# **Essays on Industrial Organization**

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# Chapter 1

# Introduction

In this thesis, I present four papers in two different areas of industrial organization. First, we study a model with boundedly rational consumers. In many markets, at least some share of consumers shows different behavioral biases while making their decisions. As the firms are also aware of these biases, their marketing strategies often aim to exploit naive consumers. Understanding the differences in consumers' behaviors is very important for designing consumer protection policies. There exist a growing literature which analyze the impact of consumer naivety on market prices and consumer welfare. The second chapter of this dissertation contributes to this literature.

Chapter 2 is titled "Advertising and Price Competition in Online Markets." This chapter studies how firms' advertising and pricing strategies, as well as consumer surplus, depend on consumers' search behavior in online markets. Nowadays, online retailers and price comparison websites provide lists of prices for many products. Many (if not most) of these platforms allow advertisements on their sites and give prominent positions to the advertised firms in the search results. We consider a sequential search model in which all prices are observable and consumers look for products which meet their needs. Consumers are different in their search orders. A group of "sophisticated consumers" always searches products in an increasing price order, while "naive consumers" start their search from the advertised products. We show that if the share of naive consumers is relatively low, only one firm advertises in the market. Otherwise, both firms advertise. Advertisements increase market prices, but the prices are the highest if the firms adopt asymmetric advertising strategies, *i.e.*, when only one firm advertises. We also show that an increase in the share of naive consumers can decrease market prices and increase consumer surplus depending on their existing share. In an extension of the main model, we introduce a positive fraction of shoppers who have zero search costs. We show that depending on the level of search costs, all consumers may benefit, suffer, or not be affected from the existence of naive consumers. This study suggests that regulating online retailers' advertising policies, which can influence the share of naive consumers, can improve consumer welfare.

In the second part of this dissertation, we analyze different aspects of vertically related markets. First, we re-visit Nash bargaining problem between an upstream supplier and a downstream manufacturer and derive a simple and instructive formula for profit-sharing rule. In Chapter 4, we analyze downstream firms' merger and FDI incentives when they bargain with plant-specific input suppliers. We focus on the role of platform (modular) production concept which allows the firms to produce different product variants at all plants they own. In Chapter 5, we consider a sequential bargaining between a downstream firm and two upstream suppliers. We study the upstream firms' merger incentives and socially desirable market structure depending on the substitutablity of the inputs.

Chapter 3 is a joint work with Markus Dertwinkel-Kalt and Christian Wey and titled "The Nash Bargaining Solution in Vertical Relations With Linear Input Prices.". We re-examine the Nash bargaining solution when an upstream and a downstream firm bargain over a linear input price. We show that the profit sharing rule is given by a simple and instructive formula which depends on the parties' disagreement payoffs, the profit weights in the Nash-product, and the derived demand elasticity. A downstream firm's profit share increases in the equilibrium derived demand elasticity which in turn depends on the final goods' demand elasticity. We show by example that the total profit of the downstream firm can increase if the final demand becomes more elastic. Our simple formula generalizes to bargaining with N downstream firms when bilateral contracts are unobservable.

Chapter 4 is a joint work with Markus Dertwinkel-Kalt and Christian Wey and titled "Multi-plant Firms, Production Shifting, and Countervailing Power." In this chapter, we study how multi-plant firms' ability to shift the production of differentiated products across their plants shapes union-firm bargaining relations and affects consumer welfare. So far, the economic literature has focused on multi-plant and cross-border mergers where firms' products are plant-specific, *i.e.*, each product variant can be produced only at its respective plant. In our analysis, we consider a merger with plant-specific products as a benchmark. In our main model, we focus on the case in which production shifting is possible. We show that a merger decreases input prices under both scenarios. In the benchmark case, a merger never increases social welfare. However, a merger which gives the possibility of production shifting increases both consumer surplus and social welfare if products are not close substitutes and the capacity constraint of the plants are not very restrictive. We also study a firm's investment incentives to open a new plant for producing a differentiated product. We show that the firm has higher incentives to invest abroad than in the home country. Incentives to invest abroad are even stronger if the firm can produce both product variants at both plants. Our findings suggest that having the option to shift production creates considerable countervailing power in supplier-manufacturer bargaining relations with important implications for both merger control and FDI policy.

Chapter 5, published in *Journal of Institutional and Theoretical Economics*, is a joint work with Christian Wey and titled "Multiunion Bargaining: Tariff Plurality and Tariff Competition." We study (efficient) sequential bargaining between two unions and a single firm. We consider a firm which gets labor inputs from two different unions to produce a single product. If labor unions represent substitutable worker groups, then there is a first-mover advantage and the second union is foreclosed. If unions represent complementary work forces, then there is a second-mover advantage, such that the wage bill of the union who bargains first is smaller than the wage bill of the second union. We also study the merger incentives of the unions under both cases. Unions have strong merger incentives when they are perfectly substitutable. On the contrary, unions prefer to bargain separately if their work forces are complementary. We also consider a multi-product firm case in which each unions labor force produces a differentiated good. Qualitatively, unions' merger incentives are the same as in single-product firm case. Moreover, we also show that these incentives always stay in conflict with socially desirable market structure.

## Chapter 2

# Advertising and Price Competition in Online Markets

### 2.1 Introduction

In many cases, consumers have to incur search costs to find the product which meets their needs. Nowadays, price comparison websites and online retailers provide lists of prices for many products. Usually, online retailers also allow advertisements on their websites. When a consumer visits an online retailer, or a price comparison site, she faces a list of products which order is determined by the pricing and advertising decisions of the firms.

In this paper, we analyze a model in which firms producing homogeneous products compete in prices and advertisements to attract consumers' attention.<sup>1</sup> Firms offer homogeneous products to consumers which meet their needs with a positive probability. Although consumers can observe all the prices in the market, they have to incur a search cost to find the true match value of each product.<sup>2</sup> We consider two groups of consumers who differ with their search order. A group

<sup>&</sup>lt;sup>1</sup> For example, if one searches a hotel on booking.com, she faces a list of offers which is initially sorted according to the site's recommendation. Although the prices of the hotels are transparent, one have to change the order of the search to get the list in an increasing price order. Even in this case, there are some offers with special labels, like "value deal" or "genius", which can influence the consumers' search order.

<sup>&</sup>lt;sup>2</sup> For instance, a consumer who search a three-stars hotel in a city center can get a list of such hotels in an increasing price order by using an online retailer or a price comparison site. Although the offers are fairly homogeneous, the consumer has to invest some time to investigate each product by checking additional services or by reading consumer comments.

of "sophisticated consumers" starts their search from the lowest priced product, while the group of "naive consumers" starts their search from the advertised product. So, the firms can attract consumers' attention either by pricing lower than their rivals, or by advertising their products.

We analyze the impact of the existence and increasing share of naive consumers on the equilibrium pricing and advertising strategies of the firms, as well as on consumer welfare. We show that if the share of naive consumers is relatively large, then both firms advertise their products. However, if their share is relatively small, then there exists a unique stable equilibrium in which only one firm advertises. The expected prices increase when at least one firm advertises in the market. Moreover, the prices are even higher when only one firm advertises its product compared to the case in which both firms advertise. Given the firms' advertising decisions, consumers suffer from an increasing share of naive consumers. However, there is a critical level, such that a small increase in the share of naive consumers increases the equilibrium number of advertisements from one to two. In this case, expected prices in the market decrease and consumer welfare increases.

In an extension of our model, we consider a third group of consumers who are fully informed about the products. The existence of informed consumers allows the firms to charge prices higher than the searchers' reservation price. If the uninformed consumers' search costs are very high, then the firms always charge prices higher than their reservation price. In this case, the firms do not serve searchers and market prices do not depend on the share of naive consumers. Oppositely, if the search costs are very low then the firms choose prices from the lower interval and serve all consumers. In this case, an increasing share of naive consumers increases the expected prices in the market and harms all consumers. If the search cost is at an intermediate level, then firms randomize their price choices between high and low price levels. When the share of naive consumers increases, then the firms more likely charge prices from the lower price interval and the group of uninformed consumers are more likely active in the market. Thus, it decreases expected prices and increases consumer welfare.

**Related Literature.** There is an extensive literature on sequential search models. Wolinsky (1986) and Stahl (1989) analyze sequential search models in which consumers visit firms randomly. Stahl (1989) analyzes a model in which firms

#### 2.1. INTRODUCTION

provide homogeneous products. He shows that an increase in the number of competitors raises obfuscation level and prices. Wolinsky (1986) consider a random search model with differentiated products. He shows that imperfect information may turn an oligopolistic market into a monopolistically competitive one.

There are several recent papers which show how market prices depend on consumers' search order. Arbatskaya (2007) considers a model in which firms sell homogenous products and consumers visit the firms in a predetermined order. She finds that the prices decrease in consumers' search order, *i.e.*, the more prominent firms charge higher prices. While Zhou (2011) shows that the prices are increasing in the order of search when the firms produce differentiated products. Armstrong, Vickers and Zhou (2009) consider a model in which only one firm is more prominent in the market and they find a similar result to Zhou (2011). In their model, the prominent firm charges lower price than its rivals, but gets more profit because of a higher demand. Note that, all the three papers analyze models in which firms' decisions which influence consumers' search orders are exogenously given.

Our paper is closely related with the following models in which firms can influence the consumers' search order. Bagwell and Ramey (1994) and Haan and Moraga-González (2011) study models where consumers' search order depends on the relative advertising levels of the firms. In our model, "naive consumers" behave similarly, *i.e.*, the advertised products are more prominent for these consumers. Haan and Moraga-González (2011) show that if the firms are ex-ante symmetric, then all firms advertise their products with the same intensity. As a result, the consumers search in a random order and the firms' advertising costs are sunk. "Sophisticated consumers" in our model behave similarly to consumers in the models of Zhang (2009) and Armstrong and Zhou (2011, Section 2). Both papers analyze price-directed search models in which consumers visit the firms in an increasing price order. Armstrong and Zhou (2011) consider two firms competing in a Hotelling framework with differentiated products. They show that the prices in the market decrease in the search costs, as the higher search costs increase the benefit of being searched first. Zhang (2009) studies price comparison sites in a different framework. He considers a model in which the firms offer products which match the consumers' needs only with some probability. Zhang (2009) shows that the equilibrium price in the market does not monotonically depends on search cost and firms can get a higher profit if the search cost of the consumers is lower.

In our paper, we analyze a model in which both groups of consumers are present in the market and we show how the firms' advertising and pricing strategies depend on the relative shares of these consumers. We show that, although the firms are ex-ante symmetric, the advertising strategies of the firms are asymmetric when the share of sophisticated consumers is relatively high. In this case, only one firm advertises its product in the only stable equilibrium. However, if the share of sophisticated consumers is relatively low or these consumers are not present in the market, then we get similar results to Haan and Moraga-González (2011), *i.e.*, both firms advertise in the market and naive consumers search randomly. Differently, from the literature discussed above, we also analyze how consumer welfare depends on the share of naive consumers. We find that, consumers do not always suffer from an increasing share of naive consumers. In the certain range of parameters, consumers can even benefit from them.

Haan, Moraga-González and Petrikaitė (2015) also analyze a model in which firms can influence consumers' search order in two different ways. In their model, firms can get a prominent position by pricing lower than the rivals and/or by providing match-value information. Differently from our model, they consider consumers who are heterogeneous in their tastes, but homogeneous in their attentions. They show that price advertising decreases prices in the market and equilibrium prices decrease in search cost. In the equilibrium, firms advertise their prices, but they do not reveal match value information when search costs are realistically low enough.

A model of price comparison websites with homogeneous products is also analyzed by Baye and Morgan (2001). They consider two groups of consumers those who are loyal to a firm, and others who are looking for the cheapest price. To attract the price sensitive consumers, firms can advertise their prices on a price comparison site. They show that the firms' advertising decisions are random, and the firms who advertise their product on the site charge lower prices.<sup>3</sup> In their model, they consider a different framework in which consumers do not incur a search cost and firms can decide whether to advertise their prices or not.

<sup>&</sup>lt;sup>3</sup> Another related work is Baye and Morgan (2009). They analyze a model in which the firms sell homogeneous products and engage in both brand and price advertising. In contrast to Baye and Morgan (2001), they assume that the share of loyal consumers depends on the firms' brand advertising decisions.

### 2.2 Model

We analyze a search model in which two firms compete on an online platform over a unit mass of consumers. The consumers have a "need". They get utility v if their need is met, and 0 otherwise. Each firm's product meets a consumer's need with a probability  $\theta \in (0, 1)$ , which is independent and identical for both firms. The cost of production is normalized to zero.<sup>4</sup>

Consumers search the products on a price comparison site, so all the prices are observable in the market. To learn the match value (quality) of each product, a consumer has to pay a search cost of s. The firms can also advertise their products on the site at a cost c. Without any advertisement the site shows only the organic results to consumers, *i.e.*, the list of the products in an increasing price order.<sup>5</sup> If one of the two firms advertises, then the site lists the advertised firm's product on the top of the organic results. When both firms advertise, then the site lists both of the advertised products in a random order on the top of the organic results. Alternatively, if there is only one slot for advertisement, then for each consumer, the site randomly lists one of the two products on the top of the organic results.

We consider the following search strategies for the consumers. Consumers search a product only if its price is lower than their "reservation price" r, where

$$r = v - \frac{s}{\theta}.\tag{2.1}$$

Note that r solves  $\theta(v - p) \ge s$ , *i.e.*, consumers search a product only if their expected utility from that product is higher than their search costs.

As the firms offer a "risky product", consumers' willingness to pay for learning the quality (their reservation price) decreases in the search cost. In our model, we assume  $s \in [0, v\theta)$ , so the reservation price is always larger than the firms' marginal cost which we set to zero.<sup>6</sup>

 <sup>&</sup>lt;sup>4</sup> Athey and Ellison (2011), Chen and He (2011), and Zhang (2009) also analyze models in which the products offered by the firms match a consumer's need with a certain probability.
 <sup>5</sup> As the firms are ex-ante symmetric and none of them pay the advertisement fees, the

platform does not have an interest to list the firms in a different order.

<sup>&</sup>lt;sup>6</sup> Note that, we consider boundedly rational consumers in our model. Consumers make their search decision for each product by considering its expected value and they purchase a product only after learning its match value. We consider a model with fully rational consumers as an extension.

We assume that there are two groups of consumers. Formally,  $\alpha \in [0, 1]$  share of consumers are "sophisticated". Sophisticated consumers always ignore the advertisements and search products in an increasing price order. If the prices are equal, the sophisticated consumers search randomly. The remaining share of consumers,  $1 - \alpha$ , are "naive".<sup>7</sup> Naive consumers always start their search from the advertised product, which is listed on the top of the results. If there is no advertisement, all consumers behave like a sophisticated one. When both firms advertise, then naive consumers randomly search the products. In all cases, the consumers stop their search when they find a product which meets their need.

The timing of the game is as follows. In the first stage, firms simultaneously make their advertising decisions. In the second stage, firms' advertising decisions become public and they compete in prices. We solve the game backward. In Section 2.3, we analyze the pricing strategies of the firms given their advertising decisions from the first stage and compare the results from the different cases. In Section 2.4, we derive firms' equilibrium advertising strategies and discuss the platform's optimal policy.

### 2.3 Equilibrium Pricing

In this section, we derive equilibrium pricing strategies for each firm under three different advertising scenarios. First, we discuss the symmetric cases and then we analyze the asymmetric case in which one firm is more prominent than the other one. In the end of the section, we compare the results from the three cases and discuss how the equilibrium prices change in the number of advertisements.

### 2.3.1 Equilibrium Pricing without Advertisements

Without any advertisement, the site lists the products in an increasing price order and all consumers behave like a sophisticated consumer.<sup>8</sup> This model without

<sup>&</sup>lt;sup>7</sup> Many platforms which provide lists of prices for different products also allow advertisements on their sites. The impact of advertisements on the share of naive consumers depends on platform's advertising strategy, rather than firms' advertising decisions. For example, an advertisement on top of the organic results may be more effective than the advertisement on the side of the results.

<sup>&</sup>lt;sup>8</sup> One can make a less strong assumption, such that without advertisements, only a fraction of the naive consumers behaves like a sophisticated consumers, and the other fraction

advertisements is a special case of Zhang (2009) with only two firms and without informed consumers.

#### Claim 1. There exists no equilibrium in pure strategies.

It is obvious that there is no equilibrium in symmetric pure strategies. If both of the firms charge the same price p > 0, then one of the two firms can slightly undercut the rival's price and increase its expected profit. If both firms set the price p = 0, then one firm can deviate to a positive price and get a positive expected profit.

If there is an equilibrium in asymmetric pure strategies, then the firm with the higher price has to choose the reservation price, r. In this case, the other firm has to charge a price  $p \leq r(1 - \theta)$ , otherwise the firm with the higher price can deviate to  $p - \varepsilon$ , with arbitrarily small  $\varepsilon > 0$ , and increase its expected profit. But given the rival firm charges the reservation price, the firm with lower price can increase its price above  $r(1 - \theta)$ , up to the reservation price. So, there is no equilibrium in pure strategies.

Thus, we are looking for an equilibrium in mixed strategies. The following two lemmas must hold in any possible equilibrium.

**Lemma 1.** In equilibrium, i) both firms charge prices from the same interval  $[p_{min}, p_{max}]$ , and ii) there exists no price interval  $(x, y) \subseteq (p_{min}, p_{max})$ , such that firm i attaches zero probability to it, while it sets prices from the intervals  $(x-\epsilon, x]$  and  $[y, y + \epsilon)$ , for any  $\epsilon > 0$ , with positive probabilities.

According to Lemma 1, the firms have to choose the prices from the same continuous interval. The following lemma formally states that the firms do not charge any price with a positive probability, and the reservation price is always included in the support of equilibrium cumulative distribution function.

**Lemma 2.** In equilibrium, none of the firms charges a price  $p \in [p_{min}, p_{max}]$  with a positive probability. Moreover, in the equilibrium,  $p_{max} = r$ .

By using the lemmas above, we can derive the equilibrium pricing strategies as follows. Assume that, in the equilibrium, firm j chooses its price from the interval  $[p_{min}^w, r]$ , according to a cdf H(p).<sup>9</sup>

searches randomly. The results will not change qualitatively and the main results of this paper does not depend on this assumption.

<sup>&</sup>lt;sup>9</sup> The superscript "w'' stands for the "without advertisement" regime.

Given firm j's equilibrium strategy, firm i's expected profit from setting the reservation price r is equal to

$$r\theta(1-\theta).\tag{2.2}$$

As firm *i* chooses the maximum possible price, all the consumers first visit firm j, and they will buy from firm *i* only if its product is the only match. Firm *i*'s expected profit from setting a price  $p \in [p_{min}^w, r]$  is

$$p\theta[(1-\theta) + \theta(1-H(p))].$$
(2.3)

The consumers will buy firm *i*'s product, if *i*) its product is the only match which occurs with probability  $\theta(1 - \theta)$ , or *ii*) both firms' products match, but firm *i*'s price is lower than its rival's price which occurs with probability  $\theta^2(1 - H(p))$ .

In mixed strategy equilibrium, the firms have to be indifferent between any prices  $p \in [p_{min}^w, r]$ . The following cdf solves the equality of (2.2) and (2.3):

$$H(p) = \frac{1}{\theta} \left( 1 - \frac{r}{p} (1 - \theta) \right).$$
(2.4)

The support of H(p) is  $[p_{min}^w, r]$ , where  $p_{min}^w$  solves H(p) = 0, and is given by

$$p_{min}^w = r(1-\theta). \tag{2.5}$$

Note that the firms are symmetric, and there is a unique solution of H(p) which makes the rival firm indifferent between charging any price in  $[p_{min}^w, r]$ . So, there is a unique equilibrium in symmetric mixed strategies in which both firms price according to the same cdf, H(p), with the support  $[p_{min}^w, r]$ . The following proposition formalizes the firms' equilibrium pricing strategies when there is no advertisement.

**Proposition 1.** Without advertisements there exists a unique equilibrium in symmetric mixed strategies, such that each firm chooses its price according to the cdf H(p), with the support  $[p_{min}^w, r]$ , where H(p) and  $p_{min}^w$  are given by (2.4) and (2.5), respectively.

If the search cost increases then both the maximum and the minimum prices decrease. Moreover, the firms choose prices from a narrower interval, *i.e.*,  $r - p_{min}^w$  decreases in s. In the extreme cases, if the search cost is close to consumers' expected utility from the product, then the firms charge prices closer to the marginal cost 0. When the search cost is equal to 0, then the maximum possible price is equal to v. In the equilibrium, each firm's expected profit is equal to  $\pi^w = r\theta(1-\theta)$ . The firms' profits also decrease in the search cost s.

#### 2.3.2 Equilibrium Pricing when Both Firms Advertise

If both firms advertise on the price comparison site, then the site randomly lists one of the two firms on the top of the organic results. As the search order of naive consumers depends on the order of the products in the list, their search order is ex-ante random.

If the share of sophisticated consumers is equal to one, then the advertisements do not play a role and the equilibrium of the game is described in Proposition 1. If all consumers are naive, *i.e.*,  $\alpha = 0$ , then the model reduces to the random search model. In this case, there is a unique equilibrium in which both firms charge the reservation price, r, and get profit equal to  $r\theta(1 - \theta/2)$ .

**Claim 2.** If  $\alpha \in (0,1)$ , then there exists no equilibrium in pure strategies.

In this case, the behavior of the naive consumers does not depend on the firms' pricing strategies. The behavior of the sophisticated consumers does not depend on the advertisements and it is exactly the same as in the case without advertisements. So, the proof of Claim 2 as well as the proofs of the following two lemmas are analogous to those in the previous section and we skip them to keep the analysis short. We look for an equilibrium in mixed strategies.

**Lemma 3.** In equilibrium, i) both firms charge prices from the same interval  $[p_{min}, p_{max}]$ , and ii) there exists no price interval  $(x, y) \subseteq (p_{min}, p_{max})$ , such that firm i attaches zero probability, while it sets prices from the intervals  $(x - \epsilon, x]$  and  $[y, y + \epsilon)$ , for any  $\epsilon > 0$ , with positive probabilities.

**Lemma 4.** In equilibrium, none of the firms charges a price  $p \in [p_{min}, p_{max}]$  with a positive probability. Moreover, in the equilibrium,  $p_{max} = r$ .

Now we can derive the equilibrium cdf by using the previous two lemmas. Assume that firm j sets its price according to a cdf G(p), with the support  $[p_{min}^b, r]^{10}$ 

Given firm j plays its equilibrium strategy G(p), firm i's profit from setting the reservation price r is equal to

$$r\alpha\theta(1-\theta) + r(1-\alpha)\left(\theta(1-\theta) + \frac{\theta^2}{2}\right).$$
(2.6)

<sup>&</sup>lt;sup>10</sup> The superscript "b'' stands for "both advertise".

As firm *i* sets the maximum possible price, the sophisticated consumers always visit firm *j* first, and they buy from firm *i* only if its product is the only match. Expected profit from sophisticated consumers is given by the first term of (2.6). While the naive consumers first visit the firm listed on the top of the results and buy the product from firm *i* only if *i*) it is the only match, or *ii*) both products match, but firm *i*'s product is listed on top of firm *j*'s product, which happens with probability 1/2. Firm *i*'s expected profit from naive consumers is given by the second term of (2.6).

Similarly, firm *i*'s expected profit if it sets a price  $p \in [p_{min}^b, r]$  is

$$p\alpha\left(\theta(1-\theta)+\theta^2\left(1-G(p)\right)\right)+p(1-\alpha)\left(\theta(1-\theta)+\frac{\theta^2}{2}\right).$$
 (2.7)

The sophisticated consumers buy the product from firm i, only if i) firm i's product is the only match, or ii) both firms' products match but the firm i's price is lower than its rival's price. The naive consumers buy firm i's product, only if i) it is the only match, or ii) both products match, but the site lists firm i's product on the top of the list.

From the equality of the profits (2.6) and (2.7), we get the equilibrium cdf

$$G(p) = \frac{1}{2} \left[ \frac{r+p}{p} - \left( \frac{r-p}{p} \right) \left( \frac{2-\theta}{\alpha \theta} \right) \right], \qquad (2.8)$$

with the support  $[p_{min}^b, r]$ , where  $p_{min}^b$  solves G(p) = 0, and is given by

$$p_{min}^b = \frac{r(2-\theta-\alpha\theta)}{2-\theta+\alpha\theta}.$$
(2.9)

Similar to the previous case, the firms are symmetric and there is a unique solution of G(p), so firm *i*'s equilibrium pricing is also given by the same cdf G(p). The following proposition describes the unique equilibrium of the game in which both firms advertise their products.

**Proposition 2.** If both firms advertise on the site, then there exist a unique equilibrium in symmetric mixed strategies, such that each firm chooses its price according to the cumulative distribution function G(p), with the support  $[p_{min}^b, r]$ , where G(p) and  $p_{min}^b$  are given by (2.8) and (2.9), respectively.

Similar to the previous case, both the maximum and the minimum prices decrease in the search costs. The range of prices also decreases in s. The maximum

possible price in the equilibrium is equal to the reservation price, which does not depend on the share of sophisticated consumers, while the minimum price decreases in  $\alpha$ . If  $\alpha$  is close to one, then the firms choose prices from a larger interval of prices. If the share of sophisticated consumers is close to 0, then the firms charge prices closer to the reservation price. So, when both firms advertise on the site, the sophisticated consumers suffer from an increasing share of naive consumers.

The firms' expected profit is equal to  $\pi^b = r\theta(2-\theta-\alpha\theta)/2$ . The firms' profit decreases both in the search cost s and the share of sophisticated consumers  $\alpha$ .

### 2.3.3 Equilibrium Pricing when Only One Firm Advertises

If only one firm advertises on the price comparison site, then the site always lists the advertised firm's product on top of the organic results. In other words, one firm is always prominent for the naive consumers.

Similar to the previous subsection, if  $\alpha = 1$ , *i.e.*, there are only sophisticated consumers, then the advertisement does not play any role, and the equilibrium pricing is given in Proposition 1. If there are only naive consumers, then the model reduces to a search model in which one firm is more prominent for all consumers. In this case, both firms set the reservation price, r, in the unique equilibrium. The profit of the advertised firm is equal to  $r\theta$ , while the non-advertised firm's profit is  $r\theta(1-\theta)$ . For any other  $\alpha \in (0,1)$  we derive the equilibrium pricing as follows.

Note again that the behavior of the sophisticated consumers does not depend on the advertisement, whereas the naive consumers are not price sensitive unless  $p \leq r$ . So, we can use the similar arguments as we had in Section 2.2.1 to prove the following claim.

#### **Claim 3.** If $\alpha \in (0,1)$ , then there exist no equilibrium in pure strategies.

The proofs of the following two lemmas are also analogous to the proofs of Lemma 1 and the first part of Lemma 2. We skip these proofs to keep the analysis short.

**Lemma 5.** In the equilibrium, i) the upper and the lower limits of the prices are the same for both firms, and ii) there exists no price interval  $(x, y) \subseteq (p_{min}, p_{max})$ ,

such that firm i attaches zero probability, while it sets prices from the intervals  $(x - \epsilon, x]$  and  $[y, y + \epsilon)$ , for any  $\epsilon > 0$ , with positive probabilities.

**Lemma 6.** In the equilibrium, none of the firms charges a price  $p \in [p_{min}, p_{max})$ with a positive probability.

Although, in the equilibrium, the firms charge prices from the same continuous interval, the following lemma states that only advertised firm can charge the reservation price with a positive probability.

**Lemma 7.** The maximum price the firms charge in equilibrium is equal to the reservation price, i.e.,  $p_{max} = r$ . Moreover, only the advertised firm can charge the reservation price with a positive probability.

According to Lemmas 5-7, we look for an equilibrium in which the advertised firm sets the reservation price with a probability  $\beta \geq 0$ , and all other prices  $p \in [p_{min}^o, r)$  with probability  $1 - \beta$ , according to a cdf  $\tilde{F}_a(p)$ , whereas the nonadvertised firm sets prices from the interval  $[p_{min}^o, r)$  according to a cdf  $F_n(p)$ .<sup>11</sup>

Given that the non-advertised firm plays its equilibrium strategy, the advertised firm's profit from charging the reservation price r is equal to

$$r\alpha\theta(1-\theta) + r(1-\alpha)\theta. \tag{2.10}$$

The sophisticated consumers buy from the advertised firm only if it is the only match, while the naive consumers will buy from the advertised firm if its product meets their need.

The advertised firm's expected profit if it sets price  $p \in [p_{min}^o, r)$  is

$$p\alpha(\theta(1-\theta) + \theta^2(1-F_n(p))) + p(1-\alpha)\theta.$$
(2.11)

The sophisticated consumers buy from the advertised firm, if i) its product is the only match, or ii) both firms' products match but the advertised firm's price is lower than the rival's price. The naive consumers will buy from the advertised firm if its product meets their need.

From the equality of expected prices (2.10) and (2.11) we get the equilibrium cdf for the non-advertised firm,

$$F_n(p) = \frac{1}{\alpha \theta} \left( 1 - \frac{r}{p} \left( 1 - \alpha \theta \right) \right).$$
(2.12)

<sup>&</sup>lt;sup>11</sup> The superscript "o'' stands for "one firm advertises".

From the equation  $F_n(p) = 0$ , we can find the minimum price  $p_{min}^o$  as

$$p_{min}^o = r(1 - \alpha\theta). \tag{2.13}$$

In the equilibrium, the non-advertised firm will not charge the reservation price. As there is a positive probability of a tie in that case, half of the sophisticated consumers will visit the rival firm first. But if the non-advertised firm charges prices at the left limit of the reservation price, then it will get all the sophisticated consumers' attention with probability  $1 - \beta$ , and will not decrease its expected price.

Given the advertised firm plays its equilibrium strategy, the non-advertised firm's profit from charging price on the left limit of the reservation price r is equal to

$$r\alpha(\theta(1-\theta) + \beta\theta^2) + r(1-\alpha)\theta(1-\theta).$$
(2.14)

The sophisticated consumers buy from the advertised firm i) if its product is the only match, or ii) both products match but the advertised firm charges the reservation price. The naive consumers buy from the non-advertised firm only if its product is the only match.

If the non-advertised firm sets price p, then its expected profit is

$$p\alpha \left(\theta(1-\theta) + \theta^2 \left(\beta + (1-\beta)(1-\tilde{F}_a(p))\right)\right) + p(1-\alpha)\theta(1-\theta).$$
(2.15)

The sophisticated consumers buy from the non-advertised firm, if i) its product is the only match, or ii) both firms' products match but the non-advertised firm's price is lower. The naive consumers will buy from the non-advertised firm only if its product is the only product that meets their need.

From the equality of expected profits (2.14) and (2.15), we get

$$\tilde{F}_a(p) = \frac{1}{1-\beta} \left( 1 - \beta \frac{r}{p} + \frac{1-\theta}{\alpha \theta} \left( 1 - \frac{r}{p} \right) \right).$$
(2.16)

From  $\tilde{F}_a(p) = 0$ , we can find the minimum price for the advertised firm  $p_{min}^o = r(1 - \theta + \beta \alpha \theta)/(1 - \theta + \alpha \theta)$ . By using (2.13), we can find

$$\beta = \theta(1 - \alpha). \tag{2.17}$$

If we plug (2.17) in (2.16), we can find the advertised firm's equilibrium pricing

strategy as follows

$$F_{a}(p) = \begin{cases} (1-\beta)\tilde{F}_{a}(p) & \text{if } p \in [p_{min}^{o}, r); \\ 1 & \text{if } p = r \\ \\ = \begin{cases} (1-\theta(1-\alpha))\frac{1}{\alpha\theta} \left(1 - \frac{r}{p} \left(1 - \alpha\theta\right)\right) & \text{if } p \in [p_{min}^{o}, r); \\ 1 & \text{if } p = r. \end{cases}$$
(2.18)

The following proposition formalizes the equilibrium pricing of the firms if only one firm advertises on the site.

**Proposition 3.** If only one firm advertises, then there exist a unique equilibrium in mixed strategies, such that the advertised firm chooses its price according to the cumulative distribution function  $F_a(p)$  which is given by (2.18), and the nonadvertised firm chooses its price according to the cumulative distribution function  $F_n(p)$  which is given by (2.12), with the support  $[p_{\min}^o, r]$ , where  $p_{\min}^o$  is given by (2.13),  $\beta = \theta(1 - \alpha)$ , and  $r = v - \frac{s}{\theta}$  is the reservation price of the consumers.

In the equilibrium, the advertised firm takes the advantage and exploits the naive consumers with a positive probability  $\beta = \theta(1 - \alpha)$ . If the share of naive consumers  $(1 - \alpha)$  increases, then the advertised firm charges the reservation price with higher probability.  $\beta$  also increases in the probability of a match  $\theta$ . The reason is that if the probability of match increases, then it is more likely that a firm will sell its product to the consumers who start their search from its product. With the remaining probability  $1-\beta$ , the advertised firm competes with the non-advertised firm over the sophisticated consumers. Note that the range of the prices is the same for both firms, and the advertised firm's equilibrium cdf in the interval  $[p_{min}^o, r)$  is equal to  $(1 - \beta)F_n(p)$ . So, in the equilibrium, the non-advertised firm chooses any price range in  $[p_{min}^o, r)$  more frequently than the advertised firm.

**Corollary 1.** The non-advertised firm's equilibrium cdf,  $F_n(p)$ , is first-order stochastically dominated by the advertised firm's equilibrium cdf,  $F_a(p)$ .

Similar to the previous two cases, the maximum and the minimum prices, as well as the range of the possible prices decrease in search cost s. The maximum price is equal to the reservation price and does not depend on the share of naive consumers, while the minimum price increases in the share of naive consumers,  $1 - \alpha$ . Similar to the previous case, the sophisticated consumers suffer from the existence of naive consumers.

The advertised firm's expected equilibrium profit is  $\pi_a^o = r\theta(1-\alpha\theta)$ , while the non-advertised firm's expected equilibrium profit is given by  $\pi_n^o = r\theta(1-\alpha\theta)(1-\theta+\alpha\theta)$ . Both firms' expected profits decrease in search cost s. The profit of the advertised firm also decreases in the share of sophisticated consumers,  $\alpha$ .

The non-advertised firm's profit is not monotonic in  $\alpha$ . If  $\alpha < 1/2$ , then the non-advertised firm's expected demand increases in  $\alpha$ , as the positive effect of an increasing share of the sophisticated consumers is larger than the negative effect of increasing  $1 - \beta$ . Although the expected prices decrease in  $\alpha$ , the profit of the non-advertised firm increases in  $\alpha$ , as its expected demand increases. If  $\alpha \ge 1/2$ , then the negative effect of increasing  $1 - \beta$  on the non-advertised firm's expected demand is larger than the positive effect of increasing sophisticated consumers. As the expected prices also changes negatively, in this case,  $\pi_n^o$  decreases in  $\alpha$ .

#### 2.3.4 Comparative Statics

In all three cases, the maximum possible price in the market is equal to consumers' reservation price and does not depend on the number of advertisements and the share of naive consumers, while the minimum prices are different in all three cases. Moreover, they are strictly decreasing in the share of sophisticated consumers  $\alpha$ , except in the case without advertisements. Compared to the benchmark case, the minimum possible price in the market increases when at least one firm advertises on the site, *i.e.*,  $p_{min}^w < p_{min}^o$ , and  $p_{min}^w < p_{min}^b$  holds for any  $\alpha \in (0, 1)$ . Interestingly, the minimum price is higher when only one firm advertises on the site, *i.e.*,  $p_{min}^b < p_{min}^o$ . The following corollary summarizes this result.

