Chapter 3

Quantification as a Major Module of Natural Language Semantics*

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1. QUANTIFIERS

Quantification has been a challenge to the formal semantics of natural language since the very beginnings of this discipline. It has caused Russell to talk of a fundamental discrepancy between surface and underlying logical form of sentences, a dilemma for compositional semantics that began to be overcome not earlier than 1970 when Montague first presented his "Proper Treatment of Quantification in Ordinary English" (Montague 1974). Recently a major new attempt to cover more of the quantificational phenomena in a uniform manner, including logical and non-logical quantifiers, was undertaken by Barwise and Cooper (1981).[1]

Up to now all major approaches have been confined to the semantics and syntax of certain noun phrases that can be considered correlates or relatives of the quantifiers of predicate logics. In particular, the interest centred on singular count noun NPs. This might be explained by the preoccupation of formal semanticists with first order predicate logics and of linguists in general with languages such as English which exhibit a number and mass/count distinction.

Taken as a semantic phenomenon, however, quantification is by no means restricted to the cases investigated so far. It can be found in various syntactic categories, the most obvious cases being adverbs of quantification like *always* or *nowhere*, but also modal verbs, verbs with infinitive, gerund, or clausal complements, certain adjectives and several sorts of adverbs. I shall present several examples below, that may illustrate the grammatical variety of natural language quantification in the case of English. Of course, if one once starts to try to delineate the whole field in question one will soon encounter cases which are traditionally not at all covered by the term *quantification*. Having no other term at hand, I use it to refer to a seemingly very comprehensive range of phenomena which are syntactically and grammatically rather diverse but semantically closely enough related to form a class of their own.

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1.1. Duality

I follow the tradition of Montague and Barwise & Cooper in considering quantifiers semantically as one place second order predicates which take again one place predicates as arguments.[2] Any such operator has the property of possessing a correspondent d u a l operator of the same type. In fact, any quantifier is part of a duality square, as shown in the following diagram:

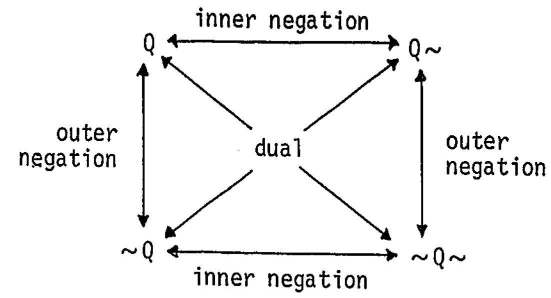

Diagram 1

The dual of a quantifier is defined as the outer negation of the inner negation. Accordingly there are three further operators given with any quantifier: its inner negation Q~, its outer negation ~Q, and its dual ~Q~.[3] Note, that the scheme is absolutely symmetrical and commutative. It is closed in itself and any of the four operators generates the whole scheme.[4] In case of self-dual quantifiers the square collapses into a binary opposition. We shall not deal with this special subclass of operators here. They are, in a way, atypical, since applied to them inner and outer negation have the same effect. In case of self-dual natural language quantifiers it is questionable whether there is any second order level involved at all.

Duality is a fundamental concept in connection with quantification, but has been neglected almost completely in the relevant linguistic literature. It is a fact that natural language quantifiers usually exist alongside others out of the same duality square. Very seldom the whole square is lexicalized but, normally, at least two elements are. Thus, any correct analysis of one element out of a duality square should at the same time hold for the other elements (provided duality can be established independently). This helps considerably judging the validity of one's analytical results.

The general duality scheme is not to be confused with the well-known Aristotelian square of opposition given in diagram 2. I have chosen the universal quantifier for Q and maintained the arrangement of diagram 1. Of course the existential quantifier could be replaced by the universal quantifier exploiting the duality relationship, but it does not matter how the four

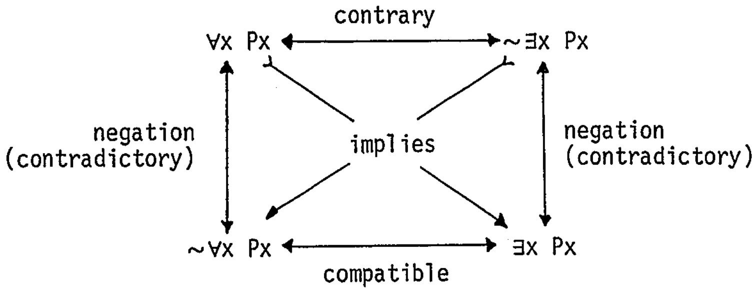

Diagram 2

statements in the square of oppositions are formulated. The relationships in the Aristotelian diagram only make complete sense if empty universes are excluded. Otherwise, again, the square collapses into a binary opposition, destroying the original structure. The essential difference between the Aristotelian square and the duality square is that the concepts of inner and outer negation and duality are third order concepts, in contrast to the second order concepts of compatibility, contrariness, and implication that constitute the square of oppositions. To see this, consider the following definitions, where A and P are any two predicates in the widest sense, including propositions (as predicates over possible worlds, situations, or whatever), and c (for "case") is a variable for whatever the predicates apply to.[5]

(1) DEFINITION:

A is *compatible* with	P	iff	$\exists c(A(c) \,\&\, P(c))$
A *implies*	P	iff	$\forall c(A(c) \rightarrow P(c))$
A is *contrary* to	P	iff	$\sim\exists c(A(c) \,\&\, P(c))$
[A does *not imply*	P	iff	$\sim\forall c(A(c) \rightarrow P(c))$]

The fourth relationship of non-implication is also involved in the constitution of the Aristotelian scheme because the asymmetry of the implication relationship is crucial in order to distinguish the elements that are opposed diagonally and also to distinguish contrariness from contradictoriness. The four concepts defined in (1) themselves form a duality square with respect to the predicate P. For example, being compatible with A and being implied by A are dual second order predicates. (Needless to say, they constitute another Aristotelian square too, implication implying compatibility and so on.) Note further, that the Aristotelian square does not exhibit all the symmetries of the duality square.

Although in some cases the Aristotelian oppositions hold for the elements of a duality square, the two schemes are in principle logically independent from each other. The following two examples illustrate this point.

On the one hand, there are instances of the Aristotelian scheme without duality, due to the lack of any second order level. Take any two real first order predicates which exclude each other, together with their respective negations, and you can establish a square of oppositions, as shown in diagram 3.

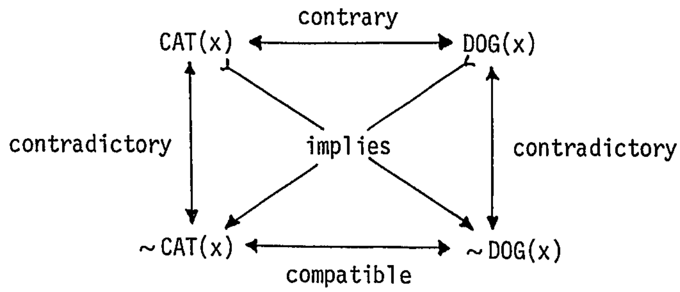

Diagram 3

On the other hand, there are dual operators for which the Aristotelian relations do not hold, such as *already* and *still. Still* and *already* span the duality scheme of diagram 4, when conceived as operators taking durative propositions. (I shall suggest an analysis below which will substantiate the duality claim involved.)

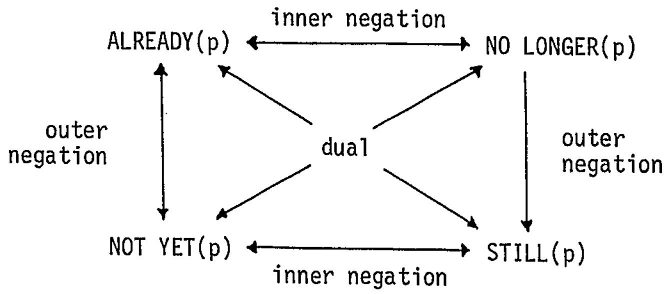

Diagram 4

The Aristotelean relations of compatability, contrariness, and implication are not even defined between the respective elements, because the statements on the left have different presuppositions from those on the right, and hence have truth values in different sets of cases. The Aristotelian concepts do not make sense in such a constellation.

1.2. Quantifiers and determiners

To come back to the general semantic conception – it is, however, not quantifiers, in the sense used here, that are the crucial operators underlying quantification, but determiners in the sense of Barwise & Cooper (1981). Barwise and Cooper postulate that any natural language quantifier has the property to "live on" a certain set:

(2) DEFINITION: A quantifier Q *lives on* the set A iff for every set P: Q(P) iff Q(P ∩ A).[6]

This set constitutes the relevant domain of objects under consideration. The standard restricted quantifiers of predicate logic, for example, live on their respective domain of quantification. This feature of natural language quantifiers shows that there is generally another predicate involved. It is therefore reasonable to consider two-place operators, namely determiners, instead of the one place quantifiers. Determiners take two predicates, one for the domain of quantification and one for the predicate quantified. Insertion of the domain predicate yields a quantifier in the sense defined above. Duality always involves the second argument, the predicate quantified.

