NONLINEAR MODULATION OF RANDOMLY DISTRIBUTED UPPER-HYBRID MODES IN PLASMAS

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A new type of modulations of an ensemble of random waves near the upper-hybrid resonance frequency is investigated. For the linearly unstable cases, the growth rates are obtained and exact nonlinear stationary distributions are presented.

It is well-known [1] that a system of random phased high-frequency waves can be unstable with respect to low-frequency perturbations. Recently [2], the linear dynamics of an ensemble of upper-hybrid waves has been studied using lower-hybrid as well as ion cyclotron modes as low-frequency perturbations. As expected, the growth rates are larger for smaller modulation frequencies. Obviously, the fastest modulation process will dominate in the system. It is therefore worthwhile to consider perturbations with even lower real frequencies. Therefore, in this letter we consider adiabatic modulations. Besides the corresponding linear growth rates the possible final nonlinear stationary distributions of the upper-hybrid turbulence are presented.

For the reason of simplicity, we use a one-dimensional model for the upper-hybrid modes propagating perpendicular (x-direction) to the external magnetic field $B_0 = B_0\hat{z}$. The dynamics of upper-hybrid turbulence is described by the wave-kinetic equation for the plasmon distribution $N_k = \langle |E_k|^2\rangle/4\pi\omega_H$, i.e.,

$$\frac{\partial N_k}{\partial t} + \alpha k \frac{\partial N_k}{\partial x} - \frac{\omega_{pe}^2}{2n_0\omega_H}\frac{\partial \delta n_e}{\partial x}\frac{\partial N_k}{\partial k} = 0. \tag{1}$$

Here, ω_{pe} is the electron plasma frequency, $\omega_H = (\omega_{pe}^2 + \omega_{Be}^2)^{1/2}$ is the upper-hybrid resonance frequency, ω_{Be} is the electron gyrofrequency, $\alpha = v_e^2 A\omega_{pe}^2/\omega_H^3$, with $A = 3\omega_H^2/(\omega_{pe}^2 - 3\omega_{Be}^2)$, and v_e is the thermal speed of the electrons. The turbulence is modulated through perturbations δn_e in the electron density ($n_e = n_0 + \delta n_e$) which are assumed to vary on a large scale compared to the phase variation of a upper-hybrid mode. We note that $\alpha > 0$ for $\omega_H > 2\omega_{Be}$ corresponding to positive group dispersion; negative group dispersion is obtained for $\omega_H < 2\omega_{Be}$.

The density changes δn_e are caused by the ponderomotive force. The latter acts on the electrons and ions and can be written in the form [3]

$$F_j = -\hat{x}\frac{\partial}{\partial x}\left[\left(\frac{\omega_{pj}/\omega_H}{\omega_H^2 - \omega_{Bj}^2}\right)^2 \frac{\langle |E|^2\rangle}{16\pi n_0}\right] - \hat{z}\frac{\partial}{\partial z}\left[\frac{\omega_{pj}^2}{\omega_H^2 - \omega_{Bj}^2}\frac{\langle |E|^2\rangle}{16\pi n_0}\right], \quad j = \mathrm{e, i}. \tag{2}$$

Here, $\langle |E|^2\rangle$ is the mean square turbulent electric field determined by the strength of the turbulence. It is related to N_k through the conservation of norm, i.e.,

$$\langle |E|^2\rangle = 2\omega_H L \int N_k \mathrm{d}k, \tag{3}$$

where L is the length of the system.

Taking into account the adiabatic response of the plasma, we obtain from (2)

$$\delta n_e \approx n_0\frac{e\varphi}{T_e} - \frac{\omega_{pe}^2}{T_e(\omega_H^2 - \omega_{Be}^2)}\frac{\langle |E|^2\rangle}{16\pi}, \qquad \delta n_i \approx -n_0\frac{e\varphi}{T_i} - \frac{\omega_{pi}^2}{T_i\omega_H^2}\frac{\langle |E|^2\rangle}{16\pi}. \tag{4a,b}$$

Assuming quasineutrality and using the cold dispersion relation for upper-hybrid waves, we can eliminate the ambipolar potential φ to obtain

$$\frac{\delta n_e}{n_o} \approx -\frac{\langle|E|^2\rangle}{16\pi n_o(T_e+T_i)}. \tag{5}$$

Inserting (5) into (1) and using (3) we get

$$\frac{\partial N_k}{\partial t} + \alpha k \frac{\partial N_k}{\partial x} + \beta\left(\frac{\partial}{\partial x}\int N_k \mathrm{d}k\right)\frac{\partial N_k}{\partial k} = 0, \tag{6}$$

with $\beta = L\omega_{pe}^2/16\pi n_o(T_e+T_i)$.

