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Chapter 1

Technology Licensing by Advertising Supported Media Platforms: An Application to Internet Search Engines

Abstract

This chapter is based on joint research with Irina Suleymanova. It has benefited a lot from helpful comments by Pio Baake, Steffen H. Hoernig, Martin Peitz, Christian Wey, participants of the ZEW Mannheim, Telecom Paris-Tech ICT Conferences and the EARIE and EEA Annual Meetings in 2009.

We develop a duopoly model with advertising supported platforms and analyze incentives of a superior firm to license its advanced technologies to an inferior rival. We highlight the role of two technologies characteristic for media platforms: the technology to produce content and to place advertisements. Licensing incentives are driven solely by indirect network effects arising from the aversion of users to advertising. We establish a relationship between licensing incentives and the nature of technology, the decision variable on the advertiser side, and the structure of platforms' revenues. Only the technology to place advertisements is licensed. If users are charged for access, licensing incentives vanish. Licensing increases the advertising intensity, benefits advertisers and harms users. Our model provides a rationale for technology-based cooperations between competing platforms, such as the planned Yahoo-Google advertising agreement in 2008.

1.1 Introduction

Many media firms function as two-sided platforms. They attract audience with content and sell advertising space to businesses. Digital technologies have created several new ways for such platforms both to compete and cooperate. While competition between media platforms is subject to much research, the technological peculiarities of these businesses and technology-based cooperation receive little attention. In this article we highlight two technologies that are of crucial importance for advertising supported platforms: The technology to produce content, and the technology to place advertisements. In addition to investing in the improvement of their own technologies, media firms often engage in cooperation agreements that involve sharing of their know-how with rivals. The competitive effects of such agreements are the focus of this article.

This article is strongly motivated by recent cooperation agreements between internet search engines. In 2008 search engine operators *Google* and *Yahoo* announced plans to cooperate in advertising. The envisaged cooperation would have let *Yahoo* use *Google*'s technology to match advertisements with search keywords in Canada and the U.S. After competition authorities expressed their doubts, the parties officially abandoned the agreement. In 2009 search engine operators *Yahoo* and *Microsoft* entered an agreement with the latter providing the underlying search technology on some of *Yahoo*'s Web sites.[1]

We consider two media platforms serving advertisers and users, with one possessing superior content producing and advertisement placing technologies and aim to answer two questions. First, what drives a platform endowed with superior technologies to improve its rival by licensing a technology? Second, what welfare effects does such cooperation have? Our results show that a purely advertising financed platform with superior capabilities licenses only its technology to place advertisements, but not the technology to produce content. Licensing incentives are driven by indirect network effects: By improving the com-

[1]See, for example, BBC News, 13 June, 2008, "Yahoo-Google agree online ad deal," and The New York Times, 30 July, 2009, "Yahoo-Microsoft Deal."

petitor on the advertiser side of the market, the superior platform increases its demand on the user side. A better technology to place advertisements makes it profitable for the competitor to place more ads, inducing some users to switch to the superior platform. A larger user base at the superior platform boosts in turn its advertiser demand, which finally increases its profit. Our results are robust to whether platforms decide on advertisement quantities or prices. However, if platforms can charge users and choose advertisement quantities, incentives for technology licensing vanish: Licensing of the advertisement placing technology decreases the superior platform's user demand. We consider welfare implications of technology licensing and find that it is likely to be beneficial for advertisers and detrimental to users. Furthermore, we show that private licensing incentives can be socially suboptimal.

This article makes three main contributions to the research. First, we establish a relationship between the incentives of an advertising supported two-sided platform to license its technologies to a competitor with *i)* the nature of a technology; *ii)* the type of a game played on the advertiser side of the market (quantity or price setting); and *iii)* the structure of platforms' revenues. Second, we provide a rationale for technology licensing that is based purely on indirect network effects. Third, we make predictions about the welfare effects of technology licensing involving advertising supported two-sided platforms, and aim to provide guidance to competition authorities for the evaluation of cases such as the 2008 *Yahoo-Google* and the 2009 *Yahoo-Microsoft* cooperation agreements.

Our article is closely related to the literature on technology licensing in oligopoly. Earlier articles explained a monopolist's incentives to licence a proprietary technology by licensing serving as a commitment device for low future prices (Farrell and Gallini, 1988)or high quality (Shepard, 1987)in a dynamic setting. We add to this strand by explaining how the two-sided nature of the market may drive licensing incentives. As in the models of Farell and Gallini (1988)and Shepard (1987), licensing serves to boost demand for the product of the licensor, which in our case corresponds to advertising space. In our baseline model, by making a more advanced advertisement placing technology available to the inferior rival, the superior platform directly increases the demand for the inferior platform's advertising space. The inferior platform responds by placing more advertisements, which induces advertising-averse users to switch to the superior platform. A larger user base, in turn, increases demand for advertising space on the superior platform as advertisers value a larger audience.

This article also fits into the literature on two-sided markets (Armstrong, 2006; Rochet and Tirole, 2003), particularly the strand focusing on advertising platforms (Anderson and Coate, 2005; Crampes et al., 2009). We contribute to this literature by analyzing asymmetric and vertically differentiated platforms and their incentives to license different technologies. An article particularly close to ours is Crampes et al. (2009). The authors present a model of competition between media platforms financed by both subscriptions and advertising receipts, highlighting the relationship between equilibrium prices, advertising levels and advertising technology. They show that advertising levels may be either too high or too low, depending on the returns to scale in audience size. Our article also highlights the role of technologies in the context of two-sided platforms, however, our main aim is to explain incentives of media platforms to cooperate in technology licensing and analyze its welfare effects.

The article proceeds as follows. The next section presents the baseline model and characterizes the equilibrium without technology licensing, assuming quantity setting on the advertiser side and no access fee for users. In Section 3 we apply our framework to the analysis of technology licensing incentives and provide a welfare analysis. In Section 4 we extend our baseline model in two directions in order to derive the results relevant to a broader range of advertising supported media platforms. In particular, we analyze licensing incentives under price setting on the advertiser side and also address the case where users are charged for access. In Section 5 we discuss some of our modelling assumptions. Section 6 concludes.

1.2 The Baseline Model And Equilibrium Analysis

We analyze a two-sided market in which two horizontally and vertically differentiated platforms $i = \{1, 2\}$ provide content to *users* and sell advertising space to *advertisers*. Our main modelling novelty is that we explicitly distinguish between content producing (CP) and advertisement placing (AP)technologies. The CP technology of a platform is responsible for the intrinsic utility a user draws from consuming content on a platform. The AP technology in turn determines the probability that an advertisement shown on a platform motivates a consumer to buy the advertised product.[2] We assume that both the AP and CP technologies of platform 1 are superior to those of platform 2. In other words, platform 1 can produce both higher quality content as well as place more relevant advertisements.[3] We will refer to platform 1 as *superior* and to platform 2 as *inferior*.

In our baseline model content consumption at the platforms is free of charge for users, while advertisers pay a price p_i^a for an advertisement slot at platform i.[4] Each platform decides on the number of advertisement slots, a_i, to place, with every advertisement requiring

[2] For example, the content producing technology corresponds to the quality of television channels' programmes and the relevance of organic search results in the case of internet search engines. The advertisement placing technologies are often proprietary in media markets. For example, U.S. patent No. 7398207 held by a television industry player relates to a technology that adjusts the volume level of an advertisement to that of the program in which the advertisement is embedded. This is to prevent a sudden volume change during advertising breaks. Other technologies held by media firms prevent viewers from disabling advertisements when recording television programmes. Similarly, internet search engines use sophisticated algorithms to match the most relevant advertisements to search keywords thus determining the probability that a click on a sponsored link will result in a successful sale of the advertised good.

[3] The assumption that one platform is superior in both technologies is not crucial for our results. In Section 5 we discuss the case where each platform is superior in one of the technologies.

[4] In Section 4.2 we relax the assumption that platforms do not charge users and analyze platforms' licensing incentives given two sources of revenues: Payments from advertisers and users.

one slot.[5] The platforms provide their services at zero marginal cost and realize profits

$$\pi_i = p_i^a a_i. \tag{1.1}$$

We assume that users single-home, i.e., every user visits only one platform. Following Peitz and Valletti (2008), we assume that every potential advertiser can place advertisements at just one or both of the platforms, or refrain from advertising. If advertiser k places an advertisement at platform i, its expected profit $E\left(\pi_i^k\right)$ is

$$E\left(\pi_i^k\right) = Pr_i\left\{Sale\right\} n_i p^k - p_i^a - c^k,$$

where n_i, $p^k$and c^k denote the user market share of platform i, the price (net of marginal cost)of advertiser k's product and its costs associated with placing an advertisement, respectively. The price of the advertised product is normalized to unity for all advertisers ($p^k = 1$). The advertising costs c^k capture the advertiser k's fixed costs associated with placing an advertisement other than the price paid for advertising space, such as the costs for designing an advertisement. Advertisers are heterogeneous with respect to costs, which are uniformly distributed on the interval $c^k \in [0, \infty)$.[6] We assume that every user of platform i becomes aware of advertiser k's product after having seen an ad and may buy exactly one unit of the advertised good. $\Pr_i\left\{Sale\right\}$ captures the level of AP technology of platform i: It denotes the probability that a user buys the product after having seen its advertisement on platform i.

We assume that $\Pr_i\left\{Sale\right\} = 1 - \rho_i$, where $\rho_i \in [0, 1)$ corresponds to platform i's handicap in ability to place high-quality advertisements (i.e., ones that result in a sure sale of the advertised good). A platform with a lower ρ_i has a better AP technology. For example, $\rho_i = 1/3$ implies that 2/3 of those who have seen an advertisement end up buying the product.

[5] In Section 4.1 we analyze licensing incentives assuming that platforms set slot prices.

[6] The analysis of the case where advertisers sell their products at different prices and have the same advertising costs is available from the authors on request. It is shown that licensing incentives in that case are the same as in the model formulated here.

There is a marginal advertiser on platform i with advertising costs $\bar{c}_i$, who is indifferent between placing an ad and not advertising. The expected profit of the marginal advertiser is $(1-\rho_i)n_i - p_i^a - \bar{c}_i = 0$. As every advertiser places one ad, the number of ads on a platform equals the advertising costs of the marginal advertiser, with $a_i = \bar{c}_i$. The inverse demand for advertisement slots at platform i is then given by

$$p_i^a = (1-\rho_i)n_i - a_i. \tag{1.2}$$

With a superior AP technology, platform 1 can display more relevant advertisements, which increase the probability of a successful sale by advertisers. This translates into a higher willingness to pay for an advertisement slot. We assume for the superior platform that $\rho_1 = 0$, while $\rho_2 \in [0,1)$ reflects the inferior platform's handicap in AP technology.

We now turn to users and the role of CP technology. Users derive a basic utility $u > 0$ and platform-specific utility $\zeta_i q > 0$ from consuming content on platform i, which increases in platform's ability to produce high-quality content. The value $\zeta_i q$ is higher, the better the CP technology of platform i becomes. We assume that content quality is (weakly)higher at platform 1 and $\zeta_1 = 1$ while $\zeta_2 \in (0,1]$. With $\zeta_2 < \zeta_1$ the platforms are vertically differentiated. For notational simplicity, in the following we will often write ζ and ρ instead of ζ_2 and ρ_2, respectively. Let finally $\Delta \geq 0$ denote the advantage of platform 1 in content quality, with $\Delta := (1-\zeta)q$. The superior platform is said to have a strict advantage in CP technology if $\zeta < 1$ ($\Delta > 0$). We say it has a strict advantage in AP technology if $\rho > 0$.

The platforms are placed on a unit circle and are assumed to be maximally differentiated from each other, such that the address of platform 1 is normalized to $s_1 = 0$ while the address of the other platform is $s_2 = 1/2$. Users are uniformly distributed along the circle with each having an address $t \in [0,1]$ reflecting the preference for the optimal platform. Visiting platform i involves quadratic transportation costs for users, which are positive if the visited platform is not located in the user's ideal position.

We assume that users dislike advertisements. This assumption is often made in the literature on advertising supported two-sided platforms and seems to apply well to most of the markets we have in mind. The user disutility from advertisements depends on the number of ads and is given by a linear function, μa_i, with $\mu > 0$ denoting the strength of disutility per advertisement.[7]

The utility of a user with address t visiting platform i, U_i^t, then takes the form

$$U_i^t = \begin{cases} u + q - [\delta_1(t)]^2 - \mu a_1, & \text{if} \quad i = 1 \\ u + \zeta q - [\delta_2(t)]^2 - \mu a_2, & \text{if} \quad i = 2, \end{cases} \tag{1.3}$$

$$\text{with } \delta_1(t) = \min\{t, 1-t\} \text{ and } \delta_2(t) = |t - 1/2|.$$

The term $\delta_i(t)$ captures the distance between user t and platform i and his transportation costs are $[\delta_i(t)]^2$. We assume that u is high enough, so that in equilibrium every user visits one of the platforms.

The timing of the game is as follows: First, the platforms determine the number of advertisement slots to display. Second, users choose their preferred platform and advertisers buy advertisement slots. We seek for the subgame-perfect Nash equilibrium and solve the game backwards.

Equilibrium Analysis

Every user chooses the platform providing higher utility. We can find two marginal users with addresses t_1 and t_2 which are indifferent between the platforms:

$$\begin{aligned} t_1(a_1, a_2; \zeta, \mu, q) &= \mu(a_2 - a_1) + 1/4 + \Delta, \\ t_2(a_1, a_2; \zeta, \mu, q) &= \mu(a_1 - a_2) + 3/4 - \Delta, \end{aligned} \tag{1.4}$$

with $t_1 < t_2$. The market shares of the platforms are then $n_1 = 1 - t_2 + t_1$ and $n_2 = t_2 - t_1$. This yields the following user demand at the platforms:

$$\begin{aligned} n_1(a_1, a_2; \zeta, \mu, q) &= 1/2 + 2\left[\Delta - \mu(a_1 - a_2)\right], \\ n_2(a_1, a_2; \zeta, \mu, q) &= 1/2 - 2\left[\Delta - \mu(a_1 - a_2)\right], \end{aligned} \tag{1.5}$$

[7] The linear specification of disutility from advertising is common in the literature (see Gal-Or and Dukes, 2003; Anderson and Coate, 2005; Peitz and Valletti, 2008).

with $\partial n_i(a_i, a_j; \cdot)/\partial a_i < 0$ and $\partial n_i(a_i, a_j; \cdot)/\partial a_j > 0$ for $i, j = \{1, 2\}$ and $i \neq j$. By plugging (1.5)into (1.2)we get platform i's profit as

$$\pi_i(a_i, a_j; \rho_i, \zeta, \mu, q) = [(1 - \rho_i)n_i(a_i, a_j; \cdot) - a_i]\, a_i. \tag{1.6}$$

Platform i maximizes its profit by choosing the number of advertisement slots, a_i. The following lemma states the condition under which both platforms are active on both sides of the market.

Lemma 1 *The necessary and sufficient condition for the platforms to be active on both sides of the market is $\Delta < \overline{\Delta}$, with $\overline{\Delta} := (1 + 3\mu)/[4(1 + \mu)]$. If this condition holds, the platforms display advertisements, serve some users and realize positive profits.*

Proof. See Appendix.

To guarantee that in equilibrium both platforms are active on both sides of the market, the superior platform's advantage in CP technology should not be too large (i.e., $\Delta < \overline{\Delta}$). The value $\overline{\Delta}$ corresponds to the minimum magnitude of content quality advantage of the superior platform which drives the inferior platform out of the market. If $\Delta = \overline{\Delta}$, then all users choose platform 1 that places advertisements, while the other platform does not advertise. It follows from the platforms' FOCs that a platform placing a positive number of advertisement slots also serves some users:

$$n_1^*(\rho, \zeta, \mu, q) = 2(1 + \mu)a_1^*(\rho, \zeta, \mu, q), \tag{1.7}$$

$$(1 - \rho)n_2^*(\rho, \zeta, \mu, q) = 2\,[1 + \mu(1 - \rho)]\, a_2^*(\rho, \zeta, \mu, q).$$

If a platform places advertisements, it also charges a positive price for them:

$$p_1^{a*}(\rho, \zeta, \mu, q) = (1 + 2\,\mu)\, a_1^*(\rho, \zeta, \mu, q), \tag{1.8}$$

$$p_2^{a*}(\rho, \zeta, \mu, q) = [1 + 2\,\mu(1 - \rho)]\, a_2^*(\rho, \zeta, \mu, q), \tag{1.9}$$

leading to positive profits. For further analysis in this section we assume $\Delta < \overline{\Delta}$. The following proposition characterizes the equilibrium without technology licensing.

Proposition 1 *The equilibrium without technology licensing has the following properties.*

i)If it has a strict advantage in at least one technology, the superior platform displays more advertisements, charges a higher price for its advertisement slots and realizes larger profits than the inferior platform.

ii)The superior platform has a larger (weakly smaller)market share among users than the inferior platform if $\Delta > \underline{\Delta}$ *(*$\Delta \leq \underline{\Delta}$*), with* $\underline{\Delta} = \mu\rho/[4(1+\mu)(1+\mu(1-\rho))]$.

Proof. See Appendix.

The superior platform places more advertisements in equilibrium and charges a higher price for them than its competitor if it has a strict advantage in at least one of the technologies. For the intuition behind this result it is helpful to consider the roles of both technologies on advertising decisions. The advantage in CP technology allows the superior platform to place more advertisements because its better content compensates users for the additional nuisance. With a more advanced AP technology, each user is more valuable to advertisers on the superior platform. For the same user market shares advertiser demand is higher at the superior platform, which makes it profitable to place more advertisements. These two insights imply that with a strict advantage in at least one technology, the superior platform displays more advertisements in equilibrium. It follows directly from the equilibrium slot prices in Expressions (1.8)and (1.9)that the price of an advertisement slot at the superior platform is higher: $p_1^{a*}(\cdot) > p_2^{a*}(\cdot)$ if $a_1^*(\cdot) > a_2^*(\cdot)$.

The superior platform has a larger market share among users if its advantage in the CP technology is large enough ($\Delta > \underline{\Delta}$). As we showed, the superior platform places more advertisements, it can therefore only have a larger market share among users if it is able to compensate users for the disutility caused by additional advertisements. The only way it can do so is by providing higher quality content. If the content quality advantage is larger than the critical value $\underline{\Delta}$, the superior platform can hold a dominant position among users even though it displays more ads. With a content quality advantage below $\underline{\Delta}$, the superior platform displays more ads than the rival, but it attracts less than half of users, despite having a better CP technology.

We note that the upper bound of the superior platform's quality advantage ($\overline{\Delta}$)depends on the user disutility per advertisement (μ)and does not depend on the inferior platform's handicap in AP technology (ρ). The upper bound is the quality advantage that makes all users prefer the superior platform when it has advertising while the inferior platform does not place any advertisements. However, $\underline{\Delta}$ depends on ρ, and is larger if the inferior platform's ability to place high-quality advertisements is lower. With a lower quality of AP technology, the inferior platform places fewer advertisements in equilibrium. Thus, the superior platform needs a larger advantage in CP technology to attract the majority of users.

Before turning to the analysis of technology licensing incentives, we briefly discuss how our modelling setup applies to the market of internet search engines. Although we omit some unique characteristics of the internet search market, our model takes into account the most important factors determining the choice of search engines by users. According to a survey conducted among internet searchers in 2008, the three most important factors driving user choice are *general search quality*, *home page appeal* and *special features*.[8] These factors are captured in our model by the vertical and horizontal differentiation between the platforms. We do not explicitly model the auction by which search engines allocate advertising space. Instead, we focus on two polar cases: In the baseline model we assume that platforms set advertisement quantities, while in Section 4.1 we investigate the case where platforms decide on the prices of an advertisement slot. We show that licensing incentives are similar in these cases. At this point it is worth noting that our model's prediction on the superior platform converting its technology advantage into higher profits by placing more advertisements is well in line with the observations in the internet search engines market. In the period from December 2008 till March 2010, *Google* placed on average more advertisements per search query than its closets competitor *Yahoo*.[9]

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[8] See Keynote Systems (2008), "Search Portals Study."

[9] According to the "Search Engine Advertiser Update-Q409", in the mentioned period *Google* displayed on average 5.24 ads per search keyword in the U.S. and abroad. In the same period *Yahoo* placed on average

1.3 Technology Licensing

We are interested in the incentives of a platform holding superior CP and AP technologies to license one or both technologies to the competitor. Technology licensing is a transaction that requires mutual consent of both platforms and we distinguish between the cases when transfers between platforms are allowed and when they are not. If transfers are not allowed, the superior platform licenses its technology (technologies)only if by doing so its individual profit increases. In this case the superior platform chooses the extent to which the competitor can access its proprietary technology by maximizing the superior platform's own profit. The inferior platform accepts any offered technology as it is costless and the improvement leads unambiguously to a higher individual profit.

In case transfers are possible, the superior platform makes a take-it-or-leave-it offer to the competitor involving a payment in exchange for the shared technology. Such an offer allows the licensor to appropriate the entire additional industry profits arising from the improvement of the inferior platform. It follows that when transfers are allowed, the superior platform determines the optimal level of licensing by maximizing joint profits of both platforms. We now formalize how licensing changes the inferior platform's technologies.

If CP technology is licensed, content quality at the inferior platform increases. We model this by assuming that parameter ζ increases. If the superior platform licenses its AP technology, the inferior platform becomes able to better place advertisements. Formally, ρ decreases and demand for advertisement slots at platform 2 grows. We introduce parameters $\rho_0 \in [0, 1)$ and $\zeta_0 \in (0, 1]$, denoting the initial handicap of the inferior platform in the quality of its AP technology and the initial quality of the inferior platform's CP technology, respectively. We assume that $(1 - \zeta_0)q < \overline{\Delta}$. We start with investigating the effects of AP technology licensing. The following lemma states the effects of a change in ρ on the equilibrium values.

4.05 ads per keyword.

Lemma 2 *As the demand for advertising space at the inferior platform gets larger (i.e., ρ decreases), the following holds:*

i)both platforms provide more advertisement slots,

ii)the superior (inferior)platform gains (loses)market shares among users,

iii)both platforms charge a higher price for advertisement slots,

iv)both platforms make larger profits, therefore, joint profits increase.

Proof. See Appendix.

Although demand for advertising space on a platform is independent of demand on the other platform, both platforms benefit from the increased demand for advertisement slots on the inferior platform due to indirect network effects. The increased advertiser demand at the inferior platform allows it to place more advertisements. In response, some users switch to the superior platform. The increased user demand, in turn, boosts demand for advertising space at the superior platform. Both platforms increase the number of advertisements with the rise in demand for advertisement slots at the inferior platform. The inferior platform displays more advertisements as it is directly affected by the change in advertiser demand. The superior platform is affected indirectly through users being driven away from the competitor due to intensified advertising and increases the number of its advertisement slots too.

We now consider how changes in platforms' advertising levels affect other equilibrium variables. It is instructive to inspect the reaction functions,
$a_1(a_2;\zeta,\mu,q)$ and $a_2(a_1;\rho,\zeta,\mu,q)$, which give the optimal number of advertisements placed by each platform in response to the number of advertisements placed by the competitor:

$$\begin{aligned} a_1(a_2;\cdot) &= \frac{1+4(\Delta+\mu a_2)}{4(1+2\mu)}, \\ a_2(a_1;\cdot) &= \max\left\{0, \frac{(1-\rho)[1+4(\mu a_1-\Delta)]}{4\,[1+2\mu(1-\rho)]}\right\}. \end{aligned}$$

Note that decisions about the number of advertisements to place are strategic complements as $\partial a_1(a_2;\cdot)/\partial a_2 = \mu/(1+2\mu) > 0$ and

$\partial a_2(a_1;\cdot)/\partial a_1 = \mu(1-\rho)/\left[1+2\mu(1-\rho)\right] > 0$ if $a_1 > \Delta/\mu - 1/\left(4\mu\right)$.

With a decrease in parameter ρ, the superior platform's reaction function remains unchanged while that of the inferior platform shifts outwards for $a_2 > 0$. Figure 1 illustrates the change in the equilibrium for two situations.[10] In the first situation the reaction function of the inferior platform, $a_2(a_1;\cdot)$, is affected by a decrease in parameter ρ in two ways: Its slope increases and it shifts upwards, resulting in $a_2^{\rho}(a_1;\cdot)$. The equilibrium point moves from F to G. In the second case, the reaction function of the inferior platform, $\widetilde{a}_2(a_1;\cdot)$, rotates around point C, with $\widetilde{a}_2^{\rho}(a_1;\cdot)$ denoting the new function.[11] The equilibrium point shifts from D to E. A decrease in ρ leads to a higher number of advertisements on both platforms.

It is the sum of two effects that determines how the equilibrium number of advertisements changes with AP technology licensing. The *direct effect* corresponds to the change in demand for advertisement slots on the inferior platform and affects only the advertising decision of the latter. The *strategic effect* relates to the fact that decisions on the number of advertisement slots are strategic complements. If one platform displays more advertisements, the other can do so as well. The two effects can be disentangled using the reaction functions:

$$\frac{\partial a_i^*(\cdot)}{\partial \rho} = \underbrace{\left.\frac{\partial a_i(a_j;\cdot)}{\partial \rho}\right|_{a_j^*(\cdot)}}_{\text{direct effect}} + \underbrace{\frac{\partial a_i(a_j;\cdot)}{\partial a_j}\frac{\partial a_j^*(\cdot)}{\partial \rho}}_{\text{strategic effect}}.$$

The inferior platform is affected directly by the AP technology licensing agreement. As its advertising space becomes more valuable, it displays more advertisements. The advertiser

[10] The two situations differ in the following way. In the situation where the equilibrium point moves from F to G, the superior platform's advantage in CP technology is not very large: $a_2(0;\cdot) > 0$ implying that $\Delta < 1/4$. In this case the inferior platform advertises even if the superior platform does not. In the case where the equilibrium point moves from D to E, the superior platform's quality advantage is larger: $a_2(0;\cdot) \leq 0$ implying that $\Delta \geq 1/4$. In this case the inferior platform does not advertise if the superior platform places sufficiently few advertisements, namely, if $a_1 \leq (4\Delta - 1)/(4\mu)$.

[11] The maximum number of advertisements the superior platform can place to drive the inferior platform out of the user market does not depend on ρ.

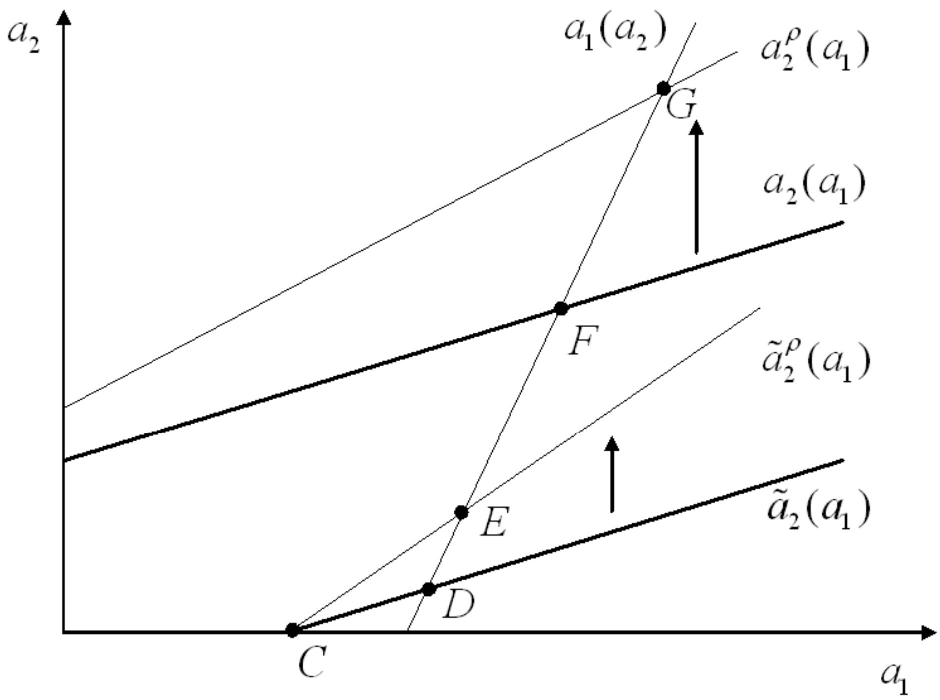

Figure 1-1: The effect of a decrease in ρ on the reaction functions of the platforms

demand at the superior platform remains unaffected by this change. The superior platform is affected only indirectly by the change in parameter ρ, through strategic effect. The strategic effect is at work at the inferior platform too and amplifies the positive direct effect. As a result, in the new equilibrium both platforms display more advertisements. Table 1 summarizes these effects.

	Direct effect	Strategic effect	Total effect
$a_1^*(\cdot)$	0	$+$	$+$
$a_2^*(\cdot)$	$+$	$+$	$+$

Table 1: The effects of a decrease in ρ on the advertising decisions of the platforms

It follows from Equations (1.8)and (1.9)that when platforms place more advertisements as parameter ρ decreases, they also set higher slot prices. Due to indirect network effects both platforms then benefit from the intensified advertiser demand at the inferior platform. It is left to note that the superior platform gains user market shares with licensing. As the inferior platform is affected both directly and through the strategic effect, it increases the number of advertisement slots more than its competitor does, losing thereby some of its users.

We now turn to the effects of licensing the CP technology. The following lemma states our results.

Lemma 3 *As the content quality of the inferior platform improves (i.e., parameter ζ increases):*

i)the superior (inferior)platform displays less (more)advertisements,

ii)the superior (inferior)platform loses (gains)user market shares,

iii)the superior (inferior)platform charges a lower (higher)price for an advertisement slot,

iv)the superior (inferior)platform makes lower (higher)profits,

v)platforms' joint profits decrease.

Proof. See Appendix.

Although the inferior platform makes higher profits with the improved CP technology, the additional profit is not sufficient to compensate the losses suffered by the superior platform. For the intuition behind this result it is useful to inspect how the licensing of CP technology alters the advertising decisions of the platforms. We can again distinguish between the *direct effect* and the *strategic effect* of the change in parameter ζ on the advertising levels:

$$\frac{\partial a_i^*(\cdot)}{\partial \zeta} = \underbrace{\left.\frac{\partial a_i(a_j;\cdot)}{\partial \zeta}\right|_{a_j^*(\cdot)}}_{\text{direct effect}} + \underbrace{\frac{\partial a_i(a_j;\cdot)}{\partial a_j}\frac{\partial a_j^*(\cdot)}{\partial \zeta}}_{\text{strategic effect}}.$$

For the licensing of CP technology, the direct effect is driven by the change in the content quality advantage of the superior platform. With ζ getting larger, the direct and strategic effects point in opposite directions at both platforms. As the content quality gap between the platforms narrows, the direct effect is positive for the inferior and negative for the superior platform. The content quality at the inferior platform increases, hence, it can place more advertisements in equilibrium without losing users. At the same time, the superior platform's advantage in content quality erodes and it has to reduce the number of advertisements to keep users from switching. In contrast, the strategic effect is negative for the inferior platform. In equilibrium the superior platform decreases its advertising level, and the strategic response of the inferior platform is to show fewer advertisements too. For the superior platform it is the other way around: As the inferior platform shows more advertisements in the new equilibrium, the superior platform displays more advertisements as well. The direct effect is stronger than the strategic effect and the inferior platform increases the number of advertisement slots in the new equilibrium while the superior platform decreases it. Table 2 summarizes these effects.

	Direct effect	Strategic effect	Total effect
$a_1^*(\cdot)$	$-$	$+$	$-$
$a_2^*(\cdot)+$		$-$	$+$

Table 2: The effects of an increase in ζ on the advertising decisions of the platforms

The changes in prices of advertisement slots can again be derived from inspecting Equations (1.8)and (1.9). Following a change in parameter ζ, equilibrium prices move in the same direction as advertising levels. Hence, the advertisement slot price rises at the inferior platform while it decreases at the superior platform. The negative effect of an increase in ζ on the superior platform's profit and the positive effect on the inferior platform's profit are then straightforward. Moreover, as the content quality improves at platform 2, it is able to attract users from platform 1 despite increasing the number of advertisements. As a result, the inferior platform increases its user market shares at the expense of the superior platform.

Using Lemmas 2 and 3 we are in position to state platforms' optimal licensing decisions.

Proposition 2 *Regardless whether transfers are allowed or not, the inferior platform gets full access to the superior AP technology, while CP technology is not licensed ($\rho^* = 0$ and $\zeta^* = \zeta_0$).*

Proof. We know from Lemma 2 that both platforms' profits increase when ρ gets smaller, hence, the full licensing of AP technology implying $\rho^* = 0$ is optimal regardless whether transfers are possible or not. We also know from Lemma 3 that both the superior platform's profit as well as joint profits decrease as ζ gets larger. This implies that no transfer exists which compensates the superior platform for its profit loss. Consequently, CP technology is not licensed, regardless whether transfers are allowed or not implying $\zeta^* = \zeta_0$. *Q.E.D.*

Our results show that while AP technology is fully licensed to the competitor, the quality of its CP technology remains unchanged. This result does not depend on the presence of transfers. Technology licensing has an interesting implication for competition policy analysis. The following corollary states our result.

Corollary 1 *Technology licensing agreement intensifies concentration at the user market: The user market shares of the superior platform increase.*

Proof. We know from Proposition 2 that both with and without transfers only AP technology is licensed, in which case $n_1^*(\cdot)$ increases as stated in Lemma 2. *Q.E.D.*

With the inferior platform improving its AP technology, the market share of the superior platform among users increases, leading to a larger concentration on the user side of the market. Our result provides an additional argument to support concerns of competition authorities about the planned advertising cooperation between *Yahoo* and *Google* in 2008. The DoJ justified its critical approach by claiming that the deal would have virtually eliminated *Yahoo* as a competitor in the advertising market (DoJ, 2008). We argue additionally that although the cooperation was aimed at advertising, it could have strengthened the already dominant position of *Google* among users.

Our results extend earlier insights on licensing incentives (Farrell and Gallini, 1988; Shepard, 1987)to a two-sided market environment. In the mentioned articles the licensor boosts demand for its products directly through technology licensing. In contrast, our model emphasizes the role of indirect network effects. By providing a better advertisement placing technology to the inferior rival, the superior platform does not directly increase demand for its own advertising space. Instead, the superior platform achieves this effect indirectly, by increasing demand for the inferior platform's advertising space. The inferior platform responds by placing more advertisements, which induces advertising-averse users to switch to the rival. The larger user audience at the superior platform, in turn, boosts demand for advertising space as advertisers prefer advertisements placed at a platform with more users.

1.3.1 Welfare Analysis

In the following we turn to the welfare effects of technology licensing. We analyze the influence of an increase in parameter ζ and a decrease in parameter ρ on advertiser and user surpluses separately and start with the advertiser side. Advertiser surplus (AS)can be derived as:

$$AS(a_1, a_2) = \sum_i \left[(1-\rho_i)n_i - p_i^a\right] a_i/2 = \sum_i a_i^2/2. \tag{1.10}$$

For user surplus, we get from Equation (1.4)that $t_1(\cdot) = 1 - t_2(\cdot)$, hence, marginal users are located symmetrically on the circle. User surplus (US)follows from Expression (1.3)as

$$\begin{aligned} US(a_1, a_2; \zeta, \mu, q, u) = \; & u + 2\int_0^{t_1(\cdot)} \left(q - \mu a_1 - [\delta_1(t)]^2\right) dt + \\ & + 2\int_{t_1(\cdot)}^{1/2} \left(\zeta q - \mu a_2 - [\delta_2(t)]^2\right) dt. \end{aligned} \tag{1.11}$$

The effects of changes in parameters ζ and ρ on advertiser and user surpluses are summarized in the following proposition.

Proposition 3 *Changes in parameters ζ and ρ have contrary effects on user and advertiser surpluses:*

i)with an increase in ζ user (advertiser)surplus increases (decreases),

ii)with a decrease in ρ user (advertiser)surplus decreases (increases).

Proof. See Appendix.

Following an increase in parameter ζ, the inferior platform expands its market shares among users as some users switch from the superior platform. In the resulting equilibrium the inferior (superior)platform advertises more (less). As users switch from the superior platform despite the fact that it places less advertisements, they enjoy a higher surplus. Users who stay with the superior platform win due to fewer advertisements. Users choosing platform 2 are also better off. Although the inferior platform advertisers more, gains from higher content quality outweigh losses from intensified advertising. As a result, users enjoy higher surplus. The effect on the advertiser surplus is straightforward from Expression (1.10). As the overall advertising intensity $(a_1^*(\cdot)+\ a_2^*(\cdot))$decreases and platforms become more symmetric in their advertising levels, advertiser surplus is reduced. In summary, an increase in the content quality of the inferior platform benefits users and effects advertisers negatively.

With a decrease in parameter ρ, the inferior platform faces higher demand for advertisement slots. As a result, both platforms show more advertisements in the new equilibrium, which affects the utility of every user negatively. At the same time, advertiser surplus increases: Advertisers benefit from intensified advertising.[12]

We now turn to the effect of technology licensing on social welfare. Due to high nonlinearity of the social welfare function we are not able to derive the socially optimal level of technology licensing. Instead, we focus on the question of whether the privately optimal extent of technology licensing (implying full licensing of AP technology and no licensing of CP technology)can be improved upon from a social welfare perspective. We address this issue by first evaluating the sign of the derivative of the social welfare function $(SW^*(\rho,\zeta,\mu,q,u))$ with respect to parameter ρ at the point $(\rho,\zeta)=(0\ ,\zeta_0)$, which corresponds to the privately optimal extent of licensing. If this derivative is positive, the privately optimal incentives to license AP technology to the full extent are socially excessive. However, the non-positive sign of the derivative does not imply that full licensing of AP technology is socially optimal. We then consider the derivative of the social welfare function evaluated at the point $\rho=0$. Using this derivative we can answer the question whether given full licensing of AP

[12] Our result contrasts with the argument made by competition authorities opposing the *Yahoo-Google* advertising agreement in 2008, which emphasized the negative effect the deal may have had on advertisers.

technology, social welfare can be improved through additional licensing of CP technology. We introduce $\Delta_0 := (1-\zeta_0)q$. The following proposition summarizes our results.

Proposition 4 *If $\mu > (1+\sqrt{7})/2$ and $\Delta_0 > \Delta_\rho$, then the privately optimal level of AP technology licensing is socially excessive, with $\Delta_\rho := (3+4\mu)(1+3\,\mu)^2 \,/\, \left[4\,(1+\mu)^2\,(3+4\mu\,(1+\mu))\right]$. If AP technology is licensed to the privately optimal extent, incentives to additionally license CP technology are insufficient compared to the socially optimal (implying $\zeta^* = 1$)if and only if*

i) $\mu \geq (1+\sqrt{7})/2$ or

ii) $\mu < (1+\sqrt{7})/2$ and $\Delta_0 < \Delta_\zeta$, with $\Delta_\zeta := (1+3\mu)^2 \,/\, [4\,(4+\mu\,(6+\mu))]$ or

iii) $\mu < (1+\sqrt{7})/2$, $\Delta_0 \geq \Delta_\zeta$ and $SW^*(0,1,\cdot) > SW^*(0,\zeta_0,\cdot)$

and they are socially optimal otherwise.

Proof. See Appendix.

If user disutility per advertisement (μ)and the asymmetry in the quality of platforms' CP technologies (Δ_0)are large enough, private incentives to license AP technology to the full extent are socially suboptimal. In this case social welfare is higher if platform 2 holds AP technology of a worse quality than the superior platform. The intuition is the following. We showed in Lemma 2 and Proposition 3 that advertisers and platforms win with ρ getting smaller. The only actors in our model who win from an increase in ρ are, therefore, users. If μ is high, users benefit strongly from a relatively low level of advertising due to a handicap of the inferior platform in AP technology. In addition, when Δ_0 is large, platforms differ significantly in their user market shares implying high user transportation costs. A positive ρ due to a less than full licensing of AP technology leads to a more symmetric allocation of users between the platforms, thus lowering transportation costs and increasing user surplus.

If the licensing of AP technology takes place to the full extent, private incentives to license CP technology can be suboptimal. If μ is high enough, the gains of users outweigh the joint losses of advertisers and platforms: Users benefit from a decrease in the overall advertising intensity following the licensing of CP technology. When μ is low, users benefit relatively little from a decrease in advertising intensity. In this case social welfare can only increase if platforms' losses are relatively low compared to user benefits. The superior platform loses less by sharing its CP technology if its initial advantage in the ability to produce content is low. Hence, social welfare increases following the licensing of CP technology if the initial asymmetry in platforms' content qualities is sufficiently small. If, however, platforms differ a lot in the quality of their CP technologies, then social welfare increases only if the condition $SW^*(0,1,\cdot) > SW^*(0,\zeta_0,\cdot)$ holds. Otherwise (if $SW^*(0,1,\cdot) \leq SW^*(0,\zeta_0,\cdot)$), given full licensing of AP technology, social welfare is maximized if CP technology is not shared.

1.4 Extensions

In this section we extend our analysis in two directions. In the first extension we analyze licensing incentives under the assumption that platforms set advertisement prices instead of choosing advertisement quantities. In the second extension we return to the assumption of a quantity game on the advertiser side, but allow platforms to charge users. The latter applies to a broader range of advertising supported media platforms including TV channels, newspapers, where users are usually charged for excess to a platform's content. To economize on notation, we omit indexing variables with respect to a particular extension.

1.4.1 Technology Licensing Under Price Setting For Advertisers

In this extension we analyze how platforms' licensing incentives change when platforms play a price game on the advertiser side compared to the quantity game analyzed above. If platforms choose prices on the advertiser side, then the choice variables become strategic substitutes: An increase in the slot price of a platform puts pressure on the competitor to reduce its price. Despite this difference, licensing incentives do not change significantly: Only AP technology is licensed. However, the incentives to license AP technology are now driven by the direct effect, and not through the strategic effect as in the quantity game. Whether licensing takes place depends on the strength of the direct effect relative to the strategic effect. The former is stronger when Δ becomes lower.

We assume that in the first stage each platform $i = \{1, 2\}$ chooses a uniform slot price p_i^a. In the second stage advertisers decide whether to place an advertisement at a particular platform, and users choose which platform to interact with. By rearranging Expression (1.2) we obtain the number of advertisements placed at platform i as a function of its audience size, n_i, and slot price, p_i^a:

$$a_i = (1 - \rho_i)n_i - p_i^a. \tag{1.12}$$

By plugging (1.12)into (1.5)and solving for n_1 and n_2 we get the user demand at each platform as a function of slot prices:

$$\begin{aligned} n_1(p_1^a, p_2^a; \rho, \zeta, \mu, q) &= \frac{1/2 + 2\Delta + 2\mu(1-\rho) + 2\,\mu(p_1^a - p_2^a)}{1 + 2\mu(2-\rho)}, \\ n_2(p_1^a, p_2^a; \rho, \zeta, \mu, q) &= \frac{1/2 - 2\Delta + 2\mu - 2\mu(p_1^a - p_2^a)}{1 + 2\mu(2-\rho)}. \end{aligned} \tag{1.13}$$

Platform i maximizes its profit

$$\pi_i(p_1^a, p_2^a; \rho, \zeta, \mu, q) = [(1 - \rho_i)n_i(p_1^a, p_2^a; \cdot) - p_i^a]\, p_i^a \tag{1.14}$$

with respect to p_i^a. The following proposition characterizes the equilibrium without licensing.

Proposition 5 *If platforms play a price game on the advertiser side, the equilibrium without licensing depends on the superior platform's advantage in CP technology. A threshold* $\overline{\Delta}_{p^a} := \left[1 + \mu(5 - 2\rho) + 4\,\mu^2(1-\rho)\right] / \left[4 + 4\mu(3 - 2\rho)\right]$ *exists, such that*

i)if $\Delta \geq \overline{\Delta}_{p^a}$, *only the superior platform is active (on both sides of the market),*

ii)if $\Delta < \overline{\Delta}_{p^a}$, *both platforms are active on both sides of the market. The superior platform charges a higher slot price, displays more advertisements and realizes higher profits than the competitor provided that it has a strict advantage in at least one of the technologies.*

Proof. See Appendix.

The equilibrium without licensing under a price game is qualitatively similar to the results derived under quantity game. The inferior platform is active on both sides of the market only if the quality advantage of the rival in CP technology is not very large ($\Delta < \overline{\Delta}_{p^a}$). The superior platform uses its technology advantage to charge a higher slot price and is able to place more advertisements. We assume that the condition $\Delta < \overline{\Delta}_{p^a}$ is satisfied under $\rho = \rho_0$ and $\Delta = \Delta_0$. In the following lemma we characterize how equilibrium variables change if a technology is licensed.

Lemma 4 *If platforms play a price game on the advertiser side of the market, technology licensing has the following effects on the equilibrium variables.*

i)If AP technology is shared, the inferior platform increases its advertisement price, places more advertisement slots and realizes higher profits. The superior platform places more advertisements, its slot price increases if $\Delta < \Delta_{p^a}$ *and (weakly)decreases otherwise, with* $\Delta_{p^a} := (1+\mu)/\left[4\left(1+3\mu\right)\right]$*. The superior platform's profit increases (weakly decreases) if* $\Delta < \Delta_\pi(\mu, \rho)$ $(\Delta \geq \Delta_\pi(\cdot))$*, with*

$$\begin{aligned}
\Delta_\pi(\cdot) &:= \ \chi(\mu,\rho)/\theta(\mu,\rho), \ \textit{where} \\
\chi(\mu,\rho) &:= 24\ \mu^5(1-\rho)^2 + 6\mu^4(1-\rho)(13-7\rho) + 6\ \mu^3(3-\rho)(5-4\rho) + \\
&\quad + \mu^2(17+\sqrt{105}-4\rho)(17-\sqrt{105}-4\rho)/4 + \mu(11-4\rho) + 1\ , \\
\theta(\mu,\rho) &:= 24\ \mu^4(1-\rho)(5-3\rho) + \ \mu^3(29+\sqrt{105}-16\rho)(29-\sqrt{105}-16\rho)/4 + \\
&\quad + 8\mu^2(3-2\rho)(5-\rho) + 4\ \mu(9-4\rho) + 4\ .
\end{aligned}$$

It holds that $\Delta_{p^a} < \Delta_\pi(\cdot)$.

ii)If CP technology is licensed, the inferior (superior)platform raises (reduces)its advertisement price, places more (less)slots and realizes higher (lower)profits.

Proof. See Appendix.