**Corollary 2.** Minimum Prices. Advertisement increases the minimum prices in the market. Moreover, the minimum price in the market is higher when there is only one advertisement on the site compare to the case with two advertisements. The order of the possible minimum prices in the market is as follows:  $p_{min}^{o} > p_{min}^{b} > p_{min}^{w} > p_{min}^{b} > p_{min}^{w}$ .

Note that the ranges of the prices,  $r - p_{min}$ , has the reverse order. The range of prices is the widest if there is no advertisement. When at least one firm advertises,



Figure 2.1: Equilibrium cdfs G(p) (dashed) and  $F_n(p)$  (dotted), for  $\alpha = 0.9, 0.5,$ and 0.1.

then the share of the consumers who are price sensitive decreases to  $\alpha$ , as the naive consumers are not price sensitive anymore. This decreases the intensity of competition, and the minimum price in the market increases, while the range of prices decreases.

The intensity of competition is even lower if there is only one advertisement on the site compare to the two advertisements. When there is only one advertisement in the market, then the advertised firm takes the advantage and charge the monopoly price r with a positive probability  $\beta$ . As a result, it competes for the share of the sophisticated consumers only with probability  $1-\beta$ . As the intensity of competition reduces for the same share of the consumers, *i.e.*, share of sophisticated consumers, compared with the two advertisements case, the minimum price increases, as the range of the prices decreases.

The following corollary compares the equilibrium cdfs from the three different cases.

**Corollary 3.** The ordering of the equilibrium cumulative distribution functions are as follows:  $F_a(p) \leq F_n(p) \leq G(p) \leq H(p)$ .

Following the discussion in this section, when there is only one advertisement then the advertised firm has strong market power and it sets the highest price more frequently. As the competition for the sophisticated consumers is less intense in this case, this also allows the non-advertised firm to charge higher prices more frequently compared to the two-advertisements case. If there is no advertisement on the site, then competition is most intense, and the firms have to charge the lower prices more frequently. So, the equilibrium cdf in the case without advertisement stochastically dominates all other cdfs.

Figure 2.1 shows the equilibrium cdfs for three different values of  $\alpha$ , given specific parameter values v = 1,  $\theta = 0.5$ , and s = 0.1. The dashed lines show the cases with two advertisements, while the dotted lines are the cdfs of the non-advertised firm when there is only one advertisement. One can observe that, when  $\alpha$  increases the price competition also increases and firms charge prices from a larger interval. In the extreme case, when all consumers are sophisticated (*i.e.*,  $\alpha = 1$ ), both cases reduce to the model without any advertisement. The opposite is true when the share of naive consumers increases. If  $\alpha$  is close to 0, then all consumers are naive and the firms charge only the prices close to the reservation price. In the extreme case, when  $\alpha = 0$ , both firms charge only the reservation price r.

### 2.4 Advertising Strategies and Consumer Welfare

In this section, we derive the firms' optimal advertising strategies by comparing their expected profits from the three cases discussed in the previous section. We assume that naive and sophisticated consumers are always present in the market, *i.e.*,  $\alpha \in (0, 1)$ . First, we assume that the advertising cost is zero, *i.e.*, c = 0, and we show how consumer welfare changes depending on firms' advertising decisions. Then, we consider positive advertising costs and derive optimal strategies for the general case. In the end of the section, we also discuss the platform's optimal policy.

First, we compare the two symmetric cases. Without advertisement, the firms get the expected profit  $\pi^w = r\theta(1-\theta)$ . When both firms advertise, then each firm's expected profit is equal to  $\pi^b = [r\theta(2-\theta-\alpha\theta)]/2$ . The difference between the two cases is  $\pi^w - \pi^b = -[r\theta^2(1-\alpha)] < 0$  for any  $\alpha < 1$ . The firms' profits increase when both of them advertise on the site.

When there is only one advertisement on the site, the advertised firm gets larger profit than its rival, *i.e.*,  $\pi_a^o - \pi_n^o = r\theta^2(1 - \alpha\theta)(1 - \alpha) > 0$ . Given the rival does not advertise on the site, the firm always prefers to advertise its product, *i.e.*,  $\pi_a^o > \pi^w$ . The non-advertised firm is also better off when its rival advertises compare to the case without advertisement, *i.e.*,  $\pi_n^o > \pi^w$ . So, the firms are always better off if at least one firm advertises its product on the site.

The advertised firm's profit always decreases when its rival also starts to advertise its product, *i.e.*,  $\pi^b - \pi_a^o = -[r\theta^2(1-\alpha)]/2 < 0$ . Given the rival firm advertises, the non-advertised firm's decision depends on the share of naive consumers and the probability of a product match. If  $\alpha\theta < 1/2$ , then the non-advertised firm also prefers to advertise its product, *i.e.*,  $\pi_n^o < \pi^b$ . When  $\alpha\theta > 1/2$ , then the non-advertised firm's decreases if it also advertises its product, *i.e.*,  $\pi_n^o > \pi^b$ . If  $\alpha\theta = 1/2$ , then the non-advertised firm is indifferent, *i.e.*,  $\pi_n^o = \pi^b$ .

If advertising costs are equal to zero, c = 0, then firms prefer the market with at least one advertisement to the one without advertisement. The equilibrium number of advertisements depends on the share of sophisticated consumers and the probability of a match. Note that  $\alpha\theta$  is the expected demand of the sophisticated consumers for the lower priced product. If this value is smaller than 1/2, then the advantage of being the lowest pricing firm is not very high, so the nonadvertised firm prefers to advertise its product. If  $\alpha\theta > 1/2$ , then the expected demand from the sophisticated consumers is relatively high. In this case, the non-advertised firm prefers to use this advantage and do not advertise its product, as its rival competes for the sophisticated consumers only with probability  $1 - \beta$ . The following proposition summarizes advertising decisions of the firms in the online market.

#### Proposition 4. Advertising strategies given c = 0.

(i) If  $\alpha \theta < 1/2$ , then there is a unique equilibrium, in which both firms advertise their products on the site.

(ii) If  $\alpha \theta > 1/2$ , then there are stable equilibria in pure strategies, in which only one of the two firms advertises. There is also an unstable equilibrium in mixed strategies, in which the firms advertise their products with the probability  $2/(2\alpha\gamma + 1)$ .

Note that, the mixed equilibrium in the second case is  $unstable^{12}$ . In our

 $<sup>^{12}</sup>$   $\,$  If  $\alpha\theta>1/2$  holds, then advertising game between the firms is a  $2\times2$  coordination game.



Figure 2.2: Expected prices of firms in the equilibrium.

analysis, we will focus only on the stable equilibria.

Figure 2.2 shows the firms' expected prices in the market depending on the share of naive consumers,  $(1 - \alpha)$ , given the parameter values v = 1,  $\theta = 0.75$ , and s = 0.1. If  $(1 - \alpha) > 1/3$ , then there is a unique equilibrium in which both firms advertise their products. The solid line depicts the expected prices of each firm. If the share of naive consumers is not very high (*i.e.*,  $(1 - \alpha) < 1/3$ ) then both firms are better off if only one of them advertises in the market. The dashed line shows the advertised firm's expected prices, while the dotted line shows the non-advertised firm's expected price. Note that, in all cases, the expected prices in the market increase in the share of naive consumers. But if  $\alpha \theta = 1/2$  (*i.e.*,  $1 - \alpha = 1/3$ ), then the consumers can benefit from a small increase in the share of naive consumers increases the equilibrium number of advertisements. In other words, the non-advertised firm also starts advertising its product. As a result, expected prices in the market decreases.

If the advertising cost on the site is positive,  $c \in (0, \hat{c} = \pi_a^o - \pi^w)$ , then the

According to Echenique and Edlin (2004, Corollary 1), if the mixed equilibrium of a  $2 \times 2$  game is not unique, then it is unstable for a broad class of learning dynamics.

firms' advertising decisions are as follows.<sup>13</sup>

#### Corollary 4. Advertising strategies for any $c \in (0, \hat{c})$ .

(i) If  $\pi^b - \pi_n^o > c$ , then there is a unique equilibrium, in which both firms advertise their products in the online market.

(ii) If  $\pi^b - \pi_n^o < c$ , then there are two stable equilibria in pure strategies, in which only one of the two firms advertises its product. There is also an unstable equilibrium in mixed strategies, in which the firms advertise their products with the probability  $\gamma = \frac{2}{2\alpha\theta+1} - \frac{c}{(\pi_a^o + \pi_n^o) - (\pi^b + \pi^w)}$ .

Given the rival firm advertises in the market, the firm advertises its product only if its gain from advertisement,  $\pi^b - \pi_n^o$ , is larger than the advertising cost c. In this case, there is only one equilibrium in which both firms advertise their products. Otherwise, there are three equilibria. In pure strategies, only one firm advertises its product. In addition, there is also an unstable equilibrium in which the firms advertise their products with a positive probability  $\gamma$ .<sup>14</sup> Similarly to the previous case, we will only consider stable equilibria in our analysis.

The advertising decision of the firms depends on the share of naive consumers. If the share of naive consumers is relatively high, then both firms advertise their products. Otherwise  $\pi^b - \pi_n^o < c$  holds, and only one firm advertise its product. In general, consumers suffer from the existence of naive consumers, but there is a critical level such that a small increase in their share increases number of advertisements and decreases market prices.

**Platform's Optimal Policy.** As the main focus of our paper is to show how the firms' strategies and consumer welfare depend on consumers' search behavior, until now we have analyzed the model in an exogenously given platform environment. But from Corollary 4 we can observe that beside other parameters, the advertising cost plays an important role in firms' decisions. So, in the rest of this section, we will derive the optimal policy for the platform. For this purpose, we consider stage 0 in which platform chooses the number of allowed advertisements in the market and advertising fees. The following two stages of the game is the

<sup>&</sup>lt;sup>13</sup> If  $c > \pi_a^o - \pi^w$  then the cost of advertisement is larger than the maximum possible gain from it. So, none of the firms will advertise its product.

<sup>&</sup>lt;sup>14</sup> Note again that, when  $\pi^b - \pi_n^o < c$ , then the advertising game between the firms is a 2 × 2 coordination game and according to Echenique and Edlin (2004) is unstable.

same as described in Section 2.2. We assume that, the platform has zero marginal costs and its profit is equal to the sum of advertising fees received from the firms.

First we consider the case in which the platform allows only one firm to advertise its product. The platform can choose the prices from the interval  $[0, \pi_a^o - \pi^w]$ , as none of the firms will advertise its product when  $c > \pi_a^o - \pi^w$ . In this case, the platform will charge the maximum possible price, *i.e.*,  $c^o = \pi_a^o - \pi^w$ , and will get equilibrium profit equal to  $\Pi^o = c^o = \pi_a^o - \pi^w$ .

Now we consider the second case in which the platform does not restrict the number of possible advertisements. If it chooses the fee  $c \leq \pi^b - \pi_n^o$ , then both firms will advertise their products. In this case, the platform will choose  $c^b = \pi^b - \pi_n^o$  and get  $\Pi^b = 2c^b = 2(\pi^b - \pi_n^o)$ . If it will charge the fee  $c \geq \pi^b - \pi_n^o$ , then there exist unique stable equilibria in which only one firm advertise its product. In this case, the optimal level of advertising fee is equal to  $c^o = \pi_a^o - \pi^w$ and platform gets  $\Pi^o = c^o = \pi_a^o - \pi^w$ .

If we compare platform's expected profits under different scenarios, we can conclude that the platform's profit is maximal when only one firm advertises on the platform and the optimal level of advertising fees is  $c^o = \pi_a^o - \pi^w$ . The platform's optimal strategy forces the firms to adopt the asymmetric advertising strategies. As we discussed above, consumer welfare is the lowest under this scenario. We can also conclude that, consumer welfare can be improved by regulating advertising fees and platform's policy.<sup>15</sup>

### 2.5 Market with Fully Informed Consumers

In many cases, there exists a positive share of consumers who have no search costs or fully informed about the offers. In this section, we extend our model by considering a positive share of informed consumers. Formally, we assume that  $\lambda \in (0, 1)$  share of the consumers are shoppers who have search costs of 0. Accordingly,  $1 - \lambda$  is the share of the consumers who have to incur search costs of s to learn the quality of each product. Similar to the previous model,  $\alpha$  share of the searchers are sophisticated. They start their search from the lowest priced product. The remaining  $1 - \alpha$  share of the searchers are naive who start their

 $<sup>^{15}</sup>$   $\,$  We have discussed possible regulations in the conclusion of this paper.

search from the advertised firm.<sup>16</sup>

In this section, we focus only on the case in which both firms advertise in the market; *i.e.*, the naive consumers visit the first firm randomly. Note that the reservation price of the searchers is equal to r, which is given by (2.1). As the shoppers do not incur any search costs, they can buy a product even at a price of v. This allows the firms to charge prices above the searchers' reservation price, but not larger than v. If the uninformed consumers' search costs are very high then firms do not serve them.

#### 2.5.1 Equilibrium Analysis

Similar to the previous case, one can show that there is no equilibrium in pure strategies. We look for the symmetric Nash equilibrium in mixed strategies. Note also that, in the equilibrium, none of the firms will charge one price with a positive probability.

We introduce the following notation. In the equilibrium, the firms charge prices  $p \leq r$  with probability  $\beta$ , according to a cdf  $G_l(p)$ , and prices  $v \geq p > r$ , with probability  $1 - \beta$ , according to a cdf  $G_h(p)$ .<sup>17</sup>

Given the rival firm plays its equilibrium strategy, firm i's expected profit from charging price v is equal to

$$\lambda v \theta (1 - \theta). \tag{2.19}$$

In this case, the shoppers will buy from firm i only if its product is the only match.

If firm *i* charges price  $p \in (r, v)$ , then its expected profit is

$$\lambda p\theta((1-\theta) + \theta(1-\beta)(1-G_h(p))). \tag{2.20}$$

The shoppers will buy from firm i if it has the only product which matches their need, or if both products match, but firm i has the lower price.

When firm i charges price r, then its expected profit is

$$(\lambda + (1 - \lambda)\alpha)r\theta((1 - \theta) + \theta(1 - \beta)) + (1 - \lambda)\frac{(1 - \alpha)}{2}r\theta((1 - \theta) + \theta(1 - \beta)) + (1 - \lambda)\frac{(1 - \alpha)}{2}r\theta.$$
(2.21)

<sup>&</sup>lt;sup>16</sup> If the share of the naive searchers is  $\alpha = 0$ , then our model replicate the results of Zhang (2009) for the case of two firms.

<sup>&</sup>lt;sup>17</sup> Subscripts l and h stands for "lower level of prices" and "high level of prices", respectively.

In this case, shoppers and sophisticated searchers will buy from firm i if firm i's product is the only match, or if both products match but the other firm's price is larger than r. With probability 1/2 the website will advertise the rival firm's product. Thus, the naive searchers will buy from firm i, only if its product is the only match, or if both products match but the advertised firm's price is higher than the reservation price r. With probability 1/2, the naive searchers will start their search from firm i, and they will buy from firm i if its product matches their need.

Similarly, when firm *i* charges a price p < r, then its expected profit is given by

$$(\lambda + (1 - \lambda)\alpha)p\theta((1 - \theta) + \theta((1 - \beta) + \beta(1 - G_l(p)))) + + (1 - \lambda)\frac{(1 - \alpha)}{2}p\theta((1 - \theta) + \theta(1 - \beta)) + (1 - \lambda)\frac{(1 - \alpha)}{2}p\theta.$$
 (2.22)

The behavior of the shoppers and the sophisticated searchers is the same: they buy from firm i, either if firm i's product is the only match or both products match but the rival's price is larger than firm i's price. With probability 1/2 the website will advertise the rival firm's product. In this case, the naive consumers will buy from firm i, either if it is the only match or if both products match but the advertised firm's price is higher than their reservation price. With probability 1/2, the naive consumers will start their search from firm i, and they will buy from firm i if its product matches their need.

From the equations (2.19) and (2.21), we can find the probability of choosing prices from the interval  $[p_{min,l}, r]$ , which is given by

$$\beta = \frac{2}{\theta(1+\lambda+\alpha(1-\lambda))} \left(1 - \frac{v}{r}\lambda(1-\theta)\right).$$
(2.23)

Note that  $\beta \in [0, 1]$  only if  $s \in [\underline{s}(\alpha), \overline{s}]$ , where

$$\underline{s}(\alpha) = \frac{v\theta(1-\lambda)(2-\theta-\alpha\theta)}{2-\theta(1+\lambda+\alpha-\alpha\lambda)} \text{ and } \bar{s} = v\theta(1-\lambda(1-\theta)).$$
(2.24)

If  $s < \underline{s}(\alpha)$  then  $\beta = 1$  and the firms choose prices only from the interval  $[p_{min,l}, r]$ . Otherwise, *i.e.*,  $s > \overline{s}$ ,  $\beta = 0$  and the firms choose prices only from the interval  $[p_{min,h}, v]$ .

A firm charging the price from the lower interval competes for all consumers. While if it charges a price from the higher price range it serves only the informed consumers. In the first case, the firm has higher expected demand, but lower margin, while the opposite holds for the second case. When  $s < \underline{s}(\alpha)$ , the searchers' reservation price is high enough to prevent the firms to charge the prices larger than r. Conversely, when  $s > \overline{s}$ , then the searchers reservation price is too low, so that the firms decide not to serve the searchers at all. For intermediate values of the search costs, the firms randomize their prices between the two intervals.

We analyze different cases separately depending on the level of search costs. We start from the case  $s \in (\underline{s}(\alpha), \overline{s})$ . In equilibrium, each firm has to be indifferent between choosing any price from the support of the equilibrium cdf. From (2.19) and (2.20), we can find the equilibrium price distribution over the interval  $[p_{min,h}, v]$  as

$$G_h(p) = 1 - \frac{1-\theta}{\theta(1-\beta)} \frac{v-p}{p},$$
 (2.25)

such that  $p_{min,h}$  solves  $G_h(p_{min,h}) = 0$  and is equal to

$$p_{\min,h} = v \frac{1-\theta}{1-\theta\beta}.$$
(2.26)

Similarly, we can find the equilibrium price distribution over the interval  $[p_{min,l}, r]$  from (2.19) and (2.22) which give

$$G_l(p) = \frac{1}{\theta\beta(\lambda + \alpha - \alpha\lambda)} \left( 1 - \frac{v}{p}\lambda(1 - \theta) - \frac{\theta\beta(1 - \alpha)(1 - \lambda)}{2} \right), \qquad (2.27)$$

where  $p_{min,l}$  solves  $G_l(p_{min,l}) = 0$  and is equal to

$$p_{min,l} = \frac{2v\lambda(1-\theta)}{2-\theta\beta(1-\alpha)(1-\lambda)}.$$
(2.28)

If  $s < \underline{s}(\alpha)$ , then the firms charge only the prices  $p \leq r$ . We can find the equilibrium cdf from equations (2.21) and (2.22), with  $\beta = 1$ , as

$$\tilde{G}_l = \frac{1}{\theta(\lambda + \alpha(1 - \lambda))} \left( 1 - \frac{r}{p}(1 - \theta) - \frac{\theta(1 - \lambda)(1 - \alpha)}{2} \left( 1 + \frac{r}{p} \right) \right), \quad (2.29)$$

which is defined on the interval  $[\tilde{p}_{min,l}, r]$ , where  $\tilde{p}_{min,l}$  solves  $G_l(p) = 0$  and is equal to

$$\tilde{p}_{min,l} = \frac{r(2 - \theta(1 + \lambda + \alpha - \alpha\lambda))}{2 - \theta(1 - \lambda)(1 - \alpha)}.$$
(2.30)

If  $s > \bar{s}$ , then the firms charge only the prices  $v \ge p > r$ . We can find the equilibrium cdf from equations (2.19) and (2.20), with  $\beta = 0$ , as

$$\tilde{G}_h(p) = 1 - \frac{1-\theta}{\theta} \frac{v-p}{p}, \qquad (2.31)$$


Figure 2.3: Equilibrium pricing intervals depending on  $\alpha$  and s.

which is defined on the interval  $[\tilde{p}_{min,h}, v]$ , where  $\tilde{p}_{min,h}$  solves  $\hat{G}_h(p_{min,h}) = 0$ and is equal to

$$\tilde{p}_{min,h} = v(1-\theta). \tag{2.32}$$

The following proposition summarizes the equilibrium pricing strategies of the firms when both advertise and a positive share of the consumers are shoppers.

#### Proposition 5. Equilibrium pricing with shoppers

i) If  $s < \underline{s}(\alpha)$ , then there is a unique symmetric equilibrium, in which the firms choose their prices from the interval  $[\tilde{p}_{min,l}, r]$  according to the cdf  $G_l(p)$  given by (2.29).

ii) If  $s \in [\underline{s}(\alpha), \overline{s}]$ , there is a unique symmetric equilibrium, such that the firms choose their prices according to the following cdf

$$G(p) = \begin{cases} \beta G_{l}(p) & \text{if } p \in [p_{\min,l}, r], \\ \beta & \text{if } p \in (r, p_{\min,h}), \\ \beta + (1 - \beta)G_{h}(p) & \text{if } p \in [p_{\min,h}, v], \end{cases}$$
(2.33)

with  $G_l(p)$  and  $G_h(p)$  given by (2.25), and  $\beta$  is given by (2.23).

iii) If  $s > \bar{s}$ , then there is a unique symmetric equilibrium, in which the firms choose their prices from the interval  $[\tilde{p}_{min,h}, v]$  according to the cdf  $\tilde{G}_h(p)$  which is given by (2.25), where  $\beta = 0$ .



Figure 2.4: Equilibrium cdfs, given v = 1,  $\theta = 0.5$ ,  $\lambda = 0.5$ .

Given both firms advertise their products, an increasing share of naive consumers has different effects on the equilibrium prices compare to the previous model without informed consumers. In the previous model, we have shown that given the number of advertisements, consumers always suffer from the existence of naive consumers. In the next section, we show that in the presence of shoppers, the effect of naive consumers on the expected prices and consumer welfare can be positive, negative, or zero depending on the level of search cost.

### 2.5.2 Comparative Statics

In this section, we analyze how the equilibrium pricing strategies of the firms depend on the share of sophisticated consumers,  $\alpha$ , as well as on the search cost s in the presence of a positive share of shoppers. Figure 2.3 shows firms' equilibrium pricing intervals depending on the share of the sophisticated searchers and the

level of search cost.

We distinguish the following four cases depending on the level of search cost s. If  $s \geq \bar{s}$ , then for any  $\alpha \in [0, 1]$ , the firms choose prices only from the interval  $[\tilde{p}_{min,h}, v]$ , according to the cdf  $\tilde{G}_h(p)$ . When  $s \in (\underline{s}_{max}, \bar{s})$ , then for any  $\alpha \in [0, 1]$  the firms charge prices from the intervals  $[p_{min,l}, r]$  and  $[p_{min,h}, v]$ , with probabilities  $\beta$  and  $1 - \beta$ , and according to the cdfs  $G_l(p)$  and  $G_h(p)$ , respectively. If  $s \in [\underline{s}_{min}, \underline{s}_{max}]$ , then there is an  $\hat{\alpha}(s)$  such that the firms choose prices only from the interval  $[\tilde{p}_{min,l}, r]$  when  $\alpha < \bar{\alpha}(s)$ , and randomize between the intervals  $[p_{min,l}, r]$  and  $[p_{min,h}, v]$  when  $\alpha > \bar{\alpha}(s)$ . In the last case, if  $s \in [0, \underline{s}_{min}]$ , then the firms choose prices from the interval  $[\tilde{p}_{min,l}, r]$  according to the cdf  $\tilde{G}_l(p)$ .

Figure 2.4 depicts the firms' equilibrium pricing strategies for different intervals of search costs s and three different values of  $\alpha$ , given the parameter values v = 1,  $\theta = 0.5$ , and  $\lambda = 0.5$ . If  $s \in [0, \underline{s}_{min} = 0.25]$ , then the firms charge only the prices smaller than the searchers' reservation price and all consumers are active in the market. Figure 2.4a shows also that if the share of naive consumers increases, then the firms charge prices from a narrower interval. If  $s \in (\underline{s}_{min} = 0.25, \underline{s}_{max} = 0.3)$ , then firms charge prices only from lower price interval if the share of the naive consumers are less than the threshold value, otherwise they randomize prices between high and low price intervals. Figure 2.4b reflects this case. If  $s \in (\underline{s}_{max} = 0.3, \overline{s} = 0.375)$ , then, as shown in Figure 2.4c, the firms randomize their prices between low and high price levels for any value of  $\alpha$ . In this case, the searchers do not consider the product priced higher than their reservation price r. Figure 2.4d depicts the last case, *i.e.*,  $s > \overline{s} = 3/8$ , in which the firms do not serve searchers and charge only the prices higher than r.

Now we analyze how the increasing share of the naive searchers affects the firms' equilibrium pricing strategies in the four different cases mentioned above. As  $\bar{s}$  and the firms' equilibrium pricing strategies do not depend on  $\alpha$ , the increasing share of naive searchers does not change the expected prices and consumer welfare when  $s \geq \bar{s}$ .

If  $s \in [0, \underline{s}_{min}]$ , then in the equilibrium, firms choose prices only from the interval  $[\tilde{p}_l, r]$ , according to the cdf  $\tilde{G}_l$  which is given by (2.29). The derivative of  $\tilde{p}_l$  with respect to  $\alpha$  is

$$\frac{\partial \tilde{p}_l}{\partial \alpha} = -\frac{2r\theta(1-\lambda)(2-\theta)}{(2-\theta(1-\lambda)(1-\alpha))^2} < 0,$$

so the minimum price in the market decreases in the share of the sophisticated



Figure 2.5: Expected prices depending on  $\alpha$ , given  $v = 1, \theta = 0.5, \lambda = 0.5$ .

searchers. On the other hand, the first derivative of  $\tilde{G}_l$  with respect to  $\alpha$  is

$$\frac{\partial \hat{G}_l}{\partial \alpha} = \frac{(1-\lambda)(2-\theta)(r-p)}{2p\theta(\lambda+\alpha-\lambda\alpha)^2} > 0,$$

*i.e.*, when the share of sophisticated consumers increases then new equilibrium cumulative distribution functions of the firms are first-order stochastically dominated by their previous ones. Thus, the expected prices in the market decrease in the share of sophisticated consumers. In other words, all consumers suffers from the increasing share of naive consumers. Figure 2.5a depicts how the expected prices change in the share of the naive searchers, given the specific parameter values.

If  $s \in [\underline{s}_{max}, \overline{s})$ , then for any  $\alpha \in [0, 1]$ , the firms charge the prices from both intervals,  $[p_{min,l}, r]$  and  $[p_{min,h}, v]$  with the probabilities,  $\beta$  and  $1 - \beta$ , respectively. Note that  $\beta$  is decreasing in the share of the sophisticated searchers,  $\alpha$ . In other words, if the share of naive consumers increases then the firms charge low prices with a higher probability. This increases the expected demand as the searchers are more likely active in the market. Note also that the firms expected profit is equal to  $v\theta(1-\theta)$  and independent of  $\alpha$ . As the firms' expected demands increase, while their expected profits remain constant, the expected prices in the market decrease in the share of naive consumers. Figure 2.5c shows how the expected prices change in  $1 - \alpha$  for four different values of  $s \in [\underline{s}_{max}, \overline{s})$ .

If  $s \in (\underline{s}_{min}, \underline{s}_{max})$ , then the firms' pricing strategies depends on the share of naive consumers. If the share of the sophisticated searchers fulfills  $\alpha < \hat{\alpha}$ , then the firms choose prices only from the interval  $[\tilde{p}_{min,l}, r]$ , where  $\hat{\alpha} \in (0, 1)$ solves  $\underline{s}(\hat{\alpha}) = s$ . In this case, the expected prices decrease in the share of the sophisticated searchers. If  $\alpha > \hat{\alpha}$ , then the firms' equilibrium strategies are analogous to the case in which  $s \in [\underline{s}_{max}, \overline{s}]$ , and the expected prices in the market increase in the share of naive consumers. As it is shown in Figure 2.5b, the consumers benefit from the increasing share of naive consumers when  $\alpha > \hat{\alpha}$ , and the opposite holds when  $\alpha < \hat{\alpha}(s)$ . We summarize these results as follows:

#### Corollary 5. Consumer welfare

i) If  $s \in [0, \underline{s}_{min}]$ , then the consumers suffer from the increasing share of naive searchers.

ii) If  $s \in (\underline{s}_{min}, \underline{s}_{max})$  and  $\alpha \leq \hat{\alpha}(s)$ , then the consumers suffer from the increasing share of naive searchers, if  $\alpha > \hat{\alpha}(s)$  then the consumers benefit from the increasing share of naive searchers.

iii) If  $s \in [\underline{s}_{min}, \overline{s}]$ , then the consumers benefit from the increasing share of naive searchers.

iv) If  $s > \bar{s}$ , then the consumer welfare and the expected prices do not depend on the existence of naive searchers.

## 2.6 Conclusion

In this paper, we have studied how the firms' advertising and pricing strategies, as well as consumer welfare, depend on the consumers' search behavior. We find that, although the firms are ex-ante symmetric, their advertising strategies are asymmetric if the share of naive consumers who pay attention to the advertisements is relatively low. Consumer welfare does not monotonically depend on the share of naive consumers. Depending on the other parameters and the existing share of naive consumers, a small increase in their share can be beneficial for all consumers.

Usually, the share of the consumers who are directed by the advertisements largely depends on platform's advertising strategies, rather than an individual firm's advertising decision. For example, placing an advertisement on top of the organic results may attract more consumers' attention compared to the advertisements on the right or on the left side of the results. Similarly, labeling the advertised products as "best deal" or "special offer" may attract more consumers' attention. This paper shows that, consumer welfare can be improved by regulating advertising policy of the online retailers or price comparison sites. These regulations might include restrictions on the position of the advertisements or the usage of special labels for the advertised products, as well as regulating advertising fees. But note also that, the same regulation does not always positively affect consumer welfare. The positive effect of these regulations largely depends on the other parameters and the existing share of naive consumers.

## 2.7 Appendix

**Proof of Lemma 1**. First we prove the second part of the lemma.

*ii)* Suppose that there is an interval (x, y), such that firm *i* attaches zero probability in the equilibrium, while it sets prices from the intervals  $(x - \epsilon, x]$  and  $[y, y + \epsilon)$ , for any  $\epsilon > 0$ , with positive probabilities.

First, we show that if it is the case, then firm j will not charge prices from the interval (x, y), as well.

Note that, firm *i* cannot charge the price *y* with a positive probability. If both firms charge the price *y* with a positive probability, then firm *i* can strictly be better off by charging price  $y - \varepsilon$ , with an arbitrarily small  $\varepsilon > 0$ , with the same probability. If only firm *i* charges the price *y* with a positive probability, then the difference in firm *j*'s expected demand on the left and the right limits of *y* is  $\theta^2 \gamma > 0$ . So, there always exists an  $\varepsilon > 0$  such that firm *j* will not charge prices from the interval  $[y, y + \varepsilon]$ , as long as firm *i* chooses *y* with a positive probability. If firm *j* does not charge prices from  $[y, y + \varepsilon]$ , then firm *i*'s expected demand is the same at all prices from this interval and it will be better of charging only the price on the left limit of  $y+\varepsilon$ . This contradicts the assumption that firm *i* charges prices from the interval  $[y, y + \varepsilon]$ , for any  $\varepsilon > 0$ , with a positive probability. As

#### 2.7. APPENDIX

firm j's expected demand will be the same at all prices  $p \in (x, y]$  and firm i does not charge y with a positive probability, firm j will prefer to charge price y to any other price from the interval (x, y).

Now, we show that this strategy cannot be an equilibrium.

In the equilibrium, both of the firms cannot charge the price x with a positive probability. Otherwise, one of the firms can charge price  $x - \varepsilon$ , with arbitrarily small  $\varepsilon > 0$ , with the same probability it used to charge x, and can increase its expected profit by attracting all the sophisticated consumers who used to visit the other firm before him. Although the firm's expected demand increases, its expected price decreases only infinitesimally.

If only one firm charges the price x with a positive probability, then it would be profitable for that firm to deviate from x to  $y - \varepsilon$ , with an arbitrarily small  $\varepsilon$ , as it will not lose any demand but will increase its price.

If none of the firms charge price x with a positive probability, then again one of the two firms can deviate from its equilibrium pricing strategy to  $y - \varepsilon$ . Note that, firms' expected profits have to be the same at all prices from the equilibrium price range. As x is also in the equilibrium range of the prices and firm's expected demands are the same at both x and  $y - \varepsilon$ , firm's expected profit is higher at the price  $y - \varepsilon$  compared to the equilibrium profit.

In the equilibrium, there cannot be an interval (x, y), such that at least one of the two firms attaches zero probability. Thus, both firms' equilibrium cdfs have to be strictly increasing in their supports.

i) Assume that, firm i's minimum price  $p_{min,i}$  is smaller than firm j's minimum price  $p_{min,j}$ . Then there is an interval  $[p_{min,i}, p_{min,j})$ , from which firm i chooses prices, while firm j does not. As firm i's expected demand is the same at all prices  $p \in [p_{min,i}, p_{min,j})$ , in the equilibrium, it will not charge any price from the interval  $[p_{min,i}, p_{min,j})$ .

Analogously, if firm *i*'s maximum price  $p_{max,i}$  is larger than firm *j*'s maximum price  $p_{max,j}$ , then there is an interval  $(p_{max,j}, p_{max,i}]$ , such that firm *j* attaches zero probability in the equilibrium. As firm *i*'s expected demand is the same at all prices  $p \in (p_{max,j}, p_{max,i}]$ , firm *i* will prefer to charge the price  $p_{max,i}$  to any price  $p \in (p_{max,j}, p_{max,i})$ . But, this is contradictory to the strict monotonicity of equilibrium cdfs.

**Proof of Lemma 2**. First of all, we show that, none of the firms charges p < p

 $p_{max}$  with a positive probability, *i.e.*, there is no atom in equilibrium cdfs. Consider the opposite holds. Firm *i* sets price  $p \in [p_{min}, p_{max})$  with a positive probability  $\gamma$ . Note that, the sophisticated consumers' demand is price sensitive when the prices are lower than their reservation price and both products match their needs. So, the difference in firm *j*'s demand on the left and right limits of *p* is equal to  $\theta^2 \gamma > 0$ . This means, there is always an  $\varepsilon > 0$ , such that firm *j* will not charge any price from the interval  $[p, p + \varepsilon)$ , as long as firm *i* attaches a positive probability to *p*. But this is contradictory to the second part of Lemma 1.

Note also that, in the equilibrium, both of the firms cannot charge the maximum price  $p_{max}$  with a positive probability. Because one of the two firms can charge price  $p_{max} - \varepsilon$ , with the same probability as it used to charge  $p_{max}$  and can attract all the indifferent sophisticated consumers.

If  $p_{max} < r$  and none of the firms charges the maximum price with a positive probability, then one of the firms can profitably deviate from  $p_{max}$  to r without losing any demand. Similarly, if only one firm charges  $p_{max} < r$  with a positive probability, then that firm can charge the reservation price with the same probability without losing any demand. So, in the equilibrium, the maximum price cannot be smaller than the reservation price.

Now we show that, in the equilibrium, none of the firms charges the reservation price with a positive probability.