Eq. (6) governs the dynamics of upper-hybrid turbulence in a nonlinear dispersive medium. Nonlinear equations of that type have been analysed by many authors, [e.g. 4.]. We briefly summarize the main results.

A *linear* analysis, using

$$N_k = Nf_k^0 + N_k^1 \exp(\mathrm{i}Kx - \mathrm{i}\Omega t), \tag{7}$$

yields the dispersion relation

$$1 + \frac{\omega_H(\omega_{pe}^2 - 3\omega_{Be}^2)NL}{48\pi n_o(T_e+T_i)v_e^2}\int \frac{\partial f_k^0/\partial k}{k - \Omega\omega_H(\omega_{pe}^2 - 3\omega_{Be}^2)/3v_e^2\omega_{pe}^2 K}\,\mathrm{d}k. \tag{8}$$

For positive dispersive upper-hybrid modes unstable long-wavelength perturbations exist. The corresponding growth rate γ is

$$\gamma = \sqrt{3}\,Kv_e\omega_{pe}^2[\langle|E|^2\rangle/32\pi n_o(T_e+T_i)]^{1/2}/\omega_H(\omega_{pe}^2 - 3\omega_{Be}^2)^{1/2}. \tag{9}$$

Negative dispersive upper-hybrid modes can be unstable with respect to short-wavelength perturbations. The corresponding growth rate is

$$\gamma = \frac{3\sqrt{2}v_e^2\omega_{pe}^2 K\Delta}{\sqrt{\pi}\omega_H(\omega_{pe}^2 - 3\omega_{Be}^2)}\left[1 - \frac{96\pi v_e^2\Delta^2 n_o(T_e+T_i)}{(\omega_{pe}^2 - 3\omega_{Be}^2)\langle|E|^2\rangle}\right], \tag{10}$$

where a Gaussian of width Δ has been assumed for the zeroth order plasmon distribution.

Possible *nonlinear* final states of these instabilities can be represented by trapped BGK solutions. One obtains

$$N_k = \frac{16n_o(T_e+T_i)}{L\omega_{pe}^2}\left[\frac{3v_e^2\omega_{pe}^4\langle|E|^2\rangle}{16\pi n_o(T_e+T_i)\omega_H^2(\omega_{pe}^2 - 3\omega_{Be}^2)} - \frac{9v_e^4\omega_{pe}^4 k^2}{\omega_H^2(\omega_{pe}^2 - 3\omega_{Be}^2)^2}\right]^{1/2}, \tag{11}$$

for positive dispersion and $k^2\lambda_e^2 < (1 - 3\omega_{Be}^2/\omega_{pe}^2)\langle|E|^2\rangle/48\pi n_o(T_e+T_i)$. The space dependence of the electric field energy density is still arbitrary and can be chosen to be of the form of an evelope soliton.

For negative dispersion one obtains

$$N_k = \frac{\sqrt{3}\,v_e\omega_{pe}H^{1/2}}{\sqrt{2}\pi\,\omega_H^{1/2}(3\omega_{Be}^2 - \omega_{pe}^2)^{1/2}L}\left[\frac{\langle|E|^2\rangle_\infty}{2H\omega_H} - \frac{32\pi n_o(T_e+T_i)}{\omega_{pe}^2}\right], \tag{12}$$

with

$$H = \omega_{pe}^2(\langle|E|^2\rangle - \langle|E|^2\rangle_\infty)/32\pi n_o(T_e+T_i)\omega_H - 3v_e^2\omega_{pe}^2 k^2/2\omega_H(3\omega_{Be}^2 - \omega_{pe}^2).$$

Now, $\langle|E|^2\rangle$ can be chosen to have a space dependence like an envelope hole.

In conclusion, we have investigated modulations of upper-hybrid turbulence treating the plasma response as adiabatic. It turns out that the growth rates of the corresponding instabilities can be larger than those obtained previously. Stationary nonlinear solutions in the linear unstable region are trapped BGK solutions.

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