Collective contributions to the electric microfield distribution in a turbulent plasma

K. -H. Spatschek

Institut für Theoretische Physik, Ruhr-Universität Bochum, D-463 Bochum, Germany

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The electric microfield distribution for weakly turbulent electron–ion systems is calculated using an individual–collective approach. The possibility of separation into collective ion, collective electron, and individual contributions due to electrons and ions is discussed. Analytical expressions for the collective electron and the collective ion microfield distribution are presented in terms of the turbulent spectrum. The results for the total distribution in the case of ion–sound turbulence show significant deviations compared with those of plasmas in thermodynamic equilibrium.

I. INTRODUCTION

The electric microfield distribution in turbulent plasmas is of great interest for line broadening calculations.[1–4] A first attempt to formulate the turbulent electric microfield distribution was performed by Sholin.[5] However, his results do not follow from a rigorous theoretical treatment but are based on several assumptions concerning the factorization into known individual and collective contributions. In a previous paper[6] the electric microfield distribution was calculated for a weakly turbulent electron system using an individual–collective approach.[7] The restriction in these papers to a one-component electron plasma limits its applicability for line broadening calculations, however. Since the practical importance of the momentary microfield distribution is centered in the range of the quasistatic approximation, the knowledge of the contribution of the ions is of dominant importance. This contribution must be calculated taking into account the interaction of ions with electrons.

The well-known theories of the electric microfield distribution in thermodynamic equilibrium show that the inclusion of collective long-range effects shifts the results calculated on the individual approach only toward the correlation-free Holtsmark distribution.[8] The effect disappears with the increasing number of particles in the Debye sphere. However, even at most favorable density and temperature values, it is less than 10%. It can be expected that an increase in the excitation of the collective degree of freedom due to turbulence produces a stronger effect on the microfield distribution and the electric microfield distribution may be expected to differ significantly from the previous results in nonturbulent plasmas.[7]

II. OUTLINE OF THE PROCEDURE

In this paper we discuss the conditions under which the electric microfield distribution of a plasma with electrostatic field fluctuations may be factorized into electron and ion, as well as into individual and collective contributions. For the case of separation into individual and collective effects the analytical forms of the collective parts are given, whereas a separation of the individual component into electron and ion contributions is not discussed here. For the individual microfield distribution we have used the results of Branger and Mozer.[9,10] The error introduced by this procedure may be considered to be small due to the predominance of the collective contributions in turbulent plasmas.

We start with the general formula of the electric microfield distribution

$$W(\mathbf{E}) = \int \delta(\mathbf{E} - \sum_{j\epsilon e,i} \mathbf{E}_j) P\, d^e\mathbf{r}_1\, d^e\mathbf{p}_1 \cdots d^e\mathbf{r}_{N_e}\, d^e\mathbf{p}_{N_e} \times d^i\mathbf{r}_1\, d^i\mathbf{p}_1 \cdots d^i\mathbf{r}_{N_i}\, d^i\mathbf{p}_{N_i}. \tag{1}$$

$W(\mathbf{E})\, d\mathbf{E}$ gives the probability of finding an electric field $\mathbf{E}$ in the region $(\mathbf{E}, d\mathbf{E})$ at the point of observation. P is the Γ space probability density whereas the space coordinates and momenta of N_e electrons and N_i ions are designated by ${}^e\mathbf{r}_\nu$, ${}^e\mathbf{p}_\nu$ ($\nu = 1, 2, \cdots, N_e$) and ${}^i\mathbf{r}_\nu$, ${}^i\mathbf{p}_\nu$ ($\nu = 1, 2, \cdots, N_i$), respectively. $\mathbf{E}_j$ is the field contribution of the jth particle contained in the electron–ion system at the neutral point of observation.