Assume that firm *i* charges the reservation price with a probability  $\gamma$ . Then its expected profit from charging the reservation price is equal to  $r\theta(1-\theta)$ , while its expected profit at the minimum price is  $p_{min}\theta$ . As the firms' expected profit has to be the same at all prices in the equilibrium strategies,  $p_{min} = r(1-\theta)$ . The firm *j*'s expected profit at the reservation price is equal to  $r\theta(1-\theta+\gamma\theta/2)$ , while its expected profit at the minimum price is equal to  $p_{min}\theta$ . From the equality of profits, we get the minimum price  $p_{min} = r(1-\theta+\gamma\theta/2)$ . According to Lemma 2,  $p_{min} = r(1-\theta) = r(1-\theta+\gamma\theta/2)$ , but this can be true only if  $\theta = 0$ , or  $\gamma = 0$ .

**Proof of Lemma 7.** The first part of the proof is analogous to the proof of Lemma 2. In the equilibrium, both of the firms cannot charge the maximum price  $p_{max}$  with a positive probability. Because one of the two firms can charge price  $p_{max} - \varepsilon$ , with the same probability as it used to charge  $p_{max}$ , and can attract all the indifferent sophisticated consumers.

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If  $p_{max} < r$  and none of the firms charges the maximum price with a positive probability, then one of the firms can profitably deviate from  $p_{max}$  to r without losing any demand. Similarly, if one of the two firms charges  $p_{max} < r$  with a positive probability then that firm can charge the reservation price with the same probability without losing any demand. So, in the equilibrium, the maximum price cannot be smaller than the reservation price.

Now we show that, in the equilibrium, only the advertised firm charges the reservation price with a positive probability.

Assume that the non-advertised firm charges the reservation price with a probability  $\gamma$ . Then its expected profit from charging the reservation price is equal to  $r\theta(1-\theta)$ , while its expected profit from charging the minimum price is  $p_{min}^{o}\theta(\alpha + (1-\alpha)(1-\theta))$ . As the firms' expected profit has to be the same at all prices in the equilibrium strategies,  $p_{min}^{o} = r(1-\theta)/(1-\theta+\alpha\theta)$ . Similarly, the advertised firm's expected profit at the reservation price is equal to  $r\theta\alpha(1-\theta+\gamma\theta/2) + r(1-\alpha)\theta$ , while its expected profit from charging the minimum price is equal to  $p_{min}^{o}\alpha\theta$ . From the equality of profits, we get the minimum price  $p_{min}^{o} = r(2-2\alpha\theta+\gamma\alpha\theta)/2\alpha$ . According to Lemma 6 the firms charge their prices from the same interval, *i.e.*,  $r(1-\theta)/(1-\theta+\alpha\theta) = r(2-2\alpha\theta+\gamma\alpha\theta)/2\alpha$ . But there is no positive solution for  $\gamma$ .

## Chapter 3

# The Nash Bargaining Solution in Vertical Relations With Linear Input Prices

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## 3.1 Introduction

We investigate the properties of the Nash bargaining solution when an upstream supplier bargains with a downstream firm over a linear wholesale price. The Nash bargaining solution is given by equating the slopes of the bargaining frontier and the Nash product. The slope of the Nash product depends directly on the parties' disagreement payoffs and the profit weights.<sup>1</sup> It is well understood that a better disagreement payoff and a higher profit weight in the Nash product increase a party's bargaining power, and hence, the profit share it gets.<sup>2</sup> Our focus, in contrast, is on the slope of the bargaining frontier which is different from -1

<sup>&</sup>lt;sup>1</sup> In case of the so-called asymmetric Nash bargaining solution, the profit weights differ from one half.

<sup>&</sup>lt;sup>2</sup> See Binmore, Rubinstein and Wolinsky (1986) for the primitives which determine the weights in the Nash product.

when bargaining is over a linear input price.<sup>3</sup> This is a direct result of assuming that profits can only be transferred with a linear input price which leads to the well-known double mark-up problem.<sup>4</sup> An increase of the wholesale price (so as to shift profits to the upstream firm) necessarily reduces the overall surplus available. Intuitively, the "steeper" the slope of the bargaining frontier, the harder it is to shift profits to the upstream firm so that the profit share of the downstream firm increases.

Our analysis of the bargaining frontier confirms this basic intuition and we derive a simple and instructive formula which comprises all three determinants of parties' bargaining powers according to the Nash bargaining solution; namely, the disagreement payoffs, the weights in the Nash product, and the slope of the bargaining frontier. The critical step in our analysis is to show that the slope of the bargaining frontier is equal to the total value of 1 plus the *derived demand elasticity* of the downstream firm for the input. The *derived demand elasticity* is the elasticity of the optimal order quantity with respect to the price of the input good. Its absolute value must be between zero and one to ensure the existence of a Nash bargaining solution in case of a linear transfer price. It then follows that a more elastic equilibrium derived demand goes hand in hand with an increasing share of the total profit the downstream firm gets. This is driven by the fact that the total profits (i.e., gains from trade) decrease the more with a marginal increase of the input price the more elastic the derived demand becomes.

In addition, we present a generalization of our model with N downstream firms. If contracts are unobservable, a downstream firm's quantity is not affected by the rival firms' input prices. Then, the relation between the profit sharing rule and the equilibrium demand elasticity which we have derived for the bilateral monopoly setting stays true. If contracts are observable, however, also cross-input-price elasticities affect the sharing rule and dilute the relation between profit shares and demand elasticity.

Finally, we extend our setup toward a "flexible" production technology of a downstream manufacturer which combines two inputs in a cost minimizing way. The simple relation between the equilibrium elasticity of the derived demand and

<sup>&</sup>lt;sup>3</sup> With efficient bargaining (which requires more than one price; e.g. a two-part tariff) total surplus is always maximal and the slope of the bargaining frontier is -1.

<sup>&</sup>lt;sup>4</sup> Note that the double mark-up problem can also occur with a two-part tariff when there is uncertainty and risk aversion on the downstream firm's side (see Rey and Tirole (1986)).

the profit sharing rule holds true also in this setting. A more elastic derived demand is now directly the result of the technological possibility to substitute between different inputs which strengthens the downstream firm's bargaining position. This insight applies to many setups, for instance, if an international firm bargains with national unions over wages, where a union's bargaining position is the weaker the easier it is to shift production between countries (see Chapter 4 of this thesis).

In particular, our analysis can be important for empirical studies on bargaining power and on profit sharing in vertical markets as we provide a structural model which directly links up- and downstream profits with equilibrium (final and derived) demand elasticities, disagreement payoffs, and firms' exogenous Nash profit weights. Thus, our approach allows to estimate a party's Nash profit weight if profits are observed and if the derived demand elasticity (or the final good elasticity) is estimated.

The concept of Nash bargaining over linear input prices is widely used to solve the bilateral bargaining problem between up- and downstream firms, both theoretically (Horn and Wolinsky, 1988a; Dobson and Waterson, 1997; von Ungern-Sternberg, 1996; Naylor, 2002; Symeonidis, 2010; Iozzi and Valletti, 2014; Gaudin, 2015, 2016) and empirically (Gowrisankaran, Nevo and Town, 2015; Draganska, Klapper and Villas-Boas, 2008; Grennan, 2013, 2014).<sup>5</sup>

The chapter proceeds as follows. In Section Section 3.2 we present the analysis of the bilateral bargaining problem and we derive the central profit sharing formula. In Section 3.3 we analyze several applications of our main formula to show how the downstream firm's profit share and profit level depends on the demand elasticities. Section 3.4 provides one extension with N downstream firms and another extension dealing with a flexible production technology. Finally, Section 3.5 concludes.

<sup>&</sup>lt;sup>5</sup> Nash bargaining over linear input prices has been widely assumed in labor economics where input prices are workers' wages. For instance, Dowrick (1990) and Conlin and Furusawa (2000) compare inefficient bargaining over wages with efficient bargaining over input prices *and* employment. They derive conditions such that the employer is better off under inefficient bargaining.

## **3.2** Model and Analysis

#### 3.2.1 The Model Setup

We refer to a successive monopoly problem with an upstream and a downstream firms. The input is produced at marginal cost c = 0 and transformed one to one by the downstream firm into the final good. Consumer demand for the final good is given by x(p), where p is the final good price, and p(x) gives the inverse demand. The game proceeds in two stages. In the first stage both firms bargain over a linear wholesale price w. In the second stage, the downstream firm sets the final good price (or, equivalently, the quantity of the final good).

We impose the standard assumption

$$p''x + p' < 0, (3.1)$$

which guarantees the existence of a unique equilibrium. We abstract from all downstream costs other than the procurement costs  $w \cdot x$ , such that the downstream firm's profit is given by

$$\pi := p(x)x - w \cdot x.$$

while the upstream firm maximizes  $L := w \cdot x$ . In equilibrium the downstream firm chooses quantity  $x^*$  such that the first-order condition

$$p'(x^*)x^* + p(x^*) = w ag{3.2}$$

holds. For any given w, Equation (3.2) determines a well-defined function, the *derived demand*  $x^*(w)$  of the downstream firm when bargaining with the upstream firm. Taking the total derivative of (3.2) gives the slope of the derived demand function:

$$\frac{dx^*}{dw} = \frac{1}{p''x + 2p'},$$
(3.3)

such that  $dx^*/dw < 0$ . Due to (3.1), the downstream firm's second-order condition

$$\frac{d^2\pi}{dx^2} = p''x + 2p' < 0$$

holds, which ensures that the derived demand function is strictly downward sloping. We can write the downstream firm's profit as a function of its derived demand, that is

$$\pi(w) = p(x^*(w))x^*(w) - wx^*(w).$$
(3.4)

The upstream firm's profit function can be written as

$$L(w) = wx^{*}(w). {(3.5)}$$

## 3.2.2 The Bargaining Frontier

As  $d\pi/dw < 0$  and  $dx^*/dw < 0$  hold, there is a one-to-one relation between wage levels and profit levels. Thus, the supplier's profit can be written as a well-defined function of the downstream firm's profit,  $L = L(\pi(w))$ , which assigns each profit level of the downstream firm the according profit level of the upstream firm. We denote  $L = L(\pi(w))$  the *bargaining frontier*. The chain rule yields

$$\frac{dL(\pi(w))}{dw} = \frac{dL(\pi(w))}{d\pi(w)} \cdot \frac{d\pi(w)}{dw}.$$

Rearranging gives the slope of the bargaining frontier

$$\frac{dL(\pi(w))}{d\pi(w)} = \frac{dL(\pi(w))}{dw} \left(\frac{d\pi(w)}{dw}\right)^{-1}.$$
(3.6)

Denote the *derived demand elasticity* as

$$\epsilon := \frac{dx^*(w)}{dw} \frac{w}{x^*(w)}.$$

Using  $dL/dw = x + w \cdot dx/dw$  (which follows from Equation (3.5)) and  $d\pi/dw = \partial \pi/\partial w = -x$  (which follows from Equation (3.4) and the Envelope Theorem), the slope of the bargaining frontier can be written as a function of the derived demand elasticity,

$$\frac{dL(\pi(w))}{d\pi(w)} = -(1+\epsilon). \tag{3.7}$$

This formula reflects that the transferability of utility between the retailer and the supplier depends crucially on the derived demand elasticity. The more inelastic derived demand is in equilibrium the larger is the loss the retailer has to bear in order to shift one unit of utility to the supplier. We will speak of a bargaining frontier effect when a change in the economic environment changes the derived demand elasticity  $\epsilon$  and thus the slope of the bargaining frontier.

Next, we describe the curvature of the bargaining frontier  $L(\pi(w))$ . A necessary condition for a local maximum of  $L(\pi)$  is  $dL/d\pi = 0$ . With formula (3.7) it is straightforward to check that there is a unique optimum at  $\epsilon = -1$ . If derived demand is elastic,  $\epsilon < -1$ , then  $dL/dw = x^*(w) + w \ dx^*(w)/dw < 0$ . As



Figure 3.1: Bargaining frontier

 $d\pi/dw < 0$ , it follows that  $dL/d\pi > 0$ , that is, the bargaining frontier is positively sloped. If derived demand is inelastic,  $\epsilon > -1$ , then  $dL/dw = x^*(w) + w$  $dx^*(w)/dw > 0$ . As  $d\pi/dw < 0$ , it follows that  $dL/d\pi < 0$ , that is, the bargaining frontier is negatively sloped.

Figure 3.1 depicts the bargaining frontier. If the derived demand is elastic, that is,  $\epsilon < -1$ , then dL/dw < 0 and  $d\pi/dw < 0$  hold such that both the supplier and the retailer can obtain a higher payoff with a lower input price w. Therefore, due to Pareto-optimality, the Nash bargaining solution has to lie in the domain where the derived demand is inelastic, that is,  $\epsilon \ge -1$ .

## 3.2.3 Nash Bargaining

We investigate under which conditions a Nash bargaining solution exists.

**Lemma 8.** A bargaining problem  $(X, (\pi_0, L_0))$  is defined by the set of feasible payoff combinations  $X = \{(\pi, L) \in \mathbb{R}^2 | L \leq L(\pi)\}$  and the profits  $(\pi_0, L_0)$  obtained if negotiation breaks down. Suppose that (I)  $L(\pi)$  is a concave function and (II) there exist  $(\pi, L) \in X$  with  $\pi > \pi_0$  and  $L > L_0$ . Then there exists a unique solution  $(\pi(w^*), L(w^*))$  to the Nash bargaining problem which is given by

$$\arg\max_{(\pi,L)} \{ (\pi - \pi_0)^{\alpha} (L - L_0)^{1-\alpha} | L \le L(\pi) \},\$$

where parameter  $\alpha \in [0, 1]$  gives the downstream firm's profit weight.<sup>6</sup>

**Proof:** See for instance Eichberger (1993), Theorem 9.2

In order to apply Lemma 8, we investigate under which conditions the bargaining frontier is concave.

**Lemma 9.** A necessary condition for the bargaining frontier to be concave is that the upstream firm's second-order condition

$$\frac{d^2L}{dw^2} < 0 \tag{3.8}$$

holds. A sufficient condition for the bargaining frontier to be concave is that the derived demand is concave, that is,

$$\frac{d^2x^*}{dw^2} < 0. (3.9)$$

**Proof:** We investigate under which conditions  $d^2L/d\pi^2 < 0$  holds. The chain rule gives

$$\frac{d\left[\frac{dL(\pi(w))}{d\pi(w)}\right]}{dw} = \frac{d\left[\frac{dL(\pi(w))}{d\pi(w)}\right]}{d\pi(w)} \cdot \frac{d\pi(w)}{dw}$$

such that

$$\frac{d^{2}L(\pi(w))}{d\pi^{2}} = \frac{d\left[\frac{dL(\pi(w))}{d\pi(w)}\right]}{d\pi(w)} = \frac{d\left[\frac{dL(\pi(w))}{d\pi(w)}\right]}{dw} \left(\frac{d\pi(w)}{dw}\right)^{-1} \\
= \frac{d\left[\frac{dL}{dw}\left(\frac{d\pi}{dw}\right)^{-1}\right]}{dw} \left(\frac{d\pi}{dw}\right)^{-1} \\
= \left[\frac{d^{2}L}{dw^{2}} \cdot \left(\frac{d\pi}{dw}\right)^{-1} + \frac{dL}{dw}(-1)\left(\frac{d\pi}{dw}\right)^{-2}\frac{d^{2}\pi}{dw^{2}}\right] \left(\frac{d\pi}{dw}\right)^{-1} \\
= \left[\frac{d^{2}L}{dw^{2}} - \frac{dL}{dw}\left(\frac{d\pi}{dw}\right)^{-1}\frac{d^{2}\pi}{dw^{2}}\right] \left(\frac{d\pi}{dw}\right)^{-2}, \quad (3.10)$$

where

$$\frac{d^2\pi}{dw^2} = \frac{d(d\pi/dw)}{dw} = \frac{d(-x)}{dw} = \frac{-dx}{dw} > 0$$

Thus,  $d^2L/dw^2 < 0$  is a necessary condition for the bargaining frontier to be concave.  $d^2L/d\pi^2 < 0$  is equivalent to

$$2\frac{dx}{dw} + w\frac{d^2x}{dw^2} < \left(x + w\frac{dx}{dw}\right)\frac{1}{x}\frac{dx}{dw} = (1+\epsilon)\left(\frac{dx}{dw}\right),$$

<sup>&</sup>lt;sup>6</sup> Strictly speaking,  $\alpha$  denotes the weight on the downstream firm's gain from trade.

or

$$\frac{d^2x}{dw^2} < \frac{dx}{dw} \frac{\epsilon - 1}{w}$$

This holds if the derived demand is not too convex and, in particular, if  $d^2x/dw^2 < 0$ .

Assumption. The derived demand is concave, that is,  $d^2x^*/dw^2 < 0$  holds.

Given the preceding Assumption holds (and given that the outside options for both firms are sufficiently small) the Nash bargaining solution is given by the maximum of the Nash product  $N = (\pi - \pi_0)^{\alpha} \cdot (L - L_0)^{1-\alpha}$  subject to  $L \leq L(\pi)$ .

The slope of the iso-Nash-product lines is given by the total differential of the Nash product

$$dN = \alpha (\pi - \pi_0)^{\alpha - 1} \cdot (L - L_0)^{1 - \alpha} d\pi + (\pi - \pi_0)^{\alpha} \cdot (1 - \alpha) (L - L_0)^{-\alpha} dL = 0.$$

Rearranging gives the slope of the objective function as

$$\frac{dL}{d\pi} = -\frac{\alpha}{(1-\alpha)} \frac{(L-L_0)}{(\pi-\pi_0)}$$
(3.11)

Pareto-optimality implies  $L = L(\pi)$  and thus  $dL/d\pi = dL(\pi)/d\pi$ . Using (3.11) and (3.7), we can equate the slopes of the objective function and of the bargaining frontier and obtain

$$L - L_0 = \frac{1 - \alpha}{\alpha} \cdot (1 + \epsilon) \cdot (\pi - \pi_0). \tag{3.12}$$

**Theorem.** Suppose a retailer and a supplier bargain over a linear input price via Nash bargaining while the retailer sets quantities in the final goods market. Then, in equilibrium the relation between profit shares, profit weights and the derived demand elasticity is given by (3.12).

Note that this formula gives non-negative profits for the upstream and the downstream firm as  $\epsilon \geq -1.^7$  It gives an equilibrium condition and states that the higher the derived demand elasticity in equilibrium is, the larger is the profit share of the downstream firm. In principle, it can be used to estimate empirically the profit weights of the different parties: if the firms' profits can be observed and if the derived demand elasticity was known, then the parties' bargaining power could be estimated. The derived demand elasticity, however, is typically unknown. Therefore, we show in the following that it is closely related to the final good demand elasticity which is often determined in empirical studies.

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<sup>&</sup>lt;sup>7</sup> Other papers such as Grennan (2013, 2014) have derived similar interpretations of the Nash bargaining solution (see also Gaudin, 2016). None of these papers, however, stresses the relation to the derived demand elasticity which we focus on.

#### 3.2.4 Demand Elasticity and Derived Demand Elasticity

In order to derive a relationship between the demand elasticity and the derived demand elasticity, we distinguish between the *demand function* x(p) and the *derived demand function* x(w). Demand elasticity is defined by

$$\eta = \left(\frac{dp}{dx}\right)^{-1} \frac{p}{x}.$$

We say that (derived) demand elasticity increases if  $\eta$  ( $\epsilon$ , resp.) increases in absolute value. Equations (3.2) and (3.3) yield the following relationship between the demand and the derived demand elasticity:

$$\epsilon = \frac{1+\eta}{\frac{p''}{p'}x+2}.$$
(3.13)

For instance, for linear final demand, the relation between the elasticities is linear, such that with known profits and known equilibrium final demand elasticity, (3.12) and (3.13) allow to estimate the parties' bargaining power.

Symmetric Nash Bargaining. Under symmetric Nash bargaining where both parties have no outside option we can derive a more explicit relation between  $\epsilon$  and  $\eta$ . Under symmetric Nash bargaining, the Nash product

$$[(p(x(w)) - w)x(w)] \cdot [w \cdot x(w)].$$
(3.14)

is maximized. Considering the first-order conditions and applying equations (3.2) and (3.3) gives  $3p'p + xp''p + x(p')^2 = -2xp' \cdot (p''x + 2p')$ . Using this as well as the equations (3.2) and (3.3), we can re-write the sum of the two elasticities as

$$\epsilon + \eta = \frac{3p'p + xp''p + x(p')^2}{xp'(p''x + 2p')} = -2.$$
(3.15)

Thus, under symmetric bargaining the two elasticities add up in equilibrium to -2. As the derived demand elasticity lies between -1 and 0, final demand is always elastic in equilibrium, that is,  $\eta \in (-2, -1)$ .

## 3.3 Applications

In this section we provide several applications of our Theorem. In particular, we reveal how demand elasticities and profit shares are related in equilibrium and provide instances where a higher demand elasticity increases the profits of the downstream firm.

#### 3.3.1 Linear Demand

Suppose that the downstream firm faces a linear inverse demand function p(x) = a - bx. The outside options are zero for both firms,  $\pi_0 = L_0 = 0$ . The downstream firm's profit is equal to  $\pi = (a - bx - w)x$ , while the supplier gets L = wx. In equilibrium, the downstream firm's first-order condition a - 2bx - w = 0 holds, which gives rise to the derived demand

$$x^*(w) = \frac{a-w}{2b}.$$
 (3.16)

As Condition (3.9) holds, the bargaining frontier is concave and a unique Nash bargaining solution exists. Substituting (3.16) into the downstream firm's and the supplier's profit functions gives  $\pi(w) = (a - w)^2 / (4b)$ , and L(w) = w(a-w) / (2b), respectively. Moreover, from (3.16) we obtain the derived demand elasticity

$$\epsilon = -\frac{w}{a-w}.$$

Using (3.12), we obtain the bargaining solution  $w^* = a(1 - \alpha)/2$  and  $x^* = a(1 + \alpha)/(4b)$ . In particular, both the derived demand and the demand elasticity depend only on firms' bargaining power and equal

$$\epsilon = -\frac{1-\alpha}{1+\alpha},$$
  
$$\eta = -\frac{3-\alpha}{1+\alpha}.$$

In equilibrium, the downstream firm's share is also independent of the demand function's parameters a and b as

$$\frac{\pi}{\pi + L} = \frac{1 + \alpha}{2(1 - \alpha)}$$

holds. Thus, the sharing rule between the up- and the downstream firm is independent from the exact specification of the linear demand function. Note that, under symmetric bargaining, in particular Equation (3.15) is satisfied, that is, the demand elasticities sum up to -2 in equilibrium.



(a) Derived demand elasticity as a function of the Nash profit weight



(b) Derived demand elasticity as a function of final demand elasticity

Figure 3.2: Derived demand elasticity in equilibrium

### 3.3.2 Non-Linear Demand

Suppose that the downstream firm's inverse demand function is given by  $p(x) = 1 - x^{\beta}$ . As in the previous example, the firms' outside options are zero, that is,  $\pi_0 = L_0 = 0$ . From  $p''(x) = \beta(1 - \beta)x^{\beta-2}$ , it follows that demand is concave (convex) for  $\beta < 1$  ( $\beta > 1$ ) and linear for  $\beta = 1$ . The demand elasticity equals

$$\eta = \frac{1}{-\beta x^{\beta-1}} \frac{1-x^{\beta}}{x}$$

As

$$\partial \eta / \partial \beta = \frac{1}{x^{\beta} \beta^2} \left( \beta \ln x - x^{\beta} + 1 \right) < 0 \tag{3.17}$$

holds,<sup>8</sup> the demand elasticity is increasing in  $\beta$ . The downstream firm's and the supplier's profits are  $\pi = (1 - x^{\beta} - w)x$ , and L = wx, respectively. From the downstream firm's first-order condition we obtain the derived demand

$$x^*(w) = ((1-w)/(\beta+1))^{1/\beta}.$$

and the derived demand elasticity

$$\epsilon = -\frac{w}{\beta(1-w)}.$$

In order to show that a unique Nash bargaining solution exists, we show that  $\frac{d^2L}{d\pi^2} < 0$  holds. The first and second derivatives of the firms' profits with respect

<sup>&</sup>lt;sup>8</sup> This holds as  $z := x^{\alpha} < 1$  and  $\alpha \ln x - x^{\alpha} + 1 = \ln x^{\alpha} - x^{\alpha} + 1 = \ln z - z + 1 < 0$ .

to w are given as

$$\begin{aligned} \frac{dL}{dw} &= \frac{1}{\beta} \frac{\left(-\frac{1}{\beta+1} \left(w-1\right)\right)^{\frac{1}{\beta}}}{w-1} \left(w-\beta+w\beta\right), \\ \frac{d^2L}{dw^2} &= \frac{1}{\beta^2} \frac{\left(-\frac{1}{\beta+1} \left(w-1\right)\right)^{\frac{1}{\beta}}}{\left(w-1\right)^2} \left(w-2\beta+w\beta\right), \\ \frac{d\pi}{dw} &= -\left(((1-w)/(\beta+1))^{1/\beta}, \\ \frac{d^2\pi}{dw^2} &= \frac{\left(-\frac{1}{\beta+1} \left(w-1\right)\right)^{\frac{1}{\beta}}}{\beta-w\beta}. \end{aligned}$$

Substituting these into (3.10) shows that  $d^2L/d\pi^2 < 0$  holds. Thus, the generalized Nash bargaining solution is given by

$$w^* = \frac{\beta(1-\alpha)}{1+\beta},$$
  
$$x^* = \left(\frac{\alpha\beta+1}{(1+\beta)^2}\right)^{1/\beta}$$

Using our central equation (3.12) and re-arranging yields

$$\epsilon = -\frac{1-\alpha}{1+\alpha\beta}.\tag{3.18}$$

The equilibrium derived demand elasticity is increasing in the demand elasticity parameter  $\beta$  and decreasing in the Nash profit weight  $\alpha$ . Figure 3.2a shows how equilibrium derived demand elasticity depends on the bargaining power of the downstream firm for a given  $\beta$ . Figure 3.2b gives the relationship between the derived demand elasticity and the final goods demand elasticity for a fixed  $\alpha$ .

Equation (3.12) yields also the sharing rule between the up- and the down-stream firms' profits:

$$\frac{\pi^*}{\pi^* + L^*} = \frac{1 + \alpha\beta}{2 + \beta - \alpha}.$$
(3.19)

The downstream firm's share is strictly decreasing in the demand parameter  $\beta$ . Figure 3.3 shows bargaining frontiers and Nash bargaining solutions for different values of  $\beta$ . The bargaining frontier for demand parameter  $\beta_1$  Pareto-dominates the bargaining frontier for demand parameter  $\beta_2$  if  $\beta_1 > \beta_2$ . In equilibrium, the downstream firm's profit  $\pi^*$  is increasing in  $\beta$  and therefore also in the final demand elasticity, see (3.17), while its profit share is decreasing in  $\beta$  according to (3.19).



Figure 3.3: Bargaining frontiers and Nash bargaining solutions for different values of  $\beta$ 

## 3.3.3 Competition With a Competitive Fringe Producing a Differentiated Good

In this section, we show that a downstream firm's profit can increase when the final demand becomes more elastic. Let downstream firm 1 bargain with an upstream input supplier over a linear input price w, and compete with a fringe of firms in the final product market. While all firms of the competitive fringe produce a homogenous good, it is differentiated from firm 1's product. The inverse demand of firm 1 is given by  $p_1 = A - x_1 - \gamma X_{-1}$ , where  $x_1$  gives firm 1's output and  $X_{-1}$  is the total output of the fringe firms. Similarly, the inverse demand for the fringe product is  $p_f = A - X_{-1} - \gamma x_1$ . Marginal costs for all fringe firms are given by c < A.

The competitive fringe acts as a Stackelberg follower. Given  $x_1$ , the total output of the fringe firms follows from equating the residual demand with marginal costs; i.e.,  $A - X_{-1} - \gamma x_1 = c$ . This yields

$$X_{-1}^* = \max\{A - c - \gamma x_1, 0\}.$$
(3.20)

Let  $\pi_0 = L_0 = 0$ , and let Nash bargaining be symmetric ( $\alpha = 0.5$ ). Suppose that  $X_{-1}^* > 0$  holds in equilibrium (we abstract from a corner solution). We can then

write the profit function of firm 1 as

$$\pi_1 = (A - x_1 - \gamma (A - c - \gamma x_1) - w) x_1,$$

Re-arranging firm 1's first-order condition gives the derived demand

$$x_1(w) = \frac{A(1-\gamma) - w + c\gamma}{2(1-\gamma^2)}.$$

and the elasticity of the derived demand

$$\epsilon = \frac{dx_1(w)}{dw}\frac{w}{x_1} = -\frac{w}{A - w - A\gamma + c\gamma}$$

As

$$\frac{d\epsilon}{d\gamma} = -w \frac{A-c}{\left(A-w-A\gamma+c\gamma\right)^2} < 0, \qquad (3.21)$$

the derived demand elasticity increases in the product differentiation parameter  $\gamma.$ 

While the upstream firm's profit equals  $L(w) = wx_1(w)$ , firm 1's profit, written as a function of w, is

$$\pi_1 = \frac{(A - w - A\gamma + c\gamma)^2}{4(1 - \gamma^2)}.$$

Next, we show that the bargaining frontier is concave  $(d^2L/d\pi^2 < 0)$ . The first and the second derivatives of firm 1's and the upstream firm's profits are

$$\frac{dL}{dw} = \frac{1}{2} \frac{A}{c(c-2A)} \left(c^2 - 2Ac + 2Aw\right),$$

$$\frac{d^2L}{dw^2} = \frac{A^2}{c(c-2A)},$$

$$\frac{d\pi}{dw} = -\frac{A^2}{2c^2 - 4Ac},$$

$$\frac{d^2\pi}{dw^2} = \frac{\left(-\frac{1}{\alpha+1} \left(w-1\right)\right)^{\frac{1}{\alpha}}}{\alpha - w\alpha}.$$

Substituting these values into (3.10) yields

$$\frac{d^2L}{d\pi^2} = \left(\frac{1}{2}\frac{A^2}{c^2 - 2Ac + Aw}\right) \left(\frac{A}{2c^2 - 4Ac}\left(c^2 - 2Ac + Aw\right)\right)^{-2} < 0.$$

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which proves the existence of a unique Nash bargaining solution. Inserting our results into  $L = (1 + \epsilon)\pi_1$  (which follows from (3.12)),<sup>9</sup> we obtain the bargaining solution

$$w^* = \frac{1}{4}(A - A\gamma + c\gamma).$$

This implies

$$x_1^* = \frac{3A(1-\gamma) + 3c\gamma}{8(1-\gamma^2)} \text{ and} \pi_1^* = \frac{9(A(1-\gamma) + c\gamma)^2}{64(1-\gamma^2)}.$$

Solving  $d\pi_1^*/d\gamma = 0$  yields the solution  $\gamma^* := \frac{1}{A}(A-c) < 1$ . As  $d^2\pi_1^*/d\gamma^2 > 0$  holds at  $\gamma^*$ , profit  $\pi_1^*$  has a local minimum at  $\gamma^*$ . Hence, firm 1's profit increases in  $\gamma$  as long as  $\gamma > \gamma^*$  holds and it decreases in  $\gamma$  as long as  $0 < \gamma < \gamma^*$  holds.

We have to ensure that  $X_{-1}^* > 0$  holds at  $x_1^*$ . Substituting  $x_1^*$  into (3.20), we get the condition

$$X_{-1}^* = \frac{-8c + 8A - 3A\gamma - 5A\gamma^2 + 5c\gamma^2}{8(1 - \gamma^2)} > 0.$$
(3.22)

Denote the numerator of (3.22) by  $\Psi$  with  $\Psi := -8c + 8A - 3A\gamma - 5A\gamma^2 + 5c\gamma^2$ . Note that  $\Psi(\gamma = 0) > 0$ , while  $\partial \Psi / \partial \gamma < 0$  always holds. Hence, we search for the critical value  $\overline{\gamma}$  which ensures (3.22) to hold for all  $\gamma < \overline{\gamma}$ . Solving  $\Psi = 0$ , we obtain one root which is possibly non-negative:

$$\overline{\gamma} := \frac{-3A + \sqrt{-320Ac + 169A^2 + 160c^2}}{10A - 10c}$$

Thus, assuming  $\gamma < \overline{\gamma}$  ensures that the total output of the competitive fringe is strictly positive in equilibrium. Furthermore,  $\gamma^* < \overline{\gamma}$  holds as evaluating  $\Psi$  at  $\gamma = \gamma^*$  gives

$$\Psi(\gamma^*) = \frac{5}{A^2} c \left( 2A^2 - 3Ac + c^2 \right) > 0.$$

We have therefore derived the following result.

<sup>&</sup>lt;sup>9</sup> The analysis of the monopoly case in the preceding section can be easily extended to the case with *n* competitive fringe firms. It is straightforward to check that the analysis extends to the case in which the inverse demand depends on a function  $f : \mathbb{R}^{n+1} \to \mathbb{R}_+$ , with  $\mathbf{x} := (x, x_1, ..., x_n) \to f(\mathbf{x})$ , which is linear in *x*. That is, the derivation of formula (3.12) holds if  $p = p(f(\mathbf{x}))$  as long as df/dx = 1.

**Proposition 6.** Assume  $\gamma < \overline{\gamma}$  (to ensure an interior equilibrium outcome). In the linear model with a competitive fringe, firm 1's profit increases in  $\gamma$ , whenever  $\gamma > \gamma^*$ , while the opposite holds for  $\gamma < \gamma^*$ . Note that  $\gamma^* < \overline{\gamma}$ .

Provided  $\overline{\gamma} > \gamma > \gamma^*$ , a larger  $\gamma$  implies a higher equilibrium elasticity of derived demand (see (3.21)). Therefore, a larger  $\gamma$  also implies a higher profit share for the downstream firm (as  $L = (1 + \epsilon)\pi_1$ ) and also a higher absolute profit level for the downstream firm (see the definition of  $\gamma^*$ ), even though a higher  $\gamma$  implies more homogenous goods.

## **3.4** Extensions

We show that our equilibrium condition holds also in more general setups, that is, for instance, if there are N > 1 downstream firms or if the downstream firm uses a flexible production technology which allows to produce with two input goods.

### 3.4.1 N Downstream Firms and Unobservable Contracts

We extend our model toward N downstream firms facing a single upstream firm U. As in the previous section, we normalize U's marginal production cost to zero and assume that all firms have the same production technology which transforms one unit of input to one unit of output. Firm  $i \in \{1, \ldots, N\}$  produces quantity  $x_i$  of a homogeneous product. Demand is given by the inverse demand function  $p(x_1, \ldots, x_N)$  for which the N-firm analogon of condition (3.1) is assumed to hold.

In the first stage of the game, U bargains simultaneously with the N downstream firms. We follow the literature on simultaneous Nash bargaining (see, for instance, Inderst and Wey (2003)) and assume that U bargains which the downstream firms through sales agents, that is, for each downstream firm there is one sales agent representing firm U in the negotiation. In the second stage, downstream firms compete  $\dot{a}$  la Cournot.

As contracts are unobservable, the sales agents and the downstream firms cannot observe outcomes in the other negotiations and therefore have to form beliefs on them. Most common in the economic literature on multilateral contracting are "passive beliefs" according to which it is assumed that all unobservable bargaining outcomes are equilibrium outcomes, even if it receives an out-of-equilibrium offer (Hart and Tirole, 1990; O'Brien and Shaffer, 1992; Inderst and Ottaviani, 2012)).<sup>10</sup> In order to guarantee that the Nash product is well-defined, we assume that a sales agent and the downstream firm he bargains with have the same beliefs on the outcomes of all simultaneous negotiations. Denote  $\hat{w}_j$  firm *i*'s and the respective sales agent's belief about firm *j*'s negotiated input price.