I do not use the term *determiner* in a syntactic sense. Syntactically the roles of determiner, domain predicate, and predicate quantified can be distributed in many different ways. In case of nominal quantifiers of the form determiner (in a syntactic sense) plus noun, the determiner functions as a determiner in the semantic sense, the noun serves as the domain predicate, and the rest of the sentence – as long as it does not contain any higher operators – serves as the predicate quantified. There are, on the other hand, many cases of quantifiers that cannot be decomposed into determiner and domain predicate, such as *everything, nobody, sometimes* or pronominal *all*. The adverbs mentioned above, like *already*, are of the same kind, along with modal verbs and other more remote instances of quantification. Polar adjectives exhibit yet another constellation.

In what follows, I shall list several examples of natural language quantification, discuss some representative cases and finally try to extract a universal form which might underlie all cases of quantification considered.

2. SOME EXAMPLES

In the following examples the determiners and quantifiers are given in groups of four, each constituting a duality group. The elements of each group are listed in a fixed order to which I shall refer as type 1, 2, 3, and 4 respectively. The analysis I am going to suggest will yield type 1 throughout as existential quantification, type 2 as universal quantification, type 3 as the negation of type 1 and type 4 as the negation of type 2. Accordingly, type 1 and 2 are dual, as well as type 3 and 4.[7]

In the groups of four only those elements are listed which are lexicalized. Dots indicate elements that can be composed by means of a negative for either inner or outer negation. A stroke indicates a gap that cannot be filled with any expression of the required meaning in the given syntactic construction. I first present the examples as a whole and discuss them later in more detail.

(A1) $\text{He likes} \left\{ \begin{array}{l} \text{SOME} \\ \text{ALL} \\ \text{NO} \\ \ldots \end{array} \right\} \text{books by Günter Grass.}$

(A2) $\text{She spends} \left\{ \begin{array}{l} \text{SOME} \\ \text{ALL her} \\ \text{NO} \\ \ldots \end{array} \right\} \text{money on cat food.}$

(A3) $\text{He} \left\{ \begin{array}{l} \text{SOMETIMES} \\ \text{ALWAYS} \\ \text{NEVER} \\ \ldots \end{array} \right\} \text{manages to be friendly.}$

(A4) $\text{In China you can buy Coca-Cola} \left\{ \begin{array}{l} \text{SOMEWHERE} \\ \text{EVERYWHERE} \\ \text{NOWHERE} \\ \ldots \end{array} \right\}.$

(A5) $\text{If she is tired, she reads} \left\{ \begin{array}{l} \text{comics (TOO)} \\ \text{ONLY comics} \\ \text{NO comics} \\ \ldots \end{array} \right\}.$

(B1) $\text{It is} \left\{ \begin{array}{l} \text{POSSIBLE} \\ \text{CERTAIN} \\ \text{IMPOSSIBLE} \\ \ldots \end{array} \right\} \text{that that man will be reelected.}$

(B2) $\text{That man will} \left\{ \begin{array}{l} \text{POSSIBLY} \\ \text{CERTAINLY} \\ \text{IN NO CASE/WAY} \\ \quad ? \end{array} \right\} \text{be reelected.}$

$$\text{(B3)}\quad \text{Statement A4.1 is} \left\{\begin{array}{l}\text{SATISFIABLE}\\ \text{TAUTOLOGICAL}\\ \text{CONTRADICTORY}\\ \text{DISPUTABLE}\end{array}\right\}.$$

$$\text{(B4)}\quad \text{I} \left\{\begin{array}{l}\text{THINK IT (IS) POSSIBLE}\\ \text{BELIEVE}\\ \text{RULE OUT}\\ \text{DOUBT}\end{array}\right\} \text{that the butler is the murderer.}$$

$$\text{(B5)}\quad \text{His claim} \left\{\begin{array}{l}\text{is COMPATIBLE with}\\ \text{IMPLIES}\\ \text{is CONTRARY to}\\ \ldots\end{array}\right\} \text{yours.}$$

$$\text{(B6)}\quad \text{He will} \left\{\begin{array}{l}\text{ACCEPT}\\ \text{CLAIM}\\ \text{REFUSE}\\ \text{RENOUNCE}\end{array}\right\} \text{compensation.}$$

$$\text{(B7)}\quad \text{She} \left\{\begin{array}{l}\text{LET him pay}\\ \text{MADE him pay}\\ \text{KEPT him from paying}\\ \ldots\end{array}\right\} \text{the bill.}$$

$$\text{(B8)}\quad \text{He} \left\{\begin{array}{l}\text{CAN}\\ \text{MUST}\\ \ldots\\ \text{NEED NOT}\end{array}\right\} \text{accept that deal.}$$

$$\text{(B9)}\quad \left\{\begin{array}{l}\text{GO}\\ \text{GO}\\ \text{DON'T GO}\\ -\end{array}\right\} \text{to that party.}$$

(B10) The doctor $\left\{\begin{array}{l}\text{ALLOWed}\\ \text{ORDERed}\\ \text{FORBADE}\\ \ldots\end{array}\right\}$ him to eat meat.

(B11) You have the $\left\{\begin{array}{l}\text{RIGHT}\\ \text{DUTY}\\ -\\ -\end{array}\right\}$ to vote.

(C1) The dollar is $\left\{\begin{array}{l}\text{ALREADY}\\ \text{STILL}\\ \text{NOT YET}\\ \text{NO LONGER}\end{array}\right\}$ high.

(C2) This house is $\left\{\begin{array}{l}\text{big ENOUGH}\\ \ldots\\ \ldots\\ \text{TOO big}\end{array}\right\}$ for us.

(C3) It is $\left\{\begin{array}{l}\text{BIG}\\ \text{SMALL}\\ \ldots\\ \ldots\end{array}\right\}$ and has $\left\{\begin{array}{l}\text{MANY}\\ \text{FEW}\\ \ldots\\ \ldots\end{array}\right\}$ rooms.

(C4) In the weather forecast they said it will $\left\{\begin{array}{l}\text{CONTINUE to rain}\\ \text{START raining}\\ \text{STOP raining}\\ \ldots\end{array}\right\}$.

2.1.1. Plain quantifiers

The examples of group A are obvious correspondents of the standard predicate logic quantifiers. In spite of considerable efforts there is not yet any theory which covers singular and plural count noun and mass noun quantification in a fully satisfactory way, although recent works such as Link (1983) promise a breakthrough to a uniform treatment. Nevertheless plural and mass noun quantification should be kept in view whenever quantificational phenomena are studied. One remark might be in place concerning (A5), the group around *only*. This word can occur in a noun preceding position but it is not a determiner in the syntactic sense. This is obvious because it can only occur in what looks like a determiner position when the following noun (in fact noun phrase) can be used without any determiner. *Only* can at best be considered preceding NPs in certain cases. It is, in fact, a focussing particle that can take NPs as well as all sorts of other expressions as focus elements. *Only* has two meanings which can roughly be paraphrased as "nothing but" and "no more than" and can give rise to ambiguity, though they might be closely related and even be instances of a uniform general meaning.[8] In example (A5) the intended reading is the "nothing but" variant. In this reading *only* functions as an inversion of *all*: it changes the roles of the domain of quantification predicate and the predicate quantified. The same holds for the other elements of the group, as is shown by the following equivalences:

(2)	She reads	comics (TOO)	≈	SOME of what	she reads is comics.
		ONLY comics		ALL	
		NO comics		NONE of what	
		NOT ONLY comics		NOT ALL	

Accordingly, duality applies to the predicate provided by the noun, because this is the predicate quantified. This could be more easily demonstrated if there were a proper noun negation. Take the following sentences for a demonstration of the dualities in this group:

(3) NOT ONLY members are allowed. = *Non*members are allowed (TOO).
ONLY *non*members are excluded. = NO members are excluded.

2.1.2. *Possibility and necessity*

The examples (B1) – (B11) all belong to the realm of possibility and necessity. It is generally agreed that these two concepts are instances of existential and universal quantification respectively, with a range of possibilities as domain of quantification which is given by certain characteristic conditions. The domain of quantification is usually implicit but can be made explicit by means of adverbial or conditional constructions:

(4) *If you want to catch the train*, you must leave now.

(5) *According to the recent polls* it is possible that he wins the elections.

The modal verbs form several duality groups. Another one would be *may*/

must/must not/need not. It all depends on the range of alternatives considered. Often an epistemic and a deontic reading of the modal verbs is distinguished. Kratzer (1977) has shown that there are as many readings – in this sense – as there are possible ranges of possibilities, and how to treat them all in a uniform way.