Regardless whether platforms choose advertisement quantities or prices, the equilibrium variables change exactly in the same way when the superior CP technology is licensed. The inferior platform benefits from the resulting competitive scenario, while the superior loses. The former raises its slot price and places more advertisements, and, consequently, its profit increases. We analyze in detail the effect of CP technology licensing on advertisement prices. To do so, consider the reaction functions $p_i^a(p_j^a; \rho, \zeta, \mu, q)$ $(i, j = \{1, 2\}$ and $i \neq j)$, which state the optimal advertisement price of a platform given the competitor' slot price:

$$\begin{aligned} p_1^a(p_2^a; \cdot) &= \max \left\{ \frac{1/4 + \Delta + \mu(1-\rho) - \mu p_2^a}{1 + 2\mu(1-\rho)}, 0 \right\}, \\ p_2^a(p_1^a; \cdot) &= \max \left\{ \frac{(1-\rho)[1/4 - \Delta + \mu(1 - p_1^a)]}{1 + 2\mu}, 0 \right\}. \end{aligned}$$

Advertisement prices are strategic substitutes as $\partial p_i^a(p_j^a; \cdot)/\partial p_j^a < 0$: When a platform raises the price of its advertisement slot, the other platform responds by lowering its own price. A higher advertisement price of a platform implies that it displays fewer advertisements, which in turn allows it to attract some users from the competitor. As the competitor's audience gets smaller, advertisers find that platform less attractive, which puts pressure on it to decrease the slot price. When CP technology is licensed, changes in the equilibrium slot prices are determined by the joint influence of the direct and strategic effects:

$$\frac{\partial p_i^{a*}(\rho, \zeta, \mu, q)}{\partial \zeta} = \underbrace{\left. \frac{\partial p_i^a(p_j^a; \cdot)}{\partial \zeta} \right|_{p_j^{a*}(\cdot)}}_{\text{direct effect}} + \underbrace{\frac{\partial p_i^a(p_j^a; \cdot)}{\partial p_j^a} \frac{\partial p_j^{a*}(\rho, \zeta, \mu, q)}{\partial \zeta}}_{\text{strategic effect}}.$$

Similar to the quantity game, the direct effect is positive for the inferior platform and negative for the superior platform. As the advantage of the superior platform erodes, the inferior platform can attract more users, which in turn strengthens the demand from advertisers and allows it to set a higher advertisement price. Different from the quantity game, the inferior platform benefits not only from the direct effect, but also from the strategic effect. The latter also drives the inferior platform's slot price upwards following a decrease in the competitor's equilibrium price.

We now turn to the intuition behind the result on AP technology licensing. Different from the setup where platforms decide on advertisement quantities, the direct effect is now positive for both platforms. Access to a better AP technology increases the inferior platform's advertiser demand, which is equivalent to more advertisements for any given slot prices. More advertisements at the inferior platform drive users to the competitor, which in turn increases the superior platform's slot price. The direct effect is, therefore, positive for the superior platform. Although user demand at the inferior platform decreases with licensing of AP technology, the term $(1-\rho)\, n_2(p_1^a, p_2^a; \cdot)$ in Expression (1.14)gets larger for any advertisement prices. Therefore, the direct effect at the inferior platform is also positive.

In contrast, the strategic effect is always negative for the superior platform: The increase of the competitor's equilibrium advertisement price puts a negative pressure on its own slot price. Whether the superior platform's equilibrium slot price gets larger or smaller following a decrease in ρ, depends on the relative magnitudes of the direct and strategic effects. The positive direct effect is stronger when the superior platform's advantage in CP technology is relatively small. Indeed, a decrease in ρ is equivalent to more advertisements placed by the inferior platform, this in turn may only lead to a comparatively large increase in the superior platform's user demand if Δ is small. This is why the superior platform's slot price increases only if $\Delta < \Delta_{p^a}$. The superior platform's profit may, however, increase with licensing of AP technology even if its slot price decreases, which explains the inequality $\Delta_{p^a} < \Delta_{\pi}(\cdot)$ in Lemma 4. This result is different from the one obtained under the quantity game where, following AP technology sharing, platforms' profits move always in the same direction with the number of slots and slot prices. The difference is that under the price game both platforms are influenced directly by AP technology licensing. Being equivalent to an increase of the user demand at the superior platform, the direct effect allows the superior platform's profit to grow even if its slot price decreases as the superior platform always places more advertisements when ρ decreases.

Comparing licensing incentives under the quantity game and the price game we can conclude that the main mechanism which drives the superior platform's incentives to license its AP technology is same in the two setups and works through increasing user demand due to indirect network effects. The difference is, however, that this increase is due to the strategic effect under the quantity game and due to the direct effect under the price game.

We now turn to optimal licensing decisions and focus on the case where transfers are not feasible. We later discuss how the results would change if transfers were possible. It is useful to consider the function $\Delta_\pi(\cdot)$, with $sign\left\{\partial\pi_i^*(\cdot)/\partial\rho\right\} = -sign\left\{\Delta_\pi(\cdot) - \Delta\right\}$, as follows from Lemma 4. It holds that $\partial\Delta_\pi(\cdot)/\partial\rho < 0$, therefore, $\Delta_\pi(\cdot)$ increases when ρ gets smaller. If ρ_0 and Δ_0 are initially small, which is the case when platforms hold similar technologies, then the difference $\Delta_\pi(\cdot,\rho_0) - \Delta_0$ is likely to be positive. In the latter case the superior platform shares its AP technology with the rival to the full extent as its profit increases with a decrease in ρ. If, however, the initial asymmetry in platforms' technologies is large enough, then the difference $\Delta_\pi(\cdot,\rho_0) - \Delta_0$ is likely to be negative. In that case the licensing of AP technology may either take place or not as stated in the following proposition.[13]

Proposition 6 *Assume that platforms play a price game on the advertiser side of the market and transfers are not admissible. In this case CP technology is not licensed ($\zeta^* = \zeta_0$). Whether AP technology is licensed depends on the initial asymmetries in technological capabilities of the platforms in the following manner.*

i)If the initial asymmetries in technological capabilities are small (i.e., $\Delta_0 < \Delta_\pi(\cdot,\rho_0) < \Delta_\pi(\cdot,0)$), AP technology is fully licensed ($\rho^* = 0$).

ii)If the initial asymmetries in technological capabilities are moderate (i.e., $\Delta_\pi(\cdot,\rho_0) < \Delta_0 < \Delta_\pi(\cdot,0)$), AP technology is fully licensed ($\rho^* = 0$)if $\pi_1^*(\rho = 0,\cdot) > \pi_1^*(\rho = \rho_0,\cdot)$, while it is not licensed ($\rho^* = \rho_0$)if $\pi_1^*(\rho = 0,\cdot) \leq \pi_1^*(\rho = \rho_0,\cdot)$.

iii)If the initial asymmetries in technological capabilities are large (i.e., $\Delta_\pi(\cdot,\rho_0) < \Delta_\pi(\cdot,0) < \Delta_0$), AP technology is not licensed ($\rho^* = \rho_0$).

Proof. See Appendix.

Regardless whether platforms decide on advertisement quantities or prices, the superior platform does not share its CP technology with the rival. However, different from the quantity game, the superior platform only licenses its AP technology if the initial asymmetries in platforms' technologies are not too large. Following the licensing of AP technology, the superior platform's profit is influenced positively by the direct effect and negatively by

[13] We omit from the analysis the case where $\Delta_0 = \Delta_\pi(\cdot,\rho_0)$.

the strategic effect. Initial asymmetries in platforms' technologies determine the relative strengths of these effects. For instance, the condition $\Delta_\pi(\cdot, \rho_0) < \Delta_\pi(\cdot, 0) < \Delta_0$ is likely to hold if both Δ_0 and ρ_0 are large as $\Delta_\pi(\cdot)$ decreases in ρ, implying a sufficiently large technological advantage of the superior platform. In the latter case the positive direct effect of a decrease in ρ is too small to compensate the negative strategic effect.

It can be shown that CP technology is not licensed in the presence of transfers either: The losses of the superior platform are always larger than the gains of the inferior platform. The presence of transfers, however, is likely to strengthen the incentives for AP technology licensing, such that the latter could be also possible under larger technological asymmetries.

Comparing licensing incentives under quantity and price settings on the advertiser side, we conclude that our main results are valid in both setups: While CP technology is not shared even with transfers, AP technology can be licensed even without transfers. The price game, however, brings an additional dimension into the licensing incentives problem. It points out that besides the difference between CP and AP technologies, the initial asymmetries in technological capabilities of the platforms also matter. If technological capabilities are sufficiently asymmetric, no licensing takes place. Our results imply that in reality we are more likely to observe differences in the quality of advertisement placing technologies of competing platforms in industries, in which these firms set slot prices.

1.4.2 Technology Licensing With User Prices

In this extension we return to the assumption that platforms decide on advertisement quantities, but allow for the possibility that users are charged for access. We focus on the question of how licensing incentives change when platforms can rely on both groups of customers as sources of revenues. We show that in the presence of user prices licensing incentives vanish. When AP technology is licensed, the superior platform benefits from the positive strategic effect. However, different from the setup without user prices, the direct effect is now negative for the superior platform. When ρ gets smaller at the inferior platform, it can charge a higher slot price. The price increase is larger the more users the platform serves. The latter effect induces the inferior platform to attract users away from the superior platform through setting a lower user price, which gives rise to the negative direct effect.

The game proceeds as follows: In the first stage, each platform $i = \{1, 2\}$ decides on the number of advertisements, a_i, and the user access price, p_i^u. In the second stage, advertisers decide whether to place an advertisement at a particular platform, and users choose which platform to interact with. The utility of a user with address t takes the form

$$U_i^t = \begin{cases} u + q - [\delta_1(t)]^2 - \mu a_1 - p_1^u, & \text{if} \quad i = 1 \\ u + \zeta q - [\delta_2(t)]^2 - \mu a_2 - p_2^u, & \text{if} \quad i = 2, \end{cases} \tag{1.15}$$

with $\delta_1(t)= \min\{t, 1-t\}$ and $\delta_2(t)= |t - 1/2|$.

Using (1.15)we obtain user demand at each platform:

$$n_1(a_1, p_1^u, a_2, p_2^u; \zeta, \mu, q)= 1/2 + 2\left[\Delta - (p_1^u - p_2^u) - \mu(a_1 - a_2)\right], \tag{1.16}$$

$$n_2(a_1, p_1^u, a_2, p_2^u; \zeta, \mu, q)= 1/2 - 2\left[\Delta - (p_1^u - p_2^u) - \mu(a_1 - a_2)\right]. \tag{1.17}$$

Given the demand function $n_i(\cdot)$, platform i maximizes its profit

$$\pi_i(a_1, p_1^u, a_2, p_2^u; \rho_i, \zeta, \mu, q)= \left[(1 - \rho_i)n_i(\cdot) - a_i\right] a_i + p_i^u n_i(\cdot)$$

by choosing a_i and p_i^u. In the following proposition we characterize the equilibrium without technology licensing.

Proposition 7 *If platforms can charge users, the equilibrium absent licensing depends on the user disutility per advertisement (μ)and the CP technology advantage of the superior platform (Δ)in the following way.*

i)If $\mu \geq 1$ and $\Delta \geq 3/4$, only the superior platform serves users. It displays no advertisements and relies only on revenues from users.

ii)If $\mu \geq 1$ and $\Delta < 3/4$, both platforms serve users, place no advertisements and rely only on revenues from users.

iii)If $1 - \rho \leq \mu < 1$ and $\Delta \geq \overline{\Delta}_{p^u}$, only the superior platform serves users, with $\overline{\Delta}_{p^u} := [1 + 2\mu(2 - \mu)]/4$. It charges users and places advertisements.

iv)If $1-\rho \leq \mu < 1$ *and* $\Delta < \overline{\Delta}_{p^u}$*, both platforms serve users. The superior platform charges both sides of the market, while the inferior platform relies on revenues from users only.*

v)If $\mu < 1-\rho$ *and* $\Delta \geq \overline{\Delta}_{p^u}$*, only the superior platform serves users. It charges users and places advertisements.*

vi)If $\mu < 1-\rho$ *and* $\Delta < \overline{\Delta}_{p^u}$*, both platforms serve users and charge both sides of the market.*

Proof. See Appendix.

If platforms can charge both users and advertisers, depending on the parameters, two different combinations of equilibrium sources of revenues arise. Platforms either rely on revenues from users only or they charge both sides. If users are strongly averse to advertisements (i.e., μ is high), platforms refrain from placing advertisements and rely solely on revenues from users. For each platform there is a threshold for user disutility per advertisement, below which a platform places advertisements. This threshold is larger for the superior platform: It displays ads if $\mu > 1$, while the inferior platform advertisers if $\mu > 1-\rho$. Due to its advantage in AP technology, placing advertisements is more profitable for the superior platform and it is ready to sacrifice some users in order to generate revenues from advertisers. For the same reason, the critical value of disutility per advertisement which makes it profitable to place advertisements for the inferior platform $(1-\rho)$is lower, the larger its handicap in AP technology becomes. While parameters μ and ρ are decisive for platforms' decisions to place advertisements, parameter Δ determines whether the inferior platform is active at the market. If Δ is sufficiently large, the inferior platform becomes unattractive to users even if it places no advertisements. In the following we restrict attention to the case where the platforms are active on both sides of the market, such that $\mu < 1-\rho_0$ and $\Delta_0 < \overline{\Delta}_{p^u}$. The following lemma characterizes the effects of technology licensing on equilibrium variables.

Lemma 5 *Assume that platforms charge users and advertisers. Technology licensing has* *the following effects:*

i)Following the licensing of AP technology the inferior platform reduces (increases)its user (slot)price and displays more advertisements. The superior platform charges a lower price to both users and advertisers, it places less advertisements and its market share among users decreases.

ii)Following the licensing of CP technology the inferior (superior)platform charges both users and advertisers a higher (lower)price, it places more (less)advertisements and its market share among users increases (decreases).

In both cases the inferior (superior)platform's profit increases (decreases)and joint profits decrease.

Proof. See Appendix.

Different from the baseline model where platforms receive revenues from advertisers only, with user prices the incentives to share AP technology vanish regardless whether transfers are allowed or not. To see the intuition behind this result it is again helpful to consider platforms' reaction functions, $a_1(a_2;\rho,\zeta,\mu,q)$ and $a_2(a_1;\rho,\zeta,\mu,q)$:

$$\begin{aligned} a_1(a_2;\cdot) &= \frac{(1-\mu)}{[1+\mu(2-\mu)]}\left[\frac{1}{4}+\Delta+\frac{a_2(\mu+\rho)(2-\rho-\mu)}{1-\rho-\mu}\right], \\ a_2(a_1;\cdot) &= \max\left\{\frac{(1-\rho-\mu)}{\left[2-(1-\rho-\mu)^2\right]}\left[\frac{1}{4}-\Delta+\frac{a_1\mu(2-\mu)}{1-\mu}\right],0\right\}. \end{aligned}$$

As in the baseline model, decisions on the number of advertisement slots are strategic complements: $a_i(a_j;\cdot)$ is an increasing function of a_j (with $i,j=\{1,2\}$ and $i\neq j$). However, both $a_1(a_2;\cdot)$ and $a_2(a_1;\cdot)$ are now functions of parameter ρ, the inferior platform's disadvantage in AP technology. The decrease in ρ due to the licensing of AP technology affects then both platforms directly. The direct effect is negative for the superior platform and positive for the inferior platform: $\partial\left[(\mu+\rho)(2-\rho-\mu)/\left(1-\rho-\mu\right)\right]/\partial\rho>0$ implies that as ρ gets smaller for any given number of slots at the competitor, the superior platform places fewer advertisements. Similarly, $\partial\left((1-\rho-\mu)/\left[2-(1-\rho-\mu)^2\right]\right)/\partial\rho<0$ implies the opposite relation for the inferior platform. The superior platform, however, benefits from a positive strategic effect due to the increased equilibrium number of advertisements at the inferior platform, $a_2^*(\cdot)$. As the direct effect is stronger than the strategic effect, $a_1^*(\cdot)$ decreases.

Why does the direct effect appear for the superior platform following AP technology licensing when platforms charge users? Each platform has now two sources of revenues: Users and advertisers. Depending on user disutility per advertisement (μ)and the efficiency of its AP technology $(1-\rho_i)$, platform i balances its revenues from the two sides of the market. The superior platform chooses a_1^* and p_1^{u*} such that $p_1^{u*} = a_1^*\mu/(1-\mu)$, while the inferior platform chooses a_2^* and p_2^{u*} such that $p_2^{u*} = a_2^*\left[1-(1-\rho)(1-\rho-\mu)\right]/(1-\rho-\mu)$. Since for any given a_2 we have $\partial\left(\left[1-(1-\rho)(1-\rho-\mu)\right]/(1-\rho-\mu)\right)/\partial\rho > 0$, the inferior platform charges a lower user price when its AP technology improves. With a decrease in ρ, for any given a_2 the inferior platform can charge a higher price for an advertisement slot, while this increase is larger, the more users are served by the platform. This creates incentives for the inferior platform to attract more users by charging a lower user price. A lower user price at the inferior platform in turn reduces user demand at the superior platform giving rise to the negative direct effect.

As all the other equilibrium variables at platform 1 move in the same direction with the number of advertisement slots, the superior platform reduces its prices for both advertisers and users and its market share among users becomes smaller, leading to lower profits. Although the inferior platform reduces the user price, the higher advertisement price, increased number of slots and larger user market shares allow it to realize a higher profit.

The effects of licensing of CP technology on equilibrium variables are the same as in the baseline model, except for an additional effect related to the changes in user prices. While the superior platform has to reduce its user price, the inferior platform charges users a higher price when the quality of its content improves. The following proposition characterizes the licensing incentives of the platforms when users are charged for access.

Proposition 8 *Assume that platforms charge both users and advertisers. Regardless* *whether transfers are allowed or not, none of the technologies is licensed* ($\rho^* = \rho_0$ *and* $\zeta^* = \zeta_0$*).*

Proof. As $\partial\pi_1^*(\cdot)/\partial\rho > 0$ and $\partial\pi_1^*(\cdot)/\partial\zeta < 0$, there is no licensing without transfers. As $\sum_i \partial\pi_i^*(\cdot)/\partial\rho > 0$ and $\sum_i \partial\pi_i^*(\cdot)/\partial\zeta < 0$, there is no licensing in the presence of transfers either. *Q.E.D.*

Our analysis shows that the incentives of a superior media platform to license its AP technology to the inferior rival depend strongly on the sources of revenues available to the platforms. If platforms cannot charge users, the superior platform licenses its AP technology to the full extent. However, the opportunity of user access charges makes licensing incentives vanish. Our results imply that in media industries where advertising supported platforms rely on both sources of revenues (user payments and advertising receipts)the asymmetries in platforms' technological capabilities will persist more than in those industries where platforms charge only advertisers.

1.5 Discussion

In this section we discuss the implications of some of our modelling assumptions.

Inelastic user demand: We assume throughout the article that user demand is inelastic. In this case licensing has only a business-stealing effect, any potential market expansion (or in case of increased advertising levels, market contraction)effects are ruled out. Allowing user demand to be elastic would most likely weaken the incentives to license AP technology. The fact that some users may refrain from visiting any of the platforms in response to intensified advertising would weaken incentives to advertise more in the new equilibrium. This, in turn, would make licensing of both AP and CP technologies less attractive. As licensing of CP technology does not take place under the inelastic demand, the results regarding CP technology would not change. Our assumption of inelastic user demand applies well to media markets in which changes in advertising levels are likely to have only a moderate influence on the size of the user market. This is the case when users derive a relatively high basic utility from consuming platforms' products. In many media markets, such as search engines, radio or television, it seems realistic that only a relatively small fraction of consumers would decide to completely give up using these media due to excessive advertising. Advertising intensity is likely to be mainly decisive for consumers' choices which platform to use. In most media industries the business-stealing effect of platforms' advertising decisions seems to be more important than the market expansion (contraction)effect.

Single-homing: The assumption that platforms compete for users is crucial for our results. The incentives to license a technology in our model are determined by users switching from one platform to the other depending on the platforms' relative attractiveness. Single-homing seems to be a realistic assumption for media markets where consumers pay an access price. There is also evidence that in the case of free platforms such as internet search engines where users can costlessly multi-home, many users tend to interact with only one platform.[14]

One platform is superior in both technologies: We derived our results under the assumption that one of the platforms is superior in both technologies. We could instead assume that one platform has an advantage in AP technology, while the other platform is superior in CP technology.[15] We show that there is one difference in the effects of technology licensing in this scenario compared to our scenario: Joint profits may increase following the licensing of CP technology. This is the case when platforms hold AP technologies of strictly different qualities and the asymmetry in CP technologies is not very large. Redistribution of users between the platforms following CP technology licensing (from a platform with an inferior AP technology to a platform with a superior AP technology)may increase joint profits as for any given user market share the platform with a superior AP technology is able to charge a higher slot price than the competitor. In the presence of transfers it can be in the

[14] Evans (2008)provides evidence of user single-homing in the internet search market.

[15] A formal analysis of this scenario is available from the authors upon request. We thank an anonymous referee for urging us to think along these lines.

interest of a platform with an inferior AP technology to let the competitor 'use' the audience in a more productive way through licensing its advanced CP technology, as a platform with a better AP technology can convert each user into higher revenues from advertisers. The licensing of CP technology in this case serves as a device to redistribute users to the rival in exchange for a transfer. Furthermore, we show that the losses of the licensor of CP technology are proportional to Δ and they are small when platforms' CP technologies are similar. This creates a potential for CP technology licensing in the presence of transfers if platforms do not differ a lot in content quality. CP technology licensing, however, does not take place in equilibrium. Following AP technology licensing (which always takes place), platforms hold AP technologies of the same quality, such that the redistribution of users between the platforms cannot further increase joint profits.

Expectations: We assumed that advertisers are able to correctly predict platforms' user market shares under any announced advertising levels. Users expect the number of slots at each platform to correspond to the announced one (if under this number the slot price is non-negative). We could alternatively assume that before platforms announce advertising levels, advertisers form expectations about their user market shares such that these expectations are not influenced by the announcements. This assumption would imply that platforms have strong reputations for holding particular market shares among users. For instance, in the case of search engines, *Google* is known to be the dominant firm in many markets. Our results on licensing incentives are robust to an alternative formulation of advertisers' expectations: While AP technology is licensed without transfers, CP technology is not licensed even with transfers.[16]

[16] A formal analysis of licensing incentives under the assumption that advertisers form expectations about user market shares before platforms' announcements is available from the authors on request. We thank an anonymous referee for urging us to think along these lines.

1.6 Conclusion

We develop a parsimonious model with two horizontally and vertically differentiated advertising supported media platforms which differ in their technologies to produce content (e.g., TV programs or organic search results)and place advertisements. Our main purpose is to derive conditions under which a platform endowed with a superior technology licences its knowledge to an inferior rival. We highlight the role of two technologies on licensing incentives, which are characteristic for most media platforms. First, the content producing technology, responsible for the utility users draw from consuming content on a platform. Second, the advertisement placing technology, determining the ability of a platform to show advertisements resulting in a high probability of users' purchase of the advertised product. We show that the superior platform licenses its technology to place advertisements, but not its technology to create content. Our explanation is based purely on indirect network effects: By improving the competitor on the advertiser side of the market through licensing its AP technology, the superior platform enhances its own user demand. However, when platforms rely on revenues from users in addition to their advertising receipts, licensing incentives vanish. Our model provides a non-cooperative rationale for technology-based agreements between competing platforms, such as the planned and abandoned *Yahoo-Google* advertising deal in 2008.

Empirical verification of the predictions of our model could be a potentially fruitful avenue for further research, and the market for internet search engines is a good candidate for such an analysis. Data on search engines usage as well as advertising levels for a long period of time is publicly available from sources such as *Comscore* or *Hitwise.* Several cooperation agreements between various search engine operators took place in the recent past (or were planed). The empirical analysis of the effects of these agreements is likely to attract interest from economists as well as policy makers.

Our article provides a tool for competition authorities to analyze the competitive effects of technology-based cooperations in media industries and their welfare implications. Another track for further research could be to extend our model to include more than two platforms. With three active platforms, the model would allow to analyze how competition among three asymmetric search engines is affected if two of them - perhaps the smaller ones - enter into a technology-based cooperation. Such cooperation agreement was recently struck between search engine operators *Yahoo* and *Microsoft* and was approved by competition authorities worldwide. The model could be calibrated based on the operators' real user market shares and advertising levels to be then applied to quantify the competitive and welfare effects of an agreement.

Appendix

Proof of Lemma 1. We first derive the equilibrium advertisement levels, $a_i^*(\rho, \zeta, \mu, q)$, $i = \{1, 2\}$. Solving the FOCs of the platforms with respect to a_1 and a_2 simultaneously yields $a_1^*(\cdot)$ and $a_2^*(\cdot)$ as

$$\begin{aligned} a_1^*(\cdot) &= \frac{1 + 3\mu(1-\rho) + 4\Delta\left[1 + \mu(1-\rho)\right]}{4[3\mu^2(1-\rho) + 2\,\mu(2-\rho) + 1]} \\ a_2^*(\cdot) &= \frac{(1-\rho)(1+\mu)(\overline{\Delta} - \Delta)}{3\mu^2(1-\rho) + 2\,\mu(2-\rho) + 1}. \end{aligned} \tag{1.18}$$

Given the restriction $0 \leq \rho < 1$, it holds that $a_i^*(\cdot) > 0$ if and only if $0 \leq \Delta < \overline{\Delta}$. The SOCs with respect to a_1 and a_2 are fulfilled as

$$\left.\frac{\partial^2 \pi_1(a_1, a_2; \zeta, \mu, q)}{\partial (a_1)^2}\right|_{a_1^*(\cdot), a_2^*(\cdot)} = -2 - 4\mu < 0,$$

$$\left.\frac{\partial^2 \pi_2(a_1, a_2; \rho, \zeta, \mu, q)}{\partial (a_2)^2}\right|_{a_1^*(\cdot), a_2^*(\cdot)} = -2 - 4\mu(1-\rho) < 0.$$

It follows that both platforms place advertisements in equilibrium if $0 \leq \Delta < \overline{\Delta}$. Using $a_1^*(\cdot)$ and $a_2^*(\cdot)$ we obtain the equilibrium advertisement prices, $p_i^{a*}(\rho, \zeta, \mu, q)$. The difference between the prices can be written as

$$\begin{aligned} & p_1^{a*}(\cdot) - p_2^{a*}(\cdot) = \\ & \quad = \frac{\rho[1 + 2\mu(1 + (3\mu+1)(1-\rho))] + 4\Delta[2 - \rho + 2\mu(1 + (1+\mu)(1-\rho)(2-\rho))]}{4\left[3\mu^2(1-\rho) + 2\,\mu(2-\rho) + 1\right]}. \end{aligned}$$

It follows that $p_1^{a*}(\cdot) \geq p_2^{a*}(\cdot)$, holding with equality if $\Delta = \rho = 0$. The equilibrium price of platform 2 is

$$p_2^{a*}(\cdot) = \frac{\left[1 + 2\mu(1-\rho)\right](1-\rho)(1+\mu)(\overline{\Delta} - \Delta)}{3\mu^2(1-\rho) + 2\,\mu(2-\rho) + 1}.$$

Given $0 \leq \rho < 1$, $p_2^{a*}(\cdot)$ is positive if and only if $0 \leq \Delta < \overline{\Delta}$. As $p_2^{a*}(\cdot) > 0$ and $p_1^{a*}(\cdot) \geq p_2^{a*}(\cdot)$, it must hold that $p_1^{a*}(\cdot) > 0$, hence, both platforms set positive prices for advertisements provided that $0 \leq \Delta < \overline{\Delta}$. As $a_i^*(\cdot) > 0$ and $p_i^{a*}(\cdot) > 0$, both platforms realize positive profits. Turning to the market shares of the platforms among users, we can write platform i's FOC as

$$(1-\rho_i)n_i^*(\rho,\zeta,\mu,q)= \\ =\left[2-(1-\rho_i)\frac{\partial n_i(a_1,a_2;\zeta,\mu,q)}{\partial a_i}\right]a_i^*(\cdot)= 2\left[1+\ \mu(1-\rho_i)\right]a_i^*(\cdot),$$

which implies that given $0\leq\rho_i<1$ every platform serves some users when it places a positive number of advertisements. *Q.E.D.*

Proof of Proposition 1. i)We shoved in the proof of Lemma 1 that $p_1^{a*}(\cdot)\geq p_2^{a*}(\cdot)$, holding with equality only if $\Delta=\rho=0$. Using equations in (1.18)we can write the difference between $a_1^*(\cdot)$ and $a_2^*(\cdot)$ as

$$a_1^*(\cdot)-a_2^*(\cdot)=\ \frac{\rho+4\Delta\left[2\mu(1-\rho)+2\ -\rho\right]}{4\left[3\mu^2(1-\rho)+2\ \mu(2-\rho)+1\right]},$$

which is non-negative, and equals zero if $\Delta=\rho=0$. The superior platform having a strict advantage in at least one technology places more advertisements, charges a higher price for them and realizes higher profits.

ii)We turn to the equilibrium user market shares. From Expression (1.2)we get $n_1^*(\cdot)=p_1^{a*}(\cdot)+a_1^*(\cdot)$ and $n_2^*(\cdot)=\left[\ p_2^{a*}(\cdot)+\ a_2^*(\cdot)\right]/(1-\rho)$. Both platforms serve users as $p_i^{a*}(\cdot),a_i^*(\cdot)>0$, $i=\{1,2\}$. Comparing equilibrium user market shares we get

$$n_1^*(\cdot)-n_2^*(\cdot)=\ \frac{4(1+\mu)[1+\ \mu(1-\rho)]\,(\Delta-\underline{\Delta})}{3\mu^2(1-\rho)+2\ \mu(2-\rho)+1},$$

which is positive if $\Delta>\underline{\Delta}$, negative if $0\leq\Delta<\underline{\Delta}$ and zero when $\Delta=\underline{\Delta}$. The superior platform has a larger market share among users only if its advantage in content quality is large enough. *Q.E.D.*

Proof of Lemma 2. *i)* We start with the effect of a change in ρ on the number of advertisements displayed in equilibrium, $a_i^*(\cdot)$, $i=\{1,2\}$. Taking derivative of the expressions in (1.18)with respect to ρ we get

$$\frac{\partial a_1^*(\cdot)}{\partial\rho}=-\frac{\mu(1+\mu)(\overline{\Delta}-\Delta)}{\left[3\mu^2(1-\rho)+2\ \mu(2-\rho)+1\right]^{\,2}},\tag{1.19a}$$

$$\frac{\partial a_2^*(\cdot)}{\partial\rho}=-\frac{(1+\mu)(1+2\mu)(\overline{\Delta}-\Delta)}{\left[3\mu^2(1-\rho)+2\ \mu(2-\rho)+1\right]^{\,2}}.\tag{1.19b}$$

Both derivatives are negative for $0 \leq \Delta < \overline{\Delta}$. Thus, if ρ decreases, both platforms show more advertisements.

ii) We proceed with the effect of ρ on the equilibrium market shares, $n_i^*(\cdot)= \ n_i(a_1^*(\cdot), a_2^*(\cdot); \cdot)$, by inspecting the derivative $\partial n_i(a_1^*(\cdot), a_2^*(\cdot); \cdot)/\partial\rho$, with $i, j \in \{1, 2\}$, $i \neq j$:

$$\frac{\partial n_i(a_1^*(\cdot), a_2^*(\cdot); \cdot)}{\partial \rho} = \frac{\partial n_i(a_1, a_2; \cdot)}{\partial a_i}\frac{\partial a_i^*(\cdot)}{\partial \rho} + \frac{\partial n_i(a_1, a_2; \cdot)}{\partial a_j}\frac{\partial a_j^*(\cdot)}{\partial \rho}. \tag{1.20}$$

From (1.5)we have $\partial n_i(a_1, a_2; \cdot)/\partial a_i = -\partial n_i(a_1, a_2; \cdot)/\partial a_j < 0$. By rearranging Expression (1.20)we get

$$\frac{\partial n_i(a_1^*(\cdot), a_2^*(\cdot); \cdot)}{\partial \rho} = \frac{\partial n_i(a_1, a_2; \cdot)}{\partial a_j}\left(\frac{\partial a_j^*(\cdot)}{\partial \rho} - \frac{\partial a_i^*(\cdot)}{\partial \rho}\right). \tag{1.21}$$

Note that $\partial n_1^*(\cdot)/\partial\rho = - \ \partial n_2^*(\cdot)/\partial\rho$. We evaluate the sign of the derivative $\partial(a_2^*(\cdot) - a_1^*(\cdot))/\partial\rho$ by subtracting Expression (1.19a)from Expression (1.19b)to get

$$\frac{\partial a_2^*(\cdot)}{\partial \rho} - \frac{\partial a_1^*(\cdot)}{\partial \rho} = -\frac{(1+\mu)^2(\overline{\Delta} - \Delta)}{[3\mu^2(1-\rho) + 2\,\mu(2-\rho) + 1]^{\ 2}}, \tag{1.22}$$

which is negative for $0 \leq \Delta < \overline{\Delta}$. It follows that $\partial n_1^*(\cdot)/\partial\rho < 0$ and $\partial n_2^*(\cdot)/\partial\rho > 0$.

iii) We turn to the effect of a change in ρ on the prices of advertisement slots and inspect $dp_1^a(n_1(a_1^*(\cdot), a_2^*(\cdot); \cdot), a_1^*(\cdot))/d\rho$ and
$dp_2^a(n_2(a_1^*(\cdot), a_2^*(\cdot); \cdot), a_2^*(\cdot); \rho)/d\rho$. Using Expression (1.21)these derivatives can be rearranged as

$$\frac{dp_1^a(n_1(a_1^*(\cdot), a_2^*(\cdot); \cdot), a_1^*(\cdot))}{d\rho} = \frac{\partial n_1(a_1, a_2; \cdot)}{\partial a_2}\left(\frac{\partial a_2^*(\cdot)}{\partial \rho} - \frac{\partial a_1^*(\cdot)}{\partial \rho}\right) - \frac{\partial a_1^*(\cdot)}{\partial \rho} \tag{1.23}$$

and

$$\begin{aligned} &\frac{dp_2^a(n_2(a_1^*(\cdot), a_2^*(\cdot); \cdot), a_2^*(\cdot), \rho)}{d\rho} \\ &= (1-\rho)\frac{\partial n_2(a_1, a_2; \cdot)}{\partial a_1}\left(\frac{\partial a_1^*(\cdot)}{\partial \rho} - \frac{\partial a_2^*(\cdot)}{\partial \rho}\right) - n_2^*(\cdot) - \frac{\partial a_2^*(\cdot)}{\partial \rho}. \end{aligned} \tag{1.24}$$

Taking derivatives of the expressions in (1.5)with respect to a_1 and a_2 yields

$$\frac{\partial n_i(a_1, a_2; \cdot)}{\partial a_j} = 2\mu. \tag{1.25}$$

We can now plug Expressions (1.25), (1.22)and (1.19a)into Expression (1.23)to get

$$\frac{\partial p_1^{a*}(\cdot)}{\partial \rho} = -\frac{\mu(1+2\mu)(1+\mu)(\overline{\Delta} - \Delta)}{[3\mu^2(1-\rho) + 2\,\mu(2-\rho) + 1]^{\ 2}},$$

which is negative for $0 \leq \Delta < \overline{\Delta}$. Plugging Expressions (1.25), (1.22), (1.19b)and $n_2^*(\cdot)$ into Expression (1.24)yields

$$\frac{\partial p_2^{a*}(\cdot)}{\partial \rho} = -\frac{[1 + 2\mu\,(1 + \mu\,(1 - \rho))(1 + (3\,\mu + 2)(1 - \rho))]\,(1 + \mu)(\overline{\Delta} - \Delta)}{[3\mu^2(1 - \rho) + 2\,\mu(2 - \rho) + 1]^{\,2}}.$$

This derivative is negative for $0 \leq \Delta < \overline{\Delta}$.

iv) Finally, to analyze the influence of a change in ρ on the platforms' profits, we inspect the derivative $\partial\pi(p_i^{a*}(\cdot), a_i^*(\cdot))/\partial\rho$:

$$\frac{\partial\pi(p_i^{a*}(\cdot), a_i^*(\cdot))}{\partial\rho} = \frac{\partial p_i^{a*}(\cdot)}{\partial\rho} a_i^*(\cdot) + \; p_i^{a*}(\cdot)\frac{\partial a_i^*(\cdot)}{\partial\rho}.$$

We know from *i)* and *iii)* that the derivatives $\partial p_i^{a*}(\cdot)/\partial\rho$ and $\partial a_i^*(\cdot)/\partial\rho$ are negative and $a_i^*(\cdot), p_i^{a*}(\cdot) > 0$ if $0 \leq \Delta < \overline{\Delta}$, hence, $\partial\pi_i^*(\rho, \zeta, \mu, q)/\partial\rho < 0$ and the profits of both platforms increase with a decrease in parameter ρ. Note that $\partial p_i^{a*}(\cdot)/\partial\rho < 0$, $\partial a_i^*(\cdot)/\partial\rho < 0$ and $a_i^*(\cdot), p_i^{a*}(\cdot) > 0$ hold for any $0 \leq \rho < 1$ and $0 \leq \Delta < \overline{\Delta}$. *Q.E.D.*

Proof of Lemma 3. Note first that if $(1 - \zeta_0)\,q < \overline{\Delta}$, then following an increase in ζ the condition $\Delta < \overline{\Delta}$ is again fulfilled. *i)* We start with the effect of a change in ζ on the number of advertisements displayed in equilibrium by taking derivatives of $a_1^*(\cdot)$ and $a_2^*(\cdot)$ with respect to ζ:

$$\begin{aligned} \frac{\partial a_1^*(\cdot)}{\partial\zeta} &= -\frac{1 + \mu(1 - \rho)}{3\mu^2(1 - \rho) + 2\,\mu(2 - \rho) + 1} q < 0, \\ \frac{\partial a_2^*(\cdot)}{\partial\zeta} &= \frac{(1 - \rho)(1 + \mu)}{3\mu^2(1 - \rho) + 2\,\mu(2 - \rho) + 1} q > 0. \end{aligned} \tag{1.26}$$

The superior (inferior)platform displays less (more)advertisements in equilibrium with an increase in ζ, which holds for any $0 \leq \rho < 1$ and $0 \leq \Delta < \overline{\Delta}$.

ii) Turning to the effect of ζ on the equilibrium market shares, $n_i^*(\cdot)$, we inspect the derivative $dn_i(a_1^*(\cdot), a_2^*(\cdot); \cdot)/d\zeta$. With
$\partial n_i(a_1, a_2; \cdot)/\partial a_i = -\partial n_i(a_1, a_2; \cdot)/\partial a_j$ holding, we have

$$\begin{aligned} \frac{dn_i(a_1^*(\cdot), a_2^*(\cdot); \cdot)}{d\zeta} &= \frac{\partial n_i(a_1, a_2; \cdot)}{\partial a_i}\frac{\partial a_i^*(\cdot)}{\partial\zeta} + \frac{\partial n_i(a_1, a_2; \cdot)}{\partial a_j}\frac{\partial a_j^*(\cdot)}{\partial\zeta} + \frac{\partial n_i(a_1, a_2; \cdot)}{\partial\zeta} \\ &= \frac{\partial n_i(a_1, a_2; \cdot)}{\partial a_j}\left(\frac{\partial a_j^*(\cdot)}{\partial\zeta} - \frac{\partial a_i^*(\cdot)}{\partial\zeta}\right) + \frac{\partial n_i(a_1, a_2; \cdot)}{\partial\zeta}. \end{aligned} \tag{1.27}$$

Note that $dn_1(a_1^*(\cdot), a_2^*(\cdot); \cdot)/d\zeta = -dn_2(a_1^*(\cdot), a_2^*(\cdot); \cdot)/d\zeta$ follows from the covered user market assumption. We focus on the sign of $dn_1(a_1^*(\cdot), a_2^*(\cdot); \cdot)/d\zeta$. From (1.26)we obtain

$$\frac{\partial a_2^*(\cdot)}{\partial\zeta} - \frac{\partial a_1^*(\cdot)}{\partial\zeta} = \frac{1+(1-\rho)(1+2\mu)}{3\mu^2(1-\rho)+2\,\mu(2-\rho)+1}q. \tag{1.28}$$

We now turn to the value of $\partial n_1(a_1, a_2; \cdot)/\partial\zeta$. Taking derivative of the first expression in (1.5)with respect to ζ yields

$$\frac{\partial n_1(a_1, a_2; \cdot)}{\partial\zeta} = -2q. \tag{1.29}$$

By plugging Expressions (1.25), (1.28)and (1.29)into Expression (1.27)we get

$$\frac{dn_1(a_1^*(\cdot), a_2^*(\cdot); \cdot)}{d\zeta} = -\frac{2\,(\mu+1)[\,\mu(1-\rho)+1]}{3\mu^2(1-\rho)+2\,\mu(2-\rho)+1}q. \tag{1.30}$$

It follows that $\partial n_1^*(\cdot)/\partial\zeta < 0$ and $\partial n_2^*(\cdot)/\partial\zeta > 0$ for any $0 \le \rho < 1$ and $0 \le \Delta < \overline{\Delta}$.

iii) We now turn to the effect of an increase in ζ on the equilibrium prices:

$$\begin{aligned}\frac{\partial p_1^a(n_1^*(\cdot), a_1^*(\cdot))}{\partial\zeta} &= \frac{\partial n_1^*(\cdot)}{\partial\zeta} - \frac{\partial a_1^*(\cdot)}{\partial\zeta},\\ \frac{\partial p_2^a(n_2^*(\cdot), a_2^*(\cdot))}{\partial\zeta} &= (1-\rho)\frac{\partial n_2^*(\cdot)}{\partial\zeta} - \frac{\partial a_2^*(\cdot)}{\partial\zeta}.\end{aligned}$$

The covered user market assumption yields $\partial n_1^*(\cdot)/\partial\zeta = -\,\partial n_2^*(\cdot)/\partial\zeta$. Using (1.26)and (1.30)we get

$$\begin{aligned}\frac{\partial p_1^{a*}(\cdot)}{\partial\zeta} &= -\frac{(2\mu+1)[1+\,\mu(1-\rho)]}{3\mu^2(1-\rho)+2\,\mu(2-\rho)+1}q < 0,\\ \frac{\partial p_2^{a*}(\cdot)}{\partial\zeta} &= \frac{(1-\rho)(\,\mu+1)[1+2\,\mu(1-\rho)]}{3\mu^2(1-\rho)+2\,\mu(2-\rho)+1}q > 0.\end{aligned} \tag{1.31}$$

With an increase in parameter ζ, the superior (inferior)platform chargers a lower (higher) price for advertisements for any $0 \le \rho < 1$ and $0 \le \Delta < \overline{\Delta}$.

iv) Finally, to analyze the influence of a change in ζ on platforms' profits we inspect the derivative $\partial\pi(p_i^{a*}(\cdot), a_i^*(\cdot))/\partial\zeta$:

$$\frac{\partial\pi(p_i^{a*}(\cdot), a_i^*(\cdot))}{\partial\zeta} = \frac{\partial p_i^{a*}(\cdot)}{\partial\zeta}a_i^*(\cdot) + \,p_i^{a*}(\cdot)\frac{\partial a_i^*(\cdot)}{\partial\zeta}.$$

Using (1.26)and (1.31), we conclude that $\partial\pi_1^*(\cdot)/\partial\zeta < 0$ and $\partial\pi_2^*(\cdot)/\partial\zeta > 0$ for any $0 \le \rho < 1$ and $0 \le \Delta < \overline{\Delta}$. With an increase in parameter ζ, the superior (inferior)platform makes lower (higher)profits.

v) The total effect of a change in ζ on platforms' joint profits is non-positive if $|\partial\pi_1^*(\cdot)/\partial\zeta| \geq |\partial\pi_2^*(\cdot)/\partial\zeta|$. This is equivalent to

$$\left|\frac{\partial p_1^{a*}(\cdot)}{\partial\zeta}\right| a_1^*(\cdot) + p_1^{a*}(\cdot)\left|\frac{\partial a_1^*(\cdot)}{\partial\zeta}\right| \geq \left|\frac{\partial p_2^{a*}(\cdot)}{\partial\zeta}\right| a_2^*(\cdot) + p_2^{a*}(\cdot)\left|\frac{\partial a_2^*(\cdot)}{\partial\zeta}\right|. \tag{1.32}$$

Consider next the differences $|\partial p_1^{a*}(\cdot)/\partial\zeta| - |\partial p_2^{a*}(\cdot)/\partial\zeta|$ and $|\partial a_1^*(\cdot)/\partial\zeta| - |\partial a_2^*(\cdot)/\partial\zeta|$:

$$\begin{aligned} \left|\frac{\partial p_1^{a*}(\cdot)}{\partial\zeta}\right| - \left|\frac{\partial p_2^{a*}(\cdot)}{\partial\zeta}\right| &= \frac{[1 + 2\mu(1 + (1+\mu)(1-\rho))]\,\rho}{3\mu^2(1-\rho) + 2\,\mu(2-\rho) + 1} q, \\ \left|\frac{\partial a_1^*(\cdot)}{\partial\zeta}\right| - \left|\frac{\partial a_2^*(\cdot)}{\partial\zeta}\right| &= \frac{\rho}{3\mu^2(1-\rho) + 2\,\mu(2-\rho) + 1} q. \end{aligned} \tag{1.33}$$

Note that $|\partial p_1^{a*}(\cdot)/\partial\zeta| \geq |\partial p_2^{a*}(\cdot)/\partial\zeta|$ and $|\partial a_1^*(\cdot)/\partial\zeta| \geq |\partial a_2^*(\cdot)/\partial\zeta|$, holding with equality if $\rho = 0$. Furthermore, $a_1^*(\cdot) \geq a_2^*(\cdot)$ and $p_1^{a*}(\cdot) \geq p_2^{a*}(\cdot)$, holding with equality if $\Delta = \rho = 0$. It follows that Inequality (1.32)is fulfilled for any $\zeta \in (0, 1]$ and $\rho \in [0, 1)$ and it holds with equality if $\Delta = \rho = 0$. As ζ cannot be further increased if $\zeta = 1$ ($\Delta = 0$), platforms' joint profits decrease with an increase in ζ. *Q.E.D.*

Proof of Proposition 3. *i)* We first turn to the influence of changes in parameters ρ and ζ on the advertiser surplus. It follows from Expression (1.10)that $AS(a_1^*(\cdot), a_2^*(\cdot))$ increases in $a_1^*(\cdot)$ and $a_2^*(\cdot)$. In Lemma 2 we showed that $a_1^*(\cdot)$ and $a_2^*(\cdot)$ increase with a decrease in ρ. It follows that advertiser surplus gets larger as ρ decreases. To analyze the effect of a change in parameter ζ on the advertiser surplus, we first take derivative of Expression (1.10)evaluated at equilibrium values with respect to ζ:

$$\frac{\partial AS(a_1^*(\cdot), a_2^*(\cdot))}{\partial\zeta} = a_1^*(\cdot)\frac{\partial a_1^*(\cdot)}{\partial\zeta} + a_2^*(\cdot)\frac{\partial a_2^*(\cdot)}{\partial\zeta}.$$

We showed in Lemma 3 that $\partial a_1^*(\cdot)/\partial\zeta < 0$ and $\partial a_2^*(\cdot)/\partial\zeta > 0$. From Expression (1.33) we have that $|\partial a_1^*(\cdot)/\partial\zeta| \geq |\partial a_2^*(\cdot)/\partial\zeta|$, which is fulfilled with equality if $\rho = 0$. As stated in Proposition 1, $a_1^*(\cdot) \geq a_2^*(\cdot)$, holding with equality only if $\Delta = \rho = 0$. It follows that $\partial AS(a_1^*(\cdot), a_2^*(\cdot))/\partial\zeta \leq 0$, holding with equality only if $\Delta = \rho = 0$ (in which case ζ cannot be further increased). Hence, advertiser surplus decreases as parameter ζ gets larger.

ii) We now turn to the user surplus. It is useful to distinguish between two groups of users: Those who do not switch from the original platform in response to a change in parameters ρ or ζ and those who do. We will refer to the former group of users as *switchers* and to the latter group as *non-switchers.* We start with the effect of a change in ρ on the utility of switchers. Let t_1^ρ and t_2^ρ denote the locations of the marginal users (i.e., those who are indifferent between the two platforms)and $U_i^{\rho t}$ the utility of a user t choosing platform i after the change in parameter ρ. We showed in Lemma 2 that $n_1^*(\cdot)$ increases in response to a reduction in ρ, hence, $t_1^\rho > t_1$ and $t_2^\rho < t_2$. Due to symmetry, we can restrict attention to switchers with locations $t \in [t_1, t_1^\rho]$. For these users we have $U_1^t < U_2^t$ as before the change in ρ they preferred the superior platform. We also know from Lemma 2 that $a_1^*(\cdot)$ increases with a decrease in ρ. It follows that $U_1^{\rho t} < U_1^t$ for any t. Combining the two inequalities we get $U_1^{\rho t} < U_1^t < U_2^t$ for $t \in [t_1, t_1^\rho]$. The utility of switchers decreases as ρ gets smaller.