We solve the game via backward induction. If downstream firm i has negotiated input price  $w_i$ , it expects to get a profit of

$$\pi_i(w_i) = [p(x_1^*(\hat{w}_1), \dots, x_{i-1}^*(\hat{w}_{i-1}), x_i^*(w_i), x_{i+1}^*(\hat{w}_{i+1}), \dots, x_N^*(\hat{w}_N)) - w_i]x_i^*(w_i),$$
(3.23)

while the upstream firm U expects to get

$$L(w_i) = w_i x_i^*(w_i) + \sum_{i \neq j} \hat{w}_j x_j^*(\hat{w}_j).$$
(3.24)

The best-response function of firm i solves the first-order condition

$$p - w_i = -\frac{\partial p}{\partial x_i} x_i. \tag{3.25}$$

Firm i's equilibrium quantity choice can be written as

$$x_i^*(w_i) = x_i(\hat{w}_1, ..., \hat{w}_{i-1}, w_i, \hat{w}_{i+1}, ..., \hat{w}_N).$$

In particular, firm *i*'s equilibrium quantity  $x_i^*$  depends only on its own and not on its rivals' input prices.

When bargaining with firm i, U's outside option is  $L_{i,0} = L(\hat{w}_1, \ldots, \hat{w}_{i-1}, 0, \hat{w}_{i+1}, \ldots, \hat{w}_N)$  while we set the disagreement point of the downstream firms to zero. Thus, we can write the Nash bargaining problem between the supplier and firm i as

$$N_i(w_i) = (\pi_i(w_i))^{\alpha} \cdot (L(w_i) - L_{i,0})^{1-\alpha}$$

If the Nash product is maximized, the first-order condition

$$\frac{dL(w_i)/dw_i}{d\pi_i(w_i)/dw_i} = -\frac{\alpha}{(1-\alpha)} \frac{(L(w_i) - L_{i,0})}{\pi_i(w_i)}$$
(3.26)

<sup>&</sup>lt;sup>10</sup> Besides passive beliefs, also symmetric and wary beliefs are analyzed in the literature (Rey and Vergé, 2004). If firm *i* has symmetric beliefs, negotiated prices are assumed to be identical for all firms, that is,  $w_j(w_i) = w_i$ . With wary beliefs, if firm *i* negotiates an input price  $w_i \neq w^*$  with the upstream firm, then firm *i* believes that  $w_j(w_i)$  maximizes the Nash product of *U* bargaining with firm *j*, conditional on *U* knowing  $w_i$ .

holds. From (3.24) we obtain

$$\frac{dL(w_i)}{dw_i} = x_i^*(w_i) + \frac{dx_i^*(w_i)}{dw_i}w_i.$$
(3.27)

Using firm *i*'s derived demand elasticity  $\epsilon_i = \frac{dx_i^*(w_i)}{dw_i} \frac{w_i}{x_i^*(w_i)}$ , we can rewrite (3.27) as

$$\frac{dL(w_i)}{dw_i} = x_i^*(w_i) (1 + \epsilon_i).$$
(3.28)

Similarly, (3.23) yields

$$\frac{d\pi_i(w_i)}{dw_i} = \frac{\partial p}{\partial x_i} \frac{dx_i^*(w_i)}{dw_i} x_i^*(w_i) - x_i^*(w_i) + \frac{dx_i^*(w_i)}{dw_i} (p - w_i).$$

Using (3.25) we then obtain

$$\frac{d\pi_i(w_i)}{dw_i} = -x_i^*(w_i).$$
(3.29)

Inserting (3.28) and (3.29) into (3.26) yields the equilibrium profit of the downstream firm i as

$$\pi_i^* = (L^* - L_{i,0}) \frac{\alpha}{(1-\alpha)} \frac{1}{(1+\epsilon_i)}.$$
(3.30)

where, with passive beliefs,  $L_{i,0} = \sum_{j \neq i} w_j^* x_j^*$  and  $L^* - L_{i,0} = w_i^* x_i^*$ . Thus, the equilibrium condition (3.12) derived for the bilateral monopoly case generalizes to N downstream firms if contracts are not observable. Note, however, that  $\epsilon$  in (3.12) stands for the overall demand's elasticity with respect to input prices while  $\epsilon_i$  denotes the elasticity of firm *i*'s derived demand with respect to its input price.

#### 3.4.2 Flexible Production Technology

We again refer to a successive monopoly problem with an upstream and a downstream firms. In contrast to our previous analysis, the downstream firm is now interpreted as a manufacturer rather than a retailer. The input x is produced at marginal cost c = 0 and transformed by the downstream manufacturer with a production technology f(x, y) into the final good q. Denote the partial derivative of f with respect to variable x or y by indexing the respective variable, such that for instance  $f_x := \partial f/\partial x$  and  $f_{xy} := \partial^2 f/(\partial x \partial y)$ . The input y is supplied under conditions of perfect competition with inverse supply function  $w_y(y)$ .<sup>11</sup> The

<sup>&</sup>lt;sup>11</sup> It is straightforward to generalize our setup to the case in which the supply of input y is also negotiated with unobservable contracts.

market for the final good is perfectly competitive, so that its price p is given for the downstream firm. The game proceeds in two stages. In the first stage, both firms bargain over a linear wholesale price  $w_x$ . In the second stage, the downstream firm makes its purchasing decisions concerning inputs x and y and sells the output in the final good market.

The downstream firm's profit function is

$$\pi^M = pf(x, y) - w_x x - w_y y.$$

Maximization with respect to the input quantities gives two first-order conditions

$$pf_x - w_x = 0 \tag{3.31}$$

and

$$pf_y - w_y = 0. (3.32)$$

This implies that the optimal solution  $(x^*, y^*)$  fulfills

$$\frac{f_x(x^*, y^*)}{f_y(x^*, y^*)} = \frac{w_x}{w_y}.$$
(3.33)

The left-hand side is the absolute value of the slope of the isoquant (that is the technical rate of substitution) which must be equal to the inverse of the relative input prices. We assume that the optimal solution always exists and is unique. For this to be the case, it is sufficient to require  $f_x, f_y > 0$  and  $f_{xx}, f_{yy} < 0$ . In addition we assume that  $D^2 \pi^M(x, y)$  is a negative semi-definite matrix which ensures that the second order condition holds. Production functions with decreasing returns to scale satisfy these conditions. In order to obtain a well-defined Nash bargaining problem, we also need to assume that inputs are sufficiently complementary, otherwise derived demand is very elastic and the downstream firm gets the entire surplus.

Noticing (3.33) gives the downstream manufacturer's derived inverse demand for input x as

$$w_x(x,y) = w_y \frac{f_x}{f_y}.$$
(3.34)

For any given  $w_x$ , Equations (3.31) and (3.34) determine a well-defined function, the *derived demand*  $x^*(w_x)$  of the downstream firm when bargaining with the upstream firm. Note that p and  $w_y$  are fixed in our setup and  $x^*$  is strictly decreasing in  $w_x$ . Differentiating  $w_x(x, y)$  with respect to x and rearranging gives the slope of the derived demand

$$\frac{dw_x}{dx} = w_y \frac{f_{xx}f_y - f_x f_{xy}}{f_y^2}$$

which yields

$$\frac{dx}{dw_x} = \frac{1}{w_y} \frac{f_y^2}{f_{xx}f_y - f_x f_{xy}}$$
(3.35)

Using relations (3.33) and (3.35) gives

$$\epsilon = \frac{f_x f_y}{f_{xx} f_y - f_x f_{xy}} \frac{1}{x}.$$
(3.36)

We can write the downstream firm's profit as a function of its derived demand, that is,

$$\pi^{M}(w_{x}) = pf(x(w_{x}), y(w_{x})) - w_{x}x(w_{x}) - w_{y}y(w_{x})$$

and the upstream firm's profit function as

$$L(w_x) = w_x x^*(w_x).$$

Analogously to Section 2,

$$L - L_0 = \frac{1 - \alpha}{\alpha} \cdot (1 + \epsilon) \cdot (\pi - \pi_0)$$

can be derived, such that the profit sharing rule depends on parties' threat points, on the parties' profit weights and the elasticity of derived demand as given by (3.36).

## 3.5 Conclusion

We have provide a simple and an instructive link between the profit shares and the demand elasticity in vertical relations if up- and downstream firms bargain over linear input prices. Besides the disagreement payoffs and the weights of firms' profits in the Nash product, our formula singles out the slope of the bargaining frontier as an additional determinant of bargaining power. The slope of the bargaining frontier is equal to the total value of one plus the downstream firm's derived demand elasticity. We have provided various examples in which a more elastic equilibrium demand benefits the downstream firm through a change of the slope of the bargaining frontier. Our model should be instructive also for empirical studies which seek to determine the bargaining power of the different parties based on observables such as absolute profit levels and equilibrium demand elasticity.

## Appendix: N Firms and Observable Contracts

We repeat our analysis from Section 4.1 with observable contracts. Profits are given by  $\pi_i := p(x_1, ..., x_N)x_i - w_ix_i$  and  $L := \sum_{i=1}^N w_ix_i$ , and the first-order condition (3.25) holds. As quantities are observable, firm *i*'s equilibrium quantity choice can be written as  $x_i^*(w_1, ..., w_N)$ . We assume that the second order condition holds.

We can write the downstream firm i's profit as

$$\pi_i(w_1, ..., w_N) = [p(x_1^*(w_1, ..., w_N), ..., x_N^*(w_1, ..., w_N)) - w_i]x_i^*(w_1, ..., w_N), \quad (3.37)$$

while the upstream firm's profit equals

$$L(w_1, ..., w_N) = \sum_{i=1}^N w_i x_i^*(w_1, ..., w_N).$$
(3.38)

The general Nash bargaining problem between the supplier and firm i is given by

$$N_{i}(w_{i}) = (\pi_{i}(w_{1},...,w_{N}) - \pi_{i,0})^{\alpha} \cdot (L(w_{1},...,w_{N}) - L_{i,0}(w_{1},...w_{i-1},w_{i+1},...w_{N}))^{1-\alpha},$$
(3.39)

where  $\pi_{i,0}$  is firm *i*'s the outside option and  $L_{i,0}(w_j)$  is the outside option of the upstream firm when it bargains with firm *i*. As before,  $\pi_{i,0} = 0$ .

If the Nash product is maximized, the first-order condition

$$\frac{dL(w_1, ..., w_N)/dw_i}{d\pi_i(w_1, ..., w_N)/dw_i} = -\frac{\alpha}{(1-\alpha)} \frac{(L-L_{i,0})}{\pi_i}.$$
(3.40)

holds. Formula (3.38) gives

$$\frac{dL(w_1, ..., w_N)}{dw_i} = x_i + \frac{dx_i(w_1, ..., w_N)}{dw_i} w_i + \sum_{j \neq i} \frac{dx_j(w_1, ..., w_N)}{dw_i} w_j.$$
(3.41)

Using firm *i*'s elasticity of derived demand,  $\epsilon_i = \frac{dx_i}{dw_i} \frac{w_i}{x_i}$ , and the cross-price elasticity of derived demand,  $\epsilon_{ji} = \frac{dx_j}{dw_i} \frac{w_i}{x_j}$ , we can rewrite (3.41) as

$$\frac{dL(w_1, \dots, w_N)}{dw_i} = x_i \left( 1 + \epsilon_i + \sum_{j \neq i} \epsilon_{ji} \frac{x_j w_j}{x_i w_i} \right).$$
(3.42)

Similarly, (3.37) yields

$$\frac{d\pi_i(w_1, \dots, w_N)}{dw_i} = \frac{\partial p(x_1, \dots, x_N)}{\partial x_i} \frac{dx_i}{dw_i} x_i + \sum_{j \neq i} \frac{\partial p(x_1, \dots, x_N)}{\partial x_j} \frac{dx_j}{dw_i} x_i - x_i + \frac{dx_i}{dw_i} (p - w_i).$$
(3.43)

Inserting (3.25) into the preceding equation gives

$$\frac{d\pi_i(w_1, \dots, w_N)}{dw_i} = \sum_{j \neq i} \frac{\partial p(x_1, \dots, x_N)}{\partial x_j} \frac{dx_j}{dw_i} x_i - x_i, \qquad (3.44)$$

or,

$$\frac{\partial \pi_i(w_1, \dots, w_N)}{\partial w_i} = -x_i \left( 1 - \sum_{j \neq i} \frac{p}{w_i} \eta_j \epsilon_{ji} \right), \qquad (3.45)$$

where  $\eta_j = \frac{\partial p(x_1,...,x_N)}{\partial x_j} \frac{x_j}{p}$  gives firm j's elasticity of demand. Inserting (3.42) and (3.45) into (3.40) yields

$$\frac{\alpha}{(1-\alpha)} \frac{(L-L_{i,0})}{\pi_i} = \frac{\left(1+\epsilon_i + \sum_{j\neq i} \epsilon_{ji} \frac{x_j w_j}{x_i w_i}\right)}{\left(1-\sum_{j\neq i} \frac{p}{w_i} \eta_j \epsilon_{ji}\right)}.$$
(3.46)

As the downstream firms are symmetric, we assume a symmetric Nash solution in which  $w_i^* = w_j^*$  and  $x_i^* = x_j^*$ , for any  $i, j \in 1, ..., N$ . We can write equilibrium profit of the downstream firm i as

$$\pi_i^* = (L - L_{i,0}) \frac{\alpha}{(1 - \alpha)} \frac{\left(1 - (N - 1)\frac{p}{w_i} \eta_j \epsilon_{ji}\right)}{\left(1 + \epsilon_i + (N - 1)\epsilon_{ji}\right)}.$$
(3.47)

As a consequence, with observable contracts, the profit sharing rule does not only depend on the elasticity of derived demand, but also on the cross-price elasticities of the derived demand.

## **Declaration of Contribution**

Hereby I, Hamid Aghadadashli, declare that the chapter "The Nash Bargaining Solution in Vertical Relations With Linear Input Prices" is coauthored by Dr. Markus Dertwinkel-Kalt and Prof. Dr. Christian Wey.

My contributions to this chapter are as follows:

- I have contributed substantially to the Model and Analysis.
- I have contributed to the Applications.
- I have derived part of the Extensions.

Signature of coauthor 1 (Dr. Markus Dertwinkel Kalt): <u>Markup Druce</u> Signature of coauthor 2 (Prof. Dr. Christian Wey): <u>C. Wey</u>

## Chapter 4

# Multi-plant Firms, Production Shifting, and Countervailing Power

Co-authored by Markus Dertwinkel-Kalt and Christian Wey

## 4.1 Introduction

We examine the bargaining effects of manufactures' ability to shift the production of differentiated products among the plants they operate. Large corporations' ability to shift production among plants has been steadily increasing in the last decades. International car producers like Volkswagen AG and Daimler AG, for instance, have heavily invested into new production sites outside Germany (in particular, in Eastern Europe and BRIC countries).<sup>1</sup> At the same time, the integration of internationally dispersed value chains within corporations has increased, and the use of platform-production concepts has made it possible to

<sup>&</sup>lt;sup>1</sup> See, for instance, Forbes, "Shifting Production To Central And Eastern Europe Could Boost Profits Of German Automakers," online edition, 23 June 2014, (retrieved from: www.forbes.com). Volkswagen AG provides on its website (www.volkswagenag.com) a list of all its production plants in the world and the brands it produces at each of those sites.

produce virtually all product variants at all plants which a corporation operates.  $^{2,3}\,$ 

We provide an analytical framework of multi-plant firms' ability to shift the production of their product variants among their plants. In particular, we investigate the consequences of having the option to shift production on the bargaining outcomes in the input markets when suppliers have seller power. We show that this option creates considerable countervailing power in supplier-manufacturer bargaining relations with important implications for both merger control and FDI policy. In the former case, new efficiencies have to be taken into account by antitrust authorities and in the latter case, the possibility of socially excessive investment incentives into new production sites abroad have to be considered.

We develop a model to study the efficiencies which arise from a multi-plant firm's ability to shift its production across plants. In the basic setup, we consider two manufacturers offering differentiated products which get labor inputs through bilateral monopoly relations with plant-specific unions. In order to mirror capacity constraints we assume that production costs at each plant are convex. Wages are negotiated between each firm and its plant specific labor union(s). A downstream merger yields two opportunities: first, it allows the integrated firm to hire workers from both unions, and second, it allows to shift production of the different brands between the two plants. Due to capacity constraints, production shifting is costly and will not occur in the equilibrium.<sup>4</sup> However, the threat to shift production increases the bargaining power of the integrated manufacturer

<sup>&</sup>lt;sup>2</sup> Recent trends of the production patterns in the automobile industry are presented in Sturgeon, Memedovic, Van Biesebroeck and Gereffi (2009). An early account of the merits of production shifting within multinational firms is Kogut and Kulatilaka (1995). In their work the ability to shift production among internationally dispersed plants creates an option value in the presence of exchange rate fluctuations.

<sup>&</sup>lt;sup>3</sup> In 2012, Volkswagen Group introduced the Modular Transverse Matrix (MQB) technology for the Volkswagen, Audi, Škoda, and SEAT brands which will theoretically allow VW Group to produce different models of these brands on the same production line. See, http://www.volkswagenag.com/content/vwcorp/info\_center/en/themes/2012/02/MQB.html

<sup>&</sup>lt;sup>4</sup> In case of Volkswagen, this also mirrors practice. It produces different models at different production sites (see www.volkswagenag.com/content/vwcorp/content/de/the\_group/production\_plants.htm). But, as different models are based on the same platforms, it can shift production accross plants if it needs to do so (for instance, if production breaks down because of strikes).
and exerts downward pressure on wages. Formally, we single out the threat point effect which increases the multi-plant firm's bargaining power against monopolistic labor unions. Not surprisingly, in the course of bargaining an integrated manufacturer may threaten to close a plant altogether if the union does not back down.<sup>5</sup> Similar effects have been shown in the literature, but they are all based on the fact that labor unions at different production sites produce goods which are close to perfect substitutes. The novelty of our contribution is to show that even when goods are very much differentiated, the ability of production shifting makes the different labor supplies substitutable again, from the integrated firm's perspective.

First, we consider the case in which the products are plant-specific, *i.e.*, production cannot be shifted between plants. Note that, a merger between the downstream firms creates a multi-product monopolist which can internalize its demand linkages. The downstream merger is always profitable for the merging parties, but it decreases both consumer surplus and total welfare. Next, we analyze the case that a merged manufacturer can shift the production of both products across the plants it owns. Such a merger creates considerable countervailing power by improving the integrated firm's disagreement payoff. As a consequence, input prices decrease. This effect depends on the plants' capacity constraints. If the constraints are not too restrictive and the products are not close substitutes, then a multi-plant merger is socially desirable, as the positive effect of decreasing input prices is larger than the negative effect of monopolization.

We also discuss two extensions of our model which focus on FDI. We analyze a firm's investment decision to open a new plant, either in the home country or in a foreign country, when it bargains with the country-specific labor unions.<sup>6</sup>

<sup>&</sup>lt;sup>5</sup> For instance, the Swedish multinational company Electrolux bought the Italian producer Zanussi in 1984. In 1997 Electrolux-Zanussi announced to shut down plants located in Italy. However, the company decided to continue its production in Italy after it agreed with the metalworker's union on lower wages (Paparella (1997)). In 2014, once more Electrolux threatened to shift the production to the low-wage country Poland in order to close Italian plants. Later in the same year, the company offered wage levels as paid in Poland and Hungary as an alternative solution to the plant closure (Rustico (2014)).

<sup>&</sup>lt;sup>6</sup> Zhao (1995) discusses a firm's FDI decision when labor is unionized. He finds that crosshauling FDI increases (decreases) employment and national welfare if the union is wage (employment) oriented. Eckel and Egger (2009) analyze a similar scenario and find that the manufacturer has incentives for FDI in order to improve the bargaining position vis-à-vis the input suppliers.

Specifically, we examine a firm's decision about the location of a second plant (domestic vs. foreign) at which a differentiated product is to be produced. Even if investment costs for a production plant in a foreign country are higher than for a plant in the home country, the manufacturer may strictly prefer the FDI. Moreover, the ability to produce differentiated products in different plants plays an important role in the investment decision. If the firm opens the new plant in the foreign country, then it has an incentive to design the new brand such that its production can be shifted between the plants as this improves the manufacturer's breakdown profit. If the second plant is located in the home country, the labor union is not affected by the possibility to shift production in this scenario.

We also analyze a firm's FDI decision when the country-specific labor unions have different levels of bargaining power. A firm's investment decision depends on the domestic union's bargaining power. In particular, the domestic union may have a strict preference to commit itself not to exert its full bargaining power in order to prevent the firm from investing abroad.<sup>7</sup> As the union may not be able to commit itself, the manufacturer and the union may face a variant of the prisoner's dilemma: the union is strictly better off if it does not exert its full bargaining power in order to prevent the manufacturer from FDI.<sup>8</sup> As the union cannot commit itself to this action, however, the firm will invest abroad, such that the union earns lower wages and the firm earns a lower profit due to the high investment costs.<sup>9</sup>

<sup>&</sup>lt;sup>7</sup> Egger and Etzel (2014) study a model with two countries that differ in the centralization of union-wage setting. They show that in the long run, capital outflows from the country with centralized wage-setting make the two countries more dissimilar and a decentralization of the wage setting can prevent capital outflows and the export of jobs.

<sup>&</sup>lt;sup>8</sup> Aloi, Leite-Monteiro and Lloyd-Braga (2009) analyze a two-country scenario, in which labor is competitive in one country while it is unionized in the other. They also show that unionized workers do not have to have a strict preference for an increase in their relative bargaining power.

<sup>&</sup>lt;sup>9</sup> Closely related is the following recent case. In Germany, Amazon runs several logistic centers to distribute parcels. While the labor union Verdi went on strike at Amazon's logistic centers several times to obtain better wage agreements, Amazon denied the wage requests and instead decided to scale up its capacity in Poland where workers are less well organized (see "Amazon baut drei riesige Logistikzentren in Polen" (Amazon builds three giant logistic centers in Poland), Die Welt, 7 October 2013, online edition, www.welt.de). Thus, Amazon undertook large FDIs and the German workers were not able to make any

Our analysis of the countervailing power effects of a multi-plant merger contributes to the controversy about whether and to what extent efficiencies should be taken into account when an antitrust authority decides about a proposed merger. The efficiency defense has by now been explicitly recognized in the regulatory guidelines.<sup>10</sup> For instance, the revised Section 10 on efficiencies in the Horizontal Merger Guidelines issued by the U.S. Department of Justice and the Federal Trade Commission defines explicitly the set of cognizable efficiencies. Traditionally, efficiency gains (resulting in potentially lower prices or better products) are thought to be realized i) by achieving an optimal allocation across different plants (rationalization), ii) through realization of economies of scale and scope, or iii) by enhancing technological progress. All these factors could render a horizontal merger desirable from a social welfare perspective. In our model a merger increases the purchasing power of a downstream firm vis-à-vis powerful input suppliers. A formal analysis of countervailing power has been provided by von Ungern-Sternberg (1996) and Dobson and Waterson (1997). Both papers show instances where a downstream merger creates countervailing power which reduces the input price. However, social welfare is always reduced through a merger. While these two papers assume bargaining over constant unit prices, Inderst and Wey (2003, 2011) have considered the case of efficient bargaining. It is shown that a downstream merger creates buyer power if the supplier's costs are convex. Moreover, a merger increases the supplier's investment incentives which leads to lower consumer prices.<sup>11</sup>

There are some related papers which investigate how a merger of firms which produce differentiated goods affects input prices in the presence of plant-specific input suppliers. For this case, Lommerud, Straume and Sorgard (2005) show

use of their strong domestic bargaining position.

<sup>&</sup>lt;sup>10</sup> Starting with the seminal contributions by Stigler (1950a,b) and Williamson (1968), the trade-off between monopolization and efficiency gains of mergers has been extensively discussed in the economic literature (see, e.g., Farrell and Shapiro (1990), Perry and Porter (1985)).

<sup>&</sup>lt;sup>11</sup> Dobson and Waterson (1997) and von Ungern-Sternberg (1996) have only considered cost functions with constant unit costs. Inderst and Wey (2003, 2011) also allow for multiple suppliers, multilateral relations, and horizontal mergers in either stage. The issue of downstream firms' countervailing power is also discussed without reference to downstream mergers. Chen (2003) applies the asymmetric Nash bargaining solution, while Inderst and Shaffer (2009) and Dertwinkel-Kalt, Haucap and Wey (2015) use an outside option approach.

that a merger is always profitable for the participants unless the goods are close to perfect substitutes and the unions' objective is to achieve higher employment level rather than higher wages. Lommerud, Straume and Sorgard (2006) analyze national and international mergers in a unionized oligopoly, where input suppliers are country-specific. They find that international mergers are always socially preferable over any market structure which involves national mergers. This is driven by the fact that international mergers decrease, while national mergers increase the wages. In a closely related setup, Symeonidis (2008, 2010) finds that downstream mergers always decrease input prices. Such mergers increase social welfare if goods are close substitutes and if the upstream suppliers have significant bargaining power.

While this literature emphasizes how multi-plant mergers make (plant-specific) monopolistic suppliers more substitutable through bargaining power, interestingly, all these papers assume that products are plant-specific such that the merged firm can produce each product only at one specific plant. Therefore, in the case of a high wage demand or a breakdown of bargaining at one plant, a merger allows to scale down the production at the respective plant and to increase the output of the substitutable product at the other plant. Adding to this literature, we are interested in a multi-plant firms' ability to shift the production of a *specific* product between the plants it owns.

The paper proceeds as follows. In Section 4.2, we present the basic model and analyze the duopoly case, where each manufacturer owns a single plant. Section 4.3 analyzes the case of an integrated manufacturer which operates two plants. Section 4.3.1 solves the benchmark case in which production cannot be shifted to another plant. Sections 4.3.2 and 4.3.3 provide the analysis of the production shifting case. Section 4.3.4 compares the equilibrium outcomes before and after a merger depending on whether or not the merged firm can shift the production between plants. Section 4.4 presents extensions, in particular with respect to FDI incentives, before Section 4.5 concludes.

# 4.2 The Model

Two downstream manufacturers, located in two different countries, produce a (differentiated) product and compete in Cournot fashion in the integrated final product market. Each manufacturer operates one production plant and receives



Figure 4.1: Merger with and without production shifting.

inputs at plant *i* from a monopolistic supplier *i*, where  $i \in \{1, 2\}$ .<sup>12</sup> If firms merge, the merged firm runs both plants. Throughout the paper, input prices are negotiated for each plant separately, *i.e.*, between the firm which operates plant *i* and supplier *i*. The main application of a monopolistic input supplier is a labor union which bargains with the manufacturer over a wage rate on behalf of the employees. We pose that union *i* maximizes the wage bill of the employees at plant *i*. If the downstream firms are merged, both input suppliers bargain simultaneously with a single firm.

We denote the total output of downstream firm *i*'s product by  $x_i = x_{i1} + x_{i2}$ , for  $i \in \{1, 2\}$ , where  $x_{ij}$  denotes the output of product *i* at plant *j*. If the manufacturers are independent, then each of them can only produce its own brand, and the total output at plant *i*, denoted  $X_i$ , equals  $x_{ii}$ , while  $x_{ij} = 0$ for  $i, j \in \{1, 2\}$  and  $i \neq j$ . If the two plants are run by the same firm, one of the following two cases emerges. If production shifting is not feasible, *i.e.*, the products are plant-specific, then the production at each plant *i* equals  $x_{ii}$ and  $x_{ij} = 0$  for  $i, j \in \{1, 2\}$  and  $i \neq j$ . This case is illustrated in Figure 4.1a. If the production of each product can be shifted across plants, then the total production at plant *i* equals  $X_i = x_{1i} + x_{2i}$ . Figure 4.1b reflects the merger case with production shifting.

The inverse demand for product *i* is given by  $p_i = 1 - x_i - \gamma x_j$ , for  $i, j \in \{1, 2\}$ and  $i \neq j$ , where  $\gamma \in [0, 1]$  measures the degree of product differentiation. For

<sup>&</sup>lt;sup>12</sup> Typically, the supplier is a country-specific labor union, such that in the following we refer to labor unions instead of suppliers.

 $\gamma = 1$ , both brands are perfect substitutes while  $\gamma = 0$  indicates that both brands are independent and each manufacturer is a monopolist.

The marginal costs of producing the inputs are normalized to zero. Moreover, we assume that each unit of input produces exactly one unit of the final good. Total production costs at plant i are given by

$$c_i = \left(w_i + \frac{t \cdot X_i}{2}\right) X_i$$

with t > 0, where  $w_i$  stands for the wage paid at plant *i*, and  $X_i$  denotes the total production of both products at plant *i*. If the plants are independent, then  $X_i = x_i = x_{ii}$ .

The production costs are linear in the input price,  $w_i$ , and convex in the output  $X_i$ . Parameter t measures to what extent capacity constraints limit a further expansion of production. If the manufacturers have merged, the parameter t also measures the ease of shifting production between the plants. The lower t is, the easier it is to shift production of a product to the other plant, and therefore, the stronger the bargaining position of the merged firm becomes. With a large value of t, a shift of production comes at relatively large costs.<sup>13</sup>

We solve the following two-stage game for subgame perfect Nash equilibria. In the first stage, each manufacturer bargains separately with its labor union over input price  $w_i$ . We apply the symmetric Nash-bargaining solution to solve for the equilibrium in each union-manufacturer negotiation. Moreover, both pairs bargain simultaneously. In the second stage, both manufacturers determine their production quantities non-cooperatively. If the manufacturers are independent, then the payoff function  $R_i$  of manufacturer *i* is strictly concave in  $x_i = x_{ii}$  and given by

$$R_i = (1 - x_i - \gamma x_j)x_i - \left(w_i + \frac{tx_i}{2}\right)x_i$$
, for  $i, j \in \{1, 2\}$  and  $i \neq j$ .

The payoff of union i is

$$L_i = X_i w_i = x_{ii} w_i.$$

<sup>&</sup>lt;sup>13</sup> As production costs are convex, we have to require that a downstream manufacturer does not find it optimal to open another plant. This may follow from large upfront investments for setting up another plant and learning-by-doing effects at the plant level, which grow over-proportionally with total output. We analyze the firm's incentives to open a new plant in the extension below.

**Duopoly with two single-plant manufacturers.** As the benchmark case, we assume that each of the two manufacturers operates exactly one plant. We solve the game backwards. In the second stage, the manufacturers take the negotiated input prices  $(w_1, w_2)$  as given and compete in quantities in the integrated final product market.<sup>1415</sup> Each firm *i* chooses the quantity  $x_i$  which maximizes its payoff  $R_i$ . The first-order conditions yield the following best-response functions for each firm *i*:

$$x_i^{\circ} = \frac{1 - \gamma x_j - w_i}{2 + t}$$
 for  $i, j = 1, 2, i \neq j$ .

Since input prices  $w_1$  and  $w_2$  are assumed to be observable, each manufacturer i's optimal strategy in the second stage,  $\hat{x}_i$ , can be written as a function of the input prices. This yields

$$\widehat{x}_i(w_i, w_j) = \frac{(2+t-\gamma) + \gamma w_j - (2+t)w_i}{(2+t)^2 - \gamma^2} \text{ for } i, j = 1, 2, i \neq j,$$
(4.1)

if both supplies are strictly positive, *i.e.*,  $w_i < 1 - \gamma(1 - w_j)/(2 + t)$  for  $i, j = 1, 2, i \neq j$ . If the negotiated wage for firm i exceeds  $1 - \gamma(1 - w_j)/(2 + t)$ , then firm i's output equals  $\hat{x}_i(w_i, w_j) = 0$ , while firm j produces  $\hat{x}_j(w_i, w_j) = (1 - w_j)/(2 + t)$  as long as  $w_j \leq 1$  holds. Otherwise, both firm's output will be equal to zero, *i.e.*,  $\hat{x}_i(w_i, w_j) = 0, i \in \{1, 2\}$ .

Wages are determined in the first stage of the game as a result of a simultaneous bilateral bargaining process between firm i and union i for  $i \in \{1, 2\}$ . Given the optimal production levels in the second stage, firm i's payoff if it agreed with union i on wage  $w_i$  equals  $\hat{R}_i(w_i, w_j) = (1 - \hat{x}_i(w_i, w_j) - \gamma \hat{x}_j(w_j, w_i))\hat{x}_i - w_i \hat{x}_i(w_i, w_j) - \frac{t}{2}(\hat{x}_i(w_i, w_j))^2$ , while union i's payoff is  $\hat{L}_i(w_i, w_j) = \hat{x}_i(w_i, w_j) \cdot w_i$ . As there is no outside option for the firm and the union, their disagreement points are equal to zero. We can formalize the symmetric Nash bargaining problem between firm i and union i as

$$\max_{w_i} \widehat{R}_i(w_i, w_j) \cdot \widehat{L}_i(w_i, w_j) = \left[ (1 - \widehat{x}_i - \gamma \widehat{x}_j) \widehat{x}_i - w_i \widehat{x}_i - \frac{t}{2} (\widehat{x}_i)^2 \right] \left[ \widehat{x}_i w_i \right].$$
(4.2)

Each firm *i* and union *i* choose the equilibrium wage rate  $w_i^*$  which maximizes the Nash product and is a best response to the negotiated wage rate,  $w_j$ , between firm *j* and union *j*. As the firms and the unions are symmetric, the equilibrium

<sup>&</sup>lt;sup>14</sup> We can safely restrict consideration to those subgames where bargaining at both firms has been successful such that both goods can be provided.

<sup>&</sup>lt;sup>15</sup> We assume that the bargaining outcomes of the first stage are observable for both firms.

wage rates which maximize the Nash products (4.2) are identical and given by

$$w_1^* = w_2^* = \frac{2+t-\gamma}{8+4t-\gamma}.$$
(4.3)

Substituting (4.3) into (4.1) yields the symmetric output levels

$$x_1^* = x_2^* = \frac{3(2+t)}{(2+t+\gamma)\left(8+4t-\gamma\right)} \tag{4.4}$$

Given these quantities, each firm's profit equals

$$R^* = \frac{9(2+t)^3}{2(2+\gamma+t)^2(8-\gamma+4t)^2},$$

while each union's wage bill is given by

$$L^* = \frac{3(2+t)(2-\gamma+t)}{(2+\gamma+t)(8-\gamma+4t)^2}$$

## 4.3 The Case of Integration

A merger of downstream manufacturers yields a multi-plant firm. We investigate such a merger in different scenarios. In Section 4.3.1, we analyze the benchmark case where each brand can only be produced at one specific plant. In this case, the downstream merger improves a firm's bargaining position against the unions, unless the products are independent. If bargaining breaks down with union i, the manufacturer will increase the supply of good  $j \neq i$ , which can partially compensate the foregone profits from not supplying brand i as long as  $\gamma > 0$ . In Section 4.3.2 and Section 4.3.3, we introduce the option to shift the production of each product between the plants, that is, both brands can be produced at both plants. The possibility of production shifting makes demand for each input more elastic, such that a merger enhances the bargaining position of the merged firm and induces, as we will show, lower input prices. Finally, in Section 4.3.4, we compare the outcomes under the different regimes, which summarizes our results and provides more intuition for them.

To sum up, the manufacturers' benefits from the merger are threefold. First, if goods are (imperfect) substitutes, *i.e.*,  $\gamma > 0$ , then there is the standard monopolization effect on the final goods market. Second, the merger implies a shift in the breakdown payoff. The last effect is more pronounced if the firm can shift the production across its plants and exists for all  $\gamma \geq 0$ .