Some readers might be surprised by the double type assignment for the imperative in (B9). The imperative is usually used for commands, that is type 2 statements. But there seem to be cases, where it can be used to express a permission rather than a command. Imagine a young girl asking her reluctant mother to allow her to go to a party. Finally the mother could give in, saying (B9) in the type 1 interpretation.

In epistemic logic, *believe* is usually treated alongside *know*. The two verbs, however, do not belong to the same duality group defined with respect to the embedded proposition. Both verbs are of the same type 2 according to the consistency criterium discussed in the next section. The standard uses of the verbs require consistency of the respective propositions. You cannot at the same time believe p and not-p, similarly you cannot know both p and not-p. If two operators are dual they can not however both fulfil the consistency requirement unless they are identical i.e. self-dual.[9] But clearly neither *know* nor *believe* are self-dual. Hence, they must belong to two different duality schemes because they are neither identical, nor inner or outer negations of each other, nor duals. It seems that they are operators of the same kind but drawing on different evidence. There is a principal difference between those facts one can know and those one can at best believe, depending on whether one has authentic access to the relevant information. Some languages, such as Japanese, draw a clear distinction between these two sorts of facts. For example, the Japanese do not express the fact that one himself is happy in the same way as the fact that somebody else is happy. The latter is expressed obligatorily in the sense of somebody seeming or looking happy (cf. Kuroda 1973 for details).

The remaining four groups of quantifiers, presented in examples (C1) – (C4) will be discussed in detail below.

2.2. Type assignment and type assymmetry

One generalization that is obvious from the examples cited above is a clear asymmetry among the four types of quantifiers as to their lexicalization. Type 1 is lexicalized throughout and so is type 2, but there are many languages which exhibit considerable gaps in the lexicalization of type 2. Japanese and Chinese, for example, use complex expressions in most cases of universal quantification. Type 3 is synthesized in some cases of English. With respect to type 3, Indoeuropean languages seem to be exceptional in that they possess proper lexical units such as *no, never, none, neither,* *nothing*, etc. and even in these cases the respective words are historically compounds containing a negative prefix. Type 4 is lexicalized with a single word only in four examples out of twenty above. In two cases (B8 and C1) negative polarity items are used to fill the gap.

The absence of type 4 in the lexicon has been stated under more limited perspectives by other authors before. Barwise and Cooper (1981) postulate as a natural language universal that there are no determiners of type 4. I shall discuss their postulates in more detail after introducing independent criteria for the type assignment. Horn (1972) makes a similar claim referring to a wider class of expressions including modal verbs, modal adverbs and adjectives, and connectives besides the usual quantifiers.

Any asymmetry hypothesis, of course, is as strong as the type assignment is independent. We therefore need independent criteria for the distinction of the respective types. This is a nontrivial task because of the total symmetry of the duality scheme. Even if we could start from the Aristotelian square of oppositions there would still be no way of distinguishing quantifiers from their inner negations (note the left-right symmetry of the configuration in diagram 2). Intuitively, however, there are differences associated with the type distinctions prior to any analytical understanding.

First, there is a feeling that type 1 and type 2 are positive whereas type 3 and type 4 are negative. This distinction can be expressed in terms of what Barwise and Cooper call monotonicity (1981:184).

(6) DEFINITION: A quantifier Q is *monotone increasing (mon↑)* iff Q(P) and $P \subset P'$ implies $Q(P')$. Q is *monotone decreasing (mon↓)* iff Q(P) and $P \supset P'$ implies $Q(P')$.

In other words, in case of monotone increasing quantifiers the quantified predicate can be weakened salva veritate, whereas it can be further restricted in case of monotone decreasing quantifiers. As is easily checked, type 1 and type 2 quantifiers are mon↑ as opposed to the mon↓ quantifiers of type 3 and type 4. In case of the temporal presupposing quantifiers in (C1) and (C4) not all alternative predicates P' can be used but only those the presuppositions of which are fulfilled. The direction of monotonicity is necessarily reversed both by inner and outer negation because negation reverses implication. Hence duals have the same monotonicity direction (if any) and cannot be distinguished by means of this criterion. It is extremely useful though, because it can be used even in those cases which do not exhibit a splitting of the quantifier into determiner and domain of quantification predicate.

There are several possibly interrelated ways to distinguish between duals. One very simple criterion is the possibility that a quantifier applies to both a predicate and its negation. I feel tempted to call quantifiers which exhibit this possibility weak and those which do not strong. But as these terms are defined differently and only approximately extensionally equivalent by Barwise and Cooper (1981) let me call them tolerant and intolerant instead.

(7) DEFINITION: A quantifier Q is *tolerant* iff Q(P) and Q(~P) is possible at the same time. A quantifier is *intolerant* iff Q(P) excludes Q(~P).

Thus, a quantifier is intolerant if and only if it implies its dual. Obviously this criterion is applicable in exactly those cases where the quantifiers fit into the Aristotelian scheme of oppositions, and therefore is of no use for presuppositional quantifiers. It works, however, for all examples of the groups A and B above. A very simple proof.[10] shows that if a quantifier is intolerant then it is either self-dual or its dual is tolerant. Thus the tolerance criterion separates duals (while it is obviously blind as to the distinction between quantifiers and their inner negation). Intuitively, it separates universal quantifiers which are intolerant from the tolerant existential quantifiers. In case of universal quantifications the whole domain of quantification – or at least the greater part of it – has to be checked; they are difficult to verify, but easy to falsify, whereas for existential statements the converse is true. *Some, several, many* give rise to tolerant quantifiers, whereas *all, most,* and *no* lead to intolerance, provided empty universes are generally excluded, which is a reasonable assumption in this context, because if the quantifier lives on the empty set even the contraries *no* and *all* become indistinguishable. This criterion was first used by Laurence Horn (1972), though he does not use my terms.

Horn, investigating a wide range of logical operators which can be conceived as defining values on abstract scales – including quantifiers, modal verbs, modal adjectives and adverbs, connectives and others – states that for tolerant operators the outer negation can be lexicalized, but the inner negation can not. This statement aims at ruling out type 4 quantifiers, but needs the additional condition that it applies only to type 1 or monotone increasing operators.

Barwise and Cooper postulate two universals that exclude type 4 determiners from the lexicon of natural languages (with regard to NP quantification). One is their "monotonicity correspondence universal" (1981: 186) according to which "there is a simple NP which expresses the mon↓ quantifier ~Q if and only there is a simple NP with a weak non-cardinal determiner which expresses the mon↑ quantifier Q." Weak monotone increasing quantifiers in the realm of nominal quantification are exactly the tolerant monotone increasing ones. Thus, according to this universal, the outer negation counterparts of type 2 determiners are ruled out, because type 2 quantifiers are strong (intolerant).

The other constraint relevant here is their "persistent determiner universal" (1981: 193): "every persistent determiner of human language is mon↑ and weak." In our terms: every persistent determiner of human language is type 1. Persistent determiners are those which are monotone increasing with respect to the domain of quantification predicate. Informally this means, that a

(8) DEFINITION: A determiner D is *persistent* iff D(A, P) and $A \subset B$ implies D(B, P). D is *antipersistent* iff D(A, P) and $A \supset B$ implies D(B, P).

true statement Q(P), "living on" the domain of quantification A, remains true if the domain of quantification is enlarged: adding new individuals or quantities to those which are already considered cannot provide any counterevidence. This holds for simple existential quantifiers like the numerals, *some, several, a few, numerous* and the like which state positively and non-exclusively that there is a certain quantity of positive instances of the predicate quantified. The property of persistency does not hold of determiners which may express a certain ratio between the amounts of positive and negative evidence, such as *few* and *many* in their proportional readings. Apparently, the inner negation counterparts of persistent determiners are themselves persistent, while outer negation changes persistency into antipersistency, i.e. downwards monotonicity with respect to the domain of quantification predicate.[11] Thus, persistency provides another criterion for the separation of duals. But not all determiners are either persistent or antipersistent. For determiners which are not highly degenerate, persistency implies tolerance.[12] For that reason only type 1 and type 4 determiners can be persistent. The persistent determiner universal, then, rules out type 4 because it is generally monotone decreasing.[13] I shall come back to the property of persistency below, after the discussion of phase quantifiers (C1 – C4). So far we have got a type assignment for the A and B cases by means of independent criteria, which allows to state the asymmetry hypothesis concerning the lexicalization of natural language quantifiers:

(9) CONJECTURE: Natural language quantifiers can be classified into four types. Type 1 contains all existential quantifiers (maybe among others), type 2 contains all universal quantifiers, type 3 all negated existential quantifiers, and type 4 all negated universal quantifiers. The type assignment is unique. Natural language exhibits significant differences with respect to the extent of the lexicalization of the four subclasses and to the average complexity of the expressions used in the four subclasses. The number of lexical items decreases, and the complexity of the expressions increases from type 1 through type 4 with each step.