We now turn to the effect of a change in ρ on the utility of non-switchers. From Equation (1.3)we can distinguish three components of the user utility (apart from the basic utility): Content quality ($\zeta_i q$), disutility from advertisements (μa_i)and transportation costs (δ_i^2). For non-switchers, only the disutility from advertisements is affected by a change in ρ. We showed in Lemma 2 that both $a_1^*(\cdot)$ and $a_2^*(\cdot)$ increase with a decrease in ρ, which results in a reduction of the utility of non-switchers. It follows that both switchers and non-switchers are worse-off due to a decrease in parameter ρ.

We now consider the effect of an increase in ζ on the utility of switchers. Let t_1^ζ and t_2^ζ denote the locations of the marginal users and $U_i^{\zeta t}$ the utility of a user t choosing platform i after a change in parameter ζ. We showed in Lemma 3 that $n_1^*(\cdot)$ decreases in response to an increase of ζ, hence, $t_1^\zeta < t_1$ and $t_2^\zeta > t_2$. Due to symmetry, we restrict attention to switchers with locations $t \in [t_1^\zeta, t_1]$. A user switches if doing so increases his utility, hence, for $t \in [t_1^\zeta, t_1]$ we have $U_2^{\zeta t} > U_1^{\zeta t}$. We also know from Lemma 3 that $a_1^*(\cdot)$ decreases with an increase in ζ, so that $U_1^{\zeta t} > U_1^t$ holds for any t. Combining the two inequalities we get $U_2^{\zeta t} > U_1^{\zeta t} > U_1^t$ for $t \in [t_1^\zeta, t_1]$. The utility of switchers increases due to an increase of ζ.

We finally turn to the effect of a change in ζ on the utility of non-switchers. Non-switchers on platform 1 benefit from an increase in ζ as $a_1^*(\cdot)$ decreases in the new equilibrium. The utility of non-switchers at platform 2 also increases as

$$\frac{\partial U_2^{t*}}{\partial \zeta} = q - \mu \frac{\partial a_2^*(\cdot)}{\partial \zeta} = \frac{(1+2\mu)[1+\ \mu(1-\rho)]}{3\mu^2(1-\rho)+2\ \mu(2-\rho)+1} q > 0.$$

We get, hence, that non-switchers benefit from an increase in ζ. As switchers also benefit, we conclude that user surplus increases in response to an increase in parameter ζ. *Q.E.D.*

Proof of Proposition 4. Summing up advertiser surplus, user surplus and platforms' profits given in (1.10), (1.11)and (1.1), respectively, evaluated at equilibrium values, yields social welfare in equilibrium ($SW^*(\rho, \zeta, \mu, q, u)$). Taking derivative of $SW^*(\cdot)$ with respect to ρ and evaluating at $(\rho, \zeta)= (0\ , \zeta_0)$ gives

$$\left.\frac{\partial SW^*(\cdot)}{\partial \rho}\right|_{(\rho,\zeta)=(0,\zeta_0)} = -\frac{[4\mu\,(\mu+1)+3]\,(\Delta_0-\overline{\Delta})(\Delta_0-\Delta_\rho)}{(1+3\mu)^3}. \tag{1.34}$$

The comparison of $\overline{\Delta}$ and Δ_ρ yields

$$\overline{\Delta} - \Delta_\rho = \frac{\mu\,(3\mu+1)\left[\mu - \left(1+\sqrt{7}\right)/2\right]\left[\mu - \left(1-\sqrt{7}\right)/2\right]}{(\mu+1)^2\,[3+4\mu(1+\mu)]},$$

such that $\overline{\Delta} > \Delta_\rho$ if $\mu > \left(1+\sqrt{7}\right)/2$ and $\overline{\Delta} \leq \Delta_\rho$ if $\mu \leq \left(1+\sqrt{7}\right)/2$. Consider first $\mu > \left(1+\sqrt{7}\right)/2$. It follows from (1.34)that $\partial SW^*(\cdot)/\partial\rho|_{(\rho,\zeta)=(0,\zeta_0)} > 0$ if $\Delta_0 > \Delta_\rho$. If $\mu \leq \left(1+\sqrt{7}\right)/2$, then there is no such Δ_0 for which $\partial SW^*(\cdot)/\partial\rho|_{(\rho,\zeta)=(0,\zeta_0)} > 0$ holds. We next take derivative of social welfare with respect to ζ and evaluate it at $\rho = 0$:

$$\left.\frac{\partial SW^*(\cdot)}{\partial \zeta}\right|_{\rho=0} = \frac{1}{2\Delta_\zeta}\,(\Delta_\zeta - \Delta)\,q.$$

Comparing $\overline{\Delta}$ and Δ_ζ we obtain

$$\overline{\Delta} - \Delta_\zeta = -\frac{(1+3\mu)\left[\mu - \left(1+\sqrt{7}\right)/2\right]\left[\mu - \left(1-\sqrt{7}\right)/2\right]}{2\,(\mu^3+7\mu^2+10\mu+4)},$$

such that $\overline{\Delta} > \Delta_\zeta$ if $\mu < \left(1+\sqrt{7}\right)/2$ and $\overline{\Delta} \leq \Delta_\zeta$ if $\mu \geq \left(1+\sqrt{7}\right)/2$. Consider first $\mu < \left(1+\sqrt{7}\right)/2$. Then $\partial SW^*(\cdot)/\partial\zeta|_{\rho=0} > 0$ if $\Delta < \Delta_\zeta$ and $\partial SW^*(\cdot)/\partial\zeta|_{\rho=0} \leq 0$ if $\Delta \geq \Delta_\zeta$. Assume first that $\Delta_0 < \Delta_\zeta$. Then with an increase in ζ (decrease in Δ)social welfare increases and the socially optimal amount of CP technology licensing implies $\zeta^* = 1 > \zeta_0$ such that private incentives are insufficient. Assume next that $\Delta_0 \geq \Delta_\zeta$. Then social welfare decreases with an increase in ζ (decrease in Δ)on the interval $\Delta > \Delta_\zeta$ and increases on the interval $\Delta < \Delta_\zeta$. The socially optimal amount of technology licensing implies $\zeta^* = 1 > \zeta_0$ if $SW^*(0,1,\cdot) > SW^*(0,\zeta_0,\cdot)$ and $\zeta^* = \zeta_0$ if $SW^*(0,1,\cdot) \leq SW^*(0,\zeta_0,\cdot)$. It follows that in the former case private incentives are insufficient, while they are optimal in the latter case. Consider finally $\mu \geq \left(1+\sqrt{7}\right)/2$. Then for any $\Delta \leq \Delta_0$ it holds that $\partial SW^*(\cdot)/\partial\zeta|_{\rho=0} > 0$ and socially optimal amount of technology licensing implies $\zeta^* = 1 > \zeta_0$ such that private incentives are insufficient. *Q.E.D.*

Proof of Proposition 5. *i)* We first derive the equilibrium slot prices. Maximizing profits in (1.14)with respect to advertisement prices and assuming interior solutions yields

$$\begin{aligned} p_1^{a*}(\rho,\zeta,\mu,q) &= \frac{[1+\mu(3-\rho)]\left[\Delta + \frac{4\mu^2(1-\rho)+\mu(5-3\rho)+1}{4(1+\mu(3-\rho))}\right]}{3\mu^2(1-\rho)+2\,\mu(2-\rho)+1}, \\ p_2^{a*}(\rho,\zeta,\mu,q) &= \frac{(1-\rho)[1+\,\mu(3-2\rho)]\left(\overline{\Delta}_{p^a}-\Delta\right)}{3\mu^2(1-\rho)+2\,\mu(2-\rho)+1}. \end{aligned} \tag{1.35}$$

$p_1^{a*}(\cdot) > 0$ always holds, while $p_2^{a*}(\cdot) > 0$ if $\Delta < \overline{\Delta}_{p^a}$. The SOCs are fulfilled, with

$$\partial^2\pi_1(p_1^a,p_2^a;\cdot)/\partial(p_1^a)^2 = -\left[4\mu(1-\rho)+2\right]\,/\left[2\mu(2-\rho)+1\right]\,<0$$

and

$$\partial^2\pi_2(p_1^a,p_2^a;\cdot)/\partial(p_2^a)^2 = -\left[2(2\mu+1)\right]/\left[2\mu(2-\rho)+1\right]\,<0.$$

Plugging $p_i^{a*}(\cdot)$ into (1.13)gives the equilibrium user market shares

$$\begin{aligned} n_1^*(\rho,\zeta,\mu,q) &= \frac{2\left[1+\mu(3-2\rho)\right]}{1+2\mu(2-\rho)}p_1^{a*}(\cdot), \\ n_2^*(\rho,\zeta,\mu,q) &= \frac{2\left[1+\mu(3-\rho)\right]}{\left[1+2\mu(2-\rho)\right](1-\rho)}p_2^{a*}(\cdot). \end{aligned}$$

By plugging $n_i^*(\cdot)$ and $p_i^{a*}(\cdot)$ into (1.12)we obtain the equilibrium numbers of advertisement slots

$$\begin{aligned} a_1^*(\rho,\zeta,\mu,q) &= \frac{1+2\mu(1-\rho)}{1+2\mu(2-\rho)}p_1^{a*}(\cdot), \\ a_2^*(\rho,\zeta,\mu,q) &= \frac{1+2\mu}{1+2\mu(2-\rho)}p_2^{a*}(\cdot). \end{aligned} \tag{1.36}$$

Finally, we plug $n_i^*(\cdot)$ and $p_i^{a*}(\cdot)$ into (1.14)to get the equilibrium profits

$$\begin{aligned} \pi_1^*(\rho,\zeta,\mu,q) &= \frac{1+2\mu(1-\rho)}{1+2\mu(2-\rho)}\left[p_1^{a*}(\cdot)\right]^2, \\ \pi_2^*(\rho,\zeta,\mu,q) &= \frac{1+2\mu}{1+2\mu(2-\rho)}\left[p_2^{a*}(\cdot)\right]^2. \end{aligned} \tag{1.37}$$

Provided $p_2^{a*}(\cdot) > 0$, it holds that $n_i^*(\cdot), a_i^*(\cdot), \pi_i^*(\cdot) > 0$. Hence, both platforms are active on both sides of the market if $\Delta < \overline{\Delta}_{p^a}$. If $\Delta \geq \overline{\Delta}_{p^a}$, only the superior platform is active at the market.

ii) The comparison of equilibrium slot prices in (1.35)yields

$$p_1^{a*}(\cdot) - p_2^{a*}(\cdot) = \\ = \frac{4\Delta\left[2\mu\rho^2 + 6\mu(1-\rho) + (2-\rho)\right] + 2\mu\rho\left[2-\rho+2\mu(1-\rho)\right] + \rho}{4\left[3\mu^2(1-\rho) + 2\,\mu(2-\rho) + 1\right]} \geq 0,$$

holding with equality if $\rho = \Delta = 0$. By comparing the equilibrium numbers of advertisement slots in (1.36)we get

$$a_1^*(\cdot) - a_2^*(\cdot) = \frac{\Delta f(\mu,\rho) + g(\mu,\rho)}{\left[1+2\mu(2-\rho)\right]\left[3\mu^2(1-\rho) + 2\,\mu(2-\rho) + 1\right]}, \text{ with}$$

$$f(\mu,\rho) := 6\,\mu^2(1-\rho)(2-\rho) + 10\,\mu(1-\rho) + (2-\rho) + 2\,\mu\rho^2 > 0,$$

$$g(\mu,\rho) := \mu^2\rho(1-\rho)/2 + \mu\rho(2-\rho)/2 + \rho/4 > 0.$$

This implies that $a_1^*(\cdot) \geq a_2^*(\cdot)$, holding with equality if $\rho = \Delta = 0$. Inequalities $p_1^{a*}(\cdot) \geq p_2^{a*}(\cdot)$ and $a_1^*(\cdot) \geq a_2^*(\cdot)$ yield $\pi_1^*(\cdot) \geq \pi_2^*(\cdot)$, holding with equality if $\rho = \Delta = 0$. *Q.E.D.*

Proof of Lemma 4. Note that $\overline{\Delta}_{p^a}$ increases when ρ gets smaller. It follows that if ρ_0 and ζ_0 are such that $0 \leq \Delta_0 < \overline{\Delta}_{p^a}\big|_{\rho=\rho_0}$, then for any $\rho \leq \rho_0$ and $\zeta \geq \zeta_0$ the condition $0 \leq \Delta < \overline{\Delta}_{p^a}$ is fulfilled. *i)* We start with the effect of an AP technology licensing on the inferior platform. Taking derivative of $p_2^{a*}(\cdot)$ with respect to ρ yields

$$\frac{\partial p_2^{a*}(\cdot)}{\partial \rho} = -\frac{\eta(\mu,\rho)\left[\phi(\mu,\rho)/\eta(\mu,\rho) - \Delta\right]}{4\left[3\mu^2(1-\rho) + 2\,\mu(2-\rho) + 1\right]^2},$$

with

$$\eta(\mu,\rho) := 24\,\mu^3(1-\rho)^2 + 4\mu^2(4+\sqrt{2}-2\rho)(4-\sqrt{2}-2\rho) + 4\,\mu(7-4\rho) + 4\,,$$

$$\phi(\mu,\rho) := 12\,\mu^4(1-\rho)^2 + 2\mu^3(1-\rho)(15-7\rho) + \\ + \mu^2(6+\sqrt{10}-2\rho)(6-\sqrt{10}-2\rho) + \mu(9-4\rho) + 1\,.$$

It holds that $\eta(\cdot), \phi(\cdot) > 0$ for any $\mu > 0$ and $\rho \in [0,1)$. Comparing $\phi(\cdot)/\eta(\cdot)$ and $\overline{\Delta}_{p^a}$ we get

$$\frac{\phi(\cdot)}{\eta(\cdot)} - \overline{\Delta}_{p^a} = \frac{\mu^3(1-\rho)\left[3\mu^2(1-\rho)+2\,\mu(2-\rho)+1\right]}{\left[1+\mu(3-2\rho)\right]\left[6\mu^3(1-\rho)^2+\mu^2(4+\sqrt{2}-2\rho)(4-\sqrt{2}-2\rho)+\,\mu(7-4\rho)+1\right]},$$

such that $\phi(\cdot)/\eta(\cdot) > \overline{\Delta}_{p^a}$ for any $\mu > 0$ and $\rho \in [0,1)$, implying $\partial p_2^{a*}(\cdot)/\partial\rho < 0$ for any admissible parameters (μ,ρ,Δ, with $0 \leq \Delta < \overline{\Delta}_{p^a}$ and $\rho \in [0,1)$). Taking derivative of $a_2^*(\cdot)$ with respect to ρ yields

$$\frac{\partial a_2^*(\cdot)}{\partial\rho} = -\frac{(1+2\mu)\,\lambda(\mu,\rho)[\,\xi(\mu,\rho)/\lambda(\mu,\rho)-\Delta]}{4\,[1+2\mu(2-\rho)]^2\,[3\mu^2(1-\rho)+2\,\mu(2-\rho)+1]^{\,2}},$$

where

$$\begin{aligned}\xi(\mu,\rho) :=\ & 24\,\mu^5(1-\rho)^2+2\mu^4(1-\rho)(35-19\rho)+\\ & +4\mu^3(11+\sqrt{21}-5\rho)(11-\sqrt{21}-5\rho)/5+\\ & +4\mu^2(4+\sqrt{5}-\rho)(4-\sqrt{5}-\rho)+\,\mu(11-4\rho)+1\,,\\ \lambda(\mu,\rho) :=\ & 24\,\mu^4(1-\rho)^2+16\mu^3(2-\rho)(4-3\rho)+\\ & +16\mu^2(3+\sqrt{2}-\rho)(3-\sqrt{2}-\rho)+4\,\mu(9-4\rho)+4\,,\end{aligned}$$

such that $\xi(\cdot), \lambda(\cdot) > 0$ for any $\mu > 0$ and $\rho \in [0,1)$. It follows that $\partial a_2^*(\cdot)/\partial\rho < 0$ provided that $\Delta < \xi(\cdot)/\lambda(\cdot)$. The comparison of $\xi(\cdot)/\lambda(\cdot)$ and $\overline{\Delta}_{p^a}$ yields

$$\frac{\xi(\cdot)}{\lambda(\cdot)} - \overline{\Delta}_{p^a} = \frac{4\mu^3\,(1-\rho)\,\sigma(\mu,\rho)}{\lambda(\mu,\rho)[1+\,\mu(3-2\rho)]},$$

where

$$\begin{aligned}\sigma(\mu,\rho) :=\ & 6\,\mu^3(1-\rho)(2-\rho)+\,\mu^2(19+\sqrt{57}-8\rho)(19-\sqrt{57}-8\rho)/16+\\ & +4\mu(2-\rho)+1\,.\end{aligned}$$

It holds that $\sigma(\cdot) > 0$ for any $\mu > 0$ and $\rho \in [0,1)$, hence, $\xi(\cdot)/\lambda(\cdot) > \overline{\Delta}_{p^a}$ and for any admissible parameters $\partial a_2^*(\cdot)/\partial\rho < 0$. Inequalities $\partial p_2^{a*}(\cdot)/\partial\rho < 0$ and $\partial a_2^*(\cdot)/\partial\rho < 0$ imply that $\partial\pi_2^*(\cdot)/\partial\rho < 0$.

We next consider the superior platform and start with $a_1^*(\cdot)$:

$$\frac{\partial a_1^*(\cdot)}{\partial \rho} = -\frac{1}{4}\frac{\mu\alpha(\mu,\rho)[\beta(\mu,\rho)/\alpha(\mu,\rho)-\Delta]}{[2\mu(2-\rho)+1]^{2}[3\mu^2(1-\rho)+2\mu(2-\rho)+1]^{2}},$$

where

$$\begin{aligned}
\alpha(\mu,\rho) &:= 48\,\mu^4(1-\rho)^2+16\mu^3(2-\rho)(2-3\rho)+\\
&\quad +16\mu^2(1-\rho)(4-\rho)+4\,\mu(7-4\rho)+4\,,\\
\beta(\mu,\rho) &:= 48\,\mu^5(1-\rho)^2+4\mu^4(1-\rho)(35-19\rho)+\\
&\quad +4\mu^3(20+2\sqrt{19}-9\rho)(20-2\sqrt{19}-9\rho)/9+\\
&\quad +\mu^2(11+\sqrt{57}-2\rho)(11-\sqrt{57}-2\rho)+\mu(13-4\rho)+1\,,
\end{aligned}$$

provided that $\alpha(\mu,\rho)\neq 0$. If $\alpha(\mu,\rho)=0$, then

$$\frac{\partial a_1^*(\cdot)}{\partial \rho} = -\frac{1}{4}\frac{\mu\beta(\cdot)}{[2\mu(2-\rho)+1]^{2}[3\mu^2(1-\rho)+2\mu(2-\rho)+1]^{2}}.$$

For any $\mu>0$ and $\rho\in[0,1)$ it holds that $\beta(\cdot)>0$, hence, $\partial a_1^*(\cdot)/\partial\rho<0$ if $\alpha(\mu,\rho)=0$. Assume now that $\alpha(\mu,\rho)\neq 0$. The sign of $\alpha(\cdot)$ is ambiguous. The comparison of $\beta(\cdot)/\alpha(\cdot)$ and $\overline{\Delta}_{p^a}$ yields

$$\frac{\beta(\cdot)}{\alpha(\cdot)}-\overline{\Delta}_{p^a}=\frac{4\mu\tau(\mu,\rho)}{\alpha(\cdot)[1+\mu(3-2\rho)]},$$

where

$$\begin{aligned}
\tau(\mu,\rho) &:= 12\,\mu^5(1-\rho)^2(2-\rho)+\\
&\quad +\mu^4(1-\rho)(43+\sqrt{129}-20\rho)(43-\sqrt{129}-20\rho)/20+\\
&\quad +\mu^3(2-\rho)(13+\sqrt{65}-4\rho)(13-\sqrt{65}-4\rho)/2+\\
&\quad +\mu^2(53+\sqrt{265}-24\rho)(53-\sqrt{265}-24\rho)/48+6\mu(2-\rho)+1\,.
\end{aligned}$$

For any $\mu>0$ and $\rho\in[0,1)$ it holds that $\tau(\cdot)>0$. Depending on the sign of $\alpha(\cdot)$ two cases are possible. If $\alpha(\cdot)<0$, then $\beta(\cdot)/\alpha(\cdot)<0$ and $\alpha(\cdot)(\beta(\cdot)/\alpha(\cdot)-\Delta)>0$, such that $\partial a_1^*(\cdot)/\partial\rho<0$. If $\alpha(\cdot)>0$, then $\beta(\cdot)/\alpha(\cdot)>\overline{\Delta}_{p^a}$ and for any $\Delta<\overline{\Delta}_{p^a}$ it holds $\alpha(\cdot)[\beta(\cdot)/\alpha(\cdot)-\Delta]>0$, such that $\partial a_1^*(\cdot)/\partial\rho<0$.

We next turn to the superior platform's price. Taking derivative of $p_1^{a*}(\cdot)$ with respect to ρ we get

$$\frac{\partial p_1^{a*}(\cdot)}{\partial \rho} = -\frac{\mu\,(1+2\mu)(1+3\,\mu)\,(\Delta_{p^a}-\Delta)}{[3\mu^2(1-\rho)+2\,\mu(2-\rho)+1]^{\,2}},$$

such that $\partial p_1^{a*}(\cdot)/\partial\rho < 0$ if $0 \le \Delta < \Delta_{p^a}$. The comparison of Δ_{p^a} and $\overline{\Delta}_{p^a}$ yields

$$\overline{\Delta}_{p^a} - \Delta_{p^a} = \frac{\mu\left[3\mu^2(1-\rho)+2\,\mu(2-\rho)+1\,\right]}{(1+3\mu)[1+\,\mu(3-2\rho)]} > 0,$$

which holds for any $\mu > 0$ and $\rho \in [0,1)$. We conclude that if $\Delta < \overline{\Delta}_{p^a}$ ($\Delta \ge \overline{\Delta}_{p^a}$), then $\partial p_1^{a*}(\cdot)/\partial\rho < 0$ ($\partial p_1^{a*}(\cdot)/\partial\rho \ge 0$). Using the expression for $\pi_1^*(\cdot)$ in (1.37)and taking derivative of $\pi_1^*(\cdot)$ with respect to ρ we obtain

$$\frac{\partial \pi_1^*(\cdot)}{\partial\rho} = -\frac{4\mu^2\left[p_1^{a,*}(\cdot)\right]^2}{[1+2\mu(2-\rho)]^2} + \frac{2\,[1+2\mu(1-\rho)]\,p_1^{a*}(\cdot)}{1+2\mu(2-\rho)}\frac{\partial p_1^{a*}(\cdot)}{\partial\rho}.$$

It holds that $sign\,\{\partial\pi_1^*(\cdot)/\partial\rho\} = sign\,\{[\partial\pi_1^*(\cdot)/\partial\rho]\,/p_1^{a*}(\cdot)\}$. Dividing $\partial\pi_1^*(\cdot)/\partial\rho$ with $p_1^{a*}(\cdot)$ we get

$$\frac{\partial\pi_1^*(\cdot)/\partial\rho}{p_1^{a*}(\cdot)} = -\frac{\mu\theta(\cdot)\,[\Delta_\pi(\cdot)-\Delta]}{2\,[1+2\mu(2-\rho)]^2\,[3\mu^2(1-\rho)+2\,\mu(2-\rho)+1]^{\,2}}.$$

For any $\mu > 0$ and $\rho \in [0,1)$ it holds that $\theta(\cdot), \chi(\cdot) > 0$, implying $\Delta_\pi(\cdot) > 0$. We next compare $\Delta_\pi(\cdot)$ and $\overline{\Delta}_{p^a}$:

$$\Delta_\pi(\cdot) - \overline{\Delta}_{p^a} = -\frac{4\mu^3\,(1-\rho)\,\omega(\mu,\rho)}{[1+\mu(3-2\rho)]\,\theta(\mu,\rho)},$$

where

$$\begin{aligned}\omega(\mu,\rho) :=\; & 6\,\mu^3(2-\rho)(1-\rho)+ \\ & +\mu^2(19+\sqrt{57}-8\rho)(19-\sqrt{57}-8\rho)/16+4\mu(2-\rho)+1\,.\end{aligned}$$

Note, that $\omega(\cdot)$ is positive for any $\mu > 0$ and $\rho \in [0,1)$, hence, $\Delta_\pi(\cdot) < \overline{\Delta}_{p^a}$. It follows that $\partial\pi_1^*(\cdot)/\partial\rho < 0$ ($\partial\pi_1^*(\cdot)/\partial\rho \ge 0$)if $\Delta < \Delta_\pi(\cdot)$ ($\Delta \ge \Delta_\pi(\cdot)$). We finally compare $\Delta_\pi(\cdot)$ and Δ_{p^a}:

$$\Delta_{p^a} - \Delta_\pi(\cdot) = -\frac{4\mu(1+2\mu)\left[3\mu^2(1-\rho)+2\,\mu(2-\rho)+1\,\right]^2}{(1+3\mu)\theta(\cdot)} < 0,$$

which holds for any $\mu > 0$ and $\rho \in [0,1)$.

ii) With $\Delta = (1-\zeta)q$, we have $\partial\Delta/\partial\zeta = -q$. It is straightforward that $\partial p_2^{a*}(\cdot)/\partial\zeta, \partial a_2^*(\cdot)/\partial\zeta > 0$, while $\partial p_1^{a*}(\cdot)/\partial\zeta, \partial a_1^*(\cdot)/\partial\zeta < 0$ for any $\rho \in [0,1)$ and $\zeta \in (0,1]$. This in turn implies that $\partial\pi_2^*(\cdot)/\partial\zeta > 0$ and $\partial\pi_1^*(\cdot)/\partial\zeta < 0$. *Q.E.D.*

Proof of Proposition 6. We showed in part *ii)* of the proof of Lemma 4 that $\partial\pi_1^*(\cdot)/\partial\zeta < 0$ for any $\rho \in [0,1)$ and $\zeta \in (0,1]$. The superior platform has no incentives to share its CP technology. *i)* We showed in part *i)* of the proof of Lemma 4 that $sign\{\partial\pi_1^*(\cdot)/\partial\rho\} = sign\{\Delta - \Delta_\pi(\cdot)\}$. Taking derivative of $\Delta_\pi(\cdot)$ with respect to ρ yields

$$\frac{\partial\Delta_\pi(\cdot)}{\partial\rho} = -\frac{96\mu^5(1-\rho)(2\mu+1)^2\left[3\mu^2(1-\rho)+2\mu(2-\rho)+1\right]}{[\theta(\cdot)]^2} < 0.$$

If $\Delta_0 < \Delta_\pi(\cdot,\rho_0)$, then due to $\partial\Delta_\pi(\cdot)/\partial\rho < 0$ for any $\rho \in [0,\rho_0]$ it holds that $\Delta_0 < \Delta_\pi(\cdot,\rho_0) \leqslant \Delta_\pi(\cdot)$, which implies that $\Delta_0 - \Delta_\pi(\cdot) < 0$ and $\partial\pi_1^*(\cdot)/\partial\rho < 0$. Hence, on the interval $\rho \in [0,\rho_0]$, $\pi_1^*(\cdot)$ increases with a decreases in ρ and the superior platform licenses fully its AP technology, so that $\rho^* = 0$. Note finally that with $\partial\Delta_\pi(\cdot)/\partial\rho < 0$ it holds that $\Delta_\pi(\cdot,\rho_0) < \Delta_\pi(\cdot,0)$.

ii) Assume $\Delta_\pi(\cdot,\rho_0) < \Delta_0 < \Delta_\pi(\cdot,0)$ and let $\widehat{\rho}$ be such that $\Delta_\pi(\cdot,\widehat{\rho}) = \Delta_0$. It must be that $0 < \widehat{\rho} < \rho_0$. Then for any $\rho \in [\widehat{\rho},\rho_0]$ it holds that $\Delta_0 \geq \Delta_\pi(\cdot)$, hence, $\pi_1^*(\cdot)$ (weakly) increases in ρ on the interval $\rho \in [\widehat{\rho},\rho_0]$. However, for any $\rho \in [0,\widehat{\rho})$ it holds that $\Delta_0 < \Delta_\pi(\cdot)$, hence, $\pi_1^*(\cdot)$ decreases in ρ on the interval $\rho \in [0,\widehat{\rho})$. Then depending on the relation between $\pi_1^*(\rho=0,\cdot)$ and $\pi_1^*(\rho=\rho_0,\cdot)$ two cases are possible. If $\pi_1^*(\rho=0,\cdot) > \pi_1^*(\rho=\rho_0,\cdot)$, AP technology is fully licensed, while there is no licensing if $\pi_1^*(\rho=0,\cdot) \leq \pi_1^*(\rho=\rho_0,\cdot)$.

iii) Assume finally that $\Delta_\pi(\cdot,\rho_0) < \Delta_\pi(\cdot,0) < \Delta_0$. As $\partial\Delta_\pi(\cdot)/\partial\rho < 0$, for any $\rho \in [0,\rho_0]$ it holds that $\Delta_\pi(\cdot) < \Delta_0$ and $\partial\pi_1^*(\cdot)/\partial\rho < 0$, no licensing then takes place. *Q.E.D.*

Proof of Proposition 7. Maximizing profits with respect to the number of advertisements and user prices yields the following FOCs

$$n_1^* - 2\mu\left(a_1^* + p_1^{u*}\right) - 2a_1^* \leq 0,\ a_1^* \left.\frac{\partial \pi_1(\cdot)}{\partial a_1}\right|_{a_1^*,p_1^{u*},a_2^*,p_2^{u*}} = 0, \tag{1.38}$$

$$n_1^* - 2\left(p_1^{u*} + a_1^*\right) \leq 0,\ p_1^{u*} \left.\frac{\partial \pi_1(\cdot)}{\partial p_1^u}\right|_{a_1^*,p_1^{u*},a_2^*,p_2^{u*}} = 0, \tag{1.39}$$

$$(1-\rho)\left(n_2^* - 2\mu a_2^*\right) - 2\left(\mu p_2^{u*} + a_2^*\right) \leq 0,\ a_2^* \left.\frac{\partial \pi_2(\cdot)}{\partial a_2}\right|_{a_1^*,p_1^{u*},a_2^*,p_2^{u*}} = 0, \tag{1.40}$$

$$n_2^* - 2p_2^{u*} - 2(1-\rho)a_2^* \leq 0,\ p_2^{u*} \left.\frac{\partial \pi_2(\cdot)}{\partial p_2^u}\right|_{a_1^*,p_1^{u*},a_2^*,p_2^{u*}} = 0. \tag{1.41}$$

We first show that there is no equilibrium with $p_1^{u*} = 0$. To see this, assume that there is an equilibrium with $p_1^{u*} = 0$. Condition (1.39)implies that $n_1^* - 2a_1^* \leq 0$. Assume that in this equilibrium $a_1^* > 0$. For any $a_1^* > 0$ it holds that $n_1^* - 2\mu a_1^* - 2a_1^* < n_1^* - 2a_1^*$, yielding $n_1^* - 2\mu a_1^* - 2a_1^* < 0$. In this case Condition (1.38)requires $a_1^* = 0$, which is a contradiction. Hence, if $p_1^{u*} = 0$, then $a_1^* = 0$, leading to zero profits for the superior platform, which cannot be in equilibrium.

We next show that $\mu \geq 1$ implies $a_1^* = 0$, while $\mu < 1$ implies $a_1^* > 0$. Assume $a_1^* > 0$. Condition (1.38)requires that $n_1^* - 2\mu\left(a_1^* + p_1^{u*}\right) - 2a_1^* = 0$. As $p_1^{u*} > 0$, Condition (1.39) implies $n_1^* - 2\left(p_1^{u*} + a_1^*\right) = 0$. From the latter equalities we get $p_1^{u*}(1-\mu) = \mu a_1^*$, such that $a_1^* > 0$ if and only if $\mu < 1$ (as $p_1^{u*} > 0$). Assume next that $a_1^* = 0$. As $p_1^{u*} > 0$, Condition (1.39)then yields $p_1^{u*} = n_1^*/2$, which we plug into the inequality of Condition (1.38)to get $(1-\mu)n_1^* \leq 0$. The latter inequality is fulfilled if and only if $\mu \geq 1$ (as $p_1^{u*} > 0$ implies $n_1^* > 0$).

We now turn to the inferior platform and show that if it is active on the user side, then it places advertisements if and only if $\mu < 1 - \rho$. Note that if $p_2^{u*} = 0$, then $a_2^* = 0$, and if $p_2^{u*} > 0$, then $a_2^* \geq 0$. Assume an equilibrium with $p_2^{u*} > 0$ and $a_2^* = 0$. From Condition (1.41)we get $p_2^{u*} = n_2^*/2$, which implies that $n_2^* > 0$. Plugging p_2^{u*} into the inequality of Condition (1.40)yields $(1-\rho-\mu)n_2^* \leq 0$, which holds if and only if $\mu \geq 1-\rho$ (as $n_2^* > 0$). Assume an equilibrium where $p_2^{u*}, a_2^* > 0$. Conditions (1.40)and (1.41)yield $p_2^{u*}(1-\rho-\mu) = a_2^*\left[1-(1-\rho)(1-\rho-\mu)\right]$. $p_2^{u*} > 0$ and $a_2^* > 0$ can hold together only if and only if $\mu < 1-\rho$.

It follows that depending on μ the following six equilibria are possible:

Case 1a) $\mu \geq 1$: $a_1^* = 0$, $p_1^{u*} > 0$, $a_2^* = 0$, $p_2^{u*} = 0$.

Case 1b) $\mu \geq 1$: $a_1^* = 0$, $p_1^{u*} > 0$, $a_2^* = 0$, $p_2^{u*} > 0$.

Case 2a) $1-\rho \leq \mu < 1$: $a_1^* > 0$, $p_1^{u*} > 0$, $a_2^* = 0$, $p_2^{u*} = 0$.

Case 2b) $1-\rho \leq \mu < 1$: $a_1^* > 0$, $p_1^{u*} > 0$, $a_2^* = 0$, $p_2^{u*} > 0$.

Case 3a) $\mu < 1-\rho$: $a_1^* > 0$, $p_1^{u*} > 0$, $a_2^* = 0$, $p_2^{u*} = 0$.

Case 3b) $\mu < 1-\rho$: $a_1^* > 0$, $p_1^{u*} > 0$, $a_2^* > 0$, $p_2^{u*} > 0$.

We first assume $\mu \geq 1$ and consider *Case 1b*. Plugging $a_1^* = 0$ and $a_2^* = 0$ into Conditions (1.39)and (1.41)yields $p_1^{u*} = n_1^*/2$ and $p_2^{u*} = n_2^*/2$, respectively. Plugging p_1^{u*} and p_2^{u*} into Equation (1.16)and using $n_1^* + n_2^* = 1$ yields $n_1^*(\zeta, q) = 1/2 + 2\Delta/3$. Moreover, $n_2^*(\zeta, q) = 1/2 - 2\Delta/3$, $p_1^{u*}(\zeta, q) = 1/4 + \Delta/3$ and $p_2^{u*}(\zeta, q) = 1/4 - \Delta/3$. It holds that $n_1^*(\cdot) < 1$ and $p_2^{u*}(\cdot) > 0$ if $\Delta < 3/4$. Plugging the equilibrium values into Conditions (1.38) and (1.40)yields $(1/2 + 2\Delta/3)(1-\mu) \leq 0$ and $(1/2 - 2\Delta/3)(1-\rho-\mu) \leq 0$, which are fulfilled if $\Delta < 3/4$. It is straightforward to show that if $\Delta \geq 3/4$, the equilibrium in *Case 1a* applies, with $p_1^{u*} = \Delta - 1/4$ and $n_1^* = 1$.

We next consider $1-\rho \leq \mu < 1$ and focus on the equilibrium in *Case 2b*. In this equilibrium the weak inequalities in Conditions (1.38), (1.39)and (1.41)hold as equalities. Together with Equation (1.16)they imply that

$$
\begin{aligned}
a_1^*(\zeta, \mu, q) &= (1-\mu)(3+4\Delta)/\left[8+4\mu(2-\mu)\right], \\
p_1^{u*}(\zeta, \mu, q) &= \mu(3+4\Delta)/\left[8+4\mu(2-\mu)\right], \\
p_2^{u*}(\zeta, \mu, q) &= \left(\overline{\Delta}_{p^u} - \Delta\right)/\left[2+\mu(2-\mu)\right], \\
n_1^*(\zeta, \mu, q) &= (4\Delta+3)/\left[4+2\mu(2-\mu)\right], \\
n_2^*(\zeta, \mu, q) &= 2\left(\overline{\Delta}_{p^u} - \Delta\right)/\left[2+\mu(2-\mu)\right].
\end{aligned}
$$

It holds that $a_1^*(\cdot), p_1^{u*}(\cdot), n_1^*(\cdot) > 0$, while $p_2^{u*}(\cdot), n_2^*(\cdot) > 0$ if $\Delta < \overline{\Delta}_{p^u}$. By plugging $n_2^*(\cdot)$ and $p_2^{u*}(\cdot)$ into the inequality of Condition (1.40)we obtain

$$
2(1-\mu-\rho)\left(\overline{\Delta}_{p^u} - \Delta\right)/\left[2+\mu(2-\mu)\right] \leq 0,
$$

which holds if $\Delta < \overline{\Delta}_{p^u}$. It is straightforward to check that if $\Delta \geq \overline{\Delta}_{p^u}$, then only the superior platform is active on both sides of the market and we have the equilibrium of *Case 2a*, with $p_1^{u*}(\zeta, \mu, q) = (4\Delta - 1)/\left[4(2-\mu)\right]$, $a_1^*(\zeta, \mu, q) = (4\Delta - 1)(1-\mu)/\left[4\mu(2-\mu)\right]$ and $n_1^* = 1$.

We finally turn to $\mu < 1 - \rho$ and focus on *Case 3b*. In this equilibrium the weak inequalities in all the FOCs hold as equalities. From the FOCs and Equation (1.16)we get the equilibrium values

$$\begin{aligned} p_1^{u*}(\rho, \zeta, \mu, q) &= \frac{2\mu\left(\Delta + \overline{\Delta}_{p^u}\right) - \mu\rho\left[\rho + 2(\mu - 1)\right]}{2\left[1 + 2\mu(2-\mu)\right] - 2\rho\left[\rho + 2(\mu-1)\right]}, \\ a_1^*(\rho, \zeta, \mu, q) &= \frac{(1-\mu)\left[2\left(\Delta + \overline{\Delta}_{p^u}\right) - \rho(\rho + 2(\mu-1))\right]}{2\left[1 + 2\mu(2-\mu)\right] - 2\rho\left[\rho + 2(\mu-1)\right]}, \\ p_2^{u*}(\rho, \zeta, \mu, q) &= \frac{\left[\rho(2-\rho) + \mu(1-\rho)\right]\left(\overline{\Delta}_{p^u} - \Delta\right)}{1 + 2\mu(2-\mu) - \rho\left[\rho + 2(\mu-1)\right]}, \\ a_2^*(\rho, \zeta, \mu, q) &= \frac{(1-\rho-\mu)\left(\overline{\Delta}_{p^u} - \Delta\right)}{1 + 2\mu(2-\mu) - \rho\left[\rho + 2(\mu-1)\right]}. \end{aligned} \tag{1.42}$$

Note that $\rho + 2(\mu - 1) < 0$ provided $\mu < 1 - \rho$. Hence, $p_1^{u,*}(\cdot), a_1^*(\cdot) > 0$. Moreover, $p_2^{u,*}(\cdot), a_2^*(\cdot) > 0$ if $0 \leq \Delta < \overline{\Delta}_{p^u}$. We next compute platforms' equilibrium market shares among users. Plugging equilibrium values into Equations (1.16)and (1.17)yields

$$\begin{aligned} n_1^*(\rho, \zeta, \mu, q) &= \frac{2\left(\Delta + \overline{\Delta}_{p^u}\right) - \rho\left[\rho + 2(\mu-1)\right]}{1 + 2\mu(2-\mu) - \rho\left[\rho + 2(\mu-1)\right]}, \\ n_2^*(\rho, \zeta, \mu, q) &= \frac{2\left(\overline{\Delta}_{p^u} - \Delta\right)}{1 + 2\mu(2-\mu) - \rho\left[\rho + 2(\mu-1)\right]}. \end{aligned} \tag{1.43}$$

$n_1^*(\cdot) > 0$, while $n_2^*(\cdot) > 0$ if $0 \leq \Delta < \overline{\Delta}_{p^u}$. Hence, if $0 \leq \Delta < \overline{\Delta}_{p^u}$, then both platforms are active on both sides of the market. Platforms realize profits $\pi_i^*(\rho, \zeta, \mu, q)$:

$$\begin{aligned} \pi_1^*(\cdot) &= \frac{\left[2\left(\Delta + \overline{\Delta}_{p^u}\right) - \rho(\rho + 2(\mu-1))\right]^2\left[1 + \mu(2-\mu)\right]}{4\left[2 - (1-\rho-\mu)^2 + \mu(2-\mu)\right]^2}, \\ \pi_2^*(\cdot) &= \frac{\left(\Delta - \overline{\Delta}_{p^u}\right)^2\left[2 - (1-\rho-\mu)^2\right]}{\left[2 - (1-\rho-\mu)^2 + \mu(2-\mu)\right]^2}. \end{aligned}$$

It is straightforward to check that if $\Delta \geq \overline{\Delta}_{p^u}$, then the equilibrium of *Case 3a* emerges, with

$$p_1^{u*}(\zeta,\mu,q)= \frac{4\Delta-1}{4(2-\mu)},$$
$$a_1^{*}(\zeta,\mu,q)= \frac{(4\Delta-1)(1-\mu)}{4\mu(2-\mu)},$$
$$n_1^{*}=1.$$

Q.E.D.

Proof of Lemma 5. Note first that if $0 \leq \rho_0 < 1-\mu$, then $1-\mu-\rho > 0$ holds for any $\rho \leq \rho_0$. Also, if $0 \leq \Delta_0 < \overline{\Delta}_{p^u}$, then any $\zeta > \zeta_0$ also fulfills $\Delta < \overline{\Delta}_{p^u}$. i)We start with the superior platform. Taking derivatives of $p_1^{u*}(\cdot)$, $a_1^{*}(\cdot)$ and $n_1^{*}(\cdot)$ stated in (1.42)and (1.43) in the Proof of Proposition 7 with respect to ρ yields

$$\frac{\partial p_1^{u*}(\cdot)}{\partial \rho} = \frac{2\mu(1-\mu-\rho)(\overline{\Delta}_{p^u}-\Delta)}{[1+2\mu(2-\mu)-\rho(\rho+2(\mu-1))]^2} > 0, \tag{1.44}$$
$$\frac{\partial a_1^{*}(\cdot)}{\partial \rho} = \frac{2(1-\mu)(1-\mu-\rho)(\overline{\Delta}_{p^u}-\Delta)}{[1+2\mu(2-\mu)-\rho(\rho+2(\mu-1))]^2} > 0,$$
$$\frac{\partial n_1^{*}(\cdot)}{\partial \rho} = \frac{4(1-\mu-\rho)(\overline{\Delta}_{p^u}-\Delta)}{[1+2\mu(2-\mu)-\rho(\rho+2(\mu-1))]^2} > 0.$$

By plugging $n_1^{*}(\cdot)$ and $a_1^{*}(\cdot)$ into Expression (1.2)we get the equilibrium slot price

$$p_1^{a*}(\rho,\zeta,\mu,q)= \frac{(\mu+1)\left[2(\Delta+\overline{\Delta}_{p^u})-\rho(\rho+2(\mu-1))\right]}{2\left[1+2\mu(2-\mu)-\rho(\rho+2(\mu-1))\right]}.$$

We take derivative of $p_1^{a*}(\cdot)$ with respect to ρ to obtain

$$\frac{\partial p_1^{a*}(\cdot)}{\partial \rho} = \frac{2\,(\mu+1)(1-\mu-\rho)\left(\overline{\Delta}_{p^u}-\Delta\right)}{[1+2\mu(2-\mu)-\rho(\rho+2(\mu-1))]^2} > 0.$$

From $\partial p_1^{u*}(\cdot)/\partial\rho, \partial n_1^{*}(\cdot)/\partial\rho, \partial p_1^{a*}(\cdot)/\partial\rho, \partial a_1^{*}(\cdot)/\partial\rho > 0$ it is immediate that the derivative $\partial \pi_1^{*}(\cdot)/\partial\rho$ is negative.