# 4.3.1 Analysis of the Benchmark Case when Production Shifting is Not Feasible

Suppose that manufacturers are merged and that brand i can only be produced at plant plant i, that is, it is not possible to shift the production of a brand between plants. We can write the payoff function of the multi-plant firm as

$$R^{M} = (1 - x_{1} - \gamma x_{2})x_{1} + (1 - x_{2} - \gamma x_{1})x_{2} - w_{1}X_{1} - w_{2}X_{2} - \frac{t}{2}(X_{1})^{2} - \frac{t}{2}(X_{2})^{2}.$$
 (4.5)

Similar to the no merger case, the total production at plant *i* is equal to the total production of brand *i*, *i.e.*,  $X_i = x_i = x_{ii}$ .

In the second stage, the merged firm takes the input prices  $(w_1, w_2)$  as given and maximizes its profit (4.5) by producing

$$\widehat{x}_i(w_1, w_2) = \frac{(2+t)(1-w_i) - 2\gamma(1-w_j)}{(2+t)^2 - 4\gamma^2},$$
(4.6)

if both supplies are strictly positive, *i.e.*, if  $w_i < 1 - 2\gamma(1 - w_j)/(2 + t)$  for  $i, j = 1, 2, i \neq j$ . If the latter condition does not hold for product *i*, then we obtain  $\hat{x}_i(w_1, w_2) = 0$ , while  $\hat{x}_j(w_1, w_2) = (1 - w_j)/(2 + t)$  if  $w_j \leq 1$  and  $\hat{x}_j(w_1, w_2) = \hat{x}_i(w_1, w_2) = 0$  otherwise.

We now set up the bargaining problem. Similar to the previous case, bargaining with the two unions proceeds simultaneously and separately.<sup>16</sup> If the firm reaches agreements with both unions then its payoff equals

$$\widehat{R}^{M}(w_{1}, w_{2}) = \sum_{i, j \in \{1, 2\}, i \neq j} \left[ (1 - \widehat{x}_{i} - \gamma \widehat{x}_{j}) \cdot \widehat{x}_{i} - (w_{i} + t \cdot \widehat{x}_{i})/2 \right) \cdot \widehat{x}_{i} \right],$$

and union *i* gets  $\widehat{L}_i(w_1, w_2) = w_i \cdot \widehat{x}_i$ 

In contrast to the previous analysis, the breakdown payoff of the manufacturer does not have to be zero anymore. Suppose thus that there is a breakdown of the negotiation with union *i*. Then, the merged firm can only produce product *j* at plant *j* for the given input price  $w_j$  such that  $\hat{x}_i^D = 0$ . The manufacturer maximizes its profit  $R^M = (1 - x_j)x_j - w_jx_j - \frac{t}{2}(x_j)^2$  by choosing the optimal

<sup>&</sup>lt;sup>16</sup> When a downstream firm bargains with two independent upstream suppliers simultaneously, the disagreement payoffs can be constructed in several ways. We assume that if firm *i* bargains with the representative union *i*, then it takes the expected outcome at the other plant  $j \neq i$  as given. This way of formalizing disagreement points is also used by Horn and Wolinsky (1988a) and Eckel and Egger (2009).

output level  $\hat{x}_j^D(w_j) = (1 - w_j)/(2 + t)$  if  $w_j \leq 1$  and  $\hat{x}_j^D(w_j) = 0$  otherwise. If the negotiation between firm and union *i* fails, then firm *i* makes profit

$$\widehat{R}_{i}^{MD}(w_{j}) = \max\left\{\frac{(1-w_{j})^{2}}{2(2+t)}, 0\right\}.$$
(4.7)

As the unions do not have an outside option, they get a payoff of zero in the case of disagreement. Proceeding analogously to Section 4.2, the merged firm and union i maximize the symmetric Nash product, *i.e.*, they agree on

$$\max_{w_i} \left[ \widehat{R}^M(w_i, w_j) - \widehat{R}_i^{MD}(w_j) \right] \cdot \widehat{L}_i(w_i, w_j).$$

The unique equilibrium is given by the following input prices and production quantities

$$w_1^M = w_2^M = \frac{2+t-2\gamma}{8+4t-2\gamma},\tag{4.8}$$

$$x_1^M = x_2^M = \frac{3(2+t)}{(2+t+2\gamma)\left(8+4t-2\gamma\right)}.$$
(4.9)

**Lemma 10.** The benchmark case with integrated manufacturers and no production shifting has a unique equilibrium where the equilibrium output levels  $x_i^M$  are given by (4.9) for both goods  $i \in \{1, 2\}$ .

The firm's equilibrium profit is then given by

$$R^{M} = \frac{9(2+t)^{2}}{4(2+2\gamma+t)(4-\gamma+2t)^{2}},$$
(4.10)

while each union earns a wage bill of

$$L^{M} = \frac{3(2+t)(2-2\gamma+t)}{4(2+2\gamma+t)(4-\gamma+2t)^{2}}.$$
(4.11)

If the products are independent, *i.e.*,  $\gamma = 0$ , then merging without the ability to shift production of the different brands between the plants does not affect the manufacturers' joint bargaining power as (4.8) and (4.3) give rise to the same input prices. In that case, the equilibrium production quantities also stay unaffected by the merger. There are two separate final goods markets, which are monopolized both before and after the merger.

However, if goods are (imperfect) substitutes, *i.e.*,  $\gamma > 0$ , then a merger improves the firms' bargaining position as the equilibrium post-merger wages

(4.8) are lower than the pre-merger wages (4.3). Such a merger (which does not allow manufacturers to shift the production of the brands) also lowers the output of each product and, therefore, consumer surplus. While unions receive a lower wage bill, *i.e.*,  $w_i^M \cdot x_i^M < w_i^* \cdot x_i^*$ , the merged firms' profit exceeds the firms' joint pre-merger profits. In this case, the downstream merger monopolizes the final goods market.

Lemma 11. If the products are plant-specific, i.e., shifting the production of one brand to another plant is not possible, then a downstream merger decreases consumer surplus, unless products are independent. In the latter case, the merger does not affect equilibrium outcomes.

## 4.3.2 Optimal Production Plans with Production Shifting

In this section, we allow the manufacturer to shift the production of each product between the two plants. First, we derive for each plant the optimal quantities given the wages  $(w_1, w_2)$ . We denote the aggregate quantity of the goods by  $X = X_1 + X_2 = x_1 + x_2$ , where  $X_i$  denotes the overall output at plant *i* and  $x_i$ denotes, as before, the overall output of brand *i*. The manufacturer's production costs for given quantities  $X_1$  and  $X_2$  are given by

$$C(X_1, X, w_1, w_2) = -w_1(X_1) - w_2(X - X_1) - \frac{t}{2}(X_1)^2 - \frac{t}{2}(X - X_1)^2,$$

where we used  $X = X_1 + X_2$ . If  $0 \le w_i \le 1$  with i = 1, 2 holds, then the firm will choose the following output levels at the plants in order to minimize its production costs:

$$X_{1} = \begin{cases} 0, & \text{if } w_{1} - w_{2} \ge tX \\ X, & \text{if } w_{2} - w_{1} \ge tX \\ \frac{tX - w_{1} + w_{2}}{2t}, & \text{otherwise} \end{cases}$$
(4.12)  
$$X_{2} = X - X_{1}.$$

Given these quantities, the total cost of the merged firm,

$$C(X, w_1, w_2) = \min_{\{0 \le X_1 \le X\}} C(X_1, X, w_1, w_2),$$

can be written as

$$C(X, w_1, w_2) = \begin{cases} \frac{t}{2}X^2 + Xw_2, & \text{if } w_1 - w_2 \ge tX\\ \frac{t}{2}X^2 + Xw_1, & \text{if } w_2 - w_1 \ge tX\\ \frac{t}{2}X^2 + Xw_2 - \frac{1}{4t}(tX - w_1 + w_2)^2, & \text{otherwise} \end{cases}$$

which is differentiable in all three parameters. The optimal production levels of each brand can be determined by maximizing the merged firm's profit (4.5), which equals

$$R^{M} = (1 - x_{1} - \gamma x_{2})x_{1} + (1 - x_{2} - \gamma x_{1})x_{2} - C(x_{1} + x_{2}, w_{1}, w_{2}).$$
(4.13)

**Lemma 12.** For  $\gamma \in [0, 1)$  and given input prices  $(w_1, w_2)$ , the integrated manufacturer chooses the unique production levels

$$x_{1} = x_{2} = \tilde{x}(w_{1}, w_{2}) = \begin{cases} \frac{1-w_{2}}{2(1+t+\gamma)}, & \text{if } w_{1} - w_{2} \ge \frac{t(1-w_{2})}{1+t+\gamma} \\ \frac{1-w_{1}}{2(1+t+\gamma)}, & \text{if } w_{2} - w_{1} \ge \frac{t(1-w_{1})}{1+t+\gamma} \\ \frac{2-w_{1}-w_{2}}{2(2+t+2\gamma)}, & \text{otherwise} \end{cases}$$
(4.14)

If  $\gamma = 1$  the aggregate supply of both goods is uniquely determined and equal to  $2\tilde{x}(w_1, w_2)$ .

*Proof.* Suppose  $\gamma < 1$ . We first show that it is optimal to set  $x_1 = x_2$ . For a given X, the manufacturer chooses  $x_1$  to maximize

$$x_1(1 - x_1 - \gamma(X - x_1)) + (X - x_1)(1 - (X - x_1) - \gamma x_1).$$

Differentiation yields the first-order condition  $-4x_1(1-\gamma)+2X(1-\gamma)=0$ , which together with strict concavity proves the assertion. The optimization problem thus reduces to choosing x to maximize  $R^M = 2x(1-x(1+\gamma)) - C(2x, w_1, w_2)$ . This problem is strictly concave and calculating the first order conditions for each of the different cases yields (4.14).

By substituting (4.14) in (4.12) we get the optimal level of production at plant i (*i.e.*, the derived demand for labor at plant i) as a function of the wages:

$$\tilde{X}_i(w_i, w_j) = \begin{cases} 0, & \text{if } w_i - w_j \ge \frac{t(1-w_j)}{1+t+\gamma} \\ \frac{1-w_i}{1+t+\gamma}, & \text{if } w_j - w_i \ge \frac{t(1-w_i)}{1+t+\gamma} \\ \frac{t-w_i(1+t+\gamma)+w_j(1+\gamma)}{t(2+t+2\gamma)}, & \text{otherwise.} \end{cases}$$

Given  $\tilde{x}$  and  $\tilde{X}_i$ , the firm's reduced profit function is denoted by  $\tilde{R}^M(w_1, w_2) = 2\tilde{x} \cdot (1 - \tilde{x} - \gamma \tilde{x}) - C(2\tilde{x}, w_1, w_2)$  and that of union *i* is given by  $\tilde{L}_i(w_1, w_2) = w_i \cdot \tilde{X}_i(w_1, w_2)$ .

## 4.3.3 Union-Firm Bargaining with Production Shifting

In the first stage of the game the merged firm bargains simultaneously with the two unions. If the firm reaches agreements with both unions then its payoff equals  $\tilde{R}^{M}(w_{1}, w_{2})$ , while union *i* gets  $\tilde{L}_{i}(w_{1}, w_{2})$ . We assume that the merged firm can fully shift the production between its plants, *i.e.*, it can produce both brands at both plants even if it fails to reach an agreement with union *i*.

Suppose that the manufacturer does not find an agreement with union *i*. Then the total production of goods 1 and 2 equal  $x_1 = x_{1j}$  and  $x_2 = x_{2j}$ . The optimal production levels which maximize a firm's disagreement profit  $(1 - x_1 - \gamma x_2)x_1 + (1 - x_2 - \gamma x_1)x_2 - (x_1 + x_2)w_2 - \frac{t}{2}(x_1 + x_2)^2$  can be determined from the first-order conditions and are given by

$$x_1 = x_2 = \frac{1 - w_j}{2(1 + t + \gamma)} \tag{4.15}$$

as long as  $w_j \leq 1$  and  $\gamma < 1$ . If  $w_j > 1$ , then  $x_1 = x_2 = 0$ , and if the products are perfect substitutes ( $\gamma = 0$ ), then the aggregate output X equals  $2x_1$ . Thus, if bargaining with union *i* fails the disagreement payoff equals

$$\tilde{R}_{i}^{0} = \max\left\{\frac{(1-w_{j})^{2}}{2(1+t+\gamma)}, 0\right\}.$$
(4.16)

Provided  $w_j < 1$ , the breakdown payoff (4.16) strictly exceeds the breakdown payoff if production shifting is not possible (see (4.7)), unless the two products are perfect substitutes. The symmetric Nash bargaining problem between firm and union *i* can be formalized as

$$\max_{w_i} \left[ \tilde{R}^M(w_i, w_j) - \tilde{R}^0_i(w_j) \right] \cdot \tilde{L}_i(w_i, w_j).$$

With integration and unrestricted production shifting, analogous to the previous subsection we obtain the following equilibrium wages and quantities:

$$w^{M,W} = \frac{t}{3+4t+3\gamma},$$
(4.17)

$$x^{M,W} = \frac{3(1+t+\gamma)}{(3+4t+3\gamma)(2+t+2\gamma)}.$$
(4.18)

**Lemma 13.** If manufacturers are merged and the production of the brands can be shifted between the plants, then  $\gamma < 1$  yields a unique equilibrium where quantities are given by (4.18). If  $\gamma = 1$ , the aggregate supply of the non-differentiated products equals  $2x^{M,W}$ . The merged firm's profit in the equilibrium is given by

$$R^{M,W} = \frac{9(1+\gamma+t)^2}{(2+2\gamma+t)(3+3\gamma+4t)^2},$$
(4.19)

while each union realizes a wage bill of

$$L^{M,W} = \frac{3t(1+\gamma+t)}{(2+2\gamma+t)(3+3\gamma+4t)^2}.$$
(4.20)

### 4.3.4 Comparison of the Results

We compare the equilibrium outcomes before and after a merger depending on whether or not the merged firm can shift the production of the brands between plants. A merger which gives the opportunity to shift production increases the firms' bargaining position vis-à-vis the suppliers. As a result, the bargaining process yields the equilibrium wages (4.17) which are strictly below the pre-merger wages (4.3). Moreover, as long as goods are no perfect substitutes, a merged firm which can shift production between the plants negotiates strictly lower wages than a merged firm without this ability, *i.e.*,  $w_i^{M,W} < w_i^M$  for any  $\gamma < 1$ . If the products are perfect substitutes, the ability to shift production does not affect the equilibrium wages. Intuitively, the ability to shift production between the plants increases the merged firm's bargaining power and therefore exerts downward pressure on the equilibrium wages as long as goods are heterogeneous. If products are perfect substitutes, the scenarios in which production shifting is feasible and in which it is not are indistinguishable such that equilibrium wages are the same.

A merger which allows for production shifting affects equilibrium quantities as follows. If the products are close substitutes, *i.e.*,  $\gamma \in [\sqrt{13} - 3, 1] \approx [0.61, 1]$ , then a merger which allows for production shifting decreases the output levels and consumer surplus. This effect is grounded in the merged firm's market power: as in Cournot markets with homogeneous products, the equilibrium outputs are lower under a monopoly than under a duopoly. If the two brands are close to independent, *i.e.*,  $\gamma \in [0, 0.25]$ , then the opposite holds. A merger which allows for production shifting increases the equilibrium outputs and consumer surplus. This effect is grounded in the merged firm's strong outside option, which induces lower input prices and larger output levels. The concentration of market power through a merger is less important in this scenario as (close to) independent goods imply that in the pre-merger case both markets were also (nearly) monopolized. For the intermediate cases, *i.e.*,  $\gamma \in (0.25, \sqrt{13} - 3)$ , the effect of a merger which allows for production shifting on consumer surplus depends on the following trade-off. While merging increases market power and tends to harm consumers, production shifting enhances the manufacturer's bargaining power vis-à-vis the supplier, which reduces equilibrium wages and therefore increases output. Which of these effect dominates depends on the costs of production shifting. The second effect outweighs the first effect and the merger enhances consumer surplus whenever t is rather low, that is.,  $t < \bar{t}(\gamma)$  holds, where the threshold value is given by

$$\bar{t}(\gamma) = \frac{4(1-\gamma^2) - 3\gamma(3-\sqrt{5-4\gamma})}{2(4\gamma-1)}.$$

If t is relatively large, *i.e.*,  $t > \bar{t}(\gamma)$ , then the first effect dominates the second such that the merger reduces joint output and consumer surplus.

Note that the threshold value  $\bar{t}(\gamma)$  is a decreasing function of the product differentiation parameter  $\gamma \in (0.25, \sqrt{13} - 3)$ . While a higher substitutability between the products enhances a merger's concentration effect, a lower t tends to strengthen the manufacturer's outside option, to decrease input prices and to raise outputs. Therefore, if goods are better substitutes a merger's negative effect on consumer surplus is larger, but this effect can be counterbalanced by a lower capacity constraint parameter t.

Next, we compare the profits the manufacturer earns in the different scenarios. Note first that as long as  $\gamma < 1$ , the equilibrium wages are lower, the equilibrium output levels are larger and therefore a merger is more profitable if production shifting is feasible than if it is not. If  $\gamma = 1$ , the possibility to shift production does not affect wages, equilibrium quantities and profits. Second, if production shifting is feasible, then the post-merger wages are lower than the pre-merger wages and the merged firm's equilibrium profit exceeds the joint profit of the two independent firms. An overview over the equilibrium wages, quantities and profits is provided in Table 4.1.

## Proposition 7. Effects of a merger when production shifting is feasible Suppose products are not perfect substitutes (i.e., $\gamma < 1$ ).

i) A merger which allows for production shifting decreases the wage rates, i.e.,  $w_i^{M,W} < w_i^*$ , for  $i \in \{1, 2\}$ .

- ii) A merger which allows for production shifting has different effects on the equilibrium quantities which depend on the degree of product differentiation,  $\gamma$ , and the capacity constraints of the plants, t. If the goods are close substitutes,  $\gamma \in [\sqrt{13} 3, 1)$ , then the merger strictly decreases outputs for any positive value of t. If the goods are close to independent,  $\gamma \in [0, 0.25]$ , then the merger strictly increases outputs for any positive value of t. For the intermediate cases,  $\gamma \in (0.25, \sqrt{13} 3)$ , the merger strictly decreases (increases) the output if t is above (below) the threshold value  $\bar{t}(\gamma)$ . For  $t = \bar{t}(\gamma)$ , the merger does not affect the output levels.
- iii) The post-merger profit is higher if product shifting is possible than if it is not. If the goods are substitutable, i.e.,  $\gamma \in (0,1)$ , then the merged firm's profit strictly exceeds the firms' joint pre-merger profits. If the goods are independent, then the merged firm's profit equals the firms' joint pre-merger profit.

Finally, we compare the social welfare outcomes in the different scenarios. Note that in each scenario,  $x := x_1 = x_2 = X_1 = X_2$  holds in equilibrium. Then, social welfare SW is a function of the equilibrium quantity x, the product substitutability  $\gamma$  and the capacity constraint parameter t. In particular,

$$SW(x,\gamma,t) = 2\left((1-(\gamma+1)x)x + \frac{1}{2}x^2 - \frac{t}{2}x^2\right) = 2x - (t+2\gamma+1)x^2$$

Straightforward calculations yield the following corollary which shows that consumer surplus and social welfare are aligned.

Manufacturer's profit	equilibrium wage	equilibrium quantity
$R^* = \frac{9(2+t)^3}{2(2+\gamma+t)^2(8-\gamma+4t)^2}$	$w_1^* = w_2^* = \frac{2+t-\gamma}{8+4t-\gamma}$	$x_1^* = x_2^* = \frac{3(2+t)}{(2+t+\gamma)(8+4t-\gamma)}$
$R^{M} = \frac{9(2+t)^{2}}{4(2+2\gamma+t)(4-\gamma+2t)^{2}}$	$w_1^M = w_2^M = \frac{2+t-2\gamma}{2(4+2t-\gamma)}$	$x_1^M = x_2^M = \frac{3(2+t)}{(8+4t-2\gamma)(2+t+2\gamma)}$
$R^{M,W} = \frac{9(1+\gamma+t)^2}{(2+2\gamma+t)(3+3\gamma+4t)^2}$	$w^{M,W} = \frac{t}{3+4t+3\gamma}$	$X_1 = X_2 = x^{M,W} = \frac{3(1+t+\gamma)}{(3+4t+3\gamma)(2+t+2\gamma)}$

Table 4.1: An overview of the equilibria in the different scenarios.

# Corollary 6. Effects of a merger on SW when production shifting is feasible

i) A merger which does not allow for production shifting decreases total welfare if the products are not independent and does not affect welfare if the products are independent.

#### 4.4. EXTENSIONS - PRODUCTION SHIFTING AND FDI

- ii) A merger which allows for production shifting has different effects on total welfare. If the goods are close substitutes, γ ∈ [√13 - 3, 1], then the merger strictly decreases total welfare for any positive value of t. If the goods are close to independent, γ ∈ [0, 0.25], then the merger strictly increases total welfare for any positive value of t. For the intermediate cases, γ ∈ (0.25, √13 - 3), the merger strictly decreases (increases) total welfare if the cost parameter is above (below) a threshold value t̄(γ). The merger does not affect total welfare if the cost parameter equals t̄(γ).
- *iii)* If the goods are not perfect substitutes, then total welfare is higher if production shifting is possible than if it is not. Otherwise, total welfare is the same in both scenarios.

Our results can be used to gain interesting further insights into the effects of downstream mergers. If firms 1 and 2 are asymmetric with respect to their capacity constraint t (where we can interpret a lower t indicates a higher efficiency) and merge, then wages are lowered further at the inefficient plant. This is driven by the fact that the threat to shift production to the efficient plant improves the inefficient manufacturer's bargaining position a lot, whereas the chance to shift production from the efficient to the inefficient plant cannot improve the efficient manufacturer's bargaining position by much. Therefore, it is especially the inefficient firm which has an incentive to merge with "efficient" partners.

## 4.4 Extensions - Production Shifting and FDI

In this section, we study a firm's incentives to invest in a foreign country in the presence of country-specific labor unions. In Section 4.4.1, we assume that a firm can open a new plant in order to produce a differentiated product. We analyze under which conditions the firm invests in the domestic and under which it invests in a foreign country. Hereby, we distinguish two cases depending on the firm's ability to shift production. In Section 4.4.2, we assume homogeneous products, but heterogeneous bargaining power of the country-specific unions and analyze firm's FDI incentives.

### 4.4.1 Domestic vs. Foreign Investments

Suppose a manufacturer produces one product at a domestic plant with inputs it receives through bilateral bargaining with a supplier. We analyze the following three stage game. In the first stage, the manufacturer can open a new production plant to produce a differentiated product. There is a fixed investment cost H for opening a new plant in the home country and a fixed cost F > H for opening the plant in a foreign country. In the second stage, the manufacturer negotiates over input prices for each of its plants with the country-specific supplier(s). If the manufacturer did not open a new plant at the first stage, then it bargains with the domestic input supplier over the input price  $w_1$ . If it invested in the home country, it bargains with the domestic union over the input prices  $w_1$  and  $w_2$ , where  $w_i$  is the input price at plant *i*. If it invested in the foreign country, it bargains simultaneously with the domestic supplier over input price  $w_1$  and the foreign supplier over input price  $w_2$ . In the third stage, the firm produces the products and sells them in the integrated final goods market.

The demand and cost specifications are as in the previous sections. The manufacturer transforms one unit of each input into one unit of the output. The total production costs of the firm at plant i equal  $c_i = (w_i + t \cdot X_i/2) \cdot X_i$ , where  $X_i$  is the total production at plant i. The inverse demand for the firm's product i is given by  $p_i = 1 - x_i - \gamma x_j$ , where  $x_i, i \in \{1, 2\}$ , denotes the total output of product i and  $0 \leq \gamma \leq 1$  denotes the products' substitutability. The firm can produce in a country only if it reached an agreement with the monopolistic supplier in that country.

As before, we distinguish between two scenarios. In the first case, the products are plant-specific, *i.e.*, product i can only be produced at plant i. In the second scenario, production shifting is feasible and both brands can be produced at both plants. Depending on the firm's investment decision at the first stage, we obtain three subgames which we solve separately via backward induction. Then, we compare the equilibrium profits to determine the optimal investment decision for the firm. Finally, we compare social welfare levels under the different scenarios.

If the manufacturer does not invest at the first stage, we indicate equilibrium outcomes with index  $^{S}$ , while we use index  $^{H}$  to denote a home and index  $^{F}$  to denote a foreign investment. The additional index  $^{W}$  indicates that production shifting is feasible.

#### 4.4. EXTENSIONS - PRODUCTION SHIFTING AND FDI

If the manufacturer decides not to invest, then it owns a single plant only. In the last stage of the game, it maximizes its profit  $R^S = (1-x_1)x_1 - (w_1+t \cdot x_1/2) \cdot x_1$ for a given wage rate  $w_1$ . Thus, the optimal production level equals  $\hat{x}_1(w_1) = (1-w_1)/(2+t)$  if  $w_1 < 1$ , and  $\hat{x}_1(w_1) = 0$  otherwise. In the second stage, the firm bargains with the union over the wage  $w_1$ . If the firm reaches an agreement with the union then it makes profit  $\hat{R}^S(w_1) = (1-\hat{x}_1(w_1))\hat{x}_1(w_1) - (w_1 + t \cdot \hat{x}_1(w_1)/2) \cdot \hat{x}_1(w_1)$ , while the supplier earns  $\hat{L}^S(w_1) = w_1 \hat{x}_1^S(w_1)$ . In the case of a disagreement both players get payoff zero. We can formalize the symmetric Nash bargaining as

$$\max_{w_1} \hat{R}^S(w_1) \cdot \hat{L}^S(w_1). \tag{4.21}$$

The equilibrium wage rate maximizes the Nash product and equals  $w^S = 1/4$ . Given this, the firm's equilibrium output is  $x^S = 3/(8 + 4t)$ , and its respective profit is  $R^S = 9/(64 + 32t)$ , while the supplier earns  $L^S = 3/(32 + 16t)$ .

Next, we assume that the firm invests H > 0 and opens a new plant in the home country to produce a differentiated product. As suppliers are countryspecific, the firm bargains with the same supplier at both plants. Therefore, the ability to shift production across plants does not play a role in the bargaining process and the equilibrium outcomes are the same if shifting is possible and if it is not. Consequently, without loss of generality, we assume that products are plant-specific.

In the final stage, the manufacturer chooses  $x_1$  and  $x_2$  to maximize its profit,

$$R^{H}(x_{1}, x_{2}, w_{1}, w_{2}) = \sum_{i, j \in \{1, 2\}, i \neq j} \left[ (1 - x_{i} - \gamma x_{j}) x_{i} - (w_{i} + t \cdot x_{i}/2) \cdot x_{i} \right] - H.$$

The optimal production levels are given by (4.14). In the second stage, the firm negotiates both input prices with the same supplier in the home country. If the firm can reach an agreement with the union then it earns a profit of

$$\hat{R}^{H}(w_{1}, w_{2}) = R^{H}(\hat{x}_{1}(w_{1}, w_{2}), \hat{x}_{2}(w_{1}, w_{2}), w_{1}, w_{2}),$$

while the union earns  $\hat{L}^{H}(w_1, w_2) = w_1 \hat{x}_1(w_1, w_2) + w_2 \hat{x}_2(w_1, w_2)$ . In the case of disagreement, both the firm and the union get a payoff of zero. Therefore, the firm and the union maximize the Nash product

$$\max_{w_1, w_2} \hat{R}^H(w_1, w_2) \cdot \hat{L}^H(w_1, w_2).$$
(4.22)

The equilibrium wage levels which solve (4.22) are given by  $w_1^H = w_2^H = 1/4$ . In equilibrium, the manufacturer produces  $x_1^H = x_2^H = 3/(8+4t+8\gamma)$  and gets profit  $R^H = 9/(32+16t+32\gamma) - H$ , while the supplier earns  $L^H = 3/(16+8t+16\gamma)$ .

**Lemma 14.** For any  $\gamma \in [0,1]$  and for any t > 0, there is a  $H^C > 0$  such that the manufacturer will not invest in the home country if  $H > H^C$ . In contrast, it earns a strictly higher profit if it invests in the home country as long as  $H < H^C$ .

If the firm invests F and opens a new plant in a foreign country, then the analysis of the production and the bargaining stages are the same as in Section 4.3.1 and Section 4.3.2, depending on the possibility to shift production. If the manufacturer cannot shift production of its brands between its plants, the equilibrium wages, outputs and union profits are given by (4.8), (4.9) and (4.11). If shifting is feasible, then the equilibrium is characterized by (4.17), (4.18) and (4.20). The manufacturer's profit is given by  $R^F = R^M - F$  if production shifting is not possible and  $R^{F,W} = R^{M,W} - F$  if production shifting is possible.

#### Lemma 15. Firm's FDI incentives

- i) If production shifting is not feasible, then for any  $\gamma \in (0,1]$  and for any t > 0, there is a positive  $F^C > H^C$ , such that the manufacturer will not invest in the foreign country if  $F > F^C$ . If  $\gamma = 0$ , then  $F^C = H^C$ . As long as  $F < F^C$  holds, the firm earns a strictly higher profit if it invest in the foreign country.
- ii) If production shifting is feasible, then for any  $\gamma \in [0,1)$  and for any t > 0, there is a positive  $F^{C,W} > F^{C}$ , such that the firm which can shift the production will not invest in the foreign country if  $F > F^{C,W}$ . If  $\gamma = 1$ , then  $F^{C,W} = F^{C}$ . As long as  $F < F^{C,W}$  holds, the firm earns a strictly higher profit if it invest in the foreign country.

Given the results in Lemma 14 and Lemma 15, we can summarize the firm's investment incentives in the first stage. Therefore, we define the threshold values  $D := \frac{9}{16} \frac{\gamma}{(4t-\gamma+4)^2} \frac{4t-\gamma+8}{t+2\gamma+2}$  and  $D^W := \frac{9}{16} \frac{\gamma+1}{(4t+3\gamma+3)^2} \frac{8t+7\gamma+7}{t+2\gamma+2}$ .

#### Proposition 8. Domestic vs. foreign investments

#### 4.4. EXTENSIONS - PRODUCTION SHIFTING AND FDI

- i) If production shifting is not feasible, the manufacturer invests in the foreign country if  $F < F^C$  and F - H < D hold. It invests in the home country if  $H < H^C$  and F - H > D hold. Otherwise, it does not open a second plant.
- ii) If production shifting is feasible, the manufacturer invests in the foreign country if  $F < F^{C,W}$  and  $F H < D^W$  hold. It invests in the home country if  $H < H^C$  and  $F H > D^W$  hold. Otherwise, it does not open a second plant.

There are different incentives for a firm to open a new plant. First of all, if  $\gamma < 1$ , the firm introduces a new differentiated product in the second plant and thereby increases overall demand. Second, a new plant reduces production costs as plants' production functions are convex and as the overall production quantity can be optimally split among the plants. Third, if the firm makes a foreign direct investment, then it improves its bargaining position vis-à-vis the country-specific unions and reduces input prices. Note that this effect is absent for domestic investments. Lemma 15 and Proposition 8 also imply that a firm's investment incentives are even higher if production shifting is possible. Therefore, for the manufacturer it is desirable to design a new product in such a way that it can be produced at various plants.

Note that threshold value D is increasing in  $\gamma$ , *i.e.*,  $\partial D/\partial \gamma > 0$ . If production shifting is not possible, a foreign investment is especially desirable if the goods are substitutes as this strengthens the firm's bargaining position by increasing its breakdown profit. If goods are complements, however, threshold value D is rather low as foreign investments cannot improve the firms' bargaining power by much, such that the difference in profitability between foreign and home investments vanishes. Note furthermore that foreign investments raise the manufacturer's breakdown profit for all  $\gamma$  if production shifting is feasible, such that the threshold value  $D^W$  is not monotonic in  $\gamma$ .

Finally, we investigate the effects of FDI on domestic social welfare  $SW^D$ , that is, the sum of domestic firms' profits, domestic unions' wages and the domestic consumers' surplus. Hereby, we abstract from the investment costs F and Hand assume that half of the consumers are domestic consumers, that is, domestic consumer surplus equals  $x^2/4$ . Then, social welfare after opening a new plant in the home country equals  $SW^{D,H} := 2x(1 - \gamma x - x) - tx^2 + x^2/4$  where x denotes the equilibrium output which is produced at each plant. Under FDI the domestic



Figure 4.2: The effects of FDI vs. home investment on domestic welfare.

social welfare is given by  $SW^{D,F} := 2x(1-\gamma x-x)-tx^2+x^2/4-wx$ , where x is the equilibrium output and w is the wage paid by the domestic firm to workers in the foreign country. If production shifting is not feasible, then domestic social welfare is always lower if the firm invests in a foreign country instead of at home. While foreign investments cannot enhance the firm's bargaining position by much, wages paid to foreign workers diminish domestic social welfare. If production shifting is feasible, however, the effect of FDI on domestic social welfare depends on firms' capacity constraint t. For small t, foreign investments enable the firm to shift a large share of its output rather cheaply, such that the firm and consumers benefit from lower input prices. If the downward pressure on input prices is rather low, *i.e.*, if t is very large or if t is at an intermediate level, but goods are rather independent, then domestic social welfare suffers from FDI due to the wages paid to foreign workers. Figure 4.2 illustrates the difference in domestic social welfare  $\Delta SW = SW^{D,F} - SW^{D,H}$  as a function of  $\gamma$  for three different values of t if production shifting is feasible.

Our results can be used to gain insights into the effects of mergers in related scenarios. Suppose, for instance, that domestic and foreign firms, which get inputs from their plant-specific suppliers, offer differentiated products and compete in oligopolistic fashion. Then, mergers of domestic firms can increase domestic social welfare if the merged firms can enforce lower input prices and thereby steal market shares of their foreign competitors.

## 4.4.2 Foreign Investments Under Asymmetric Bargaining

So far, we have assumed that countries are symmetric. In this chapter, however, we suppose that unions differ between countries with respect to their bargaining power. Indeed, labor organizations are more professional, for instance, in Western European than in Eastern European countries. We investigate the effects of unions' bargaining power on foreign direct investments (FDI) and on equilibrium wages if a domestic firm can decide to invest in a country where workers' organization is weaker.

Our analysis yields the interesting result that a domestic union's high bargaining power may result in particularly low wages for the domestic workers. If a firm faces a powerful union, it has higher incentives to invest in production plants abroad. Such investments would be unprofitable if labor organization was weaker in the domestic country. However, they strengthen the bargaining position of the domestic firm vis-à-vis the domestic union. The opening of a production plant in the foreign country gives the firm a better outside option such that it can negotiate lower wages in the domestic country.

Our analysis proceeds as follows. A party's bargaining power can be measured via the exponent with which this party's objective enters the Nash product. Let R denote the firm's profits and L the overall wage bill of workers in the respective country. Whereas bargaining in the foreign country is assumed to be symmetric, *i.e.*, the parties maximize  $\max_w R \cdot L$ , we say that the domestic union has bargaining power  $\alpha \in [0, 1]$  if the parties maximize  $\max_w R^{1-\alpha} \cdot L^{\alpha}$ .