From the Fourier transformation of $W(\mathbf{E})$,

$$W(\mathbf{q}) = \int W(\mathbf{E}) \exp[-i\mathbf{q}\cdot\mathbf{E}]\, d\mathbf{E},$$

we recognize that $W(\mathbf{E})$ may be factorized into an individual and a collective part only if one introduces a new coordinate system in such a way that (a) the electric field exerted by all particles may be written as a sum of a collective (C) and an individual (I) part, i.e.,

$$\sum_{j\epsilon e,i} \mathbf{E}_j = \mathbf{E}_{i,e;C} + \mathbf{E}_{i,e;I}, \tag{2a}$$

(b) the probability density P factorizes into an individual and a collective contribution, i.e.,

$$P = P_C P_I, \tag{2b}$$

(c) the functional determinant of the coordinate transformation also factorizes, i.e.,

$$d^e\mathbf{r}_1\, d^e\mathbf{p}_1 \cdots d^e\mathbf{r}_{N_e}\, d^e\mathbf{p}_{N_e}\, d^i\mathbf{r}_1\, d^i\mathbf{p}_1 \cdots d^i\mathbf{r}_{N_i}\, d^i\mathbf{p}_{N_i} \rightarrow d\{\cdots\}_C \cdot d\{\cdots\}_I. \tag{2c}$$

III. THE INDIVIDUAL–COLLECTIVE APPROACH

To solve this problem we use the individual–collective approach which enables us to formulate a simple kinetic description presented in the following section.

First, we transform the Hamiltonian of an electron–ion system,

$$H=\sum_{j\epsilon e}\frac{{}^e\mathbf{p}_j^2}{2m}+\sum_{j\epsilon i}\frac{{}^i\mathbf{p}_j^2}{2M}+\tfrac{1}{2}\sum_{\substack{j,l\epsilon e\\ \mathbf{k}}}\frac{4\pi e^2}{Vk^2}\exp[i\mathbf{k}\cdot({}^e\mathbf{r}_j-{}^e\mathbf{r}_l)]$$

$$+\tfrac{1}{2}\sum_{\substack{j,l\epsilon i\\ \mathbf{k}}}\frac{4\pi z^2e^2}{Vk^2}\exp[i\mathbf{k}\cdot({}^i\mathbf{r}_j-{}^i\mathbf{r}_l)]$$

$$-\sum_{\substack{j\epsilon i,l\epsilon e\\ \mathbf{k}}}\frac{4\pi ze^2}{Vk^2}\exp[i\mathbf{k}\cdot({}^i\mathbf{r}_j-{}^e\mathbf{r}_l)],\qquad(3)$$

using the formalism of Kurasawa and Matsuura[11] which is based on the general outline of Bohm and Pines.[12,13] Here, charge and mass of an electron are designated by $-e$ and m, whereas charge and mass of an ion are characterized by ze and M, respectively. V is the volume of the system.

The present formalism may be characterized by the following steps:

(1) A first canonical transformation introduces the collective coordinates and relates them to the long wavelength part of the density fluctuations. The limiting wavenumbers for the electrons,

$$k_c\approx\left(\frac{4\pi n_e e^2}{\kappa T_e}\right)^{1/2},\qquad(4a)$$

and ions,

$$k_c'\approx\left(\frac{4\pi n_i z^2e^2}{\kappa T_i}\right)^{1/2},\qquad(4b)$$

are introduced. Here, the collective coordinates for electrons and ions are only defined for $k<k_c$ and $k<k_c'$, respectively. T_e, n_e represent the temperature and the density of the electrons, whereas T_i, n_i are the temperature and the density of the ions in the system. Furthermore, since we are considering ion-sound turbulence,

$$T_e\gg T_i$$

holds from which the relation

$$k_c<k_c'$$

follows if $n_i\approx n_e$.

(2) A second canonical transformation yields a separation into electron and ion contributions despite the individual electron–ion interaction term. The result is a shielding of the ions due to the electrons.

The mathematical description of the shielding in the long wavelength region is different from that in the intermediate regime $k_c<k<k_c'$. Then, after the second transformation ion and electron coordinates are restricted by subsidiary conditions separately.

(3) A third canonical transformation enables a separation of the Hamiltonian into an individual and a collective part. This corresponds to the second canonical transformation of Bohm and Pines and has similar conditions for its applicability.