I have not, so far, provided criteria for the last four examples which group them together with the other ones. Instead of subclassifying the operators of these examples with general criteria I shall provide an explicit analysis for them.

3. PHASE QUANTIFICATION

3.1. Analysis of the examples

3.1.1. already, still, not yet, no longer

For reasons which will become apparent later I refer to the last four examples as phase quantifications. Let me start with the group of operators around *already*. There is a considerable amount of literature about this topic, but I am not going to discuss any other approaches because of the limited space here.

In the following analysis I treat only those uses of *already* and the other three adverbs, in which they can be understood as operators taking time-dependent durative propositions. Statements containing these adverbs are evaluated with respect to a certain temporal reference point t°, at which it is already/still/... the case that p. The adverbs carry with them certain presuppositions. Before I discuss them, let me first establish the duality relationships between the four operators.

Whatever the exact presuppositions of *already p* are, they are the same as those of *not yet p*. Dialogues as the following show that *already* and *not yet* are used as outer negations of each other, in the strong, presupposition preserving sense of negation:

(9) Has the train already arrived? – No, not yet.

(10) The train has not yet arrived. – You're wrong, it is already here.

In order to check the relationships concerning inner negation, let us assume for the sake of simplicity that *she is asleep* is the negation of *she is awake* (a simplification which will not affect the validity of the subsequent analysis). Then, the sentences (11) and (12), and (13) and (14) mean the same, respectively:

(11) She is already asleep. = already p.

(12) She is no longer awake. = no longer ~p.

(13) She is not yet asleep. = not yet p.

(14) She is still awake. = still ~p.

Consequently, *no longer* is used as the inner negation of *already*, and *not yet* functions as the inner negation of *still.*[14] *Still*, then, is the inner negation of the outer negation of *already*, i.e. its dual. From this it follows that *still* is also the outer negation of *no longer*, which is apparently the case:

(15) Is he still angry? – No, no longer.

This yields the duality relations of diagram 4 above.

I assume that *already(p, t°)* and *not yet(p, t°)* have the same presupposition, that there is a phase of not-p which has started before t° and might be followed by at most one phase of p which reaches till t°. Then the point of the alternative "already p or not yet p" is whether the endpoint of the presupposed preceding negative phase is reached until t° or not. Starting from such a negative phase before t°, t° may fall into that very phase – in case of *not yet(p, t°)* – or else it falls into the following positive phase. Both statements are undefined if there is no negative preceding phase to start with. The semantics of *already* are rather subtle. *Already(p, t°)* states the transition from ~p to p in the immediate neighbourhood of t°, not more, "immediate neighbourhood" being meant in the topological sense (ruling out the relevance of any transition points earlier than the latest one). Pragmatic requirements of relevance change that topological closeness condition to a metrical one in most cases: the farther ago the transition point lies the less probable is the relevance of a statement that the transition has taken place.[15] Hence the feeling that *already(p, t°)* is normally used when p has just begun, and *not yet(p, t°)* when ~p is about to end. *already(p, t°)* is wrong if the previous state of ~p continues to prevail at t°. In many cases the expectation that this is so may be the reason for uttering *already(p, t°)*. But contrary expectations need not necessarily play a role for such statements. Nothing is wrong about a sentence like:

(16) As I/you expected, the train has already/not yet arrived.

The meaning of *already* and its counterpart *not yet* is shown informally in diagram 5, the two arrows starting from t° symbolizing the two possibilities that t° either falls into the positive or the negative semiphase.

Being the dual of *already, still* carries a presupposition which derives from that of *already* by means of the negation of the embedded proposition:

(17) presupposition of *no longer(p, t°)*
= presupposition of *still(p, t°)*
= presupposition of *~already(~p, t°)* (by duality)
= presupposition of *already(~p, t°)*

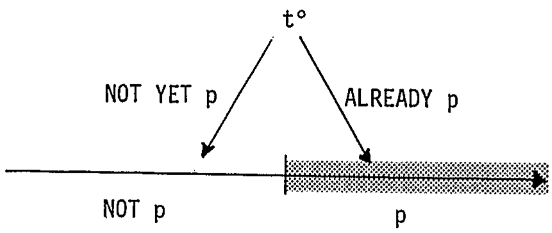

Diagram 5

Thus, the sentences *still(p, t°)* and *no longer(p, t°)* presuppose that there is a phase of p which has started before t° and might be followed by at most one phase of not-p till t°. *Still(p, t°)* is true if that phase of p includes t°, while *no longer(p, t°)* states that that phase has ended before t° and t° lies within the negative phase following it. Graphically we get the following picture of the meanings of the latter two operators in the spirit of diagram 5:

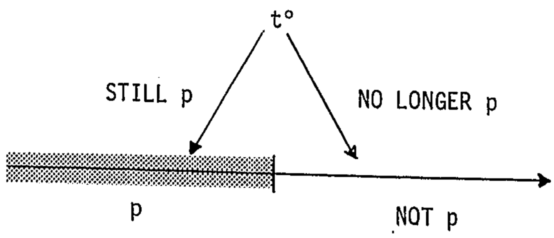

Diagram 6

Inner negation results in exchanging the positive and the negative semiphases, while outer negation concerns the decision whether the parameter t° falls into the first or the second semiphase. The middle point, in both cases is meant to belong to the positive phase. The starting and the end point of the whole interval considered are to be excluded.

3.1.2. *Enough* and *too*

As a pair of related operators *enough* and *too* take as operands any scaling adjectives or adverbs.[16] Scaling adjectives provide a specific scale of values, possibly context-dependent, e.g. a scale of size in case of *big* and *small*. *Enough* presupposes a range of admissible values on the scale with a lower bound, *too* a range of admissible values with an upper bound. *A is ADJ enough* means that the value for A on the scale provided by ADJ lies above the critical lower bound of admissibility (and is hence admissible), whereas *A is too ADJ* means that the value for A lies above the critical upper bound of admissibility and is hence not admissible. The meanings of the operators of the *enough*-group are thus completely analogous to those of the *already*-group. There is even a proper paraphrase relationship between both cases:

(18)	a is quick ENOUGH	= a is ALREADY admissible in speed
	a is NOT TOO quick	= a is STILL admissible in speed
	a is NOT quick ENOUGH	= a is NOT YET admissible in speed
	a is TOO quick	= a is NO LONGER admissible in speed

Of course the operators on the right side are not interpreted temporally in this case. Diagram 7 displays a picture of the respective meanings:

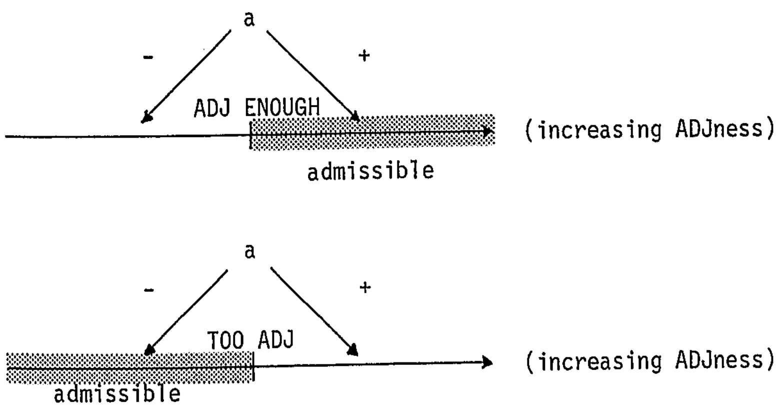

Diagram 7

We only need to replace the time scale by the scale provided by the adjective or adverb and its polarity and the proposition p by the admissibility predicate. The duality relationships obtain with respect to the implicit admissibility predicate and can therefore not be demonstrated at the surface. The otherwise inexpressible inner negation is expressed by the pair *enough/too*. If the adjective or the adverb in the focus is replaced by its antonym, the scale and the admissibility range remain the same but the order is reversed. The result is an exchange of the first and the second semiphase together with the corresponding relocation of the parameter: what is an admissible value remains an admissible value. The effect, thus, is that of inner plus outer negation: *big enough* and *small enough* are duals and so are *too big* and *too small*. For that reason the following equivalence holds, which looks like a duality but is none:

(19) a is big enough = a is not too small

The expression on the right is the dual of *a is not too big* and hence the inner negation of *a is too big*, which in turn is the inner negation of the left side, according to (18). The following diagram is an illustration of the equivalence (19).