In the benchmark case, we investigate the equilibrium outcomes if the firm bargains exclusively with the domestic union. Computations which are analogous to those in the previous subsection show that the parties agree on wage  $w^{S,\alpha} = \alpha/2$  and output  $x^{S,\alpha} = (2 - \alpha)/(4 + 2t)$ . The firm makes a profit of  $R^{S,\alpha} := (1 - x^{S,\alpha})x^{S,\alpha} - (w^{S,\alpha} + tx^{S,\alpha}/2)x^{S,\alpha}$ , while the worker's wages amount to  $L^{S,\alpha} = x^{S,\alpha} \cdot w^{S,\alpha}$ . Note that the firm's profit is strictly monotonic decreasing in  $\alpha$ , while the union's wage bill is increasing in  $\alpha$ .

Second, we determine equilibrium wages and output levels if the manufacturer invests in a second production plant which is located in a foreign country. Now, the firm has a higher breakdown profit than in the benchmark case which allows the manufacturer to hold wages low. Straightforward computations yield that the domestic workers get wage  $w_1^{F,\alpha} = (2t\alpha + 5\alpha)t/(4t^2 + 16t - 2\alpha + 16)$  and produce  $X_1^{F,\alpha} = (2t+5)(a+2)(2-\alpha)/((2t^2+8t+8-\alpha)(8+2t))$ , while foreign workers earn  $w_2^{F,\alpha} = t(t+\alpha+2)/(4t^2+16t-2\alpha+16)$  and produce  $X_2^{F,\alpha} = 3(t+2)(t+\alpha+2)/((2t^2+8t+8-\alpha)(2t+8))$ . The manufacturer earns  $R^{F,\alpha} := (1-X_1^{F,\alpha}-X_2^{F,\alpha})(X_1^{F,\alpha}+X_2^{F,\alpha}) - X_1^{F,\alpha}(w_1^{F,\alpha}+tX_1^{F,\alpha}/2) - X_2^{F,\alpha}(w_2^{F,\alpha}+tX_2^{F,\alpha}/2)$ and the domestic union gets  $L^{F,\alpha} = w_1^{F,\alpha} \cdot X_1^{F,\alpha}$ .

The firm's maximum investment is given by the difference of its profits  $F^{C}(\alpha) := R^{F,\alpha} - R^{S,\alpha}$ . The firm's willingness to pay for a foreign direct investment is strictly monotonic increasing in  $\alpha \in [0, 1]$ . Consequently, a higher bargaining power of a domestic union may render foreign investments profitable.

Given the manufacturer has opened a new plant in the foreign company, the domestic workers earn, compared to the benchmark scenario, strictly less for all  $\alpha \in (0, 1]$ , both in terms of wages (for all  $\alpha \in (0, 1]$ , we have  $w^{S,\alpha} - w_1^{F,\alpha} > 0$ ) as in terms of total wage bills (for all  $\alpha \in (0, 1]$ , we have  $L^{S,\alpha} - L^{F,\alpha} > 0$ ). For  $\alpha = 0$ , wages and total wage bills are identical in the two scenarios.

Consider the following game. Let  $\alpha \in [0,1]$  denote the domestic union's bargaining power and F > 0 the costs of a foreign direct investment. Bargaining in the foreign country is symmetric (which is equivalent to assuming that the union in the foreign country has bargaining power of 1/2). At the first stage, the firm decides if to invest in the foreign country. Second, the firm bargains (simultaneously) with the union(s) over the wages  $w^{*,\alpha}$ . Third, the firms produce equilibrium quantities  $X_i^{*,\alpha}$  at plant *i*. The domestic union earns a wage bill of  $L^{*,\alpha} = w^{*,\alpha}X_1^{*,\alpha}$ .

#### Proposition 9. Prisoner's dilemma of wage bargaining

For all  $\alpha \in (0,1]$ , there is a  $\varepsilon > 0$  s.t.  $L^{S,\alpha-\varepsilon} > L^{F,\alpha}$ . If  $F^C(\alpha) > F$ , but  $F^C(\alpha-\varepsilon) < F$ , then  $L^{*,\alpha-\varepsilon} > L^{*,\alpha}$ .

The first part follows from the fact that  $L^{S,\alpha} - L^{F,\alpha} > 0$  for all  $\alpha \in (0, 1]$ , while  $L^{S,\alpha}$  is continuous and decreasing in  $\alpha$ . The second part follows from the fact that  $F^C(\alpha)$  is strictly monotonic increasing on  $\alpha \in [0, 1]$ . If  $F^C(\alpha) > F$  holds, then the manufacturer opens a new plant in the foreign country such that the domestic union earns  $L^{F,\alpha}$ . If the union's bargaining power, however, was only  $\alpha - \varepsilon$ , then it could be that the FDI would not pay off for the firm, such that it would not invest and pay the union  $L^{S,\alpha-\varepsilon}$ .



Figure 4.3:  $\varepsilon^{\text{max}}$  as a function of the domestic union's bargaining power  $\alpha$ .

The manufacturer and the domestic union face a variant of the prisoner's dilemma. If the union could commit not to exert its full bargaining power, but to forego a substantial share and be a rather weak negotiant, the manufacturer would abstain from FDI and both the union and the manufacturer could be better off. If the manufacturer, however, does not engage in FDI, the union has an incentive to deviate and exert its full bargaining power. As there is no commitment device for the union, the manufacturer anticipates that the union will use its entire bargaining power. Therefore, a marginal increase in the domestic union's bargaining power  $\alpha$  may render FDI profitable, such that both the manufacturer's and the domestic union's payoff have a discrete drop in payoffs compared to the scenario without FDI. If the domestic union's bargaining power had been lower in the beginning, then in certain parameter ranges the manufacturer would have had abstained from FDI and the domestic union would have realized a higher wage.

In order to investigate quantitatively how much of its bargaining power the domestic union would be willing to forego, we give an example for the case t = 1. The largest parameter  $\varepsilon$  for which  $L^{S,\alpha-\varepsilon} \ge L^{F,\alpha}$  holds we denote  $\varepsilon^{\max}$ . Figure 4.3 shows that the share of bargaining power the domestic union was willing to forego strictly increases with  $\alpha$ . For  $\alpha = 1/2$ , the union would be willing to accept a bargaining power of  $\alpha - \varepsilon^{\max} \approx 0.11$  as long as this could prevent the firm from investing in the foreign country. For  $\alpha = 1$ , the union would be even willing to

accept  $\alpha - \varepsilon^{\max} \approx 0.17$  in order to prevent the firm from FDI. Consequently, the high bargaining power of a domestic union makes domestic workers *much worse* off if it induces the firm to FDI. Workers would be willing to forego most of their power in order to prevent the firm from outsourcing, in which case the union's payoff suffers heavily.

# 4.5 Conclusion

In this paper, we analyze how merger and investment decisions of multi-plant firms, which receive inputs from plant-specific suppliers, depend on the ability to shift production across plants. So far, the economic literature has focused on multi-plant and cross-border mergers where firms' products were plant-*specific*. While we analyzed such a setup in our benchmark model, in the main part we analyzed a multi-plant firm's ability to shift the production of all brands across the plants. The benchmark may reflect a merger's short-term analysis as relocating the production of a specific brand may not be feasible in the short run. In the long run, however, the ability to shift production seems natural. Our analysis shows that a merger decreases input prices in general. In the benchmark case, a merger never increases consumer surplus or social welfare. But if the multi-plant firm can shift its production between the plants, then a merger may increase consumer surplus and social welfare unless the firm's products are close substitutes. In this case, the merger's monopolization effect outweights the benefits from enforcing lower input prices.

Our findings are relevant for merger control. If antitrust authorities decide on international merger proposals, they should take into account if production shifting is feasible or not as this ability crucially impacts on equilibrium welfare outcomes. The ability to shift production may be a substantial part of the merger's efficiency defense: production shifting allows the firms to countervail unions' bargaining power and therefore to increase consumer surplus and social welfare. Without the ability to shift production, however, a merger's monopolization effects are likely to detriment welfare outcomes.

As an extension, we studied a firm's investment decision for opening a new plant. A firm has a strong incentive to design a new differentiated product either at a home or a foreign plant. Due to the improvement in bargaining power, however, a firm has typically higher incentives to invest abroad than in the home country.

In a second extension, we investigate the scenario that domestic and foreign unions have heterogeneous bargaining power. We find that the domestic union may be strictly worse of through an increase of its bargaining power as this may render FDI profitable, such that, due to investment costs, also the manufacturer is worse off in equilibrium. The firm and the union face a variant of the prisoner's dilemma, which could not be overcome as long as the union cannot commit to relinquishing its strong bargaining position.

# **Declaration of Contribution**

Hereby I, Hamid Aghadadashli, declare that the chapter "Multi-plant Firms, Production Shifting, and Countervailing Power" is coauthored by Dr. Markus Dertwinkel-Kalt and Prof. Dr. Christian Wey.

My contributions to this chapter are as follows:

- I have contributed to the Introduction.
- I have written major parts of the Model and the Analysis.
- I have derived part of the Extensions.
- I have contributed to the Conclusion.

Signature of coauthor 1 (Dr. Markus Dertwinkel Kalt): \_\_\_\_\_

Signature of coauthor 2 (Prof. Dr. Christian Wey):

C. Wey

# Chapter 5

# Multiunion Bargaining: Tariff Plurality and Tariff Competition

Co-authored by Christian Wey

# 5.1 Introduction

We study multi-union bargaining where a single employer must negotiate with two unions about working conditions. The groups of workers represented by the unions can be substitutable or complementary depending on the labor inputs they provide. In German labor law parlance, the former case is referred to as "tariff competition" and the latter case as "tariff plurality."<sup>1,2</sup>

In Germany, legal practice vis-à-vis craft unionism changed dramatically with

<sup>&</sup>lt;sup>1</sup> Taking a labor law perspective, Rieble (1996) surveys the German system of collective bargaining and the possibility of multi-union representation at the firm-level.

<sup>&</sup>lt;sup>2</sup> In contrast to Germany (which was dominated by a unified labor movement under the roof of the *Deutscher Gewerkschaftsbund*, DGB), countries like France, Belgium, the Netherlands, and Italy have a long history of trade union pluralism with trade unions being divided along ideological and sometimes confessional lines. The most important direct legal consequence in pluralistic labor markets is the confining of certain rights to "most representative" unions, so that only the "most representative" union of the workforce in question is eligible for concluding enterprise-level collective agreements (see Forde (1984), for an account of the representative criterion in France).

the decision of the Federal Labor Court (*Bundesarbeitsgericht*) of 23 June 2010 to give up the tariff unity principle of "one firm - one tariff contract." Since that time more than one tariff contract can coexist in a single firm if each tariff agreement deals with different types of labor. An example are hospitals in which case hospital doctors are (mainly) represented by the *Marburger Bund* (a craft union), while the remaining workers are represented by the German Trade Union (*Verdi*). Such a situation is called "tariff plurality" ("Tarifpluralität"). In contrast to "tariff competition," tariff plurality is characterized by unions representing workers which provide complementary services. Typically one workforce (as hospital doctors) is represented by a fully specialized craft union.<sup>3</sup>

In Germany, craft unionism and multi-union bargaining are on the rise in other industries as well (see Bachmann and Schmidt (2012)). The *Deutsche Bahn* (the dominant railway operator) must bargain with the German Train Drivers Union (*Gewerkschaft Deutscher Lokomotivführer*; GDL) and the Railway and Transport Union (*Eisenbahn- und Verkehrsgewerkschaft*; EVG). Again, the GDL is a craft union which is complementary to workers represented by the other union EVG. The former one takes care of the train drivers' employment conditions and the latter one represents the remaining railway workers' interests.<sup>4</sup>

Behind this background, we explore the consequences of multi-union bargaining, where a single employer bargains with two unions, each one representing either substitutable or complementary worker groups. A salient feature of multiunion bargaining is that each union not only stands in opposition to the employer but also "competes" with the other union over the joint surplus. Our main research question is, therefore, how labor bargaining outcomes are affected by unions' rent-shifting incentives when unions compete both with the employer and another union about the joint surplus. Closely related, we ask whether unions have incentives to merge into a single union which includes all workers.

In our main analysis we suppose that a firm produces a homogenous product using labor inputs from both unions. The unions represent either perfectly substi-

<sup>&</sup>lt;sup>3</sup> In general, collective bargaining in Germany was dominated by industry unions which are organized in the German association of unions (*Deutscher Gewerkschaftsbund*; DGB). The newly established craft unions are not members of the DGB, so that firms must determine employment condition with two unions in those instances (the craft union and the traditional industry union of the DGB).

<sup>&</sup>lt;sup>4</sup> Other examples include airlines (where pilots are represented by *Vereinigung Cockpit*) and airports (where air traffic controllers are organized in the *Gewerkschaft der Flugsicherung*).

tutable or perfectly complementary workers.<sup>5</sup> We suppose an efficient contracting setting where the union and the employer bargain over both the employment level and the wage rate. If bargaining is simultaneous when unions are separated, then we replicate existing results of the literature which show merger incentives in case of substitutable unions and disintegration incentives in case of complementary workforces. At the same time, labor contracts are always efficient; i.e., the employment level maximizes the joint surplus of all workers and the firm. Our main findings relate to the rent-shifting incentives among unions under sequential multi-union bargaining, where one union contracts first with the firm and the other union following.<sup>6</sup> When unions are perfectly substitutable, then the first union concludes a contract which forecloses the second union. In contrast, when the unions represent perfectly complementary labor groups, then there is a second-mover advantage, so that the first union's wage bill is smaller than the second union's wage bill.

If the two workforces are substitutable, then both unions prefer to form a single union and bargain jointly. A union merger increases the union's bargaining power and each workforce realizes a higher wage bill. If the workforces are complementary, then the total wage bill is lower under joint bargaining when compared with independent sequential bargaining. Interestingly, the unions' interests are not aligned in the case of complements. At least, one of the two unions is worse off under joint bargaining.<sup>7</sup>

In an extension we also explore the case of where each union's labor force produces a differentiated (either complementary or substitutable) good ("twoproducts" case). This setting mirrors the bargaining problem the Deutsche Bahn is facing. The Deutsche Bahn offers both rail journeys and an intercity bus carrier services. While train drivers are represented by the union GDL, all other employees (including bus drivers) of the Deutsche Bahn are represented by the rival union EVG. Bus and train journey services can be substitutable and com-

<sup>&</sup>lt;sup>5</sup> In an extension we also consider the where the firm produces two differentiated products using labor inputs from the respective labor union.

<sup>&</sup>lt;sup>6</sup> Assuming sequential contracting in the realm of organized labor is adequate as tariff contracts are observable and immune against renegotiation for the agreed upon contract duration.

<sup>&</sup>lt;sup>7</sup> These results mirror the lobbying activities of the incumbent monopoly unions in Germany (organized within the DGB) against craft unionism, where the workers organized in craft unions have been able to raise their wage bills at the cost of the established unions' workers.

plementary. Moreover, this relationship may not be perfect which gives rise to additional rent-shifting issues under sequential bargaining that we highlight in the two-products extension.<sup>8</sup> If unions are substitutable, then the firm employs more workers from the first union than from the second union, while in case of complementary unions the opposite is true. That means that "tariff competition" leads to overemployment (i.e., the joint surplus maximizing employment level is smaller).<sup>9</sup>

In contrast, when the unions are complementary, "tariff plurality" (or, craft unionism) leads to underemployment (i.e., the joint surplus maximizing employment level is larger). The first union does not internalize the negative externality of reducing its employment level on the second union. At the same time, the second union benefits from a worsened disagreement payoff of the firm which allows it to settle on a larger employment level than the first union. These results show additional distortions through unions' rent-shifting incentives which cannot be obtained in the one-product model. The reason is that in the one-product model unions are restricted to be either perfectly substitutable or perfectly complementary, while the two-products model also allows for imperfect substitution patterns.

Qualitatively, we obtain the same results of unions' merger incentives as in the one-product model. Unions' merger incentives under tariff competition eliminate the overemployment outcome of sequential bargaining. In contrast, in case of tariff plurality (i.e., workers are complementary) unions prefer to stay independent which implies that the underemployment result persists. Taking a total welfare perspective by including consumer surplus as well, it follows that the unions' merger decisions always stay in conflict with social welfare maximization; i.e., when unions merge in case of substitutable groups of workers, this reduces total welfare, while their preference for separate bargaining in case of complementary workforces leads also to a reduction in total welfare.<sup>10</sup>

<sup>&</sup>lt;sup>8</sup> Under simultaneous and joint bargaining, the level of employment is always joint surplus maximizing, but not under sequential bargaining.

<sup>&</sup>lt;sup>9</sup> Our overemployment result for the case of substitutable unions mirrors a similar result obtained in Stole and Zwiebel (1996) who use the Shapley value to calculate workers' wages.

<sup>&</sup>lt;sup>10</sup> Note that our model assumes a symmetric setting (except for the sequential timing of bargaining). If unions were asymmetric (e.g., because of different outside options), then the merger results may change.

We contribute to the labor market literature which deals with powerful unions and the issue of collective bargaining.<sup>11</sup> Closely related to our work is Horn and Wolinsky (1988b) who analyze bargaining between two labor unions and an employer. The labor unions represent different worker groups which are either substitutable or complementary. The unions maximize the wage bill of their workers and the firm maximizes its profit. The efficient employment levels are exogenously fixed and a union bargains with the firm over the wage rate.<sup>12</sup> In such an efficient bargaining setting, the authors show that unions benefit from joint bargaining when their labor inputs are substitutable, while unions prefer to bargain separately when the labor inputs are complementary. The result can be explained as follows. Note first that the joint surplus is shared equally between the workers and the firm if there is a single union representing all workers. Because of symmetric worker groups, each worker group then gets one-forth of the joint surplus. If, in contrast, unions are independent and bargaining is simultaneous, then a union's share of the joint surplus depends on its marginal contribution to the joint surplus, *given* that the other bargaining outcome is successful. If the two worker groups are substitutable, then the marginal contribution of each union must be less than one-half of the joint surplus (in the extreme case for perfectly substitutable worker groups the marginal contribution must go to zero). It follows that unions want to merge when they are substitutable to get together one-half of the joint surplus. In case of complementary union the opposite holds. Now a union's marginal contribution is larger than one-half, so that the sum of both unions' wage bills exceeds one-half of the joint surplus.

Our model of sequential union-firm bargaining model is related to Marx and Shaffer (1999) where two suppliers bargain sequentially with a single retailer over two-part tariff contracts. Two-part tariffs imply that bargaining is bilaterally efficient as rents can be shifted between the parties without affecting the joint

<sup>&</sup>lt;sup>11</sup> For an overview of the literature of collective bargaining in labor markets see Cahuc and Zylberberg (2004, chap. 7) and Boeri and van Ours (2013, chap. 3). Interestingly, both references do not touch on the issue of multi-union bargaining.

<sup>&</sup>lt;sup>12</sup> The case of bargaining over linear contracts with an elastic labor demand function was analyzed in Horn and Wolinsky (1988a). While the results concerning unions' merger incentives remain valid, double marginalization problems provoke additional inefficiencies. Jun (1989) presents a fully specified extensive form game to solve the "two-unions one-firm" bargaining problem. It is shown that asymmetries among unions make separation more likely.

surplus. Sequential bargaining leads to "predatory accommodation" such that the contract between the firm and the first supplier specifies a wholesale price which is below the supplier's marginal production costs. This allows the first supplier to attract some of the rents created by the second supplier. Intuitively, the firm's profits from reaching an agreement with the second supplier critically depend on the firm's disagreement point. A relatively low wholesale price specified in the contract with the first supplier improves the disagreement point of the firm which implies higher profits for the firm. The first supplier is now able extract some of these higher profits of the firm with the fixed transfer payment. We also focus on the bargaining externalities between independent suppliers (unions in our case), but in contrast to Marx and Shaffer (1999), we examine employment-wage contracts and we consider both substitutable and complementary (labor) inputs.<sup>13</sup>

Cai (2000) analyzes a model where a buyer bargains sequentially with two perfectly complementary sellers where the order of reaching an agreements is endogenously determined.<sup>14</sup> He shows the existence of a delay equilibrium which is driven by the fact that the "last" seller obtains a larger share of the joint surplus.

# 5.2 The Model

We consider a firm (employer) which has to bargain with two labor unions Xand Y to produce a single product. Each union represents different types of labor which can be substitutable or complementary. Let q(x, y) be the production function which gives the total quantity produced as a function of the workers hired from union X and from union Y, denoted by x and y, respectively. Depending on whether the labor inputs of the two unions are substitutable or complementary, we consider the following two expressions of the firm's production function (see,

<sup>&</sup>lt;sup>13</sup> The idea of rent-shifting in sequential contracting goes back to Aghion and Bolton (1987) where the first contract is an exclusive contract which stipulates a costly exit clause.

<sup>&</sup>lt;sup>14</sup> Chongvilaivan, Hur and Riyanto (2013) consider the case of a firm bargaining with a union and an input supplier over linear input prices, where the union and the supplier are perfectly complementary. Depending on the firm's Nash bargaining power, it may prefer to bargain sequentially or simultaneously, or it may prefer to integrate with the supplier before bargaining with the union.

e.g., Varian (2010, p. 334ff)):

$$q(x,y) = \begin{cases} \frac{x+y}{2}, & \text{if labor inputs are substitutes,} \\ \min\{x,y\}, & \text{if labor inputs are complements.} \end{cases}$$
(5.1)

In the former case, output is determined by the sum of the labor inputs, so that both labor inputs are perfectly substitutable. The latter case reflects the perfect complements case, where output is given by the smaller value of both labor inputs.<sup>15</sup> Form (5.1) it follows that for all pairs of inputs (x, y) with x = y, the same isoquant q = x = y is reached in the substitutes case and in the complements case.<sup>16</sup>

The inverse demand function for the final good is assumed to be linear and given by p(q(x,y)) = 1 - q(x,y).<sup>17</sup> The firm is supposed to have monopoly power in the final product market which creates profits that are at stake in the union-firm negotiations. We assume efficient bargaining between the firm and a union, so that an employment contract specifies the wage and the employment level.<sup>18</sup> Hence, the firm bargains with the union X over the wage w and the employment level x. In the same way, the firm bargains with union Y over the wage r and the employment level y. If both types of labor are represented by a single union ("union merger"), then the union bargains over both wages (w and r) and both employment levels (x and y). If negotiations are successful, then the

<sup>&</sup>lt;sup>15</sup> In both specifications of the production function we supposed a specific relationship between labor input and the firm's output. We could have used a more general production function of the form  $q = \alpha x + \beta y$  ( $q = \min\{\alpha x, \beta y\}$ ) with  $\alpha, \beta > 0$ , for the case of substitutes (complements). Aside from introducing asymmetries between both unions, that approach would not affect our main results.

<sup>&</sup>lt;sup>16</sup> For instance, if x = y = 1/2, then q = 1/2 holds for the complements and the substitutes cases. As a consequence, the same inputs will maximize the joint surplus in both cases.

<sup>&</sup>lt;sup>17</sup> All our results remain valid under a general demand function p(q) which is monotonically decreasing.

<sup>&</sup>lt;sup>18</sup> The efficient bargaining framework mirrors the fact that a tariff contract between a union and a firm often not only specifies wage rates but also employment levels (for instance, in the form of job guarantees and commitments to avoid layoffs). However, employment levels cannot be monitored perfectly through labor tariff contracts, and this is why we also consider the right-to-manage approach in an extension below. Theoretically, the efficient bargaining approach allows us to single out the rent-shifting incentives of the unions while keeping the joint surplus fixed at the efficient level.

profit of the firm is given by

$$\pi(x, w, y, r) = [1 - q(x, y)] q(x, y) - wx - ry,$$

while union X and union Y realize a wage bill of  $L_X = wx$  and  $L_Y = ry$ , respectively.<sup>19</sup> In the first stage of the game, union-firm bargaining occurs and in the second stage the firm makes production decision depending on the bargaining outcomes. For the case of two separate unions, we consider both simultaneous and sequential bargaining. While there can be instances where two unions bargain simultaneously (and independently) with a single employer, the rule in reality appears to be that tariff contracts are negotiated and concluded sequentially. The sequential timing is supported by the fact that tariff contracts are observable and fixed for their life span.

To provide a benchmark, we first solve the bilateral bargaining problem between the firm and a single union which represents all workers. If all workers are represented by a single union, then the wages and employment levels of both worker groups are determined in a single bargaining problem. Suppose that both unions X and Y merge to form a single union XY. The merged union bargains with the firm over both wages and employment levels. When the unions X and Y are substitutable, then the joint labor union represents a homogeneous worker group. Hence, the merged union and the firm negotiate over a uniform wage wand the total number of workers x + y. If the unions X and Y are complementary, then the firm negotiates with the joint union over wages, w and r, and the number of workers from each type of workers, x and y. As both worker groups are symmetric and perfectly complementary, the bargaining solution will also be symmetric; i.e., we can set w = r and x = y. We can further simplify the problem by using z := x + y for the substitutes and z/2 = x = y for the complements case, respectively. Given (5.1), the firm produces q(x, z - x) = z/2 in the case of agreement, and realizes profits of  $\pi(z, w) = [1 - q(x, z - x)] q(x, z - x) - wz$ . The joint union gets a total wage bill of  $L_{XY} := wz$ . In the case of disagreement, both

<sup>&</sup>lt;sup>19</sup> We assume that each union maximizes the wage bill of its members (see Dunlop (1944)) which can be justified by the fact that union membership fees are a fixed percentage share of the worker's monthly gross income (the industrial union, *IG Metall*, for example, charges 1% of gross income). The Stone-Geary union utility function considers relative weights for the wage and the employment level. The utilitarian approach refers to the utility levels of the union's members (for overviews, see Booth (1995) and Cahuc and Zylberberg (2004)).
#### 5.2. THE MODEL

the firm and the union get a payoff of zero. The generalized Nash bargaining problem then becomes

$$\max_{z,w \ge 0} [\pi(z,w)]^{\alpha} [zw]^{(1-\alpha)}$$

$$= \left[ \left( 1 - \frac{z}{2} \right) \frac{z}{2} - wz \right]^{\alpha} [wz]^{(1-\alpha)}.$$
(5.2)

The following employment level and wage solve the joint bargaining problem (5.2)

$$w^{joi} = \frac{1}{4}(1-\alpha) \text{ and } z^{joi} = 1,$$
 (5.3)

where the superscript "joi" denotes the joint bargaining case.<sup>20</sup> The employment level under joint bargaining is at the efficient level; i.e, it maximizes the joint surplus  $\Pi = \pi + zw$ .<sup>21</sup> The firm's equilibrium profit is  $\pi^{joi} = \alpha/4$  and the union's equilibrium wage bill is  $L_{XY} = (1 - \alpha)/4$ . The shares of the joint surplus are  $\alpha$ , and  $1 - \alpha$ , for the firm and the merged union, respectively.

We next analyze bargaining outcomes when the firm bargains with two independent labor unions each maximizing its workers' wage bill. We start with substitutable labor unions which mirrors the case of tariff competition. Then, we examine complementary labor unions which stands for the tariff plurality case. We analyze both simultaneous bargaining and sequential bargaining.

Figure 5.1 illustrates the timing of the model which is basis of our union merger analysis.

For that purpose we consider an initial stage 0 in which both unions can decide to merge or not. If unions merge, then the joint union bargains with the firm over both labor contracts. If unions do not merge, then first union X bargains with the firm and union Y bargains with the firm thereafter.

<sup>&</sup>lt;sup>20</sup> If the unions are substitutes, then the joint union has homogeneous workers and the firm negotiates only on the total number of workers, z. Hence, any combination  $x+y=z^{joi}=1$ could be the outcome. If, however, the unions are complements, then the firm needs both kind of workers in a strict one-to-one proportion, and the number of different types of workers has to be equal with  $x = y = z^{joi}/2 = 1/2$ .

<sup>&</sup>lt;sup>21</sup> Joint surplus maximization requires that the revenue maximizing output level q = 1/2 is realized, because we set the opportunity cost of labor equal to zero. It is then immediate, that x + y = 1 must hold in the substitutes case and x = y = 1/2 in the complements case, respectively.

Figure 5.1: Timing of the Model



### 5.2.1 Substitutable Labor Unions: Tariff Competition

We analyze the case in which two labor unions represent substitutable worker groups. In this case, the firm can transform labor input into output according to the production function q(x, y) = (x + y)/2.

Simultaneous Bargaining. First, we consider simultaneous bargaining between the firm and two independent unions. If the firm can reach agreements with both unions, then its profit is  $\pi(x, w, y, r) = [1 - (x + y)/2] (x + y)/2 - wx - ry$ . If the firm fails to reach an agreement with union X, or union Y, then its profit is given by its disagreement points  $\pi^{DX} := \pi(0, 0, y, r) = (1 - y/2)y/2 - ry$ , or  $\pi^{DY} := \pi(x, w, 0, 0) = (1 - x/2)x/2 - wx$ , respectively. The unions do not have any outside options and realize a profit of zero in case of disagreement. The generalized Nash bargaining problem between the firm and union X can then be written as<sup>22</sup>

$$\max_{y,r \ge 0} [\pi(x,w,y,r) - \pi^{DX}]^{\alpha} [wx]^{(1-\alpha)}$$

$$= \left[ \left( \left( 1 - \frac{(x+y)}{2} \right) \frac{(x+y)}{2} - wx - ry \right) - \left( \left( 1 - \frac{y}{2} \right) \frac{y}{2} - yr \right) \right]^{\alpha} [wx]^{(1-\alpha)},$$
(5.4)

while the generalized Nash bargaining problem between the firm and union Y

<sup>&</sup>lt;sup>22</sup> Each bargaining pair takes the wage-employment contract of the other bargaining pair as given when maximizing their Nash product. By that we solve for a Nash equilibrium of two simultaneous bargaining problems (see Chipty and Snyder (1999), for a formalization). Inderst and Wey (2003) assume contracts which condition on the fact whether or not the other bargaining problem is successful. They show that this protocol gives rise to the Shapley value (see also Stole and Zwiebel (1996)). Our results are easily shown to be qualitatively robust in this regard.

can be stated as

$$\max_{x,w \ge 0} [\pi(x,w,y,r) - \pi^{DY}]^{\alpha} [ry]^{(1-\alpha)}$$

$$= \left[ \left( \left( 1 - \frac{(x+y)}{2} \right) \frac{(x+y)}{2} - wx - ry \right) - \left( \left( 1 - \frac{x}{2} \right) \frac{x}{2} - xw \right) \right]^{\alpha} [ry]^{(1-\alpha)}.$$
(5.5)

The parameter  $\alpha$  measures the relative bargaining power of the firm, while  $1 - \alpha$  stands for the union's relative bargaining power. As the unions are symmetric, in equilibrium the optimal number of employed workers and the wages are the same for both unions. The following contracts solve the bargaining problems (5.4) and (5.5) (the superscript "*sim*" stands for simultaneous bargaining):

$$w^{sim} = r^{sim} = \frac{1}{8}(1-\alpha)$$
 and  $x^{sim} = y^{sim} = \frac{1}{2}$ 

The equilibrium employment levels are independent of the bargaining power of the firm and they maximize the joint surplus  $\Pi := \pi + wx + yr$ . The wage levels decrease when the firm's bargaining power parameter  $\alpha$  increases. The firm's equilibrium profit is equal to  $\pi^{sim} = (1 + \alpha)/8$  and each union's total wage bill is  $L_X^{sim} = L_Y^{sim} = (1 - \alpha)/16$ . It follows that for any value of  $\alpha > 0$ , the firm gets a strictly larger share of the joint surplus (namely,  $\pi^{sim}/\Pi = (1 + \alpha)/2$ ) than both unions together. Moreover, the firm's share of the joint surplus increases even further when  $\alpha$  increases. Hence, each union's total wage bill and its share of the joint surplus is decreasing in  $\alpha$ .

Comparing these results with the joint bargaining solution, we get that the total wage bill increases with a union merger as long as the firm's bargaining power is not perfect; i.e., whenever  $\alpha < 1$  holds. Only if the firm has all the bargaining power,  $\alpha = 1$ , then the firm and the unions are indifferent between simultaneous and joint bargaining. It is noteworthy that the union structure (either independent unions or a single merged union) does not affect the employment level which is always joint surplus maximizing.

Sequential Bargaining. We now examine sequential bargaining between the firm and the labor unions. In the first stage, the firm negotiates a contract with union X over both the employment level, x, and the wage, w. In the second stage, the bargaining outcome of the first stage becomes public and the firm bargains with union Y over the employment level, y, and the wage,  $r^{23}$  If the

 $<sup>^{23}</sup>$  The ordering is without loss of generality because both unions are symmetric.

negotiations between the firm and the unions are successful then the firm gets a payoff of  $\pi(x, w, y, r) = [1 - q(x, y)]q(x, y) - wx - ry$ , and unions X and Y realize wage bills of  $L_X = wx$  and  $L_Y = ry$ , respectively.

Suppose the bargaining outcome in the first stage gives rise to a labor contract (w, x). Then, in the second stage, the firm and the union Y bargain over the contact (r, y), taking the outcome of the first stage as given. If, in the second stage, bargaining is successful, then the firm gets the profit  $\pi(x, w, y, r)$ , while union Y gets  $L_Y = ry$ . In case of disagreement, the firm can still produce x units of goods and makes profits of  $\pi^{DY} := \pi(x, w, 0.0) = (1 - x/2)x/2 - xw$ . Union Y gets zero payoff in case of disagreement. Accordingly, the generalized Nash bargaining problem between the firm and union Y can be written as

$$\max_{y,r \ge 0} [\pi(x,w,y,r) - \pi^{DY}]^{\alpha} [yr]^{(1-\alpha)} = \left[ \left( \left(1 - \frac{(x+y)}{2}\right) \frac{(x+y)}{2} - xw - yr \right) - \left( \left(1 - \frac{x}{2}\right) \frac{x}{2} - xw \right) \right]^{\alpha} [yr]^{(1-\alpha)}.$$

The optimal contract  $(\hat{r}(x), \hat{y}(x))$  depends on the number of workers x employed from union X and the bargaining power of the firm,  $\alpha$ :

$$\hat{r}(x) = \frac{1}{4} (1-x) (1-\alpha) \text{ and } \hat{y}(x) = 1-x.$$
 (5.6)

In the first stage, the firm and union X take the optimal strategies (5.6) as given and bargain over the labor contract (w, x). If the negotiation is successful, then the firm's expected profit is  $\pi(x, w, \hat{y}(x), \hat{r}(x))$ , and union X obtains the wage bill  $L_X = wx$ . In case of disagreement, the firm's expected profit is  $\pi^{DX} :=$  $\pi(0, 0, \hat{y}(0), \hat{r}(0)) = \alpha/4$  and union X gets zero. The generalized Nash bargaining problem between the firm and union X is, therefore, given by

$$\max_{x,w\geq 0} [\pi(x,w,\hat{y}(x),\hat{r}(x)) - \pi^{DX}]^{\alpha} [yr]^{(1-\alpha)}.$$
(5.7)

The optimal contract  $(w^*, x^*)$  (asterisks stand for optimal values under sequential bargaining) which solves the generalized Nash bargaining problem (5.7) is

$$w^* = \frac{1}{4}(1-\alpha)^2$$
 and  $x^* = 1.$  (5.8)

Substituting (5.8) into (5.6), we get the equilibrium contract in which  $y^* = 0$  for union Y. If the bargaining is sequential and the unions are perfectly substitutable, then the firm hires workers only from the first union, while none of the workers of union Y is employed. The number of workers hired from union X maximizes the total joint surplus. The firm's profit is equal to  $\pi^* = \alpha(2 - \alpha)/4$ , and union X's total wage bill is  $L_X = (1 - \alpha)^2/4$ , while union Y gets a wage bill of zero. Intuitively, the firm prefers to strike a bargain with the first union X in which case it can threaten to obtain a deal with the second union Y. If, in contrast, negotiations are not successful with the first union X, then the firm would have to split the joint surplus with the second union Y according to the bargaining power parameter  $\alpha$ . As unions do not coordinate their demands, the first union agrees to a labor contract which ultimately excludes the second union Y.