The following treatment is valid to lowest order in the expansion parameters:

$$\langle(\mathbf{k}\cdot{}^e\mathbf{p}_j/m\omega_\mathbf{k})^2\rangle_{\mathrm{Av}},\quad\langle(\mathbf{k}\cdot{}^i\mathbf{p}_j/M\Omega_\mathbf{k})^2\rangle_{\mathrm{Av}},\quad\Omega_\mathbf{k}/kv_{te},\quad kv_{ti}/\Omega_\mathbf{k}.$$

The angular brackets $\langle\cdots\rangle_{\mathrm{Av}}$ indicate ensemble averaging. The plasma frequencies ω_{pe} and ω_{pi} are connected with the Debye lengths λ_{De} and λ_{Di} via $\omega_{pi,e}\lambda_{Di,e}=v_{ti,e}$ where v_{te} is the thermal velocity of the electrons and v_{ti} that of the ions.

The expansion parameters are small for an electron–ion system with ion-sound turbulence. After all these transformations we end up with the Hamiltonian

$$H=\sum_{j\epsilon e}\frac{{}^e\mathbf{p}_j'^2}{2m}+\sum_{j\epsilon i}\frac{{}^i\mathbf{p}_j'^2}{2M}+\tfrac{1}{2}\sum_{k<k_c}\left(P_\mathbf{k}P_{-\mathbf{k}}+\frac{\omega_\mathbf{k}^2}{V}Q_\mathbf{k}Q_{-\mathbf{k}}\right)$$

$$+\tfrac{1}{2}\sum_{k<k_c'}\left(p_\mathbf{k}p_{-\mathbf{k}}+\frac{\Omega_\mathbf{k}^2}{V}q_\mathbf{k}q_{-\mathbf{k}}\right)+\tfrac{1}{2}\sum_{\substack{k_c<k<k_c'\\ j,l\epsilon e}}\frac{4\pi e^2}{Vk^2}$$

$$\times\exp[i\mathbf{k}\cdot({}^e\mathbf{r}_j'-{}^e\mathbf{r}_l')]+\tfrac{1}{2}\sum_{\substack{k_c'<k\\ j,l}}\left\{\frac{4\pi e^2}{Vk^2}\right.$$

$$\times\exp[i\mathbf{k}\cdot({}^e\mathbf{r}_j'-{}^e\mathbf{r}_l')]+\frac{4\pi z^2e^2}{Vk^2}$$

$$\times\exp[i\mathbf{k}\cdot({}^i\mathbf{r}_j'-{}^e\mathbf{r}_l')]-2\frac{4\pi ze^2}{Vk^2}$$

$$\left.\times\exp[i\mathbf{k}\cdot({}^i\mathbf{r}_j'-{}^e\mathbf{r}_l')]\right\}.\qquad(5)$$

In Eq. (5) ${}^e\mathbf{r}_j'$, ${}^e\mathbf{p}_j'$, ${}^i\mathbf{r}_j'$, and ${}^i\mathbf{p}_j'$ are the new individual coordinates and $P_\mathbf{k}$, $Q_\mathbf{k}$, $p_\mathbf{k}$, $q_\mathbf{k}$ are the new collective coordinates for electrons and ions, respectively. In the following we omit the primes for simplicity. The frequencies $\omega_\mathbf{k}$ and $\Omega_\mathbf{k}$ are given through the dispersion relations

$$\omega_\mathbf{k}^2\approx\omega_{pe}^2+3k^2v_{te}^2\qquad(6)$$

and

$$\Omega_\mathbf{k}^2\approx\frac{\omega_{pi}^2}{1+k^{-2}\lambda_{De}^{-2}}+3k^2v_{ti}^2.\qquad(7)$$

The subsidiary conditions, written in individual coordinates only, are not considered here because the number of collective modes is small as compared with the number of individual modes.[14] It does not mean that, in general, only a small excitation of the collective degree of turbulence exists.

