions are polynomial-time solvable, whereas all item and subset questions are intractable. Most set questions are intractable. A focus on social welfare was taken by Aziz et al. [2016c]. They assumed that agents' additive utility functions are known and considered two types of decision problems. The first asks whether there is a policy from a class of policies such that a given welfare bound is achieved, whereas the second problem asks whether for every policy from a class of policies social welfare attains at least a given bound. For arbitrary policies and utilitarian social welfare in the possible case, this is solvable in polynomial time. However, for egalitarian social welfare, the problem is strongly NP-complete. Under balanced policies in the possible case and utilitarian social welfare, the problem remains polynomial-time solvable. However, the necessary case is coNP-complete. This also holds for egalitarian social welfare. For recursively balanced and balanced alternating policies, all problems become intractable.

Other procedures are, e.g., the adjusted winner procedure by Brams and Taylor [1996] (also see the work by Dall'Aglio and Mosca [2007] for optimizations of the adjusted winner procedure in terms of running time) and the undercut procedure by Brams et al. [2012]. For two players, it returns an envy-free allocation under an assumption of the agents' ordinal preferences over the contested pile. This procedure was then improved by Aziz [2015b] to work with the more general assumption of separability (instead of responsiveness). The sequential algorithm by Brams et al. [2015] works with ordinal preferences over single goods. It always returns a Pareto-optimal allocation. If there are only two players, it will also be envy-free. Brams et al. [2014] proposed the "proportional algorithm," where players have additive utility functions and submit a complete ranking of certain bundles. It is a procedure to compute maximally proportional allocations that maximize the rank among the so-called minimal bundles of the worst-off individual and works for an arbitrary number of agents. Since proportional allocations may not exist, the number of agents who receive a proportional bundle is maximized in this case.

An important question pertaining procedures is whether they are strategy-proof, that is, whether it is profitable to misrepresent one's preferences. Unfortunately, for allocation rules only a restricted class of procedures is strategy-proof. Pápai [2001] studied a model where agents may receive multiple objects. She showed that the only rules that are strategy-proof, nonbossy, and satisfy citizen sovereignty are sequential dictatorships. In a sequential dictatorship the order of the agents is not fixed a priori because the order may depend on the chosen goods instead. Ehlers and Klaus [2003] and Hatfield [2009] showed similar results when assuming responsive and separable preferences or when assuming a quota and responsive preferences, respectively. Sequential dictatorships are also characterized by Pareto optimality and coalitional strategy-proofness under the assumption of responsiveness, separability, and strict preferences (Ehlers and Klaus [2003]). Strategy-proofness, population monotonicity, and Pareto optimality characterize serial dictatorships under separability and strict preferences (Klaus and Miyagawa [2002]). Recently, Amanatidis et al. [2017] characterized deterministic truthful mechanisms for two players under additive utility functions and the assumption of completeness. These results are not applicable to the model in Chapter 4 because it deals with allocation correspondences.

Mackin and Xia [2016] considered a model where goods are categorized. Each agent is supposed to receive exactly one good from each category. Preferences are over each possible combination. The authors first characterized serial dictatorship in this domain based on the axioms of strategy-proofness, nonbossiness, and category-wise neutrality. Then they proposed sequential procedures where not only the order of the agents is fixed but also the category. So agents cannot take goods from arbitrary categories. Furthermore, they provided lower and upper bounds on the rank efficiency of the proposed procedures.

A problem related to strategy-proofness and manipulability is control. The study of electoral control was initiated by Bartholdi et al. [1992] (see the book chapters by Faliszewski and Rothe [2016] and Baumeister and Rothe [2015] for an overview). For the study of control in weighted voting games and judgment aggregation, see the works by Rey and Rothe [2016] and Baumeister et al. [2013], respectively. Aziz et al. [2016e] studied control actions in the context of fair division (see Dorn et al. [2017] for follow-up work). They suggested control actions on the set of items (addition, deletion, or replacement), on the set of agents, and partitioning cases. The goal is to perform a control action such that, for example, fairness is achieved. Hardness results with respect to checking the existence of fair allocations translate to hardness results with respect to control actions. The authors focused on envy-freeness and control actions on the set of items. They considered ordinal preferences over single goods and the responsive set extension. For an unbounded number of agents, the control actions are NP-hard. This follows from previous results by Bouveret et al. [2010]. For two agents, the problem of adding or deleting the minimum number of items can be decided in polynomial time. Moreover, the authors proved that deciding whether a complete envy-free allocations exists is NP-complete even for three agents. Therefore, the problems of performing the corresponding control actions are NP-hard.

Related Models

Apart from models with homogeneous divisible goods (see, e.g., the works by Feldman and Kirman [1974], Varian [1974], and Pazner and Schmeidler [1978]), related models are, for example, housing markets (see, e.g., the papers by Shapley and Scarf [1974], Roth and Postlewaite [1977], Abdulkadiroğlu and Sönmez [1998], Roth [1982], Pápai [2007], Sönmez [1999], Sonoda et al. [2014], Fujita et al. [2015], and Damamme et al. [2015]), where agents already own a good and perform trades; the division of a heterogeneous infinitely divisible good (see, e.g., the papers by Steinhaus [1948, 1949], Dubins and Spanier [1961], Aziz and Mackenzie [2016] and the works by Brams and Taylor [1996] and Procaccia [2016]); fairly allocating computing resources (see, e.g., the studies by Ghodsi et al. [2011], Gutman and Nisan [2012], and Parkes et al. [2015]); and combinatorial auctions (Cramton et al. [2006]).

Online problems where agents arrive over time were studied by Kash et al. [2014]. The case of goods arriving over time that have to be allocated immediately was modeled by Aleksandrov et al. [2015]. They considered a food bank problem where goods arrive at every time step. Agents then bid on this good. They proposed two mechanisms: The Like mechanism where goods are distributed uniformly among all agents that bid for that good and the Balanced Like mechanism that distributes goods uniformly among all agents that have received the fewest goods so far. They established that the Like mechanism is strategy-proof, but the Balanced Like mechanism can be manipulated. Then they studied the loss in social welfare when agents behave strategically.