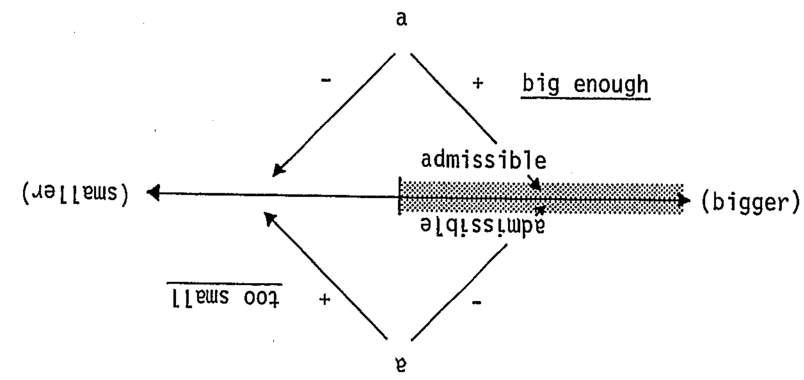

Diagram 8

3.1.2. Scaling adjectives

Scaling adjectives (and adverbs) themselves represent another example of this type of meanings, adding a whole syntactic class to the realm of natural language quantification. I regard the predicative use of adjectives as basic in the following. Scaling adjectives refer to a range on a scale into which the value of their argument falls. *A is big* says that the size of a falls into a range of possible values on the scale of size which are considered high. Scaling adjectives and adverbs require a tripartition of the respective scale[17], consisting in a marked lower third, a neutral middle part, and a marked upper third as shown in diagram 9. The choice of the scale itself and the exact partition of the scale into marked and unmarked values depend on the context in a complex way which need not concern us here.

Pairs of antonymous adjectives are asymmetrical in several regards. There is one, intuitively positive, which exhibits more general possibilities of use, in contrast to the other, which is more specific. *Big*, for example,

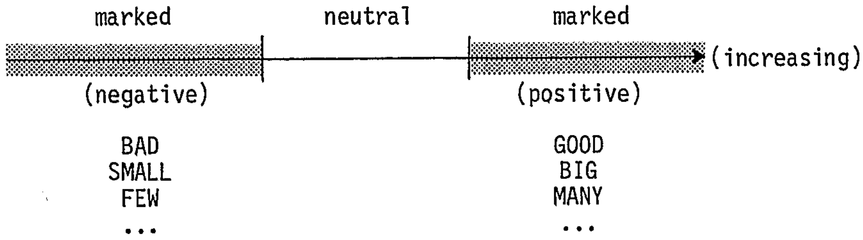

Diagram 9

being the positive pole, can be used in a neutral sense in connection with *how* or *so*, while its negative antonym *small* keeps its specific meaning in such phrases. Likewise the corresponding nouns referring to the dimension in a neutral way, such as *size, length, thickness*, and so on belong to the positive pole, often being derived from it, whereas the derivations from the negative pole cannot be taken neutral: *shortness, narrowness*, etc. In many cases no nominal derivations exist at all. These are only two differences out of several more which point to the same direction: the negative antonyms are more restricted, or more specialized, in use. This tendency is another aspect of the general type asymmetry observed above, as the positive antonyms will be analyzed as type 1 and the negative ones as type 2. Type 3 adjectives are rare and type 4 adjectives do not seem to exist at all.[18]

The meaning of scaling polar adjectives, again, is an example of phase quantification, the quantified predicate this time being the property of having a marked value on the given scale. Positive antonyms state that the value lies higher than what is considered unmarked and negative ones state that the value lies lower than that.

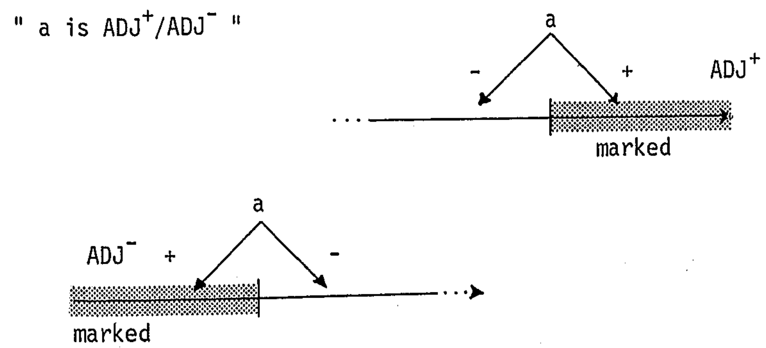

Diagram 10

Thus, antonymous scaling adjectives are duals with respect to the implicit markedness predicate.

One instance of this sort of quantifiers of particular interest is the pair *many* and *few*. They behave essentially like scaling adjectives: they can be used both predicatively and attributively, they take the same modifiers (*very, enough*[19], etc.), they have comparative and superlative forms, and they admit definite determiners preceding them in their attributive use. Semantically, they are intersective and relative – *many children* applies to those collections of children of a relatively high number just the same way as *intelligent child* applies to those children of a relatively high intelligence. The only difference between *many* and *few* on the one hand and adjectives like *thick* on the other is that the latter are distributive, whereas the former are collective. Used as quantifiers in the sense of Barwise & Cooper – the noun following *many* or *few* representing the domain of quantification and the VP the predicate quantified – the resultant meaning of $(many/few\ N)_{NP}\ VP$ is that the number of those "Ns" to which the VP applies is relatively high or low, respectively, i.e. in set-theoretical terms the cardinality of the intersection of the extensions of the noun and the VP is marked as high or low. It is left to the context to provide the criterion for markedness. The so-called proportional and absolute meanings[20] need not be distinguished semantically. Needless to state, that *many* is type 1 and few is type 2. The two operators are therefore not negations of each other[21], which is correct. They are contraries with a non-empty range of neutral cases possible between "many" and "few".

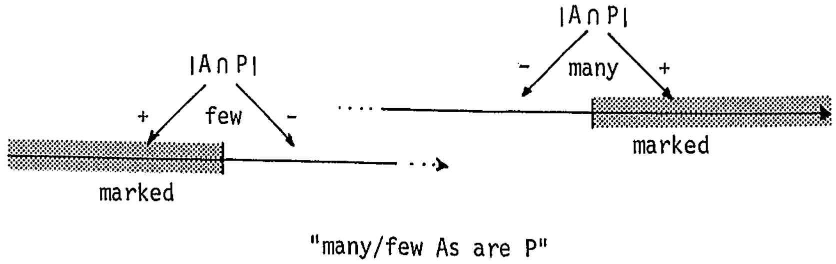

Diagram 11

3.1.4. continue, begin, stop

In what follows I treat these verbs for the sake of simplicity as propositional operators, again taking durative propositions as arguments. The fact that these verbs are quantifiers, too, suggests that verbal aspect belongs to the realm of quantification, because they just represent the standard aspects *durative, ingressive,* and *perfective*. Again, the type asymmetry observation is confirmed by the fact that there is no aspect of *not-beginning*.

The duality relationships here can easily be checked. If something stops, the contrary begins, and vice versa. Hence, *begin* and *stop* are inner negations of each other. Furthermore, *stop* is the outer negation of *continue*, as a given state either stops or continues. This renders *continue* and *begin* duals.

The verbs under consideration, too, refer to an implicit time parameter t°. In contrast to the adverbs *already, still, not yet,* and no *longer* – which tell something about the recent past – these verbs tell something about the close future, how things go on from t° with respect to the proposition embedded[15]. The time stretch under consideration again is a double phase of not-p and p which contains t°. In case of *continue(p, t°)* and *stop(p, t°)* the first semiphase is p and has started before t°. If t° is the last point of this semiphase, *stop(p, t°)* is true, otherwise *continue(p, t°)* holds.

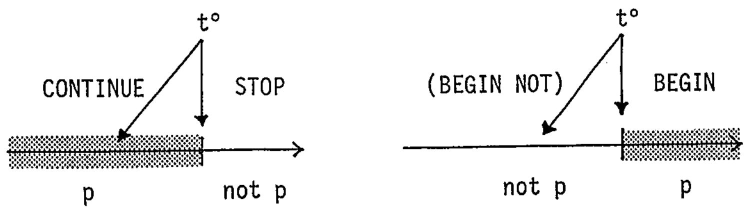

Diagram 12

The correspondence of these verbs and the adverbs around *already* becomes apparent if the course of events till t° for the latter ones and the course of events from t° on for the former ones is compared, as in the next diagram:

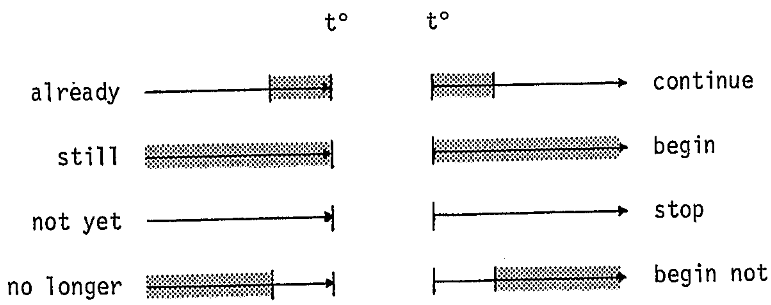

Diagram 13

Of course, in case of the statements on the right side the future course of events can only be treated as possible not as factual, because of the general asymmetry of past and future.