The electric field in the new coordinate system is given by

$$\mathbf{E}(\mathbf{r})=i\frac{4\pi e}{V}\sum_{\substack{j\epsilon e\\ k>k_c}}\frac{\mathbf{k}}{k^2}\exp[i\mathbf{k}\cdot(\mathbf{r}-{}^e\mathbf{r}_j)]-i\frac{4\pi ze}{V}$$

$$\times\sum_{\substack{j\epsilon i\\ k>k_c'}}\frac{\mathbf{k}}{k^2}\exp[i\mathbf{k}\cdot(\mathbf{r}-{}^i\mathbf{r}_j)]+i\left(\frac{4\pi}{V}\right)^{1/2}$$

$$\times\sum_{k<k_c}\frac{\mathbf{k}}{k}P_\mathbf{k}\exp(i\mathbf{k}\cdot\mathbf{r})-i\left(\frac{4\pi}{V}\right)^{1/2}$$

$$\times\sum_{k<k_c'}\frac{\mathbf{k}}{k(1+\beta_k)^{1/2}}p_\mathbf{k}\exp(i\mathbf{k}\cdot\mathbf{r})-i\left(\frac{4\pi}{V}\right)^{1/2}$$

$$\times\sum_{k_c<k<k_c'}\frac{\mathbf{k}}{k}p_\mathbf{k}(1+\beta_k)^{1/2}\exp(i\mathbf{k}\cdot\mathbf{r}),\qquad(8)$$

where we have introduced the abbreviation

$$\beta_k=k^{-2}\lambda_{De}^{-2}=\kappa T_e/4\pi n_e e^2k^2.$$

The splitting of the electric field into contributions depending separately on collective electron and ion as well as individual electron and ion coordinates now follows from Eq. (8).

IV. KINETIC DESCRIPTION

Using the Hamiltonian, Eq. (5), we formulate the generalized Klimontovich equations. A generalization is necessary because we have to consider a system of particles and waves. Essentially, the particles are described by space- and momentum-coordinates, whereas the waves are specified by collective coordinates which are generated from the Fourier components of the fields and the vector potential.[6]

Without making use of any transformation the phase-space density in an arbitrary Γ space which represents $2(N_e+N_i)$ individual and all the collective coordinates may be written as

$$F_\Gamma = \prod_{\alpha,j} \delta[{}^\alpha \mathbf{r}^j - {}^\alpha \mathbf{r}_j(t)]\delta[{}^\alpha \mathbf{p}^j - {}^\alpha \mathbf{p}_j(t)] \times \prod_{\alpha,\mathbf{k}} \delta[{}^\alpha A^{\mathbf{k}} - {}^\alpha A_{\mathbf{k}}(t)]\delta[{}^\alpha E^{\mathbf{k}} - {}^\alpha E_{\mathbf{k}}(t)].$$

The index α characterizes the different species, $\alpha = e, i$. The indices j and $\mathbf{k}$ at the upper right designate the various Γ space coordinates whereas the indices j and $\mathbf{k}$ at the lower right designate the particle and wave coordinates. $A_{\mathbf{k}}$ and $E_{\mathbf{k}}$ characterize the Fourier components of the vector potential and the electric field in quasistationary and current-free approximation. Since all the transformations are canonical and because of their special form, the phase density in the transformed Γ space is also represented by Dirac functions. Beyond that, all space and momentum coordinates are transformed according to similar transformation equations so that we can use the following representation of the density ${}^\alpha F_{jc}$ for the jth particle of species α and the waves in the enlarged μ space:

$${}^\alpha F_{jc} = \delta(\mathbf{r} - {}^\alpha \mathbf{r}_j)\delta(\mathbf{p} - {}^\alpha \mathbf{p}_j) \prod_{\substack{k<k_c \\ k'<k_c'}} \delta(Q^{\mathbf{k}} - Q_{\mathbf{k}}) \times \delta(P^{\mathbf{k}} - P_{\mathbf{k}})\delta(q^{\mathbf{k}'} - q_{\mathbf{k}'})\delta(p^{\mathbf{k}'} - p_{\mathbf{k}'}). \quad (9)$$