Most of the mentioned procedures so far are centralized. A decentralized setting makes sense when agents have an initial endowment and can negotiate and perform trades. Building on a work by Sandholm [1998], Endriss et al. [2006] considered such a model with possible side payments. A key result by Sandholm [1998] is that any sequence of individually rational deals will eventually converge to an allocation that maximizes utilitarian social welfare. Note that the number of steps until convergence can become exponential in the number of goods (Endriss and Maudet [2005]). Endriss et al. [2006] showed that additive utility functions are sufficient to guarantee convergence when deals contain only one good. Chevaleyre et al. [2010] extended this result to modular utility functions and showed that this class is maximal. Endriss et al. [2006] also considered a setting without money. There, individual rationality is too strong. Agents have to agree to deals that are local Pareto improvements. Then Endriss et al. [2006] introduced equitable deals to show that any sequence of these deals will maximize egalitarian social welfare. Under stronger conditions, a Lorenz optimal condition can be reached. Instead of maximizing social welfare, Chevaleyre et al. [2017] looked for conditions that guarantee convergence to proportional or envy-free allocations by choosing appropriate payment schemes. Dunne et al. [2005] studied the computational complexity of decision problems concerning reachability under an individual rationality constraint.

Airiau and Endriss [2014] examined a similar model but assumed that resources are shareable. The utility function of an agent is the value that an agent derives from a bundle minus the sum of the delays that are incurred by sharing resources with other agents. They showed that the general convergence result on reaching an optimal allocation using only individually rational deals still holds. Moreover, the result for modular valuation functions and deals that involve one resource holds as well. They analyzed further restrictions on deal types and the existence of pure Nash equilibria. Other models of fair division of indivisible goods with money are by Svensson [1983], Quinzii [1984], Maskin [1987], Aragones [1995], Beviá [1998] and Gal et al. [2016]. Gal et al. [2016] studied the rent-division problem where agents receive exactly one good and money is involved. They proved that the maxmin solution subject to envy-freeness can be computed in polynomial time. Moreover, the maxmin solution implies equitability. In both theoretical and empirical analyses they showed (the potential) that the maxmin solution improves on social disparity (the utility difference between the best and worst-off agent). Furthermore, they performed a user study where a higher satisfaction rating was reported for the maxmin solution than for an arbitrary envy-free solution. Alkan et al. [1991] studied a model where there can be more goods than agents. They showed that a strongly envy-free allocation always exists.

Bouveret et al. [2017] considered a centralized setting with a constrained set of allocations using an item graph. Feasible bundles have to be connected in the underlying graph. Despite strong restrictions of the item graph, proportional and envy-free allocations are still hard to find. However, for trees, finding a max-min share allocation is tractable. In addition, the authors provided a small instance that admits no max-min share allocation.

Instead of requiring deterministic solutions, one can relax the assumption to randomized solutions. Usually, these are lotteries over deterministic solutions. Hylland and Zeckhauser [1979] introduced the competitive equilibrium from equal incomes solution in this context. A seminal work is by Bogomolnaia and Moulin [2001] who considered random assignment with strict preferences, introduced ordinal efficiency (which is between ex ante efficiency and ex post efficiency), and characterized it. Indeed, a random assignment is ex ante efficient for some von Neumann-Morgenstern utilities if and only if it is ordinally efficient (necessity was proven by McLennan [2002]). Bogomolnaia and Moulin [2001] proposed the probabilistic serial procedure, where agents "eat" probability shares of their best object at a given speed until it is eaten up. It is envy-free and ordinally efficient but only weakly strategy-proof, whereas random serial dictatorship (see, e.g., the work by Abdulkadiroğlu and Sönmez [1998]) is weakly envy-free and strategy-proof. However, both mechanisms become equivalent under certain conditions (Che and Kojima [2010], Liu and Pycia [2016]). Aziz et al. [2013a] and Saban and Sethuraman [2015] showed that computing the probability that an alternative (assignment) is chosen under random serial dictatorship is #P-complete, but Aziz and Mestre [2014] provided parameterized algorithms for computing these probabilities.

Bogomolnaia and Moulin [2001] gave an impossibility result that shows that there is no mechanism for at least four agents that is ordinally efficient, strategy-proof and satisfies equal treatment of equals. When von Neumann-Morgenstern utilities are elicited, ex ante efficiency, strategy-proofness, and equal treatment of equals are incompatible (Zhou [1990]). Allowing for indifferences, Katta and Sethuraman [2006] showed that there is a mechanism that is envy-free and ordinally efficient. Weak-strategy-proofness, however, is impossible in combination with envy-freeness and ordinal efficiency. Strategic behavior for the probabilistic serial procedure was studied by Aziz et al. [2015b]. They concluded that there is a pure Nash equilibrium and that best-responses can cycle. Checking whether a profile is in a pure Nash equilibrium is coNP-complete. Experiments showed that the social welfare of equilibria is not worse in most cases. Aziz et al. [2015c] analyzed manipulation from a single agent's perspective. When agents receive multiple goods under the probabilistic serial procedure, Kojima [2009] showed that ordinal efficiency and envy-freeness are still satisfied. An application of random assignment is to allocate unused classrooms to charter schools. Kurokawa et al. [2015] adapted the leximin mechanism by Bogomolnaia and Moulin [2004] to this setting and demonstrated that the adapted mechanism is proportional, envy-free, Pareto-optimal, and group-strategy-proof.

2.4 Hedonic Games

The theory of hedonic games is set in the field of cooperative game theory. Hedonic games form a subclass of nontransferable-utility games, where the outcome of each coalition is unique (see, e.g., the overview by Chalkiadakis et al. [2012]).