We next turn to the inferior platform. Taking derivative of $a_2^{*}(\cdot)$ in (1.42)with respect to ρ yields

$$\frac{\partial a_2^{*}(\cdot)}{\partial \rho} = -\frac{\left[2(\mu\rho+1)+(1-\rho)^2\right]\left(\overline{\Delta}_{p^u}-\Delta\right)}{[1+2\mu(2-\mu)-\rho(\rho+2(\mu-1))]^2} < 0.$$

As the market is covered, $\partial n_1^{*}(\cdot)/\partial\rho > 0$ implies $\partial n_2^{*}(\cdot)/\partial\rho < 0$. To derive the change in $p_2^{u*}(\cdot)$ we consider the derivative of Expression (1.17)evaluated at equilibrium values with respect to ρ:

$$\frac{\partial n_2^*(\cdot)}{\partial \rho} = 2\frac{\partial p_1^{u*}(\cdot)}{\partial \rho} - 2\frac{\partial p_2^{u*}(\cdot)}{\partial \rho} + 2\mu\frac{\partial a_1^*(\cdot)}{\partial \rho} - 2\mu\frac{\partial a_2^*(\cdot)}{\partial \rho}.$$

As $\partial p_1^{u*}(\cdot)/\partial\rho > 0$, $\partial a_1^*(\cdot)/\partial\rho > 0$, $a_2^*(\cdot)/\partial\rho < 0$ and $\partial n_2^*(\cdot)/\partial\rho < 0$, it must hold that $\partial p_2^{u*}(\cdot)/\partial\rho > 0$. Plugging $n_2^*(\cdot)$ and $a_2^*(\cdot)$ into Expression (1.2)yields the price of an advertisement slot at platform 2:

$$p_2^{a*}(\rho,\zeta,\mu,q)= \frac{(1+\mu-\rho)\left(\overline{\Delta}_{p^u}-\Delta\right)}{1+2\mu(2-\mu)-\rho\left[\rho+2(\mu-1)\right]} > 0.$$

The derivative of $p_2^{a*}(\cdot)$ with respect to ρ is

$$\frac{\partial p_2^{a*}(\cdot)}{\partial \rho} = -\frac{\left(\overline{\Delta}_{p^u}-\Delta\right)\left[4\mu(1-\mu)+2(1-\mu\rho)+(1-\rho)^2\right]}{\left[1+2\mu(2-\mu)-\rho(\rho+2(\mu-1))\right]^2} < 0.$$

We also obtain

$$\frac{\partial \pi_2^*(\cdot)}{\partial \rho} = -\frac{2\left(\overline{\Delta}_{p^u}-\Delta\right)^2 l(\mu,\rho)}{\left[1+2\mu(2-\mu)-\rho(\rho+2(\mu-1))\right]^3},$$

where $l(\mu,\rho):= \rho^3-3\rho^2(1-\mu)+\rho(2\mu^2-4\mu+1)+1-\mu$. We next show that $l(\cdot)>0$. Taking derivative of $l(\cdot)$ with respect to μ yields $\partial l(\cdot)/\partial\mu = -4\rho(1-\mu-\rho)-1-\rho^2 < 0$. Hence, for any μ such that $\mu < 1-\rho$ it holds that $l(\cdot) > \lim_{\mu\to 1-\rho} l(\cdot)= 0$. It follows that $\partial\pi_2^*(\cdot)/\partial\rho < 0$.

We now turn to the effect of AP technology licensing on joint profits. The derivative of joint profits with respect to ρ is

$$\frac{\partial \pi_1^*(\cdot)}{\partial \rho} + \frac{\partial \pi_2^*(\cdot)}{\partial \rho} = \frac{(1-\mu-\rho)\left(\overline{\Delta}_{p^u}-\Delta\right)h(\mu,\rho)}{2\left[2-(1-\rho-\mu)^2+\mu(2-\mu)\right]^3},$$

where

$$h(\mu,\rho) := 4\Delta\left[3+2\mu(2-\mu)+\rho(1-\rho-\mu)+\rho(1-\mu)\right]+k(\mu,\rho)$$

and

$$k(\mu,\rho):= 4\,\mu^4+4\mu^3\rho-16\mu^3+2\mu^2\rho^2-12\mu^2\rho+$$
$$+12\mu^2-4\mu\rho^2+2\mu\rho+8\mu-3\rho^2+6\rho+1.$$

Taking derivative of $k(\cdot)$ with respect to ρ yields

$$\frac{\partial k(\cdot)}{\partial \rho} = 2(1-\mu-\rho)[3+2\,\mu(2-\mu)] > 0.$$

Hence, for any $\mu < 1-\rho$ and ρ it holds that $k(\cdot) > \lim_{\rho\to 0} k(\cdot)= 4\,\mu^4 + 12\mu^2(1-\mu) + 4\,\mu(2-\mu^2) + 1\ > 0$. It follows that $h(\cdot) > 0$. Consequently, $\sum_i \partial\pi_i^*(\cdot)/\partial\rho > 0$.

ii) It is straightforward that $\partial p_i^{u*}(\cdot)/\partial\zeta$, $\partial a_i^*(\cdot)/\partial\zeta$, $\partial p_i^{a*}(\cdot)/\partial\zeta$, $\partial n_i^*(\cdot)/\partial\zeta$ are negative (positive)for $i=1$ $(i=2)$. This implies that $\partial\pi_1^*(\cdot)/\partial\zeta < 0$ and $\partial\pi_2^*(\cdot)/\partial\zeta > 0$. We now consider how platforms' joint profits change:

$$\frac{\partial\pi_1^*(\cdot)}{\partial\zeta} + \frac{\partial\pi_2^*(\cdot)}{\partial\zeta} =$$
$$= -\left[\frac{4\Delta\left[(2-\rho^2) + 2\,\mu(2-\mu) + 2\,\rho(1-\mu)\right] + \rho\left[(1-\rho-\mu) + (1\ -\mu)\right]}{2\left(1+2\mu(2-\mu)-\rho(\rho+2(\mu-1))\right)^2}\right] q \le 0,$$

holding with equality if $\Delta = \rho = 0$. As ζ cannot be further increased if $\Delta = 0$, we have $\sum_i \partial\pi_i^*(\cdot)/\partial\zeta < 0$. *Q.E.D.*

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Chapter 2

Joint Customer Data Acquisition and Sharing Among Rivals

Abstract

This chapter is based on joint research with Irina Suleymanova and Nicola Jentzsch. It has benefited a lot from the helpful comments of Pio Baake, Ulrich Kamecke, Kai-Uwe Kühn, Sudipta Sarangi and Christian Wey.

It is increasingly observable that in different industries competitors jointly acquire and share customer data. We propose a modified Hotelling model with two-dimensional consumer heterogeneity to analyze the incentives for such agreements and their welfare implications. In our model the incentives of firms for data acquisition and sharing depend on the willingness of consumers to switch brands. Firms jointly collect data on transportation cost parameters when consumers are relatively immobile between brands. However, the firms are unlikely to cooperatively acquire such data, when consumers are relatively mobile. Incentives to share information depend on the portfolio of data firms hold and consumer mobility. Data sharing arises with relatively mobile and immobile consumers - it is neutral for consumers in the former case, but reduces consumer surplus in the latter. Competition authorities ought to scrutinize such cooperation agreements on a case-by-case basis and devote special attention to consumer switching behavior.

2.1 Introduction

Recent advances in information technologies allow firms to collect, analyze and share detailed information about customers and to use this information for targeted offers. The use of customer databases for price discrimination attracted the attention of regulators and privacy advocates alike. Two types of cooperation based on customer data are particularly wide-spread: i)cooperative data collection, and ii)information sharing.

There are several industries, where rivals cooperate in obtaining customer data. For example, national medical associations often provide uniform software solutions to members in order to manage patient medical records, which essentially standardizes customer data doctors acquire. Another example of cooperative data acquisition is the case of U.S. colleges, where education institutions cooperate in the College Board to jointly collect information on students for awarding institutional aid funds.

Beyond cooperation on data acquisition, the possibility to share customer data between competitors is also widely discussed in many industries. Airlines exchange detailed data on personal characteristics and travel details of passengers and target promotions to customers. Other examples include the retail industry, where firms join database cooperatives to share customer information for marketing purposes. Participants of information exchange include magazines and newspapers, which trade personal information about subscribers.

Joint customer data acquisition and information sharing initiated a heated debate between consumer privacy advocates, business groups, competition authorities and other regulators. At the same time, theoretical work on the topic is still evolving. We analyze the incentives of rival firms to cooperate on the acquisition and sharing of customer data, when firms use data to make targeted price offers. We also evaluate welfare effects of these practices in the context of a modified Hotelling model with competitive first- and third-degree price discrimination. We extend the standard model by introducing heterogeneity in consumer transportation costs. In addition, we allow firms to hold two different datasets on consumers reflecting i)brand preferences and ii)transportation cost parameters. Moreover, firms may only hold data on all consumers. We do not consider the case where firms hold data on a subset of consumers. Our approach applies well to markets, where a leading firm with a new technology is enabled to collect detailed customer profiles and to provide tailored services based upon these profiles, while competitors do not have the same ability. It also applies to newly liberalized markets, where the incumbent holds detailed purchase histories of all consumers. Depending on the data a firm holds, it offers uniform prices to all consumers, targets specific consumer groups (third-degree price discrimination)or sets individual prices (first-degree price discrimination). Firms may obtain data in addition to existing datasets and exchange data with the rival.

We are interested in three main questions: First, what type of data is acquired by both firms when firms agree to cooperatively collect data? Second, under what conditions is a firm holding a particular dataset willing to provide the competitor with access to it? Third, how does data acquisition cooperation and information sharing affect competition and welfare? To focus on the competitive effects of joint information acquisition and sharing, we assume that firms use data solely for price discrimination purposes. The important questions of collusion incentives and consumer privacy are beyond the scope of the present article.

We make the following contributions: By introducing heterogeneity in consumer transportation cost parameters into the standard Hotelling model, we show how incentives to acquire and share customer data depend on the *type* of information. Further, we allow firms to hold asymmetric data on consumers and derive incentives for partial information sharing.

Our results highlight the important role of the willingness of consumers to switch brands on the incentives of firms to jointly acquire data or to engage in information sharing. If a small price decrease can motivate a relatively large share of consumers to switch brands, cooperation between firms (holding similar types of customer data)for acquiring additional data does not take place. However, there is potential for information sharing, which is neutral for consumers and enhances social welfare. On the other hand, if consumers are generally loyal to their firms and price changes induce little switching, cooperation on data acquisition and sharing can be profitable. If such cooperation takes place, it is harmful to consumer surplus.

The main intuition of our results is as follows: If consumers are relatively mobile, a cooperation aimed at increasing the ability of firms to target individuals or specific groups is more likely to induce competition. This in turn provides little scope for using data for extracting rents, which makes cooperation unattractive for firms. Information sharing may still be profitable for firms, if it increases allocative efficiency, arising from the even allocation of consumers between firms. Equilibrium pricing strategies change with the mobility of consumers. When consumers are relatively immobile, price changes induce little switching. Firms can use customer data to extract rents from consumers, whereas the competition-intensifying effect of additional data is weak. Under these circumstances, consumers are likely to be harmed when firms cooperate by joint customer data acquisition or information sharing.

We conclude that competition authorities ought to scrutinize cooperation agreements between rival firms with respect to customer data acquisition and sharing on a case-by-case basis. Apart from the possibility that intensified information flows between rivals may facilitate collusion, a critical aspect concerning the competitive effects of a cooperation based upon customer data is whether consumers *are mobile enough* to render positive effects.

The rest of the article is organized as follows. Section 2 reviews the related literature. The model is presented in Section 3. In Section 4, we investigate the incentives of firms to cooperate in acquiring information on consumers. Section 5 turns to the analysis of information sharing, Section 6 concludes. Proofs are provided in the Appendix.

2.2 Related Literature

Despite the increasing importance of the acquisition and sharing of customer information among rivals, few theoretical articles directly addressed this issue.[1] Most relevant to our

[1] Sharing of data on customers is addressed in the banking literature, but this strand focuses on default risk of customers, whereas we consider data on consumer preferences.

work are Liu and Serfes (2006)and Chen et al. (2001), who focus on the sharing of data on customer brand preferences between rivals. Liu and Serfes employ a two-period duopoly model with horizontally and vertically differentiated firms. In the first period, firms set uniform prices and collect information about customers. In the second period, firms use the information to make personalized offers. The authors show that information sharing takes place if firms are sufficiently asymmetric in customer bases. With sufficient asymmetry, the smaller firm has an incentive to share its customer information with the larger one. We take a different approach to model information exchange: By allowing firms to distinguish between consumer brand preferences and transportation cost parameters, we are able to address the question of partial information sharing, i.e., the exchange of only one type of information. In contrast to the results of Liu and Serfes is Chen et al. (2001), who show that firms engage in the sharing of customer data only when market shares are not too asymmetric and the level of customer targetability is low. Liu and Serfes (2006)as well as Chen et al. (2001)argue that it is the market shares of firms that drives information sharing. In our setup it is the willingness of consumers to switch brands together with the portfolio of data that firms hold, which determines whether or not information sharing takes place. In contrast to the cited literature we find that information sharing may occur even with firms having perfectly symmetric market shares, depending on the consumer data firms hold. Similar to our analysis, Esteves (2009)considers price discrimination in a two-dimensional setting where firms have access to partial information on brand and product preferences of consumers. The author presents a two-dimensional Hotelling model with consumers located on a unit square, where the axes represent the two dimensions of consumer preferences. With partial information, firms can observe a consumer's location in only one of two dimensions and discriminate accordingly. Her main result is that price discrimination increases industry profits, if firms have information about the locations of consumers in the less differentiated dimension and ignore information about the more differentiated one.

This article is also related to the literature on competitive price discrimination. Earlier articles in this strand of literature focus on the question whether competition eliminates price discrimination. Borenstein (1985)presents a spatial model of monopolistic competition and shows that price discrimination prevails in a duopoly environment. He treats consumers as being heterogeneous along three dimensions: their reservation prices and brand preferences as well as the strength of the latter. The author relies on numerical simulation to determine which sorting strategy is more profitable: price discrimination based upon reservation prices or strength of brand preferences.

Thisse and Vives (1988)apply a standard Hotelling model, where firms may or may not observe the location of each consumer in the market. The authors show that price discrimination tends to intensify competition for each consumer and that discriminatory prices are usually lower than uniform prices. A similar insight is derived from a model of competitive couponing by Bester and Petrakis (1996)who analyze the sellers' incentives to offer rebates to their customers in two distinct regions. They find that offering rebates to consumers in form of coupons tends to intensify competition, which leads to lower prices and profits. In their survey on price discrimination Armstrong (2006)and Stole (2007) summarize the competitive effects of price discrimination and use the notion of best-response symmetry and asymmetry originally introduced by Corts (1998). We will rely on this concept to explain our results and discuss it in greater detail later on.

2.3 The Model

We present a duopoly pricing game between two differentiated firms, A and B, each selling a variety of the same product. Firms are situated at the two ends of a Hotelling line of unit length with firm A located at point 0 and firm B at point 1. Every consumer is characterized by an address $x \in [0, 1]$ corresponding to his brand preference for the ideal product. If the consumer buys from a firm, which does not provide the ideal product, he incurs linear transportation costs proportional to the distance to the firm. We depart from the standard Hotelling setup by introducing heterogeneity in consumer transportation costs per unit distance, which we denote by $t \in [\underline{t}, \overline{t}]$. Consumers are distributed uniformly and independently on a rectangle, where the horizontal (vertical)axis represents consumer brand preference (transportation cost parameter). The mass of consumers is normalized to one and every consumer is uniquely described by a pair (t, x). With t and x being uniformly and independently distributed, we have the following density functions: $f_t = 1/(\bar{t} - \underline{t})$, $f_x = 1$, $f_{t,x} = 1/(\bar{t} - \underline{t})$. We distinguish between two versions of the model based on the distribution of transportation cost parameters. In the first version we call consumers *relatively mobile* and assume that $\underline{t} = 0$. In the second version we assume that $\underline{t} > 0$ and $\bar{t}/\underline{t} \leq 2$ and label consumers as *relatively immobile*. When consumers are relatively mobile, switching brands is costless for some consumers (those with $\underline{t} = 0$). In the model with relatively immobile consumers, switching involves costs for every consumer, and the difference between the highest and lowest transportation cost parameter is not too large.[2] The utility of a consumer from buying at firm $i \in \{A, B\}$ is given by

$$U_i(p_i, t, x) = \upsilon - t\,|x - x_i| - p_i(t, x),$$

where υ is a basic utility from consuming the product, which is the same across all consumers, x_i is firm i's address with $x_A = 0$ and $x_B = 1$ and $p_i(t, x)$ is the price firm i offers to consumer (t, x). A consumer buys from the firm delivering higher utility. Firm A provides a strictly higher utility if the following condition holds:

$$t(1 - 2x) + p_B > p_A. \tag{2.1}$$

Assumption 1 states our tie-breaking rule.

Assumption 1: *In case both firms offer equal utilities, i.e.,*

$$t(1 - 2x) + p_B = p_A, \tag{2.2}$$

the consumer chooses the firm closer in the brand preference space (if $x = 1/2$, then w.l.o.g. the consumer visits firm A).

[2]Transportation costs are closely related to switching costs as both capture how sensitive consumers react to price changes. There is evidence that switching rates vary in different industries as well as among consumers (European Commission 2009).

In case of a price tie, consumers behave in the socially optimal manner and choose the closest firm. We say a consumer (t, x) is on *firm i's turf,* if he chooses firm i over firm j if prices are equal. The turf of firm A (B)is given by consumers with $x \leq 1/2$ ($x > 1/2$). Depending on the available data, firms can adopt the following strategies: If a firm has information on both consumer locations and transportation cost parameters, it can offer individual prices for each consumer. With information on either locations or transportation cost parameters, a firm can discriminate across groups of consumers. Without customer data, a firm sets uniform prices. Marginal costs are assumed to be zero. Firms set prices $p_i(t, x)$ to maximize their profits,

$$\Pi_i = \int_{X_i}\int_{T_i} f_{t,x} p_i(t, x) dt dx,$$

with X_i and T_i denoting the domains of locations and transportation cost parameters for consumers who buy from firm i. Next, we explain the way firms may acquire, hold and share customer data and describe the game played.

Customer Data and Timing

Let X and T be two sets containing information about the brand preferences and transportation cost parameters of *all* consumers, respectively. We refer to X and T as *datasets.* We define the union of datasets firm i holds as firm i's *information set* and denote it by I_i. Each firm may either hold information only about transportation cost parameters ($I_i = T$), only about brand preferences ($I_i = X$), complete information about consumer preferences ($I_i = X \cup T$), or no information ($I_i = \emptyset$). To simplify the notation, we write $I_i = XT$ to denote the case where firm i has complete information on consumers. We use the term *information scenario* to describe the datasets held by both firms in a pricing game, $\{I_A, I_B\}$. The superscript $I_A|I_B$ indexes values of functions and variables in the information scenario $\{I_A, I_B\}$. For example, $\Pi_A^{XT|XT}$ denotes the profit function of firm A when both firms have full information on consumers: $\{I_A, I_B\} = \{XT, XT\}$. We refer to the cases, where $I_A = I_B$ as *symmetric information scenarios.* Cases, where $I_A \neq I_B$ are referred to as *asymmetric information scenarios.* Throughout the article, we assume that firms can acquire and exchange datasets X and/or T in their entirety. We rule out the case, where data is acquired or shared for only a subset of consumers.

Firms may engage in two types of cooperation involving customer data: i) *joint information acquisition*, and ii) *information sharing.* We analyze these cooperation types separately.

In the case of joint information acquisition (JIA), the following game is played:

Stage 1 (JIA): Firms decide cooperatively whether or not to acquire dataset X or T or both from an external source, in addition to the data they already hold. Simultaneously, they decide on a distribution rule for profits realized in the next stage. Apart from this transfer, the acquisition of data is assumed to be costless. After the data is acquired, it becomes available to both firms.

Stage 2 (JIA): Firms compete in prices and realize profits, which are distributed according to the rule agreed upon in stage 1.

When firms cooperate in information sharing (IS), the game unfolds as follows:

Stage 1 (IS): The firm holding more datasets decides whether and which dataset to sell to the rival. Simultaneously, the firms decide on a distribution rule for profits realized in the next stage, which determines the price of the dataset sold. After the sale of a dataset, it is available to both firms.

Stage 2 (IS): Firms compete in prices and realize profits, which are distributed according to the rule agreed upon in stage 1.

We do not model the rule for profit distribution in Stage 1 of both games. Instead, we analyze whether a necessary condition for both types of cooperation is fulfilled, which is a strict increase of joint profits. The following Assumption relates to the timing of pricing decisions in stage 2 of both games.

Assumption 2: *In symmetric information scenarios, firms set prices simultaneously. In asymmetric information scenarios, the firm with less information moves first and the other firm follows.*

The timing structure specified in Assumption 2 is consistent with most literature on competitive price discrimination, where firms choose their targeted offers after setting uniform prices (e.g., Thisse and Vives, 1988; Shaffer and Zhang, 2000; and Liu and Serfes 2006). Furthermore, it corresponds to the observation that prices are adjusted slower, if they are applied to a larger group of consumers. In particular, it is more difficult to adjust a firm's regular (uniform)price to a large customer group compared to changing discounts (by coupons)and targeted offers to smaller groups. For the remainder of this article we assume that firm A (B)is the firm with more (less)information. To solve the pricing game in stage 2, we seek for subgame-perfect Nash equilibria in asymmetric information scenarios and Nash equilibria in symmetric information scenarios. We restrict our attention to pure strategies.

The case when firms have no data constitutes a useful benchmark for further analysis. The following lemma shows that a symmetric Nash equilibrium in pure strategies in the information scenario, where firms do not hold any customer data exists when consumers are relatively immobile and does not exist when consumers are relatively mobile.

Lemma 1. *When firms have no information about consumers (i.e.* $\{I_A, I_B\} = \{\emptyset, \emptyset\}$*), then*

i)no symmetric pure-strategy Nash equilibrium exists if consumers are relatively mobile, and

ii)there is a unique pure-strategy Nash equilibrium if consumers are relatively immobile : Both firms' prices equal the harmonic mean of the range of transportation cost parameters.

With relatively mobile consumers, for any strictly positive price of the competitor, a firm finds it profitable to undercut the rival: a small advantage in price allows to attract new consumers. Zero prices can not constitute an equilibrium either: by increasing its price slightly, a firm can attract the closest consumers with the highest transportation cost parameters and make positive profits. With relatively immobile consumers, undercutting the competitor does not constitute a profitable strategy in the equilibrium as consumers do not easily switch brands.

We next consider equilibria in other information scenarios. Proposition 1 states our results.

Proposition 1. *Equilibrium prices and profits in each information scenario are as stated in Tables 1 and 2, respectively.*

Proof. See Appendix.

Table 1: Equilibrium Prices in Different Information Scenarios

I_A	I_B	p_A^*	p_B^*
Relatively Mobile Consumers			
X	X	$\begin{cases} 2\bar{t}\left(1-2x\right)/3,\ x\leq 1/2 \\ \bar{t}\left(2x-1\right)/3,\ x>1/2 \end{cases}$	$\begin{cases} \bar{t}\left(1-2x\right)/3,\ x\leq 1/2 \\ 2\bar{t}\left(2x-1\right)/3,\ x>1/2 \end{cases}$
T	T	t	t
XT	XT	$\begin{cases} t\left(1-2x\right),\ x\leq 1/2 \\ 0,\ x>1/2 \end{cases}$	$\begin{cases} 0,\ x\leq 1/2 \\ t\left(2x-1\right),\ x>1/2 \end{cases}$
X	$\emptyset$	$\begin{cases} \bar{t}(0.73-x),\ 0\leq x<0.27 \\ 0.465\bar{t},\ 0.27\leq x\leq 0.5 \\ \bar{t}(1.47-2x),\ 0.5<x<0.62 \\ 0.24\bar{t},\ 0.62\leq x\leq 1 \end{cases}$	$0.47\bar{t}$
T	$\emptyset$	$\begin{cases} 0.85\bar{t}-t,\ t<0.28\bar{t} \\ (0.85\bar{t}+t)/2,\ t\geq 0.28\bar{t} \end{cases}$	$0.85\bar{t}$
XT	$\emptyset$	$\max\left\{0, 0.28\bar{t}+t(1-2x)\right\}$	$0.28\bar{t}$
XT	X	$\begin{cases} t\left(1-2x\right),\ x\leq 1/2 \\ (2x-1)\left(\bar{t}/2-t\right),\ x>1/2,\ t<\bar{t}/2 \\ 0,\ x>1/2,\ t\geq\bar{t}/2 \end{cases}$	$\begin{cases} 0,\ x\leq 1/2 \\ \bar{t}(x-1/2),\ x>1/2 \end{cases}$
XT	T	$\max\left\{0, t/2+t(1-2x)\right\}$	$t/2$

Continued. on the next page

Table 1 continued

I_A	I_B	p_A^*	p_B^*
		Relatively Immobile Consumers	
$\emptyset$	$\emptyset$	$H(\underline{t},\bar{t})$	$H(\underline{t},\bar{t})$
X	X	$\begin{cases} \underline{t}(1-2x),\ x \leq 1/2 \\ 0,\ x > 1/2 \end{cases}$	$\begin{cases} 0,\ x \leq 1/2 \\ \underline{t}(2x-1),\ x > 1/2 \end{cases}$
T	T	t	t
XT	XT	$\begin{cases} t(1-2x),\ x \leq 1/2 \\ 0,\ x > 1/2 \end{cases}$	$\begin{cases} 0,\ x \leq 1/2 \\ t\,(2x-1)\,,\ x > 1/2 \end{cases}$
X	$\emptyset$	$\begin{cases} \widetilde{H}(\underline{t},\bar{t})+\underline{t}(1-2x),\ x \leq \frac{1}{2} \\ \widetilde{H}(\underline{t},\bar{t})-\bar{t}(2x-1),\ \frac{1}{2} < x < \frac{1}{2}+\frac{\widetilde{H}(\underline{t},\bar{t})}{2(2\bar{t}-\underline{t})} \\ \frac{\widetilde{H}(\underline{t},\bar{t})-\underline{t}(2x-1)}{2},\ \frac{1}{2}+\frac{\widetilde{H}(\underline{t},\bar{t})}{2(2\bar{t}-\underline{t})} \leq x \leq \frac{\underline{t}+\widetilde{H}(\underline{t},\bar{t})}{2\underline{t}} \\ 0,\ x > \frac{1}{2}+\frac{\widetilde{H}(\underline{t},\bar{t})}{2\underline{t}} \end{cases}$	$\widetilde{H}(\underline{t},\bar{t})$
T	$\emptyset$	$t/2+3H(\underline{t},\bar{t})/4$	$3H(\underline{t},\bar{t})/2$
XT	$\emptyset$	$\max\{0, H(\underline{t},\bar{t})/2+t(1-2x)\}$	$H(\underline{t},\bar{t})/2$
XT	X	$\begin{cases} t(1-2x),\ x \leq 1/2 \\ 0,\ x > 1/2 \end{cases}$	$\begin{cases} 0,\ x \leq 1/2 \\ \underline{t}(2x-1),\ x > 1/2 \end{cases}$
XT	T	$\max\{0, t/2+t(1-2x)\}$	$t/2$

$A(\underline{t},\bar{t}) = (\bar{t}+\underline{t})/2$, $H(\underline{t},\bar{t}) = (\bar{t}-\underline{t})/\ln(\bar{t}/\underline{t})$, $\widetilde{H}(\underline{t},\bar{t}) = (\bar{t}-\underline{t})/\ln(2\bar{t}/\underline{t}-1)$

Note that in equilibrium firms use all available customer data for price discrimination and do not ignore any data. The equilibrium prices in Table 1 are functions of the available data that firms hold. In symmetric information scenarios, a firm's best-response function specifies the profit-maximizing price to any given price of the competitor. In this case, the only effect of not using all available customer data is to decrease the degrees of freedom in pricing. The same is true for the firm with more datasets in asymmetric information scenarios, which moves after observing the competitor's price. Perhaps less obviously, the firm with fewer datasets also maximizes its profit by using all the available customer data. Although firms maximize profits by using all the available customer data, higher profits could be reached by committing not to use some data sets. In particular, they could enjoy higher individual and joint profits by committing not to use data on consumer brand preferences.

Table 2: Equilibrium Profits in Different Information Scenarios

I_A	I_B	$\Pi_A^{I_A\|I_B}$	$\Pi_B^{I_A\|I_B}$	$\Pi_A^{I_A\|I_B}$	$\Pi_B^{I_A\|I_B}$
		Relatively Mobile Consumers		Relatively Immobile Consumers	
$\emptyset$	$\emptyset$			$H(\underline{t},\bar{t})/2$	$H(\underline{t},\bar{t})/2$
X	X	$\bar{t}/8$	$\bar{t}/8$	$\underline{t}/4$	$\underline{t}/4$
T	T	$\bar{t}/4$	$\bar{t}/4$	$A(\underline{t},\bar{t})/2$	$A(\underline{t},\bar{t})/2$
XT	XT	$\bar{t}/8$	$\bar{t}/8$	$A(\underline{t},\bar{t})/4$	$A(\underline{t},\bar{t})/4$
X	$\emptyset$	$0.32\bar{t}$	$0.12\bar{t}$	$5\widetilde{H}(\underline{t},\bar{t})/8+\underline{t}/4$	$\widetilde{H}(\underline{t},\bar{t})/4$
T	$\emptyset$	$0.53\bar{t}$	$0.23\bar{t}$	$21H(\underline{t},\bar{t})/32+A(\underline{t},\bar{t})/8$	$9H(\underline{t},\bar{t})/16$
XT	$\emptyset$	$0.32\bar{t}$	$0.05\bar{t}$	$5H(\underline{t},\bar{t})/16+A(\underline{t},\bar{t})/4$	$H(\underline{t},\bar{t})/8$
XT	X	$5\bar{t}/32$	$\bar{t}/16$	$A(\underline{t},\bar{t})/4$	$\underline{t}/4$
XT	T	$9\bar{t}/32$	$\bar{t}/16$	$9A(\underline{t},\bar{t})/16$	$A(\underline{t},\bar{t})/8$

$A(\underline{t},\bar{t})= (\bar{t}+\underline{t})/2$, $H(\underline{t},\bar{t})= (\bar{t}-\underline{t})/\ln(\bar{t}/\underline{t})$, $\widetilde{H}(\underline{t},\bar{t})= (\bar{t}-\underline{t})/\ln(2\bar{t}/\underline{t}-1)$

To understand the differences in equilibrium profits in Table 2, it is useful to recall the concepts of *best-response symmetry* and *best-response asymmetry* discussed by Corts (1998). He refers to models, where both firms set higher prices for the same group of consumers as exhibiting best-response symmetry. In contrast, best-response asymmetry exists, where one firm sets lower (higher)prices for those consumers who have a higher (lower)willingness to pay for the other firm. Prices and profits tend to be higher with best-response symmetry and lower with best-response asymmetry. To illustrate this, we first consider symmetric information scenarios. In these cases, with both relatively mobile and relatively immobile consumers, profits are the highest when both firms have data only on consumer transportation cost parameters. In contrast, if both firms hold dataset X (either alone or together with dataset T), they realize lower profits.

We obtain best-response symmetry when firms only know transportation cost parameters. All other symmetric information scenarios give rise to best-response asymmetry. When firms only hold dataset T, both set higher prices for those consumers, who are less willing to switch brands (i.e., those with higher values of t)and lower prices to those, who are ready to switch brands. In our case, best-response functions take the form

$$p_i^{T|T}(p_j|\, t)= \begin{cases} (p_j+t)/2, & p_j<3t \\ p_j-t, & p_j\geq 3t, \end{cases}$$

for $i,j\in\{A,B\}$ and $i\neq j$. Provided that $p_j<3t$, $p_i^{T|T}(p_j|\, t)$ increases in t. In contrast, if firms have information only on brand preferences and consumers are relatively immobile, the best-response functions for $x<1/2$ are

$$p_A^{X|X}(p_B|\, x \;<\; 1/2)= \begin{cases} \left[p_B+\bar{t}(1-2x)\right]/2, & p_B<\bar{t}(1-2x) \\ p_B, & p_B\geq \bar{t}(1-2x) \end{cases}$$

$$p_B^{X|X}(p_A|\, x \;<\; 1/2)= \begin{cases} \left[p_A-\underline{t}(1-2x)\right]/2, & p_A<2\bar{t}(1-2x) \\ p_A-\bar{t}(1-2x), & p_A\geq 2\bar{t}(1-2x). \end{cases}$$

If $x>1/2$, then the best-response functions take the form:

$$p_A^{X|X}(p_B|\, x \;>\; 1/2)= \begin{cases} p_B/2, & p_B<2\bar{t}(2x-1) \\ p_B-\bar{t}(2x-1), & p_B\geq 2\bar{t}(2x-1) \end{cases}$$

$$p_B^{X|X}(p_A|\, x \;>\; 1/2)= \begin{cases} \left[p_A+\bar{t}(2x-1)\right]/2, & p_A<\bar{t}(2x-1) \\ p_A, & p_A\geq \bar{t}(2x-1). \end{cases}$$

Clearly, in the case where firms have data only on brand preferences, every firm sets a higher price for consumers, who prefer its brand and lower ones for those who like the competitor more. As both groups of consumers ($x<1/2$ and $x>1/2$)have different brand preferences, the best-response functions imply best-response asymmetry. Formally, $p_A^{X|X}(p_B|\, x<1/2)>p_A^{X|X}(p_B|\, x>1/2)$, whereas $p_B^{X|X}(p_A|\, x<1/2)<p_B^{X|X}(p_A|\, x>1/2)$. If both types of information are available to the firms, best-response asymmetry is preserved. The best-response functions in this case are

$$p_A^{XT|XT}(p_B|\,x)=\begin{cases} p_B+t(1-2x), & x\leq 1/2\\ \max\{0,p_B+t(1-2x)-\epsilon\}, & x>1/2\end{cases}$$

$$p_B^{XT|XT}(p_A|\,x)=\begin{cases} \max\{0,p_A-t(1-2x)-\epsilon\}, & x\leq 1/2\\ p_A-t(1-2x), & x>1/2,\end{cases}$$

where ϵ is an infinitesimal, positive value. It is easily verified that $p_A^{XT|XT}(p_B|x \leq 1/2) > p_A^{XT|XT}(p_B|x > 1/2)$ whereas $p_B^{XT|XT}(p_A|x \leq 1/2) < p_B^{XT|XT}(p_A|x > 1/2)$, hence, the reaction functions imply best-response asymmetry.

In the asymmetric information scenarios, firms' profits are the highest in the information scenario $\{T, \emptyset\}$, in which case both firms set high prices to consumers. In contrast, profits are the lowest in the information scenario $\{XT, X\}$, which exhibits best-response asymmetry.

The concepts of best-response symmetry and asymmetry explain well why prices and profits are higher in some information scenarios than in others. In the following we demonstrate that the concepts, nevertheless, do not completely explain the incentives to jointly acquire and share customer data. In particular, they cannot be applied to situations when the market exhibits the same best-response property before and after cooperation.

By jointly acquiring customer data that neither firm holds beforehand or by making a proprietary database available to the rival, firms can influence the competitive environment. These decisions are the subject of the next sections.

2.4 Joint Acquisition of Customer Data

We now analyze the incentives of firms to cooperatively acquire customer data for price discrimination. We focus on symmetric information scenarios with firms holding identical datasets and analyze the incentives to jointly acquire additional information on consumer preferences, which (after acquisition)becomes available to both firms.

First, our results show that price discrimination may provide sufficient incentives for joint information acquisition. Only information on consumer transportation cost parameters can be jointly acquired, but not information on brand preferences. Second, incentives to jointly acquire data on transportation cost parameters depend on the consumer willingness to switch brands. Although more information potentially allows firms to extract more rents from consumers, intensified price competition may reduce prices and profits. The competition effect dominates, if consumer mobility is relatively high. If consumers are relatively loyal to their brands, acquiring data on transportation cost parameters induces little additional competition. The following proposition summarizes our insights on joint data acquisition incentives and Table 3 illustrates our results for the case of relatively mobile consumers with $\bar{t} = 1$ and the case of relatively immobile consumers with $\underline{t} = 1$ and $\bar{t} = 2$.

Proposition 2. *Firms' incentives to jointly acquire information on consumer preferences depend on the distribution of transportation cost parameters.*

i)If consumers are relatively mobile and firms have partial information on consumers (either $\{I_A, I_B\} = \{X, X\}$ or $\{I_A, I_B\} = \{T, T\}$), firms have no incentives to jointly acquire further information for price discrimination purposes. Profits across symmetric information scenarios are ranked as $\Pi_i^{XT|XT} < \Pi_i^{X|X} < \Pi_i^{T|T}$.

ii)If consumers are relatively immobile, firms do not jointly acquire dataset X, but acquire dataset T. Profits across information scenarios are ranked as $\Pi_i^{X|X} < \Pi_i^{XT|XT} < \Pi_i^{\emptyset|\emptyset} < \Pi_i^{T|T}$.

Proof. See Appendix.

Firms do not jointly acquire information on brand preferences, but only acquire information on consumer transportation cost parameters. Since additional information on consumer brand preferences always induces best-response asymmetry, firms do not jointly acquire dataset X. If firms initially have no information on consumers and acquire dataset T, they switch to best-response symmetry, which increases industry profits.

Table 3: Profits and Incentives for Joint Information Acquisition

Before Data Acquisition					Data Acquired	After Data Acquisition					Acquire?
I_A	I_B	$\Pi_A^{I_A\|I_B}$	$\Pi_B^{I_A\|I_B}$	$\Sigma\Pi$		I_A	I_B	$\Pi_A^{I_A\|I_B}$	$\Pi_B^{I_A\|I_B}$	$\Sigma\Pi$	
Relatively Mobile Consumers ($\underline{t}$= 0 and $\bar{t}=1$)											
X	X	.14	.14	.28	T	XT	XT	.13	.13	.26	No
T	T	.25	.25	.50	X	XT	XT	.13	.13	.26	No
Relatively Immobile Consumers ($\underline{t}=1$ and $\bar{t}=2$)											
$\emptyset$	$\emptyset$	.72	.72	1.44	X	X	X	.25	.25	.50	No
$\emptyset$	$\emptyset$	.72	.72	1.44	T	T	T	.75	.75	1.50	Yes
$\emptyset$	$\emptyset$	.72	.72	1.44	XT	XT	XT	.38	.38	.75	No
X	X	.25	.25	.50	T	XT	XT	.38	.38	.75	Yes
T	T	.75	.75	1.50	X	XT	XT	.38	.38	.75	No

When firms initially hold dataset X and cooperate on gathering dataset T, the concepts of best-response symmetry and asymmetry cannot be applied to explain incentives to acquire customer data. As mentioned above, both information scenarios $\{X, X\}$ and $\{XT, XT\}$ exhibit best-response asymmetry. Whether information acquisition takes place, depends on consumer mobility and is not driven by a change in the best-response property of the market. If consumers do not differ much in terms of the strength of their brand preferences (i.e., $\bar{t}/\underline{t} \leq 2$), acquiring dataset T is profitable. If, however, consumer mobility is relatively high, then complementing dataset X with T reduces industry profits.

A closer look at the two main effects at work reveals why firms do not acquire dataset T in addition to brand preference data with relatively mobile consumers and why they do acquire it if consumer mobility is low. First, the *rent-extraction effect*: more information on consumers enables firms to better target and segment consumers. Second, the *competition effect* takes account for the change in the strength of price competition between firms. Whether firms have incentives to acquire additional information on consumers depends on the sum of these two effects.

If consumers are relatively immobile, they visit the closest firm in both information scenarios $\{X, X\}$ and $\{XT, XT\}$, as shown in Figure 1. Additional information on transportation cost parameters allows firms to better target consumers. Although with the firms having both datasets X and T each consumer receives individual offers from both firms, as consumers are relatively immobile, the better targeting induces little competition and the rent-extraction effect dominates.

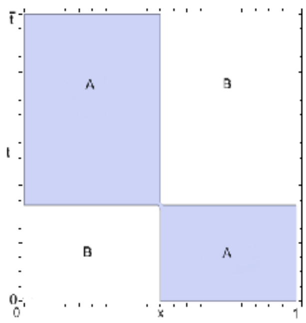
However, if consumers are mobile, firms will not complement their existing data on brand preferences with dataset T. Note that pricing strategies and, hence, equilibrium prices in the scenario where both firms have full information, do not depend on the distribution of transportation cost parameters. The reason for the altered incentives to acquire dataset T is that the pricing decisions of firms in the information scenario $\{X, X\}$ change depending on the mobility of consumers. Let us take a closer look at the strategies of the firms in this information scenario. Due to the symmetry of firms, it is sufficient to focus on the region with $x \leq 1/2$ and analyze competition on firm A's turf.

In information scenario $\{X, X\}$, if consumer mobility is low, for any given price by firm B to a group of consumers with brand preferences $x \leq 1/2$, firm A can keep all consumers in this group without significantly decreasing its price offered to them. Firm A's optimal strategy is to set a price for a group x, which allows to attract all members, even those who are most willing to switch, i.e., consumers with the lowest transportation cost parameters. The low willingness of consumers to switch brands and firm A's strategy to hold them all in turn induces firm B to price very aggressively on A's turf and to decrease its price to zero, putting a downward pressure on firm A's prices. In the end, firm A is able to keep all consumers on its own turf, but only by charging every group x a relatively low price. The same forces are at work on firm B's turf. With industry profits being relatively low, moving into the scenario with full customer data is attractive for the competitors, where they can extract more consumer surplus. If consumer mobility is high, it is expensive for firm A to hold all consumers with a given x. To achieve this, firm A must reduce its prices to prevent consumers with the lowest transportation costs from switching to firm B. It is more profitable for firm A to give up the most mobile consumers and set a price for every group x, which targets the consumers with higher values of t. Firm B is, hence, able to capture the most mobile consumers on A's turf, even with a relatively high price. In the emerging equilibrium firm A sets prices to every group x on its turf to target consumers with higher transportation cost parameters, while firm B targets those with lower values of t. With industry profits being relatively high in the information scenario $\{X, X\}$, firms do not want to acquire data on consumer transportation costs.

Figure 1: Demand Regions in $\{X, X\}$ and $\{XT, XT\}$

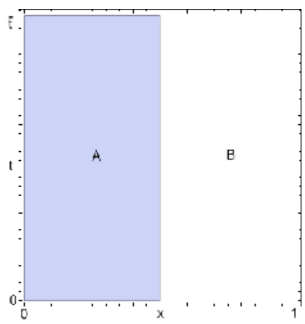
Relatively Mobile Consumers

Before (Possible)Acquisition of Dataset T



$I_A = X, I_B = X$

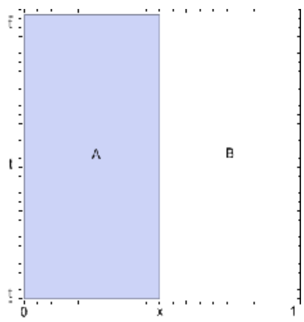
After (Possible)Acquisition of Dataset T



$I_A = XT, I_B = XT$

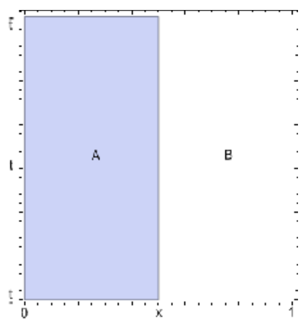
Relatively Immobile Consumers

Before Acquisition of Dataset T



$I_A = X, I_B = X$

After Acquisition of Dataset T



$I_A = XT, I_B = XT$

A (B) denotes demand region of firm A (B)

Our results show that best-response symmetry and asymmetry are not anchored in a particular type of information. The same type of information can induce both best-response symmetry and asymmetry depending on the additional data firms own. In particular, information on transportation cost parameters may induce different strategies, either best-response symmetry (if only dataset T is available)or best-response asymmetry (if dataset T is combined with dataset X). This extends the analysis in Armstrong (2006), who emphasizes that firms have an incentive to acquire information about their consumers, if firms can discriminate between consumers according to their transportation cost parameters. We show that this might not always be the case: It holds that industry profits are higher if firms can only discriminate based on T compared to the case when firms lack consumer data. However, depending on the distribution of transportation cost parameters, industry profits may either decrease or increase, when firms have access to both sets of information compared to the case, when they can only discriminate based on X.

Next, we compare consumer surplus and social welfare across information scenarios and draw conclusions about the welfare implications of joint information acquisition. The next proposition summarizes our results.

Proposition 3. *The ranking of consumer surplus (CS)and social welfare (SW)in symmetric information scenarios and welfare implications of joint customer data acquisition depend on the distribution of the transportation cost parameters.*

i)If consumers are relatively mobile, then consumer surplus and social welfare are ranked as $CS^{T|T} < CS^{X|X} < CS^{XT|XT}$ *and* $SW^{X|X} < SW^{T|T} = SW^{XT|XT}$.

ii)If consumers are relatively immobile, then consumer surplus is ranked as $CS^{T|T} < CS^{\emptyset|\emptyset} < CS^{XT|XT} < CS^{X|X}$ *and social welfare is same in all the symmetric information scenarios. Joint acquisition of dataset T reduces consumer surplus and is neutral to social welfare.*

Proof. See Appendix.

Two effects determine the ranking of consumer surplus along information scenarios: First, a competition effect capturing the level of prices, and second, an allocative effect related to the distribution of consumers between firms. Allocative efficiency requires that consumers choose the nearest firm. The only case, where allocative efficiency is distorted is the scenario where both firms hold dataset X and consumers are relatively mobile: consumers with the lowest transportation cost parameters ($t < \bar{t}/3$)then visit the firm further away, giving rise to allocative inefficiencies. When allocative efficiency is preserved, the ranking of consumer surplus is the opposite of the ranking of industry profits.

We conclude that price discrimination may provide sufficient incentives for firms to cooperatively acquire information on consumer transportation costs. With mobile consumers, firms do not acquire additional data if they already hold some, although doing so would be socially beneficial. With immobile consumers, firms cooperate on acquiring data on transportation cost parameters, regardless what data they already have. This is neutral to social welfare and decreases consumer surplus.

2.5 Sharing of Customer Data

We now analyze the incentive of a firm with more information to share it with the competitor. Our main question is under which conditions a firm possessing a particular dataset is willing to provide the competitor with access to it. The dataset(s)with information on brand preferences and/or on transportation cost parameters may be given to the rival. We call information exchange *partial*, if a firm has access to both datasets, but shares only one of them with its competitor. The following proposition summarizes our results on information sharing and Table 4 shows how information sharing alters profits using the examples with $\bar{t} = 1$ for relatively mobile consumers and $\underline{t} = 1$ and $\bar{t} = 2$ for relatively immobile consumers.

Proposition 4. *Incentives to share information depend on the distribution of consumer transportation cost parameters and the portfolio of data firms hold.*
i) With relatively mobile consumers, a firm with full information on consumers shares its data on transportation cost parameters with the competitor, if the latter holds data on customer brand preferences.
ii)If consumers are relatively immobile, then data on consumer transportation cost parameters is shared in two cases: First, if one firm has full information on consumers, whereas the

other holds data on customer brand preferences, and second, if one firm has full information on consumers, whereas the other has no data.

Proof. See Appendix.