The microscopic kinetic equations follow from

$$\frac{\partial {}^\alpha F_{jc}}{\partial t} = {}^\alpha \dot{\mathbf{r}}_j \cdot \frac{\partial {}^\alpha F_{jc}}{\partial \mathbf{r}_j} + {}^\alpha \dot{\mathbf{p}}_j \cdot \frac{\partial {}^\alpha F_{jc}}{\partial \mathbf{p}_j} + \sum_{k<k_c} \frac{\partial {}^\alpha F_{jc}}{\partial Q_{\mathbf{k}}} \dot{Q}_{\mathbf{k}} + \sum_{k<k_c} \frac{\partial {}^\alpha F_{jc}}{\partial P_{\mathbf{k}}} \dot{P}_{\mathbf{k}} + \sum_{k<k_c'} \frac{\partial {}^\alpha F_{jc}}{\partial q_{\mathbf{k}}} \dot{q}_{\mathbf{k}} + \sum_{k<k_c'} \frac{\partial {}^\alpha F_{jc}}{\partial p_{\mathbf{k}}} \dot{p}_{\mathbf{k}}, \quad (10)$$

where we have to introduce the canonical equations.

After averaging with the ensemble density we arrive at

$$\frac{\partial {}^e f_{jc}}{\partial t} = -\frac{\mathbf{p}}{m} \cdot \frac{\partial {}^e f_{jc}}{\partial \mathbf{r}} + i \frac{\partial}{\partial \mathbf{p}} \cdot \left\{ \int d\mathbf{R}' \, d\mathbf{p}' \prod_\lambda d'Q^\lambda \, d'q^\lambda \, d'P^\lambda \, d'p^\lambda \times \left[\left(\frac{4\pi e^2}{V} \sum_{k>k_c} \frac{\mathbf{k}}{k^2} \exp[i\mathbf{k}\cdot(\mathbf{r} - \mathbf{R}')] \right) \sum_l {}^{ee} f_{ljc} + \left(\frac{4\pi e^2}{V} \sum_{k>k_c'} \frac{\mathbf{k}}{k^2} \exp[i\mathbf{k}\cdot(\mathbf{r} - \mathbf{R}')] \right) \times \sum_l (-z)^{ei} f_{ljc} \right] \right\} - \sum_{k<k_c} \left(\frac{\partial^e f_{jc}}{\partial Q^{\mathbf{k}}} P^{-\mathbf{k}} - \frac{\partial^e f_{jc}}{\partial P^{\mathbf{k}}} \frac{\omega_{\mathbf{k}}^2}{V} Q^{-\mathbf{k}} \right) - \sum_{k<k_c'} \left(\frac{\partial^e f_{jc}}{\partial q^{\mathbf{k}}} p^{-\mathbf{k}} - \frac{\partial^e f_{jc}}{\partial p^{\mathbf{k}}} \frac{\Omega_{\mathbf{k}}^2}{V} q^{-\mathbf{k}} \right) \quad (11)$$

for the electron distribution function. Here, we have used the abbreviations

$${}^e f_{jc} = \int {}^e F_{jc} P \prod_{\alpha,l} d^\alpha \mathbf{r}_l \, d^\alpha \mathbf{p}_l \prod_{\mathbf{k}} dQ_{\mathbf{k}} \, dP_{\mathbf{k}} \, dq_{\mathbf{k}} \, dp_{\mathbf{k}},$$

$${}^{ee} f_{jlc} = \int {}^e F_{lc} {}^e F_{jc} P \prod_{\alpha,\nu} d^\alpha \mathbf{r}_\nu \, d^\alpha \mathbf{p}_\nu \prod_{\mathbf{k}} dQ_{\mathbf{k}} \, dP_{\mathbf{k}} \, dq_{\mathbf{k}} \, dp_{\mathbf{k}},$$

$${}^{ei} f_{jlc} = \int {}^e F_{jc} {}^i F_{lc} P \prod_{\alpha,\nu} d^\alpha \mathbf{r}_\nu \, d^\alpha \mathbf{p}_\nu \prod_{\mathbf{k}} dQ_{\mathbf{k}} \, dP_{\mathbf{k}} \, dq_{\mathbf{k}} \, dp_{\mathbf{k}}.$$