3.2. The general definition of phase quantification

The preceding four groups of examples can all be formalized in a uniform way. The formalization I present is not part of any particular framework and can certainly be given in alternative, maybe better, forms.

The four groups of operators semantically have two operands. They take a predicate quantified which defines a positive phase or range of values on a scale. The scale is the time scale in case of the *already*-group and the aspectual verbs, and the scale provided by the adjective or adverb in the other two cases. Adjacent to the positive phase defined by the predicate quantified, there is a negative contrast phase, either preceding or following the positive phase. It does not matter if there is a zone of indetermination between the positive and the negative phase.

The resulting double phase is fixed on the respective scale – which might contain several such double phases – by the additional condition that it has to contain a parameter point, t° or a in the examples above. This parameter point is the second operand. The four types of quantifiers now differ in presupposing that either the positive or the negative semiphase comes first and in stating that the parameter point falls into the first or into the second semiphase, thus resulting in four possible cases. (Minor modifications apply to the case of the aspectual verbs *begin* and *stop*.)

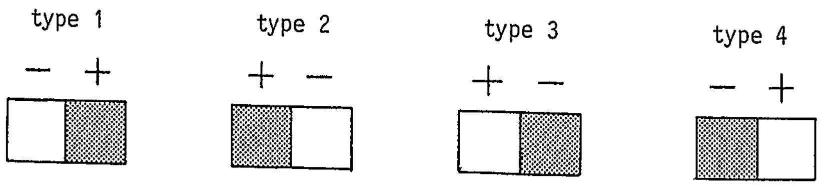

Diagram 14

Let me motivate the formalization I want to propose with a procedural description of the meanings of the four operator types. You start from within the first semiphase, no matter where but, say, from its leftmost point. This phase is either negative (type 1 and 3) or positive. You run along the scale till you reach the parameter point – which has to lie within the double phase – and check on the way whether you enter any second semiphase. If so you have true cases of type 1 or 4. Since the parameter point itself has to lie within the double phase, the starting point has to be the infimum of the l a s t positive or negative phase that starts before the parameter point. Let me call this point GSI for "greatest *s*maller *i*nfimum". The formal definition is:

(20) $\mathrm{GSI}(P, a) =_{df} \inf\{x \mid x < a \;\&\; P(x) \;\&\; \forall y(x < y \leq a \;\&\; P(y) \rightarrow \forall z(x < z < y \rightarrow P(z)))\}$

Obviously, GSI(P, a) is defined if and only if the following condition is fulfilled:

(21) $\exists x(x < a \,\&\, P(x) \,\&\, \forall y(x < y < a \,\&\, P(y) \rightarrow \forall z(x < z < y \rightarrow P(z))))$

The third conjunct, however, is redundant in case of simple durative predicates p. The existential presupposition for GSI(P, a) therefore reduces to

(22) $\exists x(x < a \,\&\, P(x))$

The use of the presuppositional term GSI renders it possible to put presupposition and assertion in one formula. In that way the duality relations become completely transparent. I shall give two equivalent formulations in order to show the latter as well as the parallelism of the phase quantifiers and the standard quantifiers of predicate logic. Type 1 can be taken as the statement that in the domain of quantification – i.e. between the relevant GSI and the parameter point – there are positive cases of the predicate quantified. From this it follows by the definition of GSI, that the parameter point itself falls into the positive semiphase. Type 2 can be expressed as universal quantification: all points within the domain of quantification fall into the positive semiphase.

(23) general format of phase quantification:

type 1: $\exists x(GSI(\sim p, a) \leq x < a \,\&\, p(x))$
type 2: $\sim\exists x(GSI(\ p, a) < x \leq a \,\&\, \sim p(x))$ (dual of type 1)
type 3: $\sim\exists x(GSI(\sim p, a) < x \leq a \,\&\, p(x))$ (outer negation)
type 4: $\exists x(GSI(\ p, a) < x \leq a \,\&\, \sim p(x))$ (inner negation)

(24) type 1: $\exists x(GSI(\sim p, a) < x \leq a \,\&\, p(x))$ (existential
type 2: $\forall x(GSI(\ p, a) < x \leq a \rightarrow p(x))$ (universal)

Note, that the inner negation of the predicate quantified affects both occurrences of *p*. The definition applies immediately to the first three cases (*already/. . .*, *enough/ . . .*, ADJ^+ /. . .):

(25)

	scale	order <	predicate p	parameter a
already(p, t°)	time	earlier	p	t°
a is ADJ enough	ADJ⁺ness	less ADJ	admissible in ADJ⁺ness	a
a is ADJ⁺/⁻	ADJ⁺ness	less ADJ⁺	marked in ADJ⁺ness	a
many/few A are P	numbers	less	marked in number	$\mid A \cap P \mid$

A format very similar to (23) applies to the interpretations of the aspectual verbs. They differ slightly in that the domain of quantification is the phase that reaches from the parameter point up to the smallest greater supremum of the appropriate phase. I will not develop the exact formulation here because it is not needed for the following considerations.

Monotonicity and persistency

With this interpretation at hand the criteria of monotonicity and persistency become applicable to the phase quantifiers, too, and correspond to well known meaning properties of these operators. Recall that a determiner is persistent if it is immune against extending the domain of quantification. Any extension of the domain of quantification – which of course has to be kept within the limitation of the given doublephase as a whole – means a shift of the parameter point to the right, while any further restriction of the domain of quantification corresponds to a shift of the parameter point to the left. This way, the persistency of type 1 and type 4 accounts for the validity of the following inferences:

(26) $\left\{\begin{matrix}\text{already} \\ \text{?no longer}\end{matrix}\right\} (p, t°) \;\&\; t^1 < t° \Rightarrow \left\{\begin{matrix}\text{already} \\ \text{no longer}\end{matrix}\right\} (p, t')$

(27) $a \text{ is } \left\{\begin{matrix}\text{ADJ enough} \\ \text{too ADJ}\end{matrix}\right\} \;\&\; b \text{ is ADJer than } a \Rightarrow b \text{ is } \left\{\begin{matrix}\text{ADJ enough} \\ \text{too ADJ}\end{matrix}\right\}$

(28) $a \text{ is } \left\{\begin{matrix}\text{ADJ}^{+} \\ \text{not ADJ}^{-}\end{matrix}\right\} \;\&\; b \text{ is ADJ}^{+}\text{er than } a \Rightarrow b \text{ is } \left\{\begin{matrix}\text{ADJ}^{+} \\ \text{not ADJ}^{-}\end{matrix}\right\}$

Antipersistency accounts for the reverse properties of the type 2 and type 3 operators.

Monotonicity, or right monotonicity, to follow van Benthems[24] terminology, makes good sense likewise. The property of upward monotonicity means immunity of the operator against any extension of the predicate quantified. Obviously, it is the "positive" type 1 and type 2 operators which have the parameter point falling into the positive semiphase that allow the phase p to be replaced by a greater phase p′ that contains p. Likewise the type 3 and type 4 operators do not allow the same change but allow for a restriction of the positive semiphase because that results in an extension of the negative one. Extension of a semiphase in the temporal cases means embedding it into a more comprehensive interval. In the cases involving adjectives it means loosening or tightening the criteria of markedness or admissability. The following inferences – all to be taken within the conditions presupposed – reflect the property of upward monotonicity for type 1 and type 2 operators:

(29) $\text{She is} \left\{\begin{matrix}\text{already}\\ \text{still}\end{matrix}\right\} \text{fast asleep.} \Rightarrow \text{She is} \left\{\begin{matrix}\text{already}\\ \text{still}\end{matrix}\right\} \text{asleep.}$

(30) $\text{He is} \left\{\begin{matrix}\text{tall for a basket ball player.}\\ \text{short for a jockey.}\end{matrix}\right\} \Rightarrow \text{He is} \left\{\begin{matrix}\text{tall}\\ \text{short}\end{matrix}\right\} \text{for a man.}$

(31) $\text{This is} \left\{\begin{matrix}\text{enough}\\ \text{not too much}\end{matrix}\right\} \text{for three days.} \Rightarrow$

$\text{This is} \left\{\begin{matrix}\text{enough for two days.}\\ \text{not too much for a week.}\end{matrix}\right\}$

According to the phase quantifier interpretation offered here, *many* is monotone increasing and persistent, *few* being monotone *in*creasing and antipersistent. Persistency and antipersistency here corresponds to upward and downward monotonicity respectively for these quantifiers taken as generalized quantifiers in the sense of Barwise and Cooper's.[23]

3.3. The standard restricted quantifiers as phase quantifiers

We have seen so far that the phase quantifiers are special instances of restricted quantifiers. If we assume that the various possibility and necessity operators of the example group B above are cases of restricted quantifiers, too – which seems highly plausible[24] – this result enables us to state the lexicalization asymmetry hypothesis for a broad class of natural language expressions, and furthermore to associate the properties of persistency and monotonicity with the four types of operators throughout. Of course, this means a substantial constraint upon possible natural language quantifiers, supposed it be valid for further cases too not yet investigated under this perspective.