The idea of hedonic coalitions was introduced by Drèze and Greenberg [1980] in the context of public and private goods production. Hedonic games were formally modeled by Banerjee et al. [2001] and Bogomolnaia and Jackson [2002]. A hedonic game is a pair consisting of a set of agents and their preferences, where an agent's preferences are only over coalitions that contain this agent. Since there are exponentially many coalitions, compact representations are crucial. One idea is that each agent assigns a value to every other agent. Then an agent prefers coalition A to coalition B if the sum of the assigned values is greater for A than for B. Hedonic games where agents' preferences can be represented in this way are called additively separable hedonic games (Bogomolnaia and Jackson [2002]). Other restrictions that were studied are, for example, to lift preferences based on the best or worst player (Cechlárová and Romero-Medina [2001], Aziz et al. [2012]), to let every agent partition the set of agents into friends and enemies (Dimitrov et al. [2006]), to consider a partition into three categories (friends, neutral agents, enemies), where friends and enemies are ranked (Lang et al. [2015]), to use the average instead of the sum of the assigned values of additively separable hedonic games (Aziz et al. [2014]), and to focus on dichotomous preferences (Aziz et al. [2016b] and Peters [2016a]). While all previous representations restrict the preference domain, Elkind and Wooldridge [2009] introduced a complete representation of hedonic games.

Coalition structures (partitions of the set of agents) are usually judged in terms of stability. Typical stability notions are core stability (no coalition can deviate profitably) and Nash stability (no single agent can deviate profitably). One can also consider various strengthenings and weakenings. See, e.g., the paper by Sung and Dimitrov [2007] who gave a framework under which they studied stability notions. They introduced new stability notions such as the contractually strict core and showed that it always exists. Karakaya [2011] introduced strong Nash stability and the weak top-choice condition, which guarantees the existence of such a coalition structure.

A strict variant of strong Nash stability, which implies strict core stability and strong Nash stability, was studied by Aziz and Brandl [2012].

Existence results are one of the main research topics in hedonic games. Banerjee et al. [2001] found the (weak) top-coalition property, a sufficient condition for the existence of core-stable coalition structures. Bogomolnaia and Jackson [2002] studied such sufficient conditions as well. They related the conditions of ordinal balancedness and consecutiveness from NTU games to the top-coalition property. Regarding individual stability notions, they proved via a potential function argument that symmetric additively separable hedonic games always admit Nash-stable coalition structures. Another domain restriction was considered by Burani and Zwicker [2003]. They introduced descending separable preferences under which a core- and Nash-stable coalition structure always exists. When preferences are strict and based on the best player, a strictly core-stable coalition structure always exists (Cechlárová and Romero-Medina [2001]). For hedonic games with appreciation of friends and aversion to enemies, Dimitrov et al. [2006] showed that (strictly) core-stable coalition structures always exist for aversion to enemies (appreciation of friends). Top responsiveness is another condition and it is known to imply in conjunction with mutuality that a strictly strong Nash-stable coalition structure always exists (Aziz and Brandl [2012]). The existence of strategy-proof mechanisms for hedonic games was studied by Pápai [2004] and Rodríguez-Álvarez [2009].

Computational Aspects

The representation influences whether it is possible to decide efficiently that, e.g., the core is nonempty. For instance, Ballester [2004] studied the complexity of problems for an individual rational list encoding. Deciding whether there is a core-stable, Nash-stable, or individually stable coalition structure is NP-complete for strict preferences. Sung and Dimitrov [2010] proved that for additively separable hedonic games deciding whether there is a (strictly) core-stable coalition structure is NP-hard. These results were improved to Σ_2^p-hardness for core stability by Woeginger [2013b] and for strict core stability by Peters [2017]. NP-hardness holds for individual and Nash stability (see the work by Olsen [2009]) as well. Although it is known that a contractually individually stable coalition structure always exists (Ballester [2004]), it is not clear that such a coalition structure can be found in polynomial time. For instance, Gairing and Savani [2010] showed that finding a Nash-stable coalition structure under symmetric additively separable hedonic games is PLS-complete. In contrast, Aziz et al. [2013c] gave a polynomial-time algorithm for finding a contractually individually stable coalition structure in additively separable hedonic games. Dimitrov et al. [2006] proved that finding a core-stable coalition structure in hedonic games of aversion to enemies is NP-hard. For appreciation of friends, the problem is polynomial-time solvable even for strict core stability. In fact, the stronger stability notions of wonderful stability and strict core stability lead to DP-hard existence problems under aversion to enemies (Rey et al. [2016]).

Allowing for ties in the preferences can make existence problems harder. For example, if preferences are based on the best agent, deciding whether there is a (strictly) core-stable coalition structure becomes NP-complete (Cechlárová and Hajduková [2003]). For fractional hedonic games, where preferences are based on the average value of a coalition, Aziz et al. [2014] showed that the existence and verification problem for core stability is NP-hard and coNP-complete, respectively. Under the assumption of simple symmetric preferences, a core-stable coalition structure always exists for certain graph classes. Results on individual-based stability were obtained by Brandl et al. [2015]. They established that the existence problem is NP-hard with respect to Nash stability and individual stability. For hedonic coalition nets, the complete representation by Elkind and Wooldridge [2009], equivalence problems and checking whether a coalition structure is in the core or whether the core is non-empty is intractable. However, Elkind and Wooldridge [2009] also showed that there is a pseudo-polynomial-time algorithm for checking core-membership when the agent graph has bounded treewidth.

The most general results on NP-hardness in hedonic games were obtained by Peters and Elkind [2015] who gave conditions on the expressiveness of preferences which are sufficient for NP-hardness of existence problems. For so-called graphical hedonic games, where agents' preferences only depend on the neighbors in some graph, Peters [2016b] showed that there is a linear-time algorithm to decide whether, e.g., a core-stable coalition structure exists when the graph has bounded treewidth and bounded degree. He also described how this can be applied to fair division problems.

When preferences are dichotomous, finding a core-stable coalition structure is FNP-hard (Peters [2016a]), although a core-stable coalition structure always exists (Aziz et al. [2016b]). Peters [2016a] considered welfare maximization as well, which is another approach to solving hedonic games, and showed that it is a hard problem. This problem was studied by Aziz et al. [2013c] for additively separable hedonic games, too. They demonstrated that the maximization problem is NP-hard for utilitarian and egalitarian social welfare. Similarly, checking whether a given coalition structure is a maximizer is coNP-complete. Moreover, they proved that it is NP-complete to decide whether there is a coalition structure that is envy-free and satisfies Nash stability. Considering Pareto optimality together with envy-freeness makes the problem Σ_2^p-complete. Here, envy-freeness is defined by replacing players. In contrast, in Chapter 6 we define envy-freeness by joining a coalition. The approximability of welfare maximization was investigated by Aziz et al. [2015a]. For fractional hedonic games, the problem is NP-hard for utilitarian social welfare, egalitarian social welfare, and Nash social welfare. However, it can be approximated within a constant-factor when preferences are from $\{0, 1\}$ and symmetric.