A similar equation holds for the ions. Now Eq. (11) may be separated into an individual and a collective part by using the ansatz ${}^e f_{jc} = {}^e f_j \cdot P_C$. For stationary solutions it can easily be shown that the constant of separation must be zero. The individual equation is

$$\frac{\partial^e f_j}{\partial t} + \frac{\mathbf{p}}{m} \cdot \frac{\partial^e f_j}{\partial \mathbf{r}} + i \frac{\partial}{\partial \mathbf{p}} \left\{ \int d\mathbf{R}' \, d\mathbf{p}' \left(\left[\frac{4\pi e^2}{V} \sum_{k>k_c} \frac{\mathbf{k}}{k^2} \times \exp[i\mathbf{k}\cdot(\mathbf{r} - \mathbf{R}')] \right] \sum_l {}^{ee} f_{jl} + \left[\frac{4\pi e^2}{V} \sum_{k>k_c'} \frac{\mathbf{k}}{k^2} \times \exp[i\mathbf{k}\cdot(\mathbf{r} - \mathbf{R}')] \right] \sum_l (-z)^{ei} f_{jl} \right) \right\} = 0, \quad (12)$$

whereas the stationary solutions of the collective equations may be written in the form

$$P_C = \prod_{k<k_c} {}^e f_{\mathbf{k}} \left(Q_{\mathbf{k}} Q_{-\mathbf{k}} \frac{\omega_{\mathbf{k}}^2}{V} + P_{\mathbf{k}} P_{-\mathbf{k}} \right) \times \prod_{k<k_c'} {}^i f_{\mathbf{k}} \left(q_{\mathbf{k}} q_{-\mathbf{k}} \frac{\Omega_{\mathbf{k}}^2}{V} + p_{\mathbf{k}} p_{-\mathbf{k}} \right), \quad (13)$$

where the explicit forms of the analytical functions ${}^e f_{\mathbf{k}}$ and ${}^i f_{\mathbf{k}}$ need not be specified at this stage. Due to the fact that for canonical transformations the functional determinant factorizes, we end up with the following results: (i) The total electric microfield distribution can be factored into an individual and a collective part. (ii) The collective part may be factorized into electron and ion contributions. (iii) The kinetic equations for the individual contributions show no simple separation into electron and ion contributions. For the evaluation of the individual microfield distribution one may follow the discussion of Baranger and Mozer;[9,10] if one assumes that for weakly turbulent situations, the individual microfield distribution is not changed significantly due to turbulence.

V. RESULTS

The main purpose of this contribution was to calculate the collective electron and ion microfield distributions. They will give the total distribution by convolution with well-known individual results.

The collective ion microfield distribution can now be defined through

$${}^i W_C(\mathbf{q}) = \int \exp(-i\mathbf{q}\cdot{}^i\mathbf{E}_C) {}^i P_C \prod_{k<k_c'} dq_{\mathbf{k}} \, dp_{\mathbf{k}} \quad (14)$$

where we have to introduce

$$ {}^i\mathbf{E}_C = -i\left(\frac{4\pi}{V}\right)^{1/2} \sum_{k<k_c} \frac{\mathbf{k}}{k(1+\beta_k)^{1/2}} p_{\mathbf{k}} \exp[i\mathbf{k}\cdot\mathbf{r}] - i\left(\frac{4\pi}{V}\right)^{1/2} \sum_{k_c<k<k_c'} \frac{\mathbf{k}}{k} p_{\mathbf{k}}(1+\beta_k)^{1/2} \exp[i\mathbf{k}\cdot\mathbf{r}] \quad (15) $$

and

$$ {}^iP_C = \prod_{k<k_c'} {}^if_{\mathbf{k}} \left(q_{\mathbf{k}} q_{-\mathbf{k}} \frac{\Omega_{\mathbf{k}}^2}{V} + p_{\mathbf{k}} p_{-\mathbf{k}} \right). \quad (16) $$