What is more informative about natural language quantification, however, is the fact that, conversely, the general cases of restricted quantification, too, fit into this considerably specific scheme – though at the cost of a slight generalization. This generalization, however, has its own merits. The standard restricted quantifications as given in (32) can be equivalently

(32) $\exists x(x \in A \;\&\; x \in P)$
$\forall x(x \in A \rightarrow x \in P)$ equivalently: $\sim\exists x(x \in A \;\&\; x \in \bar{P})$

expressed using second order quantifiers over subsets of the domain of quantification instead of first order quantifiers over its elements, rendering the formulations in (33). Note that it is essential that only non-empty subsets of the range of quantification are considered, and that duality still holds with respect to the predicate P, which is now considered to apply to its subsets.

(33) $\exists X(\emptyset \subset X \subseteq A \;\&\; X \subseteq P)$
$\forall X(\emptyset \subset X \subseteq A \rightarrow X \subseteq P)$ equivalently: $\sim\exists X(\emptyset \subset X \subseteq A \;\&\; X \subseteq \bar{P})$

Now, the empty set $\emptyset$, figuring as the excluded lower bound in the restrictive condition in (33) is the unique infimum of any set whatsoever with respect to the partial ordering of set inclusion. So it is the GSI for any set P as well as for its complement $\bar{P}$. Definition (20) above yields

(34) $$\mathrm{GSI}^*(P, A) = \inf_{\subseteq}\{X \mid X \subset A \;\&\; X \subseteq P \;\&\; \forall Y(X \subset Y \subseteq A \;\&\; Y \subseteq P \rightarrow \forall Z(X \subset Z \subset Y) \rightarrow Z \subseteq P))\}$$

The third condition is redundant, because it holds for any sets A, P, X whatsoever.

By this we get

(35) $$\mathrm{GSI}^*(P, A) = \inf_{\subseteq}\{X \mid X \subset A \;\&\; X \subseteq P\}$$

which is obviously the empty set if the term is defined at all, i.e. if the domain of quantification is not itself empty, a condition we presuppose throughout. (32) can therefore equivalently be reformulated as:

(36) $\exists X(\mathrm{GSI}^*(\bar{P}, A) \subset X \subseteq A \;\&\; X \subseteq P)$
$\forall X(\mathrm{GSI}^*(P, A) \subset X \subseteq A \rightarrow X \subseteq P)$

(36) differs from the general phase quantification scheme in two respects. First, the predication relation here is not set membership but set inclusion. This seems to be a harmless step. One should be flexible at this point. Set

membership is only an adequate interpretation of predication in case of distributive predicates applied to individuals. The more general schemes in (33) and (36) can also be applied to collective predicates. In a similar way, mass noun quantification might require a further generalization of the predication relation, replacing set inclusion by a less specific part-of-relation. Thus, this generalization is not only harmless but even necessary.

The second deviation from the phase quantification scheme is the replacement of the underlying strict ordering with the partial ordering of set inclusion. For that reason I used the term *GSI** instead of *GSI*. This modification is indeed substantial because intuitively an essential feature of what I called phase quantifiers is that they work on scales. Is there any way to conceive the restricted standard quantifiers as phase quantifiers in the narrower sense? The answer is yes, and the way it is possible is more than merely a mathematical possibility.

The set theoretical formulae expressing the standard restricted quantifications either in the individual or in the subset mode depict a static conception of quantification: "*There are* elements/subsets of the domain of quantification A to which the predicate P applies." Such a picture is natural in a semantic framework which has in view the truth conditions of sentences and does not consider the way truth or falsity comes about. This would be the task of a procedural semantics. Apart from being particularly appealing in case of quantification, procedural descriptions of meanings could provide criteria to choose among alternative formulations of truth conditions which are equivalent when viewed from their results but not from the way they come about. (32), (33), (36) and the following interpretations of quantificational statements are examples of formulations which suggest different evaluation procedures for the same results.

Any procedure to determine the truth value of a restricted quantificational statement will in one way or the other contain a step by step checking of the domain of quantification with respect to the relevant predicate P. This presupposes – or induces, if you like – an ordering among the elements of the domain of quantification. (From Barwise & Cooper's "determiner universal" (1981: 179) we learn that every natural language quantifier lives on its domain of quantification, hence no other elements of the universe are relevant for the evaluation procedure.) If we restrict our considerations to the case of finite domains of quantification[25] it is a trivial fact that the domain can be linearly ordered, in particular it can be ordered in such a way that the elements which exhibit a certain property come first. Using the well-ordering theorem this result can be carried over to arbitrary domains of quantification. Diagram 15 shows such an ordering for the finite case, each little square representing an element of A.

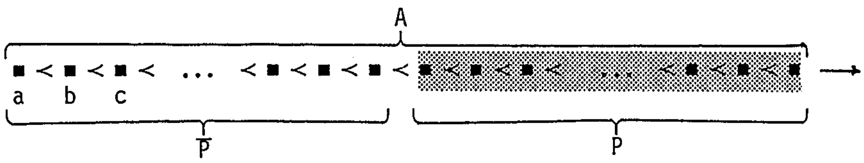

Diagram 15

If one runs through the elements of A from left to right, at the same time one runs through an ascending chain of subsets of A, starting from the empty set and gradually adding one element after the other till A is complete. The scale of elements of A in diagram 15 thus defines a scale of ascending subsets of A, represented by the dots in the next diagram.

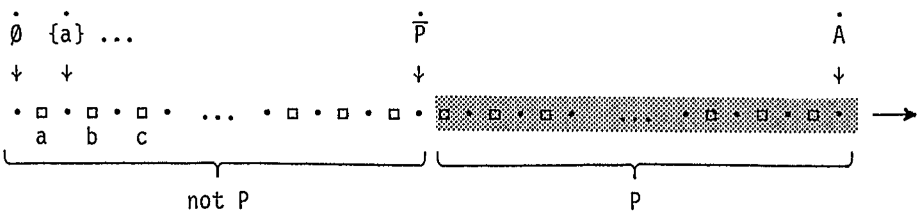

Diagram 16

I have marked the set names with a point in order to emphasize their role of being points on a linearly ordered scale. Any set can be conceived of in two different ways: as the unordered collection of its elements, and as the result of the enumeration of its elements. In the latter sense any set A marks a point on a scale of all individuals, namely the point where this set is completed. This ambivalence is directly related to the ordinal-cardinal ambivalence of natural numbers, the cardinal view corresponding to the unordered collection conception and the ordinal view to the enumeration conception. Using the ordinal set conception we can gain complete uniformity of the usual restricted quantification and the phase quantification format. This is expressed in the following formula. I use $<$ for the ordering among sets conceived as points. The application of P to a point X, written P[X] means that the point where the set X is completed falls into P, which implies that X contains elements with the property P.

(36) $\exists\dot{X}(GSI(\sim P, \dot{A}) < \dot{X} \leqq \dot{A}\ \&\ P[\dot{X}])$
$\forall\dot{X}(GSI(\ P, \dot{A}) < \dot{X} \leqq \dot{A} \rightarrow P[\dot{X}])$

Conceived in this way, the standard restricted quantifiers exhibit a striking similarity with the adverb *already* and its associates. *Already(p, t°)* means: start somewhere in the phase of not-p that immediately precedes t°, go to t°, and you will enter a phase of p – or shorter: the time till t° reaches into a phase of p. *Some A are P* means analogously: start with elements of A for which P does not hold (if there are any), run through A, and you will enter P – or shorter: A reaches into P.