Aziz et al. [2013b] analyzed Pareto optimality in the context of hedonic games. They established a connection between perfect and Pareto-optimal coalition structures. If it is hard to find a perfect coalition structure, then it is also hard to find a Pareto-optimal coalition structure. Moreover, they found a connection of these two properties in terms of relaxations of profiles. They leveraged this connection to give an algorithm, which always returns an individually rational and Pareto-optimal coalition structure, where the running time depends on the time to find a perfect coalition structure.

Price of Stability

Another research topic is to study the ratio of optimal social welfare and, for example, maximum social welfare under the constraint of Nash stability. Intuitively, this is the smallest multiplicative loss in social welfare incurred by a stability requirement. Bilò et al. [2014, 2015] considered the price of stability for fractional hedonic games. For general graphs, it is at least 2. However, the (upper) bounds can be improved under additional restrictions, e.g., for triangle-free graphs and for bipartite graphs. In the context of coalitional affinity games, which are equivalent to additively separable hedonic games, Brânzei and Larson [2009] examined the price of core stability as well. Elkind et al. [2016] studied the price of Pareto optimality. For symmetric additively separable hedonic games, it is one if the graph is acyclic. However, for symmetric fractional hedonic games, they showed that the price can be bounded quadratically in the maximum degree of the graph if the graph is unweighted. For trees, the bound becomes linear in the maximum degree. In addition, they considered modified fractional hedonic games (the average is computed differently), where they proved a constant upper bound if the graph is a tree.

Special Cases and Related Models

Since the model of hedonic games is quite general, it subsumes, e.g., the model of matching and fair division of indivisible goods. Another special case was studied by Darmann et al. [2012], which is labeled as the group activity selection problem where agents have preferences over activities and the number of participants. They showed hardness results for deciding whether an instance has a perfect assignment under various restrictions. For even stronger restrictions such as bounding the number of activities by a constant, they proved tractability results. Although in general a Nash-stable assignment does not always exist, under, for example, increasing preferences its existence can be guaranteed. Igarashi et al. [2017] studied this problem on social networks. Wright and Vorobeychik [2015] considered team formation problems, which they modeled as additively separable hedonic games with cardinality constraints on coalition sizes. They adapted random serial dictatorship and the draft mechanism and proposed two other mechanisms, one of which is experimentally good, the other having desirable theoretical guarantees. Then they compared these mechanisms with respect to social welfare and fairness. Although they adapted fairness notions from fair division of indivisible goods as we do in Chapter 6, their results are different because of the cardinality constraints. Related to Chapter 7 is the work by Brânzei and Larson [2011] who studied social distance games. Agents' preferences are determined by their proximity on a graph. They gave a ($1/2$)-approximation algorithm for optimal social welfare and studied the price of stability. Igarashi and Elkind [2016] also considered hedonic games on graphs. The key difference is that coalitions have to form connected subgraphs. Other related papers are by Feldman et al. [2015], who studied hedonic clustering games from the perspective of noncooperative game theory and by Bloch and Diamantoudi [2011], who focused on bargaining aspects.

3 Positional Scoring-Based Allocation of Indivisible Goods

This chapter deals with fair division of indivisible goods using ordinal preferences, but it also contains various aspects of social welfare optimization by using additive representations of ordinal preferences.

3.1 Summary

We studied scoring allocation rules, generalizing prior work by Brams et al. [2003]. Let N be the set of n agents and let G be the set of m indivisible goods. Each agent submits a linear order $>_i$ over the set of goods to a scoring allocation rule or scoring allocation correspondence. Rules map a profile $> = (>_1, \dots, >_n)$ to a complete allocation and correspondences map a profile to a nonempty set of complete allocations. Scoring allocation rules and correspondences are parameterized by an aggregation function and by a scoring vector $s = (s_1, \dots, s_m)$ that satisfies $s_1 \geq s_2 \geq \dots \geq s_m \geq 0$ and $s_1 > 0$. They then cardinalize each linear order $>_i$ to an additive utility function u as follows:

$$u_{>_i,s}(B) = \sum_{b \in B} s_{rank(b,>_i)},$$

where B denotes a bundle and $rank(b, >_i)$ is the position of good b in linear order $>_i$. Thus, agent i receives s_j points if a good b is ranked at position j. We considered scoring allocation rules and correspondences for example with the Borda scoring vector family $(m, m-1, \dots, 1)$ and the lexicographic scoring vector family $(2^{m-1}, 2^{m-2}, \dots, 1)$. While the absolute difference between consecutive entries is 1 for a Borda scoring vector, the entries decrease exponentially under a lexicographic scoring vector. Therefore, the utility function induced by a lexicographic scoring vector values goods better than all goods together that are ranked worse. These utility functions may not represent agents' underlying preferences correctly (but the choice can be facilitated by domain expertise or data). Alternatively, the induced utility functions can be interpreted as an external perception of agents' preferences. Sensible choices of aggregation functions are collective utility functions or the leximin preorder. We analyzed scoring allocation correspondences that return allocations that maximize

$$\sum u_{>_i,s}(\pi_i),$$

or

$$\min u_{>_i,s}(\pi_i),$$

where π_i denotes the share of agent i, or that return allocations whose utility vector maximizes the leximin preorder. By using tie-breaking relations we studied the corresponding scoring allocation rules as well. We focused on three aspects of scoring allocation rules and correspondences, axiomatic properties, computational complexity of winner determination, and approximability.