# 5.2.2 Complementary Labor Unions: Tariff Plurality

If the two labor unions represent complementary work forces, then the firm produces one unit of the final good with the use of one unit of labor from each of the two labor types; according to (5.1), the production function is then given by  $q(x, y) = \min\{x, y\}.$ 

Simultaneous Bargaining. We proceed as before. First, we consider simultaneous bargaining between the firm and and the two independent unions. If the firm can reach an agreement with both unions, then its profit is  $\pi(x, w, y, r) =$  $(1-\min\{x, y\})(\min\{x, y\}) - wx - ry$ . Because of perfect complementary, the firm can not start production unless the negotiations with both unions are successful. We assume binding contracts between the firm and the unions; i.e., if the firm can reach an agreement only with union X (union Y), but not with the other union, then it has to pay the agreed upon wage bill  $L_X = wx$  ( $L_Y = ry$ ) to union X (union Y). This assumption mirrors the possibility of losses in case of negotiation failure which is an increasing policy issue associated with craft unionism.<sup>24</sup> A strike induces (temporary) losses for the firm.

If bargaining fails either with union X or union Y, then the firm's disagreement point is  $\pi^{DY} := \pi(x, w, 0, 0) = -wx$  or  $\pi^{DX} := \pi(0, 0, y, r) = -ry$ , respectively. The unions get a payoff of zero if they can not reach an agreement with the firm. The generalized Nash bargaining problem between the firm and the union

<sup>&</sup>lt;sup>24</sup> The current strikes in the railway sector in Germany are a recent example, where a craft union (the GDL which represents train drivers) pushes for strikes that result in substantial losses for the Deutsche Bahn.

X can be written as

$$\max_{x,w\geq 0} [\pi(x,w,y,r) - \pi^{DX}]^{\alpha} [wx]^{(1-\alpha)}$$

$$= [((1 - \min\{x,y\}) \min\{x,y\} - wx - ry) - (-ry)]^{\alpha} [wx]^{(1-\alpha)},$$
(5.9)

while the generalized Nash bargaining problem between the firm and union Y can be stated accordingly as

$$\max_{y,r\geq 0} [\pi(x,w,y,r) - \pi^{DY}]^{\alpha} [ry]^{(1-\alpha)}$$

$$= [((1 - \min\{x,y\}) \min\{x,y\} - wx - ry) - (-wx)]^{\alpha} [ry]^{(1-\alpha)}.$$
(5.10)

As the unions are symmetric, in equilibrium the optimal number of employed workers and their wages are the same. The following contracts solve the simultaneous bargaining problems (5.9) and (5.10) (the superscript "sim" stands for simultaneous bargaining)

$$w^{sim} = r^{sim} = \frac{1}{2}(1-\alpha)$$
 and  $x^{sim} = y^{sim} = \frac{1}{2}$ .

The equilibrium employment levels do not depend on the bargaining power of the firm. They always maximize the joint surplus  $\Pi = \pi + wx + yr$ . The firm's equilibrium profit is  $\pi^{sim} = (2\alpha - 1)/4$ , and unions' wage bills are given by  $L_X^{sim} = L_Y^{sim} = (1 - \alpha)/4$ . If the bargaining power of the firm is less than the bargaining power of the union (i.e.,  $\alpha < 1/2$ ), then the firm will abstain from initiating the bargaining process, as its expected profits would be negative.<sup>25</sup> This outcome mirrors to some extent recent complaints by the Deutsche Bahn that a too powerful complementary union (the GDL in this case) is hardly bearable.

If the two unions are perfect complements, then the firm prefers to bargain with a merged union than to bargain simultaneously with two independent unions. The opposite is true for the unions. The unions' joint wage bill is lower if they pool their negotiations into a single one.

Sequential Bargaining. Under sequential bargaining, the firm first negotiates a contract with union X over both the employment level, x, and the wage,

<sup>&</sup>lt;sup>25</sup> This restriction on  $\alpha$  is a result of non-conditional (binding) contracts. If we assume, that the agreed upon labor contracts are conditional on a successful bargaining outcome in the other relationship, then the firm will make non-negative profits for any value of  $\alpha$ . We are not aware of such a conditionality clause in practice, so that we decided to use non-conditional contracts.

w. In the second stage, the bargaining outcome of the first stage becomes public and the firm bargains with the union Y over the employment level, y, and the wage, r. If bargaining between the firm and both unions is successful then its profit is  $\pi(x, w, y, r) = pq - wx - ry$  and union X (union Y) realizes the wage bill  $L_X = wx$  ( $L_Y = ry$ ). We solve the game by backward induction.

Suppose the bargaining outcome in the first stage is (w, x). Then, in the second stage, the firm and the union Y bargains over the contact (r, y) taking the outcome of the first stage as given. If the bargaining in the second stage is successful, then the firm gets the profit  $\pi(x, w, y, r)$ , while union Y gets ry. Because of binding contracts, the firm can not produce any product, but has to pay the wage bill wx to union X in the case of disagreement. This means the disagreement point of the firm when it bargains with union Y is negative and equal to  $\pi^{DY} := \pi(x, w, 0.0) = -xw$ . The generalized Nash bargaining problem between the firm and union Y is given by

$$\max_{y,r} [\pi(x, w, y, r) - \pi^{DY}]^{\alpha} [yr]^{(1-\alpha)}$$
  
= [((1 - min{x, y}) min{x, y} - xw - yr) - (-xw)]^{\alpha} [yr]^{(1-\alpha)}.

The optimal contract  $(\hat{r}(x), \hat{y}(x))$  depends on the number of workers x and the bargaining power of the firm,  $\alpha$ 

$$\widehat{r}(x) = \begin{cases} (1-\alpha)(1-x), & \text{if } x < \frac{1}{2}, \\ \frac{1}{2}(1-\alpha), & \text{if } x \ge \frac{1}{2}, \end{cases} \text{ and } \widehat{y}(x) = \begin{cases} x, & \text{if } x < \frac{1}{2}, \\ \frac{1}{2}, & \text{if } x \ge \frac{1}{2}. \end{cases}$$
(5.11)

In the first stage, the firm and the union X take the optimal strategies (5.11) as given and bargain over the contract (w, x). If the negotiation is successful, then the firm's expected profit is  $\pi(x, w, \hat{y}(x), \hat{r}(x))$ , and union X obtains the wage bill wx. In the case of disagreement, the firm's profit is  $\pi^{DX}(x, w, \hat{y}(0), \hat{r}(0)) := 0$ . Union X also does not have any outside options and gets a disagreement payoff of zero. The generalized Nash bargaining problem between the firm and union X is given by

$$\max_{x,w} [\pi(x,w,\hat{y}(x),\hat{r}(x)) - \pi^{DX}]^{\alpha} [wx]^{(1-\alpha)}$$

$$= [((1-\min\{x,\hat{y}(x)\})\min\{x,\hat{y}(x)\} - xw - \hat{y}(x)\hat{r}(x))]^{\alpha} [wx]^{(1-\alpha)}.$$
(5.12)

The optimal contract  $(w^*, x^*)$  which solves the last generalized Nash bargaining problem (5.12) is then given by

$$w^* = \frac{1}{2}\alpha(1-\alpha) \text{ and } x^* = \frac{1}{2}.$$
 (5.13)

Substituting (5.13) into (5.11), we get the subgame perfect equilibrium contract  $(r^* = (1 - \alpha)/2, y^* = 1/2)$  for union Y.

If the unions are complementary and the contracts are binding, then the second union Y gets a higher wage bill than the first union X. The firm's profit is  $\pi^* = \alpha^2/4$ , and the union X's and union Y's wage bills are  $L_X = \alpha(1 - \alpha)/4$  and  $L_Y = (1 - \alpha)/4$ , respectively. The firm's share and the unions' shares of the joint surplus are  $\alpha^2$  and  $1 - \alpha^2$ , respectively.

## 5.2.3 Summary and Discussion of Results

We first summarize our results under sequential bargaining in the next lemma.

**Lemma 16.** Under sequential multi-union bargaining (union X bargains first and union Y second) equilibrium employment levels always maximize the joint surplus. In addition, the following orderings hold:

i)  $w^* > r^* = 0$  ( $w^* < r^*$ ) if unions are perfect substitutes (if unions are perfect complements), with equality holding for  $\alpha = 1$ .

ii)  $x^* > y^* = 0$  ( $x^* = y^*$ ) if unions are perfect substitutes (if unions are perfect complements).

iii)  $L_X = w^* x^* > r^* y^* = L_Y = 0$  ( $L_X = w^* x^* \le r^* y^* = L_Y$ ) if unions are perfect substitutes (if unions are perfect complements), with equality holding in the complements case for  $\alpha = 1$ .

Lemma 16 highlights the extreme externalities among unions under sequential bargaining. When the worker groups represented by the unions are perfectly substitutable, then there is a strict first-mover advantage which induces foreclosure of the second union. When the worker groups are perfectly complementary, then there is a strict second-mover advantage for all  $\alpha < 1$ : the wage bill of the first union is lower than the wage bill of the second union. In addition, when the bargaining power of the unions increases, then the second mover advantage becomes even stronger. In fact, for an exogenous union bargaining power  $(1 - \alpha) \rightarrow 1$ , we get that  $L_X \rightarrow 0$ , while  $L_Y \rightarrow 1/4$ , so that the second union gets the entire surplus in the limit.

The intuition behind those results critically depends on the bargaining problem in the second round between union Y and the firm. When both unions are substitutable, then both parties have a disagreement payoff of zero and split the joint surplus according to the Nash bargaining power parameter  $\alpha$ . Given this expected outcome, the first union X can always make an offer in the first round which makes the firm slightly better off than rejecting the offer. Such an offer is always feasible for the first union because the joint surplus which can be created is the same independently whether the firm strikes an exclusive deal with union X or union Y. Hence, we get a foreclosure outcome in which only the first union X is active.

In contrast, when unions are complementary, then the firm's disagreement payoff in the second round is negative whenever the firm stroke a deal with the first union X (binding contracts assumptions). It follows that the "contribution" of the second union Y to the joint surplus when bargaining with the firm is larger than the "contribution" of the first X union. This is so because the firm's disagreement point is zero in the first round (when facing union X) but it is negative in the second round (when facing union Y), whenever the firm reached an agreement in the first round. Or, put another way, the firm can avoid a negative payoff in the second round which allows the second union to get a larger share from the joint surplus, giving rise to the second-mover advantage as stated in Lemma 16.

# 5.2.4 Union Merger Incentives

We analyze a game in which the unions X and Y can form a single union in an initial stage 0 (see the Figure 5.1). If the unions do not merge, then they bargain with the firm sequentially (without loss of generality, union X bargains before union Y). We also assume that the bargaining order in the sequential bargaining subgame is public knowledge. <sup>26</sup> We denote subgame perfect contracts under sequential bargaining by by  $(w^*, x^*)$  and  $(r^*, y^*)$ . If the unions decide to merge, then they agree on the shares each union gets from the joint wage bill to ensure participation. In this case, the optimal contract between the firm and the joint union is given by  $(w^{joi}, z^{joi})$  and the joint wage bill is  $L_{XY} = w^{joi} z^{joi}$ . We solve the entire game by backward induction.

If the unions are complementary, then the total wage bill is lower under joint

<sup>&</sup>lt;sup>26</sup> Otherwise, the unions are symmetric and each union will get an equal share from the joint wage bill. The unions will have merger incentives only if their joint wage bill increase under joint bargaining. Otherwise, they will prefer to bargain separately.

bargaining when compared with the total wage bill under sequential bargaining. It follows, that there is no room for a Pareto-improvement through a union merger. Therefore, the unions will bargain separately in equilibrium, and the optimal contracts will be the same as in the sequential bargaining game.

If the unions are perfect substitutes, then joint bargaining increases the unions' total wage bill. If the unions form a joint union, then their total wage bill will be  $w^{joi}z^{joi} = w^{joi}(x^{joi} + y^{joi})$ . As the workers from the two unions are homogeneous, and the firm negotiates with the joint union the total number of workers,  $z^{joi}$ , we can make the following assumption. In the initial stage 0, to share the total wage bill, the unions negotiates the number of workers,  $x^{joi}$  and  $y^{joi}$ , will be hired from each union, X and Y, respectively. In this case, unions X and Y will get wage bills which are equal to  $w^{joi}x^{joi}$  and  $w^{joi}y^{joi}$ , respectively. In the case of disagreement, the unions will bargain sequentially and get  $w^*x^* = (1 - \alpha)^2/4$  and  $r^*y^* = 0$ , respectively. The symmetric Nash bargaining problem between the unions can be states as

$$\max_{x \ge 0} [w^{joi} x^{joi} - w^* x^*] [w^{joi} (z^{joi} - x^{joi})].$$
(5.14)

The following employment level for union X solves the Nash bargaining problem (5.14)  $x = 1 - \alpha/2$ . Equilibrium wage bills for union X and union Y are  $L_X = w^{joi}x^{joi} = (1 - \alpha)(2 - \alpha)/8$  and  $L_Y = w^{joi}y^{joi} = \alpha(1 - \alpha)/8$ , respectively.

**Proposition 10.** Unions X and Y have strict incentives to merge if the worker groups are substitutable while they have strict incentives to stay independent when workers are complementary, with indifference holding for  $\alpha = 0$  and  $\alpha = 1$ . The following contracts are then implemented in equilibrium:

i) If the unions are substitutable (tariff competition), then wage and employment levels are given by  $(w^{joi}, x^{joi}, y^{joi})$ .

ii) If the unions are complementary (tariff plurality), then wages and employment levels are given by  $(w^*, x^*, r^*, y^*)$ .

Proposition 10 shows that unions have strong incentives to merge when they represent substitutable workers to increase their joint bargaining power. The aim of this merger was to avoid tariff competition between substitutable labor unions. A union merge deprives the firm from playing the unions off against each other which lowers the firm's share of the joint surplus to the benefit of the employed workers. In Germany, for instance, when sector boundaries in services

#### 5.3. DISCUSSION AND EXTENSION

became more and more blurred because of technological changes, in March 2001 the biggest union merger ever took place to establish the German Trade Union (*Vereinte Dienstleistungsgewerkschaft*; in short: *Verdi*) with almost 3 million members (see Haucap, Pauly and Wey (2007, p. 125 f.)).

When workers are complementary, then this logic no longer holds. To the contrary, the sum of unions' wage bills is larger under sequential bargaining than under joint bargaining, so that a union merger cannot make both unions better off. With complementary unions, the second union gets a hold on the entire surplus, while the firm's disagreement point is negative when bargaining with the second union.

From Proposition 10 it also follows that the firm prefers to bargaining with independent unions when workers are substitutable, while it prefers to bargain with a single union when worker groups are complementary. Recently, the Deutsche Bahn struggling with the bargaining power of the (independent) train drivers union GDL, invited both unions the GDL and EVG to bargaining together.<sup>27</sup> Both unions are complementary, so that this invitation to pool the bargaining problems into a single one can be interpreted as an attempt of the Deutsche Bahn to reach a better agreement than under a separate bargaining procedure.

# 5.3 Discussion and Extension

In this section we discuss how our model relates to the right-to-manage setting (in short: RTM) and how the specification of the disagreement point affects our results. We also present an extension where each union's workers produce a different final good which can be substitutable or complementary. Substitutablity and complementarity is not restricted to be perfect but can be gradual (i.e, the final goods are differentiated). This allows us to derive additional results concerning the employment levels which are not necessarily joint surplus maximizing anymore.

<sup>&</sup>lt;sup>27</sup> See "GDL-Lokführer nehmen Einladung zu Gesprächen an," Handelsblatt, online edition, November 13th, 2014.

## 5.3.1 Right-to-manage Setting

If we assume that bargaining is only about wages, then the firm retains the right to choose the employment levels.<sup>28</sup> This restriction has important consequences for the union. If labor is perfectly substitutable, then the firm will get the entire surplus both under simultaneous and sequential bargaining. This follows from a Bertrand-competition argument. Both unions are perfectly substitutable and undercut each other as the firm hires workers only from the union with the lower negotiated wage rate. A consequence is that unions always want to merge which is in line with our results under efficient bargaining.

If labor is complementary, then the results are different than under efficient bargaining. If unions bargains jointly with the firm (i.e., there is a union merger), then the wage is  $w^{joi} = (1-\alpha)/4$  according to (5.3). The total number of workers the firm hires from the merged union is  $x^{joi} = y^{joi} = (1+\alpha)/4$ . It is obvious that the optimal employment level is lower than the efficient level ( $x^{joi} = y^{joi} = 1/2$ ) because of the well-known double marginalization problem, unless the firm has all the bargaining power (i.e.,  $\alpha = 1$ ).

If the unions merge, then the firm and the merged union realize  $\pi^{joi} = (1 + \alpha)^2/4$ , and  $L_{XY} = w^{joi}x^{joi} = (1 - \alpha^2)/8$ , respectively. The total surplus is then  $\Pi^{joi} = (3 + 2\alpha - \alpha^2)/16$ , which is increasing in the firm's bargaining power because a greater firm bargaining power tends to reduce the double marginalization problem.

If labor is complementary and the unions bargain sequentially with the firm (first union X and second union Y), then the unions X and Y get wages  $w^* = (1-\alpha)/2$  and  $r^* = (1-\alpha^2)/4$ , respectively. Even though the bargaining outcome of the first stage is binding, the firm does not have a negative disagreement point when bargaining with the second union Y. As the firm has discretion about the employment level after the first bargaining stage, its disagreement point can never be negative in the second stage. If negotiations fail in the second stage, then the firm would decide to hire no workers from the first union and produce nothing. Because of the complementarity of unions' labor inputs its disagreement point is, therefore, equal to zero in the second stage. Hence, contrary to the

<sup>&</sup>lt;sup>28</sup> The labor economics literature distinguishes between the right-to-manage model (bargaining only about wage) and the efficient bargaining model (bargaining about both wage and employment level). See Oswald and Turnbull (1985) and Booth (1995) for surveys.

efficient bargaining case, the first union gets a higher wage bill than the second union, unless the firm has all the bargaining power. If  $\alpha = 1$ , then the both unions get the same wages which are equal to zero. The equilibrium level of employment is equal to  $x^* = y^* = (1 + \alpha)^2/8$ , which is lower than the efficient level of employment  $(x^{joi} = y^{joi} = 1/2)$ , unless  $\alpha = 1$ . In equilibrium, the firm's profit is equal to  $\pi^* = (1 + \alpha)^4/64$  and the union's total wage bill is  $L_{XY} = w^*x^* + r^*y^* = (1 + \alpha)^2(1 - a)(a + 3)/32$ . The total surplus given by  $\Pi^* = (1 + \alpha)^2(7 - 2\alpha - \alpha^2)/64$  which is increasing in the firm's bargaining power  $\alpha$ .

Those results give rise to a reassessment of unions' merger incentives when they are complementary. If the firm's bargaining power is relatively large (i.e.,  $\alpha > \sqrt{5} - 2 \approx 0.236$ ), then the unions prefer to bargain separately, as under efficient bargaining. But if the firm's bargaining power is relatively low (i.e.,  $\alpha < \sqrt{5} - 2 \approx 0.236$ ), then in contrast to the efficient bargaining case, the unions prefer to bargain jointly with the firm under RTM. The reason for this result is the efficiency loss associated with the double marginalization problem under RTM bargaining. While the unions' percentage share of the total surplus is always higher if they bargain separately and sequentially, the inefficiency under sequential bargaining may become so large that the unions prefer to bargain jointly. This is the more likely the smaller the firm's bargaining power  $\alpha$  becomes. If  $\alpha < \sqrt{5} - 2$ , then the total surplus is increased by approximately 38% when the unions bargain jointly. If  $\alpha = 0$ , then this increase is largest and approximately equal to 71%.

## 5.3.2 Specification of Disagreement Point

When bargaining is sequential, then an issue is whether the contract concluded with the first union remains valid if bargaining with the second union is not successful. We assumed that the first contract is binding which leads for the case of complementary unions to negative firm profits when there is disagreement in the second bargaining problem. As a consequence, there is a second mover advantage such that the first union's wage bill,  $L_X = \alpha(1-\alpha)/4$ , is smaller than the second union's wage bill,  $L_Y = (1-\alpha)/4$ . When we assume instead that the first contract is not binding in case of break down of bargaining with the second union, then the wage bills are exactly reversed. That is, there is a first-mover advantage as under substitutable labor groups (but with different wage bill levels). Of course, a non-binding contract allows the firm to abandon that contract if there is no agreement with the second union and to avoid, therefore, a negative profit level.

## 5.3.3 The Two-Products Model

As an extension, we consider a firm (the employer) which has to reach agreements with two unions X and Y to produce two goods (or services). An example is the Deutsche Bahn which offers both rail journeys and an intercity bus carrier services. The first one is based on trains and the second one is based on buses. Train drivers are represented by the union GDL and all other employees (including bus drivers) of the Deutsche Bahn are represented by the rival union EVG. Bus and train journey services can be substitutable and complementary. In contrast to (5.1), this relationship may not be perfect but imperfect which gives rise to additional substitution issues that we highlight in the following.

Assume that the production of good 1 requires only labor input from union X with constant returns to scale, i.e.;  $q_1 = x$ , where  $q_1$  is the output of good (service) 1. Similarly, the production of good 2 requires only labor input from union Y with constant returns to labor input, i.e.,  $q_2 = y$ , where  $q_2$  is the output of good (service) 2. The inverse demand for good i is assumed to be linear and given by  $p_i(q_i, q_j) = 1 - q_i - \gamma q_j$ , with i, j = 1, 2 and  $i \neq j$ , where  $p_i$  is the price of good i and the parameter  $\gamma$  describes product differentiation with  $\gamma \in (\bar{\gamma}, 1]$  and  $\bar{\gamma} := -1/2$ .<sup>29</sup> The sign of the parameter  $\gamma$  determines whether the labor inputs of the two unions are substitutes ( $\gamma > 0$ ) or complements ( $\gamma < 0$ ) (products and labor inputs are independent for  $\gamma = 0$ ).

The parameter  $\gamma$  determines also the relationship between the two workforces represented by unions X and Y, respectively. If the products (or, services) are substitutable ( $\gamma > 0$ ), then the workforces and their respective unions are also substitutable which mirrors the case of tariff competition. If, to the contrary, the products (or services) are complementary ( $\gamma < 0$ ), then the workforces and their

<sup>&</sup>lt;sup>29</sup> The upper bound  $\gamma = 1$  follows from noting that the two goods are homogeneous at this value. The lower bound  $\overline{\gamma}$  ensures that the firm's profit function is strictly positive in the (unique) interior equilibrium. A qualitatively similar restriction on the complementarity of the unions' work forces is also invoked in Horn and Wolinsky (1988b).

unions are complementary which stands for the case of tariff plurality.<sup>30</sup>

We assume again an efficient bargaining setting and we use the symmetric Nash bargaining solution to solve for optimal contracts.<sup>31</sup> The firm negotiates with union X (union Y) over the employment level x (y) and the wage rate w (r). When negotiations are successful, then the firm produces quantities  $q_1$  and  $q_2$  and realizes profits  $\pi(x, w, y, r) = \sum_{i=1}^{2} p_i(q_i, q_j)q_i - xw - yr$ .<sup>32</sup> Unions X and union Y maximize their wage bills given by  $L_X = xw$  and  $L_Y = yr$ , respectively. First, we analyze the simultaneous and joint bargaining problems and second, we examine sequential bargaining. Third, we analyze unions' incentives to merge in an initial stage "0", with the bargaining game (either joint or sequential bargaining) following thereafter.

Simultaneous and Joint Bargaining Benchmarks. Solving the Nash bargaining problems under simultaneous bargaining, we get (all calculations can be found in the Appendix)

$$w^{sim} = r^{sim} = \frac{1}{4(1+\gamma)}$$
 and  $x^{sim} = y^{sim} = \frac{1}{2(1+\gamma)}$ , (5.15)

where the superscript "sim" stands for simultaneous bargaining. The equilibrium employment levels are efficient; i.e.,  $x^{sim} = y^{sim}$  maximize the joint surplus which is given by  $\Pi := \pi + wx + ry = \sum_{i=1}^{2} p_i(q_i, q_j)q_i$ . Substituting the equilibrium values we get  $\Pi^{sim} = 1/[2(1+\gamma)]$ . The firm's equilibrium profit is given by  $\pi^{sim} = (1+2\gamma)/(4(1+\gamma)^2)$ . To understand how the relationship between both unions' workforces (as measured by the parameter  $\gamma$ ) affects the surplus sharing, it is instructive to calculate the share the firm gets from the joint surplus. We obtain

$$\frac{\pi^{sim}}{\Pi^{sim}} = \frac{2\gamma + 1}{2(1+\gamma)}$$

from which it follows that the firm's share is monotonically increasing in  $\gamma$ . When goods are independent, then the firm gets exactly one-half of the joint surplus.

<sup>&</sup>lt;sup>30</sup> We can also express the relationship between both workforces by the change of the marginal product of labor with respect to the labor input of the other workforce. Formally, we then get  $\frac{\partial^2((1-q_i-\gamma q_j))}{\partial x \partial y} = -\gamma$ , for i, j = 1, 2 and  $i \neq j$ , which is positive (negative) if  $\gamma < 0$   $(\gamma > 0)$ .

<sup>&</sup>lt;sup>31</sup> It suffices to focus on the symmetric Nash bargaining solution to highlight our main additional results which are absent in our main analysis.

<sup>&</sup>lt;sup>32</sup> In the following, we will write the firm's profit directly as a function of the employment levels as we assumed  $q_1 = x$  and  $q_2 = y$ .

Its share increases beyond one-half, when the unions' workforces become more and more substitutable (at  $\gamma = 1$ , the firm obtains 3/4 of the joint surplus). In contrast, the firm's share decreases when the two unions become less substitutable (for  $\gamma > 0$ ) or more complementary (for  $\gamma < 0$ ). In fact, for the case of complementary goods, as  $\gamma \to -1/2$  the firm's share goes to zero. Of course, the opposite relationship holds for the unions' wage bills which are monotonically decreasing in  $\gamma$ .

Those relationships are intuitive. When goods become more substitutable then each union's bargaining power decreases accordingly. For the case of complementary goods ( $\gamma < 0$ ), the unions exert their largest bargaining power. In the most complementary case (when  $\gamma \rightarrow -1/2$ ) the unions are able to pocket the entire joint surplus.

An intermediate result is that unions have incentives to form a single bargaining unit when goods are substitutes while they prefer to bargain separately when their workforces are complementary (see Horn and Wolinsky (1988b)). This follows directly from observing that the joint surplus is always maximized and shared equally when both workforces are represented by a single union which bargains with the firm (the analysis is provided in the Appendix). The firm's share must then be equal to one-half of the joint surplus. As the firm's share under separate bargaining is larger (smaller) than one-half when goods are substitutes (complements), the unions want to merge (stay separate) when the workers are substitutable (complementary).

Formally, consider that unions X and Y merge to negotiate with the firm. Then the optimal employment levels which solve the Nash bargaining problem between the firm and the merged union are the same as under simultaneous bargaining and are given by  $x^{joi} = y^{joi} = 1/[2(1+\gamma)]$ . The union's wage bill which solves the Nash bargaining problem (see the Appendix) is given by

$$w^{joi}x^{joi} + r^{joi}y^{joi} = \frac{1}{4(1+\gamma)},$$
(5.16)

which implies  $w^{joi} = 1/4$ . Accordingly, the firm's profit is  $\pi^{joi} = 1/(4(1+\gamma))$ . It follows that the merged union and the firm share the joint surplus equally for all possible values of  $\gamma$ ; i.e.,  $\pi^{joi}/\Pi^{joi} = 1/2$ , where  $\Pi^{joi} := \pi^{joi} + w^{joi}x^{joi} + r^{joi}y^{joi}$ . The following proposition summarizes these results.

**Proposition 11.** Assume the two-products model and suppose efficient bargaining solved by the symmetric Nash bargaining solution. Simultaneous bargaining as

well as joint bargaining then lead to efficient employment levels. Under joint bargaining the entire surplus is shared equally between workers and the firm, while under simultaneous bargaining the firm's share of overall surplus is monotonically increasing in  $\gamma$ ; i.e., increases when the two workforces become more substitutable or less complementary. Workers are jointly better off under joint bargaining when the two workforces are substitutable, while they do better under separate bargaining when they are complementary.

We now turn to sequential bargaining which reveals how bargaining externalities lead to inefficiencies depending on the nature of the two workforces. Those inefficiencies will have a pronounced effects on the overall employment level under separate bargaining and on the unions' incentives to integrate in the first place. **Sequential Bargaining.** The firm bargains bilaterally and sequentially with the two unions; first with union X and second with union Y (the analysis is relegated to the Appendix). Solving the Nash bargaining problem between the firm and union Y, we get

$$\hat{r}(x) = \frac{1 - 2\gamma x}{4} \text{ and } \hat{y}(x) = \frac{1}{2} - \gamma x,$$
(5.17)

so that the optimal solution depends in the employment level x. Proceeding backward we get the solution of the Nash bargaining problem between the firm and union X (asterisks indicate equilibrium values)

$$w^* = \frac{2 - \gamma}{8}$$
 and  $x^* = \frac{2 - \gamma}{2(2 - \gamma^2)}$ . (5.18)

Substituting (5.18) into (5.17) we get the optimal labor contract for union Y:

$$r^* = \frac{1-\gamma}{2(2-\gamma^2)}$$
 and  $y^* = \frac{1-\gamma}{2-\gamma^2}$ . (5.19)

The equilibrium quantities of the goods 1 and 2 are then  $q_1^* = x^*$  and  $q_2^* = y^*$ , respectively. The firm's equilibrium profit then becomes  $\pi^* = (8 - 4\gamma - \gamma^2)/[16(2 - \gamma^2)]$ , while union X's wage bill is

$$w^* x^* = \frac{(2-\gamma)^2}{16(2-\gamma^2)},\tag{5.20}$$

and union Y's wage bill is

$$r^*y^* = \frac{(1-\gamma)^2}{2(2-\gamma^2)^2}.$$
(5.21)

The firm's share of total surplus which follows from  $\pi^*/\Pi^*$ , with  $\Pi^* := \pi^* + w^*x^* + r^*y^* = (2\gamma^3 - \gamma^2 - 8\gamma + 8)/[4(2 - \gamma^2)^2]$ , is monotonically increasing in  $\gamma$ . It reaches one-half at  $\gamma = 0$  and 3/4 at  $\gamma = 1$ . However, when products are complementary and  $\gamma \to -1/2$ , then the firm's share of total surplus is roughly 37% which reveals a sharp difference to the cases of simultaneous bargaining and joint bargaining. Before we fully compare the different bargaining regimes, the next lemma summarizes the orderings of wages, employment levels, and wage bills under sequential bargaining.<sup>33</sup>

**Lemma 17.** Consider sequential multi-union bargaining (union X bargains first and union Y secondly). Then the following orderings hold:

i)  $w^* > r^*$  ( $w^* < r^*$ ) if  $\gamma > 0$  ( $\gamma < 0$ ), with equality holding for  $\gamma = 0$ . ii)  $x^* > y^*$  ( $x^* < y^*$ ) if  $\gamma > 0$  ( $\gamma < 0$ ), with equality holding for  $\gamma = 0$ . iii)  $w^*x^* > r^*y^*$  ( $w^*x^* < r^*y^*$ ) if  $\gamma > 0$  ( $\gamma < 0$ ), with equality holding for  $\gamma = 0$ .

Sequential bargaining creates externalities between the two union-firm bargaining pairs which affect the unions' wage bills differently. The sign of the externality of the first contract (x, w) on the second contract (y, r) can be seen immediately from the optimal bargaining outcome with union Y which depends on the first contract (see (5.17)). Of course, the externality is negative (positive) if both workforces are substitutable (complementary). Now note that each bargaining pair maximizes the joint surplus over the firm's disagreement point. As bargaining is about both the wage rate and the employment level, this outcome is always bilaterally efficient if successful; i.e., the *net* surplus is maximized and split equally. Note next that the firm's disagreement point when bargaining with the first union X is fixed at  $\pi^{DX*} = 1/8$  which follows from the optimal contract (5.17) which the will firm agree upon with union Y in case of a settlement. To the contrary, the firm's (equilibrium) disagreement point when bargaining with the second union Y is given by  $\pi^{DY*} := \pi(x^*, w^*, 0, 0) = (1 - x^*)x^* - x^*w^*$ . Substituting the equilibrium values (5.18) yields

$$\pi^{DY*} = (\gamma^4 + 4\gamma^3 - 18\gamma^2 + 8\gamma + 8) / \left[ 16\left(2 - \gamma^2\right)^2 \right].$$

Inspection of the difference  $\pi^{DY*} - \pi^{DX*}$  gives that

$$\frac{\partial(\pi^{DY*} - \pi^{DX*})}{\partial\gamma} > 0 \text{ with } sign\left[\pi^{DY*} - \pi^{DX*}\right] = sign\left[\gamma\right],$$

<sup>&</sup>lt;sup>33</sup> The proof follows directly from inspecting the respective equilibrium values.

and  $\pi^{DY*} - \pi^{DX*} = 0$  for  $\gamma = 0$ . Hence, the firm enjoys a better disagreement point when bargaining with union Y if the unions are substitutable (i.e.,  $\pi^{DY*} > \pi^{DX*}$  for  $\gamma > 0$ ). Because of the symmetry of the bargaining problems, it follows immediately that union Y obtains a smaller wage bill than union X. This wage bill reduction is caused by a negative externality the first union X exerts on the second union Y. It also implies a first-mover advantage for unions when workforces are substitutable.

In the case of complementary unions ("craft unionism"), the opposite is true; i.e., the firm's disagreement point is better when bargaining with the first union X (i.e.,  $\pi^{DX*} > \pi^{DY*}$  for  $\gamma < 0$ ). The first union X creates a positive externality on the second union Y, if it reaches an agreement with the firm. This implies that union X's wage bill must be smaller than union Y's wage bill which mirrors the second-mover advantage under craft unionism.<sup>34</sup>

Joint versus Separate Bargaining. We examine the incentives of the unions X and Y to form a single union before bargaining starts. If unions merge, then the bargaining outcome is given by  $(w^{joi}, x^{joi}, r^{joi}, y^{joi})$ . If the unions do not merge, then bargaining is assumed to be sequential with union X bargaining first with the firm followed by union Y (the solutions given by (5.18) and (5.19)). Before solving the entire game, it is instructive to perform some comparisons between the joint bargaining outcome and the sequential bargaining equilibrium.

Sequential bargaining has the following impact on employment levels when compared with joint bargaining.

**Lemma 18.** Consider sequential multi-union bargaining (union X first and union Y second). Comparison of the employment levels of both unions under joint bargaining and sequential bargaining gives rise to the following orderings:

i)  $x^* > x^{joi}$  ( $x^* < x^{joi}$ ) if  $\gamma > 0$  ( $\gamma < 0$ ), with equality holding for  $\gamma = 0$ . ii)  $y^* < y^{joi}$  for all  $\gamma \neq 0$ , with equality holding for  $\gamma = 0$ .

iii)  $x^* + y^* > x^{joi} + y^{joi} (x^* + y^* < x^{joi} + y^{joi})$  if  $\gamma > 0$  ( $\gamma < 0$ ), with equality holding for  $\gamma = 0$  and  $\gamma = 1$ .