Table 4: Joint Profits and Incentives for Information Sharing

Before (Possible)Data Sharing					Data Shared	After (Possible)Data Sharing					Share?
I_A	I_B	$\Pi_A^{I_A\|I_B}$	$\Pi_B^{I_A\|I_B}$	$\Sigma\Pi$		I_A	I_B	$\Pi_A^{I_A\|I_B}$	$\Pi_B^{I_A\|I_B}$	$\Sigma\Pi$	
Relatively Mobile Consumers ($\bar{t} = 1$)											
X	$\emptyset$	.32	.12	.44	X	X	X	.14	.14	.28	No
T	$\emptyset$	.53	.23	.76	T	T	T	.25	.25	.50	No
XT	$\emptyset$	.32	.05	.37	X	XT	X	.16	.06	.22	No
XT	$\emptyset$	.32	.05	.37	T	XT	T	.28	.06	.34	No
XT	X	.16	.06	.22	T	XT	XT	.13	.13	.26	Yes
XT	T	.28	.06	.34	X	XT	XT	.13	.13	.26	No
Relatively Immobile Consumers ($\underline{t} = 1$ and $\bar{t} = 2$)											
X	$\emptyset$	.82	.23	1.05	X	X	X	.25	.25	.50	No
T	$\emptyset$	1.13	.81	1.94	T	T	T	.75	.75	1.50	No
XT	$\emptyset$	.83	.18	1.01	X	XT	X	.38	.25	.63	No
XT	$\emptyset$	.83	.18	1.01	T	XT	T	.84	.19	1.03	Yes
XT	X	.38	.25	.63	T	XT	XT	.38	.38	.75	Yes
XT	T	.84	.19	1.03	X	XT	XT	.38	.38	.75	No

A conventional explanation for the incentives of firms to share information is whether doing so induces best-response symmetry in the market (Armstrong 2006). For instance, data on consumer brand preferences is never shared in our model. The reason for this is that dataset X induces best-response asymmetry (and, hence, stronger competition)if both firms have it. This offsets any benefits arising from the possibility to better target consumers. Although dataset X is never shared, it plays a decisive role for the incentives of firms whether to share the dataset T. We call this interplay between the datasets X and T the *portfolio effect*. With this label we refer to the observation that the incentives to share a particular dataset depend on what other data both firms already hold. The same dataset may or may not be shared with the competitor depending on what additional data firms already hold. In particular, the necessary condition for sharing dataset T is that the firm with more information also holds dataset X. If one firm owns data only on transportation cost parameters (while the other has no data at all), information sharing does not take place.[3]

Our results highlight the importance of consumer transportation cost parameters on the incentives of firms to share customer data. With mobile consumers, a firm with full information does not share its dataset T with the competitor who holds no data, while in the same scenario with relatively immobile consumers this data is shared even without monetary transfers. Figure 2 presents the demand regions with relatively mobile and immobile consumers for the information scenarios $\{XT, \emptyset\}$ and $\{XT, T\}$. The differences in incentives to share dataset T in the scenario $\{XT, \emptyset\}$ depend on consumer mobility and originate from the differences in pricing strategies of the firm with less information (firm B)before potential data sharing. In the scenario after information sharing (i.e., in $\{XT, T\}$)regardless of the distribution of transportation cost parameters, firm B sets $p_B = t/2$ and firm A matches this price to leave consumers indifferent whenever it can with a non-negative price. Firm A pursues the same strategy in the information scenario before potential information sharing (i.e., in $\{XT, \emptyset\}$): it matches the price of the competitor and leaves consumers indifferent whenever it can set a non-negative price. The strategy of firm B, however, depends on

[3] This result contrasts with Armstrong (2006), who shows that with simultaneous pricing decisions dataset T is shared in the information scenario, where one firm holds only dataset T, while the other does not have any customer data. It is easy to check that with simultaneous pricing decisions our model also predicts that the firm possessing dataset T shares it with the competitor both with mobile and immobile consumers.

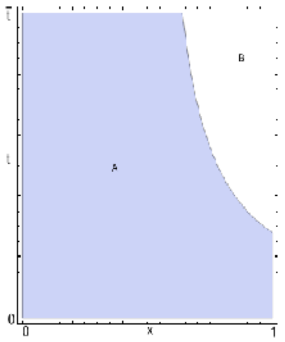
the level of consumer mobility in information scenario $\{XT, \emptyset\}$. If consumers are mobile, firm B tailors its price to target only the most loyal consumers (i.e., those who are close to it and have high transportation costs). This relatively high price serves as basis for firm A as well, resulting in high overall industry profits. In contrast, with relatively immobile consumers (given firm A's strategy), it is optimal for firm B to set a uniform price, which allows to attract some of the consumers even with the lowest transportation costs, close to firm B. The latter must decrease its price to avoid being undercut by firm A, resulting in a relatively low uniform price set by firm B. As firm A bases its prices on firm B's uniform price, all prices in the market are relatively low.

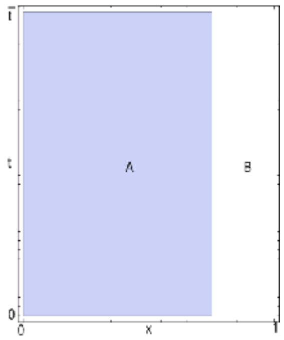
What changes, if firm B obtains database T? By being able to identify groups of consumers with the same transportation cost parameters, firm B sets lower (higher)prices to those with lower (higher)values of t. With relatively mobile consumers, firm B's uniform price is targeted at consumers with higher values of t. In this case the improved ability to price discriminate allows firm B to increase its price only for a few consumers (with nearly maximal values of transportation cost parameters), while it reduces the price for all consumers with lower t values. As firm A acts similarly, the additional information generally leads to a price decrease in the market. With relatively immobile consumers, the price of firm B is aimed to appeal even to consumers with low values of t. And with additional data on transportation cost parameters firm B can increase the price for most consumers, which drives up firm A's prices as well. Hence, with immobile consumers both firms profit from sharing dataset T.

Figure 2: Demand Regions in $\{X, X\}$ and $\{XT, XT\}$

Relatively Mobile Consumers

Before (Possible)Sharing of Dataset T | After (Possible)Sharing of Dataset T

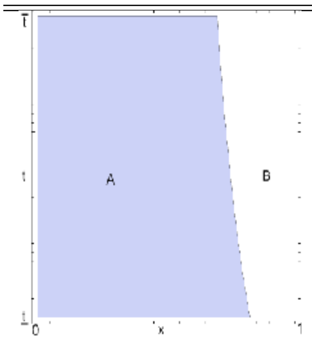


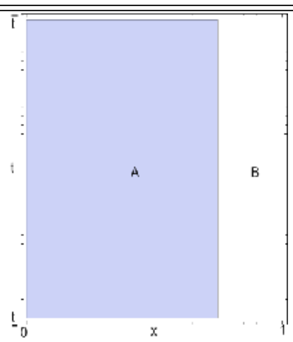


$\{XT, \emptyset\}$ | $\{XT, T\}$

Relatively Immobile Consumers

Before Sharing of Dataset T | After Sharing of Dataset T





$\{XT, \emptyset\}$ | $\{XT, T\}$

A (B)denotes demand region of firm A (B)

Finally, we turn to the welfare implications of customer information sharing. Proposition 5 summarizes our insights.

Proposition 5. *Welfare implications of customer data sharing depend on the distribution of transportation cost parameters among consumers.*

i) With relatively mobile consumers, information sharing is neutral for consumer surplus and enhances social welfare.

ii) With relatively immobile consumers, information sharing always decreases consumer surplus and social welfare either decreases or does not change.

Proof. See Appendix.

Proposition 5 highlights the importance of consumer mobility for the welfare effects of information sharing. When consumers are relatively mobile, information sharing is Pareto-optimal: it increases profits and leaves consumer surplus unchanged. However, with relatively immobile consumers information sharing harms consumers and is at best neutral to social welfare. In our setup, social welfare can only decrease due to the misallocation of consumers, which occurs if consumers do not visit their closest firm.

When consumers are mobile and a firm with full information shares its dataset T with the rival holding dataset X, social welfare increases, because it leads to a more efficient allocation of consumers among the firms. In the resulting equilibrium all consumers are served by their most preferred firm. Consumers on firm B's turf with high transportation costs lose, because firm B uses its new dataset T to extract higher rents from them. However, consumers on firm B's turf with low transportation cost parameters gain, because they are served by their preferred firm. In our setting, these two effects cancel each other out, which renders information sharing neutral for consumer surplus.

When consumers are relatively immobile between brands, information sharing takes place in two cases: a firm with full information shares its dataset T with the rival either holding dataset X or holding no information. In the former case, sharing customer data does not affect social welfare as consumers choose the closest firm both before and after the transaction. Information sharing leads here solely to a redistribution of rents from consumers to firms due to the improved targeting ability. If consumers are relatively immobile and the firm with full information shares dataset T with its rival (who initially holds no data), social welfare decreases. This result is driven by the increased misallocation of consumers between firms: Some consumers with high values of t (which previously visited their most preferred firm, B)now choose firm A. This negative effect is not compensated by the improved allocation of some consumers with low values of t, which previously visited their less preferred firm, A. Since industry profits increase due to data sharing, consumer surplus declines.

2.6 Conclusions

It is increasingly observable that competitors in different information-intensive industries coordinate on information acquisition in terms of standardization or exchange profiles of their customers with each other. These activities have raised the suspicion of consumer advocates as well as regulatory authorities. We present a modified Hotelling model with first- and third-degree price discrimination and horizontally differentiated firms, which possess different sets of data on consumer preferences (that is brand preferences and transportation cost parameters). Of particular interest to us are two kinds of agreements between rivals: joint acquisition and sharing of customer data.

We model cooperation with regard to customer data in a novel manner: We distinguish between two datasets firms may acquire and share, which encompass brand preferences and transportation cost parameters. We analyze how the incentives to engage in cooperation involving customer data depend on the type of information. Furthermore, we allow firms to hold asymmetric customer data. A firm with more datasets can decide to share its datasets with the competitor. With relatively mobile consumers, firms do not cooperate on acquiring customer data, if they already hold any of the two datasets. When consumers are immobile, firms cooperate to obtain the dataset on transportation cost parameters regardless of whether they possess data on brand preferences. In this case, information acquisition reduces consumer surplus and is neutral to social welfare. Incentives to share information depend on the portfolio of data the firms hold and the distribution of consumers with respect to their transportation cost parameters. Information sharing may arise with both relatively mobile and immobile consumers. Whereas information sharing is at best neutral for consumer surplus, it enhances social welfare with relatively mobile consumers.

Our results highlight that the evaluation of such agreements depends on the welfare standard adopted by a competition authority. Competition authorities pursuing a consumer surplus standard should be critical towards cooperation agreements between competitors involving customer data. Consumers are especially likely to be harmed, if their willingness to switch brands is low. Taking into account other potentially problematic issues such as privacy and collusion (which are not addressed herein), we are sceptical that consumers benefit overall from such agreements. However, under a social welfare standard information sharing is beneficial, if consumers are relatively mobile, in which case it improves allocative efficiency.

Appendix

Definitions and Notation. Before we proceed with the proofs, we introduce some definitions and notation. Let $t^c(p_A, p_B, x)$ denote the transportation cost parameters of those consumers with brand preference x, who are indifferent between firms A and B for given prices p_A and p_B: $t^c(\cdot)= (p_B - p_A)/(2x-1)$. It holds that $U_A(p_A, t^c(\cdot), x)= U_B(p_B, t^c(\cdot), x)$. For given p_A, p_B and x we have $\Pr\{t \geq t^c(\cdot)\} = 0$ if $t^c(\cdot) > \bar{t}$, $\Pr\{t \geq t^c\} = f(t)\left[\bar{t} - t^c(\cdot)\right]$ if $\underline{t} \leq t^c(\cdot) \leq \bar{t}$ and $\Pr\{t \geq t^c\} = 1$ if $t^c(\cdot) < \underline{t}$. As equilibrium strategies may differ on the intervals $x < 1/2$ and $x > 1/2$, it is useful to distinguish between $\underline{t}^c := t^c(\cdot, x < 1/2)$ and $\bar{t}^c := t^c(\cdot, x > 1/2)$.

Similarly, let $x^c(p_A, p_B, t)$ denote the brand preference of consumers with transportation cost parameter t indifferent between firms A and B for given prices p_A and p_B: $x^c(\cdot)= 1/2 - (p_A - p_B)/2t$. It holds that $U_A(p_A, t, x^c(\cdot))= U_B(p_B, t, x^c(\cdot))$. For given p_A, p_B and t it holds that $\Pr\{x \geq x^c(\cdot)\} = 0$ if $x^c(\cdot) > 1$, $\Pr\{x \geq x^c(\cdot)\} = 1 - x^c(\cdot)$ if $0 \leq x^c(\cdot) \leq 1$ and $\Pr\{x \geq x^c(\cdot)\} = 1$ if $x^c(\cdot) < 0$. Let $\underline{x}(p_A, p_B, \underline{t})$ and $\bar{x}(p_A, p_B, \bar{t})$ denote the brand preferences of the indifferent consumers for given prices p_A and p_B with the lowest and highest transportation cost parameters, respectively. Formally, $t^c(p_A, p_B, \underline{x})= \underline{t}$ and $t^c(p_A, p_B, \bar{x})= \bar{t}$.

We introduce $A(\underline{t}, \bar{t}):= (\bar{t} + \underline{t})/2$ and $H(\underline{t}, \bar{t}):= (\bar{t} - \underline{t})/\ln(\bar{t}/\underline{t})$ to denote the arithmetic and the harmonic mean of the transportation cost parameters $t \in \left[\underline{t}, \bar{t}\right]$ when $\underline{t} > 0$, respectively. Note that for any $\underline{t}$, $\bar{t}$ it holds that $A(\underline{t}, \bar{t}) > H(\underline{t}, \bar{t})$. We also introduce $\widetilde{H}(\underline{t}, \bar{t}):= (\bar{t} - \underline{t})/\ln\left[(2\bar{t} - \underline{t})/\underline{t}\right]$. Moreover, if $\underline{t} > 0$ we denote the ratio of the highest and the lowest transportation cost parameters as $k := \bar{t}/\underline{t}$.

We will omit the notation of information scenarios for best-response functions and equilibrium prices, which should be clear from the context.

Proof of Lemma 1. We first prove part *i)* of Lemma 1. We show that a small deviation downwards from the competitor's price is always profitable. Without loss of generality we focus on the pricing of firm A. If firm A sets $p_A = p_B > 0$, it captures half of the consumers and realize profits $\Pi_A^{\emptyset|\emptyset}(p_A = p_B, p_B)= p_B/2$. If firm A deviates downwards by setting

$p_A < p_B$, it captures all consumers on its own turf and some consumers with low transportation cost parameters on the competitor's turf. Solving $t^c(\cdot)= \bar{t}$ for x we obtain $\bar{x} = (p_B - p_A)/(2\bar{t}) + 1/2$. Firm A's profit if $p_A < p_B$ is $\Pi_A^{\emptyset|\emptyset}(p_A < p_B, p_B) = \int_0^{\bar{x}}\int_0^{\bar{t}} [f(t)p_A]\, dtdx + \int_{\bar{x}}^1\int_0^{t^c} [f(t)p_A]\, dtdx = p_A\left[(p_B - p_A)/\left(2\bar{t}\right) + 1/2 - (p_B - p_A)\ln\left((p_B - p_A)/\bar{t}\right)/2\right]$. It is helpful to introduce $\Delta = p_B - p_A$ with $\Delta \in (0, p_B]$ as the magnitude of firm A's downward deviation from firm B's price. Comparing profits with and without deviation from $p_B > 0$, we obtain that deviation is not profitable if $p_B < \Delta + \bar{t}/\left[1 - \bar{t}\ln(\Delta/\bar{t})\right]$ for any $\Delta \in (0, p_B]$. We now show that there is no such price p_B, which fulfills the latter condition. Note that the RHS of this condition is increasing in Δ, hence, it is fulfilled for any $\Delta \in (0, p_B]$ if and only if it holds for the lowest possible value of Δ. As $\lim_{\Delta \longrightarrow 0}[\Delta + \bar{t}/[1 - \bar{t}\ln(\Delta/\bar{t})]] = 0$, the condition is always violated.

It remains to consider whether $p_A = p_B = 0$ constitutes an equilibrium. This is not the case as these prices yield zero profits to both firms. With a minimal deviation upward, firm A could attract the nearest consumers with the highest transportation cost parameters and make positive profit. This completes the proof of part *i)* in Lemma 1.

We now turn to the proof of part *ii)*. Assume that $\underline{t} > 0$ and $\bar{t}/\underline{t} \leq 2$. Since firms are symmetric, we focus without loss of generality on the pricing of firm B. Consider first the case where firm B sets a (weakly)higher price than firm A: $p_B \geq p_A$. Let $\overline{d} = p_B - p_A$. Depending on the level of $\overline{d}$, the demand regions may take two possible forms: One with $\underline{x} \leq 1$ $(0 \leq \overline{d} \leq \underline{t})$and another with $\underline{x} > 1$ $(\underline{t} < \overline{d} < \bar{t})$. Let $0 \leq \overline{d} \leq \underline{t}$. In this case profits are $\Pi_A^{\emptyset|\emptyset}(p_A, p_B \geq p_A) = \int_0^{\bar{x}}\int_{\underline{t}}^{\bar{t}} [f(t)p_A]\, dtdx + \int_{\bar{x}}^{\underline{x}}\int_{\underline{t}}^{t^c} [f(t)p_A]\, dtdx$ and $\Pi_B^{\emptyset|\emptyset}(p_A, p_B \geq p_A) = \int_{\bar{x}}^{\underline{x}}\int_{t^c}^{\bar{t}} [f(t)p_B]\, dtdx + \int_{\underline{x}}^1\int_{\underline{t}}^{\bar{t}} [f(t)p_B]\, dtdx$. Maximization yields the reaction function $p_i(p_j) = \left[p_j + H(\underline{t}, \bar{t})\right]/2$ with $i, j \in \{A, B\}$ and $i \neq j$. The optimal prices are $p^* = H(\underline{t}, \bar{t})$. The corresponding profits are $\Pi_i^{\emptyset|\emptyset}(p^*, p^*) = H(\underline{t}, \bar{t})/2$. Note that these prices satisfy $\underline{x} \leq 1$. Assume next that $\underline{t} < \overline{d} < \bar{t}$, in which case $\Pi_B^{\emptyset|\emptyset}(H(\underline{t}, \bar{t}), H(\underline{t}, \bar{t}) + \overline{d}) = \int_{\underline{x}}^1\int_{t^c}^{\bar{t}} [f(t)p_B]\, dtdx = f(t)\left[\overline{d} + H(\underline{t}, \bar{t})\right]\left[\bar{t} - \overline{d} + \overline{d}\ln\left(\overline{d}/\bar{t}\right)\right]/2$. Taking the derivative with respect to $\overline{d}$ we get $\partial\Pi_B^{\emptyset|\emptyset}(H(\underline{t}, \bar{t}), H(\underline{t}, \bar{t}) + \overline{d})/\partial\overline{d} = f(t)\left[\bar{t} - \overline{d} + \left(2\overline{d} + H(\underline{t}, \bar{t})\right)\ln\left(\overline{d}/\bar{t}\right)\right]/2$, which is negative if $\bar{t}/\underline{t} \leq 2$. It follows that $\Pi_B^{\emptyset|\emptyset}(H(\underline{t},\bar{t}), H(\underline{t},\bar{t})+\ \overline{d}) < \Pi_B^{\emptyset|\emptyset}(H(\underline{t},\bar{t}), H(\underline{t},\bar{t}))$ for any $0 \leq \overline{d} \leq \bar{t}$, hence, firm B does not have an incentive to deviate upwards when firm A sets $p_A = H(\underline{t},\bar{t})$.

We next analyze deviation downwards where firm B sets a (weakly)lower price than firm A: $p_B \leq p_A$. Let $\underline{d} = p_A - p_B$. Depending on the level of $\underline{d}$, the demand regions may take two possible forms: One with $0 \leq \underline{x} \leq 1/2$ ($0 \leq \underline{d} \leq \underline{t}$)and another with $\underline{x} < 0$ ($\underline{t} < \underline{d} < H(\underline{t},\bar{t})$). Let $0 \leq \underline{d} < \underline{t}$. Note that in this case the optimization problem of firm B mirrors that of firm A when $0 \leq \overline{d} \leq \underline{t}$, and it holds that $\Pi_B^{\emptyset|\emptyset}(H(\underline{t},\bar{t}), H(\underline{t},\bar{t}) - \underline{d}) \leq \Pi_B^{\emptyset|\emptyset}(H(\underline{t},\bar{t}), H(\underline{t},\bar{t}))$, with equality if $\underline{d} = 0$. Assume next that $\underline{t} < \underline{d} < H(\underline{t},\bar{t})$. Firm B realizes $\Pi_B^{\emptyset|\emptyset}(H(\underline{t},\bar{t}), H(\underline{t},\bar{t}) - \underline{d})= \int_0^{\underline{x}} \int_{\underline{t}}^{t^c} [f(t)p_B]\, dtdx + \int_{\underline{x}}^{1} \int_{\underline{t}}^{\bar{t}} [f(t)p_B]\, dtdx = \left[\underline{d} - H(\underline{t},\bar{t})\right]\left[2\underline{t} - \underline{d} - \bar{t} + d\ln\left(\underline{d}/\bar{t}\right)\right] / \left[2(\bar{t} - \underline{t})\right]$. Taking the derivative with respect to $\underline{d}$ we get $\partial\Pi_B^{\emptyset|\emptyset}(H(\underline{t},\bar{t}), H(\underline{t},\bar{t}) - \underline{d})/\partial\underline{d} = -\left[\underline{d} + \bar{t} - 2\underline{t} + (H(\underline{t},\bar{t}) - 2\underline{d})\ln(\underline{d}/\bar{t})\right] / \left[2(\bar{t} - \underline{t})\right]$. This expression is negative with $\bar{t}/\underline{t} \leq 2$. It follows that $\Pi_B^{\emptyset|\emptyset}(H(\underline{t},\bar{t}), H(\underline{t},\bar{t}) - \underline{d}) < \Pi_B^{\emptyset|\emptyset}(H(\underline{t},\bar{t}), H(\underline{t},\bar{t}))$. Hence, for any $0 \leq \underline{d} \leq H(\underline{t},\bar{t})$ we have that $\Pi_B^{\emptyset|\emptyset}(H(\underline{t},\bar{t}), H(\underline{t},\bar{t}) - \underline{d}) < \Pi_B^{\emptyset|\emptyset}(H(\underline{t},\bar{t}), H(\underline{t},\bar{t}))$, with equality if $\underline{d} = 0$, hence, firm B does not have an incentive to deviate downwards when firm A sets $p_A = H(\underline{t},\bar{t})$. *Q.E.D.*

Proof of Proposition 1. We derive equilibrium prices and profits of the firms in different information scenarios. We first consider the symmetric information scenarios.

Claim 1. Let $\underline{t} = 0$. Consider the information scenario $\{X, X\}$. In equilibrium firm i sets $p_i^*(x)= 2\ \bar{t}\,|1 - 2x|/3$ on its own turf and $p_i^*(x)=\ \bar{t}\,|1 - 2x|/3$ on the competitor's turf. Firm i serves consumers with $t \geq \bar{t}/3$ on its own turf and consumers with $t < \bar{t}/3$ on the competitor's turf and realizes profit $\Pi_i^{X|X} = \bar{t}/8$.

Proof of Claim 1. As firms are symmetric, we only analyze pricing strategies on firm A' turf. Consider first $x < 1/2$. A consumer in this region chooses firm A if $t \geq t^c$. Both firms treat the consumer transportation cost parameter as a random variable and maximize their expected profits for a given value of x: $E\left[\Pi_A^{X|X} |x < 1/2\right] = p_A \Pr\{t \geq t^c\}$ and $E\left[\Pi_B^{X|X} |x < 1/2\right] = p_B \Pr\{t < t^c\}$. Solving the corresponding maximization problems yields equilibrium prices $p_A^*(x) = 2\,\bar{t}(1-2x)/3$ and $p_B^*(x) = \bar{t}(1-2x)/3$ for $x < 1/2$. Consider now $x = 1/2$. It follows from Assumption 1 that $E\left[\Pi_B^{X|X}|x = 1/2\right] = 0$, whenever $p_B \geq p_A$. Firm B will always undercut firm A if $p_A^*(1/2) > 0$, hence, it must be that $p_A^*(1/2) = p_B^*(1/2) = 0$. From $p_A^*(x)$ and $p_B^*(x)$ when $x \leq 1/2$ we get $t^c = \bar{t}/3$. To compute firm A's equilibrium profit we sum up the revenues across the demand regions: $\Pi_A^{X|X} = \int\limits_0^{1/2}\int\limits_{t^c}^{\bar{t}}\left[f(t)2\bar{t}(1-2x)/3\right]dtdx + \int\limits_{1/2}^{1}\int\limits_{0}^{t_c}\left[f(t)\bar{t}(2x-1)/3\right]dtdx = 5A(\underline{t},\bar{t})/18$. Since firms are symmetric, $\Pi_B^{X|X} = \Pi_A^{X|X}$. This completes the proof of Claim 1.

Claim 2. Let $\underline{t} > 0$ *and* $k \leq 2$. *Consider the information scenario* $\{X, X\}$. *In equilibrium firm* i *sets* $p_i^*(x) = \underline{t}\,|1-2x|$ *on its own turf and* $p_i^*(x) = 0$ *on the competitor's turf. Every firm serves all consumers on its own turf and realizes profit* $\Pi_i^{X|X} = \underline{t}/4$.

Proof of Claim 2. As firms are symmetric, we only analyze firms' pricing strategies on firm A' turf. A consumer in this region chooses firm A if $t \geq t^c$. Both firms treat consumer transportation cost as a random variable and maximize their expected profits for a given value of x: $E\left[\Pi_A^{X|X}|x < 1/2\right] = p_A \Pr\{t \geq t^c\}$ and $E\left[\Pi_B^{X|X}|x < 1/2\right] = p_B \Pr\{t < t^c\}$. Solving the corresponding maximization problems yields equilibrium prices $p_A^*(x) = \underline{t}(1-2x)$ and $p_B^*(x) = 0$ for $x < 1/2$. Consider now $x = 1/2$. It follows from Assumption 1 that $E\left[\Pi_B^{X|X}|x = 1/2\right] = 0$, whenever $p_B \geq p_A$. Firm B will always undercut firm A if $p_A^*(1/2) > 0$, hence, it must hold that $p_A^*(1/2) = p_B^*(1/2) = 0$. On its turf firm A serves all consumers. Equilibrium profits are: $\Pi_A^{X|X} = \int\limits_0^{1/2}\int\limits_{\underline{t}}^{\bar{t}}\left[f(t)\underline{t}(1-2x)\right]dtdx = \underline{t}/4 = \Pi_B^{X|X}$. This completes the proof of Claim 2.

Claim 3. Consider the information scenario $\{T, T\}$. *In equilibrium firm* i *sets* $p_i^*(t) = t$ *and serves all consumers on its own turf. Firms realize profits* $\Pi_i^{T|T} = A(\underline{t},\bar{t})/2$.

Proof of Claim 3. Both firms treat consumer brand preference as a random variable and maximize their expected profits: $E\left[\Pi_A^{T|T}|t\right] = f(t)p_A \Pr\{x \leq x^c\}$ and $E\left[\Pi_B^{T|T}|t\right] = f(t)p_B \Pr\{x > x^c\}$, which yields $p_A^*(t) = p_B^*(t) = t$ and $x^c = 1/2$. Firm A realizes the profit $\Pi_A^{T|T} = \int\limits_0^{x^c}\int\limits_{\underline{t}}^{\bar{t}}\left[f(t)t\right]dtdx = A(\underline{t},\bar{t})/2$. It holds that $\Pi_A^{T|T} = \Pi_B^{T|T}$. This completes the proof of Claim 3.

Claim 4. Consider the information scenario $\{XT, XT\}$. In equilibrium firm i sets $p_i^*(x,t)= t\,|1-2x|$ on its own turf and $p_i^*(x,t)= 0$ on the competitor's turf, and serves all consumers on its own turf. Firms realizes profits $\Pi_i^{XT|XT} = A(\underline{t}, \overline{t})/4$.

Proof of Claim 4. As firms are symmetric, we only consider pricing decisions in the region $x \in [0, 1/2]$. Here firm A has a cost advantage, hence, its best-response to any price of firm B is to render consumers indifferent by setting $p_A(p_B)= p_B + t(1-2x)$. Firm B's best-response is to undercut firm A's price by setting $p_B(p_A)= p_A - t(1-2x) - \varepsilon$ whenever it is feasible (i.e., $p_A - t(1-2x) > 0$), with $\epsilon > 0$. Otherwise, firm B sets $p_B = 0$. As undercutting is not possible in equilibrium, we get $p_B^*(x,t)= 0$ and $p_A^*(x,t)= t(1-2x)$. Firm A's profit is $\Pi_A^{XT|XT} = \int\limits_0^{1/2} \int\limits_{\underline{t}}^{\overline{t}} [f(t)t(1-2x)]\, dtdx = A(\underline{t}, \overline{t})/4$. Due to the symmetry, $\Pi_A^{XT|XT} = \Pi_B^{XT|XT}$. This completes the proof of Claim 4.

We now turn to the asymmetric information scenarios.

Claim 5. Let $\underline{t} = 0$. Consider the information scenario $\{X, \emptyset\}$. If $x < 1/2 - p_B^*/(2\overline{t})$, then in equilibrium firm A sets $p_A^*(x)= \left(\overline{t}\,(1-2x)+ p_B^*\right)/2$ and serves consumers with $\overline{t}/2 - p_B^*/\left[2(1-2x)\right] \leq t \leq \overline{t}$. If $1/2 - p_B^*/(2\overline{t}) \leq x \leq 1/2$, then firm A sets $p_A^*(x)= p_B^*$ and serves consumers with $t \leq \overline{t}$. If $1/2 < x < 1/2 + p_B^*/(4\overline{t})$, then firm A sets $p_A^*(x)= p_B^* - \overline{t}(2x-1)$ and serves consumers with $t \leq \overline{t}$. If $x \geq 1/2 + p_B^*/(4\overline{t})$, then firm A sets $p_A^*(x)= p_B^*/2$ and serves consumers with $t \leq p_B^*/\left[2(2x-1)\right]$. Firm B sets $p_B^* = 0.47\overline{t}$. Firms realize profits $\Pi_A^{X|\emptyset} = 0.32\overline{t}$ and $\Pi_B^{X|\emptyset} = 0.12\overline{t}$.

Proof of Claim 5. On its own turf firm A maximizes expected profit $E\left[\Pi_A^{X|\emptyset}|x < 1/2\right] = \Pr\{t \geq t^c\}\,p_A$, which yields reaction functions $p_A(p_B)= \left(\overline{t}(1-2x)+ p_B\right)/2$ if $0 \leq p_B < \overline{t}(1-2x)$ and $p_A(p_B)= p_B$ if $p_B \geq \overline{t}(1-2x)$. Moreover, $p_A(p_B)= p_B$ if $x = 1/2$. The reaction functions give $\underline{t}^c(x, p_B)= \overline{t}/2 - p_B/\left[2(1-2x)\right]$. Solving $\underline{t}^c(x, p_B)= \underline{t} = 0$ we get $\underline{x}(p_B)= 1/2 - p_B/\left(2\overline{t}\right)$. If $x \leq \underline{x}(p_B)$, firm A captures consumers with $t \geq \underline{t}^c(x, p_B)$, while it gets all consumers if $\underline{x}(p_B) < x \leq 1/2$. On the competitor's turf firm A maximizes the expected profit $E\left[\Pi_A^{X|\emptyset}|x > 1/2\right] = \Pr\{t < t^c\}\,p_A$, which yields the reaction functions $p_A(p_B)= p_B - \overline{t}(2x-1)$ if $p_B \geq 2\overline{t}(2x-1)$ and $p_A(p_B)= p_B/2$ if $0 \leq p_B < 2\overline{t}(2x-1)$.

These reaction functions give $\bar{t}^c(x,p_B) = p_B/[2(2x-1)]$. Solving $\bar{t}^c(x,p_B) = \bar{t}$ we get $\bar{x}(p_B,\bar{t}) = 1/2 + p_B/(4\bar{t})$. If $1/2 < x < \bar{x}(p_B,\bar{t})$, then firm A gets all consumers, while it captures consumers with $t < \bar{t}^c(x,p_B)$ if $x \geq \bar{x}(p_B,\bar{t})$. Given firm A's reaction functions, firm B's profit is $\Pi_B^{X|\emptyset} = \int_0^{\underline{x}}\int_0^{\underline{t}^c} [f(t)p_B]\,dtdx + \int_{\bar{x}}^{1}\int_{\bar{t}^c}^{\bar{t}} [f(t)p_B]\,dtdx$. Maximization of the latter profit yields $p_B^* = 0.47\bar{t} < \bar{t}$, which implies that, indeed, $0 < \underline{x}(p_B^*) < 1/2$ and $1/2 < \bar{x}(p_B^*) < 1$. Firm A's profit is computed as $\Pi_A^{X|\emptyset} = \int_0^{\underline{x}}\int_{\underline{t}^c}^{\bar{t}} \left[f(t)(\bar{t}(1-2x) + p_B^*)/2\right] dtdx + \int_{\underline{x}}^{1/2}\int_0^{\bar{t}} [f(t)p_B^*]\,dtdx + \int_{1/2}^{\bar{x}}\int_0^{\bar{t}} \left[f(t)\left(p_B^* - \bar{t}(2x-1)\right)\right] dtdx + \int_{\bar{x}}^{1}\int_0^{\bar{t}^c} [f(t)(p_B^*/2)]\,dtdx$. Firms realize profits $\Pi_A^{X|\emptyset} = 0.32\bar{t}$ and $\Pi_B^{X|\emptyset} = 0.12\bar{t}$. This completes the proof of Claim 5.

Claim 6. Let $\underline{t} > 0$ and $k \leq 2$. *Consider the information scenario* $\{X,\emptyset\}$. *In equilibrium, on its own turf firm A sets* $p_A^*(x) = p_B^* + \underline{t}(1-2x)$ *and serves all consumers. If* $1/2 < x < 1/2 + p_B^*/\left[2(2\bar{t}-\underline{t})\right]$, *then firm A sets* $p_A^*(x) = p_B^* - \bar{t}(2x-1)$ *and serves all consumers. If* $1/2 + p_B^*/\left[2(2\bar{t}-\underline{t})\right] \leq x \leq 1/2 + p_B^*/(2\underline{t})$, *then firm A sets* $p_A^*(x) = [p_B^* - \underline{t}(2x-1)]/2$ *and serves consumers with* $t < \underline{t}/2 + p_B^*/[2(2x-1)]$. *If* $x > 1/2 + p_B^*/(2\underline{t})$, *then firm A sets* $p_A^*(x) = 0$ *and serves no consumers. Firm B sets* $p_B^* = \widetilde{H}(\underline{t},\bar{t})$. *Firms realize profits* $\Pi_A^{X|\emptyset} = 5\widetilde{H}(\underline{t},\bar{t})/8 + \underline{t}/4$ *and* $\Pi_B^{X|\emptyset} = \widetilde{H}(\underline{t},\bar{t})/4$.

Proof of Claim 6. On its own turf firm A maximizes the expected profit $E\left[\Pi_A^{X|\emptyset}|x < 1/2\right] = \Pr\{t \geq t^c\}\, p_A$, which yields the reaction function $p_A(p_B) = p_B + \underline{t}(1-2x)$. Moreover, if $x = 1/2$, then $p_A(p_B) = p_B$. Firm A captures all consumers on its own turf. On the competitor's turf firm A maximizes the expected profit $E\left[\Pi_A^{X|\emptyset}|x > 1/2\right] = \Pr\{t < t^c\}\, p_A$, which yields the reaction functions $p_A(p_B) = p_B - \bar{t}(2x-1)$ if $p_B \geq (2\bar{t}-\underline{t})(2x-1)$, $p_A(p_B) = [p_B - \underline{t}(2x-1)]/2$ if $\underline{t}(2x-1) < p_B < (2\bar{t}-\underline{t})(2x-1)$ and $p_A(p_B) = 0$ if $p_B \leq \underline{t}(2x-1)$. These reaction functions give $t^c(x,p_B) = \underline{t}/2 + p_B/[2(2x-1)]$. Solving $t^c(x,p_B) = \bar{t}$ we get $\bar{x}(p_B,\bar{t}) = 1/2 + p_B/\left[2(2\bar{t}-\underline{t})\right]$, while $t^c(x,p_B) = \underline{t}$ yields $\underline{x}(p_B,\underline{t}) = 1/2 + p_B/(2\underline{t})$. If $1/2 < x < \bar{x}(p_B,\bar{t})$, then firm A captures all consumers; if $\bar{x}(p_B,\bar{t}) \leq x \leq \underline{x}(p_B,\underline{t})$, then firm A serves consumers with $t < t^c(x,p_B)$; finally, firm A does not get any consumers if $x > \underline{x}(p_B,\underline{t})$. Given firm A's reaction functions, firm B's profit is $\Pi_B^{X|\emptyset} = \int_{\bar{x}}^{\underline{x}}\int_{t^c}^{\bar{t}} [f(t)p_B]\,dtdx + \int_{\underline{x}}^{1}\int_{\underline{t}}^{\bar{t}} [f(t)p_B]\,dtdx$. Maximizing with respect to p_B yields $p_B^* = \widetilde{H}(\underline{t},\bar{t})$.

Under the constraint $1 < k \leq 2$ it holds that $\widetilde{H}(\underline{t}, \bar{t}) < \underline{t}$, hence, indeed, $1/2 < \overline{x}(p_B^*, \bar{t}) < \underline{x}(p_B^*, \underline{t}) < 1$. Firm A's profit is computed as $\Pi_A^{X|\emptyset} = \int_0^{1/2}\int_{\underline{t}}^{\bar{t}} [f(t)(p_B^* + \underline{t}(1-2x))]\, dtdx + \int_{1/2}^{\overline{x}}\int_{\underline{t}}^{\bar{t}} \left[f(t)\left(p_B^* - \bar{t}(2x-1)\right)\right] dtdx + \int_{\overline{x}}^{\underline{x}}\int_{\underline{t}}^{t^c} [f(t)(p_B^* - \underline{t}(2x-1))/2]\, dtdx$. Firms realize profits $\Pi_A^{X|\emptyset} = 5\widetilde{H}(\underline{t}, \bar{t})/8 + \underline{t}/4$ and $\Pi_B^{X|\emptyset} = \widetilde{H}(\underline{t}, \bar{t})/4$. This completes the proof of Claim 6.

Claim 7. Consider the information scenario $\{T, \emptyset\}$. If $\underline{t} = 0$, then in equilibrium firm A sets $p_A^*(t) = p_B^* - t$ and serves all consumers if $t < p_B^*/3$, if $t \geq p_B^*/3$, then it sets $p_A^*(t) = (p_B^* + t)/2$ and serves consumers with $x < 1/4 + p_B^*/(4t)$. Firm B sets $p_B^* \approx 0.85\bar{t}$. Firms realize profits $\Pi_A^{T|\emptyset} \approx 0.53\bar{t}$ and $\Pi_B^{T|\emptyset} \approx 0.23\bar{t}$. If $\underline{t} > 0$ and $k \leq 2$, then in equilibrium firms set $p_A^*(t) = (t + p_B^*)/2$ and $p_B^* = 3H(\underline{t}, \bar{t})/2$. Firm A serves all consumers if $x < 1/4 + p_B^*/(4\bar{t})$, serves consumers with $t < p_B^*/(4x-1)$ if $1/4 + p_B^*/(4\bar{t}) \leq x \leq 1/4 + p_B^*/(4\underline{t})$ and serves no consumers when $x > 1/4 + p_B^*/(4\underline{t})$. Equilibrium profits are $\Pi_A^{T|\emptyset} = 21H(\underline{t}, \bar{t})/32 + A(\underline{t}, \bar{t})/8$ and $\Pi_B^{T|\emptyset} = 9H(\underline{t}, \bar{t})/16$.

Proof of Claim 7. Firm A takes p_B as given and maximizes its expected profit $E\left[\Pi_A^{T|\emptyset} | t\right] = f(t)p_A \Pr\{x \leq x^c\}$, which yields firm A's equilibrium strategies as $p_A(p_B) = (p_B + t)/2$ if $p_B \leq 3t$ and $p_A(p_B) = p_B - t$ if $p_B > 3t$. From these reaction functions we get $t^c(x, p_B) = p_B/(4x-1)$. Assume that $\underline{t} > 0$ and $1 < k \leq 2$. Solving $t^c(x, p_B) = \bar{t}$ and $t^c(x, p_B) = \underline{t}$ we get $\overline{x}(p_B, \bar{t}) = 1/4 + p_B/(4\bar{t})$ and $\underline{x}(p_B, \underline{t}) = 1/4 + p_B/(4\underline{t})$. Depending on the relation between $\underline{x}(p_B^*, \underline{t})$ and 1 two cases are possible in equilibrium: $\underline{x}(p_B^*, \underline{t}) \geq 1$ if $3\underline{t} \leq p_B^* < 3\bar{t}$ and $\underline{x}(p_B^*, \underline{t}) < 1$ if $p_B^* < 3\underline{t}$. We show that $3\underline{t} \leq p_B^* < 3\bar{t}$ does not emerge in equilibrium. Assume that $3\underline{t} \leq p_B^* < 3\bar{t}$. Firm B chooses its price to maximize the profit $\Pi_B^{T|\emptyset} = \int_{\overline{x}}^{1}\int_{t^c}^{\bar{t}} f(t)p_B dtdx$. The optimal price p_B solves equation $p_B[1 + \ln(9)] - 3\bar{t} - 2p_B \ln\left(p_B/\bar{t}\right) = 0$. There is no analytical solution to this problem, the value $p_B \approx 0.85\bar{t}$ is, however, a good numerical approximation which fulfills the second order condition. Note that $0.85\bar{t} < 3\underline{t}$ given that $1 < k \leq 2$, hence, $3\underline{t} \leq p_B^* < 3\bar{t}$ cannot hold in equilibrium. Assume further that p_B^* satisfies $p_B^* < 3\underline{t}$. Firm B maximizes the profit $\Pi_B^{T|\emptyset} = \int_{\overline{x}}^{\underline{x}}\int_{t^c}^{\bar{t}} [f(t)p_B]\, dtdx + \int_{\underline{x}}^{1}\int_{\underline{t}}^{\bar{t}} [f(t)p_B]\, dtdx$, which yields $p_B^* = 3H(\underline{t}, \bar{t})/2$. Under the constraint $1 < k \leq 2$ it holds that $3H(\underline{t}, \bar{t})/2 < 3\underline{t}$, hence, $p_B^* = 3H(\underline{t}, \bar{t})/2$ is, indeed, the equilibrium price. Firm A' profits are computed as $\Pi_A^{T|\emptyset} = \int\limits_0^{\overline{x}}\int\limits_{\underline{t}}^{\overline{t}} \left[f(t)(p_B^* + t)/2\right] dtdx + \int\limits_{\overline{x}}^{\underline{x}}\int\limits_{\underline{t}}^{t^c} \left[f(t)(p_B^* + t)/2\right] dtdx$. Equilibrium profits are $\Pi_A^{T|\emptyset} = 21H(\underline{t},\overline{t})/32 + A(\underline{t},\overline{t})/8$ and $\Pi_B^{T|\emptyset} = 9H(\underline{t},\overline{t})/16$. Consider now $\underline{t} = 0$. Maximization of $\Pi_B^{T|\emptyset} = \int\limits_{\overline{x}}^{1}\int\limits_{t^c}^{\overline{t}} \left[f(t)p_B\right] dtdx$ yields $p_B^* \approx 0.85\overline{t}$. Firm A' profits are computed as $\Pi_A^{T|\emptyset} = \int\limits_0^1\int\limits_0^{\frac{p_B^*}{3}} \left[f(t)(p_B^* - t)\right] dtdx + \int\limits_0^{\overline{x}}\int\limits_{\frac{p_B^*}{3}}^{\overline{t}} \left[f(t)(p_B^* + t)/2\right] dtdx + \int\limits_{\overline{x}}^{1}\int\limits_{\frac{p_B^*}{3}}^{t^c} \left[f(t)(p_B^* + t)/2\right] dtdx$. Firms realize profits $\Pi_A^{T|\emptyset} \approx 0.53\overline{t}$ and $\Pi_B^{T|\emptyset} \approx 0.23\overline{t}$. This completes the proof of Claim 7.

Claim 8. Consider the information scenario $\{XT, \emptyset\}$. If $\underline{t} = 0$, then in equilibrium firms A and B set $p_A^*(x,t)= \max\{p_B^* + t(1-2x), 0\}$ and $p_B^* \approx 0.28\overline{t}$. If $x < 1/2 + p_B^*/(2\overline{t})$, firm A serves all consumers; if $x \geq 1/2 + p_B^*/(2\overline{t})$, firm A serves consumers with $t < p_B^*/(2x-1)$. Equilibrium profits are $\Pi_A^{XT|\emptyset} \approx 0.32\overline{t}$ and $\Pi_B^{XT|\emptyset} \approx 0.05\overline{t}$. If $\underline{t} > 0$ and $k \leq 2$, in equilibrium firms set $p_A^*(x,t)= \max\{p_B^* + t(1-2x), 0\}$ and $p_B^* = H(\underline{t},\overline{t})/2$. If $x < 1/2 + p_B^*/\left(2\overline{t}\right)$ firm A serves all consumers; if $1/2 + p_B^*/\left(2\overline{t}\right) \leq x \leq 1/2 + p_B^*/\left(2\underline{t}\right)$ firm A serves consumers with $t < p_B^*/(2x-1)$; if $x > 1/2 + p_B^*/\left(2\underline{t}\right)$ firm A serves no consumers. Equilibrium profits are $\Pi_A^{XT|\emptyset} = 5H(\underline{t},\overline{t})/16 + A(\underline{t},\overline{t})/4$ and $\Pi_B^{XT|\emptyset} = H(\underline{t},\overline{t})/8$.