Separability is a property of reducing dependencies among agents so that winner determination can be performed in a distributed manner. It requires that agents' bundles do not change, even if other agents and their assigned bundles are removed. However, this property is not satisfied by most rules and correspondences. Moreover, we studied monotonicity and global monotonicity. Monotonicity says that if agents rank a good that they receive higher ceteris paribus, then they still receive that good. All of our studied scoring allocation rules satisfy monotonicity. Global monotonicity asks that agents still receive the same bundle after the change. Unfortunately, this property is not satisfied for strictly decreasing scoring vectors. Regarding variable properties, a scoring allocation rule is object monotonic if adding a good to the sets of goods does not make any agent worse off. Note that we only have linear orders over single goods as information. Therefore, we considered possible and necessary modality. For strictly decreasing scoring vectors, none of the studied rules satisfy this property. Yet, the scoring allocation rule parameterized by utilitarian social welfare does satisfy the property for two agents and under a condition on the tie-breaking relation. The last property that we examined was duplication monotonicity. If an agent is duplicated with the same preferences, then the combined bundle of the original agent and the duplicated agent should not be worse than the bundle that was assigned before the duplication. For so-called duplication-compatible tie-breaking relations, scoring allocation rules with utilitarian social welfare satisfy this property. This also holds when using leximin and specific scoring vector families.

In terms of computational complexity, we analyzed two decision problems. The first problem is deciding whether an allocation is optimal with respect to some aggregation function. The second problem asks whether a given value of social welfare is attainable for some allocation. The value problem is in P for utilitarian social welfare and NP-complete for the lexicographic and Borda scoring vector for egalitarian social welfare and leximin. When the number of agents is constant, the problem is decidable in polynomial time for the Borda scoring vector. For the first problem of checking optimality, we had similar results.

By finding an appropriate maximum matching and greedily assigning the remaining goods, there is a ($1/2$)-approximation for the lexicographic scoring vector under egalitarian social welfare. Then we investigated the price of picking-sequence elicitation-freeness, which is the worst-case ratio of optimal social welfare and the social welfare of the sequential allocation. For regular policies and utilitarian social welfare, the price is at least $1 + {n-1}/m + \Theta(1/m^2)$ when m goes to infinity. Empirically, for both utilitarian and egalitarian social welfare, it seems that the price tends to one as the number of goods becomes large. The price is upper bounded by $2 - 1/n + \Theta(1/m)$ for both cases when the number of goods goes to infinity.

3.2 Publication – Baumeister et al. [2017]

Citation: D. Baumeister, S. Bouveret, J. Lang, N. Nguyen, T. Nguyen, J. Rothe, and A. Saffidine. Positional scoring-based allocation of indivisible goods. *Autonomous Agents and Multi-Agent Systems*, 31(3):628–655, 2017. doi: 10.1007/s10458-016-9340-x. URL `https://doi.org/10.1007/s10458-016-9340-x`

A preliminary version appeared in conference proceedings (Baumeister et al. [2014]).

4 Strategy-Proofness of Scoring Allocation Correspondences for Indivisible Goods

Chapter 3 introduced scoring allocation correspondences for ordinal preferences. We extend this model to study strategy-proofness using an approach via set extensions.

4.1 Summary

We assumed that agents have responsive preferences over bundles of goods, but submit only linear orders, as in Chapter 3, over single goods to the social planner. The main focus is on scoring-based allocation correspondences that maximize utilitarian social welfare. Outcomes of allocation correspondences are sets of allocations. Since multiple allocations may maximize social welfare, agents' preferences over bundles of goods are no longer sufficient to compare outcomes. Assuming no externalities, agents need to compare sets of bundles. Therefore, we used set extensions from social choice theory to facilitate these comparisons:

Let $\geq$ be a weak order over bundles of goods and A and B be two sets of bundles. The Kelly extension (Kelly [1977]) of $\geq$ is defined as follows:

$$A \succeq^K B \iff \forall x \in A \; \forall y \in B: \; x \geq y.$$

A weaker set extension is by Gärdenfors (Gärdenfors [1976]):

$$A \succeq^G B \iff (1) \text{ or } (2) \text{ or } (3),$$

where (1) is

$$A \subset B \text{ and } \forall x \in A \; \forall y \in B \setminus A: \; x \geq y,$$

(2) is

$$B \subset A \text{ and } \forall x \in A \setminus B \; \forall y \in B: \; x \geq y,$$

and (3) is

$$A \not\subset B \text{ and } B \not\subset A \text{ and } \forall x \in A \setminus B \; \forall y \in B \setminus A: \; x \geq y.$$

Based on these set extensions, we defined Kelly- and Gärdenfors-strategy-proofness of an allocation correspondence as the nonexistence of profitable misrepresentations of preferences. A misrepresentation is profitable if the outcome is preferred with respect to the Kelly and Gärdenfors set extension, respectively.

We showed that scoring-based allocation correspondences that maximize utilitarian social welfare are Gärdenfors-strategy-proof if there at most two different values in the scoring vector or the largest value in the scoring value appears for more than half the number of goods. We proved this statement by showing that any scoring-based allocation correspondence that satisfies a set of properties is Gärdenfors-strategy-proof.

4.2 Publication – Nguyen et al. [2018]

Citation: N. Nguyen, D. Baumeister, and J. Rothe. Strategy-proofness of scoring allocation correspondences for indivisible goods. *Social Choice and Welfare*, 50(1):101–122, 2018. doi: 10.1007/s00355-017-1075-3. URL `https://doi.org/10.1007/s00355-017-1075-3`

A preliminary version appeared in conference proceedings (Nguyen et al. [2015]).

4.3 Corrigendum

Errors occurred in Example 2, page 7. We give the whole example (changes are in bold):

Recall profile $> = (>_1, >_2)$ from Example 1:

$$\begin{aligned} >_1: &\quad e\,a\,b\,d\,c, \\ >_2: &\quad d\,a\,b\,c\,e, \end{aligned}$$

where we have

$$F_{\text{borda}}(>) = \{(\{a,b,e\},\{c,d\}),(\{b,e\},\{a,c,d\}),(\{a,e\},\{b,c,d\}),(\{e\},\{a,b,c,d\})\}$$

for the Borda scoring vector. So $F_{\text{borda}}(>)_2 = \{\{c,d\},\{a,c,d\},\{b,c,d\},\{a,b,c,d\}\}$. Now consider profile $>' = (>_1, >'_2)$ after agent 2's manipulation:

$$\begin{aligned} >_1: &\quad e\,a\,b\,d\,c, \\ >'_2: &\quad a\,b\,\boldsymbol{d}\,\boldsymbol{e}\,\boldsymbol{c}. \end{aligned}$$

Then we have

$$F_{\text{borda}}(>') = \{(\{\boldsymbol{c},e\},\{a,b,\boldsymbol{d}\}),(\{e\},\{a,b,c,d\})\}.$$

Hence, $F_{\text{borda}}(>')_2 = \{\{a,b,\boldsymbol{d}\},\{a,b,c,d\}\}$. It follows that

$$F_{\text{borda}}(>')_2 \not\succ_2^K F_{\text{borda}}(>)_2$$

because $\{a,b,c,d\} \succ_2 \{a,b,\boldsymbol{d}\}$ (by responsiveness), but

$$F_{\text{borda}}(>')_2 \succ_2^G F_{\text{borda}}(>)_2$$

because $\{a,b,\boldsymbol{d}\}$ is preferred to $\{c,d\}$, $\{b,c,d\}$, and $\{a,c,d\}$ (see Figure 2).