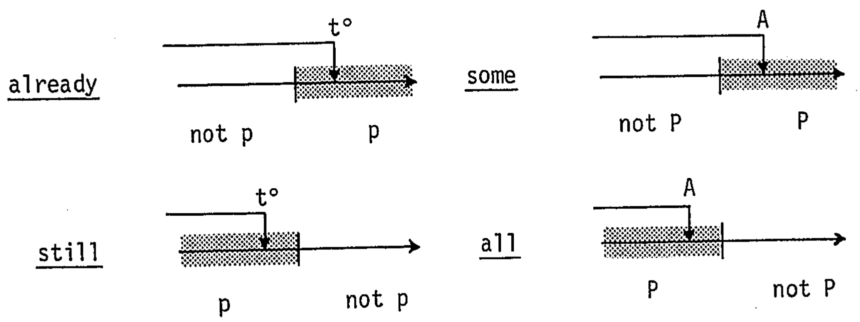

Diagram 17

One might object that this result seems artificial in that it uses an ordering which cannot be considered given in all cases, or even worse, that it is in conflict with an essential property of the quantifiers considered namely their being immune against a permutation of the elements of the respective universe.[23] This is right. The respective ordering, however, does not play an essential role. The only condition it must fulfil is the one, that the universe is divided into two halves and that the elements of one half precede those of the other half. This in turn requires not more than the possibility to distinguish between the two subsets properly. Thus the actual requirement is much weaker than it seems to be at the first glance. On the other hand, the cases of phase quantifications discussed before do not make full use of the underlying total ordering, either. It just happens, that time is totally ordered. There are uses of the *already*-group in German with spatial interpretation, working perfectly in the, of course not linearly ordered, natural three-dimensional space. A sentence like

(37) Basel liegt schon in der Schweiz.
"Basel lies already in Switzerland."

is to be interpreted as: "Walk along any relevant path to Basel and you will cross the border of Switzerland", a relevant path being any path starting outside Switzerland (the spatial region specification "Switzerland" representing a spatial predicate) and ending with Basel (conceived as the parameter point), crossing the border to Switzerland at most one time. This case resembles very much the general restricted quantification case.

3.4. Phase quantification and semantic automata

Johan van Benthem in his talk at this conference presented a new semantical approach to quantification which seems promising for the solution of the problems considered here. He suggests describing the meanings of (nominal) quantifiers by means of automata.[26] The universal and the existential quantifiers, e.g., are represented by two state finite automata with one accepting state, working on a binary alphabet. Their input consists in a tape with one entry for each element of the domain of quantification, the entry being 1 if the predicate quantified holds for that element and 0 if it does not hold. Let me call the accepting state "YES" and the refuting state "NO":[27] .

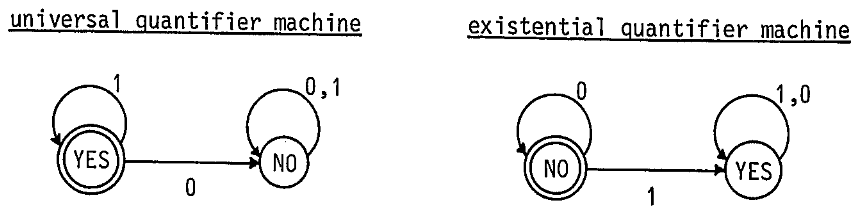

Diagram 18

The two automata are dual: you get the one out of the other if you exchange YES and NO (outer negation) and all 0s and 1s (inner negation). The two automata can be replaced by even simpler indeterministic finite automata:[28]

Diagram 19

They work for every ordering of the domain of quantification whatsoever, but clearly represent a simple notion of border-crossing (from P to not-P or the other way round) as their crucial element. Interpreted continuously, they can be considered to represent the meanings of *already(p, t°)* and *still(p, t°)*, supposing they start from the relevant GSI and end at t°. E.g. the universal automaton yields *still*: start with the truth-value YES and keep to it as long as you stay in p, but change irreversibly to NO as soon as you encounter not-p. Something similar to these automata could serve to represent the meanings of phase quantifiers in general, provided two things:

(i) a definition of generalized automata that work on continuous scale in-

tervals divided into phases out of a finite choice of states, instead of working on tapes with discrete entries,
(ii) a way to treat presuppositions properly.

The latter problem opens very interesting perspectives. The operator *already*, for one, is presuppositional. It selects certain time intervals to which it can apply either positively or negatively, precluding others, namely those which do not start with a negative half or which have more than one change between positive and negative sub-phases. This behavior could be modeled by indeterministic automata that are defined for the relevant input intervals only, yielding no truth value if they encounter other data. In this way, there could be a very elegant solution available for the problems concerning the projection of presuppositions in quantificational contexts. Apparently, automata of the kind involved here can be inserted as subroutines into others, replacing the input 1 by the acceptance of a subautomaton and 0 by its refutation. (Note, that 1 and 0 anyway stand for the complex procedures of verifying the predicate quantified for the object under consideration.) Presupposition projection, now, can just be left to the functioning of the machine as a whole. It will fail to calculate a truth value in case of presupposition failure on any of its internal levels. Or to put it the other way round: the presupposition of a complex expression will be represented by the input selective behavior of the complex automaton representing its meaning.

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NOTES

1. Van Eijck (1985) gives a comprehensive survey and discussion of the work about quantification done by linguists, logicians, and philosophers.
2. This general conception of quantification will be modified below in a way that will, however, be compatible with the considerations following now.
3. Formally, Q~ is $\{P | Q(\bar{P})\}$, ~Q is $\{P | \sim Q(P)\}$, and ~Q~ is $\{P | Q(\bar{P})\}$.
4. I.e., the set { Q, Q~, ~Q, ~Q~ } forms an operator algebra with respect to the operations of inner negation, outer negation, and dual.
5. In case of diagram 4 take the predicate variable *P* for c.
6. The original formulation of the definition (cf. Barwise and Cooper 1981: 178) is slightly more complicated.
7. I will not discuss the regularities governing the way inner and outer negation is expressed. Horn (1978) provides evidence which strongly suggests that the type-assignment used here is relevant for the occurrence of NEG-raising, which complicates the matter considerably.
8. The concept of phase quantification developed below seems to provide the basis for a uniform treatment of both meaning variants as the same operator working on different scales.
9. Cf. the remark concerning the "tolerance" criterion below.
10. Let Q be intolerant, i.e. $Q(P) \Rightarrow \sim Q(\sim P)$. Now, either the reverse holds too, or it does not hold. If it holds, Q(P) is equivalent with ~Q(~P), its dual, hence it is selfdual. If it does not hold there must be cases, where ~Q(~P) holds and Q(P) does not hold. This, in turn, means, that both ~Q~ and ~Q are tolerant.
11. Let D be persistent, then $D(A, P) \Rightarrow D(B, P)$ if $A \subset B$. But this is the same as $\sim D(B, P) \Rightarrow \sim D(A, P)$ if $A \subset B$. Hence, ~D is antipersistent.
12. Let D be persistent. If D is not highly degenerate there exist sets A, B, and P, and a universe containing them such that D(A, P) and D(B, ~P) hold. From that it follows by the persistency of D that both $D(A \cup B, P)$ and $D(A \cup B, \sim P)$ hold, rendering D tolerant.
13. I do not offer any explanation for the asymmetry described. Horn (1972) suggests that type 4 is rare because it is unnecessary, due to the fact that type 4 usually is a conversational implicature of type 1. But I presume that an explanation along this line is too weak. Often, it seems, type 4 is not only not needed but actually a v o i d e d, cf. the numerous cases of NEG-raising with type 2 (but not type 1) quantifiers, which yields type 3 in place

of type 4 (Horn 1978), or the pseudo-type-4 adjectives mentioned in note 18 below.

14. This is reflected immediately by the use of the corresponding particles in German, *still* translating *noch*, and *not yet* translating *noch nicht*.
15. Note that t° may be different from the time of utterance, due to tense, temporal adverbials or implicit dislocation. (t° is what Reichenbach 1947: 288) calls the "point of reference".) Hence, the transition point need not be recent or imminent in absolute terms, i.e. with respect to the time of utterance.
16. *enough* can also be used in the sense of **much/many enough* without taking any adjective or adverb.
17. Cf. Kitcher (1978) for that point.
18. In German, there are a few examples of lexicalized adjectives which look like type 4 but nevertheless are used with a different (type 2!) meaning: there is *gut* (1), *übel* (2), *ungut* (3), and *unübel* (pseudo 4), the latter being used only in the combination "nicht unübel" meaning just "not bad". Likewise, there is *schwer* (1) (in the sense of *difficult*), *leicht* (2) (= *easy*), *unschwer* (3), and *unleicht* (pseudo 4) which means just the same as *leicht* in phrases like "... wie man unleicht erkennt". In such cases, type 4 meanings seem not only to be rare but somehow to be blocked off.
19. With the exemption that the role of **much/many enough* is played by *enough*.
20. Cf. Blau 1983.
21. For the contrary suggestion cf. Barwise and Cooper (1981: 208). They do however not commit themselves to that view.
22. Cf. van Benthem (1984).
23. *Many A are P* means many(A, $A \cap P$) to Barwise and Cooper and many ($|A \cap P|$, M) to me. $P \subset P'$ implies both $A \cap P \subseteq A \cap P'$ and $|A \cap P| \leq |A \cap P'|$.
24. Cf. Kratzer (1981) among others.
25. As van Benthem (1984) does.
26. Cf. Benthem (this volume).
27. The starting state is marked by a double circle.
28. These automata are indeterministic, according to the terminology of Hopcroft & Ulman (1979), in so far as they are not totally defined for the second state. They stop as soon as they reach the second state, no matter what the further input would be.