Proof of Claim 8. Consider first $\underline{t} > 0$ and $k \leq 2$. Firm A maximizes its profit given p_B. Firm A's optimal strategy is $p_A(p_B)= \max\{0, t(1-2x)+ p_B\}$, which gives $t^c(x,p_B)= p_B/(2x-1)$ and $\overline{x}(p_B,\overline{t})= 1/2 + p_B/(2\overline{t})$ and $\underline{x}(p_B,\underline{t})= 1/2 + p_B/(2\underline{t})$. Depending on the relation between $\underline{x}(p_B^*,\underline{t})$ and 1 two cases are possible in equilibrium: $\underline{x}(p_B^*,\underline{t}) \leq 1$ if $p_B^* \leq \underline{t}$ and $\underline{x}(p_B^*,\underline{t}) > 1$ if $\underline{t} < p_B^* < \overline{t}$. We show first that $\underline{t} < p_B^* < \overline{t}$ cannot characterize firm B's equilibrium price. Assume that $\underline{t} < p_B^* < \overline{t}$. Firm B sets p_B to maximize the profit $\Pi_B^{XT|\emptyset} = \int\limits_{\overline{x}}^{1}\int\limits_{t^c}^{\overline{t}} \left[f(t)p_B\right] dtdx$ given firm A's optimal strategy. The optimal price p_B solves the equation $p_B\left[2\ln\left(p_B/\overline{t}\right) - 1\right] + \overline{t} = 0$. There is no analytical solution to this problem, the value $p_B \approx 0.28\overline{t}$ is, however, a good numerical approximation, which fulfills the second order condition. Note that $0.28\overline{t} < \underline{t}$ given $1 < k \leq 2$, hence, $\underline{t} < p_B^* < \overline{t}$ is not possible in equilibrium. We show next that in equilibrium $p_B^* \leq \underline{t}$. Assume this is the case. Firm B sets p_B to maximize the profit $\Pi_B^{XT|\emptyset} = \int\limits_{\overline{x}}^{\underline{x}}\int\limits_{t^c}^{\overline{t}} \left[f(t)p_B\right] dtdx + \int\limits_{\underline{x}}^{1}\int\limits_{\underline{t}}^{\overline{t}} \left[f(t)p_B\right] dtdx = \left[p_B(\overline{t} - \underline{t} - p_B\ln(\overline{t}/\underline{t}))\right] / \left[2(\overline{t} - \underline{t})\right]$, which yields $p_B^* = H(\underline{t},\overline{t})/2$. Under the constraint

$1 < k \leq 2$ it holds that $H(\underline{t},\bar{t})/2 < \underline{t}$, hence, $p_B^* = H(\underline{t},\bar{t})/2$ is indeed the equilibrium price. Firm A's profit is computed as $\Pi_A^{XT|\emptyset} = \int_0^{\bar{x}}\int_{\underline{t}}^{\bar{t}}[f(t)(p_B^* + t(1-2x))]\,dtdx + \int_{\bar{x}}^{\underline{x}}\int_{\underline{t}}^{t^c}[f(t)(p_B^* + t(1-2x))]\,dtdx$. Equilibrium profits are $\Pi_A^{XT|\emptyset} = 5H(\underline{t},\bar{t})/16 + A(\underline{t},\bar{t})/4$ and $\Pi_B^{XT|\emptyset} = H(\underline{t},\bar{t})/8$. Consider finally $\underline{t} = 0$, in which case $\Pi_B^{XT|\emptyset} = \int_{\bar{x}}^{1}\int_{\bar{t}^c}^{\bar{t}}[f(t)p_B]\,dtdx$ and $p_B^* \approx 0.28\bar{t}$. Firm A's profit is computed as $\Pi_A^{XT|\emptyset} = \int_0^{\bar{x}}\int_0^{\bar{t}}[f(t)(p_B^* + t(1-2x))]\,dtdx + \int_{\bar{x}}^{1}\int_0^{t^c}[f(t)(p_B^* + t(1-2x))]\,dtdx$. Equilibrium profits are $\Pi_A^{XT|\emptyset} \approx 0.32\bar{t}$ and $\Pi_B^{XT|\emptyset} \approx 0.05\bar{t}$. This completes the proof of Claim 8.

Claim 9. Consider the information scenario $\{XT, X\}$*. In equilibrium firm A sets* $p_A^*(x,t) = t(1-2x)$ *if* $x \leq 1/2$ *and* $p_A^*(x,t) = (2x-1)\max\{0, \bar{t}/2 - t\}$ *if* $x > 1/2$*. Firm B sets* $p_B^*(x) = 0$ *if* $x \leq 1/2$ *and* $p_B^*(x) = (2x-1)t^m$ *if* $x > 1/2$ *and serves consumers with* $x > 1/2$ *and* $t \geq t^m$*, where* $t^m = \max\{\bar{t}/2, \underline{t}\}$*. Firms realize profits* $\Pi_A^{XT|X} = 5\bar{t}/32$ *and* $\Pi_B^{XT|X} = \bar{t}/16$ *if* $\underline{t} = 0$ *and* $\Pi_A^{XT|X} = A(\underline{t},\bar{t})/4$ *and* $\Pi_B^{XT|X} = \underline{t}/4$ *if* $\underline{t} > 0$ *and* $k \leq 2$*.*

Proof of Claim 9. Firm B treats t as a random variable and maximizes its expected profits given firm A's equilibrium strategy separately in the regions $x \leq 1/2$ and $x > 1/2$. In the region $x \leq 1/2$ firm A can undercut any price set by firm B, hence, $p_B^*(x) = 0$ for $x \leq 1/2$. In the region $x > 1/2$ firm A can undercut firm B as long as it can set a non-negative price, which is the case if $t(2x-1) < p_B(x)$ holds. Firm B's expected profit in the region $x > 1/2$ is $E\left[\Pi_B^{XT|X}|x > 1/2\right] = p_B \Pr\{t(2x-1) \geq p_B|x > 1/2\}$. Maximization of the latter profit yields the optimal price of firm B: $p_B^*(x) = \bar{t}\,(x - 1/2)$ if $\bar{t} > 2\underline{t}$ and $p_B^*(x) = \underline{t}(2x-1)$ if $\bar{t} \leq 2\underline{t}$. If $\underline{t} = 0$, then $p_B^*(x) = \bar{t}\,(x-1/2)$, which yields $t^c = \bar{t}/2$ and firm B serves consumers with $t \geq t^c$ on its turf. Firms realize profits $\Pi_A^{XT|X} = \int_0^{1/2}\int_0^{\bar{t}}[f(t)t(1-2x)]\,dtdx + \int_{1/2}^{1}\int_0^{\frac{\bar{t}}{2}}\left[f(t)\left(\bar{t} - 2t\right)(x-1/2)\right]dtdx = 5\bar{t}/32$ and $\Pi_B^{XT|X} = \int_{1/2}^{1}\int_{\frac{\bar{t}}{2}}^{\bar{t}}\left[f(t)\bar{t}(x-1/2)\right]dtdx = \bar{t}/16$. If $\underline{t} > 0$ and $k \leq 2$, then $p_B^*(x) = \underline{t}(2x-1)$ and firm B serves all consumers on its turf. Firms realize profits $\Pi_A^{XT|X} = \int_0^{1/2}\int_{\underline{t}}^{\bar{t}}[f(t)t(1-2x)]\,dtdx = A(\underline{t},\bar{t})/4$ and $\Pi_B^{XT|X} = \int_{1/2}^{1}\int_{\underline{t}}^{\bar{t}}[f(t)\underline{t}(2x-1)]\,dtdx = \underline{t}/4$. This completes the proof of Claim 9.

Claim 10. Consider the information scenario $\{XT, T\}$*. In equilibrium firm A sets* $p_A^*(x,t)=\max\{t/2+t(1-2x),0\}$ *and serves consumers with* $x<3/4$*. Firm B sets* $p_B^*(t)=\ t/2$*. Firms realize profits* $\Pi_A^{XT|T}=9A(\underline{t},\bar{t})/16$ *and* $\Pi_B^{XT|T}=A(\underline{t},\bar{t})/8$*.*

Proof of Claim 10. Since firm A has full information, it can undercut the rival as long as it can set a non-negative price. This translates into firm A's equilibrium strategy as $p_A(p_B)=\max\ \{p_B+t(1-2x),0\}$. Undercutting is possible whenever $t(2x-1)<p_B(t)$. Firm B treats x as a random variable and maximizes its expected profit given firm A's equilibrium strategy: $E\left[\Pi_B^{XT|T}|t\right]=f(t)p_B\Pr\{t(2x-1)\geq p_B\}$. Solving the maximization problem for p_B yields $p_B^*(t)=\ t/2$, which gives $p_A^*=\max\{t/2+t(1-2x),0\}$, such that $t/2+t(1-2x)$ is positive whenever $x<x^c=3/4$. Firms A and B realize profits $\Pi_A^{XT|T}=\int_0^{x^c}\int_{\underline{t}}^{\bar{t}}f(t)(\,t/2+t(1-2x))\,dtdx=9A(\underline{t},\bar{t})/16$ and $\Pi_B^{XT|T}=\int_{x^c}^{1}\int_{\underline{t}}^{\bar{t}}f(t)(t/2)dtdx=A(\underline{t},\bar{t})/8$, respectively. This completes the proof of Claim 10.

The equilibrium prices and profits stated in Claims 1-10 are given in Tables 1 and 2. *Q.E.D.*

Proof of Proposition 2. With $\underline{t}=0$ the comparison of profits across different information scenarios is straightforward and yields $\Pi_i^{XT|XT}<\Pi_i^{X|X}<\Pi_i^{T|T}$. Consider now $\underline{t}>0$ and $k\leq 2$. It is straightforward that $\Pi_i^{X|X}<\Pi_i^{XT|XT}$ and $\Pi_i^{\emptyset|\emptyset}<\Pi_i^{T|T}$. By substituting in k into $\Pi_i^{\emptyset|\emptyset}-\Pi_i^{XT|XT}$ and rearranging, we get $4\ln k(\Pi_i^{\emptyset|\emptyset}-\Pi_i^{XT|XT})/\left(\bar{t}+\underline{t}\right)=4(k-1)/(k+1)-\ln k$. The second derivative of the RHS of the latter equality is negative on the interval $1<k\leq 2$, while the first derivative is positive if $k=2$, hence, the RHS increases on the interval $1<k\leq 2$. As it approaches zero if $k\to 1$, we get that $\Pi_i^{XT|XT}<\Pi_i^{\emptyset|\emptyset}$. These comparisons yield the ranking $\Pi_i^{X|X}<\Pi_i^{XT|XT}<\Pi_i^{\emptyset|\emptyset}<\Pi_i^{T|T}$. *Q.E.D.*

Proof of Proposition 3. Consider first the case $\underline{t}=0$. We use the demand regions and equilibrium prices as stated in the proof of Proposition 1 to find consumer surplus. As in the information scenarios $\{XT, XT\}$ and $\{T, T\}$ every firm serves only its own turf, we use the formula $\int_0^{1/2}\int_0^{\bar{t}}U_A(x,t)f(t)dtdx+\int_{1/2}^{1}\int_0^{\bar{t}}U_B(x,t)f(t)dtdx$ to compute $CS^{XT|XT}=v-3A(\underline{t},\bar{t})/4$ and $CS^{T|T}=v-5A(\underline{t},\bar{t})/4$. We also obtain $CS^{X|X}=\int_0^{1/2}\int_{t^c}^{\bar{t}}U_A(x,t)f(t)dtdx+$

$\int_{1/2}^{1}\int_{0}^{t^c} U_A(x,t)f(t)dtdx + \int_{0}^{1/2}\int_{0}^{t^c} U_B(x,t)f(t)dtdx + \int_{1/2}^{1}\int_{t^c}^{\bar{t}} U_B(x,t)f(t)dtdx = v - 31A(\underline{t},\bar{t})/36.$ The comparison is straightforward and yields the ranking $CS^{T|T} < CS^{X|X} < CS^{XT|XT}$. Social welfare follows immediately from adding up profits and consumer surplus as $SW^{I_A|I_B} = CS^{I_A|I_B} + \Pi_A^{I_A|I_B} + \Pi_B^{I_A|I_B}$, from where we get $SW^{XT|XT} = SW^{T|T} = v - A(\underline{t},\bar{t})/4$ and $SW^{X|X} = v - 11A(\underline{t},\bar{t})/36$.

The comparison is straightforward and yields the ranking $SW^{X|X} < SW^{XT|XT} = SW^{T|T}$.

Consider now $\underline{t} > 0$ and $k \leq 2$. Note that in all the symmetric information scenarios firms share the market equally, hence, social welfare is same and is given by $SW^{XT|XT} = SW^{T|T} = SW^{X|X} = SW^{\emptyset|\emptyset} = v - 2\int_{0}^{1/2}\int_{\underline{t}}^{\bar{t}} txf(t)dtdx = v - A(\underline{t},\bar{t})/4$. We can use the formula $CS^{I_A|I_B} = SW^{I_A|I_B} - \Pi_A^{I_A|I_B} - \Pi_B^{I_A|I_B}$ to derive consumer surplus as $CS^{T|T} = v - 5A(\underline{t},\bar{t})/4$, $CS^{\emptyset|\emptyset} = v - H(\underline{t},\bar{t}) - A(\underline{t},\bar{t})/4$, $CS^{XT|XT} = v - 3A(\underline{t},\bar{t})/4$ and $CS^{X|X} = v - (\bar{t} + 5\underline{t})/8$. Since social welfare is same in all the symmetric information scenarios, the ranking of consumer surplus follows directly from the ranking of the profits as $CS^{T|T} < CS^{\emptyset|\emptyset} < CS^{XT|XT} < CS^{X|X}$. *Q.E.D.*

Proof of Proposition 4. The comparison of joint profits in the case of mobile consumers is straightforward and shows that only dataset T is shared, in the information scenario $\{XT, X\}$. We now turn to the case of immobile consumers. Many comparisons are straightforward using $H(\underline{t},\bar{t}) < A(\underline{t},\bar{t})$. We only consider the non-trivial cases. Let $\Pi_{A+B}^{I_A|I_B}$ denote the sum of profits in the scenario $\{I_A, I_B\}$. We first show that dataset X is not shared in the scenario $\{XT, \emptyset\}$. By substituting k into $\Pi_{A+B}^{XT|\emptyset} - \Pi_{A+B}^{XT|X}$ and rearranging we get $16\ln k(\Pi_{A+B}^{XT|\emptyset} - \Pi_{A+B}^{XT|X})/\underline{t} = 7(k-1) - 4\ln k$. The LHS of the latter equation increases on the interval $1 < k \leq 2$ and approaches zero when $k \to 1$, hence, $\Pi_{A+B}^{XT|\emptyset} > \Pi_{A+B}^{XT|X}$. We next show that both datasets together are not shared in this information either. Substituting k into $\Pi_{A+B}^{XT|\emptyset} - \Pi_{A+B}^{XT|XT}$ and rearranging yields $16\ln k(\Pi_{A+B}^{XT|\emptyset} - \Pi_{A+B}^{XT|XT})/\underline{t} = 7(k-1) - 2(k+1)\ln k$. The second derivative of the RHS of the latter equation is negative on the interval $1 < k \leq 2$ and the first derivative is positive at the point $k = 2$, hence, the LHS increases on the whole

interval. Note, finally, that the RHS approaches zero when $k \to 1$, hence, $\Pi^{XT|\emptyset}_{A+B} > \Pi^{XT|XT}_{A+B}$.
There is no information sharing in the scenario $\{T, \emptyset\}$. By substituting k into $\Pi^{T|\emptyset}_{A+B} - \Pi^{T|T}_{A+B}$ and rearranging we get $32 \ln k(\Pi^{T|\emptyset}_{A+B} - \Pi^{T|T}_{A+B})/\underline{t} = 39(k-1) - 14(k+1)\ln k$. The second derivative of the RHS of the latter equation is negative on the interval $1 < k \leq 2$ and the first derivative is positive at the point $k = 2$, hence, the LHS increases on the whole interval. Note, finally, that the RHS approaches zero when $k \to 1$, it follows that $\Pi^{T|\emptyset}_{A+B} > \Pi^{T|T}_{A+B}$.
Finally, we show that dataset X is not shared in the information scenario $\{X, \emptyset\}$. Substituting k into $\Pi^{X|\emptyset}_{A+B} - \Pi^{X|X}_{A+B}$ and rearranging yields $8 \ln(2k-1)(\Pi^{X|\emptyset}_{A+B} - \Pi^{X|X}_{A+B})/\underline{t} = 7(k-1) - 2\ln(2k-1)$. The derivative of the RHS of the latter equation is positive on the interval $1 < k \leq 2$. Moreover, the RHS approaches zero when $k \to 1$, hence, it takes only positive values and $\Pi^{X|\emptyset}_{A+B} > \Pi^{X|X}_{A+B}$. *Q.E.D.*

Proof of Proposition 5. Consider first $\underline{t} = 0$. Consumer surplus in the information scenario $\{XT, X\}$ is $CS^{XT|X} = \int_0^{1/2}\int_0^{\bar{t}} U_A(x,t)f(t)dtdx + \int_{1/2}^{1}\int_0^{\bar{t}/2} U_A(x,t)f(t)dtdx + \int_{1/2}^{1}\int_{\bar{t}/2}^{\bar{t}} U_B(x,t)f(t)dtdx = v - 3\bar{t}/8$. As was shown in the proof of Proposition 3, $CS^{XT|XT} = v - 3\bar{t}/8$, hence, $CS^{XT|X} = CS^{XT|XT}$. Social welfare follows immediately from adding up firms' profits and consumer surplus such that $SW^{XT|X} \approx v - 0.16\bar{t} < SW^{XT|XT} = v - 0.13\bar{t}$.

Consider now $\underline{t} > 0$ and $k \leq 2$. Consumer surplus in the information scenario $\{XT, \emptyset\}$ is $CS^{XT|\emptyset} = \int_0^{\overline{x}}\int_{\underline{t}}^{\bar{t}} U_A(x,t)f(t)dtdx + \int_{\overline{x}}^{\underline{x}}\int_{\underline{t}}^{t^c} U_A(x,t)f(t)dtdx + \int_{\overline{x}}^{\underline{x}}\int_{t^c}^{\bar{t}} U_B(x,t)f(t)dtdx + \int_{\underline{x}}^{1}\int_{\underline{t}}^{\bar{t}} U_B(x,t)f(t)dtdx = v - \left[A(\underline{t},\bar{t}) + H(\underline{t},\bar{t})\right]/2$ and social welfare is $SW^{XT|\emptyset} = v - \left[4A(\underline{t},\bar{t}) + H(\underline{t},\bar{t})\right]/16$. Consumer surplus in the information scenario $\{XT, T\}$ is $CS^{XT|T} = \int_0^{x^c}\int_{\underline{t}}^{\bar{t}} U_A(x,t)f(t)dtdx + \int_{x^c}^{1}\int_{\underline{t}}^{\bar{t}} U_B(x,t)f(t)dtdx = v - A(\underline{t},\bar{t})$ and social welfare is $SW^{XT|T} = v - 5A(\underline{t},\bar{t})/16$. Straightforward comparison yields that $CS^{XT|\emptyset} > CS^{XT|T}$ and $SW^{XT|\emptyset} > SW^{XT|T}$.

Consumers enjoy $CS^{XT|X} = \int_0^{1/2}\int_{\underline{t}}^{\bar{t}} U_A(x,t)f(t)dtdx + \int_{1/2}^{1}\int_{\underline{t}}^{\bar{t}} U_B(x,t)f(t)dtdx = v - (\bar{t} + 3\underline{t})/8$ in the information scenario $\{XT, X\}$. We showed in the proof of Proposition 3 that $CS^{XT|XT} = v - 3A(\underline{t},\bar{t})/4$, hence, $CS^{XT|X} > CS^{XT|XT}$. As in the information scenarios $\{XT, X\}$ and $\{XT, XT\}$ every firm serves consumers on the own turf, it follows that $SW^{XT|X} = SW^{XT|XT}$. *Q.E.D.*

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Chapter 3

Bargaining, Vertical Mergers and Entry

Abstract

This paper analyzes vertical integration incentives in a bilaterally duopolistic industry where upstream producers bargain with downstream retailers on terms of supply. In the applied framework integration does not affect the total output produced, but it affects the distribution of rents among players. Vertical integration incentives depend on the strength of substitutability or complementarity between products and the shape of the unit cost function. Furthermore I demonstrate that in contrast to the widely prevailing view in competition policy, vertical integration can, under particular circumstances convey more bargaining power to the merged entity than a horizontal merger to monopoly. The model is used to analyze strategic merger incentives that influence entry decisions. Mergers can facilitate and deter entry. While horizontal mergers that deter entry are never profitable, firms at different market levels may strategically choose to integrate vertically in order to keep a potential entrant out of the market. I provide conditions for such entry-deterring vertical mergers to occur.

3.1 Introduction

Competition policy traditionally looks at vertical and horizontal mergers through different glasses. While horizontal mergers are often regarded to be motivated by the intent to reduce competition, it is frequently argued that vertical integration is driven by efficiencies, for example by eliminating double markups, reducing transaction costs or solving some variant of the holdup problem. This is explicitly stated in paragraph 11 of the EC non-horizontal merger guidelines, recognizing that *"[n]on-horizontal mergers are generally less likely to significantly impede effective competition than horizontal mergers."*[1] A similar view emerges in the U.S. Department of Justice Merger Guidelines, noting that *"...non-horizontal mergers are less likely than horizontal mergers to create competitive problems."*[2] Some of this sharp distinction between horizontal and vertical mergers may lie in the tradition of economic analysis to ignore the ability of downstream firms to influence upstream markets. Yet perhaps in most vertically related industries, supply conditions are determined through bargaining, where downstream firms have the ability to actively shape contracts with suppliers. Much research is devoted to determining how horizontal integration can tip bargaining in favor of the merging parties. This research also gave rise to the heated debate on buyer-power in the antitrust arena. The question of how vertical integration can affect bargaining outcomes is significantly less well studied.

This article intends to make a step towards closing this gap. I investigate the driving forces behind vertical integration, its effects and social desirability while taking into account that delivery conditions are shaped by bilateral bargaining. I apply a setup in which competition does not change the total industry surplus, only the distribution of rents among actors due to shifting bargaining power across firms. This framework is particularly useful because it facilitates focusing on the strategic considerations behind vertical integration

[1] Commission Guidelines on the Assessment of Non-Horizontal Mergers under the Council Regulation on the Control of Concentrations Between Undertakings, 2008 O.J. (C 265)07, par. 11.

[2] U.S. Department of Justice Merger Guidelines, 1984, Chapter 4.

in isolation. After identifying these strategic effects, I extend the analysis to incorporate efficiency. To do so, I investigate how vertical integration can be used as a device to deter entry both upstream and downstream.

I provide conditions for vertical mergers to take place regarding the strength of substitutability or complementarity between products and the shape of the unit cost function. I show that in an environment where integration is driven purely by strategic considerations, horizontal and vertical merger incentives are very closely related. In the simplest of all settings, the decision to vertically integrate can be expressed as a mix of horizontal integration incentives upstream and downstream. I also demonstrate that vertical integration can, under particular circumstances convey more bargaining power to the merged entity than a horizontal merger to monopoly. Finally, I contrast the entry-deterring potential of horizontal and vertical mergers. Contrary to the broadly prevailing view in competition policy, my results show that vertical mergers can be more detrimental to welfare than horizontal ones. This is because the former can be a more apt device to deter entry than horizontal integration. If vertical integration prevents entry, this occurs by *reducing the rival's revenues*: Vertical integration functions as a structural barrier to entry either upstream or downstream, by reducing the potential entrant's expected revenues below the level necessary to cover its setup costs.

3.2 Literature Review

Economic theories emphasize that vertical integration may have anticompetitive effects through various avenues. One such avenue is the foreclosure of necessary inputs or of market access for rivals. Salinger (1988)and Ordover, Saloner and Salop (1990)are seminal papers in this strand, demonstrating that an upstream firm (even without substantial market power)may have an incentive to integrate vertically and limit supply to downstream rivals. Martin et al (2001), and Normann (2007)provide an empirical experimental validation for theoretical predictions on vertical integration incentives.

Hart and Tirole (1990)are the first to point out that a dominant upstream firm may be unable to exert its market power if it sells a product to competing downstream firms. If the upstream firm makes secret offers and downstream firms hold passive beliefs about the offers of the competitors, the upstream firm faces a Coasian commitment problem that prevents it from reaping full monopoly profits. In this case, despite the market power of the upstream firm, quantities produced will correspond to the competitive levels. With vertical integration the upstream firm can credibly commit to supplying the monopoly quantity. This allows it to foreclose downstream rivals and reduce output, which unambiguously harms welfare.

Much of the literature on vertical integration focuses on the effect of altered ownership structures on broader-defined investment incentives (Hart and Moore 1990; Stole and Zwiebel 1996a and 1996b). In this strand, upstream and downstream competition is typically preceded by a stage during which firms make investment, capacity or technology adoption decisions. Baake et al (2004)extend the model of Hart and Tirole (1990)by allowing the upstream firm to carry out an investment that determines its marginal costs. Downstream competition weakens incentives to invest into cutting marginal costs. Furthermore, the resulting welfare loss due to underinvestment may be larger than the welfare loss arising from the restoration of monopoly power. Also in this strand of literature, Choi and Yi (2000)develop a similar argument as Hart and Tirole (1990), showing that vertical integration can serve as a commitment device, since it may create incentives for a vertically integrated upstream firm to provide a specialized input (i.e. one that can be used by only one downstream firm), although if separated it would offer an input that could be used by all firms downstream.

Inverting the question of when firms integrate vertically, Bonanno and Vickers (1988) investigate incentives of vertically integrated firms to separate. They point out that if suppliers can fully extract the rents of retailers (for example with a franchise fee), and retailers' decisions are strategic complements, then vertical separation can be profitable. A separated manufacturer can induce all retailers to price higher by increasing the wholesale price, whereas under vertical integration the wholesale price is set to equal production costs. Heavner (2004)similarly concludes that vertical separation may be profitable, when the integrated firm cannot commit to providing equal service quality to an upstream rival.

It is recognized in industrial organization theory, as well as in competition policy, that if delivery conditions between sellers and buyers are determined by bargaining, the resulting outcomes may be markedly different from usual ones (Horn and Wolinsky 1988; Campbell 2007). This article explicitly takes bargaining into account to derive the incentives for vertical mergers to occur. In particular, I follow the property rights literature (Grossman and Hart, 1986; Hart and Moore, 1990)and consider a merger as combining two otherwise independently bargaining units into one. Whereas without integration each supplier and retailer bargains separately, when integrated the negotiations of the merged entity are controlled by one common agent. Regarding the way bargaining is modelled, this article has several predecessors. I follow, among others, Hart and Moore (1990); Stole and Zwiebel (1996a, 1996b); Rajan and Zingales (1998); Inderst and Wey (2003); Segal (2003); de Fontenay and Gans (2005b)and Montez (2007), all of whom use the Shapley value to capture the outcome of bargaining between various actors.

The two articles closest to the present one are Inderst and Wey (2003)and de Fontenay and Gans (2005b)(in the following, respectively IW and dFG). Both articles focus on bargaining in an industry with two upstream and two downstream firms and use the Shapley value to capture the outcome of bargaining.[3] Under the assumption that downstream markets are independent, IW analyze horizontal merger incentives upstream and downstream, as well as the choice of a manufacturer between two technologies that influence production costs. They show that upstream merger incentives depend on whether products are substitutes or complements, whereas downstream merger incentives are determined by the shape of the unit cost function.

In turn, dFG focus on vertical merger incentives in a similar bargaining framework and compare outcomes under upstream competition and monopoly.[4] The key modelling

[3] These articles derive the Shapley value as the outcome of different bargaining procedures.

[4] In the basic model of dFG, downstream firms do not excert competitive externalities on each other.

difference between IW and dFG is the way mergers are regarded. In IW, in a merger between two firms the integrated entity bargains with other firms as a single party. This is not the case in dFG, who distinguish between the owner and the manager of a firm. After a merger takes place, the manager of a purchased entity remains indispensable in further negotiations, and acts as an independently negotiating party. This creates scope for a rich set of interaction between managers, which allows the authors to distinguish between forward and backward vertical integration.

This article can be regarded as an intersection between IW and dFG. As with the latter, I focus on vertical integration incentives, but follow IW and assume that a merged entity has one central management conducting negotiations with other parties. Doing so yields markedly different results for vertical merger incentives than those in dFG; two of which stand out. First, while in the baseline model of dFG with no downstream competitive externalities, vertical integration (either forward or backward)is always preferred to non-integration, in my model vertical integration may not be profitable. Second, different from dFG, upstream competition in my setup does not always strengthen the incentives to vertically integrate.

The article proceeds as follows: Section 3.3 introduces the model. In Section 3.4 the framework is applied to analyze vertical merger incentives. Section 3.5 compares horizontal and vertical merger incentives in greater detail and derives conditions determining which of these incentives is stronger. In Section 3.6 I introduce the possibility of entry and compare the deterring potential of horizontal and vertical mergers. Section 3.7 provides an example to illustrate and verify selected results. Finally, Section 3.8 concludes.

This setup is identical to the one in IW and is what I apply in this article. dFG introduce downstream competition in Section 4 of their article.

3.3 Model Setup

Consider an industry in which two upstream suppliers, $s \in S^0 = \{A, B\}$ produce inputs that are turned into final goods by two downstream retailers, $r \in R^0 = \{a, b\}$. The inputs are differentiated with each supplier controlling the production of one input. The input from at least one supplier is necessary for a retailer to produce the final good. The demand at the retailers is independent, hence, there are no competitive externalities downstream.[5] The indirect demand function for the good of supplier s at retailer r is denoted by $p_{sr}(q_{sr}, q_{s\prime r})$, where s' stands for the alternative supplier (similarly, r' will denote the alternative retailer) and q_{sr} denotes the quantity of input s supplied to retailer r. The total costs of supplier s for providing input quantities q_{sr} and $q_{sr\prime}$ to the retailers are given by $C_s(q_{sr} + q_{sr\prime})$. I will denote the average unit cost of supplier s for providing quantity q of the product as $\overline{C}_s(q)= C_s(q)/q$. The retailers turn inputs into final good costlessly.

Supply contracts between upstream and downstream firms are determined through bargaining. I follow other authors studying the effects of integration in a bargaining framework and adopt the Shapley value as solution concept of the bargaining game (e.g. Hart and Moore 1990, Rajan and Zingales 1998, Inderst and Wey 2003, Segal 2003 and de Fontenay and Gans 2005b).[6] As there are no competitive externalities between retailers, changes in the industry structure affect only the distribution of bargaining power, not the supplied quantities and therefore the surplus generated.

The Shapley value allocates to each independently negotiating party their expected marginal contribution to coalitions, where the expectation is taken over all coalitions in which the party may belong, with all coalitions assumed to occur with equal probability. More

[5] We can think of retailers operating in different geographic markets, or of ones turning inputs into strongly differentiated final goods.

[6] While it is an axiomatic solution concept, the theoretical literature proposes a number of justifications for the Shapley value as outcome of non-cooperative bargaining processes. See for example Gül (1989), Inderst and Wey (2003), de Fontenay and Gans (2005). Section 8 of Winter (2002)provides an extensive overview.

formally, let Ψ denote the set of independently negotiating parties and $|\Psi|$ the cardinality of this set, respectively. The payoff of firm $\psi \in \Psi$ is given by

$$U_{\psi}^{\Psi} = \sum_{\widetilde{\Psi} \subseteq \Psi | \psi \in \widetilde{\Psi}} \frac{\left(\left|\widetilde{\Psi}\right| - 1\right)! \left(|\Psi| - \left|\widetilde{\Psi}\right|\right)!}{|\Psi|!} \left[W_{\widetilde{\Psi}} - W_{\widetilde{\Psi}\backslash\psi}\right],$$

where $\widetilde{\Psi} \subseteq \Psi | \psi \in \widetilde{\Psi}$ represents a set $\widetilde{\Psi} \subseteq \Psi$, such that ψ is a member of coalition $\widetilde{\Psi}$ and $W_{\widetilde{\Psi}}$ denotes the maximum surplus achieved by the firms in coalition $\widetilde{\Psi}$. For simplicity, I write $\widetilde{\Psi}\backslash\psi$ for $\widetilde{\Psi}\backslash\{\psi\}$. I furthermore denote the set of all firms by $\Omega = \{A, B, a, b\}$ and define W_{Ω} as the maximum industry profit. In the terminology of cooperative game theory $W(\cdot)$ is often referred to as the *characteristic function.* Importantly, since at least one supplier and retailer is necessary for production, $W_{\widetilde{\Psi}} = 0$ if $\widetilde{\Psi}$ contains less than one firm of each kind. Before proceeding with the analysis, I make some additional definitions and assumptions.

Definition 1 *The cost function $C_s(\cdot)$ is said to exhibit strictly increasing (decreasing)unit costs if the unit cost function $\overline{C}_s(q)$ is strictly increasing (decreasing)on $q > 0$.*

Definition 2 *Take any $s, s' \in S^0$ with $s \neq s'$ and $r \in R^0$. The two goods are said to be strict substitutes if $q''_{s'r} > q'_{s'r}$ and $p_{sr}(q_{sr}, q_{s\prime r}) > 0$ imply $p_{sr}(q_{sr}, q_{s'r}) > p_{sr}(q_{sr}, q''_{s'r})$. They are strict complements if $q''_{s'r} > q'_{s'r}$ and $p_{sr}(q_{sr}, q''_{s'r}) > 0$ imply $p_{sr}(q_{sr}, q'_{s'r}) < p_{sr}(q_{sr}, q''_{s'r})$.*

Definition 3 *Let $\Delta_C^{\Omega'} := \overline{C}_s(2q_{sr}^{\Omega'}) - \overline{C}_s(q_{sr}^{\Omega'})$ and $\Delta_p^{\Omega'} := p_{sr}(q_{sr}^{\Omega'}, q_{s'r}^{\Omega'}) - p_{sr}(q_{sr}^{\Omega'}, 0)$, with $\Omega' \subseteq \Omega$. From definition 1 unit costs are strictly increasing (decreasing)if $\Delta_C^{\Omega'} > 0$ ($\Delta_C^{\Omega'} < 0$). From Definition 2 products are strict complements (substitutes)if $\Delta_p^{\Omega'} > 0$ ($\Delta_p^{\Omega'} < 0$).*

Assumption 1 *(superadditivity) $W_{(\cdot)}$ is superadditive: $W_{\Omega'} \geq W_{\Omega''}$ for every Ω' and $\Omega''$with $\Omega'' \subset \Omega' \subseteq \Omega$.*

Assumption 2 *(symmetry)Suppliers and retailers are symmetric: $C_s(\cdot)= C_{s'}(\cdot)= C(\cdot)$, $q_{sr} = q_{\widetilde{s}\widetilde{r}}$ and $p_{sr}(\cdot)= p_{\widetilde{s}\widetilde{r}}(\cdot)$ for any $s, \widetilde{s}, r, \widetilde{r} \in S^0 \times R^0$.*

Definitions 1 and 2 are borrowed from IW. While Assumption 1 is put forward throughout this article, Assumption 2 will be necessary only for some results and will be explicitly invoked only during specific segments of the text.

3.4 Vertical Merger Incentives

Throughout this paper I refer to a merger as a transaction that combines the merging firms into one bargaining unit. This is a realistic way to think about mergers in which the merged firms are united under common management, which then conducts negotiations with other entities. It would happen, for example, if the key executives of the acquired company were replaced by the new owner.

We now calculate equilibrium payoffs under different market structures. Throughout this article I use the notation $\{s, s', r, r'\}$ to denote a market structure, where the commas separate non-merged and therefore individually negotiating entities. For example, $\{AB, a, b\}$ stands for the market structure with an upstream monopoly facing a duopoly of retailers. Similarly, $\{Aa, B, b\}$ denotes the market structure consisting of supplier A being vertically integrated with retailer a, and supplier B as well as retailer b negotiating independently. I focus on the following market structures: $\{A, B, a, b\}$ (*full separation*), $\{AB, a, b\}$ (*upstream monopoly*), $\{A, B, ab\}$ (*downstream monopoly*), $\{ABa, b\}$ (*vertically integrated upstream monopoly*), $\{Aab, B\}$ (*vertically integrated downstream monopoly*), $\{ABab\}$ (*full integration*), $\{Aa, B, b\}$ (*single vertical integration*), $\{Aa, Bb\}$(*double vertical integration*).

Lemma 1 The payoffs of the actors under the different market structures are as given by Table (1).

Proof: The proof is immediate by applying the Shapley value for the various market structures.

Before proceeding with the analysis of vertical merger incentives I provide a brief interpretation of the payoffs generated by the Shapley value in Table (1). In a well-known interpretation of the Shapley value, players are randomly ordered in a sequence. Since several randomizations are possible, each assumed to be equally probable. Every player gets as payoff its marginal contribution to the coalition formed by the preceding players in the sequence. The Shapley value is the expected payoff taken over all possible sequences.

Take for example the industry structure of upstream monopoly, with $\Psi = \{AB, a, b\}$.

In this case six orderings are possible, those displayed in Table (2). I focus on the payoff of supplier AB.

		Marginal contribution		
	Ordering	AB	a	b
1	AB, a, b	0	$W_{\Omega\backslash b}$	$W_\Omega - W_{\Omega\backslash b}$
2	AB, b, a	0	$W_\Omega - W_{\Omega\backslash a}$	$W_{\Omega\backslash a}$
3	a, AB, b	$W_{\Omega\backslash b}$	0	$W_\Omega - W_{\Omega\backslash b}$
4	b, AB, a	$W_{\Omega\backslash a}$	$W_\Omega - W_{\Omega\backslash a}$	0
5	a, b, AB	W_Ω	0	0
6	b, a, AB	W_Ω	0	0

Table (2): Marginal contributions in various orderings.

In orderings 1 and 2, supplier AB comes first. Its marginal contribution is zero, because without a retailer preceding it the supplier cannot produce. It comes second in orderings 3 and 4. In ordering 3 supplier AB's contribution is to enable production with retailer a, together creating $W_{\Omega\backslash b}$ of surplus. This is the surplus that can be created without retailer b. Similarly, in ordering 4 supplier AB enables production with retailer b, and therefore generates $W_{\Omega\backslash a}$ of surplus. In orderings 5 and 6 supplier AB comes last. Since the retailers preceding it are not able to generate value absent a supplier, firm AB receives the entire industry surplus W_Ω in these orderings. Taking expectations about the orderings, the Shapley value yields as payoff for the supplier

$$U_{AB} = \frac{1}{6}(0) + \frac{1}{6}(0) + \frac{1}{6}W_{\Omega\backslash b} + \frac{1}{6}W_{\Omega\backslash a} + \frac{1}{6}W_\Omega + \frac{1}{6}W_\Omega = \frac{1}{6}\left[W_{\Omega\backslash b} + W_{\Omega\backslash a} + 2W_\Omega\right].$$

The payoffs of the retailers can be determined in a similar manner.

We can now compare vertical merger incentives for various pre-merger market structures.

Proposition 1 *Whether a vertical merger between supplier $s \in S^0$ and retailer $r \in R^0$ increases their joint payoff depends on the pre-merger market structure the following way:*

(i) *If suppliers and retailers are non-integrated ($\Psi = \{A, B, a, b\}$), the joint profit of supplier s and retailer r weakly increases by vertically merging if*

$$\left(W_{\Omega\backslash s'r'} - W_{\Omega\backslash sr}\right) + W_{\Omega\backslash s} + W_{\Omega\backslash r} \geq W_{\Omega}, \tag{3.1}$$

whereas it decreases if the opposite holds.

(ii) *If suppliers are integrated and retailers are separated ($\Psi = \{AB, a, b\}$), the joint profit of supplier AB and retailer r weakly increases by vertically merging if*

$$W_{\Omega\backslash r} + W_{\Omega\backslash r'} \geq W_{\Omega},$$

whereas it decreases if the opposite holds.

(iii) *If suppliers are separated and retailers are integrated ($\Psi = \{A, B, ab\}$), the joint profit of supplier s and retailer ab weakly increases by vertically merging if*

$$W_{\Omega\backslash s} + W_{\Omega\backslash s'} \geq W_{\Omega},$$

whereas it decreases if the opposite holds.

Proof: See Appendix.

The results formulated in Proposition 1 stand somewhat in contrast to the conventional stance of competition policy on vertical mergers that regards these as markedly different from horizontal ones. In my model, where integration affects only the distribution of surplus among the actors, horizontal and vertical mergers are very closely related. Cases *(ii)* and *(iii)* (pre-merger upstream and downstream monopoly, respectively)have particularly interesting implications if they are compared to horizontal merger incentives. These are derived by IW using the same framework employed here. Proposition 1 implies that, with a monopolist supplier facing competing retailers, vertical merger incentives are identical to horizontal merger incentives between retailers if all firms are initially independent. Similarly, with a monopoly at the retail level facing competing suppliers, vertical integration incentives are identical to horizontal merger incentives between suppliers if prior to the merger all firms are independent. The following corollary proves useful for providing further intuition on these insights.

Corollary 1 *Vertical merger incentives depend on the initial market structure, the level of substitutability/complementarity between the products and the shape of the unit cost function in the following way:*

(i) *With suppliers integrated and retailers separated ($\Psi = \{AB, a, b\}$), a vertical merger between supplier AB and retailer r takes place (does not take place)if both retailers have strictly increasing (decreasing)units costs.*

(ii) *With suppliers separated and retailers integrated ($\Psi = \{A, B, ab\}$), a vertical merger between supplier s and retailer ab takes place (does not take place)if the products are strict substitutes (complements).*

(iii) *Invoke Assumption 2 (symmetry)and take the scenario with all firms separated ($\Psi = \{A, B, a, b\}$). Supplier s and retailer r merge (stay separated)if for all $\Omega' \in \Omega$ we have $\Delta_p^{\Omega'} < \Delta_C^{\Omega'}$ ($\Delta_p^{\Omega'} > 0$ and $\Delta_C^{\Omega'} < 0$).*

Proof: See Appendix.

Corollary 1 links vertical integration incentives expressed in Proposition 1 to the primitives of the model. I now provide some additional intuition on vertical merger incentives. Take first the pre-merger case of a monopolist retailer facing separated suppliers. In this situation, vertical integration between the retailer and one supplier is profitable for the merging parties if products are substitutes. Why is this so? It is convenient to focus on the effects of integration on the non-merged supplier: Since only the distribution of payoffs are affected, not overall output, any gains of the merging parties must exactly correspond to the losses of the non-merged supplier.

If products are substitutes, each supplier wants to be first to reach an agreement with the retailer. This is so, because bargaining between a supplier and the retailer revolves around the sharing of the marginal rent generated by the negotiating parties: With products being substitutes, the additional rent generated by the first supplier to reach an agreement with the retailer is larger than that generated by the second supplier. Substitutability of the products implies that the latter generates negative price externalities for the first supplier. Therefore, suppliers prefer negotiating on infra-marginal quantities to bargaining "on the margin." This explains why with substitutes the non-merging supplier loses if the other market actors integrate vertically. With vertical integration between the retailer and the rival upstream firm, negotiations between the merged parties cannot break down. Hence, the non-merging supplier cannot be the first to reach an agreement with the retailer, because vertical integration guarantees that an agreement between the rival and the retailer is in place. The non-merging supplier is left with having to bargain at the margin, i.e. about the lower surplus it generates by coming second to the retailer.

The same logic holds if goods are complements. In that case, each supplier prefers to be second in reaching an agreement with the retailer: Complementary products imply that the additional surplus generated by the second supplier to reach an agreement with the retailer is larger than that generated by the first one. Vertical integration with complements would only ensure that the integrated supplier cannot be second to reach an agreement with the retailer. This would benefit the non-merging party and therefore harm the firms considering integration.

Take now the situation in which pre-merger a monopoly supplier negotiates with two retailers. Vertical integration between the supplier and a retailer takes place if unit costs are strictly increasing. The reason is as follows: If unit costs are strictly increasing, each retailer prefers to be first in reaching an agreement with the supplier, i.e. to negotiate about infra-marginal quantities. The retailer coming second faces higher unit costs and is therefore left with a smaller surplus to negotiate about with the supplier. Vertical integration corresponds to a sure agreement between the integrating upstream and downstream firms, leaving the non-merging retailer with the only option to be second. This erodes the bargaining power of the second retailer and therefore benefits the merging parties. If unit costs are strictly decreasing, each retailer prefers to be second in reaching an agreement with the supplier and to negotiate about marginal quantities. Once a supplier-retailer agreement is in place, the additional rent generated by an other retailer is larger, since unit costs are lower. In this case a vertical merger is not attractive, since it forces the integrated supplier to be first.

It was noted above that vertical merger incentives in the pre-merger market structures of downstream and upstream monopoly are identical to horizontal merger incentives between retailers and suppliers respectively, if initially all firms are independent (see IW). I now provide further intuition for why this is the case. Take the initial market structure of full separation and retailer horizontal merger incentives. IW show that retailers merge if suppliers have strictly increasing unit costs. In this case, each retailer prefers to be first to reach an agreement with a supplier. The retailer coming second to any supplier generates lower marginal surplus, since unit costs for the additional output to be delivered are higher with strictly increasing unit costs. A horizontal merger between retailers ensure that the merged entity can always come first to each supplier. As explained above, this is the same logic which drives vertical merger incentives in the downstream monopoly pre-merger market structure. The intuition behind why vertical merger incentives in an upstream monopoly correspond to upstream horizontal merger incentives under full separation is analogous.

I now explain the intuition behind vertical integration incentives under pre-merger full separation. I focus on the most instructive case, namely when all firms are symmetric as assumed in Corollary 1 and postpone discussing the role of asymmetry in the verticals for later. Under such circumstances, vertical integration incentives correspond to a mix of horizontal integration incentives upstream and downstream. IW show that upstream horizontal mergers depend on whether goods are substitutes or complements, while downstream mergers depend on whether unit costs are strictly increasing or decreasing. My results show that vertical merger incentives are very similar and can be expressed as a mix of horizontal merger incentives. In particular, whether a vertical merger is profitable if initially all firms are separated depends on *how strong* complements or substitutes the products are compared to *how strongly* unit costs increase or decrease. This relationship is illustrated in Figure (1). The strength of complementarity/substitutability is captured by $\Delta_p^{\Omega'}$ while the speed with which unit costs increase or decrease is measured by $\Delta_C^{\Omega'}$.

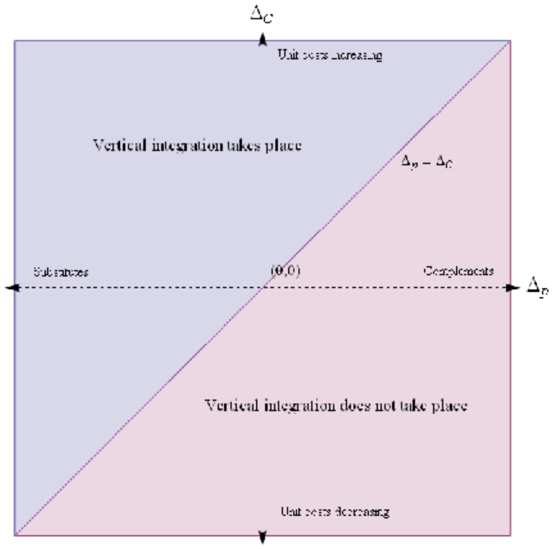

Figure 1: Vertical integration incentives

A vertical merger implies for the integrating firms that they are always first to reach an agreement with each other. If this is what they would want in the absence of the merger, than integration is unambiguously profitable. This is the case when products are substitutes ($\Delta_p^{\Omega'} < 0$)and unit costs are increasing ($\Delta_C^{\Omega'} > 0$). If unit costs are increasing, retailers want to be first to reach an agreement with each supplier. Being second means having to negotiate about the distribution of a lower surplus, because unit costs are higher for the additional output to be supplied. If products are substitutes, the suppliers also prefer to be first in striking an agreement with retailers. The supplier coming second must take into account the negative price externality it imposes on the other supplier already having an agreement in place with the same supplier, and is hence left to negotiate about a lower surplus. Putting these together, with substitute products and strictly increasing unit costs both retailers as well as suppliers prefer to be first to reach an agreement with the other firms. This is exactly what a vertical merger guarantees with the merging partner, and is therefore profitable. The logic is the same for why vertical mergers are not preferred if products are complements ($\Delta_p^{\Omega'} > 0$)and unit costs are strictly decreasing ($\Delta_C^{\Omega'} < 0$).