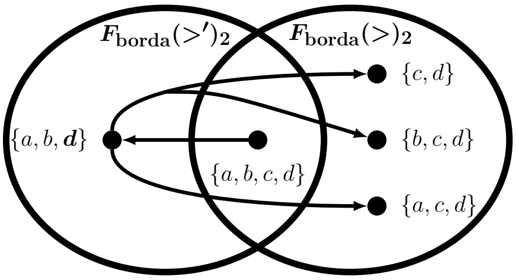

Figure 2: Illustration of the Gärdenfors extension for Example 2. An arrow expresses weak preference (e.g., the arrow from node $\{a,b,\boldsymbol{d}\}$ to node $\{c,d\}$ indicates that $\{a,b,\boldsymbol{d}\}$ is weakly preferred to $\{c,d\}$).

5 Fairness and Rank-Weighted Utilitarianism in Resource Allocation

In this chapter we consider cardinal preferences and work with both fairness criteria and collective utility functions.

5.1 Summary

We studied the hierarchy proposed by Bouveret and Lemaître [2016] for k-additive utility functions, which are able to capture synergies between goods with increasing k. Let N be the set of agents and G the set of indivisible goods. A utility function maps subsets of G to the rational numbers. We say a utility function is k-additive if there are weights $w(H)$ for each bundle H with $|H| \leq k$ such that

$$u(B) = \sum_{C \subseteq B, |C| \leq k} w(C),$$

for every $B \subseteq G$. The scale by Bouveret and Lemaître [2016] was originally studied for 1-additive utility functions. It says that competitive equilibrium from equal incomes, envy-freeness, min-max share, proportionality, and max-min share form a linear scale. They are defined as follows. The max-min share of an agent i is

$$\max_{\pi} \min_{j \in N} u_i(\pi_j).$$

The proportional share of an agent i is

$$\frac{1}{|N|} u_i(G).$$

The min-max share of agent i is

$$\min_{\pi} \max_{j \in N} u_i(\pi_j).$$

An allocation is a max-min/proportional/min-max share allocation if every agent receives at least their max-min/proportional/min-max share. A competitive equilibrium from equal incomes is pair consisting of an allocation and a price vector $p \in [0, 1]^{|G|}$ such that for every agent i

$$\pi_i \in \arg\max_{B \subseteq G} \left\{ u_i(B) \middle| \sum_{b \in B} p_b \leq 1 \right\}.$$

For k-additive utility functions, $k \geq 2$, the hierarchy no longer holds. For instance, it may be the case that an allocation satisfies min-max share but not proportionality. However, competitive equilibrium from equal incomes still implies envy-freeness, which implies min-max share.

Bouveret and Lemaître [2016] left open the complexity of the decision problem that asks whether a given instance admits a min-max share allocation. We provided a partial answer and showed that the problem is NP-hard. They also wondered whether envy-freeness and Pareto optimality implies competitive equilibrium from equal incomes when agents have additive utility functions that have distinct values for every bundle because the converse is known to be true. We gave a counterexample and showed that this direction does not hold.

Then we analyzed rank-weighted utilitarian social welfare which is defined as

$$\sum_{i \in N} w_i v_i^*(\pi),$$

where $w = (w_1, \ldots, w_n)$ is a normalized weight vector and $v^*(\pi) = (v_1^*(\pi), \ldots, v_n^*(\pi))$ is the nondecreasingly sorted utility vector under allocation π. For example, $v_1^*(\pi)$ denotes the utility of the worst-off agent. When the weight vector has a 1-entry at position j and 0 everywhere else, we have the special case of a j-rank dictator function. Therefore, a 1-rank dictator function corresponds to egalitarian social welfare. We studied the approximability for constant j and show that an $f(|N|, |G|)$-approximation for optimizing the $(j+1)$-rank dictator implies an $f(|N|+1, |G|)$-approximation for optimizing the j-rank dictator. A similar result also holds for the optimization of the median-rank dictator. Moreover, hardness results for the j-rank dictator imply the same hardness results for maximizing egalitarian social welfare. Both results show that the problems are closely related and approximation algorithms for maximizing egalitarian social welfare are also applicable to rank dictator problems. When j is not a constant, j-rank dictator also inherits the hardness from the problem of maximizing egalitarian social welfare.

For strictly decreasing weight vectors, we showed that the decision problem whether there is an allocation where the rank-weighted social welfare is at least some given value is strongly NP-complete when the number of agents is not fixed, even for identical utility functions. Therefore, there is no FPTAS for the optimization problem, unless P = NP.

In the last section we reported on computational experiments to study in an exploratory manner how rank-weighted utilitarianism is connected to the max-min share. For strictly decreasing weight vectors, most maximizers also satisfied the max-min share. For other weight vectors, we saw that the number of max-min share maximizers decreased.

5.2 Publication – Heinen et al. [2015]

Citation: T. Heinen, N. Nguyen, and J. Rothe. Fairness and rank-weighted utilitarianism in resource allocation. In *Proceedings of the 4th International Conference on Algorithmic Decision Theory*, pages 521–536. Springer-Verlag *Lecture Notes in Artificial Intelligence #9346*, September 2015. doi: 10.1007/978-3-319-23114-3_31. URL https://doi.org/10.1007/978-3-319-23114-3_31

6 Local Fairness in Hedonic Games via Individual Threshold Coalitions

This chapter transfers ideas from fair division of indivisible goods to hedonic games.