If the workforces are substitutable, then the firm hires more workers from the first union than the efficient level. The second union Y optimally responds by reducing its employment level below the efficient level. The overall effect on

<sup>&</sup>lt;sup>34</sup> This result is in line with the finding of Cai (2000) that equilibrium delay is possible when two sellers are complementary as each one prefers to be the last bargaining partner.

employment by the firm is positive and overemployment is the outcome (part iii) of Lemma 18). If both workforces are complementary, then the first union X reduces its employment level below the efficient level which induces the second union Y to reduce its employment level also below the efficient level. Part iii) of Lemma 18 shows that these changes lead to an inefficiently low employment level when workers are complementary. In fact, both unions reduce their employment levels below the efficient level so that underemployment occurs unambiguously under tariff plurality (craft unionism).

We next compare the wage rates under joint bargaining and under sequential bargaining.

**Lemma 19.** Consider sequential multi-union bargaining (union X first and union Y second). Comparison of the wage rates of both unions under joint bargaining and sequential bargaining gives rise to the following orderings:

i)  $w^* < w^{joi}$  ( $w^* > w^{joi}$ ) if  $\gamma > 0$  ( $\gamma < 0$ ), with equality holding for  $\gamma = 0$ .

ii)  $r^* < w^{joi}$   $(r^* > w^{joi})$  if  $\gamma > 0$   $(\gamma < 0)$ , with equality holding for  $\gamma = 0$ .

iii)  $\partial(w^{joi} - w^*)/\partial\gamma > 0$  and  $\partial(w^{joi} - r^*)/\partial\gamma > 0$  for all  $\gamma \in (\bar{\gamma}, 1]$ .

Both unions' wage rates are lower under sequential bargaining than under joint bargaining when workers are substitutable ( $\gamma > 0$ ). The opposite holds, when workers are complementary ( $\gamma < 0$ ). Part *iii*) of Lemma 19 shows that the relationships between the wages under sequential bargaining when compared with the wage under joint bargaining are positively monotone; i.e., the differences  $w^{joi} - w^*$  and  $w^{joi} - r^*$  are strictly increasing over the range of admissible values of  $\gamma$ .

The following proposition summarizes both unions' merger decision in the initial stage "0" and the subgame perfect equilibrium outcomes for all values of  $\gamma$ .

**Proposition 12.** Unions X and Y have strict incentives to merge if workers are substitutable ( $\gamma > 0$ ) while they have strict incentives to stay independent when workers are complementary ( $\gamma < 0$ ), with indifference holding for  $\gamma = 0$ . The following contracts are then implemented in equilibrium:

i) If  $\gamma \geq 0$ , then wages and employment levels are given by  $(w^{joi}, x^{joi}, r^{joi}, y^{joi})$ . ii) If  $\gamma < 0$ , then wages and employment levels are given by  $(w^*, x^*, r^*, y^*)$ .

The proof of Proposition 12 follows directly from comparing the wage bill under joint bargaining (5.16) with the sum of the wage bills under sequential

#### 5.3. DISCUSSION AND EXTENSION

bargaining (5.20) and (5.21). If the goods are substitutes ( $\gamma > 0$ ), the total wage bill is larger under joint bargaining than the sum of both unions' wage bills under sequential bargaining. The opposite is true if both workforces are complementary ( $\gamma < 0$ ) in which case unions prefer to bargain independently.

When workforces are substitutable, then there is no conflict of interest between the unions even if they split the surplus equally under joint bargaining; i.e.,

$$\frac{w^{joi}x^{joi} + r^{joi}y^{joi}}{2} - r^*y^* > \frac{w^{joi}x^{joi} + r^{joi}y^{joi}}{2} - w^*x^* > 0, \text{ for } \gamma > 0$$

It follows that both unions agree to merge when being substitutable, so that the efficient outcome is achieved. Bargaining independently would lead to negative externalities (in particular, overemployment) with an overall lower wage bill.

In contrast, if workers are complementary, then the unions do not find it jointly attractive to integrate both workforces into a single union even though joint bargaining would increase the entire surplus available for the workers and the firm. However, interests are not as cleanly aligned as in the case of substitutes. To see this, suppose that both workforces share the wage bill equally under joint bargaining. It is then true that union X would benefit from integration while union Y would be worse off; i.e.,

$$\frac{w^{joi}x^{joi} + r^{joi}y^{joi}}{2} - w^*x^* > 0 > \frac{w^{joi}x^{joi} + r^{joi}y^{joi}}{2} - r^*y^* \text{ for } \gamma < 0.$$

Union X (which bargains first) prefers one-half of the total wage bill realized under joint bargaining. In contrast, union Y's wage bill is higher under sequential bargaining when compared with one-half of the overall wage bill under joint bargaining. Both unions, therefore, must disagree about the question whether or not to integrate their workers into a single union.<sup>35</sup> In total, unions realize a higher wage bill under sequential bargaining when compared with the wage

<sup>&</sup>lt;sup>35</sup> In fact, the incumbent monopoly unions (organized in the *Deutscher Gewerkschaftsbund*; DGB) heavily lobby against craft unionism so as to re-integrate "renegade" workers. In contrast, of course, craft unions as the *Marburger Bund* or the *Gewerkschaft Deutscher Lokomotivführer* (GDL) have been fighting for recognition in the last years. In our model, union X (which is disadvantaged as a first-mover) would have an incentive to lobby for integration while union Y (which benefits from the second-mover advantage) would oppose such demands. In that sense, newly formed craft unions may benefit from a second-mover advantage which allows them to obtain a large wage bill at the cost of the wage bill of the incumbent union's workers.

bill realized under joint bargaining. It follows that the possibility of a union merger does not eliminate the underemployment inefficiency associated with craft unionism.

Until now we have focused on the joint surplus of the bargaining parties as our measure of efficiency (i.e., the sum of the firm's profit and both unions' wage bills). If we consider also consumer surplus to take a total welfare perspective, we obtain the following result.

**Corollary 7.** Consider the entire game where the unions can first decide to merge and then either bargain jointly with the firm or independently and sequentially. It is then always true that the unions' merger decision is in conflict with total welfare maximization; i.e., the unions' decision to merge when workers are substitutable reduces total welfare which is also true for the unions' decision to stay separated when workers are complementary.

The proof of the Corollary 7 follows from observing that total welfare is given by the sum of consumer surplus and the sum of the firm's profit plus workers' wage bills. Of course, this welfare measure is monotonically increasing in the employment levels of both workforces in the relevant range. As the unions' decisions to merge under substitutable workforces and not to merge under complementary workforces both reduce the employment levels both decisions must also reduce social welfare. We conclude that Corollary 7, therefore, mirrors the often mentioned assessment that (powerful) unionism in general (and not only craft unionism) is a challenge to a society's well-being (see, e.g., Simon (1944)).

# 5.4 Conclusion

We have analyzed multi-union bargaining which is an issue in countries with a fragmented labor movement. In countries like France, Italy, or Belgium, and also more recently, in Germany trade union pluralism is a fact which has not been much analyzed in the existing literature on union-firm bargaining. In our main model we considered a firm producing a single final good using labor inputs from two different labor unions. We also analyzed in an extension the two-product setting, where the firm produces two different products using labor inputs from two different labor unions. In both cases, even if unions can merge freely their businesses such an outcome is not likely in the presence of craft unionism. When unions' workforces are complementary (tariff plurality), then they can achieve a higher surplus when bargaining separately.

In our main analysis (with a single final good), we derived extreme externalities among unions under sequential bargaining. When unions are perfectly substitutable, then the first union has a strict first-mover advantage and the second union is fully foreclosed. Under perfect complementarity, a second-mover advantage emerges and the first union's wage bill is smaller then the second union's wage bill.

In the two-products case we obtain additional results concerning labor substitution effects. Sequential bargaining leads to overemployment under substitutable unions which yields additional incentives to form a single union. In contrast, under craft unionism the sequential bargaining outcome is characterized by underemployment. As unions prefer to stay independent when workers are complementary, the underemployment inefficiency can be expected to persist. In the one-product case with efficient bargaining, we do not observe any distortions in employment levels.

The relevance of our model is underlined by the recent bargaining between the *Deutsche Bahn* (the dominant railway operator in Germany) and the German Train Drivers Union (*Gewerkschaft Deutscher Lokomotivführer*; GDL) and the Railway and Transport Union (*Eisenbahn- und Verkehrsgewerkschaft*; EVG). While the latter union (which is part of the DGB) reached an agreement with the Deutsche Bahn in 2013 over employment security issues, the craft union GDL delayed negotiations until today to obtain a better contract for the train drivers by ripping off its second-mover advantage.<sup>36</sup>

Unions' merger incentives are exactly opposite to the social welfare maximizing union structure. If the represented workforces are substitutable, then "tariff competition" would be socially desirable, but unions' incentive to monopolize the labor market prevail. If, to the contrary, the workforces of the unions are complementary, then "tariff plurality" is socially inferior to joint bargaining, but unions' rent-shifting incentive back union plurality.

Many labor laws are extremely defensive against a fragmented union structure at the firm level. In Germany, for instance, tariff competition at the firm-level

<sup>&</sup>lt;sup>36</sup> See the newspaper article "Von Mitte Januar an drohen Zugausfälle - Lokführergewerkschaft stellt nach gescheiterten Tarifverhandlungen Streiks in Aussicht" published in the Frankfurter Allgemeine Zeitung, 3 January 2014, p. 11.

is directly fought by several instruments as the tariff-unity principle and entrydeterring strategies which assign the privilege of collective bargaining exclusively to a single union and (last but not least) extension rules which make the dominant tariff contract generally binding for all workers of a particular type in a certain industry. While those measures have been quite successful in the past, they mainly help to monopolize the labor supply and to protect it against competition.

Interestingly, tariff pluralism (or, craft unionism) is on the rise as labor institutions are less restrictive in this regard. Recently legal practice in Germany has been reassuring that tariff pluralism cannot be eliminated by the tariff-unity principle, so that firms must come to terms with powerful craft unions. A fragmented union structure is likely to persist as craft unions have strong incentives to stay independent. From a social point of view that trend is likely to induce underemployment which harms social welfare.

# 5.5 Appendix

In this Appendix we provide the calculations for the right-to-manage extension for the complementary unions case of our main model. We also provide the missing analytical steps for the two-products extension.

# 5.5.1 Right-to-manage Model: Complementary Unions

The firm and the unions X and Y bargain over only wages w and r, respectively. In the last stage, the firm decides about the employment levels x and y and takes the wage rates w and r as given. The firm's optimization problem in this stage can be stated as

$$\max_{x,y\geq 0} \pi(x,w,y,r) = (1 - \min\{x,y\}) \min\{x,y\} - wx - ry.$$

The optimal level of employment in this case is

$$\hat{x}(w,r) = \hat{y}(w,r) = (1-w-r)/2,$$
(5.22)

if the firm reaches agreements with both unions X and Y. If bargaining with at least one of the unions fails, then the firm does not hire any worker; i.e.,  $\hat{x} = \hat{y} = 0$  holds.

#### 5.5. APPENDIX

First, we consider the joint bargaining benchmark, where the unions form a joint union which bargains with the firm over a wage rate w = r. In the case of agreement, the firm's profit is  $\pi(\hat{x}, w, \hat{y}, w)$  and the union gets a total wage bill of  $L_{XY} = 2w\hat{x}$ . In case of disagreement, both the firm and the joint union get payoff of zero. The generalized Nash bargaining problem between the firm and the joint union can be stated as

$$\max_{w \ge 0} \left[ \pi(\hat{x}, w, \hat{y}, w) \right]^{\alpha} (2w\hat{x})^{1-\alpha}.$$
(5.23)

The optimal wage rate which solves the bargaining problem (5.23) is  $w^{joi} = r^{joi} = (1 - \alpha)/4$ . Substituting these values into (5.22), we get the optimal employment levels:  $x^{joi} = y^{joi} = (1 + \alpha)/4$ .

Second, we analyze the sequential bargaining case, where the firm bargains first with union X and then with union Y. We solve the game by backward induction. In the second stage, the firm bargains with union Y over the wage r, by taking the wage rate w as given. In the case of agreement, the firm gets the profit  $\pi(\hat{x}, w, \hat{y}, r)$ , while the union Y gets  $L_Y = r\hat{y}$ . In the case of disagreement, both the firm and the union get a payoff of zero. The generalized Nash bargaining problem between the firm and the union Y can be stated as

$$\max_{r>0} \left[ \pi(\widehat{x}, w, \widehat{y}, r) \right]^{\alpha} (r\widehat{y})^{1-\alpha}$$

Solving this bargaining problem gives  $\hat{r}(w) = (1-w)(1-a)/2$ , so that the optimal wage is a function of the wage w concluded in the first stage with union X. In the first stage, the firm and union X bargain over wage w. In case of agreement, the firm gets the payoff  $\pi(\hat{x}, w, \hat{y}, \hat{r}(w))$  and the union X gets  $L_X = w\hat{x}$ . The disagreement payoffs are equal to zero for both agents. The generalized Nash bargaining problem between the firm and the union X can be stated as

$$\max_{r\geq 0} \left[\pi(\widehat{x}, w, \widehat{y}, \widehat{r}(w))\right]^{\alpha} (w\widehat{x})^{1-\alpha}.$$
(5.24)

The optimal wage that solves the bargaining problem (5.24) is  $w^* = (1 - \alpha)/2$ . The optimal wage rate for union Y is then given by  $r^* = (1 - \alpha^2)/4$ , while the optimal employment level is given by  $z^* = (1 + \alpha)^2/8$ .

## 5.5.2 The Two-products Extension

We first provide the analysis for the simultaneous and joint bargaining benchmarks. Simultaneous Bargaining. Under simultaneous bargaining the firm bargains with each union separately. If the firm reaches an agreement with both unions, then its profit is  $\pi(x, w, y, r)$ . If the firm fails to reach an agreement with union X or union Y, then its profit is given by the disagreement points  $\pi^{DX} :=$  $\pi(0, 0, y, r) = (1 - y)y - ry$  or  $\pi^{DY} := \pi(x, w, 0, 0) = (1 - x)x - wx$ , respectively. Hence, the firm has a positive disagreement point when bargaining with each union, while the unions do not have a similar valuable outside option at hand. The Nash bargaining problem between the firm and union X can then be written as

$$\max_{x,w} [\pi(x, w, y, r) - \pi^{DX}]wx$$

$$= [((1 - x - \gamma y)x + (1 - y - \gamma x)y - wx - yr) - ((1 - y)y - ry)]xw,$$
(5.25)

while the Nash bargaining problem between the firm and union Y can be stated similarly as

$$\max_{r,y} [\pi(x, w, y, r) - \pi^{DY}]yr$$

$$= [((1 - x - \gamma y)x + (1 - y - \gamma x)y - wx - yr) - ((1 - x)x - wx)]yr.$$
(5.26)

As the unions are symmetric, in equilibrium the optimal number of employed workers and their wages are the same for both unions. The contracts (5.15) solve the simultaneous bargaining problems (5.25) and (5.26).

**Joint Bargaining.** Consider that unions X and Y join in a single encompassing union to negotiate with the firm. We consider the Nash bargaining problem between the firm and the encompassing union over wages and employment levels of worker groups X and Y. In the case of agreement over the contract (w, x, r, y), the firm's profit is given by  $\pi(x, w, y, r) = \sum_{i=1}^{2} p_i q_i - xw - yr$ , while the encompassing union receives wx + ry. In case of disagreement, the firm must shut down and gets zero profit. The union does not have an outside option, i.e., its disagreement payoff is zero. The Nash bargaining problem, therefore, can be written as

$$\max_{w,x,r,y} \left[ \sum_{i=1}^{2} p_i q_i - xw - yr \right] [xw + yr].$$
 (5.27)

The optimal levels of employment which solve this problem are the same as under simultaneous bargaining. Obviously, under efficient bargaining the employment levels must maximize the parties' joint surplus. The production quantities are then given by  $q_1^{joi} = x^{joi}$  and  $q_2^{joi} = y^{joi}$  for good 1 and good 2, respectively. The union's wage bill which solves the Nash bargaining problem (5.27) is given by (5.16).

Sequential Bargaining. In the first stage, the firm negotiates a contract with union X over both the employment level, x, and the wage rate, w. In the second stage, the bargaining outcome of the first stage becomes public and the firm negotiates a contract with union Y which specifies employment level, y, and a wage rate, r. With that, the firm also determines its production quantities  $q_1$ and  $q_2$  and realizes its profit  $\pi(x, w, y, r) = \sum_{i=1}^{2} p_i q_i - xw - yr$  if bargaining is successful. The unions receive their wage bills at the end of the game. The total wage bill for union X is wx, and for union Y it is yr. For each bargaining problem we use the Nash bargaining solution. <sup>37</sup> We solve the game by backward induction.

Suppose bargaining was successful in the first stage which resulted in a contract (w, x). Then, in the second stage, the firm and union Y take the contract (w, x) as given when bargaining over the wage r and the employment level y. If the firm reaches an agreement with union Y over a contract (r, y), then it gets the profit  $\pi(x, w, y, r)$ . In this case, union Y realizes the wage bill ry. In the case of disagreement, the firm can only produce good 1 and realizes the profit  $\pi^{DY} := \pi(x, w, 0, 0) = (1 - x)x - xw$  which defines the firm's disagreement point when bargaining with union Y. Again, we assume that union Y's disagreement point is zero. The Nash bargaining problem between the firm and union Y is then given by

$$\max_{y,r} [\pi(x, w, y, r) - \pi^{DY}]yr$$

$$= [(1 - x - \gamma y)x + (1 - y - \gamma x)y - xw - yr) - ((1 - x)x - xw)]yr.$$
(5.28)

The optimal contract  $(\hat{r}(x), \hat{y}(x))$  which depends on the employment level x the firm agreed upon with union X, follows from the first-order conditions of the Nash Bargaining problem (5.28). Straight forward calculations yield the solution (5.17). Unless the unions' workforces are independent, the optimal contract between the firm and the union Y depends on the quantity of workers employed from union X. If labor unions produce substitutable goods, i.e.,  $\gamma > 0$ , then both the

<sup>&</sup>lt;sup>37</sup> Our approach to sequential bargaining (in particular, the application of the Nash bargaining solution and the specification of the firm's disagreement points) builds on Marx and Shaffer (1999) where supplier-retailer bargaining over two-part tariffs is analyzed.

employment level and the wage rate decrease in the employment level x. The opposite is true for complementary unions.

In the first stage, the firm and union X take the optimal strategies (5.17) as given when they bargain over employment x and the wage w. If the firm reaches an agreement with union X over the contract (w, x), then its expected profit is  $\pi(x, w, \hat{y}(x), \hat{r}(x))$ . In this case, union X obtains the wage bill wx. In the case of disagreement with union X, the firm can only produce good 2 and realizes an expected profit of  $\pi^{DX} := \pi(0, 0, \hat{y}(0), \hat{r}(0))$  which gives  $\pi^{DX} = 1/8$  as  $\hat{y}(0) = 1/2$  and  $\hat{r}(0) = 1/4$  follow from (5.17). Hence,  $\pi^{DX} = 1/8$  is the firm's (fixed) disagreement point when bargaining with union X. Union X realizes a wage bill of zero when bargaining is not successful.

The Nash bargaining problem between the firm and union X can be stated as

$$\max_{x,w} [\pi(x,w,\hat{y}(x),\hat{r}(x)) - \pi^{DX}]xw$$

$$= [(1 - x - \gamma\hat{y}(x))x + (1 - \hat{y}(x) - \gamma x)\hat{y}(x) - wx - \hat{r}(x)\hat{y}(x)) - 1/8]xw$$

$$= \left[-\frac{1}{2}\gamma x + \frac{1}{2}\gamma^{2}x^{2} + x - x^{2} - xw\right]xw,$$
(5.29)

where the last equality follows from substituting (5.17) into (5.29). The contract  $(w^*, x^*)$  which solves the Nash bargaining problem (5.29) is given by (5.18). Substituting (5.18) into (5.17) we obtain the equilibrium contract  $(r^*, y^*)$  of union Y which is given by (5.19).

# **Declaration of Contribution**

Hereby I, Hamid Aghadadashli, declare that the chapter "Multiunion Bargaining: Tariff Plurality and Tariff Competition" is coauthored by Prof. Dr. Christian Wey.

My contributions to this chapter are as follows:

- I have contributed substantially to the Introduction.
- I have derived the Model.
- I have written the major part of the Analysis.
- I have contributed substantially to the Extension.

Signature of coauthor 2 (Prof. Dr. Christian Wey):

C. Way

# Chapter 6

# Conclusion

In this thesis, I presented four papers on industrial organizations.

Chapter 2 analyzes a directed search model in which firms compete in advertisements and prices to get a prominent position in consumers' search order. If the share of the naive consumers who start their search from the advertised products is relatively high then both firms advertise their products. Otherwise, only one firm advertises in the market. Advertisement(s) increases market prices. Moreover, the prices are even higher when only one firm advertises its product compared to the case when both firms advertise. Given the firms' advertisement strategies, an increasing share of naive consumers raises prices. However, there is a threshold level such that a small increase in their share changes firms' advertisement strategies and decreases prices. In the end of the chapter, I also introduce a third group of consumers who are fully informed about the products offered by the firms. I show that, in this case, naive consumers can increase, decrease, or not affect consumer welfare depending on the level of search costs, as well as their existing shares. This study suggests that consumer welfare can be improved in online markets by regulating online platforms' advertising policies. But the same policy which decreases the share of naive consumers may not always increase consumer welfare.

In Chapter 3, we re-examine the Nash bargaining problem between an upstream and a downstream firm over a linear input price. We provide a simple and an instructive link between the profit shares and the demand elasticities. Our formula shows the role of the slope of the bargaining frontier as an additional determinant of bargaining power, besides the disagreement payoffs and the bargaining power parameter in the Nash product. We show the relationship between the slope of the bargaining frontier and the derived demand elasticity. In examples, we have also shown that a more elastic equilibrium demand can be beneficial for the downstream firm as it changes the slope of the bargaining frontier. Our results can be used in empirical studies to determine the bargaining power of the firms based on observables such as absolute profit levels and equilibrium demand elasticity.

In Chapter 4, first, we analyze merger decision between two firms located in two different countries and get inputs from their respective country-specific input suppliers. We distinguish two post-merger scenarios. In the benchmark case, we consider plant-specific products. We show that, although merger is beneficial for the merging parties, it reduces consumer welfare and is not socially desirable. In the second case, we consider a merger which allows the firms to produce different product variants in different plants they own. In this case, merger is again profitable for the firms, but it also increases consumer surplus and social welfare if the goods are not close substitutes and the capacity of the plants are not too restrictive. In the second part of the chapter, we analyze firms' investment incentives for producing a differentiated product. We show that the firm has stronger incentives to invest in a foreign country compared to the investment in the home country. This incentive is even higher if the new multi-plant firm has the production-shifting ability. This study shows that having the option to shift production creates considerable countervailing power and has important implications for both merger control and FDI policy.

In Chapter 5, we study sequential bargaining between two unions and a single firm. First, we consider a single-product firm which gets inputs from the two upstream unions to produce a single product. We show that, if workforces from the two labor unions are substitutable, then there is a first-mover advantage. The union which bargains earlier with the firm gets a positive profit, while the second union is foreclosed. In the other case, if the workforces are complementary, then the second union gets higher wage bill compared to the first union. We also study the merger incentives of the unions under both cases. Unions prefers to bargain jointly when they represents substitutable workforces. Otherwise, *i.e.*, when their workforces are complementary, the unions prefer to bargain separately. Second, we consider a case in which each union's labor force produces a differentiated good. Unions' merger incentives are qualitatively the same as in single-product firm case. Additionally, we show that unions' merger decisions always stay in conflict with social welfare maximization.

# Bibliography

- AGHION, P. and BOLTON, P. (1987). Contracts as a barrier to entry. *The American Economic Review*, **77** (3), pp. 388–401.
- ALOI, M., LEITE-MONTEIRO, M. and LLOYD-BRAGA, T. (2009). Unionized labor markets and globalized capital markets. *Journal of International Eco*nomics, 78 (1), 149–153.
- ARBATSKAYA, M. (2007). Ordered search. The RAND Journal of Economics, 38 (1), 119–126.
- ARMSTRONG, M., VICKERS, J. and ZHOU, J. (2009). Prominence and consumer search. The RAND Journal of Economics, 40 (2), 209–233.
- and ZHOU, J. (2011). Paying for prominence. *The Economic Journal*, 121 (556), F368–F395.
- ATHEY, S. and ELLISON, G. (2011). Position auctions with consumer search. The Quarterly Journal of Economics, **126** (3), 1213–1270.
- BACHMANN, R. and SCHMIDT, C. M. (2012). Im zweifel für die freiheit: Tarifpluralität ohne chaos. *Wirtschaftsdienst*, **92**, pp. 291–294.
- BAGWELL, K. and RAMEY, G. (1994). Advertising and coordination. *The Review* of *Economic Studies*, **61** (1), 153–171.
- BAYE, M. R. and MORGAN, J. (2001). Information gatekeepers on the internet and the competitiveness of homogeneous product markets. *American Economic Review*, **91** (3), 454–474.
- and (2009). Brand and price advertising in online markets. Management Science, 55 (7), 1139–1151.

- BFA (2013). Arbeitsmarktberichterstattung: Der Arbeitsmarkt in Deutschland, Zeitarbeit in Deutschland - Aktuelle Entwicklungen. Tech. rep., Bundesanstalt für Arbeit, Nuremberg.
- BINMORE, K., RUBINSTEIN, A. and WOLINSKY, A. (1986). The nash bargaining solution in economic modelling. *The Rand Journal of Economics*, **17** (2), 176–188.
- BOERI, T. and VAN OURS, J. (2013). *The Economics of Imperfect Labor Markets*. Princeton: Princeton University Press.
- BOOTH, A. L. (1995). *The Economics of the Trade Union*. Cambridge and New York: Cambridge University Press.
- CAHUC, P. and ZYLBERBERG, A. (2004). *Labor Economics*. Cambridge, Mass.: MIT Press.
- CAI, H. (2000). Delay in multilateral bargaining under complete information. Journal of Economic Theory, 93 (2), 260–276.
- CHEN, Y. and HE, C. (2011). Paid placement: Advertising and search on the internet. *The Economic Journal*, **121** (556), F309–F328.
- CHEN, Z. (2003). Dominant retailers and the countervailing-power hypothesis. The RAND Journal of Economics, **34** (4), 612–625.
- CHIPTY, T. and SNYDER, C. M. (1999). The role of firm size in bilateral bargaining: A study of the cable television industry. *The Review of Economics* and Statistics, **81** (2), 326–340.
- CHONGVILAIVAN, A., HUR, J. and RIYANTO, Y. E. (2013). Labor union bargaining and firm organizational structure. *Labour Economics*, **24**, 116–124.
- COMMITTEE, I. C. P. A. (2000). Final Report to the Attorney General and the Assistant Attorney General for Antitrust. Tech. rep.
- CONLIN, M. and FURUSAWA, T. (2000). Strategic delegation and delay in negotiations over the bargaining agenda. *Journal of Labor Economics*, 18 (1), 55–73.
- DERTWINKEL-KALT, M., HAUCAP, J. and WEY, C. (2015). Raising rivals' cost through buyer power. *Economic Letters*, **126**, 181–184.
- DOBSON, P. and WATERSON, M. (1997). Countervailing power and consumer prices. *The Economic Journal*, **107**, 418–430.
- DOWRICK, S. (1990). The relative profitability of nash bargaining on the labour demand curve or the contract curve. *Economic Letters*, **33** (2), 121–125.
- DRAGANSKA, M., KLAPPER, D. and VILLAS-BOAS, S. B. (2008). A larger slice or a larger pie? an empirical investigation of bargaining power in the distribution channel. *Marketing Science*, **29** (1), 57–74.
- DUNLOP, J. (1944). Wage Determination Under Trade Unions. New York: Macmillan.
- ECHENIQUE, F. and EDLIN, A. (2004). Mixed equilibria are unstable in games of strategic complements. *Journal of Economic Theory*, **118** (1), 61 79.
- ECKEL, C. and EGGER, H. (2009). Wage bargaining and multinational firms. Journal of International Economics, 77 (2), 206–214.
- EGGER, H. and ETZEL, D. (2014). Union wage-setting and international trade with footloose capital. *Regional and Urban Economics*, 48, 56–67.
- EICHBERGER, J. (1993). *Game Theory for Economists*. San Diego: Academic Press.
- FARRELL, J. and SHAPIRO, C. (1990). Horizontal mergers: An equilibrium analysis. The American Economic Review, 80 (1), 107–126.
- FORDE, M. (1984). Trade union pluralism and labour law in france. International and Comparative Law Quarterly, 33 (1), 134–157.
- GAUDIN, G. (2015). Vertical bargaining and retail competition: What drives countervailing power? *DICE Discussion Paper*, **195**.
- (2016). Pass-through, vertical contracts, and bargains. *Economics Letters*, 139, 1–4.

- GOWRISANKARAN, G., NEVO, A. and TOWN, R. (2015). Mergers when prices are negotiated: Evidence from the hospital industry. *The American Economic Review*, **105** (1), 172–203.
- GRENNAN, M. (2013). Price discrimination and bargaining: Empirical evidence from medical devices. *The American Economic Review*, **103** (1), 145–177.
- (2014). Bargaining ability and competitive advantage: Empirical evidence from medical devices. *Management Science*, **60** (12), 3011–3025.
- HAAN, M. A. and MORAGA-GONZÁLEZ, J. L. (2011). Advertising for attention in a consumer search model. *The Economic Journal*, **121** (552), 552–579.
- —, MORAGA-GONZÁLEZ, J. L. and PETRIKAITĖ, V. (2015). Price and matchvalue advertising with directed consumer search. *mimeo*.
- HART, O. and TIROLE, J. (1990). Vertical integration and market foreclosure. The Brookings Papers on Economic Activity: Microeconomics, pp. 205–286.
- HAUCAP, J., PAULY, U. and WEY, C. (2007). Das deutsche tarifkartell: Entstehung, stabilität und aktuelle reformvorschläge aus sicht der wettbewerbstheorie. In R. Ohr (ed.), Arbeitsmarkt und Beschäftigung, Berlin: Duncker und Humblot, pp. 93–143.
- HORN, H. and WOLINSKY, A. (1988a). Bilateral monopolies and incentives for merger. The RAND Journal of Economics, 19, 408–419.
- and (1988b). Worker substitutability and patterns of unionization. The Economic Journal, 98, 484–497.
- INDERST, R. and OTTAVIANI, M. (2012). Competition through commissions and kickbacks. The American Economic Review, 102 (2), 780–809.
- and SHAFFER, G. (2009). Market power, price discrimination, and allocative efficiency in intermediate-goods markets. *The RAND Journal of Economics*, 40 (4), 658672.
- and WEY, C. (2003). Bargaining, mergers, and technology choice in bilaterally oligopolistic industries. The RAND Journal of Economics, 34 (1), 1–19.

- and (2011). Countervailing power and dynamic efficiency. Journal of the European Economic Association, 9 (4), 702–720.
- IOZZI, A. and VALLETTI, T. (2014). Vertical contracting and countervailing power. American Economic Journal: Microeconomics, 6 (3), 106–135.
- JUN, B. H. (1989). Non-cooperative bargaining and union formation. *The Review* of *Economic Studies*, **56** (1), 59–76.
- KOGUT, B. and KULATILAKA, N. (1995). Operating flexibility, global manufacturing, and the option value of a multinational network. *Management Science*, 40 (1), 123–139.
- LOMMERUD, K., STRAUME, O. and SORGARD, L. (2005). Downstream mergers with upstream market power. *European Economic Review*, **49** (3), 717–743.
- —, and (2006). National versus international mergers in unionized oligopoly. *The Rand Journal of Economics*, **37** (1), 212–233.
- MARX, L. and SHAFFER, G. (1999). Predatory accommodation: Below-cost pricing without exclusion in intermediate goods markets. *The RAND Journal of Economics*, **30**, 22–43.
- NAYLOR, R. (2002). Industry profits and competition under bilateral oligopoly. *Economics Letters*, 77 (2), 169–175.
- O'BRIEN, D. P. and SHAFFER, G. (1992). Vertical control with bilateral contracts. *The RAND Journal of Economics*, **23** (3), 299–308.
- OSWALD, A. and TURNBULL, P. (1985). Pay and employment determination in britain: What are labour 'contracts' really like? Oxford Review of Economic Policy, 1 (2), 80–97.
- PAPARELLA, D. (1997). Agreement on productivity and employment signed at zanussi. European Observatory of Working Life, online available at: eurofound.europa.eu.
- PERRY, M. and PORTER, R. (1985). Oligopoly and the incentive for horizontal merger. The American Economic Review, 75 (1), 219–227.

- REY, P. and TIROLE, J. (1986). The logic of vertical restraints. The American Economic Review, 76 (5), 921–939.
- and VERGÉ, T. (2004). Bilateral control with vertical contracts. The RAND Journal of Economics, 35 (4), 728–746.
- RIEBLE, V. (1996). Arbeitsmarkt und Wettbewerb. Berlin: Spring Verlag.
- RUSTICO, L. (2014). Furore over major restructuring plan for electrolux. *European Observatory of Working Life, online available at: eurofound.europa.eu*.
- SIMON, H. (1944). Some reflections on syndicalism. Journal of Political Economy, 52, 1–25.
- STAHL, D. O. (1989). Oligopolistic pricing with sequential consumer search. The American Economic Review, 79 (4), 700–712.
- STIGLER, G. (1950a). The development of utility theory i. Journal of Political Economy, 58 (4), 307–327.
- (1950b). The development of utility theory ii. Journal of Political Economy, 58 (5), 373–396.
- STOLE, L. and ZWIEBEL, J. (1996). Organizational design and technology choice under intrafirm bargaining. *The American Economic Review*, **86**, 195–222.
- STURGEON, T., MEMEDOVIC, O., VAN BIESEBROECK, J. and GEREFFI, G. (2009). Globalisation of the automotive industry: Main features and trends. International Journal of Technological Learning, Innovation and Development, 2 (1), 7–24.
- SYMEONIDIS, G. (2008). Downstream competition, bargaining and welfare. Journal of Economics & Management Strategies, 17 (1), 247–270.
- (2010). Downstream merger and welfare in a bilateral oligopoly. International Journal of Industrial Organization, 28 (3), 230–243.
- VARIAN, H. R. (2010). Intermediate Microeconomics: A Modern Approach. New York: W.W. Norton & Co.

- VON UNGERN-STERNBERG, T. (1996). Countervailing power revisited. International Journal of Industrial Organization, 14 (4), 507–519.
- WILLIAMSON, O. (1968). Economies as an antitrust defense: The welfare tradeoffs. *The American Economic Review*, **58** (1), 18–36.
- WOLINSKY, A. (1986). True monopolistic competition as a result of imperfect information. *The Quarterly Journal of Economics*, **101** (3), 493–512.
- ZHANG, T. (2009). Price-directed consumer search. Discussion Papers in Economics, Working Paper No. 09-08, University of Colorado.
- ZHAO, L. (1995). Cross-hauling direct foreign investment and unionized oligopoly. European Economic Review, 39 (6), 1237–1253.
- ZHOU, J. (2011). Ordered search in differentiated markets. *International Journal* of *Industrial Organization*, **29** (2), 253 – 262.

## Appendix

Ich versichere an Eides statt, dass die vorliegende Dissertation von mir selbstständig und ohne unzulässige fremde Hilfe unter Beachtung der "Grundsätze zur Sicherung guter wissenschaftlicher Praxis an der Heinrich-Heine-Universität Düsseldorf" erstellt worden ist.

Accelle

Düsseldorf, März 2016

Hamid Aghadadashli