Under such circumstances retailers as well as suppliers prefer to negotiate with firms of the other type once the bargaining partner already has an agreement in place. A vertical merger undermines this opportunity as it in effect guarantees being first to reach agreement, and is therefore not desired.

Interesting situations arise when products are substitutes (complements)and unit costs are strictly decreasing (increasing). In these cases the interests of the suppliers and retailers are not aligned with respect to the desired order to reach an agreement. For example, maintaining the assumption of firms being symmetric, with substitute goods and strictly decreasing unit costs suppliers prefer being first to reach an agreement with retailers, whereas retailers want to be second to agree with suppliers. Since vertical integration implies a sure agreement between the merged parties, it benefits the merging supplier but is contrary to the involved retailer's interests. The profitability of such a merger, therefore, depends on whether the gains of the former exceed the losses of the latter. This is the case if products are sufficiently strong substitutes while unit costs are sufficiently slowly decreasing (i.e. if $\Delta_p^{\Omega'} < \Delta_C^{\Omega'} < 0$). The same logic applies if products are complements and unit costs are strictly increasing.

In the discussion of vertical integration incentives under pre-merger full separation I remained silent on the role of asymmetry between firms. I address this issue now. While all of what has been said so far stays valid, asymmetry between firms has some implications for vertical merger incentives. According to Claim (i)of Proposition 1, vertical integration between supplier s and retailer r is profitable if

$$\left(W_{\Omega\backslash s'r'} - W_{\Omega\backslash sr}\right) + W_{\Omega\backslash s} + W_{\Omega\backslash r} \geq W_{\Omega}. \tag{3.2}$$

Under symmetry the terms inside the brackets cancel out, but it does not do so under asymmetry. Expression (3.2)connotes that vertical integration is more likely to take place if the merging vertical is relatively large compared to the non-merging one, (i.e. if the difference $W_{\Omega\backslash s'r'} - W_{\Omega\backslash sr}$ is larger). This is the case if the vertically integrating firms s and r are able to produce a relatively large surplus on their own compared to the surplus produced by the non-merging firms s' and r' relying solely on each other. The reason is that infra-marginal rents are greater if the merging vertical is larger. Vertical integration ensures that the merging parties receive a larger share of the infra-marginal rents. A vertical merger is therefore more likely to take place in a larger vertical.

Finally, it remains to note that, in my setup, incentives to integrate vertically are not unambiguously greater under upstream competition than under monopoly. This is in contrast to the results derived by dFG, who find that vertical integration incentives are always stronger with competition upstream. To see this, we can compare the conditions for vertical integration in both market structures as given in Claims (i)and (ii)of Proposition 1. Vertical integration incentives are greater under upstream monopoly than under competition if

$$W_{\Omega\setminus r} + W_{\Omega\setminus r'} > \left(W_{\Omega\setminus s'r'} - W_{\Omega\setminus sr}\right) + W_{\Omega\setminus s} + W_{\Omega\setminus r}, \tag{3.3}$$

whereas they are smaller if the opposite holds. To demonstrate that arrangements exists in which vertical integration incentives under upstream monopoly are stronger than under competition, I focus on the case of full symmetry. Condition (3.3)then reduces to $W_{\Omega\setminus r} > W_{\Omega\setminus s}$, which holds if an additional retailer increases total surplus by a relatively large amount, while the marginal contribution of a supplier is rather small. This is likely to be the case for example if unit costs are strongly increasing while goods are relatively weak complements. Upstream competition can thus either enhance or reduce the prospective of strategic vertical integration.

3.5 Comparison of Horizontal and Vertical Merger Incentives

In this section I aim to compare horizontal and vertical merger incentives in more detail. To create a benchmark I assume that one firm, either upstream or downstream, is available for sale by means of an auction. This firm will be referred to as the target firm. The other firms in the market bid to acquire the target, which is sold to the highest bidder. Horizontal integration incentives are said to be stronger (weaker)than vertical integration incentives, if the bidder on the same market level as the target has a higher (lower)willingness to pay for merging with the target than a bidder from the other market level. I consider a very simple two-stage game, where in the first stage firms submit sealed bids for the target. At the end of the stage the highest bidder merges with the latter. In the second stage, the acquirer pays out its bid and supply contracts are negotiated. I assume that the target firm is sold without a reservation price, i.e. it does not have the opportunity to refuse an offer and remain unsold. This is a convenient simplification that allows us to focus on horizontal and vertical merger incentives arising from the altered bargaining power of the bidders.

In what follows I analyze the first stage of the game, i.e. the auctioning of the target firm. I first turn to the case where a supplier is available for sale. I then consider the auctioning off of a retailer.

Assume w.l.o.g. that supplier A is available for sale and supplier B and retailer a submit bids β_B and β_a respectively for acquisition. Let $U^{\Psi}_{\psi\in\Psi}$ denote the profit of firm ψ in market structure Ψ resulting after stage 1. Then, depending on the outcome of the auction in stage 1, firms make the following profits in stage 2:

(i)Retailer a wins in stage 1 and merges with A: $U_A = \beta_a$, $U_B = U_B^{\{Aa,B,b\}}$, $U_a = U_{Aa}^{\{Aa,B,b\}} - \beta_a$.

(ii)Supplier B wins in stage 1 and merges with A: $U_A = \beta_B$, $U_B = U_{AB}^{\{AB,a,b\}} - b_B$, $U_a = U_a^{\{AB,a,b\}}$.

To determine the winner of the auction, I first derive the maximum possible bids, i.e. those, that leave the bidders indifferent between acquiring the target and not bidding. The indifference conditions take the form

$$U_B^{\{Aa,B,b\}} = U_{AB}^{\{AB,a,b\}} - \beta_B,$$
$$U_{Aa}^{\{Aa,B,b\}} - \beta_a = U_a^{\{AB,a,b\}}.$$

Rearranging yields the maximum bids for acquiring supplier A as

$$\begin{aligned} \beta_a &= U_{Aa}^{\{Aa,B,b\}} - U_a^{\{AB,a,b\}} = \frac{1}{6}\left[W_{\Omega\backslash B} + 2W_{\Omega\backslash Bb} - 2W_{\Omega\backslash Aa} + 2W_{\Omega\backslash a}\right], \\ \beta_B &= U_{AB}^{\{AB,a,b\}} - U_B^{\{Aa,B,b\}} = \frac{1}{6}\left[2W_{\Omega\backslash B} + W_{\Omega\backslash Bb} - W_{\Omega\backslash Aa} + W_{\Omega\backslash a}\right]. \end{aligned} \tag{3.4}$$

Assume now that retailer a is available for sale and supplier A and retailer b submit bids β_A and β_b for acquisition, respectively. Depending on the outcome of the auction in stage 1, the profits in stage 2 are:

(i)Retailer A wins in stage 1 and merges with a: $U_a = \beta_A$, $U_A = U_{Aa}^{\{Aa,B,b\}} - \beta_A$, $U_b = U_b^{\{Aa,B,b\}}$.

(ii)Supplier b wins in stage 1 and merges with a: $U_a = \beta_B$, $U_A = U_A^{\{A,B,ab\}}$, $U_b = U_{ab}^{\{A,B,ab\}} - \beta_b$.

Bidders are indifferent between acquiring the target and not bidding if the following conditions hold:

$$\begin{aligned} U_{Aa}^{\{Aa,B,b\}} - \beta_A &= U_A^{\{A,B,ab\}}, \\ U_b^{\{Aa,B,b\}} &= U_{ab}^{\{A,B,ab\}} - \beta_b. \end{aligned}$$

By rearranging we obtain the maximum bids for acquiring retailer a as

$$\begin{aligned} \beta_A &= U_{Aa}^{\{Aa,B,b\}} - U_A^{\{A,B,ab\}} = \frac{1}{6}\left[2W_{\Omega\backslash Bb} - 2W_{\Omega\backslash Aa} + 2W_{\Omega\backslash A} + W_{\Omega\backslash b}\right], \\ \beta_b &= U_{ab}^{\{A,B,ab\}} - U_b^{\{Aa,B,b\}} = \frac{1}{6}\left[W_{\Omega\backslash Bb} - W_{\Omega\backslash Aa} + W_{\Omega\backslash A} + 2W_{\Omega\backslash b}\right]. \end{aligned} \tag{3.5}$$

The following proposition sums up the results on the outcome of the auction.

Proposition 2 The auction for takeover has the following outcome:

(i) *Assume that supplier A is the target firm. The acquiring firm is supplier B if*

$$W_{\Omega\backslash a} - W_{\Omega\backslash Aa} < W_{\Omega\backslash B} - W_{\Omega\backslash Bb},$$

whereas it is retailer a if the opposite holds.

(ii) *Assume that retailer a is the target firm. The acquiring firm is supplier A if*

$$W_{\Omega\backslash A} - W_{\Omega\backslash Aa} < W_{\Omega\backslash b} - W_{\Omega\backslash Bb}.$$

whereas it is retailer b if the opposite holds.

Proof: Comparing maximum bids in (3.4)and (3.5)yields the conditions stated in Proposition 2.

Before providing some intuition to these results, it is helpful to investigate when the conditions stated in Proposition 2 hold. For the case of symmetric firms, Corollary 2 relates incentives to acquire a firm to the primitives of the model.

Corollary 2 *Under Assumption 2 (symmetry), regardless whether the target firm is a retailer or a supplier, the acquiring firm is a retailer if $-\Delta_p^{\Omega'} < \Delta_C^{\Omega'}$ for all $\Omega' \subseteq \Omega$, whereas it is a supplier if the opposite holds.*

Proof: See Appendix.

Corollary 2 implies that no matter whether a retailer or a supplier is for sale, the acquiring firm is a supplier if products are substitutes and unit costs are strictly decreasing. It is a retailer if goods are complements and unit costs are strictly increasing. In every other case, which firm acquires the target depends on the relative magnitudes of $\Delta_p^{\Omega'}$ and $\Delta_C^{\Omega'}$, capturing the strength of complementarity/substitutability between products and the speed at which unit costs increase or decrease. For example, if products are complements and unit costs are decreasing, the acquirer is a retailer if complementarity between products are relatively strong, while unit costs are not decreasing too rapidly. In case products are substitutes and unit costs increase, the acquiring firm is the retailer if unit costs increase quickly while products are relatively weak substitutes.

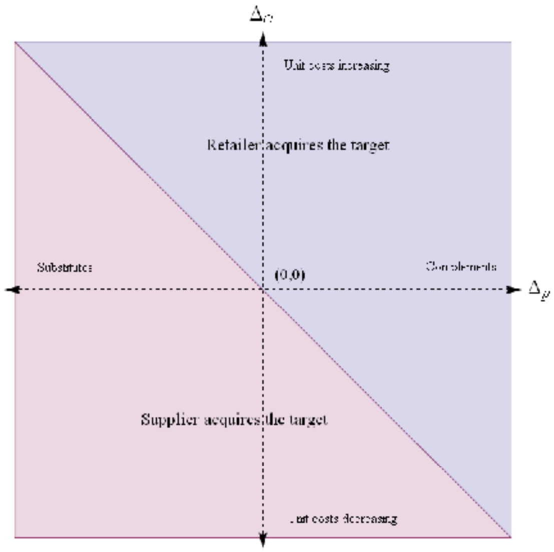

Figure 2: Outcome of the auction for acquisition.

The intuition is as follows. Take first the case where supplier A is for sale, and supplier B as well as retailer a bid for acquisition. The bidders' incentive to acquire the target stems from the fact that doing so can improve their bargaining power in the subsequent negotiations on delivery conditions. Which firm is willing to pay most for the target therefore depends on whether a merged horizontal supplier or a vertical chain is able to convey more bargaining power to the merged entity. Note that since the target is sold for any positive bid, submitting a bid is always a dominant strategy: acquiring the target for a small but positive bid is always more attractive than giving up on it. As it was explained above, a supplier prefers to merge horizontally if goods are substitutes because bargaining jointly with the other supplier allows them to move away from the margin. Similarly, a retailer prefers to integrate with a supplier if unit costs are strictly increasing. A supplier therefore has a relatively high (low)willingness to pay for its competitor if goods are substitutes (complements)At the same time, the bidding retailer has a relatively high (low)valuation for the target if unit costs are strictly increasing (decreasing). This implies, for example, that with goods being complements and unit costs increasing, the retailer can outbid the supplier, i.e. vertical merger incentives are stronger than horizontal ones.

The contrary holds if goods are substitutes and unit costs decrease. Since in this case upstream horizontal merger incentives are strong while the retailer dislikes vertically merging, the target goes to supplier B. If unit costs are increasing and products are substitutes, both bidders have relatively strong incentives to acquire supplier A. In this case, which firm gains more by buying the target depends on the relative magnitudes of $\Delta_p^{\Omega'}$ and $\Delta_C^{\Omega'}$, i.e. the relative strength of substitutability/compatibility between the products and the speed with which unit costs increase or decrease. This relationship implies that with the target being an upstream firm, vertical integration is a better instrument to extract rents from the non-merging parties compared to upstream horizontal integration, if unit costs increase relatively fast (or decrease relatively slowly)while products are relatively weak substitutes (or are relatively strong complements). In this case, the retailer can outbid the supplier for the acquisition of the upstream target firm.

The intuition behind when a vertical acquisition is preferred to a horizontal in case the target firm is a retailer follows similar lines. Take the case where retailer a is for sale, and supplier A as well as retailer b bid for acquisition, with supplier B being the non-bidding firm. All other things equal, supplier b has a stronger incentive to merge with its horizontal counterpart if unit costs increase, since negotiating jointly with the supplier prevents the retailers from being forced to the margin. At the same time, supplier A values merging vertically more if products are substitutes. In this case having a sure partner in retailer a protects it from having to negotiate about marginal quantities. This relationship reveals that with the target being a downstream firm, vertical integration is a more apt instrument to extract rents from the non-merging parties compared to upstream horizontal integration, if products are relatively strong substitutes (or relatively weak complements), while unit costs decrease relatively fast (or increase relatively slowly). In this case, the supplier can outbid the retailer for the acquisition of the downstream target firm.

To sum up, in my model integration incentives stem from the possibility to extract rents from the non-integrated parties. Acquisition incentives depend on which bidding party can improve its bargaining position more by moving towards infra-marginal quantities, given the shape of unit costs and the level of substitutability and complementarity between products. My results stand somewhat in contrast to the traditional argument that reaching a monopoly position creates strong incentives for horizontal mergers. In particular, by focusing solely on the effects of a merger to increase bargaining power, my model shows that vertical integration may under some circumstances convey more (bargaining)power to the merged entity than horizontally merging. This is the case for example, if a supplier (retailer)is for sale and unit costs are strictly increasing (decreasing)while products are complements (substitutes).

3.6 Mergers and Entry

So far I have remained silent on the effects of vertical mergers on efficiency. In this section I address this issue by analyzing how vertical integration can serve as a device to deter entry upstream or downstream. I also compare the entry deterring potential of vertical integration to horizontal mergers. In my framework, deterring entry is always harmful to total welfare. This is due to Assumption 1, which implies that total industry surplus increases in the number of firms in the market.

Consider a situation in which initially three incumbents, i_1,i_2,$i_3 \in \Omega$ are in the market. A potential entrant, $e \in \Omega$, $e \notin \{i_1, i_2, i_3\}$, considers entering at the market level on which only one firm is active. Entry is costly and involves a fixed investment I, which is sunk if e enters. The existence of e and its entry costs I are common knowledge. Incumbents i_1 and i_2 can integrate to alter the bargaining structure and influence the rents the entrant can expect. If i_1 and i_2 are of the same type, their merger is a horizontal one, whereas it is vertical if they are of different types.

The game unfolds as follows. In Stage 1, incumbents i_1 and i_2 decide whether to merge or stay separated. In Stage 2, the potential entrant e decides whether to enter the market or stay out. In Stage 3 firms bargain on the delivery conditions and payments (including the entry costs)are made. The game tree and the resulting market structures are depicted in Figure (3).

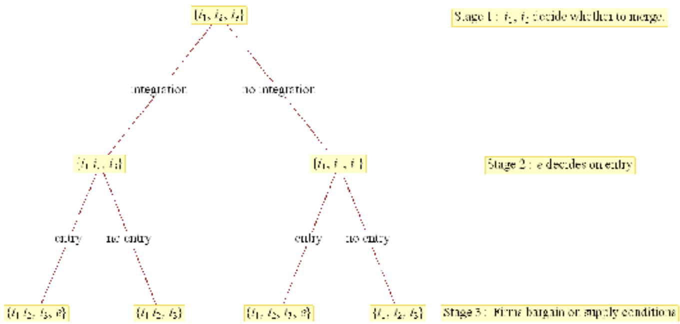

Figure 3: Merger and entry decisions.

I solve the game by backward induction. If e enters, the resulting market structure is $\{i_1 i_2, i_3, e\}$ if i_1 and i_2 merge in Stage 1, and $\{i_1, i_2, i_3, e\}$ if they stay separated. With no entry we get the industry structure $\{i_1 i_2, i_3\}$ when i_1 and i_2 integrate, and $\{i_1, i_2, i_3\}$ if they remain separated. I postpone the description of Stage 3 payoffs for later, when I assign roles to variables $i_1 - i_3$ and e.

Depending on the costs of entry, a merger can affect entry incentives in three ways: it can be irrelevant, it can deter or foster entry. In particular, if $I < \min\{U_e^{\{i_1 i_2, i_3, e\}}, U_e^{\{i_1, i_2, i_3, e\}}\}$ or $I > \max\{U_e^{\{i_1 i_2, i_3, e\}}, U_e^{\{i_1, i_2, i_3, e\}}\}$, the merger does not change entry incentives. In the former case the necessary investment is so small, that entry always occurs, irrespective of whether the incumbents have merged or not. In the latter case entry costs are prohibitive.

The most interesting cases arise if entry costs are somewhere in between these polar options. Then, we can have $U_e^{\{i_1, i_2, i_3, e\}} < I < U_e^{\{i_1 i_2, i_3, e\}}$, in which case the merger between incumbents i_1 and i_2 deters an otherwise profitable entry. Alternatively, with $U_e^{\{i_1 i_2, i_3, e\}} < I < U_e^{\{i_1, i_2, i_3, e\}}$ the merger between i_1 and i_2 enables entry, which would not occur if these firms remained separated.

Since they have perfect information in Stage 1 firms i_1 and i_2 can anticipate how their decision to merge or stay separated influences Stage 2 entry. Assume that $U_e^{\{i_1, i_2, i_3, e\}} < I < U_e^{\{i_1 i_2, i_3, e\}}$, i.e. if the merger takes place it deters entry. Firms i_1 and i_2 then choose to integrate if the profit they earn as integrated entity with e staying out is larger than the sum of their profits in the market structure where they stay separated and e enters, $\{i_1, i_2, i_3, e\}$. Formally, an entry-deterring merger is profitable, if

$$U_{i_1}^{\{i_1,i_2,i_3,e\}} + U_{i_2}^{\{i_1,i_2,i_3,e\}} < U_{i_1 i_2}^{\{i_1 i_2,i_3\}}. \tag{3.6}$$

Assume now that $U_e^{\{i_1 i_2,i_3,e\}} < I < U_e^{\{i_1,i_2,i_3,e\}}$, i.e. a merger fosters entry. Then, incumbents i_1 and i_2 merge if their integrated profit with entry is larger than the profit they realize separately if e stays out. Formally, an entry-fostering merger is profitable, if

$$U_{i_1}^{\{i_1,i_2,i_3\}} + U_{i_2}^{\{i_1,i_2,i_3\}} < U_{i_1 i_2}^{\{i_1 i_2,i_3,e\}}. \tag{3.7}$$

Having set up the general analytical framework, I am now in the position to become specific about horizontal and vertical mergers and entry. I will distinguish between the effects of horizontal and vertical mergers on entry and start with the former.

Proposition 3 *(Horizontal mergers and entry). Invoke Assumption 2 (symmetry)and let* $\underline{I}_u = (1/6)[W_{\Omega\backslash r} - W_{\Omega\backslash s}] + (1/4)W_\Omega$, $\overline{I}_u = (1/6)\left[2W_\Omega - W_{\Omega\backslash s}\right]$, $\underline{I}_d = (1/6)[W_{\Omega\backslash s} - W_{\Omega\backslash r}] + (1/4)W_\Omega$, $\overline{I}_d = (1/6)\left[2W_\Omega - W_{\Omega\backslash r}\right]$. *The following relationship holds between horizontal merger incentives and entry.*

(Upstream entry)

(i) *With* $I < \min\{\underline{I}_u, \overline{I}_u\}$ *(*$I > \max\{\underline{I}_u, \overline{I}_u\}$*), upstream entry takes place (does not take place)regardless whether retailers merge or not.*

(ii) *With* $I \in [\min\{\underline{I}_u, \overline{I}_u\}, \max\{\underline{I}_u, \overline{I}_u\}]$*, and unit costs are strictly increasing (decreasing)retailers stay separated (merge)and accommodate upstream entry.*

(Downstream entry)

(iii) *With* $I < \min\{\underline{I}_d, \overline{I}_d\}$ *(*$I > \max\{\underline{I}_d, \overline{I}_d\}$*), downstream entry takes place (does not take place)regardless whether suppliers merge or not.*

(iv) *With* $I \in [\min\{\underline{I}_d, \overline{I}_d\}, \max\{\underline{I}_d, \overline{I}_d\}]$*, and products being substitutes (complements) suppliers stay separated (merge)and accommodate downstream entry.*

Proof: See Appendix.

Note first, that a horizontal merger between incumbent suppliers or retailers never deters entry. In fact, the opposite is the case. A horizontal merger, which would in the absence of the potential entrant be unprofitable, may actually take place, precisely in order to foster entry. This can be the case if the potential entrant is a supplier, and unit costs are decreasing.

With decreasing unit costs retailers otherwise prefer to stay separated and negotiate at the margin with the supplier(s). However, to attract a supplier into the market they may merge. By doing so they reduce their own bargaining power to convey a larger share of the industry surplus to the entrant. If entry costs lie in the appropriate range (in this example $I \in [\underline{I}_u, \overline{I}_u]$), this additional surplus can motivate the supplier to enter the market. Although the bargaining power of the retailers decreases by merging, it is profitable for them to do so. They are compensated for receiving a smaller share of the pie by the increase in the size of the pie due to entry.

Similarly, if the potential entrant is a retailer and goods are complements, suppliers may choose to enter an otherwise unprofitable merger to foster entry downstream. The logic is similar as before. If the products are complements, suppliers prefer to negotiate at the margin and stay separated. However, if passing on a sufficient share of surplus to the potential entrant is necessary to allow it to cover its costs and enter, than they can achieve such a transfer by merging.

Furthermore, note that beside entering an otherwise unprofitable horizontal merger, incumbents may also refrain from an otherwise profitable horizontal merger, in order to induce entry. This can happen if the potential entrant is a supplier and unit costs increase, or if the entrant is a retailer and products are substitutes. They do so to actively reduce their own bargaining power and pass on a share of industry surplus to the entrant, which it needs to cover its entry costs.

Why are incumbents on the same market level always interested in inducing entry? Could not they gain more by increasing their bargaining power and extracting rents from the incumbent, even at the cost that entry does not occur? As it turns out, the benefits of two firms on the same horizontal level from allowing entry always exceed the rents they can extract from a single incumbent. Take for example the case with increasing unit costs and a supplier considering entry. Retailers would absent entry prefer to merge and bargain jointly with the incumbent supplier. By merging, none will be second to reach an agreement with the supplier and can avoid being marginalized. However, if they remain separated and allow entry to happen, each retailer can be first to strike an agreement with a supplier. In addition, they generate rents by being second as well. They are therefore always better off under entry. (Of course, they would be best off if entry occurred and they merged).

Horizontal competitors therefore always act in favor of entry on the other market level. This result qualifies the common claim that buyer power reduces consumer choice.[7] As I demonstrate in the following, vertical mergers are very different in this respect. In contrast to horizontal mergers, vertical integration can under circumstances profitably deter entry. The next proposition summarizes my results on this issue.

Proposition 4 (Vertical mergers and entry). Invoke *Assumption 2 (symmetry)* and let $\underline{I}_s := (1/6)[W_{\Omega\backslash r} - W_{\Omega\backslash s}] + (1/4)W_\Omega$, $\overline{I}_s := (1/6)\left[W_{\Omega\backslash r} - 2W_{\Omega\backslash s} + 2W_\Omega\right]$, $\underline{I}_r := (1/6)[W_{\Omega\backslash s} - W_{\Omega\backslash r}] + (1/4)W_\Omega$, $\overline{I}_r := (1/6)\left[W_{\Omega\backslash s} - 2W_{\Omega\backslash r} + 2W_\Omega\right]$ and $\Delta^{\Omega'} := p(q^{\Omega'}, 0) - \overline{C}(q^{\Omega'})$. The *following relationship holds between vertical merger incentives and entry.*

(Upstream entry)

(i) *With* $I < \min\{\underline{I}_s, \overline{I}_s\}$ *(*$I > \max\{\underline{I}_s, \overline{I}_s\}$*), upstream entry takes place (does not take place)regardless whether a supplier and a retailer merge or not.*

(ii) *With* $I \in [\min\{\underline{I}_s, \overline{I}_s\}, \max\{\underline{I}_s, \overline{I}_s\}]$*, vertical merger incentives and upstream entry depend on the strength of substitutability/complementarity between products and the shape of the unit cost function in the following way:*

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[7] See for example EC (1999, p.5): *"Concerns have been raised that buyer power abuses of supermarkets have long term consequences for consumers [...]. They have negative effects on (long term)consumer interests such as decreasing choice [...] of products [...]."*

(a) *If goods are complements, a vertical merger occurs and upstream entry takes place.*

(b) *If goods are substitutes and* $\Delta_C^{\Omega'} < 2\Delta_p^{\Omega'} + (1/2)\Delta^{\Omega'}$ $(\Delta_C^{\Omega'} \geq 2\Delta_p^{\Omega'} + (1/2)\Delta^{\Omega'})$ *vertical merger does not occur (occurs)and upstream entry takes place (does not take place)*

(Downstream entry)

(iii) *With* $I < \min\{\underline{I}_r, \overline{I}_r\}$ *(*$I > \max\{\underline{I}_r, \overline{I}_r\}$*), downstream entry takes place (does not take place)regardless whether a supplier and a retailer merge or not.*

(iv) *With* $I \in [\min\{\underline{I}_r, \overline{I}_r\}, \max\{\underline{I}_r, \overline{I}_r\}]$*, vertical merger incentives and downstream entry depend on the strength of substitutability/complementarity between products and the shape of the unit cost function in the following way:*

(a) *If unit costs are decreasing a vertical merger occurs and downstream entry takes place.*

(b) *If unit costs are increasing and* $\Delta_C^{\Omega'} < (1/2)\Delta_p^{\Omega'} + (1/4)\Delta^{\Omega'}$ *(*$\Delta_C^{\Omega'} \geq (1/2)\Delta_p^{\Omega'} + (1/4)\Delta^{\Omega'}$*), vertical merger does not occur (occurs)and downstream entry takes place (does not take place).*

Proof: See Appendix.

As opposed to horizontal mergers, vertical integration may actually deter the entry of a supplier and of a retailer. By vertically merging, the involved firms can shift bargaining in their own favor to an extent that renders it impossible for an entrant to cover its entry costs. In this case the merging parties trade off a *smaller pie* for a *larger slice.*

Similarly, as in some cases of horizontal mergers, vertical integration may also foster entry by passing on a share of surplus to the entrant, which enables it to cover the costs of entry. While total industry surplus is reduced if entry is deterred, the share of the reduced surplus accruing to the integrating firms can be larger than what they could capture by staying separated and accommodating entry. In the following I provide more detailed intuition for the relationship between vertical integration and entry.

Consider first the entry of a supplier, say B, while supplier A and retailers a and b are incumbent. If entry costs are in the appropriate range ($I \in [\min\{\underline{I}_s, \overline{I}_s\}, \max\{\underline{I}_s, \overline{I}_s\}]$), a vertical merger between supplier A and retailer a can affect the entry decision of supplier B. Figure (4)represents regions depending on the level of substitutability/compatibility between goods and the shape of unit costs, where a vertical merger between A and a affects entry in various ways. Table (4)provides further clarification on each region.

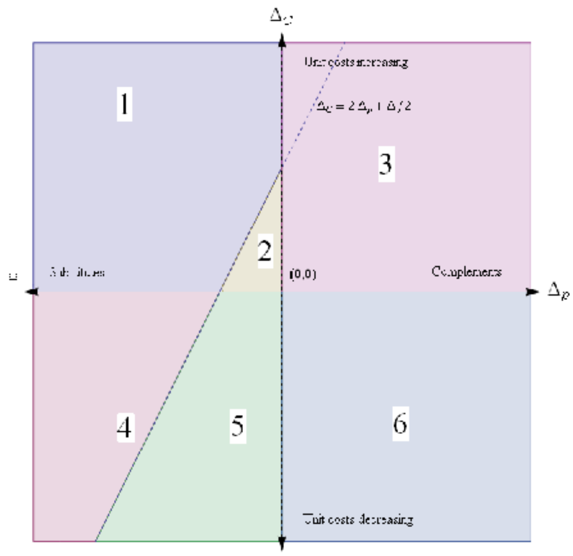

Figure 4: Vertical merger and supplier entry

Region	Merger	Entry	Merger absent potential entry
1	Yes	No	Yes
2	No	Yes	Yes
3	Yes	Yes	Yes
4	Yes	No	No
5	No	Yes	No
6	Yes	Yes	No

Table (4): Figure (4)regions, $I \in [\min\{\underline{I}_s, \overline{I}_s\}, \max\{\underline{I}_s, \overline{I}_s\}]$

It is shown in Corollary 1 that a vertical merger between a monopoly supplier and a retailer is profitable if unit costs are increasing. In this case, vertically merging allows the integrated retailer to always receive the input in the range with the lowest costs, leaving the other retailer to bargain about deliveries in the range where costs are higher. In regions 1-3 in Figure (4)therefore the merging firms can shift away rents from the other retailer by vertically integrating. However, if they do so they also shift rents away from supplier B now considering entry. If the entry costs lie in the appropriate range (namely in this case $I \in [\overline{I}_s, \underline{I}_s])$, this induces supplier B to stay out of the market. Entry not taking place is costly for the parties considering integration. A new supplier selling a new product generates additional surplus, which is forgone if entry does not take place. The benefits of increased bargaining power must therefore be weighed against the loss of extra industry surplus. I demonstrate this on each region in Figure (4). Assume in the following, that entry costs lie in the interval $I \in [\min\{\underline{I}_s, \overline{I}_s\}, \max\{\underline{I}_s, \overline{I}_s\}]$. This implies, that the merger decision of supplier A and retailer a can influence retailer B's entry incentives.

In region 1 unit costs are relatively rapidly increasing and goods are rather strong substitutes ($\Delta_C^{\Omega'} \geq 2\Delta_p^{\Omega'} + (1/2)\Delta^{\Omega'}$ and $\Delta_C^{\Omega'} > 0$, $\Delta_p^{\Omega'} < 0$). Under such circumstances vertical integration takes place and deters entry. The reason behind why this is profitable is that with unit costs rapidly increasing, the merged retailer benefits from not having to negotiate about marginal quantities with its partner. Vertical integration therefore shifts a lot of bargaining power to the merging parties. At the same time, with products being strong substitutes, the additional surplus generated by the entrant supplier is relatively low because of the strong negative cross-price effect. In this case, entry-deterring integration is profitable because it conveys a lot of bargaining power to the merging parties and the forgone increase in industry surplus is relatively low.

In region 2 products are relatively weak substitutes and unit costs are slowly increasing ($\Delta_C^{\Omega'} < 2\Delta_p^{\Omega'} + (1/2)\Delta^{\Omega'}$ and $\Delta_C^{\Omega'} > 0$, $\Delta_p^{\Omega'} < 0$). Vertical integration does not take place, and entry can occur. If unit costs are slowly increasing, vertical integration does not improve the bargaining position of the involved parties sufficiently to compensate them for their forgone share of the larger industry surplus they receive if the supplier enters. If goods are weak substitutes, entry increases total surplus by a medium amount, since the negative cross-price effect is not very strong.

In region 3 goods are complements and unit costs are increasing ($\Delta_C^{\Omega'}, \Delta_p^{\Omega'} > 0$). Although a vertical merger is profitable with increasing unit costs, and some rent is shifted away from the entrant, entry still occurs. This is because with goods being complements the entrant generates a large additional surplus due to the positive cross-price effect, and captures a sufficient portion of it to cover its entry costs. In fact, the entrant is better off if a vertical merger takes place when goods are complements. Such a merger guarantees that it can negotiate about marginal quantities with the integrated retailer, which are large if goods are complements. If entry costs are in the range $U_B^{\{A,B,a,b\}} < I < U_B^{\{Aa,B,b\}}$, the vertical merger facilitates entry: the merging parties pass on part of their profits to the entrant which allows it to cover its entry costs.

In region 4 unit costs are relatively weakly increasing while products are rather strong substitutes ($\Delta_C^{\Omega'} \geq 2\Delta_p^{\Omega'} + (1/2)\Delta^{\Omega'}$ and $\Delta_C^{\Omega'} < 0$, $\Delta_p^{\Omega'} < 0$). With unit costs decreasing, absent the possibility of upstream entry, no vertical merger would be profitable. Yet with the threat of a supplier entering, the vertical merger takes place and deters entry. The winner is primarily the integrated supplier, since the merger prevents the entry of a competitor offering a strong substitute product. With unit costs decreasing the merging parties lose a bit of bargaining power vis-à-vis the non-integrated retailer, which can always negotiate at the margin with the integrated supplier. However, since products are strong substitutes, the supplier would lose more if entry occurred.

In region 5 unit costs are relatively rapidly decreasing while products are rather week substitutes ($\Delta_C^{\Omega'} < 2\Delta_p^{\Omega'} + (1/2)\Delta^{\Omega'}$ and $\Delta_C^{\Omega'} < 0$, $\Delta_p^{\Omega'} < 0$). No merger takes place and supplier B enters. With quickly decreasing unit costs, a vertical merger erodes the merging parties' bargaining position vis-à-vis the second retailer significantly. The rather small benefit to the integrated supplier of keeping a not too strongly substitutable upstream competitor out of the market is not worth paying this price.

Finally, in region 6 goods are complements and unit costs decrease ($\Delta_C^{\Omega'} < 0$, $\Delta_p^{\Omega'} > 0$). If entry costs are in the range $U_B^{\{A,B,a,b\}} < I < U_B^{\{Aa,B,b\}}$, in this case a vertical merger takes place in order to facilitate entry. Absent potential entry this merger would not occur, and furthermore, absent the merger supplier B would not enter. If goods are complements the entrant supplier benefits from a vertical merger between incumbents, because it guarantees to bargain on marginal quantities with the integrated retailer. Goods being complements means that entry has large benefits for the incumbents as well, which exceed any rents that could be extracted from the second retailer by merging and deterring entry. Therefore, the merger is profitable and it fosters entry.

Consider next the entry of a retailer, say b, while suppliers A and B as well as retailer a are incumbent. If entry costs are in the appropriate range ($I \in [\min\{\underline{I}_r, \overline{I}_r\}, \max\{\underline{I}_r, \overline{I}_r\}]$), a merger between supplier A and retailer a can affect the entry decision of retailer b. Figure (5)depicts regions depending on the level of substitutability/compatibility between goods and the shape of unit costs, where a vertical merger between A and a affects entry in various ways.

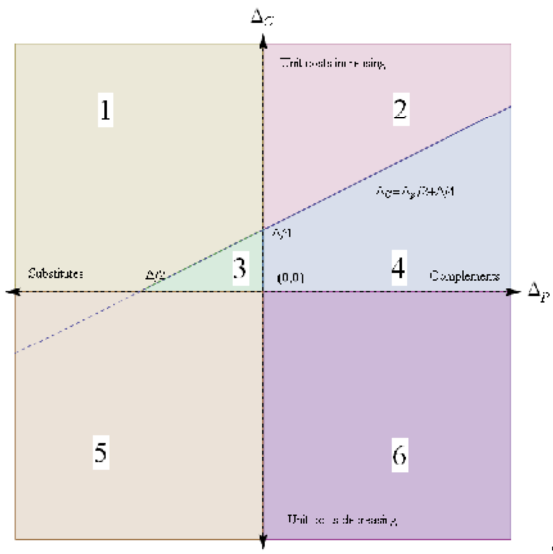

Figure 5: Vertical merger and retailer entry

Region	Merger	Entry	Merger absent potential entry
1	Yes	No	Yes
2	Yes	No	No
3	No	Yes	Yes
4	No	Yes	No
5	Yes	Yes	Yes
6	Yes	Yes	No

Table (5): Figure (5)regions, $I \in [\min\{\underline{I}_r, \overline{I}_r\}, \max\{\underline{I}_r, \overline{I}_r\}]$

I demonstrate how vertical merger incentives interact with retailer entry for each region in Figure (5). Assume that $I \in [\min\{\underline{I}_r, \overline{I}_r\}, \max\{\underline{I}_r, \overline{I}_r\}]$, so that a merger decision is not irrelevant for entry.

In region 1, unit costs are relatively quickly increasing and products are relatively strong substitutes ($\Delta_C^{\Omega'} \geq (1/2)\Delta_p^{\Omega'} + (1/4)\Delta^{\Omega'}$, $\Delta_p^{\Omega'} < 0$ and $\Delta_C^{\Omega'} > 0$). If this is the case, vertical integration takes place and it deters retailer entry. This is profitable for the following reason. With products being substitutes the integrated supplier benefits from vertical integration, since it it does not have to negotiate about marginal quantities with its partner retailer. With unit costs increasing, a second retailer generates relatively little surplus additional to the existing market configuration. Since products are sufficiently strong substitutes and unit costs are sufficiently quickly increasing ($\Delta_C^{\Omega'} \geq (1/2)\Delta_p^{\Omega'} + (1/4)\Delta^{\Omega'}$), the increased rents the merged entity can extract from the incumbent supplier absent entry exceeds the share of the parties of the increased surplus that would be realized with entry. Therefore, a vertical merger to deter the entry of a retailer is profitable.

In region 2, unit costs are strongly increasing and products are complements ($\Delta_C^{\Omega'} \geq (1/2)\Delta_p^{\Omega'} + (1/4)\Delta^{\Omega'}$, $\Delta_C^{\Omega'} > 0$ and $\Delta_p^{\Omega'} > 0$). A vertical merger takes place and it deters the entry of retailer b. Note, that this merger would not be profitable absent the threat of entry: With goods being complements the vertically merging parties actually weaken their bargaining power vis-à-vis the second supplier, because they make sure it can negotiate about marginal quantities, which it prefers. The reason why this merger is still profitable is that with strongly increasing unit costs the incumbent retailer would lose a lot if the other retailer entered. The merger is therefore primarily motivated by the incumbent retailer's choice of the lesser of two evils: slightly reduced bargaining power but keeping its downstream monopoly position instead of largely decreased bargaining power and accommodating retailer entry.

In region 3 unit costs are relatively slowly increasing and products are relatively weak substitutes ($\Delta_C^{\Omega'} < (1/2)\Delta_p^{\Omega'} + (1/4)\Delta^{\Omega'}$, $\Delta_C^{\Omega'} > 0$ and $\Delta_p^{\Omega'} < 0$). No vertical merger takes place and retailer b enters. Note, first, that under such conditions absent the threat of entry a vertical merger would be profitable, since products are (weak)substitutes. Weak substitutability also implies that by merging and deterring entry the involved parties could slightly improve their bargaining power vis-à-vis the second supplier. However, supplier A benefits a lot if retailer b enters. Since unit costs are increasing, both retailers want to be first to reach an agreement with the suppliers, which puts the latter into a comfortable bargaining position. A vertical merger is therefore not profitable, because it would make the upstream party worse off by deterring downstream entry.

In region 4 unit costs are relatively slowly increasing and products are complements ($\Delta_C^{\Omega'} < (1/2)\Delta_p^{\Omega'} + (1/4)\Delta^{\Omega'}$, $\Delta_C^{\Omega'} > 0$ and $\Delta_p^{\Omega'} > 0$). No vertical merger takes place and entry occurs. Since goods are complements a vertical merger that prevents entry does not benefit the involved parties: it guarantees that the non-merging supplier always negotiates at the margin with the retailer, which is precisely what it wants. Staying separated and allowing entry on the other hand allows firms A and a to enjoy the benefits of a larger total surplus.

In region 5 unit costs are decreasing and products are substitutes ($\Delta_C^{\Omega'} < 0$ and $\Delta_p^{\Omega'} < 0$). A vertical merger takes place and it enables entry if $I \in [\min\{\underline{I}_r, \overline{I}_r\}, \max\{\underline{I}_r, \overline{I}_r\}]$. The choice of supplier A and retailer a is between merging and inviting entry or staying separated absent entry. The merger fosters entry because it benefits retailer b by allowing it to negotiate about marginal quantities with the integrated supplier, which it prefers to do with decreasing unit costs.

Finally, in region 6 unit costs are decreasing and products are complements ($\Delta_C^{\Omega'} < 0$ and $\Delta_p^{\Omega'} > 0$). While absent the threat of entry no vertical merger would take place, now supplier A and retailer a merge in order to enable entry. In this case a new retailer increases total surplus by a relatively large amount, of which supplier A and retailer a can capture a share which exceeds what they could gain by staying separated and deterring entry.

In summary, in this section I contrasted horizontal and vertical mergers in terms of their effect on entry. In general, mergers change the bargaining position of all parties and can either deter entry or facilitate it. Entry deterrence occurs by reducing the potential entrant's expected revenues below the level sufficient to cover its entry costs. A merger can facilitate entry by credibly conveying bargaining strength to the entrant that enables it to generate sufficient revenues to cover its entry costs, which it could not do otherwise. In my framework horizontal mergers to deter entry are never profitable, and they are often entry-facilitating. However, vertical mergers that keep a potential entrant out of the market can pay off. An entry-deterring vertical merger has two effects. First, it changes the bargaining position of all parties, potentially allowing the merging firms to get a larger share of industry surplus. At the same time, by deterring entry the merger prevents the realization of a higher industry surplus. Broadly speaking, a merger to deter entry may enable the involved firms to obtain a larger slice of a smaller pie. Horizontal mergers that would deter entry are not profitable because for such mergers the size of the pie matters more, and the total surplus is larger if entry takes place. Vertical mergers are different in this respect. They may shift bargaining in favor of the merged entity so that the additional rents it extracts from the non-merging incumbent exceed the benefits from an increased pie due to entry. When this can occur depends on the level of substitution or complementarity between the products, the shape of the unit cost function and the average markup a pair of supplier and retailer can generate alone. In my setup, vertical mergers are therefore more likely to be harmful for total welfare than horizontal ones.

3.7 Example

In this section I provide a -simple, discrete example to illustrate and verify selected results derived earlier in this article. Assume that suppliers and retailers are symmetric, and each supplier can provide either one unit of the product to a retail outlet or none. Indirect demand for product s at retailer r is

$$p_{sr} = \begin{cases} p & \text{if } q_{s'r} = 0 \\ p+\gamma & \text{if } q_{s'r} = 1 \qquad \forall s, r \in S^0 \times R^0, \end{cases}$$

with $\gamma > 0$ ($\gamma < 0$)if products are complements (substitutes). The cost function of supplier s is given by

$$C_s(q_{sr} + q_{sr'}) = \begin{cases} 0 & \text{if } q_{sr} + q_{sr'} = 0 \\ c_1 & \text{if } q_{sr} + q_{sr'} = 1 \\ 2(c_1+\kappa) & \text{if } q_{sr} + q_{sr'} = 2 \qquad \forall s, r \in S^0 \times R^0. \end{cases}$$

The resulting unit cost function is then

$$\overline{C}_s(q_{sr} + q_{sr'}) = \begin{cases} 0 & \text{if } q_{sr} + q_{sr'} = 0 \\ c_1 & \text{if } q_{sr} + q_{sr'} = 1 \\ c_1+\kappa & \text{if } q_{sr} + q_{sr'} = 2 \qquad \forall s, r \in S^0 \times R^0, \end{cases}$$

where $\kappa > 0$ ($\kappa < 0$)captures increasing (decreasing)unit costs. I focus on the symmetric equilibrium, in which it is trivially always optimal for each supplier to provide one unit of the good to each retailer. The resulting industry surpluses under various market configurations are the following:

$$\begin{aligned} W_{\Omega\backslash sr} &= p - c_1, \\ W_{\Omega\backslash s} &= 2p - [2(c_1+\kappa)], \\ W_{\Omega\backslash r} &= 2(p+\gamma) - 2c_1, \\ W_{\Omega} &= 4(p+\gamma) - 2\,[2(c_1+\kappa)]. \end{aligned} \tag{3.8}$$

Assumption 1 (superadditivity)and the exclusion of corner solutions imply, that the following restrictions need to hold:

$$
\begin{aligned}
W_{\Omega\backslash sr} < W_{\Omega\backslash r} &\iff 0 < p - c_1 + 2\gamma \\
W_{\Omega\backslash sr} < W_{\Omega\backslash s} &\iff 0 < p - c_1 - 2\kappa \\
W_{\Omega\backslash s} < W_{\Omega} &\iff 0 < 2(p - c_1) - 2\kappa + 4\gamma \\
W_{\Omega\backslash r} < W_{\Omega} &\iff 0 < 2(p - c1) - 4\kappa + 2\gamma \\
0 < W_{\Omega\backslash sr} &\iff 0 < p - c_1
\end{aligned}
\tag{3.9}
$$

I first verify vertical merger incentives under full separation as stated in Proposition 1 and Corollary 1. According to Proposition 1, supplier s and retailer r merge, if $\left(W_{\Omega\backslash s'r'} - W_{\Omega\backslash sr}\right) + W_{\Omega\backslash s} + W_{\Omega\backslash r} \geq W_{\Omega}$, which after plugging in the values from (3.8)becomes simply $\kappa > \gamma$. This immediately verifies Corollary 1, according to which a vertical merger takes place if $\Delta_C^{\Omega'} > \Delta_p^{\Omega'}$. Note that from Definition 3 in this example $\Delta_p^{\Omega'} = \gamma$ and $\Delta_C^{\Omega'} = \kappa$, for every Ω'.

I next turn to vertical integration and entry, and focus on the case where entry can occur upstream, with one supplier and two retailers being incumbent. Let s' be the potential entrant and supplier s as well as retailer r consider vertical integration to deter or facilitate upstream entry. The game unfolds according to Figure (4): In Stage 1, incumbents s and r decide whether to merge or stay separated. In Stage 2, the potential entrant s' decides whether to enter the market or stay out. In Stage 3 firms bargain on the delivery conditions and payments are made.