6.1 Summary

Let N be the set of agents and denote by $\succeq_i$ agent i's preference relation. We introduced local fairness notions for coalition structures that require that every agent i is in a coalition that is at least as good as $f(\succeq_i)$, where f is a suitable function from a preference relation to a coalition. For example, for $f(\succeq_i) = \{i\}$, we have individual rationality. We studied choices of f that come from fair division of indivisible goods, namely, the max-min, the proportional, and the min-max share. Formally the min-max threshold of agent i is

$$\min_{\pi \in \Pi(N \setminus \{i\})} \max_{C \in \pi \cup \{\emptyset\}} C \cup \{i\},$$

where $\Pi(X)$ denotes the set of all partitions of X and the minimum/maximum is taken with respect to $\succeq_i$. This corresponds to a setting where an agent has no say in the coalition formation process, but can join any coalition at the end. The grand-coalition threshold of agent i is

$$\max\{\{i\}, N\}.$$

Since the grand coalition contains all agents, it can be seen as an average. The max-min threshold of agent i is

$$\max_{\pi \in \Pi(N \setminus \{i\})} \max\{\{i\}, \min_{C \in \pi} C \cup \{i\}\}.$$

This threshold corresponds to the case where an agent is allowed to choose the coalition structure but not the final coalition. Given these thresholds we can say that a coalition structure is min-max, grand-coalition, or max-min fair if every agent is in a coalition that is at least as good as the corresponding threshold.

We showed that these fairness notions form a hierarchy between the notion of perfect coalition structures and individual rationality: Max-min fairness implies grand-coalition fairness, and grand-coalition fairness implies min-max fairness, which is also implied by Nash stability. There are no other connections of this kind to the remaining stability (e.g., core stability) and social welfare notions (e.g., egalitarian social welfare) that we examined in this work.

We continued the study of the fairness notions for additively separable hedonic games. In this setting grand-coalition and max-min fairness coincide. In terms of computational complexity, in addition to existence problems, we studied threshold problems that ask whether the threshold is at least a given value. For min-max fairness, the threshold and existence problems are intractable, but there are some special cases, e.g., non-negative valuation functions or symmetric additively separable hedonic games, for which the existence problem is easy. Under grand-coalition and max-min fairness only the existence problem is NP-hard because the threshold problems are in P. Whereas we showed weak coNP-completeness for the min-max fairness threshold problem and weak NP-completeness for the grand-coalition and max-min fairness existence problems, one can show that these problems are also intractable when numbers are encoded in unary.

Then we analyzed the maximum and minimum price of fairness with respect to min-max fairness in symmetric additively separable hedonic games. For the maximum price, we obtain an upper bound of $|N| - 1$, whereas the minimum price is one.

6.2 Publication – Nguyen and Rothe [2016]

Citation: N. Nguyen and J. Rothe. Local fairness in hedonic games via individual threshold coalitions. In *Proceedings of the 15th International Conference on Autonomous Agents and Multiagent Systems*, pages 232–241. IFAAMAS, May 2016. URL `http://dl.acm.org/citation.cfm?id=2936961`

7 Altruistic Hedonic Games

This chapter considers a model of hedonic games with social preferences.

7.1 Summary

A problem of the friend-oriented extension in hedonic games is that an agent may be indifferent between two coalitions: Consider a graph that consists of a path of three agents and an isolated agent. Then an agent that is at the end of the path is indifferent between the path and the coalition that contains the neighbor and the isolated agent. This is counterintuitive because the path is connected and so the neighbor's friend is also in the coalition. We distinguished three degrees of altruism. Let

$$v_i(A) = |N| \cdot |A \cap F_i| - |A \cap E_i|$$

be the additive representation of the friend-oriented extension, where A is a coalition, F_i denotes agent i's set of friends and E_i agent i's set of enemies. Selfish-first preferences are defined as

$$A \succeq_i^{SF} B \iff Mv_i(A) + \sum_{a \in A \cap F_i} \frac{v_a(A)}{|A \cap F_i|} \geq Mv_i(B) + \sum_{b \in B \cap F_i} \frac{v_b(B)}{|B \cap F_i|},$$

where M is a large value. Equal-treatment preferences are defined as

$$A \succeq_i^{EQ} B \iff \sum_{a \in A \cap (F_i \cup \{i\})} \frac{v_a(A)}{|A \cap F_i| + 1} \geq \sum_{b \in B \cap (F_i \cup \{i\})} \frac{v_b(B)}{|A \cap F_i| + 1}.$$

Altruistic-treatment preferences are defined as

$$A \succeq_i^{AL} B \iff v_i(A) + M \sum_{a \in A \cap F_i} \frac{v_a(A)}{|A \cap F_i|} \geq v_i(B) + M \sum_{b \in B \cap F_i} \frac{v_b(B)}{|B \cap F_i|}.$$

We showed that these preference types capture the aforementioned intuition and that they are incomparable to representations from the literature. In addition, these preference types satisfy desirable properties like weak friend-orientedness, symmetry, and friend-oriented unanimity. For selfish-first preferences, we showed that monotonicity properties are satisfied as well. Stability-wise, a Nash-stable coalition structures always exists for all three preference types. For selfish-first preferences, a strictly core-stable coalition structure is guaranteed as well.

7.2 Publication – Nguyen et al. [2016]

Citation: N. Nguyen, A. Rey, L. Rey, J. Rothe, and L. Schend. Altruistic hedonic games. In *Proceedings of the 15th International Conference on Autonomous Agents and Multiagent Systems*, pages 251–259. IFAAMAS, May 2016. URL `http://dl.acm.org/citation.cfm?id=2936963`. An adapted version also appears in the proceedings of the 6th International Workshop on Computational Social Choice, 2016

7.3 Corrigendum

- Proposition 7 is not a counterexample for equal-treatment preferences.
- Under Proposition 17 it should say "trees with more than three vertices do not allow a perfect coalition structure."