The evaluation for a weakly turbulent, stationary, homogeneous, isotropic electron–ion system yields, in the limit $V \to \infty$, $N_j \to \infty$, $N_j/V = n_j = \text{const}$ $(j = e, i)$, the result for the collective ion microfield distribution

$$ {}^iW_C(\mathbf{E}) = (3\pi)^{3/2} \left(\int_{k<k_c} \frac{{}^iE_{\mathbf{k}}{}^iE_{-\mathbf{k}}}{(1+\beta_k)^2} d\mathbf{k} + \int_{k_c<k<k_c'} {}^iE_{\mathbf{k}}{}^iE_{-\mathbf{k}}\, d\mathbf{k} \right)^{-3/2} \times \exp\left(-3\pi^2 E^2 \times \left[\int_{k<k_c} \frac{{}^iE_{\mathbf{k}}{}^iE_{-\mathbf{k}}}{(1+\beta_k)^2} d\mathbf{k} + \int_{k_c<k<k_c'} {}^iE_{\mathbf{k}}{}^iE_{-\mathbf{k}}\, d\mathbf{k} \right]^{-1} \right). \quad (17) $$

The corresponding collective electron microfield distribution is likewise written as[6]

$$ {}^eW_C(\mathbf{E}) = (3\pi)^{3/2} \left(\int_{k<k_c} {}^eE_{\mathbf{k}}{}^eE_{-\mathbf{k}}\, d\mathbf{k} \right)^{-3/2} \times \exp\left\{ -3\pi^2 E^2 \left(\int_{k<k_c} {}^e\mathbf{E}_{\mathbf{k}}{}^e\mathbf{E}_{-\mathbf{k}}\, d\mathbf{k} \right)^{-1} \right\}. \quad (18) $$

Here, ${}^eE_{\mathbf{k}}$ and ${}^iE_{\mathbf{k}}$ are the Fourier components of the fields

$$ \sum_{j\epsilon e} \mathbf{E}_j \quad \text{and} \quad \sum_{j\epsilon i} \mathbf{E}_j. $$

For numerical calculations one has to introduce the turbulent spectrum $|E_{\mathbf{k}}|^2$. For the individual microfield distribution one should use the results of Branger and Mozer if assumption (iii) is fulfilled.

Then, we get the total distribution by convolution

$$ W(E) = \frac{2}{\pi} E \int W_I(\mathbf{q})\, {}^iW_C(\mathbf{q})\, {}^eW_C(\mathbf{q})\, q \sin(qE)\, dq. \quad (19) $$

VI. CONCLUDING REMARKS

The collective electric microfield distribution depends on the turbulent spectrum. For a qualitative discussion this dependence may be characterized by the parameter $\gamma^* = \int |E_{\mathbf{k}}|^2 d\mathbf{k}/n\kappa T$, i.e., the degree of turbulence. The individual microfield distribution given by Baranger and Mozer[9,10] is decisively influenced by the parameter $n\lambda_D^3$, i.e., the number of particles in the Debye sphere.

For a detailed study of the electric microfield distribution we should know these parameters. The parameter $n\lambda_D^3$ is at our disposal whereas the calculation of the parameter γ^* is a main tool in current research in turbulence theory.

For a fixed value of γ^*, on one hand, our results demonstrate the increasing influence of the collective distributions in turbulent plasmas with an increase in $n\lambda_D^3$. This may be easily understood by comparing with the corresponding equilibrium results.[7] On the other hand, for fixed values of $n\lambda_D^3$ our results show a shift of the most probable field strength to higher values of the electric field with increasing degree of turbulence.[6]

For numerical results we need the spectral dependence of the turbulent spectrum. To estimate the order of magnitude of the above-quoted shift of the most probable field strength one may use the well-known turbulent spectral distribution for ion–sound turbulence.[15] Then, one can get the order of magnitude of the effect by using the numerical results of the previous calculation for a one-component plasma.[6] From that we conclude that in some laboratory plasmas even an overthermal excitation by a factor of 100 yields significant deviations from the equilibrium results.

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