A vertical merger can deter the entry of a supplier if $U_{s'}^{\{sr,s',r'\}} < I < U_{s'}^{\{s,s',r,r'\}}$. Using Table 1 this corresponds to

$$
W_{\Omega\backslash r}/6 - W_{\Omega\backslash s}/3 + W_{\Omega}/3 < I < W_{\Omega\backslash r}/6 - W_{\Omega\backslash s}/6 + W_{\Omega}/4. \tag{3.10}
$$

Note, that this interval is non-empty if $2W_{\Omega\backslash s} > W_{\Omega}$. As was discussed, this relationship holds if products are substitutes. We can plug in the values from Expression (3.8)into (3.10)to get the interval of entry costs for which a vertical merger in Stage 1 deters entry as

$$
-2\kappa/3 - c_1 + 5\gamma/3 + p < I < -2\kappa/3 - c_1 + 4\gamma/3 + p.
$$

A vertical merger to deter the entry of a supplier is profitable if

$$U_{sr}^{\{sr,r'\}} - \left[U_s^{\{s,s',r,r\}} + U_r^{\{s,s',r,r'\}}\right] > 0. \tag{3.11}$$

Using the values from Tables (1)and (4), this equivalent to $W_{\Omega\backslash s} + W_{\Omega\backslash sr} > W_\Omega$, which after plugging in Expression (3.8)becomes

$$\kappa > (p - c_1)/2 + 2\gamma.$$

Observe that this corresponds to the condition stated in Proposition 4, Claim (II.b).

A vertical merger fosters supplier entry if $U_{s'}^{\{s,s',r,r'\}} < I < U_{s'}^{\{sr,s',r'\}}$, i.e. if $W_{\Omega\backslash r}/6 - W_{\Omega\backslash s}/6 + W_\Omega/4 < I < W_{\Omega\backslash r}/6 - W_{\Omega\backslash s}/3 + W_\Omega/3$. This interval is non-empty if $2W_{\Omega\backslash s} < W_\Omega$, and therefore, as discussed above, if products are complements. Plugging in Expression (3.8) yields the interval of entry costs in which a merger can foster entry as

$$-2\kappa/3 - c_1 + 4\gamma/3 + p < I < -2\kappa/3 - c_1 + 5\gamma/3 + p.$$

A vertical merger that enables supplier s' to enter is profitable if

$$U_{sr}^{\{sr,s',r'\}} - \left[U_s^{\{s,r,r'\}} + U_r^{\{s,r,r'\}}\right] > 0. \tag{3.12}$$

This can be simplified to get $W_{\Omega\backslash r} + W_{\Omega\backslash s} > W_{\Omega\backslash sr}$, which always holds due to superadditivity.

Table (6)contains examples for parameter values for each region in Figure (4). It is assumed that entry may occur upstream while the incumbent supplier and a retailer consider merging vertically. For the example I take $p = 1$ and $c_1 = 1/8$, implying that $\Delta = p - c_1 = 7/8$.

Figure (4)region	Potential effect of the merger on upstream entry	γ	κ	Benefit/loss from merging Expression (3.11)or (3.12)	$I \in [.,.]$
1	deter	$\frac{-1}{10}$	$\frac{1}{4}$	$\frac{1}{80}$	$\left[\frac{13}{24}, \frac{23}{40}\right]$
2	deter	$-\frac{1}{10}$	$\frac{1}{10}$	$-\frac{11}{80}$	$\left[\frac{77}{120}, \frac{27}{40}\right]$
3	foster	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{137}{240}$	$\left[\frac{113}{120}, \frac{39}{40}\right]$
4	deter	$-\frac{44}{160}$	$-\frac{1}{10}$	$\frac{1}{80}$	$\left[\frac{29}{60}, \frac{23}{40}\right]$
5	deter	$-\frac{1}{10}$	$-\frac{1}{10}$	$-\frac{27}{80}$	$\left[\frac{31}{40}, \frac{97}{120}\right]$
6	foster	$\frac{1}{10}$	$-\frac{1}{10}$	$\frac{51}{80}$	$\left[\frac{43}{40}, \frac{133}{120}\right]$

$p = 1$, $c_1 = 1/8$, $\Delta = p - c_1 = 7/8$.

Table (6): Example for upstream entry and vertical merger incentives.

The same exercise can be performed for the case where entry can occur downstream. Consider retailer r' as the potential entrant while suppliers s and s' as well as retailer r are incumbent. A vertical merger between the incumbent retailer r and supplier s is entry-deterring if $U_{r'}^{\{sr,s',r'\}} < I < U_{r'}^{\{s,s',r,r'\}}$, whereas it fosters entry if $U_{r'}^{\{s,s',r,r'\}} < I < U_{r'}^{\{sr,s',r'\}}$. The respective conditions for a vertical merger that deters or fosters entry to be profitable are

$$U_{sr}^{\{sr,s'\}} - \left[U_s^{\{s,s',r,r\}} + U_r^{\{s,s',r,r'\}}\right] > 0 \tag{3.13}$$

and

$$U_{sr}^{\{sr,s',r'\}} - \left[U_s^{\{s,s'r\}} + U_r^{\{s,s'r\}}\right] > 0. \tag{3.14}$$

Table (7)provides a similar example for each region in Figure (5), when entry may occur downstream while the incumbent supplier and a retailer consider merging vertically.

Figure (5)region	Potential effect of the merger on downstream entry	γ	κ	Benefit/loss from merging Expression (3.13)or (3.14)	$I \in [.,.]$
1	deter	$-\frac{1}{10}$	$\frac{10}{32}$	$\frac{23}{80}$	$\left[\frac{23}{80}, \frac{47}{120}\right]$
2	deter	$\frac{1}{10}$	$\frac{10}{32}$	$\frac{70}{80}$	$\left[\frac{101}{240}, \frac{21}{40}\right]$
3	deter	$-\frac{1}{10}$	$\frac{1}{10}$	$-\frac{11}{80}$	$\left[\frac{77}{120}, \frac{27}{40}\right]$
4	deter	$\frac{1}{10}$	$\frac{1}{10}$	$-\frac{27}{80}$	$\left[\frac{31}{40}, \frac{97}{120}\right]$
5	foster	$-\frac{1}{10}$	$-\frac{1}{10}$	$\frac{137}{240}$	$\left[\frac{113}{120}, \frac{39}{40}\right]$
6	foster	$\frac{1}{10}$	$-\frac{1}{10}$	$\frac{51}{80}$	$\left[\frac{43}{40}, \frac{133}{120}\right]$

$p = 1$, $c_1 = 1/8$, $\Delta = p - c_1 = 7/8$.

Table (7): Example for downstream entry and vertical merger incentives.

The benefits from merging in Tables (6)and (7)can be compared with the corresponding prediction for a vertical merger to take place in Tables (4)and (5), respectively. With possible entry occurring upstream, a vertical merger takes place for the parameter combinations in regions 1,3,4 and 5 of Figure (4). If entry can occur downstream, a vertical merger is profitable for the parameter combinations 1,2,5 and 6 of Figure (5). The example therefore complies with the general theoretical results derives in the previous Sections.

3.8 Conclusions

I propose a model of a bilaterally duopolistic industry where upstream producers bargain with downstream retailers on supply conditions. In the applied framework integration does not affect the total output produced, but it affects the distribution of rents among players. I make three contributions in this article. First, I identify conditions for vertical mergers to occur and show that in a framework in which delivery conditions are determined by bargaining, vertical integration incentives can be regarded as a mix of horizontal merger incentives downstream and upstream. Second, I directly compare the strength of horizontal and vertical merger incentives if either an upstream or a downstream firm is available for sale by means of an auction to the highest bidder. I demonstrate that - as opposed to conventional wisdom - a merger to monopoly may convey less bargaining power to the merged entity than vertical integration. Third, I compare the potential of horizontal and vertical mergers to deter entry. My results show that while horizontal mergers are never an apt device to deter entry, vertical integration can profitably induce a potential entrant to stay out of the market.

The results presented here on the effects of vertical mergers stand in sharp contrast to several prevailing views in competition policy, which strongly favors vertical mergers over horizontal ones. Taking explicitly into account that deliveries are determined by bargaining between parties, the contrast between horizontal and vertical mergers become less clear. In fact, my insights suggest that in such an environment vertical mergers are likely to be more harmful for welfare than horizontal ones because they are more likely to deter entry. This creates scope for welfare enhancing intervention into such transactions by competition policy.

While many of my results are general, this article has some limitations. In particular, some results are derived under the assumption of symmetry. Imposing this assumption helps identifying the main forces at work, but omits other effects stemming from the asymmetry between firms. Discovering these additional effects could be an interesting avenue for further research, and the first step in this direction is provided in the general formulae derived here.

A further restrictive assumption is that of no competitive externalities downstream. This assumption is necessary to ensure superadditivity, which is required for the application of the Shapley value as allocation rule. Taking into account competitive externalities downstream while maintaining the assumption that the merged firms melt into one bargaining unit, could undoubtedly provide valuable insights and extend the applicability of the model to several realistic market scenarios. This could be done for example by applying modifications of the Shapley value to determine the outcome of bargaining, which take into account externalities in the total value generated by various coalitions, as recently suggested for example by De

Clippel and Serrano (2008)and Macho-Stadler et al. (2007).

Finally, while this article confines itself to the analysis of vertical merger incentives and its comparison to horizontal ones, many possible extensions arise naturally. Moving beyond the simple bilateral duopoly setup as well as taking into account investment incentives could be fruitful topics for further research.

3.9 Appendix

Market structure	Payoffs
$\{A,B,a,b\}$	$U_A = \frac{1}{12}\begin{bmatrix} W_{\Omega\backslash Bb} + W_{\Omega\backslash Ba} + W_{\Omega\backslash b} - W_{\Omega\backslash Ab} + \\ + W_{\Omega\backslash a} - W_{\Omega\backslash Aa} + W_{\Omega\backslash B} - 3W_{\Omega\backslash A} + 3W_{\Omega} \end{bmatrix}$ $U_B = \frac{1}{12}\begin{bmatrix} -W_{\Omega\backslash Bb} - W_{\Omega\backslash Ba} + W_{\Omega\backslash b} + W_{\Omega\backslash Ab} + \\ + W_{\Omega\backslash a} + W_{\Omega\backslash Aa} - 3W_{\Omega\backslash B} + W_{\Omega\backslash A} + 3W_{\Omega} \end{bmatrix}$ $U_a = \frac{1}{12}\begin{bmatrix} W_{\Omega\backslash Bb} - W_{\Omega\backslash Ba} + W_{\Omega\backslash b} + W_{\Omega\backslash Ab} + \\ -3W_{\Omega\backslash a} - W_{\Omega\backslash Aa} + W_{\Omega\backslash B} + W_{\Omega\backslash A} + 3W_{\Omega} \end{bmatrix}$ $U_b = \frac{1}{12}\begin{bmatrix} -W_{\Omega\backslash Bb} + W_{\Omega\backslash Ba} - 3W_{\Omega\backslash b} - W_{\Omega\backslash Ab} + \\ + W_{\Omega\backslash a} + W_{\Omega\backslash Aa} + W_{\Omega\backslash B} + W_{\Omega\backslash A} + 3W_{\Omega} \end{bmatrix}$
$\{AB,a,b\}$	$U_{AB} = \frac{1}{6}\left[W_{\Omega\backslash b} + W_{\Omega\backslash a} + 2W_{\Omega}\right]$ $U_a = \frac{1}{6}\left[W_{\Omega\backslash b} - 2W_{\Omega\backslash a} + 2W_{\Omega}\right]$ $U_b = \frac{1}{6}\left[-2W_{\Omega\backslash b} + W_{\Omega\backslash a} + 2W_{\Omega}\right]$
$\{ABa,b\}$	$U_{ABa} = \frac{1}{2}\left[W_{\Omega\backslash b} + W_{\Omega}\right]$ $U_b = \frac{1}{2}\left[-W_{\Omega\backslash b} + W_{\Omega}\right]$
$\{A,B,ab\}$	$U_A = \frac{1}{6}\left[W_{\Omega\backslash B} - 2W_{\Omega\backslash A} + 2W_{\Omega}\right]$ $U_B = \frac{1}{6}\left[-2W_{\Omega\backslash B} + W_{\Omega\backslash A} + 2W_{\Omega}\right]$ $U_{ab} = \frac{1}{6}\left[W_{\Omega\backslash B} + W_{\Omega\backslash A} + 2W_{\Omega}\right]$
$\{Aab,B\}$	$U_{Aab} = \frac{1}{2}\left[W_{\Omega\backslash B} + W_{\Omega}\right]$ $U_B = \frac{1}{2}\left[-W_{\Omega\backslash B} + W_{\Omega}\right]$
$\{ABab\}$	$U_{ABab} = W_{\Omega}$
$\{Aa,B,b\}$	$U_{Aa} = \frac{1}{6}\left[2W_{\Omega\backslash Bb} + W_{\Omega\backslash b} + W_{\Omega\backslash B} - 2W_{\Omega\backslash Aa} + 2W_{\Omega}\right]$ $U_B = \frac{1}{6}\left[-W_{\Omega\backslash Bb} + W_{\Omega\backslash b} - 2W_{\Omega\backslash B} + W_{\Omega\backslash Aa} + 2W_{\Omega}\right]$ $U_b = \frac{1}{6}\left[-W_{\Omega\backslash Bb} - 2W_{\Omega\backslash b} + W_{\Omega\backslash B} + W_{\Omega\backslash Aa} + 2W_{\Omega}\right]$
$\{Aa,Bb\}$	$U_{Aa} = \frac{1}{2}\left[W_{\Omega\backslash Bb} - W_{\Omega\backslash Aa} + W_{\Omega}\right]$ $U_{Bb} = \frac{1}{2}\left[-W_{\Omega\backslash Bb} + W_{\Omega\backslash Aa} + W_{\Omega}\right]$

Table 1: Payoffs in various market structures

Proof of Proposition 1: The proof is immediate by comparing the change in payoffs of the merging parties as summarized in Table (3).

Change in market structure	Change in payoffs of vertically merging parties (ΔU)
$\{A,B,a,b\} \to \{Aa,B,b\}$	$[U_A+U_a]_{\{A,B,a,b\}} = \frac{1}{6}\begin{bmatrix} 3W_\Omega - W_{\Omega\backslash Aa} + W_{\Omega\backslash Bb} - W_{\Omega\backslash A} + \\ +W_{\Omega\backslash B} - W_{\Omega\backslash a} + W_{\Omega\backslash b} \end{bmatrix}$ $[U_{Aa}]_{\{Aa,B,b\}} = \frac{1}{6}\left[2W_{\Omega\backslash Bb} + W_{\Omega\backslash b} + W_{\Omega\backslash B} - 2W_{\Omega\backslash Aa} + 2W_\Omega\right]$ $\Delta U_{Aa} = \frac{1}{6}\left[\left(W_{\Omega\backslash Bb} - W_{\Omega\backslash Aa}\right) + W_{\Omega\backslash A} + W_{\Omega\backslash a} - W_\Omega\right]$
$\{AB,a,b\} \to \{ABa,b\}$	$[U_{AB}+U_a]_{\{AB,a,b\}} = \frac{1}{6}\left[4W_\Omega - W_{\Omega\backslash a} + 2W_{\Omega\backslash b}\right]$ $[U_{ABa}]_{\{ABa,b\}} = \frac{1}{2}\left[W_{\Omega\backslash b} + W_\Omega\right]$ $\Delta U_{ABa} = \frac{1}{6}\left[W_{\Omega\backslash a} + W_{\Omega\backslash b} - W_\Omega\right]$
$\{A,B,ab\} \to \{Aab,B\}$	$[U_A+U_{ab}]_{\{A,B,ab\}} = \frac{1}{6}\left[4W_\Omega - W_{\Omega\backslash A} + 2W_{\Omega\backslash B}\right]$ $[U_{Aab}]_{\{Aab,B\}} = \frac{1}{2}\left[W_{\Omega\backslash B} + W_\Omega\right]$ $\Delta U_{Aab} = \frac{1}{6}\left[W_{\Omega\backslash A} + W_{\Omega\backslash B} - W_\Omega\right]$

Table 3: Change in payoffs by vertical integration

Q.E.D.

Proof of Corollary 1. We proceed by proving each claim separately, starting with Claim (i).

Claim (i) With suppliers integrated and retailers separated ($\Psi = \{AB, a, b\}$), the condition for a vertical merger between supplier AB and retailer r to take place is given by Claim (ii) in Proposition 1. This is identical to the condition for a horizontal merger between retailers to take place in IW (2003). The proof of Claim (i)is immediate from Proposition 2 of the same article.

Claim (ii) With suppliers separated and retailers integrated ($\Psi = \{A, B, ab\}$), the condition for a vertical merger between supplier s and retailer ab to take place is given by Claim (iii) of Proposition 1. This is identical to the condition for a horizontal merger between suppliers to take place in IW (2003). The proof of Claim (ii)is immediate from Proposition 2 of the same article.

Claim (iii) Under Assumption 2 (symmetry)the condition for a vertical merger to take place in Claim (i)of Proposition 1 reduces to

$$W_{\Omega\backslash s} + W_{\Omega\backslash r} > W_{\Omega}. \tag{3.15}$$

I focus w.l.o.g. on a merger between supplier A with retailer a. The proof for any other supplier-retailer combination would proceed analogously. I first show that if the products are substitutes and unit costs are strictly increasing a vertical merger takes place. Let $q_{sr}^{\Omega'}$ denote the quantities of supplier s at retailer r if the subset $\Omega' \subseteq \Omega$ of firms participate. Condition (3.15)for supplier A and retailer a to merge can be written as

$$\left[\sum_{r\in R^0} p_{Br}(q_{Br}^{\Omega\backslash A}, 0)q_{Br}^{\Omega\backslash A} - C_B(q_{Br}^{\Omega\backslash A} + q_{Br'}^{\Omega\backslash A})\right] + \left[\sum_{s\in S^0} p_{sb}(q_{sb}^{\Omega\backslash a}, q_{s'b}^{\Omega\backslash a})q_{sb}^{\Omega\backslash a} - \sum_{s\in S^0} C_s(q_{sb}^{\Omega\backslash a})\right] >$$
$$\left[\sum_{s\in S^0}\sum_{r\in R^0} p_{sr}(q_{sr}^{\Omega}, q_{s'r}^{\Omega})q_{sr}^{\Omega} - \sum_{s\in S^0} C_s(q_{sr}^{\Omega} + q_{sr'}^{\Omega})\right]. \tag{3.16}$$

Note that the sum of payoffs on the LHS in Expression (3.15)does not increase if the optimal quantities $q_{rs}^{\Omega\backslash A}$ and $q_{rs}^{\Omega\backslash a}$ are replaced by q_{rs}^{Ω}. It follows, that (3.15)holds if

$$\left[\sum_{r\in R^0} p_{Br}(q_{Br}^{\Omega}, 0)q_{Br}^{\Omega} - C_B(q_{Br}^{\Omega} + q_{Br'}^{\Omega})\right] + \left[\sum_{s\in S^0} p_{sb}(q_{sb}^{\Omega}, q_{s'b}^{\Omega})q_{sb}^{\Omega} - \sum_{s\in S^0} C_s(q_{sb}^{\Omega})\right] >$$
$$\left[\sum_{s\in S^0}\sum_{r\in R^0} p_{sr}(q_{sr}^{\Omega}, q_{s'r}^{\Omega})q_{sr}^{\Omega} - \sum_{s\in S^0} C_s(q_{sr}^{\Omega} + q_{sr'}^{\Omega})\right].$$

Under *Assumption 2* (symmetry), this inequality can be written as

$$4p(q^{\Omega}, q^{\Omega})q^{\Omega} - 2C(2q^{\Omega}) > 2p(q^{\Omega}, 0)q^{\Omega} - C(2q^{\Omega}) + 2\,p(q^{\Omega}, q^{\Omega})q^{\Omega} - 2C(q^{\Omega}).$$

Dividing by $2q^{\Omega}$ and rearranging yields

$$p(q^{\Omega}, q^{\Omega}) - p(q^{\Omega}, 0) < \overline{C}(2q^{\Omega}) - \overline{C}(q^{\Omega}),$$

or identically,

$$\Delta_p^{\Omega} < \Delta_C^{\Omega}. \tag{3.17}$$

The RHS is positive by Definition 2 if unit costs are strictly increasing, while the LHS is by Definition 1 negative if the goods are substitutes. Consequently, if the products are substitutes and unit costs are strictly increasing, Condition (3.15)holds.

I next show that if products are complements and unit costs are strictly decreasing, no vertical merger takes place. A vertical merger does not occur if inequality (3.16)is reversed, such that

$$\left[\sum_{r\in R^0} p_{Br}(q_{Br}^{\Omega\backslash A},0)q_{Br}^{\Omega\backslash A} - C_B(q_{Br}^{\Omega\backslash A}+q_{Br'}^{\Omega\backslash A})\right] + \left[\sum_{s\in S^0} p_{sb}(q_{sb}^{\Omega\backslash a},q_{s'b}^{\Omega\backslash a})q_{sb}^{\Omega\backslash a} - \sum_{s\in S^0} C_s(q_{sb}^{\Omega\backslash a})\right] <$$
$$\left[\sum_{s\in S^0}\sum_{r\in R^0} p_{sr}(q_{sr}^{\Omega},q_{s'r}^{\Omega})q_{sr}^{\Omega} - \sum_{s\in S^0} C_s(q_{sr}^{\Omega}+q_{sr'}^{\Omega})\right]. \tag{3.18}$$

Under *Assumption 2* (symmetry)this can be written as

$$\left[2p(q^{\Omega\backslash A},0)q^{\Omega\backslash A} - C(2q^{\Omega\backslash A})\right] + \left[2p(q^{\Omega\backslash a},q^{\Omega\backslash a})q^{\Omega\backslash a} - 2C(q^{\Omega\backslash a})\right] <$$
$$\left[2p(q^{\Omega},q^{\Omega})q^{\Omega} - C(2q^{\Omega})\right] + \left[2p(q^{\Omega},q^{\Omega})q^{\Omega} - C(2q^{\Omega})\right].$$

Each bracket on the RHS corresponds to half of the industry surplus if all firms participate, which supplier B maximizes. Therefore, the relationship does not change if i replace q^{Ω} by $q^{\Omega\backslash A}$ and $q^{\Omega\backslash a}$ in each bracket on the RHS. Doing so yields

$$2p(q^{\Omega\backslash A},0)q^{\Omega\backslash A} - 2C(q^{\Omega\backslash a}) < 2p(q^{\Omega\backslash A},q^{\Omega\backslash A})q^{\Omega\backslash A} - C(2q^{\Omega\backslash a}).$$

By rearranging and dividing both sides by $2q^{\Omega\backslash a}$ I get

$$\left[p(q^{\Omega\backslash A},0) - p(q^{\Omega\backslash A},q^{\Omega\backslash A})\right]\frac{q^{\Omega\backslash A}}{q^{\Omega\backslash a}} < \overline{C}(q^{\Omega\backslash a}) - \overline{C}(2q^{\Omega\backslash a}), \tag{3.19}$$

which by Definition 3 is equivalent to

$$\Delta_C^{\Omega\backslash a} < \Delta_p^{\Omega\backslash A}\frac{q^{\Omega\backslash A}}{q^{\Omega\backslash a}}. \tag{3.20}$$

The LHS of (3.20)is negative if unit costs are strictly decreasing, while the RHS is positive when products are complements. We can conclude that if products are complements and unit costs are strictly decreasing no vertical merger between a supplier and a retailer takes place. *Q.E.D.*

Proof of Corollary 2. We first consider the case where supplier A is available for sale. We then turn to the case where retailer a is the target firm.

Assume that supplier A is the target firm. Retailer a can make a higher bid than supplier B if $\beta_a > \beta_B$. Analogously, supplier B can outbid retailer a if the opposite holds. Under Assumption 2 (symmetry), from Expression (3.4)we have, that $\beta_a > \beta_B$ ($\beta_a < \beta_B$) if $W_{\Omega\backslash a} > W_{\Omega\backslash B}$ ($W_{\Omega\backslash a} > W_{\Omega\backslash B}$). Consider first the condition $W_{\Omega\backslash a} > W_{\Omega\backslash B}$. This can be written as

$$\sum_{s\in S^0} p_{sb}(q_{sb}^{\Omega\backslash a}, q_{s'b}^{\Omega\backslash a})q_{sb}^{\Omega\backslash a} - \sum_{s\in S^0} C_s(q_{sb}^{\Omega\backslash a}) > \sum_{r\in R^0} p_{Ar}(q_{Ar}^{\Omega\backslash B}, 0)q_{Ar}^{\Omega\backslash B} - C_A(q_{Ar}^{\Omega\backslash B} + q_{Ar'}^{\Omega\backslash B}).$$

Under Assumption 2 (symmetry), the RHS remains unchanged if we replace the quantity $q_{Aa}^{\Omega\backslash B}$ by $q_{Bb}^{\Omega\backslash A}$. Furthermore, the LHS remains unchanged if we replace the quantities $q_{Ab}^{\Omega\backslash a}$ and $q_{Bb}^{\Omega\backslash a}$ by $q_{Ab}^{\Omega\backslash B}$ and $q_{Bb}^{\Omega\backslash A}$ respectively. Doing so and dividing both sides by $2q^{\Omega\backslash s}$ yields

$$p(q^{\Omega\backslash s}, q^{\Omega\backslash s}) - \overline{C}(q^{\Omega\backslash s}) > p(q^{\Omega\backslash s}, 0) - \overline{C}(2q^{\Omega\backslash s}),$$

which can be rearranged to get

$$-\Delta_p^{\Omega\backslash s} < \Delta_C^{\Omega\backslash s}. \tag{3.21}$$

Therefore, if the target firm is supplier A, the acquirer is retailer a if $-\Delta_p^{\Omega'} < \Delta_C^{\Omega'}$ holds for every $\Omega' \subseteq \Omega$. The argument for the condition $W_{\Omega\backslash a} < W_{\Omega\backslash B}$ is analogous.

Consider next the case where retailer a is the target firm. Retailer b can make a higher bid than supplier A if $\beta_b > \beta_A$. Analogously, supplier A can outbid retailer b if the opposite holds. Under Assumption 2 (symmetry), we have from Expression (3.5), that $\beta_b > \beta_A$ ($\beta_b < \beta_A$)if $W_{\Omega\backslash b} > W_{\Omega\backslash A}$ ($W_{\Omega\backslash b} < W_{\Omega\backslash A}$). Consider first the condition $W_{\Omega\backslash b} > W_{\Omega\backslash A}$. This can be written as

$$\sum_{s\in S^0} p_{sa}(q_{sa}^{\Omega\backslash b}, q_{s'a}^{\Omega\backslash b})q_{sa}^{\Omega\backslash b} - \sum_{s\in S^0} C_s(q_{sa}^{\Omega\backslash b}) > \sum_{r\in R^0} p_{Br}(q_{Br}^{\Omega\backslash A}, 0)q_{Br}^{\Omega\backslash A} - C_B(q_{Br}^{\Omega\backslash A} + q_{Br'}^{\Omega\backslash A}). \tag{3.22}$$

Since firms are assumed to be symmetric, Condition (3.22)is identical to Condition (3.21). Therefore, if the target firm is retailer a, the acquirer is retailer b if $-\Delta_p^{\Omega'} < \Delta_C^{\Omega'}$ holds for every $\Omega' \subseteq \Omega$. The argument for the condition $W_{\Omega \backslash b} < W_{\Omega \backslash A}$ is analogous. *Q.E.D.*

Proof of Proposition 3. Table (8)contains payoffs for the market structures $\{A, ab\}$ and $\{AB, a\}$ determined by applying the Shapley value, which will be useful in this proof.

Market structure	Payoffs
$\{A, ab\}$	$U_A = \frac{1}{2} W_{\Omega \backslash B}$ $U_{ab} = \frac{1}{2} W_{\Omega \backslash B}$
$\{AB, a\}$	$U_{AB} = \frac{1}{2} W_{\Omega \backslash b}$ $U_a = \frac{1}{2} W_{\Omega \backslash b}$

Table (8): Payoffs with only one retailer and horizontal merger.

I prove each claim separately, starting with Claim (i). Invoke Assumption 2 and assume w.l.o.g. that $i_1 = a$, $i_2 = b$, $i_3 = A$, $e = B$, i.e. supplier B is the potential entrant, while retailers a and b consider merging horizontally. Note that $\underline{I}_u = U_B^{\{A,B,a,b\}}$and $\overline{I}_u = U_B^{\{A,B,ab\}}$. If $I < \min\{U_B^{\{A,B,ab\}}, U_B^{\{A,B,a,b\}}\}$, the entrant supplier B can cover its entry costs regardless whether firms a and b merge or not. Conversely, if $I > \max\{U_B^{\{A,B,ab\}}, U_B^{\{A,B,a,b\}}\}$, firm B cannot make enough profits to cover its entry costs. Claim (i)follows immediately.

Consider next Claim (ii). Two cases are possible: either $\underline{I}_u \leq I \leq \overline{I}_u$ or $\overline{I}_u \leq I \leq \underline{I}_u$. I first investigate when each of these conditions hold. Assume that $\underline{I}_u \leq I \leq \overline{I}_u$. With $\underline{I}_u = U_B^{\{A,B,a,b\}}$ and $\overline{I}_u = U_B^{\{A,B,ab\}}$, for the interval $[\underline{I}_u, \overline{I}_u]$ to be non-empty we must have $U_B^{\{A,B,a,b\}} < U_B^{\{A,B,ab\}}$. Under symmetry, by plugging in the corresponding values from Table (1)this is equivalent to $\frac{1}{12}\left[2W_{\Omega \backslash r} - 2W_{\Omega \backslash s} + 3W_{\Omega}\right] < \frac{1}{6}\left[2W_{\Omega} - W_{\Omega \backslash s}\right]$, which can be rearranged to get $2W_{\Omega \backslash r} < W_{\Omega}$. From Proposition 2 of IW, this relationship holds if unit costs are strictly decreasing.

$U_B^{\{A,B,a,b\}} \leq I \leq U_B^{\{A,B,ab\}}$ implies that a merger between retailers a and b makes an otherwise unprofitable entry of supplier B profitable. For the horizontal merger to occur, it must also be profitable for the merging parties, i.e. we must have $U_a^{\{A,a,b\}} + U_b^{\{A,a,b\}} < U_{ab}^{\{A,B,ab\}}$. Plugging in the corresponding values from Tables 2 and 6 yields the profitability condition $W_\Omega > W_{\Omega\backslash s} - W_{\Omega\backslash sr}$. This relationship is fulfilled under Assumption 1. Therefore, with $\underline{I}_u \leq I \leq \overline{I}_u$ and unit costs strictly decreasing, retailers merge and accommodate upstream entry.

Assume now that $\overline{I}_u \leq I \leq \underline{I}_u$. With the same logic as above, the interval $[\overline{I}_u, \underline{I}_u]$ is non-empty if unit costs are strictly increasing. $U_B^{\{A,B,ab\}} \leq I \leq U_B^{\{A,B,a,b\}}$ implies that a merger between retailers a and b makes an otherwise profitable entry of supplier B unprofitable. Such a merger is therefore entry-deterring. For it to take place, it must also be profitable for the merging parties, i.e. we must have $U_{ab}^{\{A,ab\}} > U_a^{\{A,B,a,b\}} + U_b^{\{A,B,a,b\}}$. Plugging in the corresponding values from Tables 2 and 6 yields the profitability condition $W_{\Omega\backslash s} + 2W_{\Omega\backslash r} > 3W_\Omega$. This relationship violates Assumption 1. Therefore, with $\overline{I}_u \leq I \leq \underline{I}_u$ and unit costs strictly increasing, retailers stay separated and accommodate upstream entry. This completes the proof of Claim (ii).

I now turn to the case of downstream entry and Claim (iii). Assume w.l.o.g. that $i_1 = A$, $i_2 = B$, $i_3 = a$, $e = b$, i.e. retailer b is the potential entrant, while suppliers A and B consider merging horizontally. Note that $\underline{I}_d = U_b^{\{A,B,a,b\}}$and $\overline{I}_d = U_b^{\{AB,a,b\}}$. If $I < \min\{U_b^{\{AB,a,b\}}, U_b^{\{A,B,a,b\}}\}$, the entrant retailer b can cover its entry costs regardless whether firms A and B merge or not. Conversely, if $I > \max\{U_b^{\{AB,a,b\}}, U_b^{\{A,B,a,b\}}\}$, firm b cannot make enough profits to cover its entry costs. Claim (iii)follows immediately.

Consider next Claim (iv). Again two cases are possible: either $\underline{I}_d \leq I \leq \overline{I}_d$ or $\overline{I}_d \leq I \leq \underline{I}_d$. I first investigate when each of these conditions hold. Assume that $\underline{I}_d \leq I \leq \overline{I}_d$. With $\underline{I}_d = U_b^{\{A,B,a,b\}}$ and $\overline{I}_d = U_b^{\{AB,a,b\}}$, for the interval $[\underline{I}_d, \overline{I}_d]$ to be non-empty we must have $U_b^{\{A,B,a,b\}} < U_b^{\{AB,a,b\}}$. Under symmetry, by plugging in the corresponding values from Table (1)this is equivalent to $\frac{1}{12}\left[2W_{\Omega\backslash s} - 2W_{\Omega\backslash r} + 3W_\Omega\right] < \frac{1}{6}\left[2W_\Omega - W_{\Omega\backslash r}\right]$, which can be rearranged to get $2W_{\Omega\backslash s} < W_\Omega$. From Proposition 2 of IW, this relationship holds if the products are complements.

$U_b^{\{A,B,a,b\}} \leq I \leq U_b^{\{AB,a,b\}}$ implies that a merger between suppliers A and B makes an otherwise unprofitable entry of retailer b profitable. For the horizontal merger to occur, it must also be profitable for the merging parties, i.e. we must have $U_{AB}^{\{AB,a,b\}} > U_A^{\{A,B,a,b\}} +$

$U_B^{\{A,B,a,b\}}$. Plugging in the corresponding values from Tables 2 and 6 yields the profitability condition $W_\Omega > W_{\Omega\backslash r} - W_{\Omega\backslash sr}$. This relationship is fulfilled under Assumption 1. Therefore, with $\underline{I}_d \leq I \leq \overline{I}_d$ and products being complements, suppliers merge and accommodate downstream entry.

Assume now that $\overline{I}_d \leq I \leq \underline{I}_d$. With the same logic as above, the interval $[\overline{I}_d, \underline{I}_d]$ is non-empty if products are substitutes. $U_b^{\{AB,a,b\}} \leq I \leq U_b^{\{A,B,a,b\}}$ implies that a merger between suppliers A and B turns an otherwise profitable entry of retailer b unprofitable. Such a merger is therefore entry-deterring. For it to take place, it must also be profitable for the merging parties, i.e. we must have $U_{AB}^{\{AB,a\}} > U_A^{\{A,B,a,b\}} + U_B^{\{A,B,a,b\}}$. Plugging in the corresponding values from Tables 2 and 6 yields the profitability condition $2W_{\Omega\backslash r} + 4W_{\Omega\backslash s} > 6W_\Omega$. This relationship violates Assumption 1. Therefore, with $\overline{I}_d \leq I \leq \underline{I}_d$ and products being substitutes, retailers stay separated and accommodate downstream entry. *Q.E.D.*

Proof of Proposition 4. I prove each claim separately starting with upstream entry and Claim (i). Invoke Assumption 2 and assume w.l.o.g. that $i_1 = A$, $i_2 = a$, $i_3 = b$, $e = B$, i.e. supplier B is the potential entrant, while supplier A and retailer a consider merging vertically. If supplier B enters, the corresponding Stage 3 payoffs are contained in the rows $\{A, B, ab\}$ and $\{A, B, a, b\}$ of Table (1). If it does not enter, applying the Shapley value yields the following payoffs, depending on firm A's and a's merger decision:

Market structure	Payoffs
$\{A, a, b\}$	$U_A = \frac{1}{6}[2W_{\Omega\backslash B} + W_{\Omega\backslash Bb} + W_{\Omega\backslash Ba}]$
	$U_a = \frac{1}{6}[2W_{\Omega\backslash B} + W_{\Omega\backslash Bb} - 2W_{\Omega\backslash Ba}]$
	$U_b = \frac{1}{6}[2W_{\Omega\backslash B} - 2W_{\Omega\backslash Bb} + W_{\Omega\backslash Ba}]$
$\{Aa, b\}$	$U_{Aa} = \frac{1}{2}\left[W_{\Omega\backslash B} + W_{\Omega\backslash Bb}\right]$
	$U_b = \frac{1}{2}\left[W_{\Omega\backslash B} - W_{\Omega\backslash Bb}\right]$

Table (9): Payoffs with only one supplier and vertical merger.

If $I < \min\{U_B^{\{Aa,B,b\}}, U_B^{\{A,B,a,b\}}\}$, the entrant supplier B can cover its entry costs regardless whether firms A and a merge or not. Conversely, if $I > \max\{U_B^{\{Aa,B,b\}}, U_B^{\{A,B,a,b\}}\}$, firm

B cannot make enough profits to cover its entry costs. Note that $\underline{I}_s = U_B^{\{A,B,a,b\}}$ and $\overline{I}_s = U_B^{\{Aa,B,b\}}$. Claim (i)is therefore straightforward.

Consider next Claim (ii). Two cases are possible: either $\underline{I}_s \leq I \leq \overline{I}_s$ or $\overline{I}_s \leq I \leq \underline{I}_s$. I first investigate when each of these conditions hold. Assume first that $\underline{I}_s \leq I \leq \overline{I}_s$. With $\underline{I}_s = U_B^{\{A,B,a,b\}}$ and $\overline{I}_s = U_B^{\{Aa,B,b\}}$, for the interval $[\underline{I}_s, \overline{I}_s]$ to be non-empty we must have $U_B^{\{A,B,a,b\}} < U_B^{\{Aa,B,b\}}$. Under symmetry, by plugging in the corresponding values from Table (1)this is equivalent to $\frac{1}{12}\left[2W_{\Omega\backslash r} - 2W_{\Omega\backslash s} + 3W_\Omega\right] < \frac{1}{6}\left[W_{\Omega\backslash r} - 2W_{\Omega\backslash s} + 2W_\Omega\right]$, which can be rearranged to get $2W_{\Omega\backslash s} < W_\Omega$. From Proposition 2 of IW, this relationship holds if the products are strict complements.

$U_B^{\{A,B,a,b\}} \leq I \leq U_B^{\{Aa,B,b\}}$ implies that a merger between supplier A and retailer a makes an otherwise unprofitable entry of supplier B profitable. For the vertical merger to occur, it must also be profitable for the merging parties, i.e. we must have $U_A^{\{A,a,b\}} + U_a^{\{A,a,b\}} < U_{Aa}^{\{Aa,B,b\}}$. Plugging in the corresponding values from Tables (1)and (9)yields the profitability condition $W_{\Omega\backslash r} + 2W_\Omega > 3W_{\Omega\backslash s} + W_{\Omega\backslash sr}$. This relationship is fulfilled under the assumption $2W_{\Omega\backslash s} < W_\Omega$ (complements), which proves Claim (ii.a).

Assume now that $\overline{I}_s \leq I \leq \underline{I}_s$. With $\underline{I}_s = U_B^{\{A,B,a,b\}}$ and $\overline{I}_s = U_B^{\{Aa,B,b\}}$, for the interval $[\overline{I}_s, \underline{I}_s]$ to be non empty we must have $U_B^{\{A,B,a,b\}} > U_B^{\{Aa,B,b\}}$. Plugging in the corresponding values from Table (1)this relationship holds if $2W_{\Omega\backslash s} > W_\Omega$. From Proposition 2 of IW, this is the case if the products are strict substitutes. $U_B^{\{Aa,B,b\}} \leq I \leq U_B^{\{A,B,a,b\}}$ implies that a merger between supplier A and retailer a renders the otherwise profitable entry of supplier B unprofitable and is therefore entry-deterring. For the vertical merger to occur, it must also be profitable for the merging parties, i.e. we must have $U_{Aa}^{\{Aa,b\}} > U_a^{\{A,B,a,b\}} + U_A^{\{A,B,a,b\}}$. Plugging in the corresponding values from Tables (1)and (9)yields the profitability condition $W_\Omega < W_{\Omega\backslash s} + W_{\Omega\backslash sr}$. This can be written as

$$4p(q^\Omega, q^\Omega)q^\Omega - 2C(2q^\Omega) < [2p(q^{\Omega\backslash s}, 0)q^{\Omega\backslash s} - C(2q^{\Omega\backslash s})] + [p(q^{\Omega\backslash sr}, 0)q^{\Omega\backslash sr} - C(q^{\Omega\backslash sr})].$$

Note that the above relationship remains valid if on the RHS we plug in q^Ω for $q^{\Omega\backslash s}$ and $q^{\Omega\backslash sr}$. Doing so and simplifying yields $\Delta_C^\Omega \geq 2\Delta_p^\Omega + (1/2)\Delta^\Omega$. A vertical merger profitably deters upstream entry if this condition is fulfilled, where it is unprofitable if the opposite holds. Claim (ii.b)follows immediately.

Consider now the entry of a retailer. Assume w.l.o.g. that $i_1 = A$, $i_2 = a$, $i_3 = B$, $e = b$, i.e. retailer b is the potential entrant, while supplier A and retailer a consider merging vertically. If b enters, the corresponding Stage 3 payoffs are contained in the rows $\{Aa, B, b\}$ and $\{A, B, a, b\}$ of Table (1). If b does not enter, applying the Shapley value yields the following payoffs, depending on firm A's and a's merger decision:

Market structure	Payoffs
$\{A, B, a\}$	$U_A = \frac{1}{6}[2W_{\Omega\setminus b} + W_{\Omega\setminus Bb} - 2W_{\Omega\setminus Ab}]$
	$U_a = \frac{1}{6}[2W_{\Omega\setminus b} + W_{\Omega\setminus Bb} + W_{\Omega\setminus Ab}]$
	$U_B = \frac{1}{6}[2W_{\Omega\setminus b} - 2W_{\Omega\setminus Bb} + W_{\Omega\setminus Ab}]$
$\{Aa, B\}$	$U_{Aa} = \frac{1}{2}\left[W_{\Omega\setminus b} + W_{\Omega\setminus Bb}\right]$
	$U_B = \frac{1}{2}\left[W_{\Omega\setminus b} - W_{\Omega\setminus Bb}\right]$

Table (10): Payoffs with only one retailer and vertical merger.

If $I < \min\{U_b^{\{Aa,B,b\}}, U_b^{\{A,B,a,b\}}\}$, the entrant retailer b can cover its entry costs regardless whether firms A and a merge or not. Conversely, if $I > \max\{U_b^{\{Aa,B,b\}}, U_b^{\{A,B,a,b\}}\}$, firm b cannot make enough profits to cover its entry costs. Note that $\underline{I}_r = U_b^{\{A,B,a,b\}}$ and $\overline{I}_r = U_b^{\{Aa,B,b\}}$. Claim (iii)is therefore straightforward.

Consider next Claim (iv). Again two cases are possible: either $\underline{I}_r \leq I \leq \overline{I}_r$ or $\overline{I}_r \leq I \leq \underline{I}_r$. I first investigate when each of these conditions hold. Assume first that $\underline{I}_r \leq I \leq \overline{I}_r$. With $\underline{I}_r = U_b^{\{A,B,a,b\}}$ and $\overline{I}_r = U_b^{\{Aa,B,b\}}$, for the interval $[\underline{I}_r, \overline{I}_r]$ to be non- empty we must have $U_b^{\{A,B,a,b\}} < U_b^{\{Aa,B,b\}}$. Under symmetry, by plugging in the corresponding values from Table (1)this is equivalent to $\frac{1}{12}\left[2W_{\Omega\setminus s} - 2W_{\Omega\setminus r} + 3W_{\Omega}\right] < \frac{1}{12}\left[2W_{\Omega\setminus s} - 4W_{\Omega\setminus s} + 4W_{\Omega}\right]$, which can be rearranged to get $2W_{\Omega\setminus r} < W_{\Omega}$. From Proposition 2 of IW, this relationship holds if unit costs are strictly decreasing.

$U_b^{\{A,B,a,b\}} \leq I \leq U_b^{\{Aa,B,b\}}$ implies that a merger between supplier A and retailer a makes an otherwise unprofitable entry of retailer b profitable. For the vertical merger to occur, it must also be profitable for the merging parties, i.e. we must have $U_A^{\{A,B,a\}} +$

$U_a^{\{A,B,a\}} < U_{Aa}^{\{Aa,B,b\}}$. Plugging in the corresponding values from Tables (1)and (10)yields the profitability condition $W_{\Omega\backslash s} + 2W_\Omega > 3W_{\Omega\backslash r} + W_{\Omega\backslash sr}$. This relationship is fulfilled under the assumption $2W_{\Omega\backslash r} < W_\Omega$ (unit costs strictly decreasing), which proves Claim (iv.a).

Assume now that $\overline{I}_r \leq I \leq \underline{I}_r$. With $\underline{I}_r = U_b^{\{A,B,a,b\}}$ and $\overline{I}_r = U_b^{\{Aa,B,b\}}$, for the interval $[\overline{I}_r, \underline{I}_r]$ to be non empty we must have $U_b^{\{A,B,a,b\}} > U_b^{\{Aa,B,b\}}$. Plugging in the corresponding values from Table (1)this relationship holds if $2W_{\Omega\backslash r} > W_\Omega$. From Proposition 2 of IW, this is the case if the unit costs are strictly increasing. $U_b^{\{Aa,B,b\}} \leq I \leq U_b^{\{A,B,a,b\}}$ implies that a merger between supplier A and retailer a renders the otherwise profitable entry of retailer b unprofitable and is therefore entry-deterring. For the vertical merger to occur, it must also be profitable for the merging parties, i.e. we must have $U_{Aa}^{\{Aa,B\}} > U_a^{\{A,B,a,b\}} + U_A^{\{A,B,a,b\}}$. Plugging in the corresponding values from Tables (1)and (10)yields the profitability condition $W_\Omega < W_{\Omega\backslash r} + W_{\Omega\backslash sr}$. This can be written as

$$4p(q^\Omega, q^\Omega)q^\Omega - 2C(2q^\Omega) < [2p(q^{\Omega\backslash r}, q^{\Omega\backslash r})q^{\Omega\backslash s} - 2C(q^{\Omega\backslash r})] + [p(q^{\Omega\backslash sr}, 0)q^{\Omega\backslash sr} - C(q^{\Omega\backslash sr})].$$

Note that the above relationship remains valid if on the RHS we plug in q^Ω for $q^{\Omega\backslash r}$ and $q^{\Omega\backslash sr}$. Doing so and simplifying yields $\Delta_C^\Omega \geq \Delta_p^\Omega/2 + (1/4)\Delta^\Omega$. A vertical merger profitably deters downstream entry if this condition is fulfilled, where it is unprofitable if the opposite holds. Claim (iv.b)follows immediately. *Q.E.D.*

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