8 Conclusion

We have studied fairness when allocating indivisible goods and in hedonic games. In Chapter 3 we have proposed a class of allocation rules/correspondences that are simple and satisfy acceptable properties such as monotonicity. We have studied the computational complexity of said allocation rules and their performance in sequential allocation settings. The general picture is that for a constant number of goods or agents determining optimal allocations is easy, but without any restrictions it is hard for egalitarian social welfare and the leximin preorder. These negative results are a by-product of the cardinalization and should be seen in light of the simplicity of the rules and correspondences. In Chapter 4 we have examined strategy-proofness in the model of Chapter 3. Using the Kelly and Gärdenfors set extension, we characterize Kelly- and Gärdenfors-strategy-proofness for scoring-based allocation correspondences that maximize utilitarian social welfare. This is the case when the number of different values is at most two or the number of occurrences of the largest value in the scoring vector is larger than half the number of goods. As future work it would be interesting to have characterizations for other properties. Likewise, the study of strategy-proofness with respect to correspondences could also be fruitful in other fields of research. Experimentally comparing ordinal fair division procedures with cardinal fair division procedures in lab and real-world settings could shed further insight into bringing fair division into practice.

In Chapter 5 we have focused on fairness properties for k-additive utility functions, have given a partial answer to a complexity question, and have provided a counterexample to a connection between Pareto optimality, envy-freeness and competitive equilibrium from equal incomes, both raised by Bouveret and Lemaître [2016]. The approximability of rank dictator functions is similar to the approximability of egalitarian social welfare. Progress on one front would entail improvements at the other front. For rank-weighted utilitarianism, we have shown that there is no FPTAS and studied the compatibility between boolean fairness criteria (in the form of max-min share) and rank-weighted utilitarianism. For inequality-reducing weight vectors, we have found that compatibility is high. The broad scope of the paper should be contrasted to progress it makes on many fronts, complexity of min-max share allocations, relationship of CEEI allocations, and approximability of more general collective utility functions. Possibly interesting research questions would be to find a relaxation of the fairness hierarchy for k-additive utility functions (maybe parameterized by k). The complexity of the existence problem for max-min share is still open (it is only settled under the assumption that there is a max-min share allocation). So is the exact complexity for min-max share. Since there is no FPTAS for rank-weighted utilitarianism, even if agents have identical utility functions, it would be nice to have at least a PTAS for optimizing this collective utility function.

Chapter 6 has introduced local fairness notions that depend only on an agent's individual preferences. We have considered fairness notions from fair division of indivisible goods and have shown that there is a hierarchy as in the work by Bouveret and Lemaître [2016], but this hierarchy holds for arbitrary preferences. We have studied the complexity of determining thresholds and the existence of fair coalition structures for additively separable hedonic games and have given results on the price of fairness. Although these fairness notions do not always exist in general, they form a hierarchy and we have identified special cases where existence is guaranteed. One could also consider other fairness notions from settings different from fair division. Also, the fairness notions in Chapter 6 do not translate to matching settings. The exact complexity of checking whether a min-max fair coalition structure exists is open. It might also be worthwhile to find a fairness notion that is always guaranteed to exist (as in EF-1 in the work by Caragiannis et al. [2016]).

Altruistic hedonic games have been introduced in Chapter 7. The key idea is to incorporate the preferences of an agent's friends into their own preferences. We have distinguished three degrees of social preferences. In the selfish case, the original preferences are used until there is an indifference. Then the average opinion of the friends is employed as a tie-breaker. Similarly, altruistic-treatment preferences use the average preference first and the original preference as a tie-breaker. Equal-treatment preferences give equal weight to friends' preferences and an agent's original preferences. We have proposed properties and have shown that these preference types are incomparable to previously studied representations. Nash-stable coalition structures always exist and we have studied the complexity for other notions. As future work finding sufficient conditions that guarantee the existence of stable coalition structures in all three settings is important. One can also consider price of stability type of results.

For other possible future research directions, see Aziz [2016], Walsh [2016] and Aziz et al. [2017b]. In general it seems to be fruitful to take inspiration from other fields and work inter- and intradisciplinary. The study of fairness in other settings can take various forms as a constraint, as an objective, or by building other-regarding aspects into the model. An important research question is to better understand what fairness means to human beings. Although envy-freeness as a fairness notion has its merits because there is no interpersonal comparison and it is ordinal in nature, research suggests that it is not the first criterion that humans use to make fairness judgments. Finding a definition that is grounded both theoretically and empirically is highly relevant. Another concern which is largely neglected is procedural fairness. Most of the theoretical literature deals with outcome fairness, but when bringing fair division procedures into practice it is also important that people are treated fairly during the process. A different research question is considering social preferences in fair division of indivisible goods. How does it relate to fairness? In this case, intuitively, in fair division there should be more fair outcomes; in hedonic games there should be more stable outcomes. Subclasses of hedonic games such as matching have found many application areas. Finding more application areas for general hedonic games could give this area of research new challenges.

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9 Contribution

- Baumeister et al. [2014] and Baumeister et al. [2017]

  The model and writing was done jointly with my coauthors. Specifically (referring in the following to the paper “Positional scoring-based allocation of indivisible goods” (Baumeister et al. [2017]), which subsumes Baumeister et al. [2014]), the counterexample that plurality is not separable (Example 4), the study on global monotonicity, Proposition 3, and the correctness proofs of Theorem 6 and Proposition 5 are to be attributed to my contribution.

- Nguyen et al. [2018]

  The model and writing was done jointly with my coauthors. Parts of this work already appeared in a paper by the same authors (Nguyen et al. [2015]) and in my Master’s thesis. Contributions that were done for the doctoral thesis are an improved motivation, presentation, and discussion, new examples, a rewritten proof for Proposition 1, the introduction of new properties in Definition 6 and a new proof of the main result based on them in Lemma 3 and Lemma 4.

- Heinen et al. [2015]

  Writing was done jointly with my coauthors. Specifically, I adapted the counterexamples so that k is minimal (Example 1 and Example 2). Proposition 5, the study on approximability, and the computational experiments are to be attributed to my contribution.

- Nguyen and Rothe [2016]

  Writing was done jointly with Jörg Rothe. Modeling and technical parts are to be attributed to my contribution.

- Nguyen et al. [2016]

  The model and writing was done jointly with my coauthors. Specifically, Theorem 1, Theorem 2, and the study on expressiveness in comparison to other preference types are to be attributed to my contribution.